Northern Technical University

Engineering Technical College / Mosul

Mechanical Technical Engineering

Heat and Mass Transfer Lecture

Third Year

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<u>The Objective</u>: Knowing the students the main scientific principle in the field of heat transfer and its application in the Refrigeration, Cooling, and air conditioning fields.

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1	Introduction, Basic Concepts of Heat Transfer, Heat Transfer Mechanisms.
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4	Conduction through Multilayer Plane Wall, and Cylinder.
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21	Introduction to Heat Exchangers, Kinds of Heat Exchangers
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24-25	The Performances for Difference Kinds of the Heat Exchangers
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28	Radiation Heat Transfer Between Two Black Surfaces
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First Week

Basic Concepts of Heat Transfer, and Heat Transfer Mechanisms.

The Objective: Knowing the student the main scientific principle in the field of heat transfer and its mechanisms.

Introduction: Heat can be transferred in three different modes: *conduction, convection,* and *radiation.* All modes of heat transfer require the existence of a temperature difference, and all modes are from the high-temperature medium to a lower-temperature one.

Test: How does the energy move from a hotter to a colder object?

Summary:

The basic requirement for heat transfer is the presence of a *temperature difference.* There can be no net heat transfer between two mediums that are at the same temperature. The temperature difference is the *driving force* for heat transfer, just as the *voltage difference* is the driving force for electric current flow and *pressure difference* is the driving force for fluid flow. The rate of heat transfer in a certain direction depends on the magnitude of the *temperature gradient* (the temperature difference per unit length or the rate of change of temperature) in that direction. The larger the temperature gradient, the higher the rate of heat transfer.

Application Areas of Heat Transfer

Heat transfer is commonly encountered in engineering systems and other aspects of life, and one does not need to go very far to see some application area of heat transfer. In fact, one does not need to go anywhere. The human body is constantly rejecting heat to its surroundings, and human comfort is closely tied to the rate of this heat rejection. We try to control this heat transfer rate by adjusting our clothing to the environmental conditions.

Many ordinary household appliances are designed, in whole or in part, by using the principles of heat transfer. Some examples include the electric or gas range, the heating and air-conditioning system, the refrigerator and freezer, the water heater, the iron, and even the computer, the TV, and the VCR. Of course, energy-efficient homes are designed on the basis of minimizing heat loss in winter and heat gain in summer. Heat transfer plays a major role in the design of many other devices, such as car radiators, solar collectors, various components of power plants, and even spacecraft. The optimal insulation thickness in the walls and roofs of the houses, on hot water or steam pipes, or on water heaters is again determined on the basis of a heat transfer analysis with economic consideration

Engineering Heat Transfer

Heat transfer equipment such as heat exchangers, boilers, condensers, radiators, heaters, furnaces, refrigerators, and solar collectors are designed primarily on the basis of heat transfer analysis. The heat transfer problems encountered in practice can be considered in two groups: (1) *rating* and (2) *sizing* problems. The rating problems deal with the determination of the heat transfer rate for an existing system at a specified temperature difference. The sizing problems deal with the determination of the size of a system in order to transfer heat at a specified rate for a specified temperature difference. A heat transfer process or equipment can be studied either *experimentally* (testing and taking measurements) or *analytically* (by analysis or calculations). The experimental approach has the advantage that we deal with the actual physical system, and the desired quantity is determined by measurement, within the limits of experimental error. However, this approach is expensive, time-consuming, and often impractical. Besides, the system we are analyzing may not even exist. For example, the size of a heating system of a building must usually be determined *before* the building is actually built on the basis of the dimensions and specifications given. The analytical approach (including numerical approach) has the advantage that it is fast and inexpensive, but the results obtained are subject to the accuracy of the assumptions and idealizations made in the analysis. In heat transfer studies, often a good compromise is reached by reducing the choices to just a few by analysis, and then verifying the findings experimentally.

The First Law of Thermodynamics

The **first law of thermodynamics,** also known as the **conservation of energy principle,** states that *energy can neither be created nor destroyed; it can only change forms.* Therefore, every bit of energy must be accounted for during a process. The conservation of energy principle (or the energy balance) for *any system* undergoing *any process* may be expressed as follows: *The net change (increase or decrease) in the total energy of the system during a process is equal to the difference between the total energy entering and the total energy leaving the system during that process.* That is,

$$\begin{pmatrix}\text{Total energy}\\ \text{entering the}\\ \text{system}\end{pmatrix} - \begin{pmatrix}\text{Total energy}\\ \text{leaving the}\\ \text{system}\end{pmatrix} = \begin{pmatrix}\text{Change in the}\\ \text{total energy of}\\ \text{the system}\end{pmatrix}$$

Noting that energy can be transferred to or from a system by *heat*, *work*, and *mass flow,* and that the total energy of a simple compressible system consists of internal, kinetic, and potential energies.

Energy Balance for Closed Systems *(Fixed Mass)*

A closed system consists of a *fixed mass.* The total energy *E* for most systems encountered in practice consists of the internal energy *U*. This is especially the case for stationary systems since they don't involve any changes in their velocity or elevation during a process. The energy balance relation in that case reduces to

Stationary closed system: $$E_{in} - E_{out} = \Delta U = mC_v\Delta T \qquad \text{(J)}$$

where we expressed the internal energy change in terms of mass *m*, the specific heat at constant volume *Cv*, and the temperature change ΔT of the system.

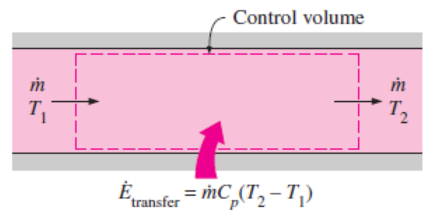
Energy Balance for Steady-Flow Systems

A large number of engineering devices such as water heaters and car radiators involve mass flow in and out of a system, and are modeled as *control volumes.* Most control volumes are analyzed under steady operating conditions. The term *steady* means *no change with time* at a specified location. The opposite of steady is *unsteady* or *transient.* Also, the term *uniform* implies *no change with position* throughout a surface or region at a specified time.

The amount of mass flowing through a cross section of a flow device per unit time is called the **mass flow rate,** and is denoted by $\dot{m}$. A fluid may flow in and out of a control volume through pipes or ducts. The mass flow rate of a fluid flowing in a pipe or duct is proportional to the cross-sectional area A_c of the pipe or duct, the density ρ, and the velocity υ. of the fluid, $\dot{m} = \rho \upsilon A_c$ (kg/s).

For a steady-flow system with one inlet and one exit, the rate of mass flow into the control volume must be equal to the rate of mass flow out of it. That is, $\dot{m}_{in} = \dot{m}_{out} = \dot{m}$ .When the changes in kinetic and potential energies are negligible, which is usually the case, and there is no work interaction, the energy balance for such a steady-flow system reduces to , $\dot{Q} = \dot{m} C_p \Delta T$

where C_p is the specific heat (kJ/kg.ºC)



Heat Transfer Mechanisms

Heat can be transferred in three different modes: conduction, convection, and radiation. ***Conduction*** is the transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones as a result of interactions between the particles, and is expressed by *Fourier's law of heat conduction* as,

$$\dot{Q}_{cond} = -kA\frac{dT}{dx}$$

where k is the *thermal conductivity* of the material, A is the *area* normal to the direction of heat transfer, and dT/dx is the *temperature gradient.* The magnitude of the rate of heat conduction across a plane layer of thickness L is given by

$$\dot{Q}_{cond} = kA\frac{\Delta T}{L}$$

where ΔT is the temperature difference across the layer.

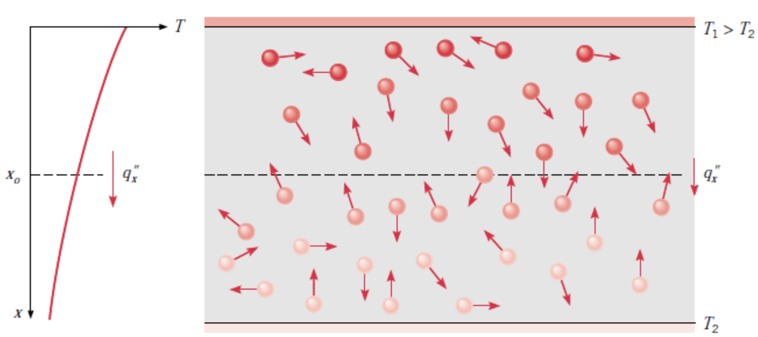


FIGURE 1.1 Association of conduction heat transfer with diffusion of energy due to molecular activity.

Convection is the mode of heat transfer between a solid surface and the adjacent liquid or gas that is in motion, and involves the combined effects of conduction and fluid motion. The rate of convection heat transfer is expressed by *Newton's law of cooling* as, $\dot{Q}_{\text{convection}} = hA_s(T_s - T_\infty)$

where h is the *convection heat transfer coefficient* in W/m^2·°C, A_s is the *surface area* through which convection heat transfer takes place, T_s is the *surface temperature*, and T_∞ is the *temperature of the fluid* sufficiently far from the surface. Convection is called **forced convection** if the fluid is forced to flow over the surface by external means such as a fan, pump, or the wind. In contrast, convection is called **natural** (or **free**) **convection** if the fluid motion is caused by buoyancy forces that are induced by density differences due to the variation of temperature in the fluid. Heat transfer processes that involve *change of phase* of a fluid are also considered to be convection because of the fluid motion induced during the process, such as the rise of the vapor bubbles during boiling or the fall of the liquid droplets during condensation.

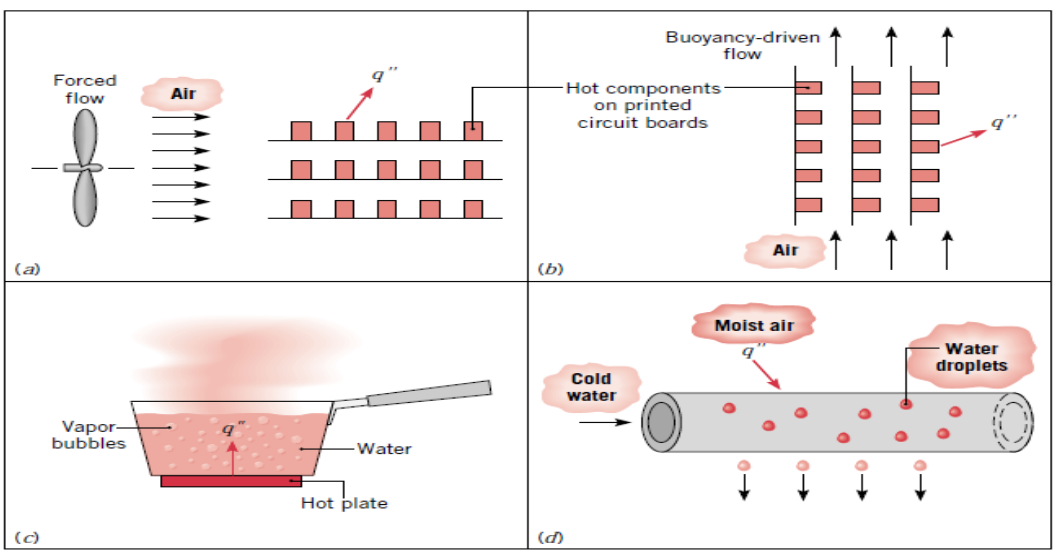


FIGURE 1.2 Convection heat transfer processes. (*a*) Forced convection. (*b*) Natural convection. (*c*) Boiling. (*d*) Condensation.

Radiation is the energy emitted by matter in the form of electromagnetic waves (or photons) as a result of the changes in the electronic configurations of the atoms or molecules. The maximum rate of radiation that can be emitted from a surface at an absolute temperature *Ts* is given by the *Stefan–Boltzmann law* as,

$\dot{Q}_{\text{emit, max}} = \sigma A_s T_s^4$, where $\sigma = 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4$

is the *Stefan Boltzmann constant.*

When a surface of emissivity and surface area A_s at an absolute temperature T_s is completely enclosed by a much larger (or black) surface at absolute temperature T_{surr} separated by a gas (such as air) that does not intervene with radiation, the net rate of radiation heat transfer between these two surfaces is given by,

$$\dot{Q}_{\text{rad}} = \varepsilon \sigma A_s (T_s^4 - T_{\text{surr}}^4)$$

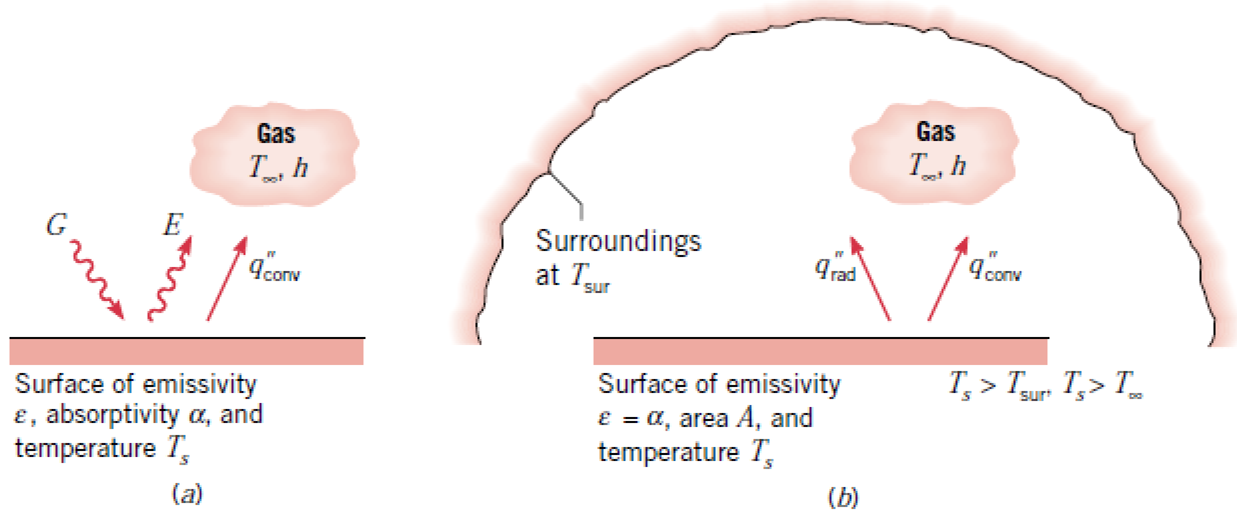


FIGURE 1.3 Radiation exchange: (a) at a surface and (b) between surface and large surroundings.

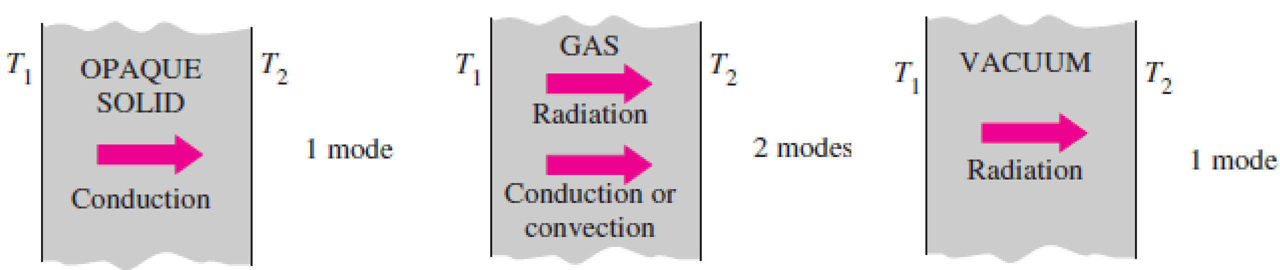
Simultaneous Heat Transfer Mechanisms

We mentioned that there are three mechanisms of heat transfer, but not all three can exist simultaneously in a medium. For example, heat transfer is only by conduction in *opaque solids*, but by conduction and radiation in *semitransparent solids*. Thus, a solid may involve conduction and radiation but not convection. However, a solid may involve heat transfer by convection and/or radiation on its surfaces exposed to a fluid or other surfaces.

Heat transfer is by conduction and possibly by radiation in a *still fluid* (no bulk fluid motion) and by convection and radiation in a *flowing fluid*. In the absence of radiation, heat transfer through a fluid is either by conduction or convection, depending on the presence of any bulk fluid motion. Convection can be viewed as combined conduction and fluid motion, and conduction in a fluid can be viewed as a special case of convection in the absence of any fluid motion.

Thus, when we deal with heat transfer through a *fluid*, we have either *conduction* or *convection*, but not both. Also, gases are practically transparent to radiation, except that some gases are known to absorb radiation strongly at certain wavelengths. Ozone, for example, strongly absorbs ultraviolet radiation. But in most cases, a gas between two solid surfaces does not interfere with radiation and acts effectively as a vacuum. Liquids, on the other hand, are usually strong absorbers of radiation.

Finally, heat transfer through a *vacuum* is by radiation only since conduction or convection requires the presence of a material medium



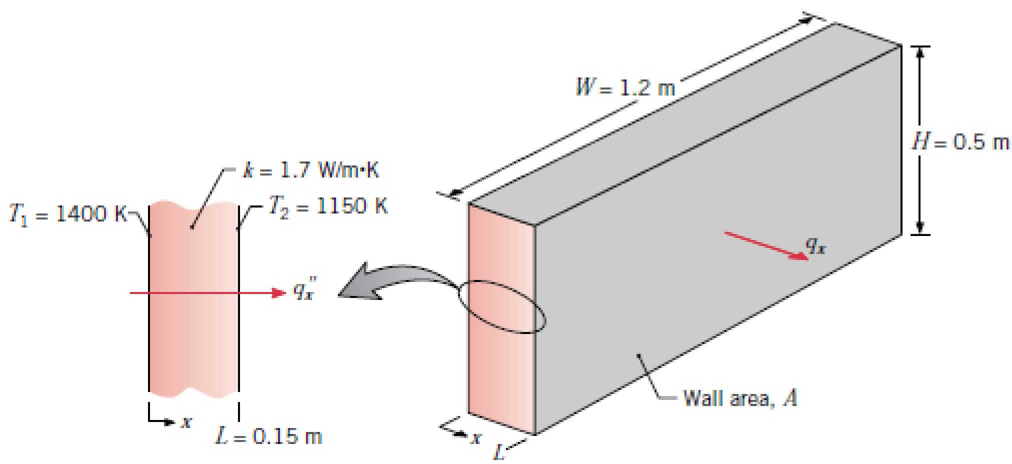
H.W 1: An ideal gas is heated from 50°C to 80°C (*a*) at constant volume and (*b*) at constant pressure. For which case do you think the energy required will be greater? Why?

EXAMPLE 1.1

The wall of an industrial furnace is constructed from 0.15-m-thick fireclay brick having a thermal conductivity of 1.7 W/m _ K. Measurements made during steady state operation reveal temperatures of 1400 and 1150 K at the inner and outer surfaces, respectively. What is the rate of heat loss through a wall that is 0.5 m by 1.2 m on a side?

SOLUTION

Known: Steady-state conditions with prescribed wall thickness, area, thermal conductivity, and surface temperatures.



Find: Wall heat loss

Assumptions:

1. Steady-state conditions.

2. One-dimensional conduction through the wall.

3. Constant thermal conductivity.

Analysis: Since heat transfer through the wall is by conduction, the heat flux may be determined from Fourier's law.

$$q''_x = k\frac{\Delta T}{L} = 1.7\ \text{W/m}\cdot\text{K} \times \frac{250\ \text{K}}{0.15\ \text{m}} = 2833\ \text{W/m}^2$$

The heat flux represents the rate of heat transfer through a section of unit area, and it is uniform (invariant) across the surface of the wall. The heat loss through the wall of area is then

$$q_x = (HW)\, q''_x = (0.5\ \text{m} \times 1.2\ \text{m})\ 2833\ \text{W/m}^2 = 1700\ \text{W}$$

EXAMPLE 1.2

An uninsulated steam pipe passes through a room in which the air and walls are at 25°C. The outside diameter of the pipe is 70 mm, and its surface temperature and emissivity are 200°C and 0.8, respectively. What is the surface emissive power and irradiation? If the coefficient associated with free convection heat transfer from the surface to the air is 15 $W/m^2.K$, what is the rate of heat loss from the surface per unit length of pipe?

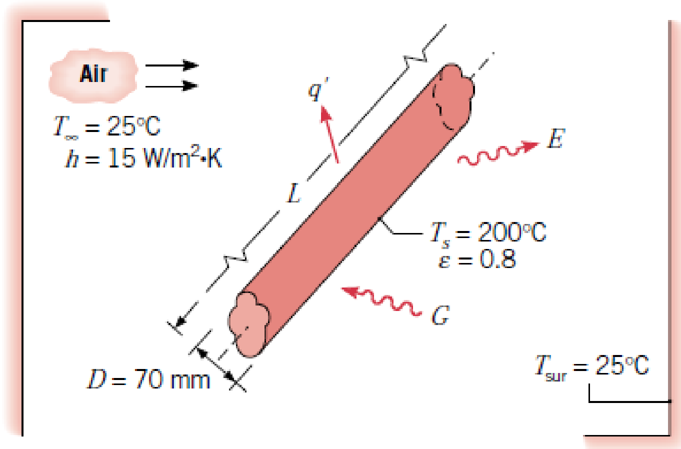
SOLUTION

Known: Uninsulated pipe of prescribed diameter, emissivity, and surface temperature in a room with fixed wall and air temperatures.

Find:

1. Surface emissive power and irradiation.

2. Pipe heat loss per unit length.



Assumptions:

1. Steady-state conditions.

2. Radiation exchange between the pipe and the room is between a small surface and a much larger enclosure.

3. The surface emissivity and absorptivity are equal.

Analysis:

1. The surface emissive power may be evaluated from Equation 1.5, while the irradiation corresponds to $G = \sigma T_{sur}^4$. Hence

$$E = \varepsilon\sigma T_s^4 = 0.8(5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)\ (473\ \text{K})^4 = 2270\ \text{W/m}^2$$

$$G = \sigma T_{sur}^4 = 5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4\ (298\ \text{K})^4 = 447\ \text{W/m}^2$$

2. Heat loss from the pipe is by convection to the room air and by radiation exchange with the walls. Hence, $q = q_{conv} + q_{rad}$ and from Equation 1.10, with $A = \pi DL$,

$$q = h(\pi DL)(T_s - T_\infty) + \varepsilon(\pi DL)\sigma(T_s^4 - T_{sur}^4)$$

The heat loss per unit length of pipe is then

$$q' = \frac{q}{L} = 15\ \text{W/m}^2\cdot\text{K}(\pi \times 0.07\ \text{m})(200 - 25)°\text{C}$$

$$+\ 0.8(\pi \times 0.07\ \text{m})\ 5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4\ (473^4 - 298^4)\ \text{K}^4$$

$$q' = 577\ \text{W/m} + 421\ \text{W/m} = 998\ \text{W/m} \quad \triangleleft$$

Example 3:

Consider steady heat transfer between two large parallel plates at constant temperatures of T_1= 290K and T_2=150K that are L=2cm apart .Assuming the surfaces to be black (Є=1) , determine the rate of heat transfer between the plates per unit surfaces area assuming the gap between the plates is (a) filled with atmospheric air *k* = 0.01979 W/m·°C (b) evacuated (c) filled with fiberglass insulation *k* = 0.036 W/m·°C and (d) filled with super insulation having an apparent thermal conductivity of 0.00015 w/m.k .

Sol:

Assumptions

1 Steady operating conditions exist since the plate temperatures remain constant.

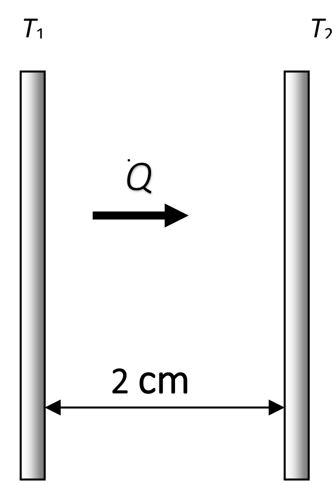
2 Heat transfer is one-dimensional since the plates are large.

3 The surfaces are black and thus ε = **1**

4 There are no convection currents in the air space between the plates.

Properties The thermal conductivities are *k* = 0.00015 W/m·°C for super insulation, *k* = 0.01979 W/m·°C at -50°C for air, and *k* = 0.036 W/m·°C for fiberglass insulation.

Analysis (*a*) Disregarding any natural convection currents, the rates of conduction and radiation heat transfer



$$\dot{Q}_{\text{cond}} = kA\frac{T_1 - T_2}{L} = (0.01979\ \text{W/m}^2.°\text{C})(1\text{m}^2)\frac{(290-150)\ \text{K}}{0.02\ \text{m}} = 139\ \text{W}$$

$$\dot{Q}_{\text{rad}} = \varepsilon\sigma A_s(T_1^{\ 4} - T_2^{\ 4})$$

$$= 1(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(1\text{m}^2)\left[(290\ \text{K})^4 - (150\ \text{K})^4\right] = 372\ \text{W}$$

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{cond}} + \dot{Q}_{\text{rad}} = 139 + 372 = \mathbf{511\ W}$$

(*b*) When the air space between the plates is evacuated, there will be radiation heat transfer only. Therefore,

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{rad}} = \mathbf{372\ W}$$

(*c*) In this case there will be conduction heat transfer through the fiberglass insulation only,

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{cond}} = kA\frac{T_1 - T_2}{L} = (0.036\ \text{W / m.}^{\text{o}}\text{C})(1\ \text{m}^2)\frac{(290-150)\ \text{K}}{0.02\ \text{m}} = \mathbf{252\ W}$$

(*d*) In the case of super insulation, the rate of heat transfer will be

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{cond}} = kA\frac{T_1 - T_2}{L} = (0.00015\ \text{W / m.}^\circ\text{C})(1\ \text{m}^2)\frac{(290-150)\ \text{K}}{0.02\ \text{m}} = \mathbf{1.05\ W}$$

H.W. 2

Hot air at 80°C is blown over a 2m x 4m flat surface at 30°C. If the average convection heat transfer coefficient is 55 W/m^2·K, determine the rate of heat transfer from the air to the plate, in kW.

Conduction Heat Transfer Through
Multilayer Plane Walls, Cylinder and Sphere

The Objective: The rate of heat transfer through a multilayer plane walls, cylinders, and spheres medium under steady conditions and surface temperatures difference.

Introduction: In heat transfer analysis, we are often interested in the rate of heat transfer through a medium under steady conditions and surface temperatures. Such problems can be solved easily without involving any differential equations by the introduction of *thermal resistance concepts* in an analogous manner to electrical circuit problems. In this case, the thermal resistance corresponds to electrical resistance, temperature difference corresponds to voltage, and the heat transfer rate corresponds to electric current. We start with *one-dimensional steady heat conduction* in a plane wall, a cylinder, and a sphere, and develop relations for *thermal resistances* in these geometries. We also develop thermal resistance relations for convection and radiation conditions at the boundaries. We apply this concept to heat conduction problems in *multilayer* plane walls, cylinders, and spheres.

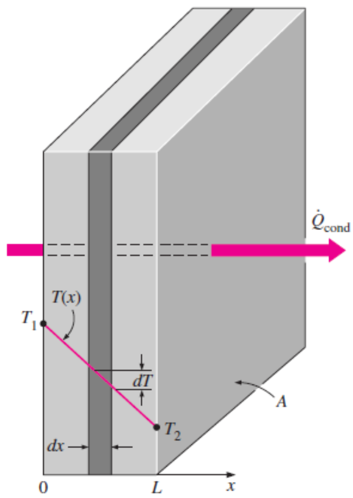
Test: What does the thermal resistance of a medium represent?

Summary:

Consider a plane wall of thickness L and average thermal conductivity k. The two surfaces of the wall are maintained at constant temperatures of T_1 and T_2. For one-dimensional steady heat conduction through the wall, we have $T(x)$. Then Fourier's law of heat conduction for the wall can be expressed as,

$$\dot{Q}_{\text{cond, wall}} = -kA\frac{dT}{dx} \qquad \text{(W)}$$

where the rate of conduction heat transfer $\dot{Q}_{\text{cond wall}}$ and the wall area A are constant. Thus we have dT/dx = constant, which means that *the temperature through the wall varies linearly with x.* That is, the temperature distribution in the wall under steady conditions is a *straight line.* Separating the variables in the above equation and integrating from $x = 0$, where $T(0) = T_1$, to $x = L$, where $T(L) = T_2$, we get



$$\int_{x=0}^{L} \dot{Q}_{\text{cond, wall}}\, dx = -\int_{T=T_1}^{T_2} kA\, dT$$

Performing the integrations and rearranging gives

$$\dot{Q}_{cond,wall} = kA\frac{T_1 - T_2}{L} \qquad (W)$$

The Thermal Resistance Concept

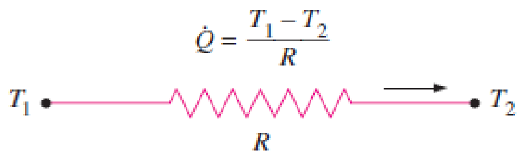
Heat conduction through a plane wall can be rearranged as

$$\dot{Q}_{cond,wall} = \frac{T_1 - T_2}{R_{wall}} \quad (W) \qquad where \quad R_{wall} = \frac{L}{kA} \quad (^{o}C/W)$$

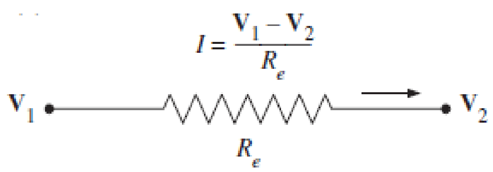
is the *thermal resistance* of the wall against heat conduction or simply the **conduction resistance** of the wall.

The equation above for heat flow is analogous to the relation for *electric current flow I*, expressed as

$$I = \frac{V_1 - V_2}{R_e}$$

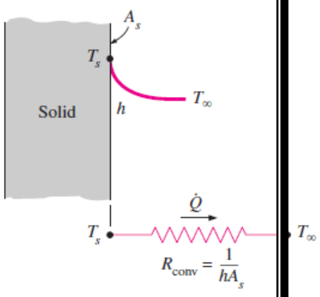


(*a*) Heat flow



(*b*) Electric current flow

Consider convection heat transfer from a solid surface of area A_s and temperature T_s to a fluid whose temperature sufficiently far from the surface is T_∞, with a convection heat transfer coefficient *h*. Newton's law of cooling for convection heat transfer rate,



$\dot{Q}_{conv} = hA_s(T_s - T_\infty)$ can be rearranged as,

$$\dot{Q}_{convl} = \frac{T_s - T_\infty}{R_{conv}} (W) \qquad where \qquad R_{convl} = \frac{1}{hA_s} (\,^oC/W)$$

is the *thermal resistance* of the surface against heat convection, or simply the **convection resistance** of the surface.

When the wall is surrounded by a gas, the *radiation effects*, which we have ignored so far, can be significant and may need to be considered. The rate of radiation heat transfer between a surface of emissivity ε and area A_s at temperature T_s and the surrounding surfaces at some average temperature T_{surr} can be expressed as,

$$\dot{Q}_{rad} = \varepsilon\sigma A_s(T_s^4 - T_{surr}^4) = h_{rad}A_s(T_s - T_{surr})$$

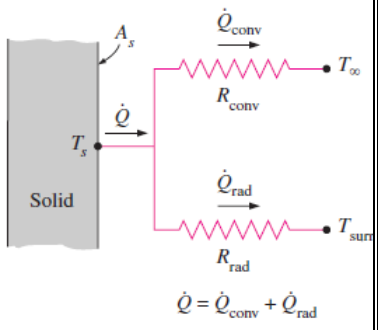
$$= \frac{T_s - T_{surr}}{R_{rad}} (W) \quad where R_{rad} = \frac{1}{h_{rad}A_s} (K/W)$$

is the *thermal resistance* of a surface against radiation, or the *radiation resistance*, and

$$h_{rad} = \frac{\dot{Q}_{rad}}{A_s(T_s - T_{surr})} = \varepsilon\sigma(T_s^2 + T_{surr}^2)(T_s + T_{surr}) \qquad (W/m^2 \cdot K)$$

is the **radiation heat transfer coefficient.** Note that both T_s and T_{surr} *must* be in K in the evaluation of h_{rad}. The definition of the radiation heat transfer coefficient enables us to express radiation conveniently in an analogous manner to convection in terms of a temperature difference. But h_{rad} depends strongly on temperature while h_{conv} usually does not.

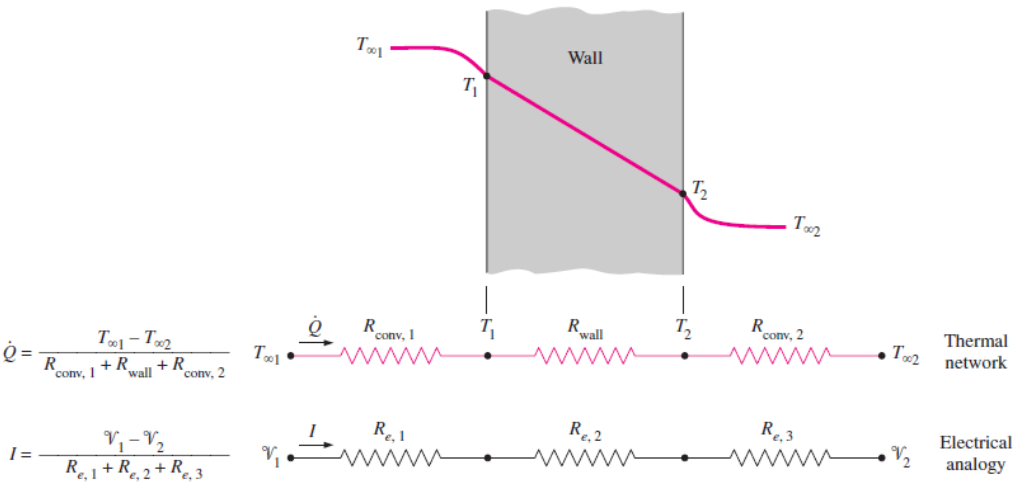
A surface exposed to the surrounding air involves convection and radiation simultaneously, and the total heat transfer at the surface is determined by adding (or subtracting, if in the opposite direction) the radiation and convection components. The convection and



radiation resistances are parallel to each other, as shown in the Fig., and may cause some complication in the thermal resistance network. When $T_{surr} = T_\infty$, the radiation effect can properly be accounted for by replacing h in the convection resistance relation by,

$$h_{combined} = h_{conv} + h_{rad} \qquad (W/m^2 \cdot K)$$

where $h_{combined}$ is the **combined heat transfer coefficient.** This way all the complications associated with radiation are avoided.



The thermal resistance network for heat transfer through a plane wall subjected to convection on both sides, and the electrical analogy.

Thermal Resistance Network

Now consider steady one-dimensional heat flow through a plane wall of thickness L, area A, and thermal conductivity k that is exposed to convection on both sides to fluids at temperatures $T_{\infty 1}$ and $T_{\infty 2}$ with heat transfer coefficients h_1

and h_2, respectively, as shown in the Figure. Assuming $T_{\infty 2} < T_{\infty 1}$, the variation of temperature will be as shown in the figure. Note that the temperature varies linearly in the wall, and asymptotically approaches $T_{\infty 1}$ and $T_{\infty 2}$ in the fluids as we move away from the wall.

Under steady conditions we have,

$$\begin{pmatrix} \text{Rate of} \\ \textit{heat convection} \\ \text{into the wall} \end{pmatrix} = \begin{pmatrix} \text{Rate of} \\ \textit{heat conduction} \\ \text{through the wall} \end{pmatrix} = \begin{pmatrix} \text{Rate of} \\ \textit{heat convection} \\ \text{from the wall} \end{pmatrix}$$

or

$$\dot{Q} = h_1 A(T_{\infty 1} - T_1) = kA \frac{T_1 - T_2}{L} = h_2 A(T_2 - T_{\infty 2})$$

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{1/h_1 A} = \frac{T_1 - T_2}{L/kA} = \frac{T_2 - T_{\infty 2}}{1/h_2 A}$$

$$= \frac{T_{\infty 1} - T_1}{R_{\text{conv, 1}}} = \frac{T_1 - T_2}{R_{\text{wall}}} = \frac{T_2 - T_{\infty 2}}{R_{\text{conv, 2}}}$$

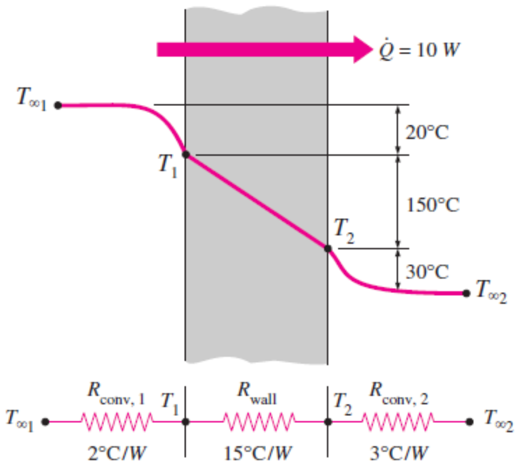
Adding the numerators and denominators yields,

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} \qquad (W)$$

Where $R_{total} = R_{conv,1} + R_{wall} + R_{conv,2} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A}$ $\qquad (^oC/W)$

$$\dot{Q} = \frac{\Delta T}{R_{total}}$$

We summarize this as *the rate of steady heat transfer between two surfaces is equal to the temperature difference divided by the total thermal resistance between those two surfaces.*



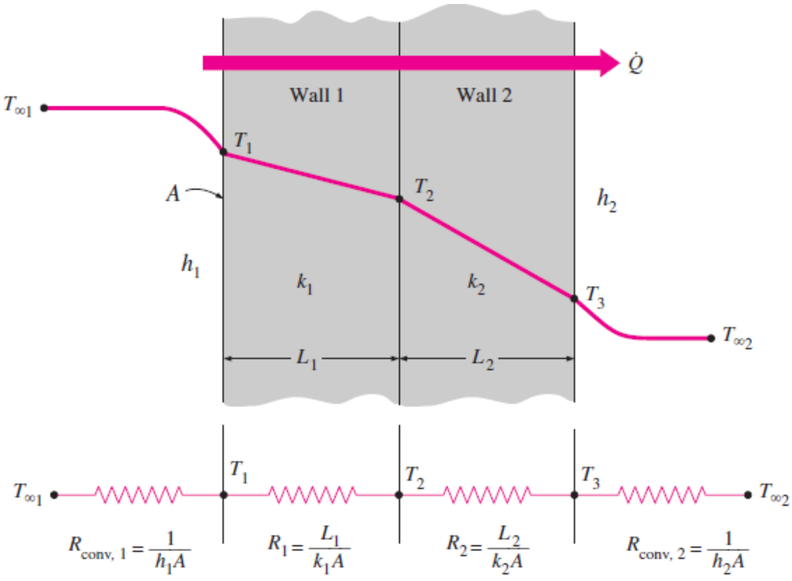
It is sometimes convenient to express heat transfer through a medium in an analogous manner to Newton's law of cooling as,

$$\dot{Q} = UA\,\Delta T \qquad (W)$$

where U is the **overall heat transfer coefficient.**

$$UA = \frac{1}{R_{total}} \qquad (W)$$

Multilayer Plane Walls



In practice we often encounter plane walls that consist of several layers of different materials. The thermal resistance concept can still be used to determine the rate of steady heat transfer through such *composite* walls.

$$\dot{Q} = \frac{T_{\infty1} - T_{\infty2}}{R_{total}} \qquad (W)$$

R_{total} = the *total thermal resistance*, expressed as

$$R_{total} = R_{conv,1} + R_{wall,1} + R_{wall,2} + R_{conv,2}$$

$$= \frac{1}{h_1A} + \frac{L_1}{k_1A} + \frac{L_2}{k_2A} + \frac{1}{h_2A} \qquad (^{\circ}C/W)$$

Once $\dot{Q}$ is *known*, an unknown surface temperature T_j at any surface or interface j can be determined from,

$$\dot{Q} = \frac{T_i - T_j}{R_{\text{total},\, i-j}}$$ where T_i is a *known* temperature at location i and $R_{\text{total},i\text{-}j}$ is the total thermal resistance between locations i and j.

To find T_1: $\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{\text{conv},1}}$

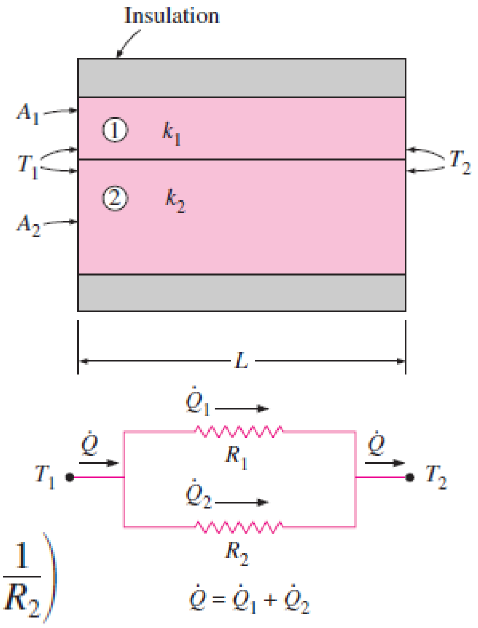
To find T_2: $\dot{Q} = \frac{T_{\infty 1} - T_2}{R_{\text{conv},1} + R_1}$

To find T_3: $\dot{Q} = \frac{T_3 - T_{\infty 2}}{R_{\text{conv},2}}$

Generalized Thermal Resistance Networks

The *thermal resistance* concept or the *electrical analogy* can also be used to solve steady heat transfer problems that involve parallel layers or combined series-parallel arrangements.

Consider the composite wall shown in the Figure, which consists of two parallel layers. The thermal resistance network, which consists of two parallel resistances, can be represented as shown in the figure. Noting that the total heat transfer is the sum of the heat transfers through each layer, we have

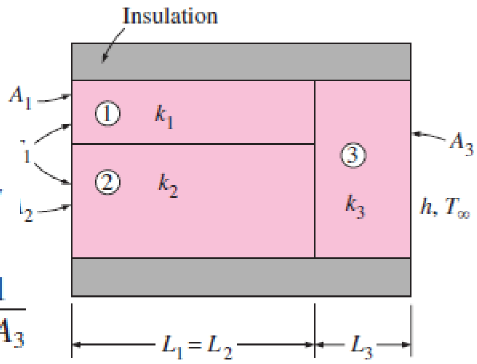


$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2 = \frac{T_1 - T_2}{R_1} + \frac{T_1 - T_2}{R_2} = (T_1 - T_2)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Utilizing electrical analogy, we get

$$\dot{Q} = \frac{T_1 - T_2}{R_{total}} \qquad where \qquad \frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} \rightarrow R_{total} = \frac{R_1 R_2}{R_1 + R_2}$$

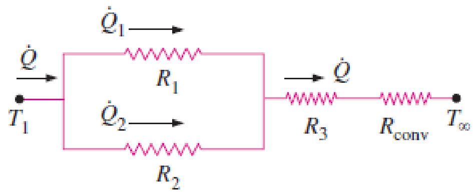
Now consider the combined series-parallel arrangement shown, the total rate of heat transfer through this composite system can again be expressed as,



where $\dot{Q} = \frac{T_1 - T_\infty}{R_{total}}$

and $$R_{total} = R_{12} + R_3 + R_{conv} = \frac{R_1 R_2}{R_1 + R_2} + R_3 + R_{conv}$$

$$R_1 = \frac{L_1}{k_1 A_1}, \quad R_2 = \frac{L_2}{k_2 A_2}, \quad R_3 = \frac{L_3}{k_3 A_3}, \quad R_{conv} = \frac{1}{hA_3}$$



Once the individual thermal resistances are evaluated, the total resistance and the total rate of heat transfer can easily be determined from the relations above.

Ex1: Find heat transfer per unit area through the composite wall Assume one-dimensional heat flow.

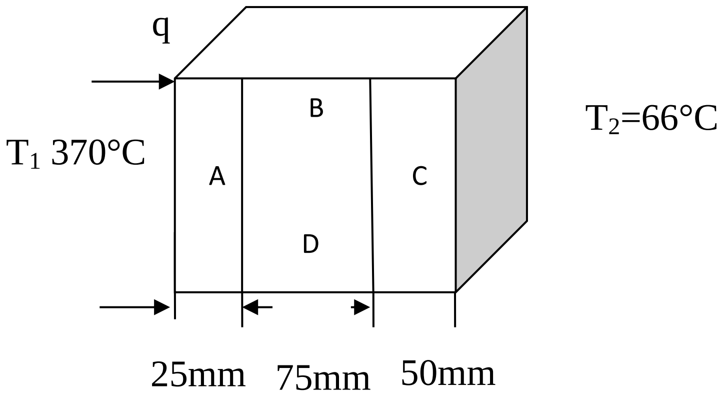
KA= 150 W/m.°c

KB=30

KC=50

KD=70

Ac=0.1m²



SOL:

$$R = \frac{\Delta x}{kA}$$

$$R_A = \frac{0.025}{(150)(0.1)} = 1.667 \times 10^{-3}$$

$$R_B = \frac{0.075}{(30)(0.05)} = 0.05$$

$$R_C = \frac{0.05}{(50)(0.1)} = 0.01$$

$$R_D = \frac{0.075}{(70)(0.05)} = 0.02143$$

$$R = R_A + R_C + \frac{1}{\frac{1}{R_B} + \frac{1}{R_D}} = 2.667 \times 10^{-2}$$

$$q = \frac{\Delta T}{R} = \frac{370 - 66}{2.667 \times 10^{-2}} = 11{,}400 \text{ W}$$

Radial system:
Heat transfer:

$$q = -kA\frac{dT}{dr} \quad A = 2\pi rL$$

$$q = -2\pi rLk\frac{dT}{dr}$$

$$q\int_{ri}^{ro} \frac{dr}{r} = -2\pi kL\int_{Ti}^{To} dT$$

$$at \;\; r = r_i \quad T = T_i$$

$$r = r_o \quad T = T_O$$

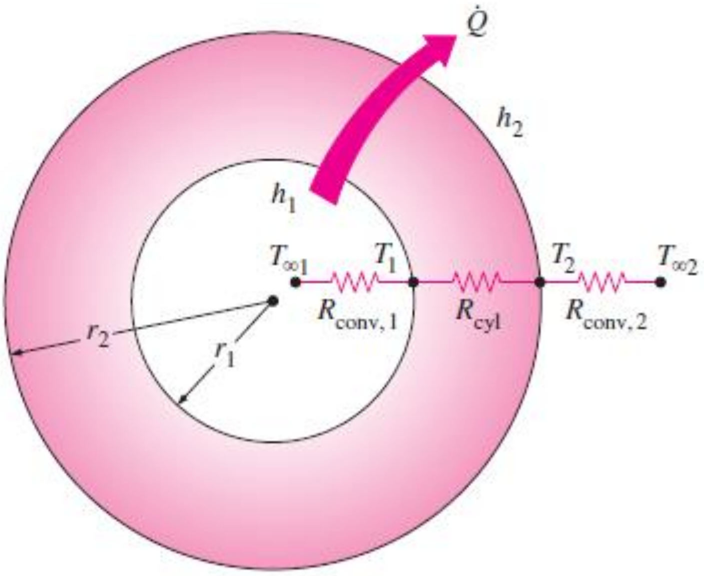
$$q\, ln\frac{r_o}{r_i} = -2\pi kL(T_o - T_i) \qquad\qquad q = \frac{2\pi kL(T_i - T_o)}{ln\frac{r_o}{r_i}}$$

HEAT CONDUCTION IN CYLINDERS AND SPHERES

$$R_{cyl} = \frac{\ln(r_2/r_1)}{2\pi Lk} = \frac{\ln(\text{Outer radius/Inner radius})}{2\pi \times (\text{Length}) \times (\text{Thermal conductivity})}$$

$$R_{sph} = \frac{r_2 - r_1}{4\pi r_1 r_2 k} = \frac{\text{Outer radius} - \text{Inner radius}}{4\pi(\text{Outer radius})(\text{Inner radius})(\text{Thermal conductivity})}$$

$$q = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}$$



$$R_{total} = R_{conv,1} + R_{cyl} + R_{conv,2}$$

where

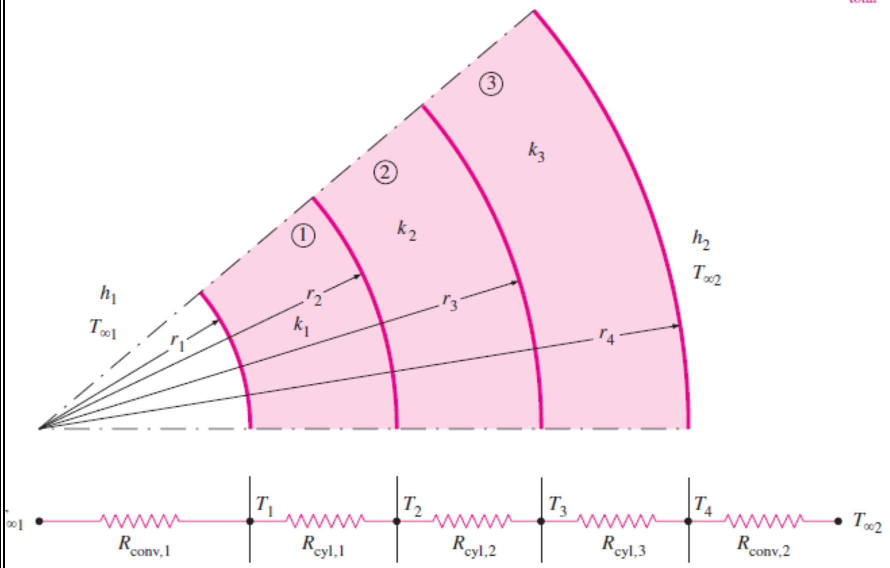
$$R_{total} = R_{conv,\,1} + R_{cyl} + R_{conv,\,2} = \frac{1}{(2\pi r_1 L)h_1} + \frac{\ln(r_2/r_1)}{2\pi L k} + \frac{1}{(2\pi r_2 L)h_2}$$

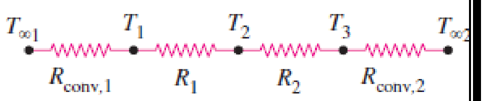
for a *cylindrical* layer, and

$$R_{total} = R_{conv,\,1} + R_{sph} + R_{conv,\,2} = \frac{1}{(4\pi r_1^2)h_1} + \frac{r_2 - r_1}{4\pi r_1 r_2 k} + \frac{1}{(4\pi r_2^2)h_2}$$

Multilayered Cylinders and Spheres

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}$$





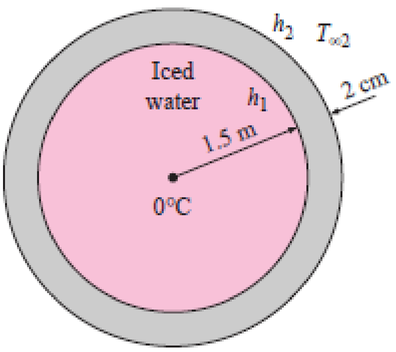
$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{conv,1}} = \frac{T_{\infty 1} - T_2}{R_{conv,1} + R_1} = \frac{T_1 - T_3}{R_1 + R_2} = \frac{T_2 - T_3}{R_2} = \frac{T_2 - T_{\infty 2}}{R_2 + R_{conv,2}}$$

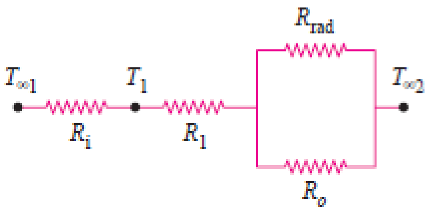
where R_{total} is the *total thermal resistance*, expressed as

$$R_{total} = R_{conv,\,1} + R_{cyl,\,1} + R_{cyl,\,2} + R_{cyl,\,3} + R_{conv,\,2} = \frac{1}{h_1 A_1} + \frac{\ln(r_2/r_1)}{2\pi L k_1} + \frac{\ln(r_3/r_2)}{2\pi L k_2} + \frac{\ln(r_4/r_3)}{2\pi L k_3} + \frac{1}{h_2 A_4}$$

Ex2: A 3-m internal diameter spherical tank made of 2-cm-thick stainless steel (k = 15 W/m.°C) is used to store iced water at $T_{\infty 1}$ = 0°C. The tank is located in a room whose temperature is $T_{\infty 2}$ = 22°C. The walls of the room are also at 22°C. The outer surface of the tank is black and heat transfer between the outer surface of the tank and the surroundings is by natural convection and radiation. The convection heat transfer coefficients at the inner and the outer surfaces of the tank are $h1$ = 80 W/m^2.°C and $h2$ = 10 W/m^2.°C, respectively. Determine the rate of heat transfer to the iced water in the tank and

SOL:





Assumptions **1** Heat transfer is steady since the specified thermal conditions at the boundaries do not change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the midpoint. **3** Thermal conductivity is constant.

Properties The thermal conductivity of steel is given to be k = 15 W/m · °C. The heat of fusion of water at atmospheric pressure is h_{if} = 333.7 kJ/kg. The outer surface of the tank is black and thus its emissivity is ε = 1.

Analysis (*a*) The thermal resistance network for this problem is given in Fig. 3–28. Noting that the inner diameter of the tank is D_1 = 3 m and the outer diameter is D_2 = 3.04 m, the inner and the outer surface areas of the tank are

$$A_1 = \pi D_1^2 = \pi(3\text{ m})^2 = 28.3\text{ m}^2$$
$$A_2 = \pi D_2^2 = \pi(3.04\text{ m})^2 = 29.0\text{ m}^2$$

Also, the radiation heat transfer coefficient is given by

$$h_{rad} = \varepsilon\sigma(T_2^2 + T_{\infty 2}^2)(T_2 + T_{\infty 2})$$

But we do not know the outer surface temperature T_2 of the tank, and thus we cannot calculate h_{rad}. Therefore, we need to assume a T_2 value now and check the accuracy of this assumption later. We will repeat the calculations if necessary using a revised value for T_2.

We note that T_2 must be between 0°C and 22°C, but it must be closer to 0°C, since the heat transfer coefficient inside the tank is much larger. Taking T_2 = 5°C = 278 K, the radiation heat transfer coefficient is determined to be

$$h_{rad} = (1)(5.67 \times 10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(295\text{ K})^2 + (278\text{ K})^2][(295 + 278)\text{ K}]$$
$$= 5.34\text{ W/m}^2\cdot\text{K} = 5.34\text{ W/m}^2\cdot\text{°C}$$

Then the individual thermal resistances become

$$R_i = R_{conv,1} = \frac{1}{h_1 A_1} = \frac{1}{(80\text{ W/m}^2\cdot\text{°C})(28.3\text{ m}^2)} = 0.000442\text{°C/W}$$

$$R_1 = R_{sphere} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{(1.52 - 1.50)\text{ m}}{4\pi(15\text{ W/m}\cdot\text{°C})(1.52\text{ m})(1.50\text{ m})}$$
$$= 0.000047\text{°C/W}$$

$$R_o = R_{conv,2} = \frac{1}{h_2 A_2} = \frac{1}{(10\text{ W/m}^2\cdot\text{°C})(29.0\text{ m}^2)} = 0.00345\text{°C/W}$$

$$R_{rad} = \frac{1}{h_{rad} A_2} = \frac{1}{(5.34\text{ W/m}^2\cdot\text{°C})(29.0\text{ m}^2)} = 0.00646\text{°C/W}$$

The two parallel resistances R_o and R_{rad} can be replaced by an equivalent resistance R_{equiv} determined from

$$\frac{1}{R_{equiv}} = \frac{1}{R_o} + \frac{1}{R_{rad}} = \frac{1}{0.00345} + \frac{1}{0.00646} = 444.7\text{ W/°C}$$

which gives

$$R_{equiv} = 0.00225\text{°C/W}$$

Now all the resistances are in series, and the total resistance is determined to be

$$R_{total} = R_i + R_1 + R_{equiv} = 0.000442 + 0.000047 + 0.00225 = 0.00274°C/W$$

Then the steady rate of heat transfer to the iced water becomes

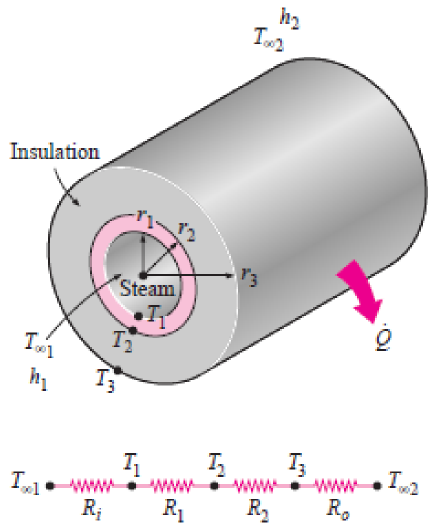
$$\dot{Q} = \frac{T_{\infty 2} - T_{\infty 1}}{R_{total}} = \frac{(22 - 0)°C}{0.00274°C/W} = \mathbf{8029\ W} \qquad (\text{or } \dot{Q} = 8.027\ \text{kJ/s})$$

To check the validity of our original assumption, we now determine the outer surface temperature from

$$\dot{Q} = \frac{T_{\infty 2} - T_2}{R_{equiv}} \longrightarrow T_2 = T_{\infty 2} - \dot{Q}R_{equiv}$$
$$= 22°C - (8029\ W)(0.00225°C/W) = 4°C$$

which is sufficiently close to the 5°C assumed in the determination of the radiation heat transfer coefficient. Therefore, there is no need to repeat the calculations using 4°C for T_2.

Ex3: Steam at $T_{\infty 1}$ = 320°C flows in a cast iron pipe (k= 80 W/m. °C) whose inner and outer diameters are D_1= 5 cm and D_2= 5.5 cm, respectively. The pipe is covered with 3cm-thick glass wool insulation with k= 0.05 W/m. °C. Heat is lost to the surroundings at $T_{\infty 2}$= 5°C by natural convection and radiation, with a combined heat transfer coefficient of h_2 =18 W/m^2.°C. Taking the heat transfer coefficient inside the pipe to be h_1= 60 W/m^2°C, determine the rate of heat loss from the steam per unit length of the pipe. Also determine the temperature drops across the pipe shell and the insulation.



SOLUTION A steam pipe covered with glass wool insulation is subjected to convection on its surfaces. The rate of heat transfer per unit length and the temperature drops across the pipe and the insulation are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivities are given to be $k = 80$ W/m · °C for cast iron and $k = 0.05$ W/m · °C for glass wool insulation.

Analysis The thermal resistance network for this problem involves four resistances in series and is given in Fig. 3–29. Taking $L = 1$ m, the areas of the surfaces exposed to convection are determined to be

$$A_1 = 2\pi r_1 L = 2\pi(0.025 \text{ m})(1 \text{ m}) = 0.157 \text{ m}^2$$
$$A_3 = 2\pi r_3 L = 2\pi(0.0575 \text{ m})(1 \text{ m}) = 0.361 \text{ m}^2$$

Then the individual thermal resistances become

$$R_i = R_{\text{conv, 1}} = \frac{1}{h_1 A} = \frac{1}{(60 \text{ W/m}^2 \cdot {}^\circ\text{C})(0.157 \text{ m}^2)} = 0.106^\circ\text{C/W}$$
$$R_1 = R_{\text{pipe}} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(2.75/2.5)}{2\pi(80 \text{ W/m} \cdot {}^\circ\text{C})(1 \text{ m})} = 0.0002^\circ\text{C/W}$$
$$R_2 = R_{\text{insulation}} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(5.75/2.75)}{2\pi(0.05 \text{ W/m} \cdot {}^\circ\text{C})(1 \text{ m})} = 2.35^\circ\text{C/W}$$
$$R_o = R_{\text{conv, 2}} = \frac{1}{h_2 A_3} = \frac{1}{(18 \text{ W/m}^2 \cdot {}^\circ\text{C})(0.361 \text{ m}^2)} = 0.154^\circ\text{C/W}$$

Noting that all resistances are in series, the total resistance is determined to be

$$R_{\text{total}} = R_i + R_1 + R_2 + R_o = 0.106 + 0.0002 + 2.35 + 0.154 = 2.61^\circ\text{C/W}$$

Then the steady rate of heat loss from the steam becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{(320 - 5)^\circ\text{C}}{2.61^\circ\text{C/W}} = \mathbf{121 \text{ W}} \qquad \text{(per m pipe length)}$$

The heat loss for a given pipe length can be determined by multiplying the above quantity by the pipe length L.

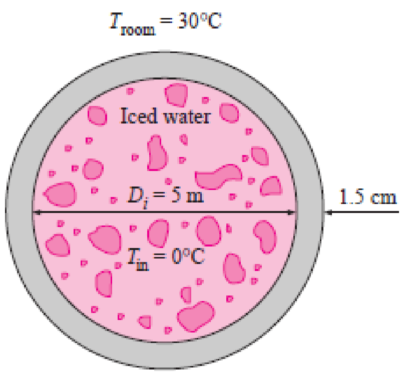
The temperature drops across the pipe and the insulation are determined from Eq. 3–17 to be

$$\Delta T_{\text{pipe}} = \dot{Q} R_{\text{pipe}} = (121 \text{ W})(0.0002^\circ\text{C/W}) = \mathbf{0.02^\circ C}$$
$$\Delta T_{\text{insulation}} = \dot{Q} R_{\text{insulation}} = (121 \text{ W})(2.35^\circ\text{C/W}) = \mathbf{284^\circ C}$$

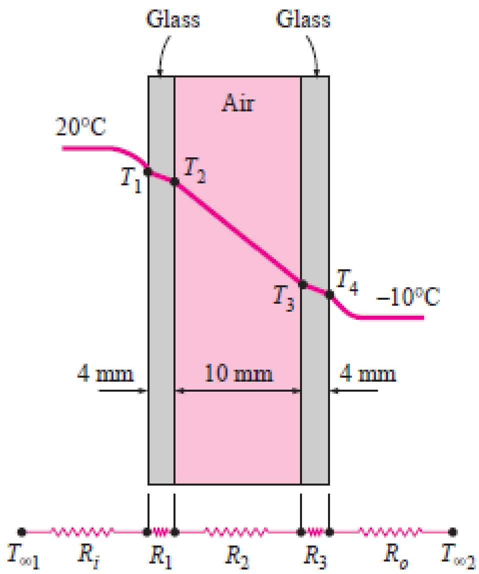
That is, the temperatures between the inner and the outer surfaces of the pipe differ by 0.02°C, whereas the temperatures between the inner and the outer surfaces of the insulation differ by 284°C.

H.W: A 5-m-internal-diameter spherical tank made of 1.5-cm-thick stainless steel (k=15 W/m.°C) is used to store iced water at 0°C. The tank is located in a room whose temperature is 30°C. The walls of the room are also at 30°C. The outer surface of the tank is black (emissivity = 1), and heat transfer between the outer surface of the tank and the surroundings is by natural convection and radiation. The convection heat transfer coefficients at the inner and the outer surfaces of the tank are 80 W/m^2. °C and 10 W/m^2. °C, respectively. Determine

(a) the rate of heat transfer to the iced water in the tank



H.W: Consider a 0.8-m-high and 1.5-m-wide double-pane window consistingof two 4-mm-thick layers of glass (k= 0.78 W/m · °C) separated by a 10mm wide stagnant air space (k = 0.026 W/m · °C). Determine the steady rate of heat



Critical Radius of Insulation,

The Objective: Investigate the steady state one dimensional heat conduction in a cylinder and sphere, and estimate the critical radius of insulation for them.

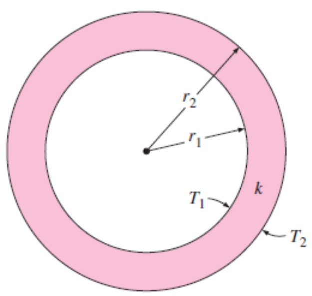
Introduction: Steady heat transfer through a cylinder or sphere, and the multilayered cylindrical or spherical shells can be handled just like plane walls by simply adding an *additional resistance* in series for each *additional layer*. The additional insulation increases the conduction resistance of the insulation layer but decreases the convection resistance of the surface because of the increase in the outer surface area for convection. The heat transfer from the pipe may increase or decrease, depending on which effect dominates. Critical radius of insulation is confirmed the idea of reducing the rate of heat loss from the cylindrical and spherical shells depending on which effect dominates.

Test: What is the critical radius of insulation? How is it defined for a cylindrical layer?

Summery:

Heat transfer through a pipe can be modeled as *steady* and *one-dimensional*. The temperature of the pipe in this case will depend on one direction only (the radial *r*-direction) and can be expressed as $T = T(r)$. The temperature is independent of the azimuthal angle or the axial distance. This situation is approximated in practice in long cylindrical pipes and spherical containers. In *steady* operation, there is no change in the temperature of the pipe with time at any point. Therefore, the rate of heat transfer into the pipe must be equal to the rate of heat transfer out of it. In other words, heat transfer through the pipe must be constant, $\dot{Q}_{conduction, cyl.} =$ constant.

Consider a long cylindrical layer (such as a circular pipe) of inner radius r_1, outer radius r_2, length L, and average thermal conductivity k. The two surfaces of the cylindrical layer are maintained at constant temperatures T_1 and T_2. There is no heat generation in the layer and the thermal conductivity is constant. For one-dimensional heat conduction through the cylindrical layer, we have $T(r)$. Then Fourier's law of heat conduction for heat transfer through the cylindrical layer can be expressed as,



$$\dot{Q}_{\text{cond, cyl}} = -kA\frac{dT}{dr} \qquad \text{(W)}$$

where $A = 2\pi rL$ is the heat transfer area at location r. Note that A depends on r, and thus it *varies* in the direction of heat transfer. Separating the variables in the above equation and integrating from $r = r_1$, where $T(r_1) = T_1$, to $r = r_2$, where $T(r_2) = T_2$, gives,

$$\int_{r=r_1}^{r_2} \frac{\dot{Q}_{\text{cond, cyl}}}{A}\, dr = -\int_{T=T_1}^{T_2} k\, dT$$

Substituting $A = 2\pi rL$ and performing the integrations give,

$$\dot{Q}_{\text{cond, cyl}} = 2\pi Lk \frac{T_1 - T_2}{\ln(r_2/r_1)} \qquad \text{(W)}$$

since $\dot{Q}_{conduction,cyl.} =$ constant. This equation can be rearranged as,

$$\dot{Q}_{cond,cyl.} = \frac{T_1 - T_2}{R_{cyl}} \qquad (W)$$

where $R_{cyl.} = \dfrac{\ln(r_2/r_1)}{2\pi Lk} = \dfrac{\ln(\text{Outer radius/Inner radius})}{2\pi \text{ x (Length) x (Thermal conductivity)}}$

is the *thermal resistance* of the cylindrical layer against heat conduction, or simply the **conduction resistance** of the cylinder layer.

We can repeat the analysis above for a *spherical layer* by taking $A = 4\pi r^2$ and performing the integrations, the result can be expressed as,

$$\dot{Q}_{cond,sph.} = \frac{T_1 - T_2}{R_{sph.}} \qquad (W)$$

where

$$R_{sph.} = \frac{r_2 - r_1}{4\pi r_1 r_2 k} = \frac{\text{Outer radius - Inner radius}}{4\pi \text{ x (outer radius) x (Inner radius) x (Thermal conductivity)}}$$

is the *thermal resistance* of the spherical layer against heat conduction, or simply the **conduction resistance** of the spherical layer.

Now consider steady one-dimensional heat flow through a cylindrical or spherical layer that is exposed to convection on both sides to fluids at temperatures $T_{\infty 1}$ and $T_{\infty 2}$ with heat transfer coefficients h_1 and h_2, respectively. The thermal resistance network in this case consists of one conduction and two convection resistances in series, just like the one for the plane wall, and the rate of heat transfer under steady conditions can be expressed as,

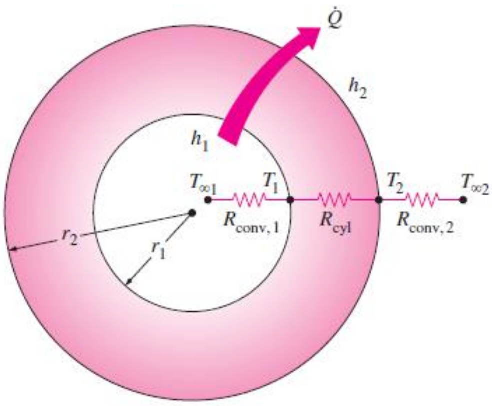
$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} \qquad (W)$$

where,

$$R_{total} = R_{conv,1} + R_{cyl.} + R_{conv,2}$$
$$= \frac{1}{(2\pi r_1 L)h_1} + \frac{\ln(r_2/r_1)}{2\pi Lk} + \frac{1}{(2\pi r_2 L)h_2} \qquad (^{o}C/W)$$

for a *cylindrical* layer, and

$$R_{total} = R_{conv,1} + R_{sph.} + R_{conv,2}$$
$$= \frac{1}{(4\pi r_1^2)h_1} + \frac{r_2 - r_1}{4\pi r_1 r_2 k} + \frac{1}{(4\pi r_2^2 L)h_2} \qquad (^{o}C/W)$$

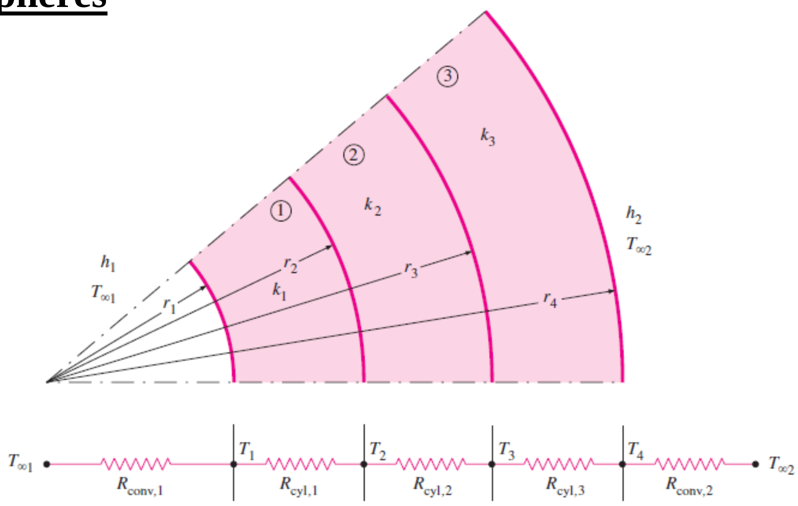


$R_{total} = R_{conv,1} + R_{cyl} + R_{conv,2}$

for a spherical layer. Note that A in the convection resistance relation $R_{conv} = 1/hA$ is the *surface area at which convection occurs*. It is equal to $A = 2\pi rL$ for a cylindrical surface and $A = 4\pi r^2$ for a spherical surface of radius r. Also note that the thermal resistances are in series, and thus the total thermal resistance is determined by simply adding the individual resistances, just like the electrical resistances connected in series.

Multilayered Cylinders and Spheres

Steady heat transfer through multilayered cylindrical or spherical shells can be handled just like multilayered plane walls discussed earlier by simply adding an *additional resistance* in series for each *additional layer*. For example, the steady heat transfer rate through the three-layered composite cylinder of length L



with convection on both sides
can be expressed as,

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} \quad (W)$$

$$R_{total} = R_{conv,1} + R_{cyl.1} + R_{cyl.2} + R_{cyl.3} + R_{conv,2}$$

$$= \frac{1}{h_1 A_1} + \frac{\ln(r_2/r_1)}{2\pi L k_1} + \frac{\ln(r_3/r_2)}{2\pi L k_2} + \frac{\ln(r_4/r_3)}{2\pi L k_3} + \frac{1}{h_2 A_4} \qquad (^oC/W)$$

where $A_1 = 2\pi r_1 L$ and $A_4 = 2\pi r_4 L$.

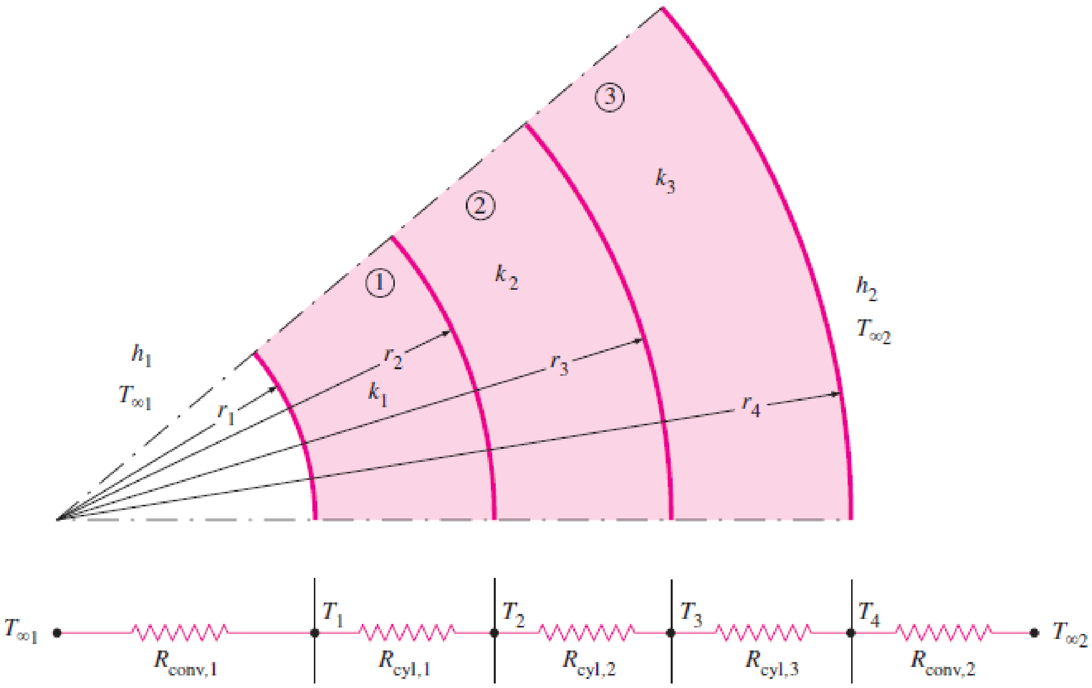
This equation can also be used for a three-layered spherical shell by replacing the thermal resistances of cylindrical layers by the corresponding spherical ones.

Once $\dot{Q}$ has been calculated, the interface temperature T_2 between the first and second cylindrical layers can be determined from,

$$\dot{Q} = \frac{T_{\infty 1} - T_2}{R_{conv,1} + R_{cyl,1}} = \frac{T_{\infty 1} - T_2}{\dfrac{1}{h_1(2\pi r_1 L)} + \dfrac{\ln(r_2/r_1)}{2\pi L k_1}}$$

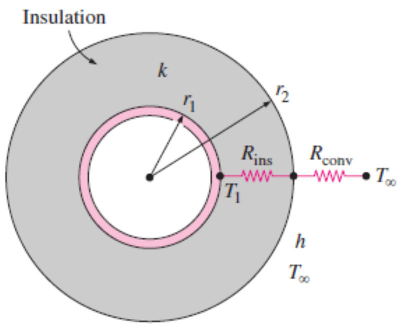
We could also calculate T_2 from

$$\dot{Q} = \frac{T_2 - T_{\infty 2}}{R_2 + R_3 + R_{conv,2}} = \frac{T_2 - T_{\infty 2}}{\dfrac{\ln(r_3/r_2)}{2\pi L k_2} + \dfrac{\ln(r_4/r_3)}{2\pi L k_3} + \dfrac{1}{h_o(2\pi r_4 L)}}$$



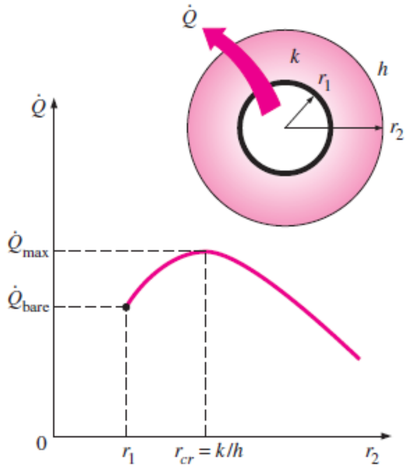
Critical Radius of Insulation

Adding more insulation to a wall always decreases heat transfer. The thicker the insulation produced the lower the heat transfer rate. This is expected, since the heat transfer area A is constant, and adding insulation always increases the thermal resistance of the wall without increasing the convection resistance.



Adding insulation to a cylindrical pipe or a spherical shell, however, is a different matter. The additional insulation increases the conduction resistance of the insulation layer but decreases the convection resistance of the surface because of the increase in the outer surface area for convection. The heat transfer from the pipe may increase or decrease, depending on which effect dominates.

Consider a cylindrical pipe of outer radius r_1 whose outer surface temperature T_1 is maintained constant. The pipe is now insulated with a material whose thermal conductivity is k and outer radius is r_2. Heat is lost from the pipe to the surrounding medium at temperature T_∞, with a convection heat transfer coefficient h. The rate of heat transfer from the insulated pipe to the surrounding air can be expressed as,



$$\dot{Q} = \frac{T_1 - T_\infty}{R_{ins} + R_{conv}} = \frac{T_1 - T_\infty}{\dfrac{\ln(r_2/r_1)}{2\pi Lk} + \dfrac{1}{h(2\pi r_2 L)}}$$

The variation of $\dot{Q}$ with the outer radius of the insulation r_2 is plotted in the Figure. The value of r_2 at which $\dot{Q}$ reaches a maximum is determined from the requirement that $d\dot{Q}/dr_2=0$ (zero slope). Performing the differentiation and solving for r_2 yields the **critical radius of insulation** for a cylindrical body to be,

$$r_{cr,cylinder} = \frac{k}{h} \qquad (m)$$

The value of the critical radius r_{cr} will be the largest when k is large and h is small. Noting that the lowest value of h encountered in practice is about 5 $W/m^2 \cdot °C$ for the case of natural convection of gases, and that the thermal conductivity of common insulating materials is about 0.05 W/m^2 · °C, the largest value of the critical radius we are likely to encounter is

$$r_{cr,\max} = \frac{k_{\max,insulation}}{h_{\min}} \approx \frac{0.05\,W/m.^oC}{5\,W/m.^oC} = 0.01m = 1cm$$

This value would be even smaller when the radiation effects are considered. The critical radius would be much less in forced convection, often less than 1 mm, because of much larger *h* values associated with forced convection. Therefore, we can insulate hot water or steam pipes freely without worrying about the possibility of increasing the heat transfer by insulating the pipes.

The discussions above can be repeated for a sphere, and it can be shown in a similar manner that the critical radius of insulation for a spherical shell is,

$$r_{cr,csphere} = \frac{2k}{h}$$

where *k* is the thermal conductivity of the insulation and *h* is the convection heat transfer coefficient on the outer surface.

Question: A pipe is insulated such that the outer radius of the insulation is less than the critical radius. Now the insulation is taken off. Will the rate of heat transfer from the pipe increase or decrease for the same pipe surface temperature?

Ex.1: A 3mm diameter and 5m long electric wire is tightly wrapped with a 2mm thick plastic cover whose thermal conductivity is k = 0.15 W/m°C. Electrical measurements indicate that a current of **10A** passes through the wire and there is a voltage drop of 8 V along the wire. If the insulated wire is exposed to a medium at T_∞=30°C with a heat transfer coefficient of h=12 W/m^2.°C, determine the temperature at the interface of the wire and the plastic cover in steady operation. Also determine whether doubling the thickness of the plastic cover will increase or decrease this interface temperature.

Analysis Heat is generated in the wire and its temperature rises as a result of resistance heating. We assume heat is generated uniformly throughout the wire and is transferred to the surrounding medium in the radial direction. In steady operation, the rate of heat transfer becomes equal to the heat generated within the wire, which is determined to be

$$\dot{Q} = \dot{W}_e = VI = (8\text{ V})(10\text{ A}) = 80\text{ W}$$

The thermal resistance network for this problem involves a conduction resistance for the plastic cover and a convection resistance for the outer surface in series, as shown in Fig. 3–32. The values of these two resistances are determined to be

$$A_2 = (2\pi r_2)L = 2\pi(0.0035\text{ m})(5\text{ m}) = 0.110\text{ m}^2$$

$$R_{\text{conv}} = \frac{1}{hA_2} = \frac{1}{(12\text{ W/m}^2\cdot{}^\circ\text{C})(0.110\text{ m}^2)} = 0.76^\circ\text{C/W}$$

$$R_{\text{plastic}} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(3.5/1.5)}{2\pi(0.15\text{ W/m}\cdot{}^\circ\text{C})(5\text{ m})} = 0.18^\circ\text{C/W}$$

and therefore

$$R_{\text{total}} = R_{\text{plastic}} + R_{\text{conv}} = 0.76 + 0.18 = 0.94^\circ\text{C/W}$$

Then the interface temperature can be determined from

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{\text{total}}} \longrightarrow T_1 = T_\infty + \dot{Q}R_{\text{total}}$$
$$= 30^\circ\text{C} + (80\text{ W})(0.94^\circ\text{C/W}) = 105^\circ\text{C}$$

Note that we did not involve the electrical wire directly in the thermal resistance network, since the wire involves heat generation.

To answer the second part of the question, we need to know the critical radius of insulation of the plastic cover. It is determined from Eq. 3–50 to be

$$r_{cr} = \frac{k}{h} = \frac{0.15\text{ W/m}\cdot{}^\circ\text{C}}{12\text{ W/m}^2\cdot{}^\circ\text{C}} = 0.0125\text{ m} = 12.5\text{ mm}$$

which is larger than the radius of the plastic cover. Therefore, increasing the thickness of the plastic cover will *enhance* heat transfer until the outer radius of the cover reaches 12.5 mm. As a result, the rate of heat transfer $\dot{Q}$ will *increase* when the interface temperature T_1 is held constant, or T_1 will *decrease* when $\dot{Q}$ is held constant, which is the case here.

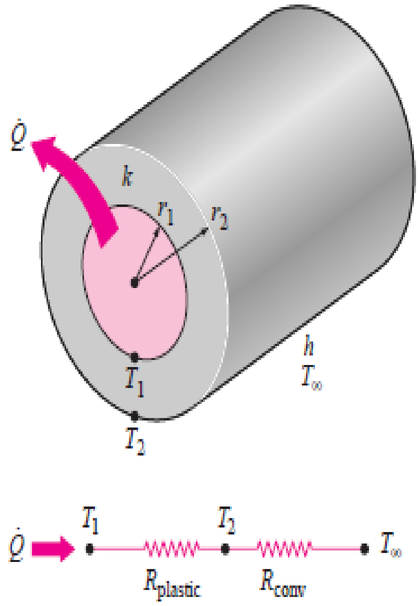
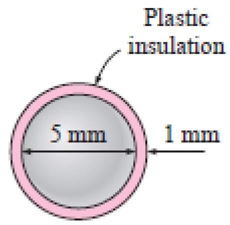


FIGURE 3–32
Schematic for Example 3–9.

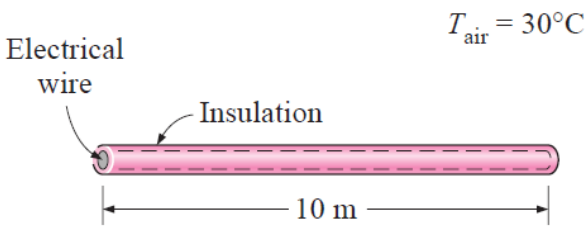
H.W.1

A 5-mm-diameter spherical ball at 50°C is covered by a 1-mm-thick plastic insulation (k =0.13 W/m.°C). The ball is exposed to a medium at 15°C, with a combined convection and radiation heat transfer coefficient of 20 W/m^2.°C. Determine if the plastic insulation on the ball will help or hurt heat transfer from the ball.



H.W.2

A 2-mm-diameter and 10 m long electric wire is tightly wrapped with a 1-mm-thick plastic cover whose thermal conductivity is k=0.15W/m.°C. Electrical measurements indicate that a current of **10 A** passes through the wire and there is a voltage drop of 8 V along the wire. If the insulated wire is exposed to a medium at T_∞ =30°C with a heat transfer coefficient of h=24 W/m^2.°C, determine the temperature at the interface of the wire and the plastic cover in steady operation. Also determine if doubling the thickness of the plastic cover will increase or decrease this interface temperature.



Steady State One Dimensional Heat Conduction in a Large Plane Wall, and in a Cylinder

The Objective: Study the heat conduction through a large plane wall and cylinder as one-dimension steady state case. This chapter deals with the theoretical and mathematical aspects of heat conduction.

Introduction: Heat conduction through a large plane wall such as the wall of a house, the glass of a single pane window, the metal plate at the bottom of a pressing iron, a cast iron steam pipe, a cylindrical nuclear fuel element, an electrical resistance wire, the wall of a spherical container, or a spherical metal ball that is being quenched or tempered. Heat conduction in these and much other geometry can be approximated as being *one-dimensional* since heat conduction through these geometries will be dominant in one direction and negligible in other directions.

Test: Why does metal feel colder than wood, if they are both at the same temperature?

Summary:

General Concepts in Conduction Heat Transfer

1- Heat conduction in a medium is three-dimensional and time dependent, $T=T(x,y,z,t)$.

2- Heat conduction in a medium is said to be steady when the temperature does not very with time, and unsteady or transient when it does very with time.

3- Heat conduction in a medium is said to be one-dimensional when conduction is significant in one dimension only and negligible in the other dimensions, two-dimensional when conduction in the third dimension is negligible and three-dimensional when conduction in all dimensions is significant.

4- Conduction can take place in liquids and gases as well as solids provided that there is no bulk motion involved in the liquid or gas.

5- Heat transfer has direction as well as magnitude, and thus it is a vector quantity.

6- In coordinate system, a positive quantity indicates heat transfer in the positive direction and a negative quantity indicates heat transfer in the negative direction.

7- The driving force for any form of heat transfer is the temperature difference. The larger the temperature difference, the larger the rate of heat transfer.

8- The specification of the temperature at a point in a medium first requires the specification of the location of that point, by choosing a suitable coordinate system such as rectangular, cylindrical or spherical coordinates.

9- The location of a point is specified as (x,y,z) in rectangular coordinates, or as (r,φ,z) in cylindrical coordinates, and or (r,φ,θ) in spherical coordinates.

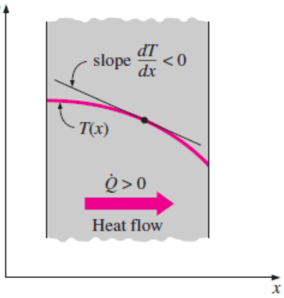
10- The best coordinate system for a given geometry is the one that describes the surfaces of the geometry best.

11- Thermal conductivity of the material is a measure of the ability of a material to conduct heat. It is in general, varies with temperature, but sufficiently accurate results can be obtained by using a constant value for thermal conductivity at the average temperature.

Fourier' Law

The rate of heat conduction through a medium in a specified direction (say, in the *x*-direction) is proportional to the temperature difference across the medium and the area normal to the direction of heat transfer, but is inversely proportional to the distance in that direction. This was expressed in the differential form by **Fourier's law of heat conduction** for one-dimensional heat conduction as,

$$\dot{Q}_{\text{cond}} = -kA\frac{dT}{dx} \qquad \text{(W)}$$



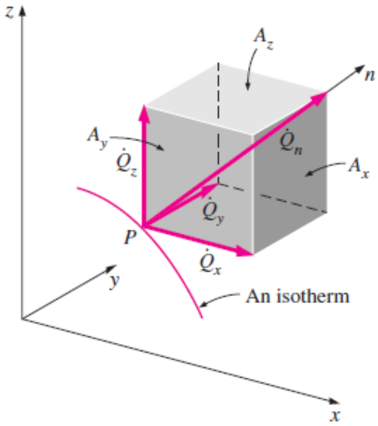
where *k* is the *thermal conductivity* of the material, which is a measure of the ability of a material to conduct heat, and *dT/dx* is the *temperature gradient,* which is the slope of the temperature curve on a *T-x* diagram.

Heat is conducted in the direction of decreasing temperature, and thus the temperature gradient is negative when heat is conducted in the positive *x*-direction. The *negative sign in the equation* ensures that heat transfer in the positive *x*-direction is a positive quantity.

General Relation for Fourier's Law

To obtain a general relation for Fourier's law of heat conduction, consider a medium in which the temperature distribution is three-dimensional. The heat flux vector at a point *P* on this surface must be perpendicular to the surface, and it must point in the direction of decreasing temperature. If *n* is the normal of the isothermal surface at point *P*, the rate of heat conduction at that point can be expressed by Fourier's law as,

$$\dot{Q}_n = -kA\frac{\partial T}{\partial n} \qquad (W)$$



In a rectangular coordinates system,

$$\vec{\dot{Q}}_n = \dot{Q}_x \vec{i} + \dot{Q}_y \vec{j} + \dot{Q}_z \vec{k}$$

where $\vec{i}$, $\vec{j}$, and $\vec{k}$ are the unit vectors,

$\dot{Q}_x$, $\dot{Q}_y$, and $\dot{Q}_z$ are the magnitudes of the heat transfer rates in the *x*-, *y*-, and *z*-directions, which again can be determined from Fourier's law as

$$\dot{Q}_x = -kA_x\frac{\partial T}{\partial x}, \quad \dot{Q}_y = -kA_y\frac{\partial T}{\partial y}, \quad and \quad \dot{Q}_z = -kA_z\frac{\partial T}{\partial z},$$

Here A_x, A_y and A_z are heat conduction areas normal to the *x*-, *y*-, and *z*-directions, respectively. Most engineering materials are *isotropic* in nature, and thus they have the same properties in all directions.

Heat Generation

A medium through which heat is conducted may involve the conversion of electrical, nuclear, or chemical energy into heat (or thermal) energy. In heat conduction analysis, such conversion processes are characterized as **heat generation.**

The rate of heat generation in a medium is usually specified *per unit volume* and is denoted by $\dot{g}$, whose unit is W/m^3.

In the special case of *uniform* heat generation, as in the case of electric resistance heating throughout a homogeneous material,

$$\dot{G} = \dot{g}V,$$

where $\dot{g}$: the constant rate of heat generation per unit volume.

V: total volume (m^3)

$\dot{G}$: total heat generation (Watt)

One-Dimensional Heat Conduction Equation

Consider a thin element of thickness x in a large plane wall. Assume the density of the wall is ρ, the specific heat is *C*, and the area of the wall normal to the direction of heat transfer is *A*. An *energy balance* on this thin element during a small time interval Δ*t* can be expressed as,

$$\begin{pmatrix}\text{Rate of heat}\\\text{conduction}\\\text{at } x\end{pmatrix} - \begin{pmatrix}\text{Rate of heat}\\\text{conduction}\\\text{at } x+\Delta x\end{pmatrix} + \begin{pmatrix}\text{Rate of heat}\\\text{generation}\\\text{inside the}\\\text{element}\end{pmatrix} = \begin{pmatrix}\text{Rate of change}\\\text{of the energy}\\\text{content of the}\\\text{element}\end{pmatrix}$$

or

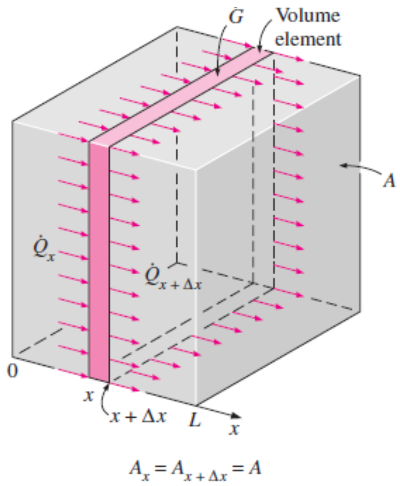
$$\dot{Q}_x - \dot{Q}_{x+\Delta x} + \dot{G}_{\text{element}} = \frac{\Delta E_{\text{element}}}{\Delta t}$$

But the change in the energy content of the element and the rate of heat generation within the element can be expressed as

$$\Delta E_{\text{element}} = E_{t+\Delta t} - E_t = mC(T_{t+\Delta t} - T_t) = \rho CA\Delta x(T_{t+\Delta t} - T_t)$$

$$\dot{G}_{\text{element}} = \dot{g}V_{\text{element}} = \dot{g}A\Delta x$$

Substitute to get,



$$\dot{Q}_x - \dot{Q}_{x+\Delta x} + \dot{g}A\Delta x = \rho CA\Delta x\frac{T_{t+\Delta t} - T_t}{\Delta t}$$

Dividing by AΔx gives,

$$-\frac{1}{A}\frac{\dot{Q}_{x+\Delta x} - \dot{Q}_x}{\Delta x} + \dot{g} = \rho C\frac{T_{t+\Delta t} - T_t}{\Delta t}$$

Taking the limit as $\Delta x \to 0$ and $\Delta t \to 0$ yields

$$\frac{1}{A}\frac{\partial}{\partial x}\left(kA\frac{\partial T}{\partial x}\right) + \dot{g} = \rho C\frac{\partial T}{\partial t}$$

or $$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \dot{g} = \rho C\frac{\partial T}{\partial t}$$ [Variable thermal conductivity]

The *thermal conductivity* in most practical applications can be assumed to remain *constant* at some average value, so that,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{g}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$ [Constant thermal conductivity]

where the property $\alpha = k/C$ is the **thermal diffusivity** of the material and represents how fast heat propagates through a material.

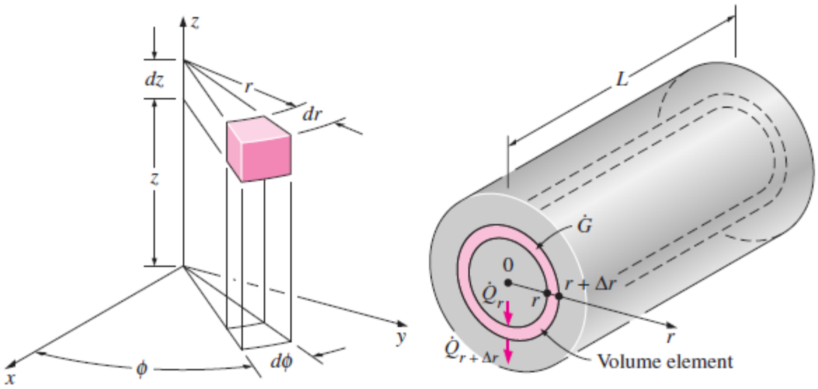
This equation represents one dimensional heat conduction equation. It reduces to the following forms under specified conditions:

In the same way the one-dimensional heat conduction equation in cylindrical and spherical coordinate systems can be found. The **rectangular**, **cylindrical**, and **spherical** coordinate systems for the case of constant thermal conductivities are expressed as,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{g}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\dot{g}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{\dot{g}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$



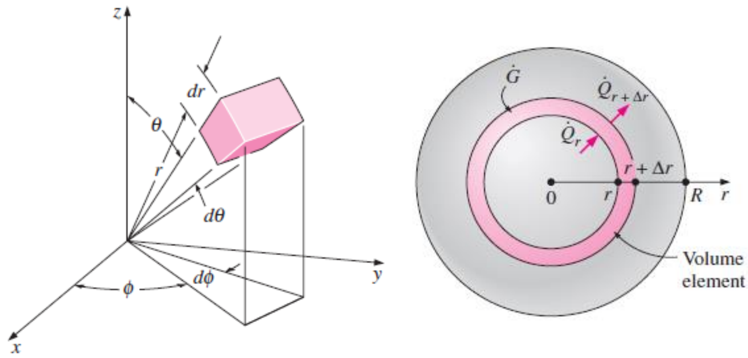
Cylindrical coordinate system

For steady state with heat generation case the above equation be,

$$\frac{\partial^2 T}{\partial T^2} + \frac{\dot{g}}{k} = 0$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\dot{g}}{k} = 0$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{\dot{g}}{k} = 0$$



Spherical coordinate system

and for steady state without heat generation case the above equation be,

$$\frac{\partial^2 T}{\partial T^2} = 0$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = 0$$

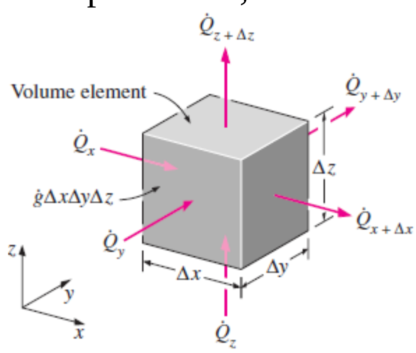
$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) = 0$$

Solving these equations with the boundary conditions give the temperature distribution and heat transfer for any problems.

General Heat Conduction Equation …... Rectangular Coordinates

Consider a small rectangular element of length Δx, width Δy, and height Δz. Assume the density of the body is ρ and the specific heat is C. An *energy balance* on this element during a small time interval Δt can be expressed as,

$$\begin{pmatrix}\text{Rate of heat}\\ \text{conduction at}\\ x, y, \text{ and } z\end{pmatrix} - \begin{pmatrix}\text{Rate of heat}\\ \text{conduction}\\ \text{at } x + \Delta x,\\ y + \Delta y, \text{ and } z + \Delta z\end{pmatrix} + \begin{pmatrix}\text{Rate of heat}\\ \text{generation}\\ \text{inside the}\\ \text{element}\end{pmatrix} = \begin{pmatrix}\text{Rate of change}\\ \text{of the energy}\\ \text{content of}\\ \text{the element}\end{pmatrix}$$



$$\dot{Q}_x + \dot{Q}_y + \dot{Q}_z - \dot{Q}_{x+\Delta x} - \dot{Q}_{y+\Delta y} - \dot{Q}_{z+\Delta z} + \dot{G}_{\text{element}} = \frac{\Delta E_{\text{element}}}{\Delta t}$$

Noting that the volume of the element is $V_{\text{element}}=\Delta x \Delta y \Delta z$, the change in the energy content of the element and the rate of heat generation within the element can be expressed as,

$$\Delta E_{\text{element}} = E_{t+\Delta t} - E_t = mC(T_{t+\Delta t} - T_t) = \rho C \Delta x \Delta y \Delta z (T_{t+\Delta t} - T_t)$$

$$\dot{G}_{\text{element}} = \dot{g} V_{\text{element}} = \dot{g} \Delta x \Delta y \Delta z$$

Substituting, we get

$$\dot{Q}_x + \dot{Q}_y + \dot{Q}_z - \dot{Q}_{x+\Delta x} - \dot{Q}_{y+\Delta y} - \dot{Q}_{z+\Delta z} + \dot{g} \Delta x \Delta y \Delta z = \rho C \Delta x \Delta y \Delta z \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

Dividing by $\Delta x \Delta y \Delta z$ gives,

$$-\frac{1}{\Delta y \Delta z} \frac{\dot{Q}_{x+\Delta x} - \dot{Q}_x}{\Delta x} - \frac{1}{\Delta x \Delta z} \frac{\dot{Q}_{y+\Delta y} - \dot{Q}_y}{\Delta y} - \frac{1}{\Delta x \Delta y} \frac{\dot{Q}_{z+\Delta z} - \dot{Q}_z}{\Delta z} + \dot{g} = \rho C \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

Noting that the heat transfer areas of the element for heat conduction in the *x*, *y*, and *z* directions are, $A_x = \Delta y \Delta z$, $A_y = \Delta x \Delta z$, $A_z = \Delta x \Delta y$ respectively, and taking the limit as Δx, Δy, Δz and $t \to 0$ yields

$$\frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k \frac{\partial}{\partial z}\right) + \dot{g} = \rho C \frac{\partial T}{\partial t}$$

In the case of constant thermal conductivity, it reduces to

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{g}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

This is the general transient three-dimension heat conduction with heat generation.

In the case of transient three-dimension heat conduction without heat generation,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$ (Called the diffusion equation)

For Steady, three dimension heat conduction with heat generation.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{g}}{k} = 0$$ (Called the Poisson equation)

And for Steady state, three-dimension heat conduction without heat generation,

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=0$$ (Called the Laplace equation)

Question: What is heat generation in a solid? Give examples. Is heat transfer a scalar or vector quantity? Explain. Same question for "temperature".

General solution of one dimensional heat conduction

1- Plane wall

$$\frac{d^2T}{d^2x}=0$$

By integration

$\frac{dT}{dx}=C_1$…….(1)

$T=C_1x+C_2$ ………..(2)

at x= 0 T=T_1 sub in equ 2

$\therefore\ C_2=T_1$

At x= L T=T_2 sub in equ 2

$\therefore C_1=\frac{T_2-T_1}{L}$

$$\therefore\ \boldsymbol{T=\frac{T_2-T_1}{L}x+T_1}$$

2-Plane wall with heat generation

$$\boldsymbol{\frac{d^2T}{d^2x}+\frac{\dot{q}}{k}=}$$

By integration

$$\boldsymbol{\frac{dT}{dx}}=-\boldsymbol{\frac{\dot{q}x}{k}}+C_1\ \ldots\ldots\ldots\ldots(1)$$

$\boldsymbol{T}=-\frac{\dot{q}x^2}{2k}+C_1x+C_2$**…………..(2)**

at x= 0 T=T_1 sub in equ 2

$\therefore\ C_2=T_1$

at x= L T=T_2 sub in equ 2

$\boldsymbol{T_2}=-\frac{\dot{q}L^2}{2k}+C_1L+T_2$

$\therefore C_1 = \frac{T_2 - T_1}{L} + \frac{\dot{q}L^2}{2k}$

$\boldsymbol{T = -\frac{\dot{q}x^2}{2k} + (\frac{T_2 - T_1}{L} + \frac{\dot{q}L^2}{2k})x + T_1}$

General solution of equation of cylinder with heat generation

$$\frac{d^2T}{d^2r} + \frac{1}{r}\frac{dT}{dr} = -\frac{\dot{q}}{k}$$

$r\frac{d^2T}{d^2r} + \frac{dT}{dr} = -\frac{\dot{q}r}{k}$ **nota that** $\boldsymbol{r\frac{d^2T}{d^2r} + \frac{dT}{dr} = \frac{d}{dr}(r\frac{dT}{dr})}$

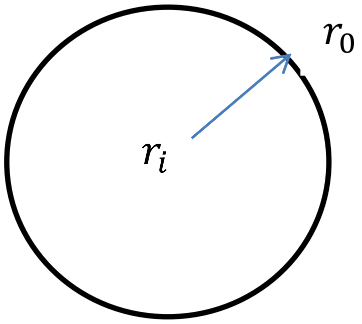
$\frac{d}{dr}(r\frac{dT}{dr}) = -\frac{\dot{q}r}{k}$

By integration

$\left(r\frac{dT}{dr}\right) = -\frac{\dot{q}r^2}{k} + C_1$

$$\frac{dT}{dr} = -\frac{\dot{q}r}{k} + \frac{C_1}{r} \dots\dots\dots\dots\dots(1)$$

$$T = -\frac{\dot{q}r^2}{4k} + C_1 lnr + C_2 \dots\dots\dots\dots(2)$$



at r=0 $\frac{dT}{dr} = 0$

$\therefore C_1 = 0$

at r=r_0 T=T_0 sub in equ 2

$$T_0 = -\frac{\dot{q}r^2}{4k} + C_2$$

$$C_2 = T_0 + \frac{\dot{q}r_0^2}{4k}$$

$$\boldsymbol{T - T_0 = \frac{\dot{q}(r_o^2 - r^2)}{4k}}$$

Final sol for temp dist. for solid cylinder

And for r=r_i $\boldsymbol{T_{center} = \frac{\dot{q}r_o^2}{4k} + T_0}$

And for hollow cylinder

$$\boldsymbol{T - T_0 = \frac{\dot{q}(r_0^2 - r^2)}{4k} + C_1 ln\frac{r}{r_0}}$$

$$\boldsymbol{C_1 = \frac{T_i - T_o + \frac{\dot{q}(r_i^2 - r_o^2)}{4k}}{ln\frac{r_i}{r_0}}}$$

General solution of equation of sphere without heat generation

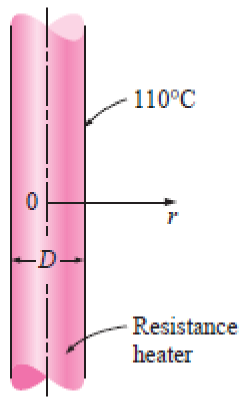
$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$$

$$r^2\frac{dT}{dr} = C_1$$

$$\frac{dT}{dr} = \frac{C_1}{r^2}$$

$$T = -\frac{C_1}{r} + C_2$$

Ex.1:- A 2-kW resistance heater wire with thermal conductivity of k = 20 W/m .°C, a diameter of D=5 mm, and a length of L =0.7 m is used to boil water. If the outer surface temperature of the resistance wire is T_s = 110°C, determine the temperature at the center of the wire.



Assumptions **1** Heat transfer is steady since there is no change with time. **2** Heat transfer is one dimensional since there is thermal symmetry about the center line and no change in the axial direction. **3** Thermal conductivity is constant. **4** Heat generation in the heater is uniform.

Properties The thermal conductivity is given to be *k* = 20 W/m·°C.

Sol.

The resistance heater converts electric energy into heat at a rate of 2 kW. The rate of heat generation per unit volume of the wire is

$$\dot{g} = \frac{\dot{Q}_{gen}}{V_{wire}} = \frac{\dot{Q}_{gen}}{\pi r_o^{\,2} L} = \frac{2000 \text{ W}}{\pi(0.0025 \text{ m})^2(0.7 \text{ m})} = 1.455\times10^8 \text{ W/m}^3$$

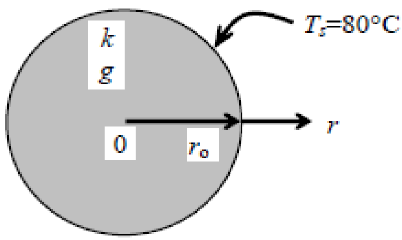
The center temperature of the wire is then determined from Eq. 2-71 to be

$$T_o = T_s + \frac{\dot{g} r_o^{\,2}}{4k} = 110°\text{C} + \frac{(1.455\times10^8 \text{ W/m}^3)(0.0025 \text{ m})^2}{4(20 \text{ W/m.°C})} = \mathbf{121.4°C}$$

Ex.2:- Consider a homogeneous spherical piece of radioactive material of radius r_0 = 0.04 m that is generating heat at a constant rate of $g\cdot$ =4 x 107 W/m^3. The heat generated is dissipated to the environment steadily. The outer surface of the sphere is maintained at a uniform temperature of 80°C and the thermal conductivity of the sphere is k =15 W/m · °C. Assuming steady one-dimensional heat transfer, (*a*) express the differential equation and the boundary conditions for heat conduction through the sphere, (*b*) obtain a relation for the variation of temperature in the sphere by solving the differential equation, and (*c*) determine the temperature at the center of the sphere.

Assumptions **1** Heat transfer is steady since there is no indication of any changes with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the midpoint. **3** Thermal conductivity is constant. **4** Heat generation is uniform.

Properties The thermal conductivity is given to be k = 15 W/m·°C.



$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)+\frac{\dot{g}}{k}=0 \quad \text{with} \quad \dot{g}=\text{constant}$$

and $$T(r_0)=T_s=80°\text{C} \quad \text{(specified surface temperature)}$$

$$\frac{dT(0)}{dr}=0 \quad \text{(thermal symmetry about the mid point)}$$

(*b*) Multiplying both sides of the differential equation by r^2 and rearranging gives

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=-\frac{\dot{g}}{k}r^2$$

Integrating with respect to r gives

$$r^2\frac{dT}{dr}=-\frac{\dot{g}}{k}\frac{r^3}{3}+C_1 \qquad (a)$$

Applying the boundary condition at the mid point,

B.C. at $r=0$: $$0\times\frac{dT(0)}{dr}=-\frac{\dot{g}}{3k}\times 0+C_1 \quad\rightarrow\quad C_1=0$$

Dividing both sides of Eq. (*a*) by r^2 to bring it to a readily integrable form and integrating,

$$\frac{dT}{dr}=-\frac{\dot{g}}{3k}r$$

and $$T(r)=-\frac{\dot{g}}{6k}r^2+C_2 \qquad (b)$$

Applying the other boundary condition at $r=r_0$,

B. C. at $r=r_0$: $$T_s=-\frac{\dot{g}}{6k}r_0^2+C_2 \quad\rightarrow\quad C_2=T_s+\frac{\dot{g}}{6k}r_0^2$$

Substituting this C_2 relation into Eq. (*b*) and rearranging give

$$T(r)=T_s+\frac{\dot{g}}{6k}(r_0^2-r^2)$$

which is the desired solution for the temperature distribution in the wire as a function of r.

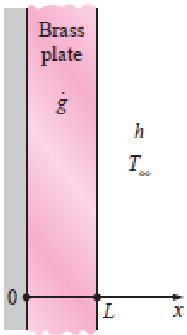
(c) The temperature at the center of the sphere ($r = 0$) is determined by substituting the known quantities to be

$$T(0) = T_s + \frac{\dot{g}}{6k}(r_0^2 - 0^2) = T_s + \frac{\dot{g}r_0^2}{6k} = 80°C + \frac{(4 \times 10^7 \text{ W/m}^3)(0.04 \text{ m})^2}{6 \times (15 \text{ W/ m.°C})} = \mathbf{791°C}$$

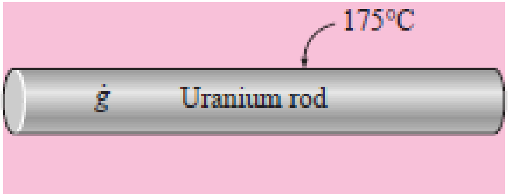
Thus the temperature at center will be about 711°C above the temperature of the outer surface of the sphere.

………………………………………………………………………………………

W.H.1:- Consider a large 5-cm-thick brass plate (k =111W/m.°C) in which heat is generated uniformly at a rate of 2= 105 W/m^3. One side of the plate is insulated while the other side is exposed to an environment at 25°C with a heat transfer coefficient of 44 W/m^2·°C. Explain where in the plate the highest and the lowest temperatures will occur, and determine their values.



W.H.2:- In a nuclear reactor, 1-cm-diameter cylindrical uranium rods cooled by water from outside serve as the fuel. Heat is generated uniformly in the rods (k= 29.5 W/m·°C) at a rate of 7x107 W/m^3. If the outer surface temperature of rods is 175°C, determine the temperature at their center.



Northern Technical University

Engineering Technical College / Mosul

Mechanical Technical Engineering

Heat Transfer Lecture

Third Year

Northern Technical University

Technical college of Engineering / Mosul
Heat transfer (third year)

Lecture No(5)

Heat Transfer from Finned Surfaces

1. Class : third Year

2. Subject : **Heat Transfer from Finned Surfaces**

3. Number of weeks: three weeks

4. Central idea: Study the Heat Transfer from Finned Surfaces
5. The Test:

HEAT TRANSFER FROM FINNED SURFACES

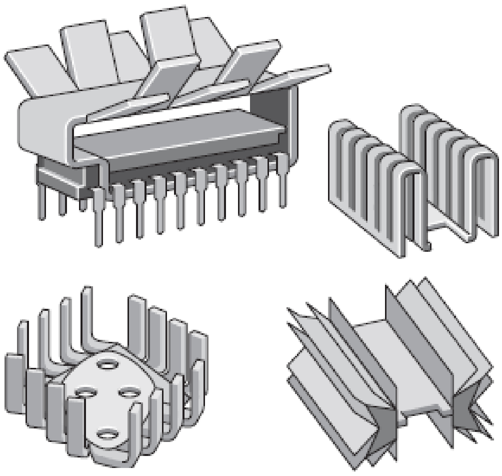
The rate of heat transfer from a surface at a temperature T_s to the surrounding medium at T_∞ is given by Newton's law of cooling as

$$q_{conv} = hA_s(T_s - T_\infty)$$

where A_s is the heat transfer surface area and h is the convection heat transfer coefficient. When the temperatures T_s and T_∞ are fixed by design considerations, as is often the case, there are *two ways* to increase the rate of heat transfer: to increase the *convection heat transfer coefficient h* or to increase the *surface area* A_s. Increasing h may require the installation of a pump or fan, or replacing the existing one with a larger one, but this approach may or may not be practical. Besides, it may not be adequate. The alternative is to increase the surface area by attaching to the surface *extended surfaces* called *fins* made of highly conductive materials such as aluminum. Finned surfaces are manufactured by extruding, welding, or wrapping a thin metal sheet on a surface. Fins enhance heat transfer from a surface by exposing a larger surface area to convection and radiation.

Finned surfaces are commonly used in practice to enhance heat transfer, and they often increase the rate of heat transfer from a surface severalfold. The car radiator shown in Fig. 10 is an example of a finned surface. The closely packed thin metal sheets attached to the hot water tubes increase the surface area for convection and thus the rate of convection heat transfer from the tubes to the air many times. There are a variety of innovative fin designs available in the market, and they seem to be limited only by imagination

In the analysis of fins, we consider *steady* operation with *no heat generation* in the fin, and we assume the thermal conductivity k of the material to remain constant. We also assume the convection heat transfer coefficient h to be *constant* and *uniform* over the entire surface of the fin for convenience in the analysis. We recognize that the convection heat transfer coefficient h, in general, varies along the fin as well as its circumference, and its value at a point is a strong function of the *fluid motion* at that point. The value of h is usually much lower at the *fin base* than it is at the *fin tip* because the fluid is surrounded by solid surfaces near the base, which seriously disrupt its motion to



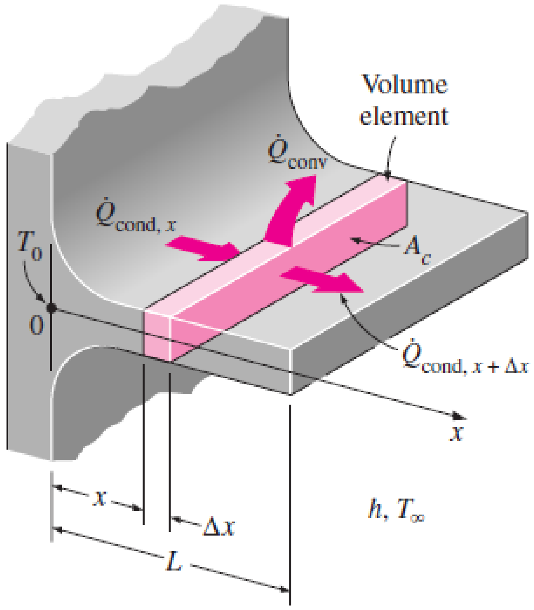


FIGURE 10

the point of "suffocating" it, while the fluid near the fin tip has little contact with a solid surface and thus encounters little resistance to flow. Therefore, adding too many fins on a surface may actually decrease the overall heat transfer when the decrease in h offsets any gain resulting from the increase in the surface area.

Consider a volume element of a fin at location x having a length of Δx, cross-sectional area of A_c, and a perimeter of p, . Under steady conditions, the energy balance on this volume element can be expressed as

$$\left(\begin{array}{c}\text{Rate of } \textit{heat} \\ \textit{conduction} \text{ into} \\ \text{the element at } x\end{array}\right) = \left(\begin{array}{c}\text{Rate of } \textit{heat} \\ \textit{conduction} \text{ from the} \\ \text{element at } x + \Delta x\end{array}\right) + \left(\begin{array}{c}\text{Rate of } \textit{heat} \\ \textit{convection} \text{ from} \\ \text{the element}\end{array}\right)$$

or

$$\boldsymbol{q_x = q_{x+\Delta x} + q_{conv}} \qquad \text{where} \quad \boldsymbol{q_{conv} = hp\Delta x(T - T_\infty)}$$

$$\frac{q_{x+\Delta x} - q_x}{\Delta x} + hp(T - T_\infty) = 0$$

Taking the limit as $\Delta x \to 0$

$$\frac{dq_x}{dx} + hp(T - T_\infty) = 0 \qquad \text{where } q_x = -kA_c\frac{dT}{dx}$$

Where A_c is the cross- sectional Area at x

$$\frac{d}{dx}\left(kA_c\frac{dT}{dx}\right) - hp(T - T_\infty) = 0$$

Divide by kA_c

$$\frac{d^2T}{d^2x} = \frac{hp}{kA}(T - T_\infty)$$

Let $\frac{hp}{kA} = m^2$ and $\Theta = (T - T_\infty)$ and $= \Theta_b = (T_b - T_\infty)$

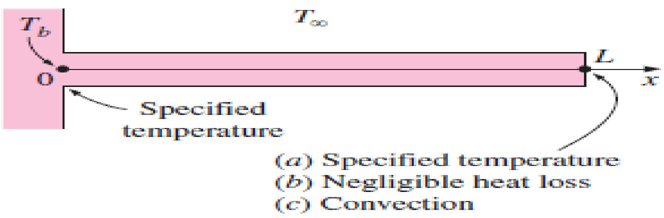
Therefore, the general solution of the differential equation l

$$(T - T_\infty) = C_1e^{mx} + C_2e^{-mx} \qquad \ldots\ldots(1)$$

B.C at x=0 T=T_b

$$(T_b - T_\infty) = C_1e^{m0} + C_2e^{-m0}$$

$$(T_b - T_\infty) = C_1 + C_2$$



1 Infinitely Long Fin ($T_{\text{fin tip}} = T_\infty$)

For a sufficiently long fin of *uniform* cross section (A_c = constant), the temperature of the fin at the fin tip will approach the environment temperature T_∞ and thus θ will approach zero. That is,

Boundary condition at fin tip: $\theta(L) = T(L) - T_\infty = 0$ as $L \rightarrow \infty$

This condition will be satisfied by the function e^{-ax}, but not by the other prospective solution function e^{ax} since it tends to infinity as x gets larger. Therefore, the general solution in this case will consist of a constant multiple of e^{-mx} The value of the constant multiple is determined from the requirement that at the fin base where $x = 0$ the value of θ will be θ_b. Noting that

$$0 = C_1 e^{m\infty} + C_2 e^{-m\infty}$$

$\therefore C_1 = 0$ from equ (1) $T_b - T_\infty = C_2$

$$\therefore \; temp\ dist\ equ \quad T - T_\infty = (T_b - T_\infty)e^{-mx}$$

Heat transfer

$$\boldsymbol{q = -kA\frac{dT}{dx} \quad at\ x = 0}$$

$$T - T_\infty = (T_b - T_\infty)e^{-mx}$$

$$\frac{\boldsymbol{dT}}{\boldsymbol{dx}} = -m(T_b - T_\infty)e^{-mx}$$

$$\frac{\boldsymbol{dT}}{\boldsymbol{dx}} = -m(T_b - T_\infty) \quad at\ \ x = 0$$

$$\boldsymbol{q} = -kA(-m(T_b - T_\infty))$$

$$\boxed{q = \sqrt{hpkA}(T_b - T_\infty)}$$

2=The fin is finite length and loses heat by convection from its end

$$(T_b - T_\infty) = C_1 + C_2$$

$$at\ x = L \qquad q = -kA\frac{dT}{dx} = hA(T_{x=L} - T_\infty)$$

$$(T - T_\infty) = C_1 e^{mx} + C_2 e^{-mx}$$

$$at\ x = L \qquad \frac{dT}{dx} = mC_1 e^{mL} - mC_2 e^{-mL}$$

After simplification and find values C1 and C2

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$$

$$Q = \sqrt{hpkA}\,(T_b - T_\infty)\frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$$

Negligible Heat Loss from the Fin Tip (Insulated fin tip, $\dot{Q}_{fin\ tip} = 0$)

Negligible Heat Loss From the Fin Tip (Insulated fin tip $q_{fin\ tip}$ =0)

Fins are not likely to be so long that their temperature approaches the surrounding temperature at the tip. A more realistic situation is for heat transfer from the fin tip to be negligible since the heat transfer from the fin is proportional to its surface area, and the surface area of the fin tip is usually a negligible fraction of the total fin area. Then the fin tip can be assumed to be insulated, and the condition at the fin tip can be expressed as

Boundary condition at fin tip: $\left.\frac{d\theta}{dx}\right|_{x=L} = 0$

$$(T - T_\infty) = C_1 e^{mx} + C_2 e^{-mx}$$

$$\frac{dT}{dx} = 0 = mC_1e^{mL} - mC_2e^{-mL}$$

$$C_2 = C_1e^{2mL}$$

From equ (1)

$$(\boldsymbol{T_b - T_\infty}) = C_1 + C_2$$

$$(\boldsymbol{T_b - T_\infty}) = C_1 + C_1e^{2mL}$$

$$C_1 = \frac{(\boldsymbol{T_b - T_\infty})}{1 + e^{2mL}}$$

$$C_2 = \frac{(\boldsymbol{T_b - T_\infty})}{1 + e^{-2mL}}$$

$$(\boldsymbol{T - T_\infty}) = \frac{(\boldsymbol{T_b - T_\infty})}{1 + e^{2mL}}e^{mx} + \frac{(\boldsymbol{T_b - T_\infty})}{1 + e^{-2mL}}e^{-mx}$$

$$\frac{(\boldsymbol{T - T_\infty})}{(\boldsymbol{T_b - T_\infty})} = \frac{e^{mx}}{1+e^{2mL}} + \frac{e^{-mx}}{1+e^{-2mL}}$$

$$\boxed{\frac{(\boldsymbol{T - T_\infty})}{(\boldsymbol{T_b - T_\infty})} = \frac{\cosh \boldsymbol{m(L - x)}}{\cosh \boldsymbol{mL}}}$$

Heat transfer

$$\boldsymbol{q = -kA\frac{dT}{dx} \quad at\ x = 0}$$

$$(\boldsymbol{T - T_\infty}) = (\boldsymbol{T_b - T_\infty})\frac{\cosh \boldsymbol{m(L - x)}}{\cosh \boldsymbol{mL}}$$

$$\boldsymbol{\frac{dT}{dx} = -(T_b - T_\infty)m\frac{\sinh m(L - x)}{\cosh mL}}$$

$$\boldsymbol{at\ x = 0 \quad \frac{dT}{dx} = -(T_b - T_\infty)m\frac{\sinh mL}{\cosh mL}}$$

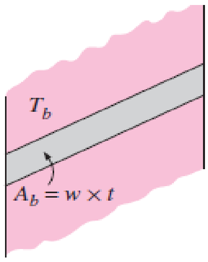
$$\frac{dT}{dx} = -m(T_b - T_\infty)\tanh mL$$

$$q = -kA * -m(T_b - T_\infty)\tanh mL$$

$$q = \sqrt{hpkA}(T_b - T_\infty)\tanh mL$$

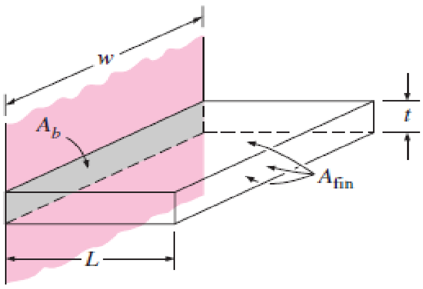
Fin Efficiency

Consider the surface of a *plane wall* at temperature T_b exposed to a medium at temperature T_∞. Heat is lost from the surface to the surrounding medium by



(*a*) Surface without fins

$$q = hA_b(T_b - T_\infty)$$



(*b*) Surface with a fin

$$A_{fin} = 2 \times w \times L + w \times t$$
$$\cong 2 \times w \times L$$

$$q_{fin.max} = hA_{fin}(T_b - T_\infty)$$

$$\eta_{fin} = \frac{actual\ heat\ transfer\ rate\ from\ the\ fin}{ideal\ heat\ transfer\ from\ the\ fin\ if\ the\ entire\ fin\ were\ at\ base\ temperature}$$

$$q_{fin} = \eta_{fin}\, q_{fin.max} = \eta_{fin} hA_{fin}(T_b - T_\infty)$$

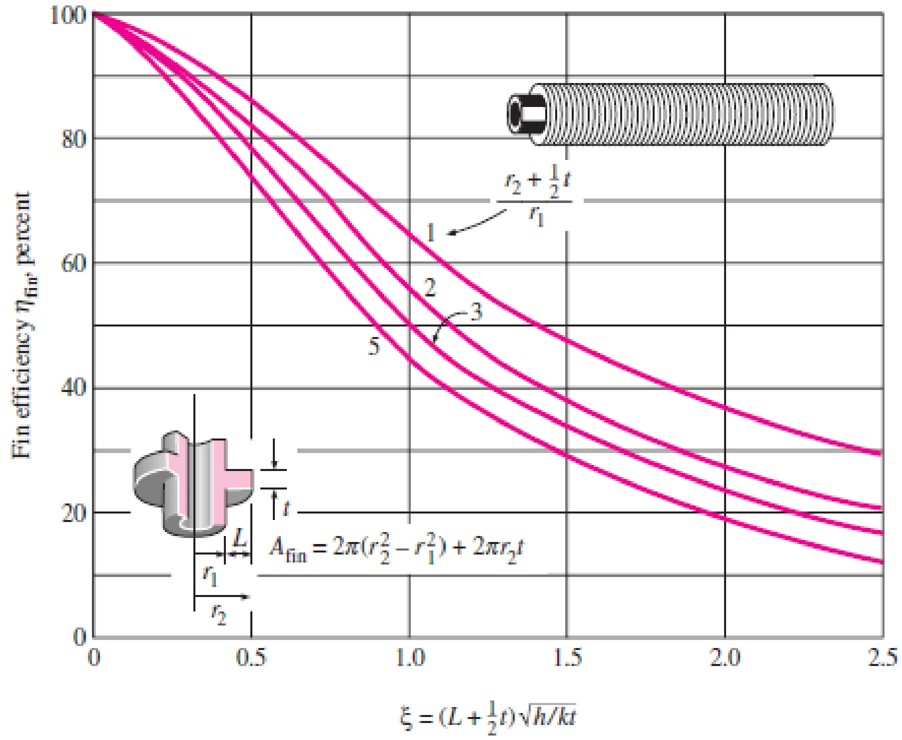
where A_{fin} is the total surface area of the fin. This relation enables us to determine the heat transfer from a fin when its efficiency is known. For the cases of constant cross section of *very long fins* and *fins with insulated tips,* the fin efficiency can be expressed as

$$\eta_{long.fin} = \frac{q = \sqrt{hpkA}(T_b - T_\infty)}{hA_{fin}(T_b - T_\infty)} = \frac{1}{L}\sqrt{\frac{kA}{hp}} = \frac{1}{mL}$$

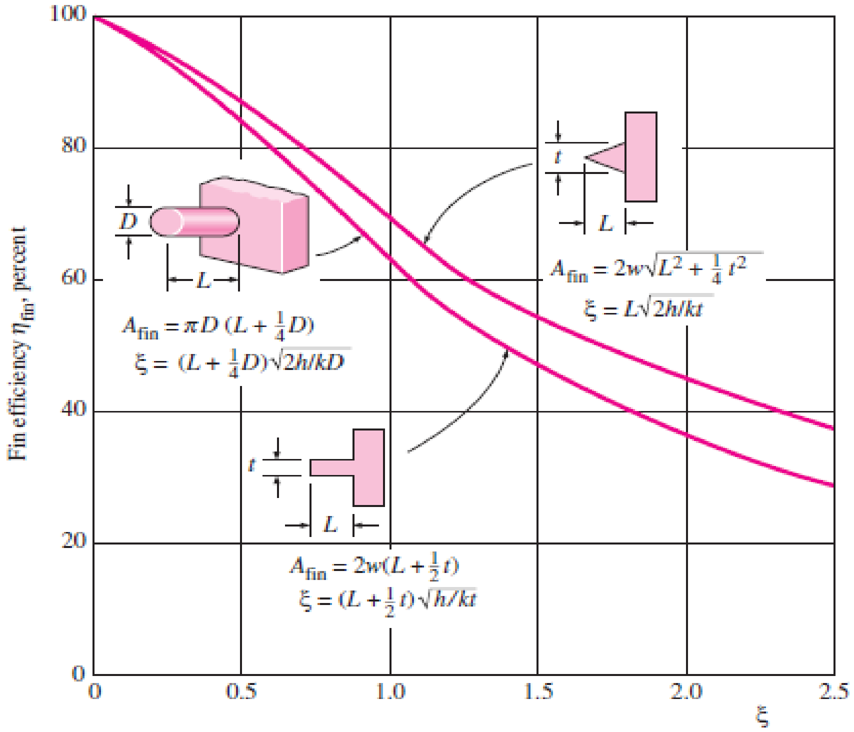
$$\eta_{insulated.tip} = \frac{q = \sqrt{hpkA}(T_b - T_\infty)tanhml}{hA_{fin}(T_b - T_\infty)} = \frac{tanhmL}{mL}$$

$$\eta_{\ finite\ length} = \frac{q=\sqrt{hpkA}(T_b-T_\infty)\frac{\cosh m(L-x)+({}^{h}/_{mk})sinhm(L-x)}{coshmL+({}^{h}/_{mk})sinhmL}}{hA_{fin}(T_b-T_\infty)} = \frac{\frac{\cosh m(L-x)+({}^{h}/_{mk})sinhm(L-x)}{coshmL+({}^{h}/_{mk})sinhmL}}{mL}$$

since $A_{fin} = pL$ for fins with constant cross section. Equation can also be used for fins subjected to convection provided that the fin length L is replaced by the corrected length L_c.



Fin efficiency relations are developed for fins of various profiles and are plotted in Fig. for fins on a *plain surface* and in Fig. for *circular fins* of constant thickness. The fin surface area associated with each profile is also given on each figure. For most fins of constant thickness encountered in practice, the fin thickness t is too small relative to the fin length L, and thus the fin tip area is negligible.

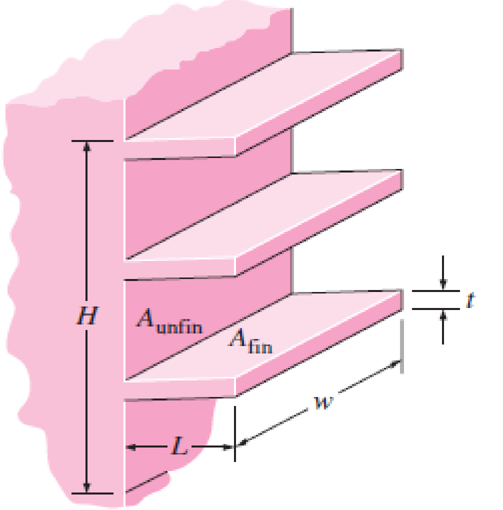


Fin Effectiveness

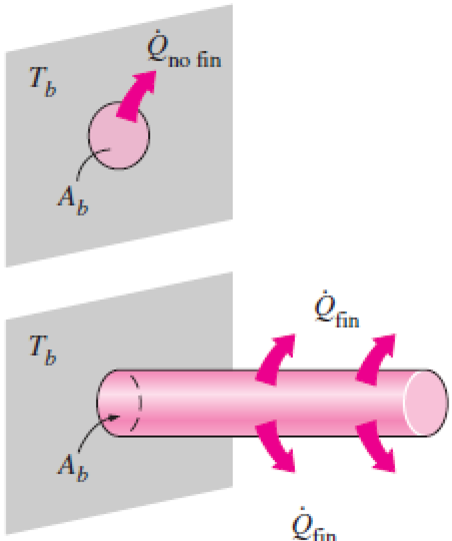
The performance of the fins is judged on the basis of the enhancement in heat transfer relative to the no-fin case. The performance of fins expressed in terms of the ***fin effectiveness*** $(\boldsymbol{\varepsilon_{fin}})$

$$\varepsilon = \frac{heat\ transfer\ from\ fin}{heat\ transfer\ from\ surface\ area\ A_b}$$

$$\varepsilon = \frac{q_{fin}}{q_{no\ fin}} = \frac{q_{fin}}{hA_b(T_b - T_\infty)}$$



$A_{no\ fin} = w \times H$
$A_{unfin} = w \times H - 3 \times (t \times w)$
$A_{fin} = 2 \times L \times w + t \times w$ (one fin)
$\approx 2 \times L \times w$



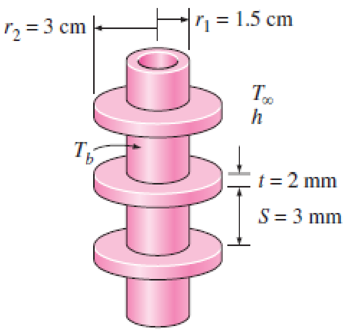
$$\varepsilon_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{no\ fin}}$$

Ex1: A very long fin copper rod (k=372 w/m.°c) 25 mm in diameter has one end maintained at 90 °C .The rod is exposed to a fluid whose temperature is 40 °C .The heat – transfer coefficient is 3.5 w/m².°c .How much heat is lost by the rod

Sol:

$$q = \sqrt{hPkA}\theta_0 = \left[\frac{(3.5)\pi(0.025)(372)\pi(0.025)^2}{4}\right]^{1/2}(90-40) = 11.2 \text{ W}$$

Ex2: Steam in a heating system flows through tubes whose outer diameter is *D*1= 3 cm and whose walls are maintained at a temperature of 120°C. Circular aluminum fins (*k* =180 W/m · °C) of outer diameter *D*2 = 6 cm and constant thickness *t*= 2 mm are attached to the tube, as shown in Fig .The space between the fins is 3 mm, and thus there are 200 fins per meter length of the tube. Heat is transferred to the surrounding air at *T*= 25°C, with a combined heat transfer coefficient of *h* =60 W/m²· °C. Determine the increase in heat transfer from the tube per meter of its length as a result of adding fins.



Sol:

$$A_{\text{no fin}} = \pi D_1 L = \pi(0.03\ \text{m})(1\ \text{m}) = 0.0942\ \text{m}^2$$

$$\dot{Q}_{\text{no fin}} = hA_{\text{no fin}}(T_b - T_\infty)$$
$$= (60\ \text{W/m}^2\cdot{}^\circ\text{C})(0.0942\ \text{m}^2)(120 - 25)^\circ\text{C}$$
$$= 537\ \text{W}$$

The efficiency of the circular fins attached to a circular tube is plotted in Fig. 3–43. Noting that $L = \frac{1}{2}(D_2 - D_1) = \frac{1}{2}(0.06 - 0.03) = 0.015$ m in this case, we have

$$\left.\begin{aligned} \frac{r_2 + \frac{1}{2}t}{r_1} &= \frac{(0.03 + \frac{1}{2}\times 0.002)\ \text{m}}{0.015\ \text{m}} = 2.07 \\ (L + \tfrac{1}{2}t)\sqrt{\frac{h}{kt}} &= (0.015 + \tfrac{1}{2}\times 0.002)\ \text{m} \times \sqrt{\frac{60\ \text{W/m}^2\cdot{}^\circ\text{C}}{(180\ \text{W/m}\cdot{}^\circ\text{C})(0.002\ \text{m})}} = 0.207 \end{aligned}\right\} \eta_{\text{fin}} =$$

$$A_{\text{fin}} = 2\pi(r_2^2 - r_1^2) + 2\pi r_2 t$$
$$= 2\pi[(0.03\ \text{m})^2 - (0.015\ \text{m})^2] + 2\pi(0.03\ \text{m})(0.002\ \text{m})$$
$$= 0.00462\ \text{m}^2$$

$$\dot{Q}_{\text{fin}} = \eta_{\text{fin}}\dot{Q}_{\text{fin, max}} = \eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty)$$
$$= 0.95(60\ \text{W/m}^2\cdot{}^\circ\text{C})(0.00462\ \text{m}^2)(120 - 25)^\circ\text{C}$$
$$= 25.0\ \text{W}$$

Heat transfer from the unfinned portion of the tube is

$$A_{\text{unfin}} = \pi D_1 S = \pi(0.03\ \text{m})(0.003\ \text{m}) = 0.000283\ \text{m}^2$$

$$\dot{Q}_{\text{unfin}} = hA_{\text{unfin}}(T_b - T_\infty)$$
$$= (60\ \text{W/m}^2\cdot{}^\circ\text{C})(0.000283\ \text{m}^2)(120 - 25)^\circ\text{C}$$
$$= 1.60\ \text{W}$$

Noting that there are 200 fins and thus 200 interfin spacings per meter length of the tube, the total heat transfer from the finned tube becomes

$$\dot{Q}_{\text{total, fin}} = n(\dot{Q}_{\text{fin}} + \dot{Q}_{\text{unfin}}) = 200(25.0 + 1.6)\ \text{W} = 5320\ \text{W}$$

Therefore, the increase in heat transfer from the tube per meter of its length as a result of the addition of fins is

$$\dot{Q}_{\text{increase}} = \dot{Q}_{\text{total, fin}} - \dot{Q}_{\text{no fin}} = 5320 - 537 = \mathbf{4783\ W} \qquad \text{(per m tube length)}$$

Discussion The overall effectiveness of the finned tube is

$$\varepsilon_{\text{fin, overall}} = \frac{\dot{Q}_{\text{total, fin}}}{\dot{Q}_{\text{total, no fin}}} = \frac{5320\ \text{W}}{537\ \text{W}} = 9.9$$

EX: 3The thermal conductivity of a metal plate measuring (1m*1m) is (20 w/m.°c) . The plate is at a uniform temperature of 80°C and losses heat to surrounding air which is at 40°C .The convection coefficient is 40 w/m².°c .it was intended to double . the heat loss by adding rectangular fins to the surface .Each fin is (1m) width (5mm) thick . the fins are spaced (45 mm) center to center . Neglect heat transfer from fin tips and calculate the required length of fins

Sol: $q_1 = h\,A(\Delta T) = 40*1*(80-40) = 1600\ W$

$q_2 = 2*q_1 = 3200\ W$

$q_2 = q_{fins} + q_{space}$

no of fins = 1 / 0.045 =22 fin

$q_{space} = h\,A_{space}\,(\Delta T) = 40((1*1) - ((0.005*1)*22))(80-40) = 1424\ W$

$q_{fins} = q_2 - q_{space} = 3200 - 1424 = 1776\ W$

$q_{one\ fin} = 1776 / 22 = 80.727\ W$

$$\boldsymbol{q} = \sqrt{hpkA}(T_b - T_\infty)\,\mathbf{tanh}\,\boldsymbol{mL}$$

$$80.727 = \sqrt{40*((0.005+1)*2)*20*(0.005*1)}(80-40)\,\mathbf{tanh}\,\mathbf{28.35}\boldsymbol{L} \qquad L = ?$$

H.W:A plane wall (7.5 cm) thick generates heat internally at rate of (10^5 W/m³) .one side of the wall is insulated and the temperature middle of the wall 115°C . In order to dissipate heat from other side of wall (10 fins) are mounted at wall . the fin are of (1m) width and (1cm) thick and (4cm) length. Consider that all the heat is dissipated through the fin only . the fins are to be considered insulated at the tip and the thermal conductivity of material of the fins and the wall is (17.3 w/m.°c) .Determine the heat transfer coefficient between the fin and the environment if the temperature of the air is 25 °C . Take the surface area of the wall is to be (1 m²).

Two-Dimensional Steady Heat Conduction

The Objective: Studying numerically the two-dimensional steady heat conduction.

Introduction: Many problems encountered in practice involve complicated geometries with complex boundary conditions or variable properties and cannot be solved analytically. In such cases, sufficiently accurate approximate solutions can be obtained by computers using a numerical method.

Test: What are the limitations of the analytical solution methods?

Summary:

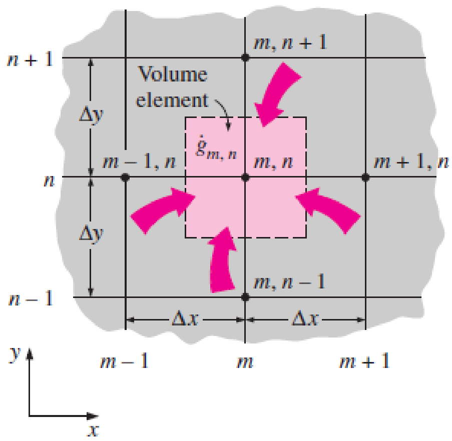
Now consider a *volume element* of size $\Delta x \times \Delta y \times 1$ centered about a general interior node (m, n) in a region in which heat is generated at a rate of $\dot{g}$ and the thermal conductivity k is constant, as shown in Figure : Again assuming the direction of heat conduction to be *toward* the node under consideration at all surfaces, the energy balance on the volume element can be expressed as

$$\begin{pmatrix}\text{Rate of heat conduction}\\ \text{at the left, top, right,}\\ \text{and bottom surfaces}\end{pmatrix} + \begin{pmatrix}\text{Rate of heat}\\ \text{generation inside}\\ \text{the element}\end{pmatrix} = \begin{pmatrix}\text{Rate of change of}\\ \text{the energy content}\\ \text{of the element}\end{pmatrix}$$

or

$$\dot{Q}_{\text{cond, left}} + \dot{Q}_{\text{cond, top}} + \dot{Q}_{\text{cond, right}} + \dot{Q}_{\text{cond, bottom}} + \dot{G}_{\text{element}} = \frac{\Delta E_{\text{element}}}{\Delta t} = 0$$

for the *steady* case. Again assuming the temperatures between the adjacent nodes to vary linearly and noting that the heat transfer area is $A_x = \Delta y \times 1 = \Delta y$ in the x-direction and $A_y = \Delta x \times 1 = \Delta x$ in the y-direction, the energy balance relation above becomes



FIGURE

The volume element of a general interior node (m, n) for two-dimensional conduction in rectangular coordinates.

$$k\Delta y \frac{T_{m-1,n} - T_{m,n}}{\Delta x} + k\Delta x \frac{T_{m,n+1} - T_{m,n}}{\Delta y} + k\Delta y \frac{T_{m+1,n} - T_{m,n}}{\Delta x} + k\Delta x \frac{T_{m,n-1} - T_{m,n}}{\Delta y} + \dot{g}_{m,n}\, \Delta x\, \Delta y = 0$$

Dividing each term by $\Delta x \times \Delta y$ and simplifying gives

$$\frac{T_{m-1,n} - 2T_{m,n} + T_{m+1,n}}{\Delta x^2} + \frac{T_{m,n-1} - 2T_{m,n} + T_{m,n+1}}{\Delta y^2} + \frac{\dot{g}_{m,n}}{k} = 0$$

for $m = 1, 2, 3, \ldots, M - 1$ and $n = 1, 2, 3, \ldots, N - 1$. This equation is identical to Eq. 5–12 obtained earlier by replacing the derivatives in the differential equation by differences for an interior node (m, n). Again a rectangular region M equally spaced nodes in the x-direction and N equally spaced nodes in the y-direction has a total of $(M + 1)(N + 1)$ nodes, and Eq. 5–33 can be used to obtain the finite difference equations at all interior nodes.

In finite difference analysis, usually a **square mesh** is used for simplicity (except when the magnitudes of temperature gradients in the x- and y-directions are very different), and thus Δx and Δy are taken to be the same. Then $\Delta x = \Delta y = l$, and the relation above simplifies to

$$T_{m-1,n} + T_{m+1,n} + T_{m,n+1} + T_{m,n-1} - 4T_{m,n} + \frac{\dot{g}_{m,n} l^2}{k} = 0$$

$$T_{\text{left}} + T_{\text{top}} + T_{\text{right}} + T_{\text{bottom}} - 4T_{\text{node}} + \frac{\dot{g}_{\text{node}} l^2}{k} = 0$$

When there is no heat generation in the medium, the finite difference equation for an interior node further simplifies to $T_{\text{node}} = (T_{\text{left}} + T_{\text{top}} + T_{\text{right}} + T_{\text{bottom}})/4$, *which has the interesting interpretation that the temperature of each interior* node *is the arithmetic average of the temperatures of the four neighboring* node*s.* This statement is also true for the three-dimensional problems except that the interior nodes in that case will have six neighboring nodes instead of four.

Solution Techniques

The nodal equation may be written as

$$a_{11}T_1 + a_{12}T_2 + \cdots + a_{1n}T_n = C_1$$

$$a_{21}T_1 + a_{22}T_2 + \cdots + a_{1n}T_n = C_2$$

$$a_{31}T_1 + a_{32}T_2 + \cdots + a_{1n}T_n = C_3$$

$$a_{n1}T_1 + a_{n2}T_2 + \cdots + a_{nn}T_n = C_n$$

$$[A] = \begin{matrix} a_{11} & a_{12} & a_{1n} \\ a_{21} & a_{22} & a_{2n} \\ a_{31} & a_{32} & a_{3n} \end{matrix} \qquad [C] = \begin{matrix} C_1 \\ C_2 \\ C_3 \end{matrix} \qquad [T] = \begin{matrix} T_1 \\ T_2 \\ T_3 \end{matrix}$$

Can be expressed as

$$[A][T] = [C]$$

And the problem is to find the inverse of $[A]$ such that

$$[T] = [A]^{-1}[C]$$

$$[A]^{-1} = \begin{matrix} b_{11} & b_{12} & b_{1n} \\ b_{21} & b_{22} & b_{2n} \\ b_{31} & b_{32} & b_{3n} \end{matrix}$$

The final solution for unknown temperature are written in expanded form

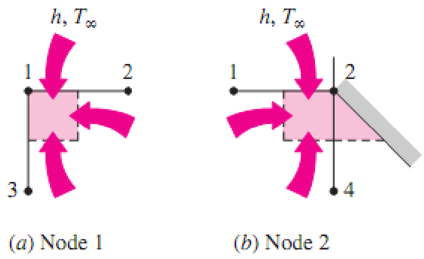
$$T_1 = b_{11}C_1 + b_{12}C_2 + \cdots + b_{1n}C_n$$

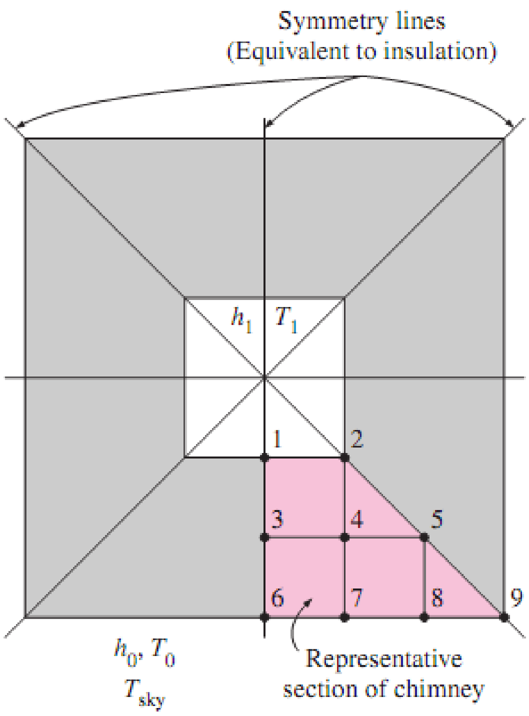
$$T_2 = b_{21}C_1 + b_{22}C_2 + \cdots + b_{2n}C_n$$

$$T_n = b_{11}C_1 + b_{n2}C_2 + \cdots + b_{nn}C_n$$

Example: Heat Loss through Chimneys

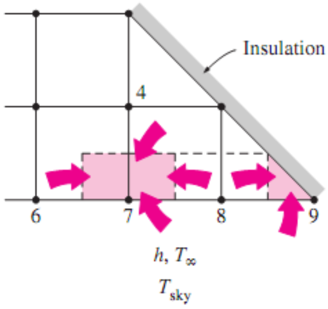
Hot combustion gases of a furnace are flowing through a square chimney made of concrete (k = 1.4 W/m · °C). The flow section of the chimney is 20 cm x 20 cm, and the thickness of the wall is 20 cm. The average temperature of the hot gases in the chimney is Ti = 300°C, and the average convection heat transfer coefficient inside the chimney is h_i= 70 W/m^2.°C. The chimney is losing heat from its outer surface to the ambient air at T_o = 20°C by convection with a heat transfer coefficient of h_o = 21 W/m^2.°C and to the sky by radiation. The emissivity of the outer surface of the wall is ε= 0.9, and the effective sky temperature is estimated to be 260 K. Using the finite difference method with Δx =Δy = 10 cm and taking full advantage of symmetry, determine the temperatures at the nodal points of a cross section and the rate of heat loss for a 1-m-long section of the chimney.





Solution:

The most striking aspect of this problem is the apparent symmetry about the horizontal and vertical lines passing through the midpoint of the chimney as well as the diagonal axes, as indicated on the figure. Therefore, we need to consider only one-eighth of the geometry in the solution whose nodal network consists of nine equally spaced nodes. No heat can cross a symmetry line, and thus symmetry lines can be treated as insulated surfaces and thus "mirrors" in the finite difference formulation. Then the nodes in the middle of the symmetry lines can be treated as interior nodes by using mirror images. Six of the nodes are boundary nodes, so we will have to write energy balances to obtain their finite difference formulations. First we partition the region among the nodes equitably by drawing dashed lines between the nodes through the middle. Then the region around a node surrounded by the boundary or the dashed lines represents the volume element of the node. Considering a unit depth and using the energy balance approach for the boundary nodes (again assuming all heat transfer into the volume element for convenience) and the formula for the interior nodes, the finite difference equations for the nine nodes are determined as follows:



(*a*) Node 1. On the inner boundary, subjected to convection, Figure 5–33*a*

$$0 + h_i \frac{\Delta x}{2}(T_i - T_1) + k\frac{\Delta y}{2}\frac{T_2 - T_1}{\Delta x} + k\frac{\Delta x}{2}\frac{T_3 - T_1}{\Delta y} + 0 = 0$$

Taking $\Delta x = \Delta y = l$, it simplifies to

$$-\left(2 + \frac{h_i l}{k}\right)T_1 + T_2 + T_3 = -\frac{h_i l}{k}T_i$$

(*b*) Node 2. On the inner boundary, subjected to convection, Figure 5–33*b*

$$k\frac{\Delta y}{2}\frac{T_1 - T_2}{\Delta x} + h_i\frac{\Delta x}{2}(T_i - T_2) + 0 + k\Delta x\frac{T_4 - T_2}{\Delta y} = 0$$

Taking $\Delta x = \Delta y = l$, it simplifies to

$$T_1 - \left(3 + \frac{h_i l}{k}\right)T_2 + 2T_4 = -\frac{h_i l}{k}T_i$$

(*c*) Nodes 3, 4, *and* 5. (Interior nodes, Fig. 5–34)

$$\text{Node 3: } T_4 + T_1 + T_4 + T_6 - 4T_3 = 0$$
$$\text{Node 4: } T_3 + T_2 + T_5 + T_7 - 4T_4 = 0$$
$$\text{Node 5: } T_4 + T_4 + T_8 + T_8 - 4T_5 = 0$$

(*d*) Node 6. (On the outer boundary, subjected to convection and radiation)

$$0 + k\frac{\Delta x}{2}\frac{T_3 - T_6}{\Delta y} + k\frac{\Delta y}{2}\frac{T_7 - T_6}{\Delta x}$$
$$+ h_o\frac{\Delta x}{2}(T_o - T_6) + \varepsilon\sigma\frac{\Delta x}{2}(T_{\text{sky}}^4 - T_6^4) = 0$$

(*e*) Node 7. (On the outer boundary, subjected to convection and radiatio
Fig. 5–35)

$$k\frac{\Delta y}{2}\frac{T_6 - T_7}{\Delta x} + k\Delta x\frac{T_4 - T_7}{\Delta y} + k\frac{\Delta y}{2}\frac{T_8 - T_7}{\Delta x} + h_o\Delta x(T_o - T_7) + \varepsilon\sigma\Delta x(T_{sky}^4 - T_7^4) = 0$$

Taking $\Delta x = \Delta y = l$, it simplifies to

$$2T_4 + T_6 - \left(4 + \frac{2h_o l}{k}\right)T_7 + T_8 = -\frac{2h_o l}{k}T_o - \frac{2\varepsilon\sigma l}{k}(T_{sky}^4 - T_7^4)$$

(*f*) Node 8. Same as Node 7, except shift the node numbers up by 1 (replace 4 by 5, 6 by 7, 7 by 8, and 8 by 9 in the last relation)

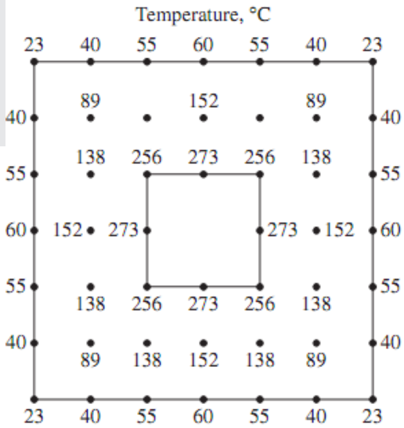
$$2T_5 + T_7 - \left(4 + \frac{2h_o l}{k}\right)T_8 + T_9 = -\frac{2h_o l}{k}T_o - \frac{2\varepsilon\sigma l}{k}(T_{sky}^4 - T_8^4)$$

(*g*) Node 9. (On the outer boundary, subjected to convection and radiation, Fig. 5–35)

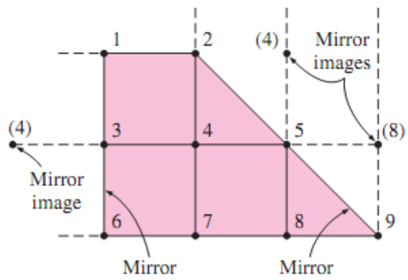
$$k\frac{\Delta y}{2}\frac{T_8 - T_9}{\Delta x} + 0 + h_o\frac{\Delta x}{2}(T_o - T_9) + \varepsilon\sigma\frac{\Delta x}{2}(T_{sky}^4 - T_9^4) = 0$$

Taking $\Delta x = \Delta y = l$, it simplifies to

$$T_8 - \left(1 + \frac{h_o l}{k}\right)T_9 = -\frac{h_o l}{k}T_o - \frac{\varepsilon\sigma l}{k}(T_{sky}^4 - T_9^4)$$



This problem involves radiation, which requires the use of absolute temperature, and thus all temperatures should be expressed in Kelvin. Alternately, we could use °C for all temperatures provided that the four temperatures in the radiation terms are expressed in the form $(T + 273)^4$. Substituting the given quantities, the system of nine equations for the determination of nine unknown nodal temperatures in a form suitable for use with the Gauss-Seidel iteration method becomes



$$T_1 = (T_2 + T_3 + 2865)/7$$
$$T_2 = (T_1 + 2T_4 + 2865)/8$$
$$T_3 = (T_1 + 2T_4 + T_6)/4$$
$$T_4 = (T_2 + T_3 + T_5 + T_7)/4$$
$$T_5 = (2T_4 + 2T_8)/4$$
$$T_6 = (T_2 + T_3 + 456.2 - 0.3645 \times 10^{-9}\, T_6^4)/3.5$$
$$T_7 = (2T_4 + T_6 + T_8 + 912.4 - 0.729 \times 10^{-9}\, T_7^4)/7$$
$$T_8 = (2T_5 + T_7 + T_9 + 912.4 - 0.729 \times 10^{-9}\, T_8^4)/7$$
$$T_9 = (T_8 + 456.2 - 0.3645 \times 10^{-9}\, T_9^4)/2.5$$

which is a system of *nonlinear* equations. Using an equation solver, its solution is determined to be

$$
\begin{aligned}
&T_1 = 545.7\text{ K} = 272.6°\text{C} \quad &&T_2 = 529.2\text{ K} = 256.1°\text{C} \quad &&T_3 = 425.2\text{ K} = 152.1°\text{C}\\
&T_4 = 411.2\text{ K} = 138.0°\text{C} \quad &&T_5 = 362.1\text{ K} = 89.0°\text{C} \quad &&T_6 = 332.9\text{ K} = 59.7°\text{C}\\
&T_7 = 328.1\text{ K} = 54.9°\text{C} \quad &&T_8 = 313.1\text{ K} = 39.9°\text{C} \quad &&T_9 = 296.5\text{ K} = 23.4°\text{C}
\end{aligned}
$$

The variation of temperature in the chimney is shown in Figure 5–36.

Note that the temperatures are highest at the inner wall (but less than 300°C) and lowest at the outer wall (but more that 260 K), as expected.

The average temperature at the outer surface of the chimney weighed by the surface area is

$$T_{\text{wall, out}} = \frac{(0.5T_6 + T_7 + T_8 + 0.5T_9)}{(0.5 + 1 + 1 + 0.5)}$$

$$= \frac{0.5 \times 332.9 + 328.1 + 313.1 + 0.5 \times 296.5}{3} = 318.6\text{ K}$$

Then the rate of heat loss through the 1-m-long section of the chimney can be determined approximately from

$$
\begin{aligned}
\dot{Q}_{\text{chimney}} &= h_o A_o (T_{\text{wall, out}} - T_o) + \varepsilon\sigma A_o (T^4_{\text{wall, out}} - T^4_{\text{sky}})\\
&= (21\text{ W/m}^2 \cdot \text{K})[4 \times (0.6\text{ m})(1\text{ m})](318.6 - 293)\text{K}\\
&\quad + 0.9(5.67 \times 10^{-8}\text{ W/m}^2 \cdot \text{K}^4)\\
&\qquad [4 \times (0.6\text{ m})(1\text{ m})][(318.6\text{ K})^4 - (260\text{ K})^4]\\
&= 1291 + 702 = \mathbf{1993\text{ W}}
\end{aligned}
$$

We could also determine the heat transfer by finding the average temperature of the inner wall, which is (272.6 + 256.1)/2 = 264.4°C, and applying Newton's law of cooling at that surface:

$$
\begin{aligned}
\dot{Q}_{\text{chimney}} &= h_i A_i (T_i - T_{\text{wall, in}})\\
&= (70\text{ W/m}^2 \cdot \text{K})[4 \times (0.2\text{ m})(1\text{ m})](300 - 264.4)°\text{C} = 1994\text{ W}
\end{aligned}
$$

The difference between the two results is due to the approximate nature of the numerical analysis.

Discussion We used a relatively crude numerical model to solve this problem to keep the complexities at a manageable level. The accuracy of the solution obtained can be improved by using a finer mesh and thus a greater number of nodes. Also, when radiation is involved, it is more accurate (but more laborious) to determine the heat losses for each node and add them up instead of using the average temperature.

EXAMPLE Steady Two-Dimensional Heat Conduction in L-Bars

Consider steady heat transfer in an L-shaped solid body whose cross section is given in Figure 5–26. Heat transfer in the direction normal to the plane of the paper is negligible, and thus heat transfer in the body is two-dimensional. The thermal conductivity of the body is $k = 15$ W/m · °C, and heat is generated in the body at a rate of $\dot{g} = 2 \times 10^6$ W/m³. The left surface of the body is insulated, and the bottom surface is maintained at a uniform temperature of 90°C. The entire top surface is subjected to convection to ambient air at $T_\infty = 25$°C with a convection coefficient of $h = 80$ W/m² · °C, and the right surface is subjected to heat flux at a uniform rate of $\dot{q}_R = 5000$ W/m². The nodal network of the problem consists of 15 equally spaced nodes with $\Delta x = \Delta y = 1.2$ cm, as shown in the figure. Five of the nodes are at the bottom surface, and thus their temperatures are known. Obtain the finite difference equations at the remaining nine nodes and determine the nodal temperatures by solving them.

(*a*) *Node 1.* The volume element of this corner node is insulated on the left and subjected to convection at the top and to conduction at the right and bottom surfaces. An energy balance on this element gives [Fig. 5–27*a*]

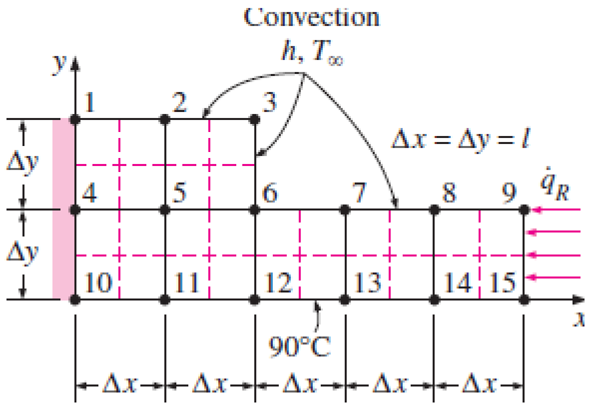
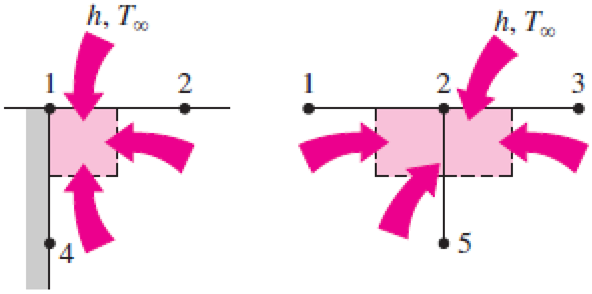


FIGURE
Schematic for Example 5–3 and the nodal network (the boundaries of volume elements of the nodes are indicated by dashed lines).



(*a*) Node 1 (*b*) Node 2

FIGURE

Schematics for energy balances on the volume elements of nodes 1 and 2.

$$0 + h\frac{\Delta x}{2}(T_\infty - T_1) + k\frac{\Delta y}{2}\frac{T_2 - T_1}{\Delta x} + k\frac{\Delta x}{2}\frac{T_4 - T_1}{\Delta y} + \dot{g}_1\frac{\Delta x}{2}\frac{\Delta y}{2} = 0$$

Taking $\Delta x = \Delta y = l$, it simplifies to

$$-\left(2 + \frac{hl}{k}\right)T_1 + T_2 + T_4 = -\frac{hl}{k}T_\infty - \frac{\dot{g}_1 l^2}{2k}$$

(*b*) *Node 2.* The volume element of this boundary node is subjected to convection at the top and to conduction at the right, bottom, and left surfaces. An energy balance on this element gives [Fig. 5–27*b*]

$$h\Delta x(T_\infty - T_2) + k\frac{\Delta y}{2}\frac{T_3 - T_2}{\Delta x} + k\Delta x\frac{T_5 - T_2}{\Delta y} + k\frac{\Delta y}{2}\frac{T_1 - T_2}{\Delta x} + \dot{g}_2\Delta x\frac{\Delta y}{2} = 0$$

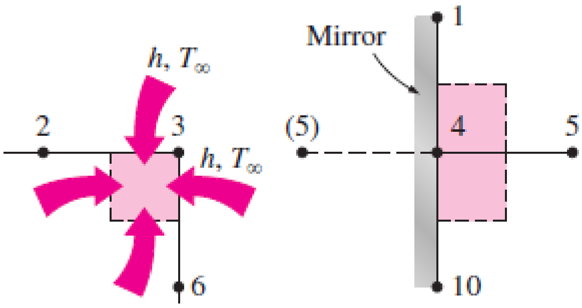
Taking $\Delta x = \Delta y = l$, it simplifies to

$$T_1 - \left(4 + \frac{2hl}{k}\right)T_2 + T_3 + 2T_5 = -\frac{2hl}{k}T_\infty - \frac{\dot{g}_2 l^2}{k}$$

(*c*) *Node 3.* The volume element of this corner node is subjected to convection at the top and right surfaces and to conduction at the bottom and left surfaces. An energy balance on this element gives [Fig. 5–28*a*]

$$h\left(\frac{\Delta x}{2} + \frac{\Delta y}{2}\right)(T_\infty - T_3) + k\frac{\Delta x}{2}\frac{T_6 - T_3}{\Delta y} + k\frac{\Delta y}{2}\frac{T_2 - T_3}{\Delta x} + \dot{g}_3\frac{\Delta x}{2}\frac{\Delta y}{2} = 0$$

Taking $\Delta x = \Delta y = l$, it simplifies to



$$T_2 - \left(2 + \frac{2hl}{k}\right) T_3 + T_6 = -\frac{2hl}{k} T_\infty - \frac{\dot{g}_3 l^2}{2k}$$

(*d*) *Node 4.* This node is on the insulated boundary and can be treated as an interior node by replacing the insulation by a mirror. This puts a reflected image of node 5 to the left of node 4. Noting that $\Delta x = \Delta y = l$, the general interior node relation for the steady two-dimensional case (Eq. 5–35) gives [Fig. 5–28*b*]

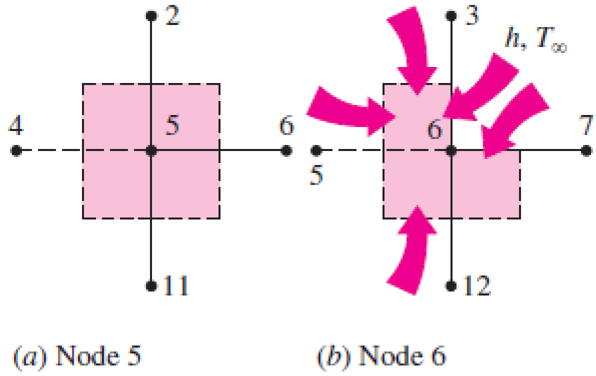
$$T_5 + T_1 + T_5 + T_{10} - 4T_4 + \frac{\dot{g}_4 l^2}{k} = 0$$

or, noting that $T_{10} = 90°$ C,

$$T_1 - 4T_4 + 2T_5 = -90 - \frac{\dot{g}_4 l^2}{k}$$

(*e*) Node 5. This is an interior node, and noting that $\Delta x = \Delta y = l$, the finite difference formulation of this node is obtained directly from Eq. 5–35 to be [Fig. 5–29*a*]

$$T_4 + T_2 + T_6 + T_{11} - 4T_5 + \frac{\dot{g}_5 l^2}{k} = 0$$



FIGURE

Schematics for energy balances on the volume elements of nodes 5 and 6.

or, noting that $T_{11} = 90°C$,

$$T_2 + T_4 - 4T_5 + T_6 = -90 - \frac{\dot{g}_5 l^2}{k}$$

(*f*) *Node 6.* The volume element of this inner corner node is subjected to convection at the L-shaped exposed surface and to conduction at other surfaces. An energy balance on this element gives [Fig. 5–29*b*]

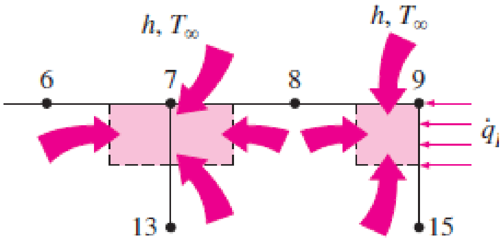
$$h\left(\frac{\Delta x}{2} + \frac{\Delta y}{2}\right)(T_\infty - T_6) + k\frac{\Delta y}{2}\frac{T_7 - T_6}{\Delta x} + k\Delta x\frac{T_{12} - T_6}{\Delta y}$$
$$+ k\Delta y\frac{T_5 - T_6}{\Delta x} + k\frac{\Delta x}{2}\frac{T_3 - T_6}{\Delta y} + \dot{g}_6\frac{3\Delta x\Delta y}{4} = 0$$

Taking $\Delta x = \Delta y = l$ and noting that $T_{12} = 90°C$, it simplifies to

$$T_3 + 2T_5 - \left(6 + \frac{2hl}{k}\right)T_6 + T_7 = -180 - \frac{2hl}{k}T_\infty - \frac{3\dot{g}_6 l^2}{2k}$$

(*g*) *Node 7.* The volume element of this boundary node is subjected to convection at the top and to conduction at the right, bottom, and left surfaces. An energy balance on this element gives [Fig. 5–30*a*]

$$h\Delta x(T_\infty - T_7) + k\frac{\Delta y}{2}\frac{T_8 - T_7}{\Delta x} + k\Delta x\frac{T_{13} - T_7}{\Delta y}$$
$$+ k\frac{\Delta y}{2}\frac{T_6 - T_7}{\Delta x} + \dot{g}_7\Delta x\frac{\Delta y}{2} = 0$$



(*h*) *Node 8*. This node is identical to Node 7, and the finite difference formulation of this node can be obtained from that of Node 7 by shifting the node numbers by 1 (i.e., replacing subscript *m* by *m* + 1). It gives

$$T_7 - \left(4 + \frac{2hl}{k}\right)T_8 + T_9 = -180 - \frac{2hl}{k}T_\infty - \frac{\dot{g}_8 l^2}{k}$$

(*i*) *Node 9*. The volume element of this corner node is subjected to convection at the top surface, to heat flux at the right surface, and to conduction at the bottom and left surfaces. An energy balance on this element gives [Fig. 5–30*b*]

$$h\frac{\Delta x}{2}(T_\infty - T_9) + \dot{q}_R\frac{\Delta y}{2} + k\frac{\Delta x}{2}\frac{T_{15} - T_9}{\Delta y} + k\frac{\Delta y}{2}\frac{T_8 - T_9}{\Delta x} + \dot{g}_9\frac{\Delta x}{2}\frac{\Delta y}{2} = 0$$

Taking $\Delta x = \Delta y = l$ and noting that $T_{15} = 90°C$, it simplifies to

$$T_8 - \left(2 + \frac{hl}{k}\right)T_9 = -90 - \frac{\dot{q}_R l}{k} - \frac{hl}{k}T_\infty - \frac{\dot{g}_9 l^2}{2k}$$

This completes the development of finite difference formulation for this problem. Substituting the given quantities, the system of nine equations for the determination of nine unknown nodal temperatures becomes

$$\begin{aligned}
-2.064T_1 + T_2 + T_4 &= -11.2\\
T_1 - 4.128T_2 + T_3 + 2T_5 &= -22.4\\
T_2 - 2.128T_3 + T_6 &= -12.8\\
T_1 - 4T_4 + 2T_5 &= -109.2\\
T_2 + T_4 - 4T_5 + T_6 &= -109.2\\
T_3 + 2T_5 - 6.128T_6 + T_7 &= -212.0\\
T_6 - 4.128T_7 + T_8 &= -202.4\\
T_7 - 4.128T_8 + T_9 &= -202.4\\
T_8 - 2.064T_9 &= -105.2
\end{aligned}$$

which is a system of nine algebraic equations with nine unknowns. Using an equation solver, its solution is determined to be

$T_1 = 112.1°C$	$T_2 = 110.8°C$	$T_3 = 106.6°C$
$T_4 = 109.4°C$	$T_5 = 108.1°C$	$T_6 = 103.2°C$
$T_7 = 97.3°C$	$T_8 = 96.3°C$	$T_9 = 97.6°C$

Note that the temperature is the highest at node 1 and the lowest at node 8. This is consistent with our expectations since node 1 is the farthest away from the bottom surface, which is maintained at 90°C and has one side insulated, and node 8 has the largest exposed area relative to its volume while being close to the surface at 90°C.

Transient Heat conduction

1. Class : third Year

2. Subject : Transient Heat conduction
3. Number of weeks: Two weeks

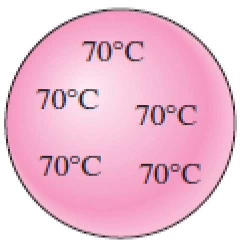
4. Central idea : Study the Transient Heat conduction
5. The Test:

TRANSIENT HEAT CONDUCTION

The temperature of a body, in general, varies with time as well as position. In rectangular coordinates, this variation is expressed as $T(x, y, z, t)$, where (x, y, z) indicates variation in the x, y, and z directions, respectively, and t indicates variation with time. In the preceding **Lecture** we considered heat conduction under *steady* conditions, for which the temperature of a body at any point does not change with time. This certainly simplified the analysis, especially when the temperature varied in one direction only, and we were able to obtain analytical solutions. In this **Lecture** we consider the variation of temperature with *time* as well as *position*

LUMPED SYSTEM ANALYSIS

In heat transfer analysis, some bodies are observed to behave like a "lump" whose interior temperature remains essentially uniform at all times during a heat transfer process. The temperature of such bodies can be taken to be a function of time only, $T(t)$. Heat transfer analysis that utilizes this idealization is known as **lumped system analysis,** which provides great simplification in certain classes of heat transfer problems without much sacrifice from accuracy.



(*a*) Copper ball

Consider a body of arbitrary shape of mass m, volume V, surface area A_s, density ρ, and specific heat C_p initially at a uniform temperature T_i (Fig.). At time $t = 0$, the body is placed into a medium at temperature T_∞, and heat transfer takes place between the body and its environment, with a heat transfer coefficient h. For the sake of discussion, we will assume that $T_\infty > T_i$, but the analysis is equally valid for the opposite case. We assume lumped system analysis to be applicable, so that the temperature remains uniform within the body at all times and changes with time only, $T = T(t)$.

During a differential time interval dt, the temperature of the body rises by a differential amount dT. An energy balance of the solid for the time interval dt can be expressed as

$$\begin{pmatrix}\text{Heat transfer into the body} \\ \text{during } dt\end{pmatrix} = \begin{pmatrix}\text{The increase in the} \\ \text{energy of the body} \\ \text{during } dt\end{pmatrix}$$

or

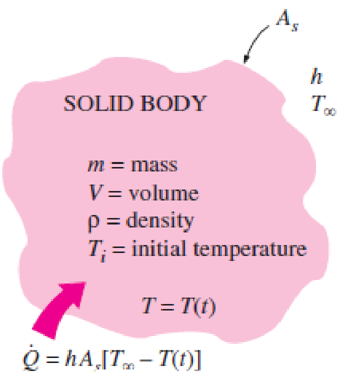
$$hA_s(T_\infty - T)\,dt = mC_p\,dT$$

Noting that $m = \rho V$ and $dT = d(T - T_\infty)$ since $T_\infty = \text{constant}$, Eq. can be rearranged as

$$\frac{d(T - T_\infty)}{T - T_\infty} = -\frac{hA_s}{\rho V C_p}\,dt$$

Integrating from $t = 0$, at which $T = T_i$, to any time t, at which $T = T(t)$, gives

$$\ln\frac{T(t) - T_\infty}{T_i - T_\infty} = -\frac{hA_s}{\rho V C_p}\,t$$



FIGURE

The geometry and parameters involved in the lumped system analysis.

Taking the exponential of both sides and rearranging, we obtain

$$\frac{T(t)-T_\infty}{T_i-T_\infty}=e^{-\frac{h\,A_s}{\rho\,c_pV}\,t}$$

$$\frac{Volume}{surface\ Area}=\frac{V}{A}=L$$ Characteristic length

$$\frac{hAt}{\rho C_pV}=\frac{ht}{\rho C_p}\frac{1}{L}=\frac{t}{\rho C_pL}\left(\frac{hL}{k}\right)*\left(\frac{k}{L}\right)=\alpha.B_i.\frac{t}{L^2}=B_i.\tau$$

$$\therefore\ \frac{T(t)-T_\infty}{T_i-T_\infty}=e^{-B_i.\tau}$$ $,where\ (\tau)\ is\ Dimensionless$ **Fourier number**

Heat transfer :

$$q=hA(T-T_\infty)dt=hA(T_i-T_\infty)\int_0^t e^{-\frac{h\,A_s}{\rho\,c_pV}\,t}$$

$$q=hA(T_i-T_\infty)\left[-\frac{\rho\,c_pV}{h\,A_s}\,e^{-\frac{h\,A_s}{\rho\,c_pV}\,t}\right]_0^t$$

$$q=\rho\,c_pV(T_i-T_\infty)\left[1-e^{-\frac{h\,A_s}{\rho\,c_pV}\,t}\right]$$

Criteria for Lumped System Analysis

The lumped system analysis certainly provides great convenience in heat transfer analysis, and naturally we would like to know when it is appropriate to use it. The first step in establishing a criterion for the applicability of the lumped system analysis is to define a **characteristic length** as

$$L_c = \frac{V}{A_s}$$

and a **Biot number** Bi as

$$\mathrm{Bi} = \frac{hL_c}{k}$$

It can also be expressed as

$$\mathrm{Bi} = \frac{h}{k/L_c}\frac{\Delta T}{\Delta T} = \frac{\text{Convection at the surface of the body}}{\text{Conduction within the body}}$$

or

$$\mathrm{Bi} = \frac{L_c/k}{1/h} = \frac{\text{Conduction resistance within the body}}{\text{Convection resistance at the surface of the body}}$$

all uncertainty, provided that it is minor. It is generally accepted that lumped system analysis is *applicable* if

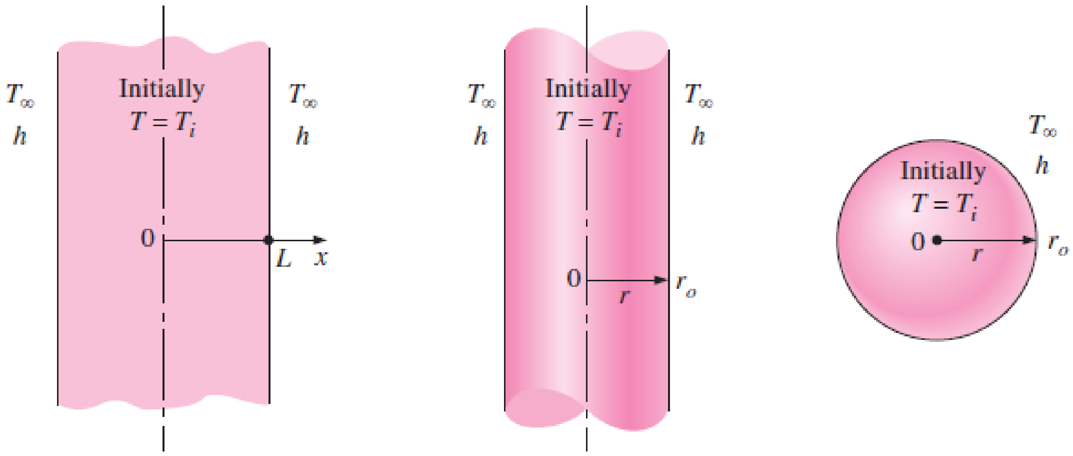
$$\mathrm{Bi} \leq 0.1$$

When this criterion is satisfied, the temperatures within the body relative to the surroundings (i.e., $T - T_\infty$) remain within 5 percent of each other even for well-rounded geometries such as a spherical ball. Thus, when $\mathrm{Bi} < 0.1$, the variation of temperature with location within the body will be slight and can reasonably be approximated as being uniform.

TRANSIENT HEAT CONDUCTION IN LARGE PLANE WALLS, LONG CYLINDERS, AND SPHERES WITH SPATIAL EFFECTS

In Section, we considered bodies in which the variation of temperature within the body was negligible; that is, bodies that remain nearly *isothermal* during a process. Relatively *small* bodies of *highly conductive* materials approximate this behavior. In general, however, the temperature within a body will change from point to point as well as with time. In this section, we consider the variation of temperature with *time* and *position* in one-dimensional problems such as those associated with a large plane wall, a long cylinder, and a sphere.

Consider a plane wall of thickness $2L$, a long cylinder of radius r_o, and a sphere of radius r_o initially at a *uniform temperature* T_i, as shown in Fig. At time $t = 0$, each geometry is placed in a large medium that is at a constant temperature T_∞ and kept in that medium for $t > 0$. Heat transfer takes place between these bodies and their environments by convection with a *uniform* and *constant* heat transfer coefficient h. Note that all three cases possess geometric and thermal symmetry: the plane wall is symmetric about its *center plane* ($x = 0$), the cylinder is symmetric about its *centerline* ($r = 0$), and the sphere is symmetric about its *center point* ($r = 0$). We neglect *radiation* heat transfer between these bodies and their surrounding surfaces, or incorporate the radiation effect into the convection heat transfer coefficient h.



(*a*) A large plane wall (*b*) A long cylinder (*c*) A sphere

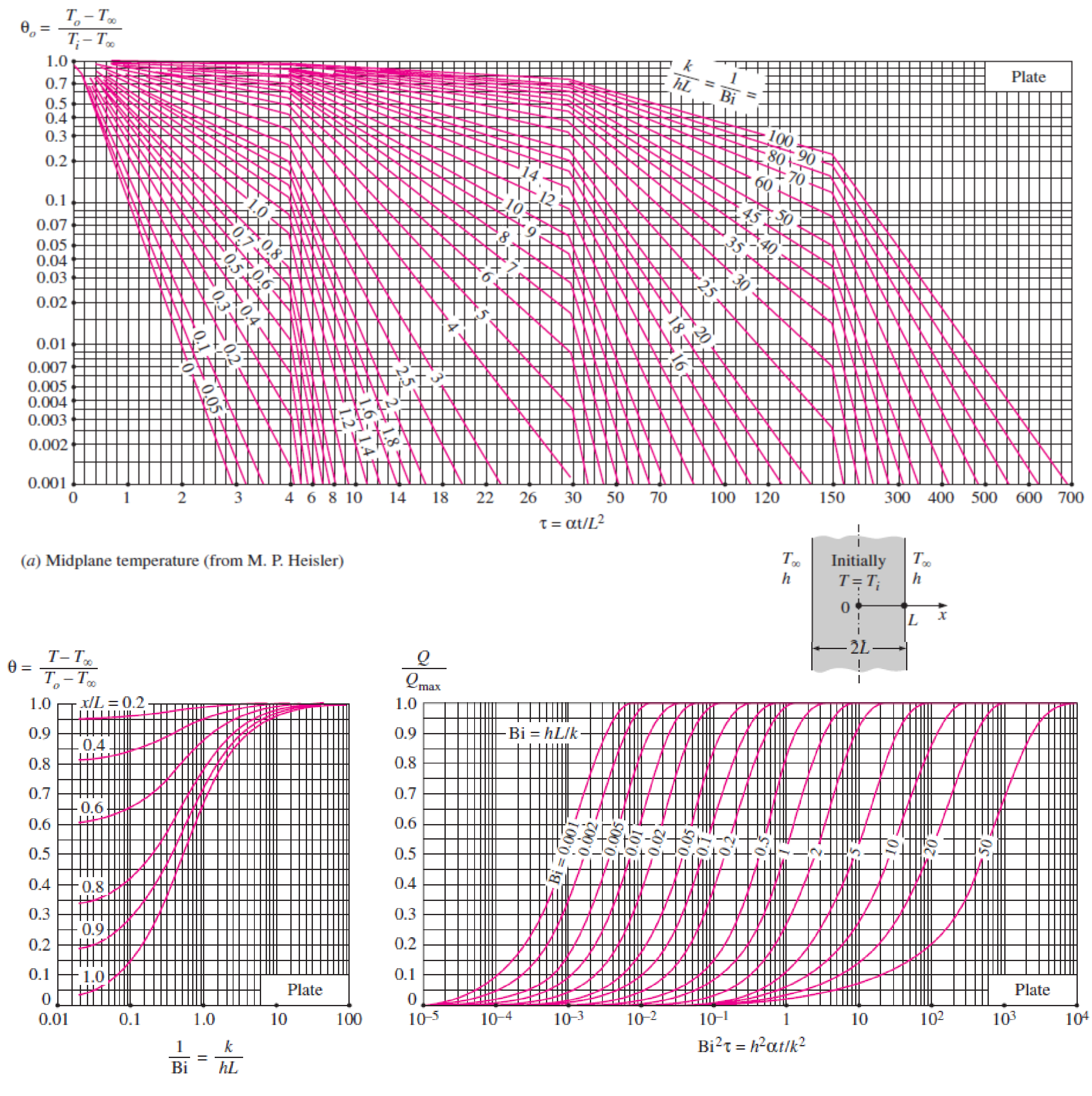
Dimensionless temperature: $\theta(x, t) = \dfrac{T(x, t) - T_\infty}{T_i - T_\infty}$

Dimensionless distance from the center: $X = \dfrac{x}{L}$

Dimensionless heat transfer coefficient: $\text{Bi} = \dfrac{hL}{k}$ **(Biot number)**

Dimensionless time: $\tau = \dfrac{\alpha t}{L^2}$ **(Fourier number)**

$$q_{max} = mC_p(T_\infty - T_i) = \rho V C_p(T_\infty - T_i) \qquad \text{(kJ)}$$



(*a*) Midplane temperature (from M. P. Heisler)

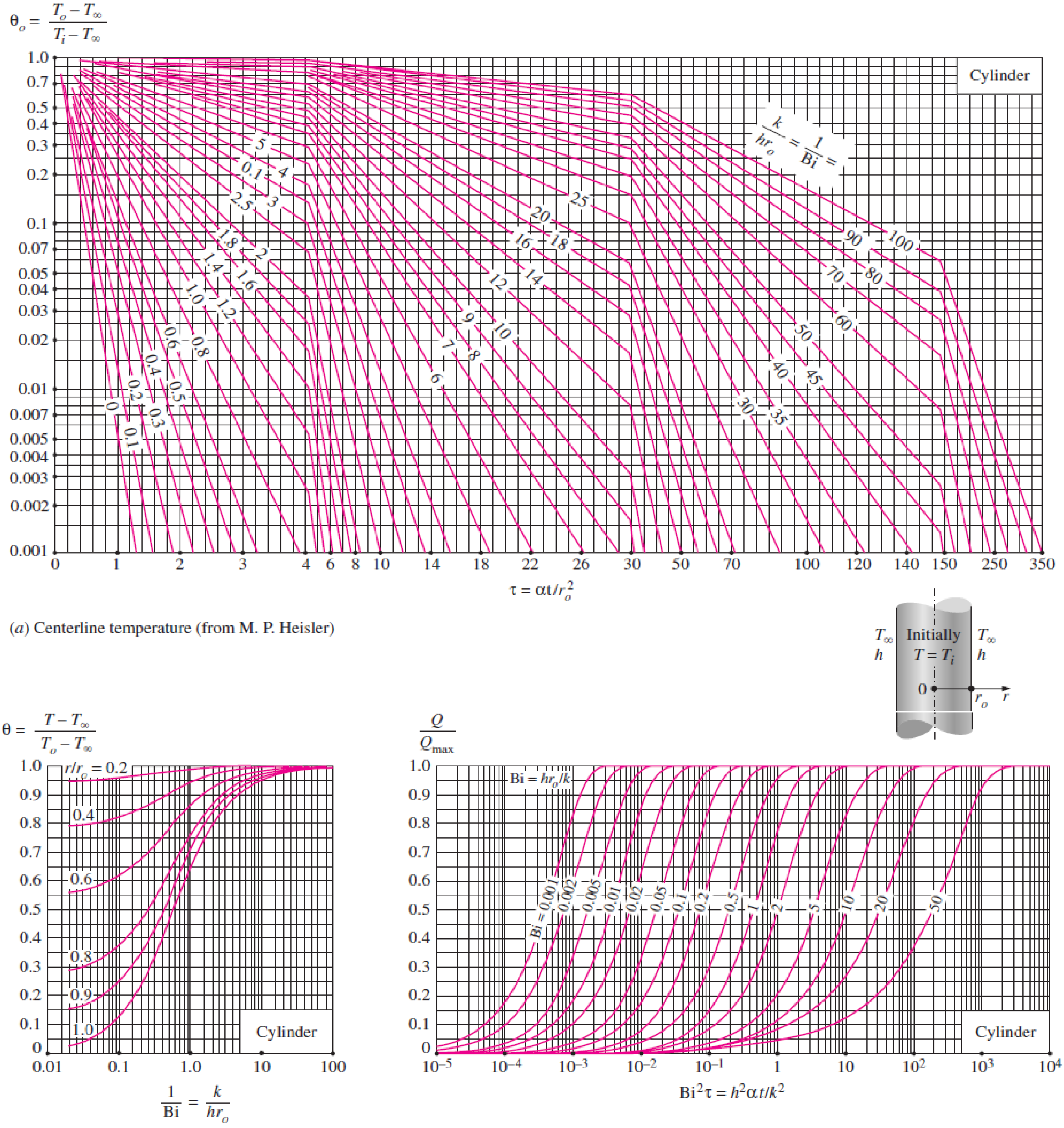
(*b*) Temperature distribution (from M. P. Heisler)

(*c*) Heat transfer (from H. Gröber et al.)

FIGURE

Transient temperature and heat transfer charts for a plane wall of thickness $2L$ initially at a uniform temperature T_i subjected to convection from both sides to an environment at temperature T_∞ with a convection coefficient of h.

where m is the mass, V is the volume, ρ is the density, and C_p is the specific heat of the body. Thus, Q_{max} represents the amount of heat transfer for $t \to \infty$. The amount of heat transfer Q at a finite time t will obviously be less than this

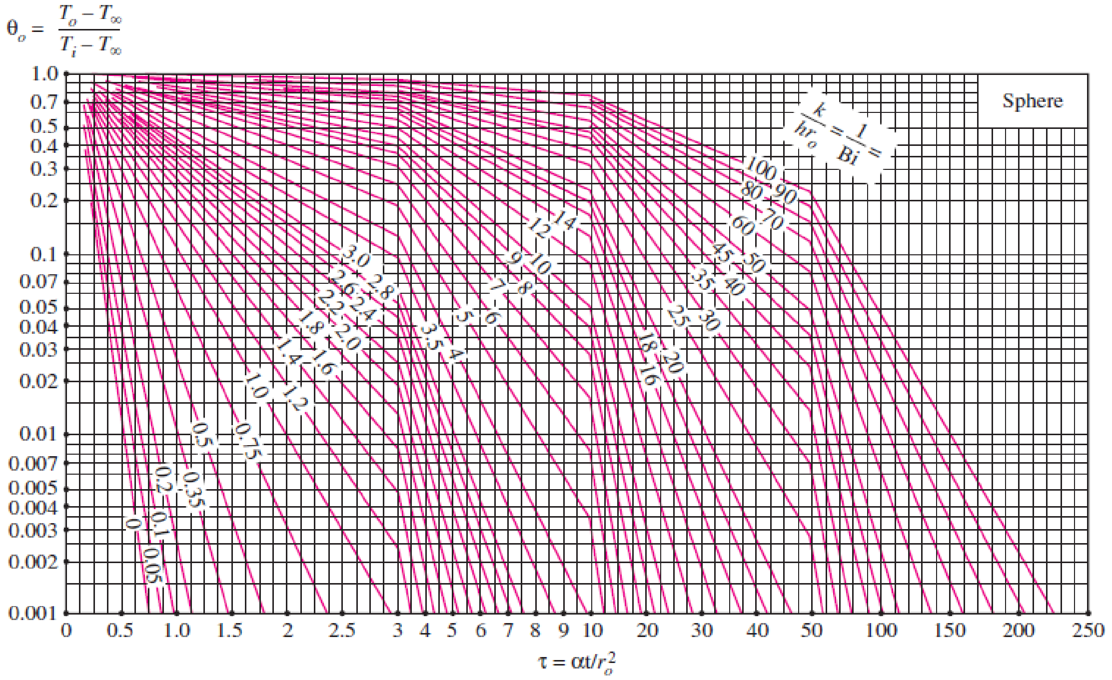


(b) Temperature distribution (from M. P. Heisler)

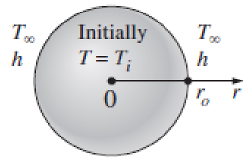
(c) Heat transfer (from H. Gröber et al.)

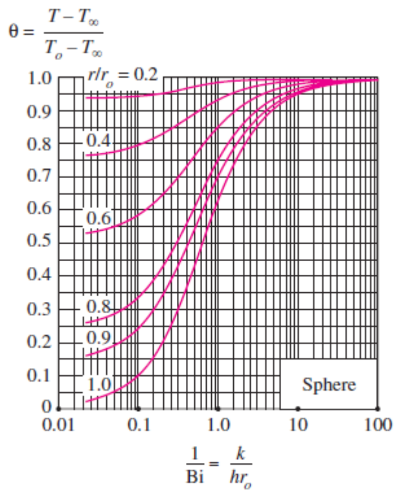
FIGURE

Transient temperature and heat transfer charts for a long cylinder of radius r_o initially at a uniform temperature T_i subjected to convection from all sides to an environment at temperature T_∞ with a convection coefficient of h.

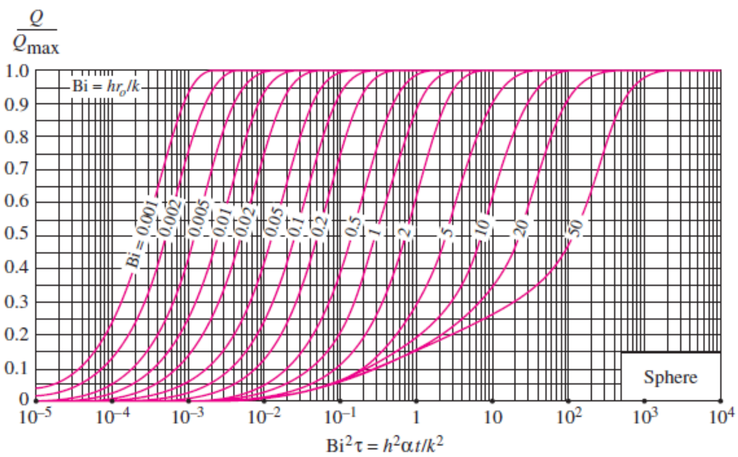


(*a*) Midpoint temperature (from M. P. Heisler)





(*b*) Temperature distribution (from M. P. Heisler)



(*c*) Heat transfer (from H. Gröber et al.)

FIGURE

Transient temperature and heat transfer charts for a sphere of radius r_o initially at a uniform temperature T_i subjected to convection from all sides to an environment at temperature T_∞ with a convection coefficient of h.

TRANSIENT HEAT CONDUCTION IN SEMI-INFINITE SOLIDS

A semi-infinite solid is an idealized body that has a *single plane surface* and extends to infinity in all directions, as shown in Fig. This idealized body is used to indicate that the temperature change in the part of the body in which we are interested (the region close to the surface) is due to the thermal conditions on a single surface. The earth, for example, can be considered to be a semi-infinite medium in determining the variation of temperature near its surface. Also, a thick wall can be modeled as a semi-infinite medium if all we are interested in is the variation of temperature in the region near one of the surfaces, and the other surface is too far to have any impact on the region of interest during the time of observation.

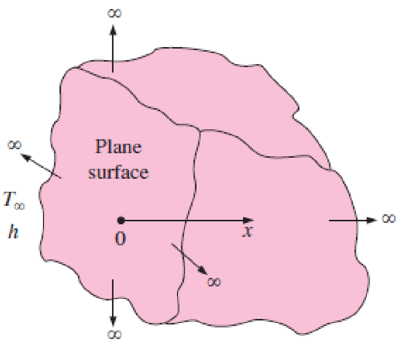


FIGURE
Schematic of a semi-infinite body.

Case 1:a sudden change in surface te ture $T_O \neq T_i$

$$\frac{T(x,t) - T_O}{T_i - T_O} = \operatorname{erf}\frac{x}{2\sqrt{\alpha t}}$$

T_0= surface temperature = $T(0,t)$

T_i = initial temperature of medium = $T(x,0)$

At surface the heat flow

$$q_0 = \frac{KA(T_O - T_i)}{\sqrt{\pi \alpha t}}$$

Table A-1 | The error function.

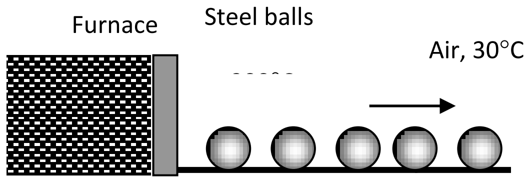
$\frac{x}{2\sqrt{\alpha\tau}}$	$\mathrm{erf}\frac{x}{2\sqrt{\alpha\tau}}$	$\frac{x}{2\sqrt{\alpha\tau}}$	$\mathrm{erf}\frac{x}{2\sqrt{\alpha\tau}}$	$\frac{x}{2\sqrt{\alpha\tau}}$	$\mathrm{erf}\frac{x}{2\sqrt{\alpha\tau}}$
0.00	0.00000	0.76	0.71754	1.52	0.96841
0.02	0.02256	0.78	0.73001	1.54	0.97059
0.04	0.04511	0.80	0.74210	1.56	0.97263
0.06	0.06762	0.82	0.75381	1.58	0.97455
0.08	0.09008	0.84	0.76514	1.60	0.97636
0.10	0.11246	0.86	0.77610	1.62	0.97804
0.12	0.13476	0.88	0.78669	1.64	0.97962
0.14	0.15695	0.90	0.79691	1.66	0.98110
0.16	0.17901	0.92	0.80677	1.68	0.98249
0.18	0.20094	0.94	0.81627	1.70	0.98379
0.20	0.22270	0.96	0.82542	1.72	0.98500
0.22	0.24430	0.98	0.83423	1.74	0.98613
0.24	0.26570	1.00	0.84270	1.76	0.98719
0.26	0.28690	1.02	0.85084	1.78	0.98817
0.28	0.30788	1.04	0.85865	1.80	0.98909
0.30	0.32863	1.06	0.86614	1.82	0.98994
0.32	0.34913	1.08	0.87333	1.84	0.99074
0.34	0.36936	1.10	0.88020	1.86	0.99147
0.36	0.38933	1.12	0.88079	1.88	0.99216
0.38	0.40901	1.14	0.89308	1.90	0.99279
0.40	0.42839	1.16	0.89910	1.92	0.99338
0.42	0.44749	1.18	0.90484	1.94	0.99392
0.44	0.46622	1.20	0.91031	1.96	0.99443
0.46	0.48466	1.22	0.91553	1.98	0.99489
0.48	0.50275	1.24	0.92050	2.00	0.995322
0.50	0.52050	1.26	0.92524	2.10	0.997020
0.52	0.53790	1.28	0.92973	2.20	0.998137
0.54	0.55494	1.30	0.93401	2.30	0.998857
0.56	0.57162	1.32	0.93806	2.40	0.999311
0.58	0.58792	1.34	0.94191	2.50	0.999593
0.60	0.60386	1.36	0.94556	2.60	0.999764
0.62	0.61941	1.38	0.94902	2.70	0.999866
0.64	0.63459	1.40	0.95228	2.80	0.999925
0.66	0.64938	1.42	0.95538	2.90	0.999959
0.68	0.66278	1.44	0.95830	3.00	0.999978
0.70	0.67780	1.46	0.96105	3.20	0.999994
0.72	0.69143	1.48	0.96365	3.40	0.999998
0.74	0.70468	1.50	0.96610	3.60	1.000000

Q1: Stainless steel ball bearings (ρ= 8085 kg/m3, k =15.1 W/m · °C, Cp = 0.480 kJ/kg · °C, and α =3.91 x$10^{-6}\, m^2/s$) having a diameter of 1.2 cm are to be quenched in water. The balls leave the oven at a uniform temperature of 900°C and are exposed to air at 30°C for a while before they are dropped into the water. If the temperature of the balls is not to fall below 850°C prior to quenching and the heat transfer coefficient in the air is 125 W/m²· °C, determine how long they can stand in the air before being dropped into the water.

Sol:

$$L_c = \frac{V}{A_s} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6} = \frac{0.012\text{ m}}{6} = 0.002\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(125\text{ W/m}^2.°\text{C})(0.002\text{ m})}{(15.1\text{ W/m.°C})} = 0.0166 < 0.1$$



Therefore, the lumped system analysis is applicable. Then the allowable time is determined to be

$$b = \frac{hA_s}{\rho C_p V} = \frac{h}{\rho C_p L_c} = \frac{125\text{ W/m}^2.°\text{C}}{(8085\text{ kg/m}^3)(480\text{ J/kg.°C})(0.002\text{ m})} = 0.01610\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{850-30}{900-30} = e^{-(0.0161\text{s}^{-1})t} \longrightarrow t = \mathbf{3.68\,s}$$

The result indicates that the ball bearing can stay in the air about 4 s before being dropped into the water.

Ex2: A stainless-steel rod (ρ = 7817 kg/m3, Cp = 460J/kg · °C) 6.4 mm in diameter is initially at a uniform temperature of 25◦C and is suddenly immersed in a liquid at 150◦C with h= 120 W/m² · ◦C. Using the lumped-capacity method of analysis, calculate the time necessary for the rod temperature to reach 120◦C.

$T_0 = 25°\text{C}$ $\quad T_\infty = 150°\text{C}$ $\quad h = 120\ \frac{\text{W}}{\text{m}^2 \cdot °\text{C}}$ $\quad T = 120°\text{C}$

$\rho = 7817$ $\quad c = 460$ $\quad d = 6.4\text{ mm}$

$$\frac{A}{V} = \frac{4}{d}$$

$$\frac{hA}{\rho c V} = \frac{(120)(4)}{(0.0064)(7817)(460)} = 0.02086$$

$$\frac{T - T_\infty}{T_0 - T_\infty} = \exp\left(-\frac{hA}{\rho c V}\tau\right)$$

$$\frac{120-25}{150-25} = 0.76 = e^{-0.02086\tau}$$

$\tau = 13.16$ sec

Ex3: A large plate of aluminum 5.0 cm thick(k = 215 W/m.◦C , α=8.4×10−5 m2/s , ρ=2700 kg/m³, c=0.9 kJ/kg · ◦C) and initially at 200◦C is suddenly exposed to the convection environment h= 525 W/m²◦C and T_∞=70◦C Calculate the temperature at a depth of 1.25 cm from one of the faces **1** min after the plate has been exposed to the environment. How much energy has been removed per unit area from the plate in this time?

Sol:

$\theta_i = T_i - T_\infty = 200-70 = 130$ $\quad$ $2L = 5.0$ cm $\quad$ L=2.5 cm

τ =1 min=60 s

k = 215 W/m · ◦C $\quad$ h = 525 W/m2 · ◦C

x = 2.5−1.25=1.25 cm

Then

$\alpha\tau / L^2 = (8.4\times10^{-5})*(60)/(0.025)^2 = 8.064$

$k/hL = 215/(525)(0.025) = 16.38$

$x/L = 1.25/2.5 = 0.5$

From Figure

$\theta_0/\theta_i = 0.61$

$\theta_0 = T_0 - T_\infty = (0.61)(130) = 79.3$

From Figure at x/L=0.5,

$\theta/\theta_0 = 0.98$

and

$\theta = T - T_\infty = (0.98)(79.3) = 77.7$

T = 77.7+70=147.7◦C

For Figure we need

$h^2\alpha\tau/k^2 = (525)^2(8.4\times10^{-5})(60)/(215)^2 = 0.03$

$hL/k = (525)(0.025)/215 = 0.061$

From Figure

$Q/Q_0 = 0.41$

For unit area

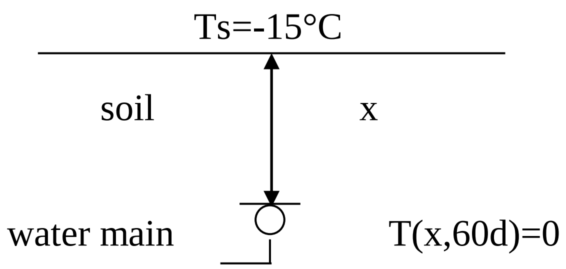
$Q_0/A = \rho cV\theta_i/A = \rho c(2L)\theta_i = (2700)(900)(0.05)(130)$

$= 15.8\times10^6$ J/m²

So that the heat removed per unit surface area is

$Q/A = (15.8\times10^6)(0.41) = 6.48\times10^6$ J/m²

Q4:Estimate the minimum depth *x* at which one must place a water main below the surface to avoid freezing .The soil is initially at a uniform temperature of 20 °C .Assume that under the worst conditions anticipated it is subjected to surface temperature of -15 °C for a period of 60 days .Use the following properties for soil(c=1840J/kg.k),(ρ=2050 kg/m³)(K=0.52w/m.k) ($\alpha = k/\rho c = 0.138 * 10^{-6}$ m²/s)



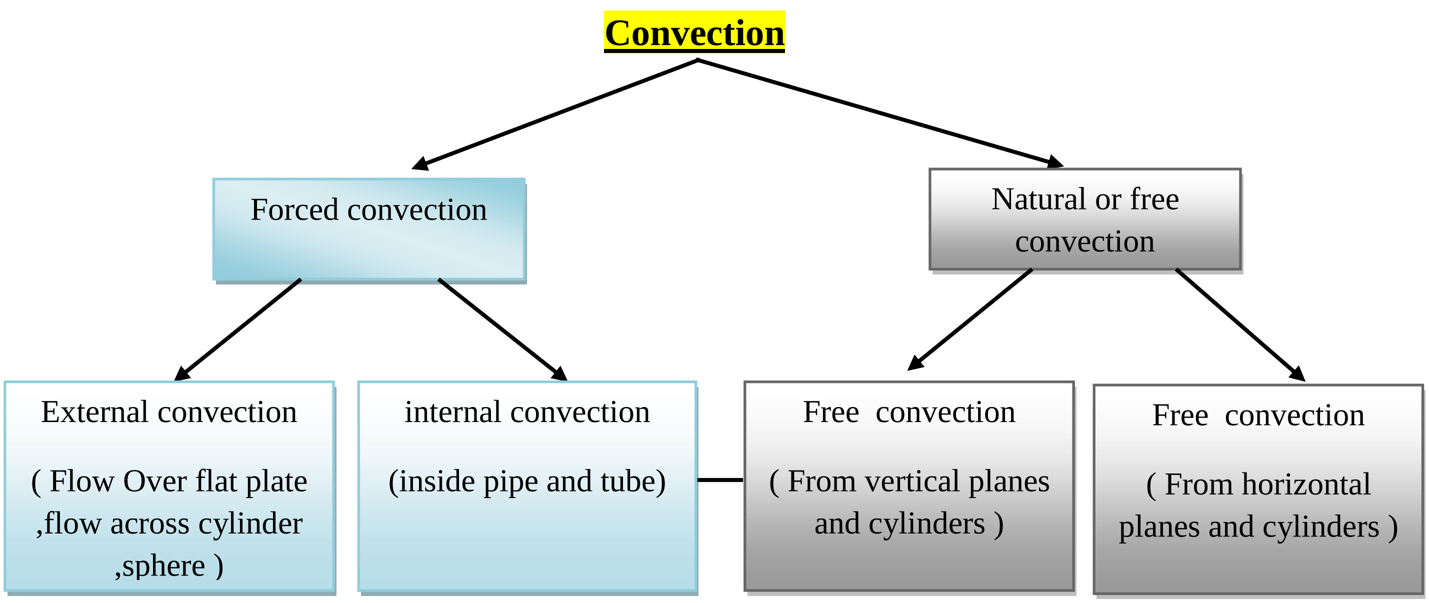
Fundamentals of Convection

The Objective: Study the mechanism of heat transfer through a fluid in the presence of bulk fluid motion.

Introduction: Heat transfer through a solid is always by conduction, since the molecules of a solid remain at relatively fixed positions. Heat transfer through a liquid or gas, however, can be by conduction or convection, depending on the presence of any bulk fluid motion. Heat transfer through a fluid is by convection in the presence of bulk fluid motion and by conduction in the absence of it.

Convection heat transfer is complicated by the fact that it involves fluid motion as well as heat conduction.

The rate of heat transfer through a fluid is much higher by convection than it is by conduction. In fact, the higher the fluid velocity, the higher the rate of heat transfers.



Summary:

Convection is classified as ***natural*** (or ***free***) and ***forced convection***, depending on how the fluid motion is initiated.

In forced convection, the fluid is forced to flow over a surface or in a pipe by external means such as a pump or a fan.

In natural convection, any fluid motion is caused by natural means such as the buoyancy effect, which manifests itself as the rise of warmer fluid and the fall of the cooler fluid.

Convection is also classified as ***external*** and ***internal***, depending on whether the fluid is forced to flow over a surface or in a channel.

Experience shows that convection heat transfer strongly depends on the fluid properties *dynamic viscosity* μ, *thermal conductivity k, density* ρ, and *specific heat Cp*, as well as the *fluid velocity* $\mathscr{V}$. It also depends on the *geometry* and the *roughness* of the solid surface, in addition to the *type of fluid flow* (such as being laminar "streamlined" or turbulent).

Convection heat transfer is observed to be proportional to the temperature difference and is conveniently expressed by **Newton's law of cooling** as:

$\dot{q}_{conv} = h(T_s - T_\infty)$ (W/m^2)

or

$\dot{Q}_{conv} = hA_s(T_s - T_\infty)$ (W)

Where:

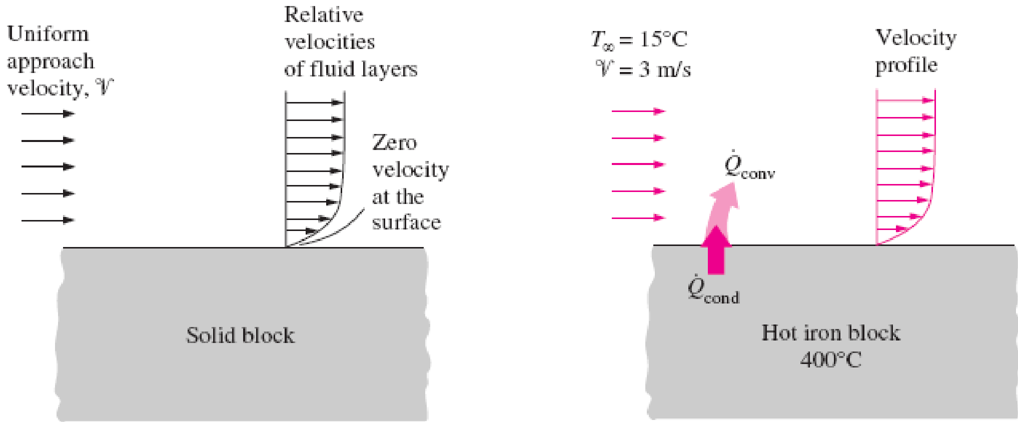
h = convection heat transfer coefficient, W/m^2.°C

A_s = heat transfer surface area, m^2

T_s = temperature of the surface, °C

T_∞ = temperature of the fluid sufficiently far from the surface, °C

Judging from its units, the **convection heat transfer coefficient** h can be defined as *the rate of heat transfer between a solid surface and a fluid per unit surface area per unit temperature difference.*



$$h = \frac{Q}{A_s(T_s - T_\infty)} \qquad (\text{W/m}^2.^\circ\text{C})$$

When a fluid is forced to flow over a solid surface, it is observed that the fluid in motion comes to a complete stop at the surface and assumes a zero velocity relative to the surface. That is, the fluid layer in direct contact with a solid surface "sticks" to the surface and there is no slip. In fluid flow, this phenomenon is known as the **no-slip condition,** and it is due to the viscosity of the fluid.

A fluid and a solid surface will have the same temperature at the point of contact. This is known as **no-temperature-jump condition.** An implication of the no-slip and the no-temperature jump conditions is that heat transfer from the solid surface to the fluid layer adjacent to the surface is by *pure conduction,* since the fluid layer is motionless, and can be expressed as:

$$\dot{q}_{conv} = \dot{q}_{cond} = -k_{fluid} \left.\frac{\partial T}{\partial y}\right|_{y=0} \qquad (\text{W/m}^2)$$

where T represents the temperature distribution in the fluid and $\left.\frac{\partial T}{\partial y}\right|_{y=o}$ is the *temperature gradient* at the surface. This heat is then *convected away* from the surface as a result of fluid motion. Note that convection heat transfer from a solid surface to a fluid is merely the conduction heat transfer from the solid surface to the fluid layer adjacent to the surface. Therefore, we can equate eq.(1) & eq. (4) for the heat flux to obtain:

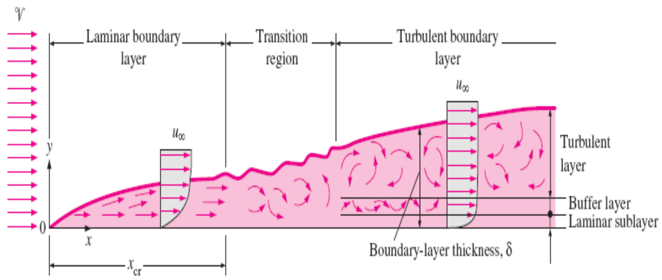
$$h = \frac{-k_{\text{fluid}}(\partial T/\partial y)_{y=0}}{T_s - T_\infty} \qquad (\text{W/m}^2 \cdot {}^\circ\text{C})$$

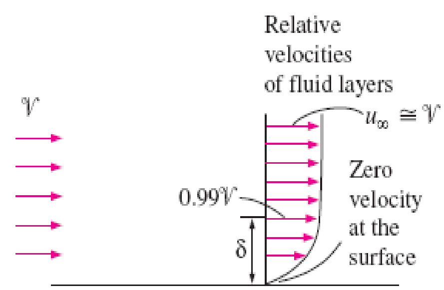
The convection heat transfer coefficient, in general, varies along the flow (or *x*-) direction. The *average* or *mean* convection heat transfer coefficient for a surface in such cases is determined by properly averaging the local convection heat transfer coefficients over the entire surface.

Velocity boundary layer

Consider the parallel flow of a fluid over a *flat plate*. The *x*-coordinate is measured along the plate surface from the *leading edge* of the plate in the direction of the flow, and *y* is measured from the surface in the normal direction. The fluid approaches the plate in the *x*-direction with a uniform upstream velocity of $\mathcal{V}$, which is practically identical to the free-stream velocity u_∞ over the plate away from the surface. the *x* component of the fluid velocity, *u*, will vary from 0 at $y = 0$ to nearly u_∞ at $y = \delta_v$.

The region of the flow above the plate bounded by δ in which the effects of the viscous shearing forces caused by fluid viscosity are felt is called the **velocity boundary layer.** The *boundary layer thickness,* δ_v, is typically defined as the distance *y* from the surface at which $u = 0.99u_\infty$. The hypothetical line of $u = 0.99\ u_\infty$ divides the flow over a plate into two regions: the **boundary layer region,** in which the viscous effects and the velocity changes are significant, and the **in viscid flow region,** in which the frictional effects are negligible and the velocity remains essentially constant.





Friction force per unit area is called **shear stress,** and is denoted by τ. Experimental studies indicate that the shear stress for most fluids is proportional to the *velocity gradient*, and the shear stress at the wall surface is as

$$\tau_s = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} \qquad (\text{N/m}^2)$$

where the constant of proportionality μ is called the **dynamic viscosity** of the fluid.

Kinematic viscosity is expressed as $\upsilon = \mu / \rho$. Two common units of kinematic viscosity are m^2/s and *stoke* (1 stoke = 1 cm^2/s = 0.0001 m^2/s).

The determination of the surface shear stress τ_s is not practical since it requires a knowledge of the flow velocity profile. A more practical approach in external flow is to relate τ_s to the upstream velocity $\mathscr{V}$ as

$$\tau_s = C_f \frac{\rho \mathscr{V}^2}{2} \qquad (\text{N/m}^2)$$

where C_f is the dimensionless **friction coefficient,** whose value in most cases is determined experimentally, and ρ is the density of the fluid. Note that the friction coefficient, in general, will vary with location along the surface. Once the average friction coefficient over a given surface is available, the friction force over the entire surface is determined from

$$F_f = C_f A_s \frac{\rho \mathscr{V}^2}{2} \qquad (\text{N})$$

where A_s is the surface area.

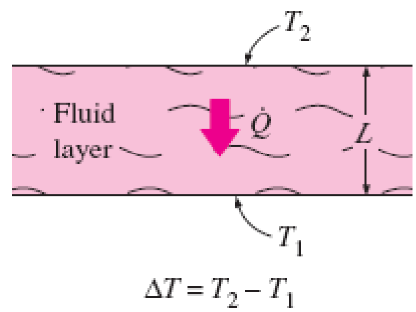
Nusselt Number

It is common practice to nondimensionalize the heat transfer coefficient *h* with the Nusselt number, defined as:

$$Nu = \frac{hL_c}{k}$$

where *k* is the thermal conductivity of the fluid and *Lc* is the *characteristic length.*

To understand the physical significance of the Nusselt number, consider a fluid layer of thickness L and temperature difference $\Delta T = T_2 - T_1$, as shown in the figure. Heat transfer through the fluid layer will be by *convection* when the fluid involves some motion and by *conduction* when the fluid layer is motionless. Heat flux (the rate of heat transfer per unit time per unit surface area) in either case will be



$$\dot{q}_{\text{conv}} = h\Delta T$$

and

$$\dot{q}_{\text{cond}} = k\frac{\Delta T}{L}$$

Taking their ratio gives

$$\frac{\dot{q}_{\text{conv}}}{\dot{q}_{\text{cond}}} = \frac{h\Delta T}{k\Delta T/L} = \frac{hL}{k} = \text{Nu}$$

which is the Nusselt number. Therefore, the **Nusselt number** represents the enhancement of heat transfer through a fluid layer as a result of convection relative to conduction across the same fluid layer. The larger the Nusselt number, the more effective the convection. A Nusselt number of Nu = 1 for a fluid layer represents heat transfer across the layer by pure conduction.

Reynolds Number

The transition from laminar to turbulent flow depends on the *surface geometry, surface roughness, free-stream velocity, surface temperature,* and *type of fluid,* among other things. **The flow regime depends mainly on the ratio of the *inertia forces* to *viscous forces* in the fluid**. This ratio is called the **Reynolds number,** which is a ***dimensionless*** quantity, and is expressed for external flow as

$$\text{Re} = \frac{\text{Inertia forces}}{\text{Viscous}} = \frac{\mathcal{V}L_c}{\nu} = \frac{\rho\mathcal{V}L_c}{\mu}$$

where $\mathcal{V}$ is the upstream velocity (equivalent to the free-stream velocity u_∞ for a flat plate), L_c is the characteristic length of the geometry, and $\upsilon = \mu/\rho$ is the kinematic viscosity of the fluid. For a flat plate, the characteristic length is the distance x from the leading edge. The Reynolds number at which the flow becomes turbulent is called the **critical Reynolds number which is equal to $5x10^5$ for flat plate.** For flow on flat plate $Re<5x10^5$ the flow is laminar, $Re>5x10^5$ the flow is turbulent.

Prandtl Number

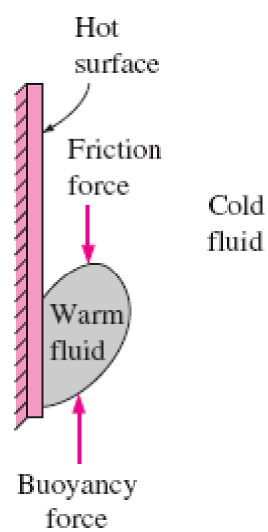
The relative thickness of the velocity and the thermal boundary layers is best described by the *dimensionless* parameter **Prandtl number,** defined as

$$Pr = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of heat}} = \frac{\nu}{\alpha} = \frac{\mu C_p}{k}$$

The Prandtl numbers of fluids range from less than 0.01 for liquid metals to more than 100,000 for heavy oils. Note that the Prandtl number is in the order of 10 for water.

The Grashof Number

The flow regime in forced convection is governed by the dimensionless *Reynolds number*, which represents the ratio of inertial forces to viscous forces acting on the fluid. The flow regime in natural convection is governed by the dimensionless ***Grashof number***, which represents the ratio of the *buoyancy force* to the *viscous force* acting on the fluid.



$$Gr = \frac{Buoyancy\ forces}{Viscous\ forces} = \frac{g\,\Delta\rho V}{\rho v^2} = \frac{g\,\beta\,\Delta T\,V}{v^2}$$

Since $\Delta\rho \approx \rho\beta\,\Delta T$,

The ***Grashof number*** for a characteristic length L_c is**,**

$$Gr_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}$$

where .

g = gravitational acceleration, m/s^2

β = coefficient of volume expansion, 1/K (β = 1/T for ideal gases)

T_s = temperature of the surface, $^{\circ}C$

T_∞ = temperature of the fluid sufficiently far from the surface, $^{\circ}C$

L_c = characteristic length of the geometry, m
ν = kinematics viscosity of the fluid, m^2/s

Rayleigh number

It is represented the product of the Grashof and Prandtl numbers,

$$Ra_L = Gr_L\, Pr = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\, Pr$$

One Dimensional Steady State Force Convection Heat Transfer on Flat Plate

External Flow

Surface Shear Stress

$$\tau_s = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} \qquad (\text{N/m}^2) \qquad 1$$

where the constant of proportionality μ is called the **dynamic viscosity** of the fluid, whose unit is kg/m · s (or equivalently, N · s/m^2, or Pa · s, or poise = 0.1 Pa · s).

The determination of the surface shear stress τ_s from Eq. is not practical since it requires a knowledge of the flow velocity profile. A more practical approach in external flow is to relate τ_s to the upstream velocity $\mathcal{V}$ as

$$\tau_s = C_f \frac{\rho \mathcal{V}^2}{2} \qquad (\text{N/m}^2) \qquad 2$$

where C_f is the dimensionless **friction coefficient,** whose value in most cases is determined experimentally, and ρ is the density of the fluid. Note that the friction coefficient, in general, will vary with location along the surface. Once the average friction coefficient over a given surface is available, the friction force over the entire surface is determined from

$$F_f = C_f A_s \frac{\rho \mathcal{V}^2}{2} \qquad (\text{N}) \qquad 3$$

where A_s is the surface area.

The friction coefficient is an important parameter in heat transfer studies since it is directly related to the heat transfer coefficient and the power requirements of the pump or fan.

$$\frac{u}{u_\infty} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 \qquad \textbf{for laminar flow} \qquad 4$$

For turbulent flow

$$\frac{u}{u_\infty} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}} \qquad (5)$$

Heat Transfer

The experimental data for heat transfer is often represented conveniently with reasonable accuracy by a simple power-law relation of the form

$$\mathrm{Nu} = C\,\mathrm{Re}_L^m\,\mathrm{Pr}^n$$

where m and n are constant exponents, and the value of the constant C depends on geometry and flow.

The fluid temperature in the thermal boundary layer varies from T_s at the surface to about T_∞ at the outer edge of the boundary. The fluid properties also vary with temperature, and thus with position across the boundary layer. In order to account for the variation of the properties with temperature, the fluid properties are usually evaluated at the so-called **film temperature**, defined as

$$T_f = \frac{T_s + T_\infty}{2}$$

Parallel Flow over flat plates

Consider the parallel flow of a fluid over a flat plate of length L in the flow direction, as shown in Figure . The x-coordinate is measured along the plate surface from the leading edge in the direction of the flow. The fluid approaches the plate in the x-direction with uniform upstream velocity $\mathcal{V}$ and temperature T_∞. The flow in the velocity boundary layer starts out as laminar, but if the plate is sufficiently long, the flow will become turbulent at a distance x_{cr} from the leading edge where the Reynolds number reaches its critical value for transition.

The transition from laminar to turbulent flow depends on the *surface geometry, surface roughness, upstream velocity, surface temperature,* and the *type of fluid,* among other things, and is best characterized by the Reynolds number. The Reynolds number at a distance x from the leading edge of a flat plate is expressed as

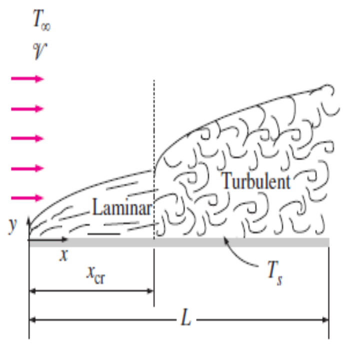


FIGURE 7–6
Laminar and turbulent regions of the boundary layer during flow over a flat plate.

$$\mathrm{Re}_x = \frac{\rho \mathcal{V} x}{\mu} = \frac{\mathcal{V} x}{\nu}$$

Note that the value of the Reynolds number varies for a flat plate along the flow, reaching $\mathrm{Re}_L = \mathcal{V} L/\nu$ at the end of the plate.

For flow over a flat plate, transition from laminar to turbulent is usually taken to occur at the *critical Reynolds number* of

$$\mathrm{Re}_{cr} = \frac{\rho \mathcal{V} x_{cr}}{\mu} = 5 \times 10^5$$

Friction Coefficient

Based on analysis, the boundary layer thickness and the local friction coefficient at location x for laminar flow over a flat plate were determined in Chapter 6 to be

$$\textit{Laminar:} \quad \delta_{v,x} = \frac{5x}{\mathrm{Re}_x^{1/2}} \quad \text{and} \quad C_{f,x} = \frac{0.664}{\mathrm{Re}_x^{1/2}}, \quad \mathrm{Re}_x < 5 \times 10^5 \qquad \text{6-7}$$

The corresponding relations for turbulent flow are

$$\textit{Turbulent:} \quad \delta_{v,x} = \frac{0.382x}{\mathrm{Re}_x^{1/5}} \quad \text{and} \quad C_{f,x} = \frac{0.0592}{\mathrm{Re}_x^{1/5}}, \quad 5 \times 10^5 \le \mathrm{Re}_x \le 10^7 \qquad \text{8-9}$$

where x is the distance from the leading edge of the plate and $\mathrm{Re}_x = \mathcal{V} x/\nu$ is the Reynolds number at location x. Note that $C_{f,x}$ is proportional to $\mathrm{Re}_x^{-1/2}$ and thus to $x^{-1/2}$ for laminar flow. Therefore, $C_{f,x}$ is supposedly *infinite* at the leading edge (x = 0) and decreases by a factor of $x^{-1/2}$ in the flow direction. The local friction coefficients are higher in turbulent flow than they are in laminar flow because of the intense mixing that occurs in the turbulent boundary layer. Note that $C_{f,x}$ reaches its highest values when the flow becomes fully turbulent, and then decreases by a factor of $x^{-1/5}$ in the flow direction.

The *average* friction coefficient over the entire plate is detennined by substituting the relations above into Eq. and performing the integrations . We get

Laminar: $C_f = \frac{1.328}{Re_L^{1/2}}$ $Re_L < 5 \times 10^5$ 10

Turbulent: $C_f = \frac{0.074}{Re_L^{1/5}}$ $5 \times 10^5 \leq Re_L \leq 10^7$ 11

The first relation gives the average friction coefficient for the entire plate when the flow is *laminar* over the *entire* plate. The second relation gives the average friction coefficient for the entire plate only when the flow is *turbulent* over the *entire* plate, or when the laminar flow region of the plate is too small relative to the turbulent flow region (that is, $x_{cr} \ll L$ where the length of the plate x_{cr} over which the flow is laminar can be determined from $Re_{cr} = 5 \times 10^5 = \mathcal{V} x_{cr}/v$).

For combined flow (laminar & turbulent)

$$C_f = \frac{0.074}{Re_L^{1/5}} - \frac{1742}{Re_L} \qquad 5 \times 10^5 \leq Re_L \leq 10^7 \qquad 12$$

Heat Transfer Coefficient

The local Nusselt number at a location x for laminar flow over a flat plate was determined by solving the differential energy equation to be

Laminar: $Nu_x = \frac{h_x x}{k} = 0.332\, Re_x^{0.5}\, Pr^{1/3}$ $Pr > 0.60$ 13

The corresponding relation for turbulent flow is

Turbulent: $Nu_x = \frac{h_x x}{k} = 0.0296\, Re_x^{0.8}\, Pr^{1/3}$ $0.6 \leq Pr \leq 60$, $5 \times 10^5 \leq Re_x \leq 10^7$ 14

Note that h_x is proportional to $Re_x^{0.5}$ and thus to $x^{-0.5}$ for laminar flow. Therefore, h_x is *infinite* at the leading edge ($x = 0$) and decreases by a factor of $x^{-0.5}$ in the flow direction. The variation of the boundary layer thickness δ and the

The *average* Nusselt number over the entire plate is determined by substituting the relations above into Eq.   and performing the integrations. We get

Laminar: $$\text{Nu} = \frac{hL}{k} = 0.664\ \text{Re}_L^{0.5}\ \text{Pr}^{1/3} \qquad \text{Re}_L < 5 \times 10^5 \qquad (15)$$

Turbulent: $$\text{Nu} = \frac{hL}{k} = 0.037\ \text{Re}_L^{0.8}\ \text{Pr}^{1/3} \qquad \begin{array}{l} 0.6 \leq \text{Pr} \leq 60 \\ 5 \times 10^5 \leq \text{Re}_L \leq 10^7 \end{array} \qquad (16)$$

The first relation gives the average heat transfer coefficient for the entire plate when the flow is *laminar* over the *entire* plate. The second relation gives the average heat transfer coefficient for the entire plate only when the flow is *turbulent* over the *entire* plate, or when the laminar flow region of the plate is too small relative to the turbulent flow region.

For combined flow (laminar & turbulent)

$$\text{Nu} = \frac{hL}{k} = (0.037\ \text{Re}_L^{0.8} - 871)\text{Pr}^{1/3} \qquad \begin{array}{l} 0.6 \leq \text{Pr} \leq 60 \\ 5 \times 10^5 \leq \text{Re}_L \leq 10^7 \end{array} \qquad (17)$$

Uniform Heat Flux

When a flat plate is subjected to *uniform heat flux* instead of uniform temperature, the local Nusselt number is given by

Laminar: $$\text{Nu}_x = 0.453\ \text{Re}_x^{0.5}\ \text{Pr}^{1/3} \qquad (18)$$

Turbulent: $$\text{Nu}_x = 0.0308\ \text{Re}_x^{0.8}\ \text{Pr}^{1/3} \qquad (19)$$

Ex1: Engine oil at 60°C flows over the upper surface of a 5-m-long flat plate whose temperature is 20°C with a velocity of 2 m/s . Determine the total friction force and the rate of heat transfer per unit width of the entire plate

Properties The properties of engine oil at the film temperature of $T_f = (T_s + T_\infty)/2 = (20 + 60)/2 = 40°C$ are (Table A–14).

$$\rho = 876 \text{ kg/m}^3 \qquad \text{Pr} = 2870$$
$$k = 0.144 \text{ W/m} \cdot °\text{C} \qquad \nu = 242 \times 10^{-6} \text{ m}^2/\text{s}$$

Analysis Noting that $L = 5$ m, the Reynolds number at the end of the plate is

$$\text{Re}_L = \frac{\mathcal{V}L}{\nu} = \frac{(2 \text{ m/s})(5 \text{ m})}{0.242 \times 10^{-5} \text{ m}^2/\text{s}} = 4.13 \times 10^4$$

which is less than the critical Reynolds number. Thus we have *laminar flow* over the entire plate, and the average friction coefficient is

$$C_f = 1.328 \text{ Re}_L^{-0.5} = 1.328 \times (4.13 \times 10^3)^{-0.5} = 0.0207$$

Noting that the pressure drag is zero and thus $C_D = C_f$ for a flat plate, the drag force acting on the plate per unit width becomes

$$F_D = C_f A_s \frac{\rho \mathcal{V}^2}{2} = 0.0207 \times (5 \times 1 \text{ m}^2) \frac{(876 \text{ kg/m}^3)(2 \text{ m/s})^2}{2} \left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right)$$
$$= \mathbf{181\ N}$$

The total drag force acting on the entire plate can be determined by multiplying the value obtained above by the width of the plate.

This force per unit width corresponds to the weight of a mass of about 18 kg. Therefore, a person who applies an equal and opposite force to the plate to keep it from moving will feel like he or she is using as much force as is necessary to hold a 18-kg mass from dropping.

Similarly, the Nusselt number is determined using the laminar flow relations for a flat plate,

$$\text{Nu} = \frac{hL}{k} = 0.664 \text{ Re}_L^{0.5} \text{ Pr}^{1/3} = 0.664 \times (4.13 \times 10^4)^{0.5} \times 2870^{1/3} = 1918$$

Then,

$$h = \frac{k}{L} \text{Nu} = \frac{0.144 \text{ W/m} \cdot °\text{C}}{5 \text{ m}} (1918) = 55.2 \text{ W/m}^2 \cdot °\text{C}$$

and

$$\dot{Q} = hA_s(T_\infty - T_s) = (55.2 \text{ W/m}^2 \cdot °\text{C})(5 \times 1 \text{ m}^2)(60 - 20)°\text{C} = \mathbf{11{,}040\ W}$$

Ex2:A 0.3-cm-thick, 12-cm-high, and 18-cm-long circuit board houses 80 closely spaced logic chips on one side, each dissipating 0.06 W. The board is impregnated with copper fillings and has an effective thermal conductivity of 16 W/m · °C. All the heat generated in the chips is conducted across the circuit board and is dissipated from the back side of the board to the ambient air at 30°C, which is forced to flow over the surface by a fan at a free-stream velocity of 400 m/min. Determine the temperatures on the two sides of the circuit board.

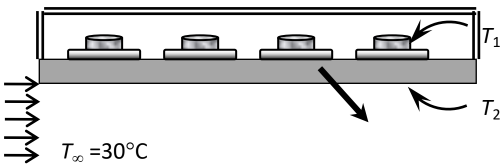
Sol:

Properties Assuming a film temperature of 40°C, the properties of air are (Table A-15)

$$k = 0.02662 \text{ W/m.°C}$$
$$\upsilon = 1.702 \times 10^{-5} \text{ m}^2\text{/s}$$
$$\Pr = 0.7255$$

Analysis The Reynolds number is

$$\text{Re}_L = \frac{V_\infty L}{\upsilon} = \frac{[(400/60)\text{ m/s}](0.18\text{ m})}{1.702\times10^{-5}\text{ m}^2\text{/s}} = 7.051\times10^4$$



which is less than the critical Reynolds number. Therefore, the flow is laminar. Using the proper relation for Nusselt number, heat transfer coefficient is determined to be

$$Nu = \frac{hL}{k} = 0.664\,\text{Re}_L^{0.5}\Pr^{1/3} = 0.664(7.051\times10^4)^{0.5}(0.7255)^{1/3} = 158.4$$

$$h = \frac{k}{L}Nu = \frac{0.02662\text{ W/m.°C}}{0.18\text{ m}}(158.4) = 23.43\text{ W/m}^2\text{.°C}$$

The temperatures on the two sides of the circuit board are

$$\dot{Q} = hA_s(T_2 - T_\infty) \rightarrow T_2 = T_\infty + \frac{\dot{Q}}{hA_s}$$

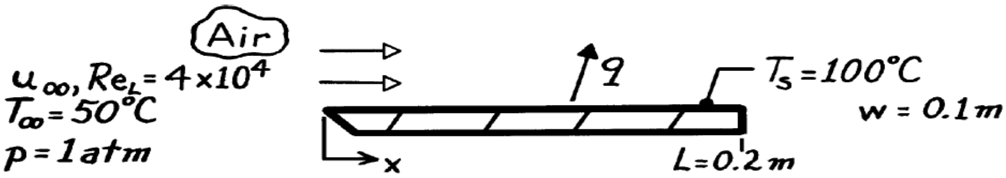
$$= 30°\text{C} + \frac{(80\times0.06)\text{ W}}{(23.43\text{ W/m}^2\text{.°C})(0.12\text{ m})(0.18\text{ m})} = \mathbf{39.48°C}$$

$$\dot{Q} = \frac{kA_s}{L}(T_1 - T_2) \rightarrow T_1 = T_2 + \frac{\dot{Q}L}{kA_s}$$

$$= 39.48°\text{C} + \frac{(80\times0.06\text{ W})(0.003\text{ m})}{(16\text{ W/m.°C})(0.12\text{ m})(0.18\text{ m})} = \mathbf{39.52°C}$$

Ex3:Air at a pressure of 1 atm and temperature of 50 °C is parallel flow over the top surface of a flat plate that is heated to uniform temperature of 100 °C .the plate has a length of 0.2 m (in the flow direction) and a width of 0.1m . the Reynolds number based on the plate length is 40000 .what is the rate of heat transfer from the plate to the air

Sol:



ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform surface temperature, (3) Negligible radiation, (4) $Re_{x_c} = 5\times10^5$.

PROPERTIES: *Table A-4*, Air (T_f = 348K, 1 atm): k = 0.0299 W/m·K, Pr = 0.70.

ANALYSIS: (a) The heat rate is

$$q = \bar{h}_L (w \times L)\ (T_s - T_\infty).$$

Since the flow is laminar over the entire plate for $Re_L = 4 \times 10^4$, it follows that

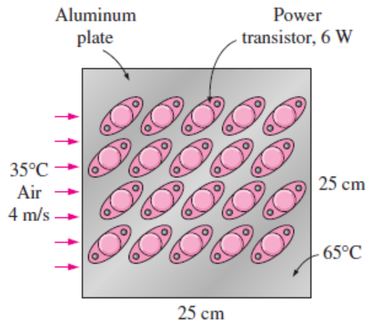
$$\overline{Nu}_L = \frac{\bar{h}_L L}{k} = 0.664\, Re_L^{1/2}\ Pr^{1/3} = 0.664(40{,}000)^{1/2}(0.70)^{1/3} = 117.9.$$

Hence $$\bar{h}_L = 117.9\frac{k}{L} = 117.9\frac{0.0299\ W/m\cdot K}{0.2m} = 17.6\ W/m^2\cdot K$$

and $$q = 17.6\frac{W}{m^2\cdot K}(0.1m \times 0.2m)\ (100-50)^\circ C = 17.6\ W.$$ <

H.W:

An array of power transistors, dissipating 6 W of power each, are to be cooled by mounting them on a 25-cm x 25-cm square aluminum plate and blowing air at 35°C over the plate with a fan at a velocity of 4 m/s. The average temperature of the plate is not to exceed 65°C. Assuming the heat transfer from the back side of the plate to be negligible and disregarding radiation, determine the number of transistors that can be placed on this plate



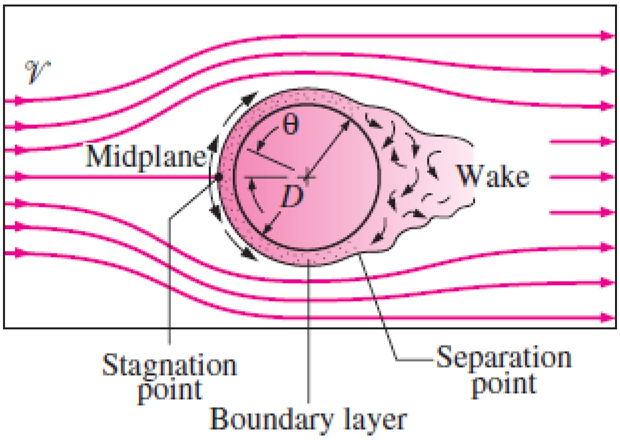
Properties of air at 1 atm pressure

Temp. T, °C	Density ρ, kg/m^3	Specific Heat c_p, J/kg·K	Thermal Conductivity k, W/m·K	Thermal Diffusivity α, m^2/s	Dynamic Viscosity μ, kg/m·s	Kinematic Viscosity ν, m^2/s	Prandtl Number Pr
−150	2.866	983	0.01171	4.158×10^{-6}	8.636×10^{-6}	3.013×10^{-6}	0.7246
−100	2.038	966	0.01582	8.036×10^{-6}	1.189×10^{-5}	5.837×10^{-6}	0.7263
−50	1.582	999	0.01979	1.252×10^{-5}	1.474×10^{-5}	9.319×10^{-6}	0.7440
−40	1.514	1002	0.02057	1.356×10^{-5}	1.527×10^{-5}	1.008×10^{-5}	0.7436
−30	1.451	1004	0.02134	1.465×10^{-5}	1.579×10^{-5}	1.087×10^{-5}	0.7425
−20	1.394	1005	0.02211	1.578×10^{-5}	1.630×10^{-5}	1.169×10^{-5}	0.7408
−10	1.341	1006	0.02288	1.696×10^{-5}	1.680×10^{-5}	1.252×10^{-5}	0.7387
0	1.292	1006	0.02364	1.818×10^{-5}	1.729×10^{-5}	1.338×10^{-5}	0.7362
5	1.269	1006	0.02401	1.880×10^{-5}	1.754×10^{-5}	1.382×10^{-5}	0.7350
10	1.246	1006	0.02439	1.944×10^{-5}	1.778×10^{-5}	1.426×10^{-5}	0.7336
15	1.225	1007	0.02476	2.009×10^{-5}	1.802×10^{-5}	1.470×10^{-5}	0.7323
20	1.204	1007	0.02514	2.074×10^{-5}	1.825×10^{-5}	1.516×10^{-5}	0.7309
25	1.184	1007	0.02551	2.141×10^{-5}	1.849×10^{-5}	1.562×10^{-5}	0.7296
30	1.164	1007	0.02588	2.208×10^{-5}	1.872×10^{-5}	1.608×10^{-5}	0.7282
35	1.145	1007	0.02625	2.277×10^{-5}	1.895×10^{-5}	1.655×10^{-5}	0.7268
40	1.127	1007	0.02662	2.346×10^{-5}	1.918×10^{-5}	1.702×10^{-5}	0.7255
45	1.109	1007	0.02699	2.416×10^{-5}	1.941×10^{-5}	1.750×10^{-5}	0.7241
50	1.092	1007	0.02735	2.487×10^{-5}	1.963×10^{-5}	1.798×10^{-5}	0.7228
60	1.059	1007	0.02808	2.632×10^{-5}	2.008×10^{-5}	1.896×10^{-5}	0.7202
70	1.028	1007	0.02881	2.780×10^{-5}	2.052×10^{-5}	1.995×10^{-5}	0.7177
80	0.9994	1008	0.02953	2.931×10^{-5}	2.096×10^{-5}	2.097×10^{-5}	0.7154
90	0.9718	1008	0.03024	3.086×10^{-5}	2.139×10^{-5}	2.201×10^{-5}	0.7132
100	0.9458	1009	0.03095	3.243×10^{-5}	2.181×10^{-5}	2.306×10^{-5}	0.7111
120	0.8977	1011	0.03235	3.565×10^{-5}	2.264×10^{-5}	2.522×10^{-5}	0.7073
140	0.8542	1013	0.03374	3.898×10^{-5}	2.345×10^{-5}	2.745×10^{-5}	0.7041
160	0.8148	1016	0.03511	4.241×10^{-5}	2.420×10^{-5}	2.975×10^{-5}	0.7014
180	0.7788	1019	0.03646	4.593×10^{-5}	2.504×10^{-5}	3.212×10^{-5}	0.6992
200	0.7459	1023	0.03779	4.954×10^{-5}	2.577×10^{-5}	3.455×10^{-5}	0.6974
250	0.6746	1033	0.04104	5.890×10^{-5}	2.760×10^{-5}	4.091×10^{-5}	0.6946
300	0.6158	1044	0.04418	6.871×10^{-5}	2.934×10^{-5}	4.765×10^{-5}	0.6935
350	0.5664	1056	0.04721	7.892×10^{-5}	3.101×10^{-5}	5.475×10^{-5}	0.6937
400	0.5243	1069	0.05015	8.951×10^{-5}	3.261×10^{-5}	6.219×10^{-5}	0.6948
450	0.4880	1081	0.05298	1.004×10^{-4}	3.415×10^{-5}	6.997×10^{-5}	0.6965
500	0.4565	1093	0.05572	1.117×10^{-4}	3.563×10^{-5}	7.806×10^{-5}	0.6986
600	0.4042	1115	0.06093	1.352×10^{-4}	3.846×10^{-5}	9.515×10^{-5}	0.7037
700	0.3627	1135	0.06581	1.598×10^{-4}	4.111×10^{-5}	1.133×10^{-4}	0.7092
800	0.3289	1153	0.07037	1.855×10^{-4}	4.362×10^{-5}	1.326×10^{-4}	0.7149
900	0.3008	1169	0.07465	2.122×10^{-4}	4.600×10^{-5}	1.529×10^{-4}	0.7206
1000	0.2772	1184	0.07868	2.398×10^{-4}	4.826×10^{-5}	1.741×10^{-4}	0.7260
1500	0.1990	1234	0.09599	3.908×10^{-4}	5.817×10^{-5}	2.922×10^{-4}	0.7478
2000	0.1553	1264	0.11113	5.664×10^{-4}	6.630×10^{-5}	4.270×10^{-4}	0.7539

FLOW ACROSS CYLINDERS AND SPHERES

Flow across cylinders and spheres is frequently encountered in practice. For example, the tubes in a shell-and-tube heat exchanger involve both *internal flow* through the tubes and *external flow* over the tubes, and both flows must be considered in the analysis of the heat exchanger. Also, many sports such as soccer, tennis, and golf involve flow over spherical balls.

The characteristic length for a circular cylinder or sphere is taken to be the *external diameter D.* Thus, the Reynolds number is defined as $\text{Re} = \mathcal{V}D/\nu$ where $\mathcal{V}$ is the uniform velocity of the fluid as it approaches the cylinder or sphere. The critical Reynolds number for flow across a circular cylinder or sphere is about $\text{Re}_{cr} \approx 2 \times 10^5$. That is, the boundary layer remains laminar for about $\text{Re} \lesssim 2 \times 10^5$ and becomes turbulent for $\text{Re} \gtrsim 2 \times 10^5$.



Drag coefficient

The nature of the flow across a cylinder or sphere strongly affects the total drag coefficient C_D. Both the *friction drag* and the *pressure drag* can be significant. The high pressure in the vicinity of the stagnation point and the low pressure on the opposite side in the wake produce a net force on the body in the direction of flow. The drag force is primarily due to friction drag at low Reynolds numbers ($Re < 10$) and to pressure drag at high Reynolds numbers ($Re > 5000$). Both effects are significant at intermediate Reynolds numbers.

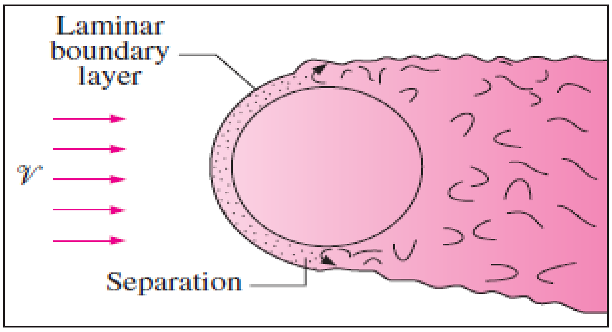
The average drag coefficients C_D for cross flow over a smooth single circular cylinder and a sphere are given in Figure . The curves exhibit different behaviors in different ranges of Reynolds numbers:

- For $Re \lesssim 1$, we have creeping flow, and the drag coefficient decreases with increasing Reynolds number. For a sphere, it is $C_D = 24/Re$. There is

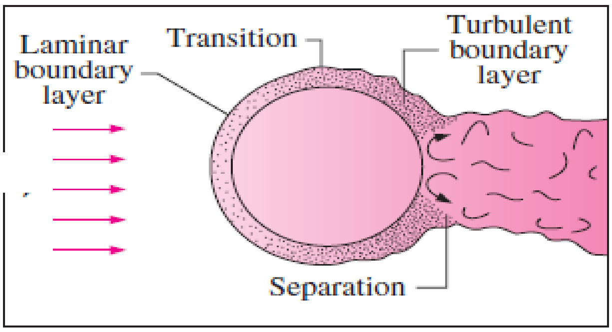
- At about $Re = 10$, separation starts occurring on the rear of the body with vortex shedding starting at about $Re \approx 90$. The region of separation increases with increasing Reynolds number up to about $Re = 10^3$. At this point, the drag is mostly (about 95 percent) due to pressure drag. The drag coefficient continues to decrease with increasing Reynolds number in this range of $10 < Re < 10^3$. (A decrease in the drag coefficient does not necessarily indicate a decrease in drag. The drag force is proportional to the square of the velocity, and the increase in velocity at higher Reynolds numbers usually more than offsets the decrease in the drag coefficient.)

- In the moderate range of $10^3 < \text{Re} < 10^5$, the drag coefficient remains relatively constant. This behavior is characteristic of blunt bodies. The flow in the boundary layer is laminar in this range, but the flow in the separated region past the cylinder or sphere is highly turbulent with a wide turbulent wake.
- There is a sudden drop in the drag coefficient somewhere in the range of $10^5 < \text{Re} < 10^6$ (usually, at about 2×10^5). This large reduction in C_D is due to the flow in the boundary layer becoming ***turbulent,*** which moves the separation point further on the rear of the body, reducing the size of the wake and thus the magnitude of the pressure drag. This is in contrast to streamlined bodies, which experience an increase in the drag coefficient (mostly due to friction drag) when the boundary layer becomes turbulent.

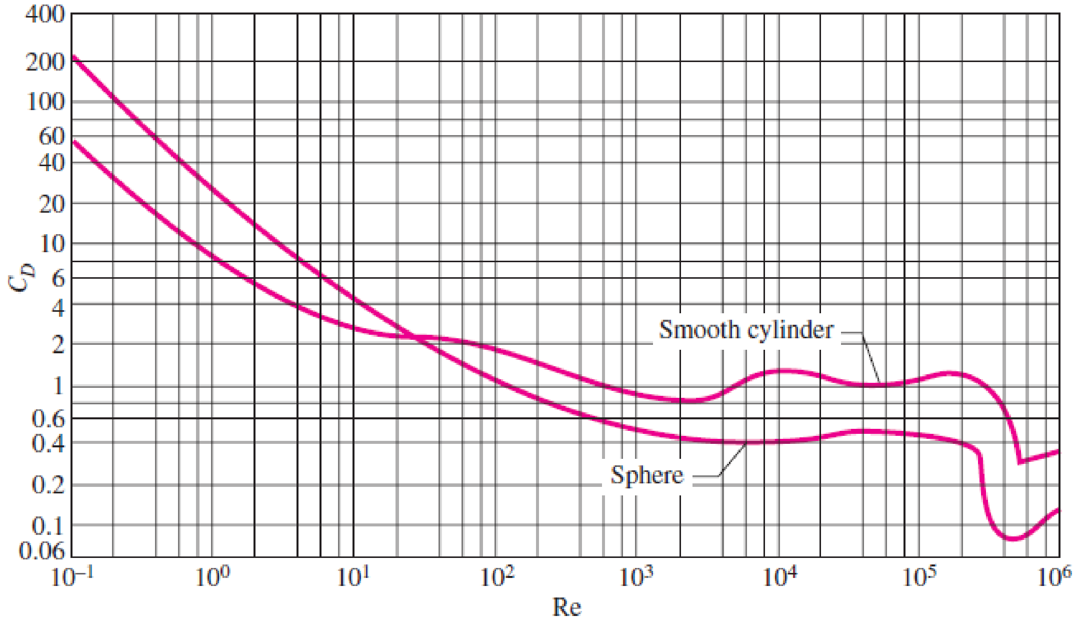
Flow separation occurs at about $\theta \approx 80°$ (measured from the stagnation point) when the boundary layer is *laminar* and at about $\theta \approx 140°$ when it is *turbulent* (Fig. 7–18). The delay of separation in turbulent flow is caused by the rapid fluctuations of the fluid in the transverse direction, which enables the turbulent boundary layer to travel further along the surface before separation occurs, resulting in a narrower wake and a smaller pressure drag. In the range of Reynolds numbers where the flow changes from laminar to turbulent, even the drag force F_D decreases as the velocity (and thus Reynolds number) increases. This results in a sudden decrease in drag of a flying body and instabilities in flight.



(a) Laminar flow ($\text{Re} < 2 \times 10^5$)



(b) Turbulence occurs ($\text{Re} > 2 \times 10^5$)



Heat Transfer Coefficient

Flows across cylinders and spheres, in general, involve *flow separation,* which is difficult to handle analytically. Therefore, such flows must be studied experimentally or numerically. Indeed, flow across cylinders and spheres has been studied experimentally by numerous investigators, and several empirical correlations have been developed for the heat transfer coefficient.

The discussions above on the local heat transfer coefficients are insightful; however, they are of little value in heat transfer calculations since the calculation of heat transfer requires the *average* heat transfer coefficient over the entire surface. Of the several such relations available in the literature for the average Nusselt number for cross flow over a cylinder, we present the one proposed by Churchill and Bernstein:

$$\text{Nu}_{\text{cyl}} = \frac{hD}{k} = 0.3 + \frac{0.62\,\text{Re}^{1/2}\,\text{Pr}^{1/3}}{[1 + (0.4/\text{Pr})^{2/3}]^{1/4}}\left[1 + \left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5} \qquad 1$$

This relation is quite comprehensive in that it correlates available data well for Re Pr > 0.2. The fluid properties are evaluated at the *film temperature* $T_f = \frac{1}{2}(T_\infty + T_s)$, which is the average of the free-stream and surface temperatures.

For flow over a *sphere,* Whitaker recommends the following comprehensive correlation:

$$\text{Nu}_{\text{sph}} = \frac{hD}{k} = 2 + [0.4\ \text{Re}^{1/2} + 0.06\ \text{Re}^{2/3}]\ \text{Pr}^{0.4} \left(\frac{\mu_\infty}{\mu_s}\right)^{1/4} \qquad 2$$

which is valid for $3.5 \leq \text{Re} \leq 80{,}000$ and $0.7 \leq \text{Pr} \leq 380$. The fluid properties in this case are evaluated at the free-stream temperature T_∞, except for μ_s, which is evaluated at the surface temperature T_s. Although the two relations above are considered to be quite accurate, the results obtained from them can be off by as much as 30 percent.

The average Nusselt number for flow across cylinders can be expressed compactly as

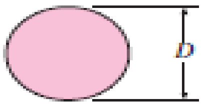
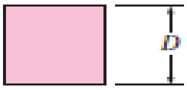
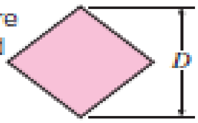
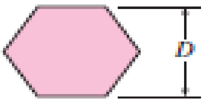
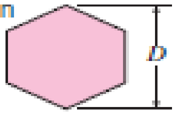
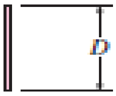
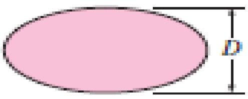
$$\text{Nu}_{\text{cyl}} = \frac{hD}{k} = C\ \text{Re}^m\ \text{Pr}^n \qquad 3$$

where $n = \frac{1}{3}$ and the experimentally determined constants C and m are given in Table -1 for circular as well as various noncircular cylinders. The characteristic length D for use in the calculation of the Reynolds and the Nusselt numbers for different geometries is as indicated on the figure. All fluid properties are evaluated at the film temperature.

The relations for cylinders above are for *single* cylinders or cylinders oriented such that the flow over them is not affected by the presence of others. Also, they are applicable to *smooth* surfaces. *Surface roughness* and the *free-stream turbulence* may affect the drag and heat transfer coefficients significantly. Eq. 3 provides a simpler alternative to Eq. 1 for flow over cylinders. However, Eq. 1 is more accurate, and thus should be preferred in calculations whenever possible.

Table 1

Empirical correlations for the average Nusselt number for forced convection over circular and noncircular cylinders in cross flow (from Zukauskas, and Jakob,

Cross-section of the cylinder	Fluid	Range of Re	Nusselt number
Circle 	Gas or liquid	0.4–4 4–40 40–4000 4000–40,000 40,000–400,000	$Nu = 0.989Re^{0.330}\ Pr^{1/3}$ $Nu = 0.911Re^{0.385}\ Pr^{1/3}$ $Nu = 0.683Re^{0.466}\ Pr^{1/3}$ $Nu = 0.193Re^{0.618}\ Pr^{1/3}$ $Nu = 0.027Re^{0.805}\ Pr^{1/3}$
Square 	Gas	5000–100,000	$Nu = 0.102Re^{0.675}\ Pr^{1/3}$
Square (tilted 45°) 	Gas	5000–100,000	$Nu = 0.246Re^{0.588}\ Pr^{1/3}$
Hexagon 	Gas	5000–100,000	$Nu = 0.153Re^{0.638}\ Pr^{1/3}$
Hexagon (tilted 45°) 	Gas	5000–19,500 19,500–100,000	$Nu = 0.160Re^{0.638}\ Pr^{1/3}$ $Nu = 0.0385Re^{0.782}\ Pr^{1/3}$
Vertical plate 	Gas	4000–15,000	$Nu = 0.228Re^{0.731}\ Pr^{1/3}$
Ellipse 	Gas	2500–15,000	$Nu = 0.248Re^{0.612}\ Pr^{1/3}$

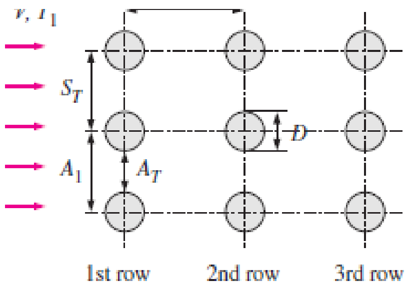
FLOW ACROSS TUBE BANKS

The tubes in a tube bank are usually arranged either *in-line* or *staggered* in the direction of flow, as shown in Figure . The outer tube diameter D is taken as the characteristic length. The arrangement of the tubes in the tube bank is characterized by the *transverse pitch* S_T, *longitudinal pitch* S_L, and the *diagonal pitch* S_D between tube centers. The diagonal pitch is determined from

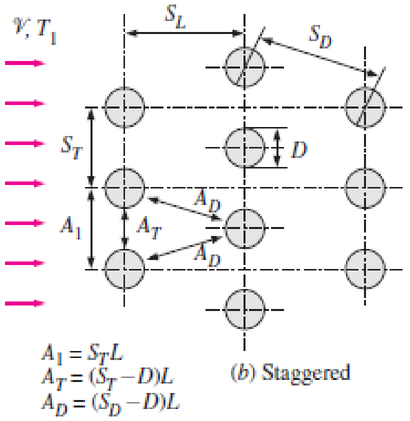
$$S_D = \sqrt{S_L^2 + (S_T/2)^2} \qquad 4$$

As the fluid enters the tube bank, the flow area decreases from $A_1 = S_T L$ to $A_T = (S_T - D)L$ between the tubes, and thus flow velocity increases. In staggered arrangement, the velocity may increase further in the diagonal region if the tube rows are very close to each other. In tube banks, the flow characteristics are dominated by the maximum velocitiy $\mathcal{V}_{max}$ that occurs within the tube bank rather than the approach velocity $\mathcal{V}$. Therefore, the Reynolds number is defined on the basis of maximum velocity as

$$\mathrm{Re}_D = \frac{\rho \mathcal{V}_{max} D}{\mu} = \frac{\mathcal{V}_{max} D}{\nu} \qquad \textbf{(7-39)}$$



(*a*) In-line



(*b*) Staggered

The maximum velocity is determined from the conservation of mass requirement for steady incompressible flow. For *in-line* arrangement, the maximum velocity occurs at the minimum flow area between the tubes, and the conservation of mass can be expressed as (see Fig. 7-26*a*) $\rho \mathcal{V} A_1 = \rho \mathcal{V}_{max} A_T$ or $\mathcal{V} S_T = \mathcal{V}_{max}(S_T - D)$. Then the maximum velocity becomes

$$\mathcal{V}_{max} = \frac{S_T}{S_T - D}\mathcal{V} \qquad 5$$

In *staggered* arrangement, the fluid approaching throug A_1 in Figure 7–26*b* passes through area A_T and then through area $2A_D$ as it wraps around the pipe in the next row. If $2A_D > A_T$, maximum velocity will still occur at A_T between the tubes, and thus the $\mathcal{V}_{max}$ relation Eq. 7-40 can also be used for staggered tube banks. But if $2A_D < A_T$ [or, if $2(S_D - D) < (S_T - D)$], maximum velocity will occur at the diagonal cross sections, and the maximum velocity in this case becomes

Staggered and $S_D < (S_T + D)/2$:

$$\mathcal{V}_{max} = \frac{S_T}{2(S_D - D)}\mathcal{V} \qquad 6$$

since $\rho \mathcal{V} A_1 = \rho \mathcal{V}_{max}(2A_D)$ or $\mathcal{V} S_T = 2\mathcal{V}_{max}(S_D - D)$.

Several correlations, all based on experimental data, have been proposed for the average Nusselt number for cross flow over tube banks. More recently, Zukauskas has proposed correlations whose general form is

$$Nu_D = \frac{hD}{k} = C\, Re_D^m\, Pr^n (Pr/Pr_s)^{0.25} \qquad 7$$

where the values of the constants C, m, and n depend on value Reynolds number. Such correlations are given in Table 7–2 explicitly for $0.7 < Pr < 500$ and $0 < Re_D < 2 \times 10^6$. The uncertainty in the values of Nusselt number obtained from these relations is ±15 percent. Note that all properties except Pr_s are to be evaluated at the arithmetic mean temperature of the fluid determined from

$$T_m = \frac{T_i + T_e}{2} \qquad 8$$

where T_i and T_e are the fluid temperatures at the inlet and the exit of the tube bank, respectively.

Table 2

Nusselt number correlations for cross flow over tube banks for N > 16 and $0.7 < \mathrm{Pr} < 500$ (from Zukauskas, Ref. 15, 1987)*

Arrangement	Range of Re_D	Correlation
In-line	0–100	$\mathrm{Nu}_D = 0.9\,\mathrm{Re}_D^{0.4}\mathrm{Pr}^{0.36}(\mathrm{Pr}/\mathrm{Pr}_s)^{0.25}$
	100–1000	$\mathrm{Nu}_D = 0.52\,\mathrm{Re}_D^{0.5}\mathrm{Pr}^{0.36}(\mathrm{Pr}/\mathrm{Pr}_s)^{0.25}$
	$1000–2 \times 10^5$	$\mathrm{Nu}_D = 0.27\,\mathrm{Re}_D^{0.63}\mathrm{Pr}^{0.36}(\mathrm{Pr}/\mathrm{Pr}_s)^{0.25}$
	$2 \times 10^5–2 \times 10^6$	$\mathrm{Nu}_D = 0.033\,\mathrm{Re}_D^{0.8}\mathrm{Pr}^{0.4}(\mathrm{Pr}/\mathrm{Pr}_s)^{0.25}$
Staggered	0–500	$\mathrm{Nu}_D = 1.04\,\mathrm{Re}_D^{0.4}\mathrm{Pr}^{0.36}(\mathrm{Pr}/\mathrm{Pr}_s)^{0.25}$
	500–1000	$\mathrm{Nu}_D = 0.71\,\mathrm{Re}_D^{0.5}\mathrm{Pr}^{0.36}(\mathrm{Pr}/\mathrm{Pr}_s)^{0.25}$
	$1000–2 \times 10^5$	$\mathrm{Nu}_D = 0.35(S_T/S_L)^{0.2}\,\mathrm{Re}_D^{0.6}\mathrm{Pr}^{0.36}(\mathrm{Pr}/\mathrm{Pr}_s)^{0.25}$
	$2 \times 10^5–2 \times 10^6$	$\mathrm{Nu}_D = 0.031(S_T/S_L)^{0.2}\,\mathrm{Re}_D^{0.8}\mathrm{Pr}^{0.36}(\mathrm{Pr}/\mathrm{Pr}_s)^{0.25}$

*All properties except Pr_s are to be evaluated at the arithmetic mean of the inlet and outlet temperatures of the fluid (Pr_s is to be evaluated at T_s).

The average Nusselt number relations in Table 2 are for tube banks with 16 or more rows. Those relations can also be used for tube banks with N_L provided that they are modified as

$$\mathrm{Nu}_{D,N_L} = F\,\mathrm{Nu}_D \quad (9)$$

where F is a *correction factor F* whose values are given in Table 3 For $\mathrm{Re}_D > 1000$, the correction factor is independent of Reynolds number.

Once the Nusselt number and thus the average heat transfer coefficient for the entire tube bank is known, the heat transfer rate can be determined from Newton's law of cooling using a suitable temperature difference ΔT. The first thought that comes to mind is to use $\Delta T = T_s - T_m = T_s - (T_i + T_e)/2$. But this will, in general, over predict the heat transfer rate. We will show in the next chapter that the proper temperature difference for internal flow (flow over tube banks is still internal flow through the shell) is the *logarithmic mean temperature difference* ΔT_{ln} defined as

$$\Delta T_{\mathrm{ln}} = \frac{(T_s - T_e) - (T_s - T_i)}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e/\Delta T_i)} \quad (10)$$

We will also show that the exit temperature of the fluid T_e can be determined from

Table 3

Correction factor F to be used in $\mathrm{Nu}_{D,N_L} = F\mathrm{Nu}_D$ for $N_L < 16$ and $\mathrm{Re}_D > 1000$ (from Zukauskas, Ref 15, 1987).

N_L	1	2	3	4	5	7	10	13
In-line	0.70	0.80	0.86	0.90	0.93	0.96	0.98	0.99
Staggered	0.64	0.76	0.84	0.89	0.93	0.96	0.98	0.99

$$T_e = T_s - (T_s - T_i)\exp\left(-\frac{A_s h}{\dot{m}C_p}\right) \qquad 11$$

where $A_s = N\pi DL$ is the heat transfer surface area and $\dot{m} = \rho\mathcal{V}(N_T S_T L)$ is the mass flow rate of the fluid. Here N is the total number of tubes in the bank, N_T is the number of tubes in a transverse plane, L is the length of the tubes, and $\mathcal{V}$ is the velocity of the fluid just before entering the tube bank. Then the heat transfer rate can be determined from

$$\dot{Q} = hA_s\Delta T_{ln} = \dot{m}C_p(T_e - T_i) \qquad 12$$

The second relation is usually more convenient to use since it does not require the calculation of ΔT_{ln}.

Pressure Drop

Another quantity of interest associated with tube banks is the *pressure drop* ΔP, which is the difference between the pressures at the inlet and the exit of the tube bank. It is a measure of the resistance the tubes offer to flow over them, and is expressed as

$$\Delta P = N_L f\chi\frac{\rho\mathcal{V}_{max}^2}{2} \qquad 13$$

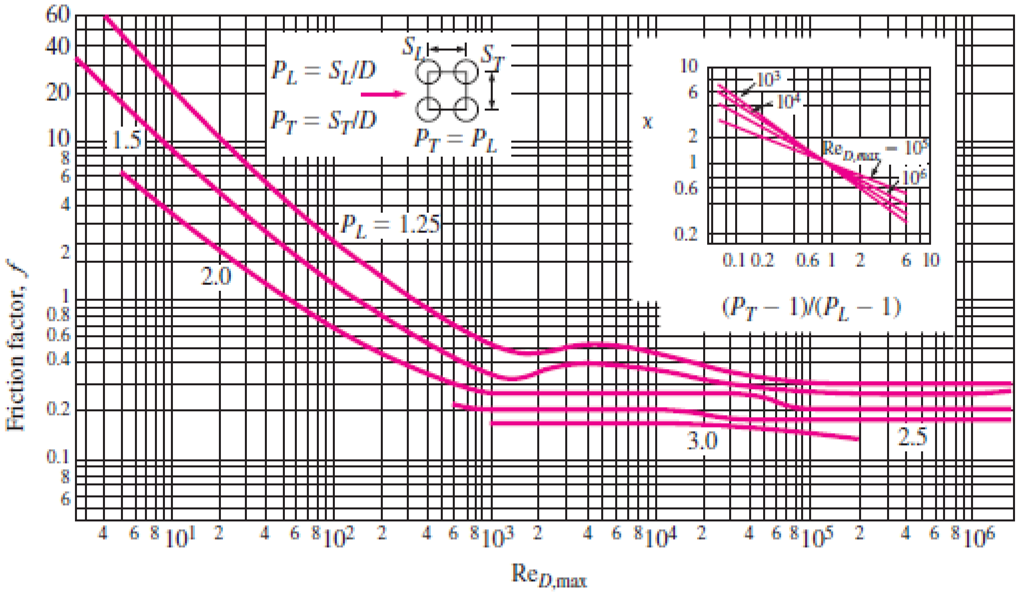
where f is the friction factor and χ is the correction factor, both plotted in Figures 7–27*a* and 7–27*b* against the Reynolds number based on the maximum velocity $\mathcal{V}_{max}$. The friction factor in Figure 7–27*a* is for a *square* in-line tube bank ($S_T = S_L$), and the correction factor given in the insert is used to account for the effects of deviation of rectangular in-line arrangements from square arrangement. Similarly, the friction factor in Figure 7–27*b* is for an *equilateral* staggered tube bank ($S_T = S_D$), and the correction factor is to account for the effects of deviation from equilateral arrangement. Note that $\chi = 1$ for both square and equilateral triangle arrangements. Also, pressure drop occurs in the flow direction, and thus we used N_L (the number of rows) in the ΔP relation.

The power required to move a fluid through a tube bank is proportional to the pressure drop, and when the pressure drop is available, the pumping power required can be determined from

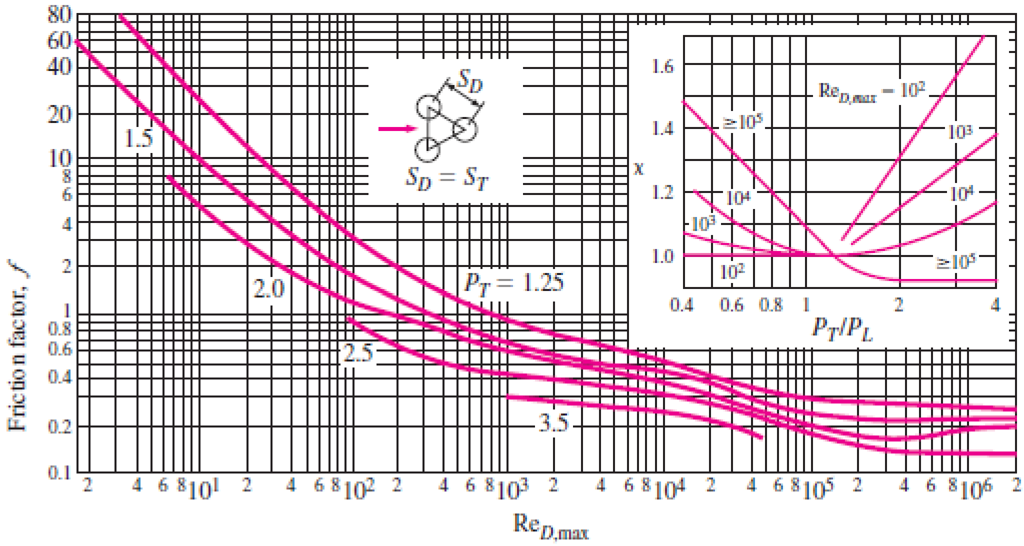
$$\dot{W}_{pump} = \dot{V}\Delta P = \frac{\dot{m}\Delta P}{\rho} \qquad 14$$

where $\dot{V} = \mathcal{V}(N_T S_T L)$ is the volume flow rate and $\dot{m} = \rho\dot{V} = \rho\mathcal{V}(N_T S_T L)$ is the mass flow rate of the fluid through the tube bank. Note that the power required to keep a fluid flowing through the tube bank (and thus the operating cost) is proportional to the pressure drop. Therefore, the benefits of enhancing heat transfer in a tube bank via rearrangement should be weighed against the cost of additional power requirements.

In this section we limited our consideration to tube banks with base surfaces (no fins). Tube banks with finned surfaces are also commonly used in practice, especially when the fluid is a gas, and heat transfer and pressure drop correlations can be found in the literature for tube banks with pin fins, plate fins, strip fins, etc.

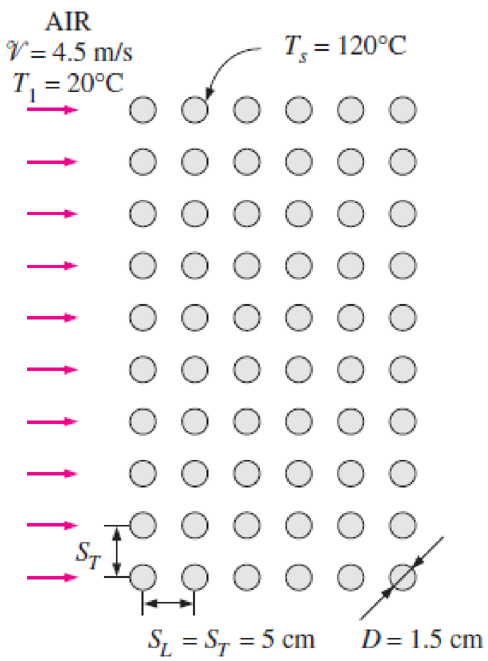


(*a*) In-line arrangement



(*b*) Staggered arrangement

Ex4: In an industrial facility, air is to be preheated before entering a furnace by geothermal water at 120ºC flowing through the tubes of a tube bank located in a duct. Air enters the duct at 20ºC and 1 atm with a mean velocity of 4.5 m/s, and flows over the tubes in normal direction. The outer diameter of the tubes is 1.5 cm, and the tubes are arranged in-line with longitudinal and transverse pitches of *SL* _ *ST* _ 5 cm. There are 6 rows in the flow direction with 10 tubes in each row, as shown in Figure . Determine the rate of heat transfer per unit length of the tubes, and the pressure drop across the tube bank



Assumptions **1** Steady operating conditions exist. **2** The surface temperature of the tubes is equal to the temperature of geothermal water.

Properties The exit temperature of air, and thus the mean temperature, is not known. We evaluate the air properties at the assumed mean temperature of 60ºC (will be checked later) and 1 atm are Table A–15):

$$k = 0.02808 \text{ W/m} \cdot \text{K}, \qquad \rho = 1.06 \text{ kg/m}^3$$
$$C_p = 1.007 \text{ kJ/kg} \cdot \text{K}, \qquad \text{Pr} = 0.7202$$
$$\mu = 2.008 \times 10^{-5} \text{ kg/m} \cdot \text{s} \qquad \text{Pr}_s = \text{Pr}_{@Ts} = 0.7073$$

Also, the density of air at the inlet temperature of 20ºC (for use in the mass flow rate calculation at the inlet) is $\rho_1 = 1.204$ kg/m^3

Analysis It is given that $D = 0.015$ m, $S_L = S_T = 0.05$ m, and $\mathcal{V} = 4.5$ m/s. Then the maximum velocity and the Reynolds number based on the maximum velocity become

$$\mathcal{V}_{\max} = \frac{S_T}{S_T - D}\mathcal{V} = \frac{0.05}{0.05 - 0.015}(4.5 \text{ m/s}) = 6.43 \text{ m/s}$$
$$\text{Re}_D = \frac{\rho \mathcal{V}_{\max} D}{\mu} = \frac{(1.06 \text{ kg/m}^3)(6.43 \text{ m/s})(0.015 \text{ m})}{2.008 \times 10^{-5} \text{ kg/m} \cdot \text{s}} = 5091$$

The average Nusselt number is determined using the proper relation from Table 7–2 to be

$$\begin{aligned}\text{Nu}_D &= 0.27 \text{ Re}_D^{0.63} \text{ Pr}^{0.36}(\text{Pr/Pr}_s)^{0.25} \\ &= 0.27(5091)^{0.63}(0.7202)^{0.36}(0.7202/0.7073)^{0.25} = 52.2\end{aligned}$$

This Nusselt number is applicable to tube banks with $N_L > 16$. In our case, the number of rows is $N_L = 6$, and the corresponding correction factor from Table 7–3 is $F = 0.945$. Then the average Nusselt number and heat transfer coefficient for all the tubes in the tube bank become

$$\text{Nu}_{D,N_L} = F\text{Nu}_D = (0.945)(52.2) = 49.3$$
$$h = \frac{\text{Nu}_{D,N_L} k}{D} = \frac{49.3(0.02808 \text{ W/m} \cdot \text{ºC})}{0.015 \text{ m}} = 92.2 \text{ W/m}^2 \cdot \text{ºC}$$

The total number of tubes is $N = N_L \times N_T = 6 \times 10 = 60$. For a unit tube length ($L = 1$ m), the heat transfer surface area and the mass flow rate of air (evaluated at the inlet) are

$$A_s = N\pi DL = 60\pi(0.015 \text{ m})(1 \text{ m}) = 2.827 \text{ m}^2$$

$$\begin{aligned}\dot{m} &= \dot{m}_1 = \rho_1 \mathcal{V}(N_T S_T L) \\ &= (1.204 \text{ kg/m}^3)(4.5 \text{ m/s})(10)(0.05 \text{ m})(1 \text{ m}) = 2.709 \text{ kg/s}\end{aligned}$$

Then the fluid exit temperature, the log mean temperature difference, and the rate of heat transfer become

$$\begin{aligned}T_e &= T_s - (T_s - T_i)\exp\left(-\frac{A_s h}{\dot{m}C_p}\right) \\ &= 120 - (120 - 20)\exp\left(-\frac{(2.827 \text{ m}^2)(92.2 \text{ W/m}^2 \cdot \text{ºC})}{(2.709 \text{ kg/s})(1007 \text{ J/kg} \cdot \text{ºC})}\right) = 29.11\text{ºC}\end{aligned}$$

$$\Delta T_{\ln} = \frac{(T_s - T_e) - (T_s - T_i)}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{(120 - 29.11) - (120 - 20)}{\ln[(120 - 29.11)/(120 - 20)]} = 95.4°C$$

$$\dot{Q} = hA_s\Delta T_{\ln} = (92.2 \text{ W/m}^2 \cdot °\text{C})(2.827 \text{ m}^2)(95.4°\text{C}) = \mathbf{2.49 \times 10^4 \text{ W}}$$

The rate of heat transfer can also be determined in a simpler way from

$$\begin{aligned} \dot{Q} &= hA_s\Delta T_{\text{in}} = \dot{m}C_p(T_e - T_i) \\ &= (2.709 \text{ kg/s})(1007 \text{ J/kg} \cdot °\text{C})(29.\ 11 - 20)°\text{C} = 2.49 \times 10^4 \text{ W} \end{aligned}$$

For this square in-line tube bank, the friction coefficient corresponding to $Re_D = 5088$ and $S_L/D = 5/1.5 = 3.33$ is, from Fig. 7–27*a*, $f = 0.16$. Also, $\chi = 1$ for the square arrangements. Then the pressure drop across the tube bank becomes

$$\begin{aligned} \Delta P &= N_L f \chi \frac{\rho V_{max}^2}{2} \\ &= 6(0.16)(1)\frac{(1.06 \text{ kg/m}^3)(6.43 \text{ m/s})^3}{2}\left(\frac{1\text{N}}{1 \text{ kg} \cdot \text{m/s}^2}\right) = \mathbf{21 \text{ Pa}} \end{aligned}$$