Chapter One

General PrinciPals

قال النبي صلى الله عليه وسلم:

تعلموا العلم، فان تعلمه لله خشية،

وطلبه عبادة،

ومذاكرته تسبيح،

والبحث عنه جهاد،

و تعليمه لمن لا يعلمه صدقة

وبذله لأهله قربة.

General Principals

1.1 Introduction

The subject of statics developed very early in history because it's principles can be formulated simply from measurements of geometry and force. ***Statics is the study of bodies that are at rest or move with constant velocity***. We can consider statics as a special case of ***dynamics***, in which the ***acceleration is zero***.

يهتم علم الإستاتيكا بدراسة و تحليل الأجسام التي هي في حالة السكون

1.2 Fundamental Concepts

Before we begin our study, it is important to understand the meaning of certain fundamental concepts and principles.

Length: Length is used to ***locate the position*** of a point in space and thereby describe the size of a physical system.

"كمية الطول": هو تعبير لقياس شيء ما ويستعمل لتحديد أماكن النقاط.

Time: Although the principles of statics are ***time independent***. This quantity plays an ***important*** role in the study of ***dynamics***.

دور "كمية الزمن" مهمة جدا في علم الديناميكا.

Mass: Mass is a ***measure of a quantity of matter***.

الكتلة عبارة عن خاصية فيزيائية للأجسام، وتعرف على إنها مقدار ما يحويه الجسم من مادة.

Force: Force is considered as a "***push***" or "***pull***" exerted by one body on another. This interaction can occur when there is direct contact between the bodies, such as a person pushing on a wall. A force is completely characterized by its ***magnitude***, ***direction***, and ***point of application***.

القوى هي كميات متجهة لها مقدار و اتجاه ونقطة تأثير.

Particle: Particle has a ***mass***, but it ***size can be neglected***.

الجسيم له كتلة و ليس له أي حجم.

Rigid Body: A rigid body can be considered as a ***combination*** of a ***large number*** of ***Particles*** .

الجسم الجاسئ هو الحالة المثالية لجسم صلب المسافة بين الجسيمات تبقى فيه ثابتة عبر الزمن بغض النظر عن القوى الخارجية المطبقة عليه. عادة ما يعتبر الجسم الجاسئ على أنه توزيع مستمر للكتلة.

Newton's first law: A ***particle originally*** at ***rest*** or ***moving*** in a straight line with ***constant velocity***, ***tends*** to ***remain*** in this ***State*** provided the particle is not subjected to an unbalanced force (Fig.1-1).

$$\sum_{i=1}^{N} F_i = 0$$

يظل الجسم في حالته الساكنة (إما السكون التام أو التحريك في خط مستقيم بسرعة ثابتة) ما لم تؤثر عليه قوة تغير من هذا الحالة. يشير هذا القانون أنه إذا كان مجموع الكميات الموجهة من القوى التي تؤثر على جسم ما صفرا، فسوف يظل هذا الجسم ساكنا. وبالمثل فإن أي جسم متحرك سيظل على حركته بسرعة ثابتة في حالة عدم وجود أية قوى تؤثر عليه مثل قوى الاحتكاك.

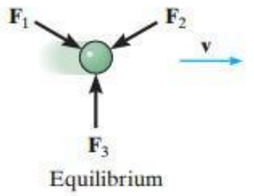


Fig. 1-1

إذا أثرت قوة أو مجموعة قوى F على جسم ما فإنها تكسبه تسارعاً (أو عجلة) a، يتناسب مع محصلة القوى المؤثرة، ومعامل التناسب هو الكتلة m للجسم.

Newton's second law:

A ***particle acted*** upon by an unbalanced ***force*** "***F***" ***experiences*** an ***acceleration*** "***a***" that has the same direction as the ***force*** and a ***magnitude*** that is directly ***proportional*** to the force (Fig. 1-2). If "F" is applied to a particle or mass "m", this law may be expressed mathematically as:

$$F = m \, . \, a$$

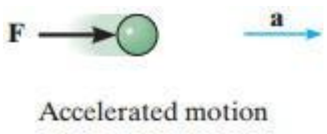


Fig. 1-2

لكل قوة فعل قوة رد فعل تساويها في المقدار و معاكسة لها في الاتجاه.

Newton's third Law:

The ***mutual forces*** of ***action*** between two particles are ***equal***, ***opposite***, and ***collinear*** (Fig. 1-3).

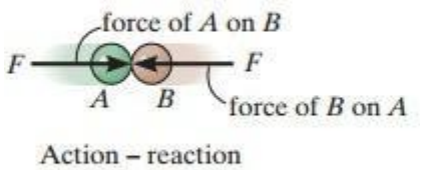


Fig. 1-3

قانون الجذب العام هو قانون فيزيائي استنباطي ينص على أنه "توجد قوة تجاذب بين أي جسمين في الكون، تتناسب طرديًا مع حاصل ضرب كتلتيهما، وعكسيًا مع مربع المسافة بينهما".

Newton's Law of Gravitational Attraction:

Shortly after formulating his three laws of motion. Newton postulated a law governing the ***gravitational attraction between any two particles***. Stated mathematically.

$$F = G\frac{m_1 m_2}{r^2}$$

Where

F: ***Force of gravitational*** between the two particles.
G: ***Universal constant of gravitation***, according to experimental evidence.

$$G = 66.73\ 10^{-12}\ \frac{\mathrm{m}^3}{\mathrm{kg\ s}^2}$$

m1, m2: mass of each of the two particles.
r: distance between the two particles.

Weight: Weight refers to the ***gravitational attraction*** of the ***earth*** on a body or quantity of mass. The weight of a particle having a mass is stated mathematically.

الوزن هو قوة جذب الأرض للجسم

$$W = mg$$

Measurements give $\boldsymbol{g = 9.8066\ \frac{m}{s^2}}$

Therefore, a body of ***mass 1 kg*** has a ***weight of 9.81*** **N**, a 2 kg body weights 19.62 N, and so on (Fig. 1-4).

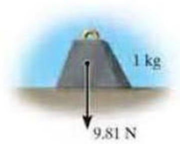


Fig. 1-4

Units of Measurement:

النظام الدولي للوحدات هو نظام وحدات القياس الأوسع انتشارا في العالم، وهو يستخدم في أغلب دول العالم. يسمى هذا النظام بالنظام المتري.

- **SI units:** The ***international System of units***. Abbreviated SI is a ***modern version*** which has received worldwide recognition. As shown in Tab 1.1. The SI system defines ***length in meters (m)***, ***time in seconds (s)***, and ***mass in kilograms (kg).*** In the SI system the unit of force, the ***Newton*** is a ***derived unit***. Thus, 1 Newton (N) is equal to a force required to give 1 kilogram of mass and acceleration of 1 $\frac{m}{s^2}$.

و هو نظام وحدات قياس يستخدم بكثرة في الولايات المتحدة الأمريكية و بريطانيا.

- **US customary:** In the ***U.S. Customary*** system of units (***FPS***) ***length*** is measured in ***feet (ft)***, ***time*** in ***seconds (s)***, and ***force*** in ***pounds (lb)***. The unit of ***mass***, called a ***slug***, ***1 slug*** is equal to the amount of ***matter*** accelerated at ***1*** $\boldsymbol{\frac{ft}{s^2}}$ when acted upon by a ***force of 1 lb*** $(1\ slug = 1\ \frac{lb\ s^2}{ft})$.

Table 1.1 Systems of Units				
Name	**Length**	**Time**	**Mass**	**Force**
International Systems of Units	*meter*	*seconds*	*kilogram*	*Newton**
SI	*m*	*s*	*kg*	*N* $\frac{kg.m}{s^2}$
US Customary	*foot*	*second*	*Slug**	*pound*
FPS	*ft*	*s*	$\frac{lb.s^2}{ft}$	*lb*
***Derived unit**				

Conversion of Units:

Table 1.2 provides a set of direct conversion factors between ***FPS*** and ***SI*** units for the basic quantities. Also in the FPS system, recall that:

1 ft = 12 in inches   1 mile = 5280 ft   1 kp kilo pound = 1000 lb   1 ton = 2000 lb

Table 1.2 Conversion factors			
Quantities	**Unit of Measurement (FPS)**	**equals**	**Unit of Measurement (SI)**
Force	lb		4.448 N
Mass	slug		14.59 kg
Length	ft		0.3048 m

Prefixes: When a ***numerical quantity*** is either very ***Large*** or very ***small***, the units used to define its size may be modified by using a ***prefix***. Some of the prefixes used in the SI system are shown in Table 1.3. Each represents a ***multiple*** or ***submultiples*** of a unit which, if applied successively, moves the decimal point of a numerical quantity to every third place. For example, 4000000N=4000kN (kilo-newton)=4MN (mega-newton), or 0.005m=5mm (milli·meter).

البادئة

(multiple) المضاعفات الأعلى
(submultiple) المضاعفات الأوطء

Table 1.3 Prefixes			
	Exponential Form	**Prefix**	**SI Symbol**
Multiple			
1 000 000 000	10^9	giga	G
1 000 000	10^6	mega	M
1 000	10^3	kilo	K
Submultiple			
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	µ
0.000 000 001	10^{-9}	nano	n

Exercise 1.1:

Convert 2 $\frac{\text{km}}{\text{h}}$ to $\frac{\text{m}}{\text{s}}$. How many $\frac{\text{ft}}{\text{s}}$ is this?

Ans: $2\frac{km}{} = 0.556\frac{m}{s} = 1.82\frac{ft}{s}$

Exercise 1.2:

Convert the quantities 300 lb. s and 52 $\frac{\text{slug}}{\text{ft}^3}$ to appropriate SI units.

Ans: $300\ lb.s = 1.33\ kN.s$ $\quad 52\frac{slug}{ft^"} = 26.8\frac{Mg}{m^3}$

Exercise 1.3:

Evaluate each of the following and express with SI units having an appropriate prefix: (a) $50\ \text{mN}\quad 6\ \text{GN}$ (b) $400\ \text{mm}\quad 0.6\ \text{MN}\ ^2$ (c) $\frac{45\ \text{MN}^3}{900\ \text{Gg}}$

Ans: $50\ mN\quad 6\ GN\ = 300\ kN^2$ $\quad 400\ mm\quad 0.6\ MN\ ^2 = 144\ Gm.N^2$ $\quad 45\frac{MN^3}{900Gg} = 50\frac{kN^3}{kg}$

Exercise 1.4:

Round off the following numbers to three significant figures:

(a) 4.65735 m (b) 55.578 s (c) 4555 N (d) 2768 kg

Ans: $a\ \ 4.66\ m$ $\quad b\ \ 55.6\ s$ $\quad c\ \ 4.56\ kN$ $\quad d\ \ = 2.77Mg$

Exercise 1.5:

Represent each of the following combinations of units in the correct SI form using an appropriate prefix:

(a) μMN (b) N/μm (c) MN/ks² (d) kN/ms.

Ans: $a\ \ N$ $\quad b\ \frac{MN}{m}$ $\quad c\ \frac{N}{s^2}$ $\quad d\ \frac{MN}{s}$

Exercise 1.6:

Represent each of the following combinations of units in the correct SI form:

(a) Mg/ms (b) N/mm (c) mN/(kg. μs).

Ans: $a\ \ \frac{\text{Mg}}{\text{ms}} = \frac{\text{Gg}}{\text{s}}$ $\quad b\ \ \frac{\text{N}}{\text{mm}} = \frac{\text{kN}}{\text{m}}$ $\quad c\ \ \frac{\text{mN}}{\text{kg}.\mu\text{s}} = \frac{\text{kN}}{\text{kg}.\text{s}}$

Exercise 1.7:

A rocket has a mass of 250 10^3 slugs on earth. Specify (a) its mass in SI units and (b) its weight in SI units. If the rocket is on the moon, where the acceleration due to gravity is g_m=5.30 ft/s², determine to 3 significant figures (c) its weight in units, and (d) its mass in SI units.

Ans: $\text{a}\ \ 3.65\ \text{Gg}$ $\quad \text{b}\ \ \text{W}_\text{e} = 35.8\ \text{MN}$ $\quad \text{c}\ \ \text{W}_\text{m} = 5.89\ \text{MN}$ $\quad \text{m}_\text{m} = \text{m}_\text{e} = 3.65\ \ \text{Gg}$

Exercise 1.8:

If a car is traveling at 55 mi/h, determine its speed in kilometers per hour and meters per second.

Ans: $a\ \ 88.514\frac{km}{}$ $\quad b\ \ 24.6\ \frac{m}{s}$

Exercise 1.9:

The Pascal (Pa) is actually a very small units of pressure. To show this, convert 1 Pa=1 N/m² to lb/ft². Atmospheric pressure at sea level is 14.7 lb/in². How many Pascals is this?

Ans: $a\ \ 1\ Pa = 20.9\ 10^{-3}\ \frac{lb}{ft^2}$ $\quad b\ \ 1\ ATM = 101.34\ kPa$

Exercise 1.10:

Two particles have a mass of 8 kg and 12 kg, respectively. If they are 800 mm apart, determine the force of gravity acting between them. Compare this result with the weight of each particle.

Ans: $a \quad F = 10.0\ nN$

Exercise 1.11:

Determines the mass in kilograms of an object that has a weight of:

(a) 20 mN (b) 150 kN (c) 60 MN

Ans: $a \quad m = 2.04\ g$ $\quad b \quad m = 15.3\ Mg$ $\quad c \quad m = 6.12\ Gg$

Chapter Two

Force Vectors

تعلم فليس المرء يولد عالما وليس أخو علم كمن هو جاهل
وإن كبير القوم لا علم عنده صغير إذا التفت عليه المحافل

Force Vectors

2.1 Scalar and vectors

A **scalar** is any **positive** or **negative physical quantity** that can be completely specified by its **magnitude**.

الكمية القياسية: تلك الكمية التي يمكن تعيينها بمعرفة مقدارها و وحدة قياسها.

A **vector** is any physical quantity that requires both a **magnitude** and **direction** for its complete description. A vector is shown **graphically** by an **arrow**. The **length** of the arrow represents the **magnitude** of the vector, and a fixed axis defines the **direction** of its line of action .The **head** of the arrow indicates the **sense** of **direction of the vector** (Fig 2-1).

الكميات المتجهة هي كميات فزيائية يمكن تعيينها بمعرفة مقدارها العددي و اتجاهها و نقطة تأثيرها و محور عملها.

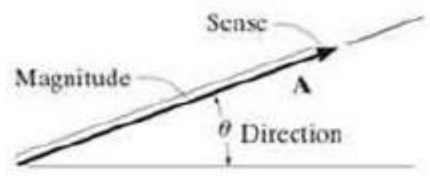


Fig. 2–1

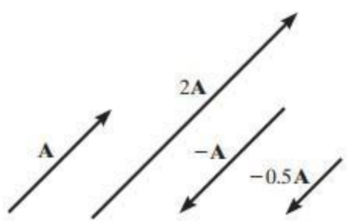
For **handwritten** work, it is often **convenient** to denote a vector quantity by simply drawing an arrow on top it $\overline{A}$.

In **print**, vector quantities are represented by **bold** face letters such as **A**, and its **magnitude** of the vector is **italicized**, *A*.

2.2 Vector operations

ضرب المتجهات

Multiplication and division of vector by a scalar:
If a vector is **multiplied** by a **positive scalar**, its **magnitude is increased** by that amount. When **multiplied by a negative scalar** it will also **change the directional** sense of the vector (Fig 2-2).



Scalar multiplication and division

Fig. 2–2

Vector addition:

All vector quantities **obey** the **parallelogram law** of addition. Fig 2-3 and Fig 2-4 and Fig 2-5 illustrates addition of vectors $\vec{A}$ and $\vec{B}$ to obtain a resultant $\vec{R}$.

جمع المتجهات بطريقة الرسم البياني

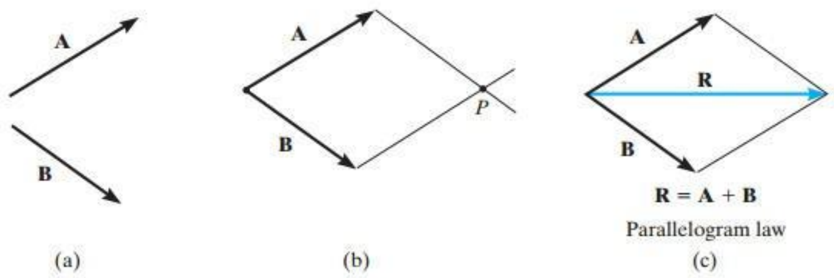


Fig. 2–3

قاعدة المتوازي الأضلاع

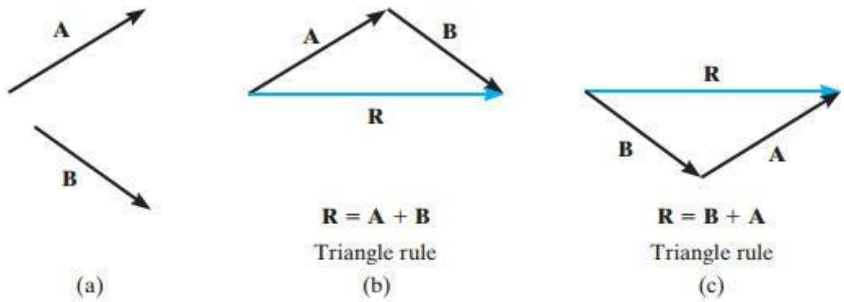
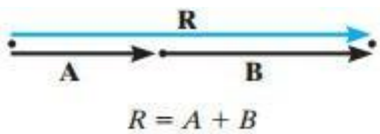


Fig. 2–4

قاعدة المثلث



Addition of collinear vectors

Fig. 2–5

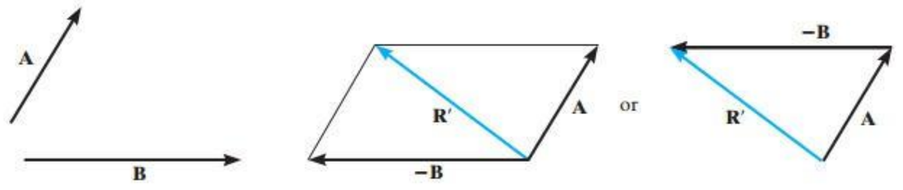
متجهات على نفس الخط

Vector subtraction:

The resultant of the difference between two vectors $\vec{A}$ and $\vec{B}$ of the same type may be expressed as:

$$\mathbf{R'} = \mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$$

Fig 2-6 illustrates subtraction of vectors **A** and **B**



Parallelogram law　　Triangle construction

Vector subtraction

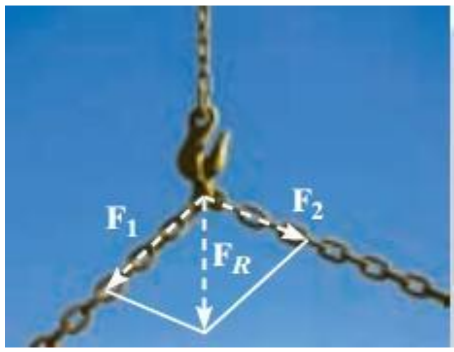
Fig. 2–6

تغيير اتجاه المتجه B

2.3 vector addition of forces:

Experimental evidence has shown that a **force** is a **vector quantity** since it has a specified **magnitude**, **direction**, and **sense** and it **adds according to the parallelogram law.**

القوى عبارة عن متجهات



تأثير قوتين على دبوس

Finding a resultant force:

The two component forces $\vec{F}_1$ and $\vec{F}_2$ acting on the pin in Fig 2-7 can be added together to form the **resultant** force

$$\vec{F}_R = \vec{F}_1 + \vec{F}_2$$

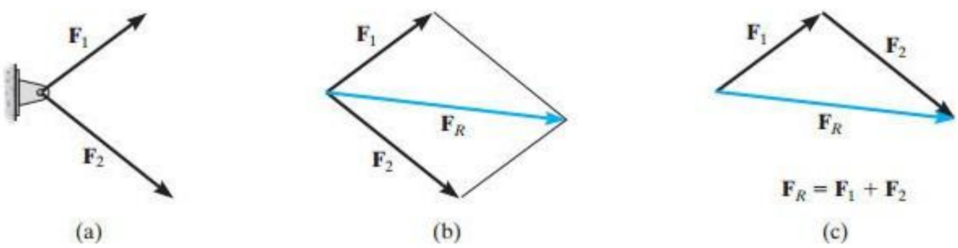


Fig. 2–7

Finding the components of a force:

Sometimes it is necessary to **resolve a force** into two components in order to **study its pulling and pushing effect** in two specific directions.

ضرورة دراسة تأثير القوة في اتجاهين مختلفين يتطلب تحويلها إلى مركبات

For example, in Fig 2.8, F is to be resolved into two components along two members, defined by *u* and *v* (Fig 2.8)

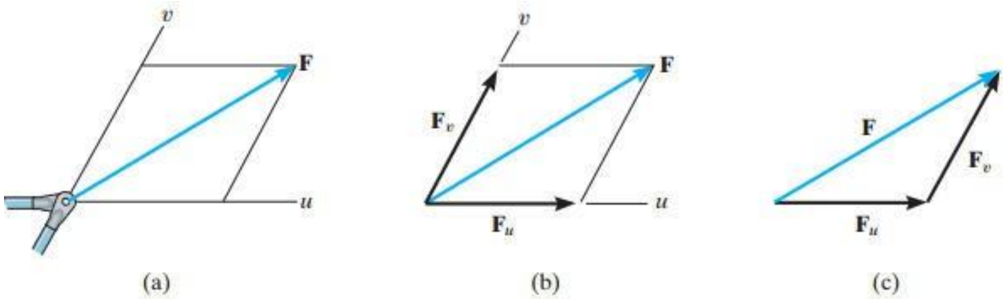


Fig. 2–8

Addition of several forces:

استعمال طريقة الرسم البياني بشكل متتالي

If **more than two forces** are to be **added successive applications of the parallelogram law** can be carried out in order to obtain the resultant force. For example if the three forces $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$ act at a point o, the **resultant** of any two of the forces is found $(\vec{F}_1 + \vec{F}_2)$ and then this resultant is **added** to the **third force** yielding the **resultant of all three forces** ($\vec{F}_R = (\vec{F}_1 + \vec{F}_2) + \vec{F}_3$) (Fig 2-9).

هنا نجمع محصلة $\vec{F}_1$ و $\vec{F}_2$ مع المتجهة $\vec{F}_3$ للحصول على المحصلة الكلية $\vec{F}_R$

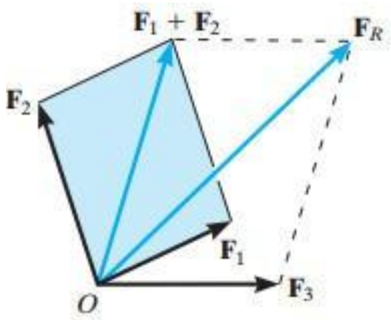


Fig. 2–9

Trigonometry analysis:

التحليل المثلثي

Redraw a half portion of the parallelogram to illustrate the **triangular** head to tail addition of the components. From this triangle, the **magnitude of the resultant force** can be determined using **the law of cosines,** and its direction is determined from the **law of sines** .

The magnitudes of two force components are determined from the law of sines. The formulas are given in Fig 2-10

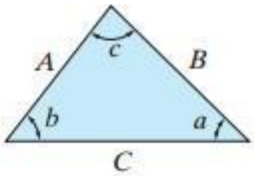
cosine law:

$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

الطريقة التحليلية تستلزم قاعدة الجيب التمام وقاعدة الجيب للمثلثات

sine law:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$



Cosine law:

$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

Sine law:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

Fig 2-10

Exercise 2.1:
The screw eye in Fig 2-11 is subjected to two forces, $\vec{F_1}$ and $\vec{F_2}$. Determine the magnitude and direction of the resultant force.

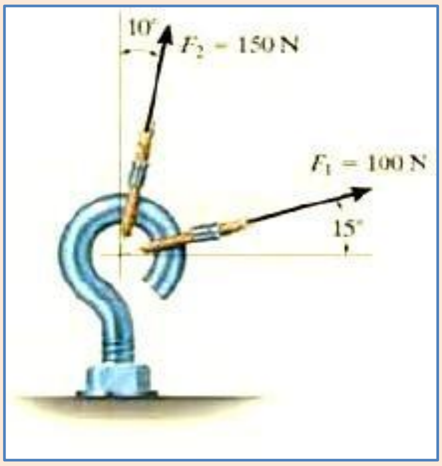


Fig 2-11

Ans: $F_R = 213$ N $\quad \emptyset = 54.7°$

Exercise 2.2:
Resolve the horizontal 600lb force in fig 2.12 into components action along the *u* and *v* axes and determine the magnitudes of these components.

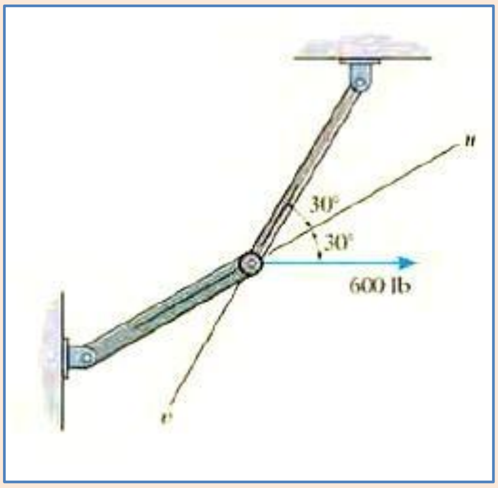


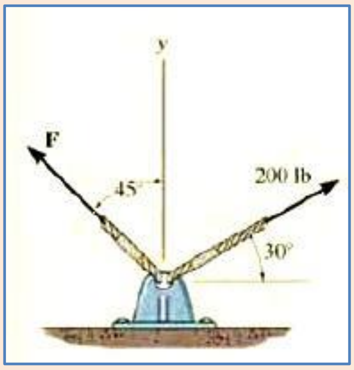
Fig 2-12

Ans: $F_u = 1039$ lb $\quad F_v = 600$ lb

Exercise 2.3:

Determine the magnitude of the component force $\vec{F}$ in Fig 2-13 and the magnitude of the resultant force $\vec{F_R}$ if $\vec{F_R}$ is directed a long the positive y axis.

Fig 2-13

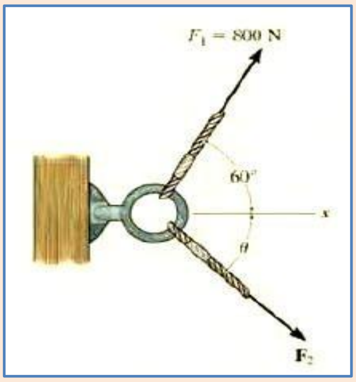


Ans: $F = 245$ lb $\quad F_R = 273$ lb

Exercise 2.4:

It is required that the resultant force acting on the eyebolt in Fig 2.14 be directed along the positive *x* axis and that $\vec{F_2}$ have a minimum magnitude. Determine this magnitude, the angle θ, and the corresponding resultant force.

Fig 2-14

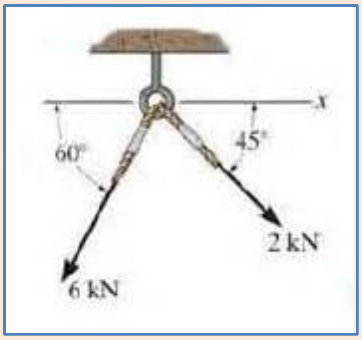


Ans: $\theta = 90°$ $\quad F_R = 400$ N $\quad F_2 = 693$ N

Exercise 2.5:

Determine the magnitude of the resultant force acting on the screw eye and its direction measured clockwise from the x axis.

Fig 2-15

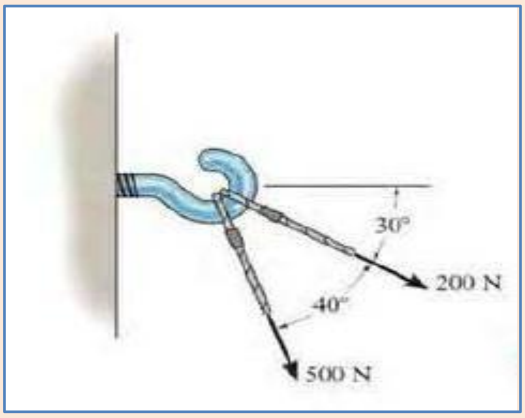


Ans: $F_R = 6.80$ kN $\quad \theta = 103°$

Exercise 2.6:

Two forces act on the hook. Determine the magnitude of the resultant force.

Fig 2-16



Ans: $F_R = 666$ N

Exercise 2.7:

Resolve the 30 lb force into components along the *u* and *v* axes and determine the magnitude of each of these components.

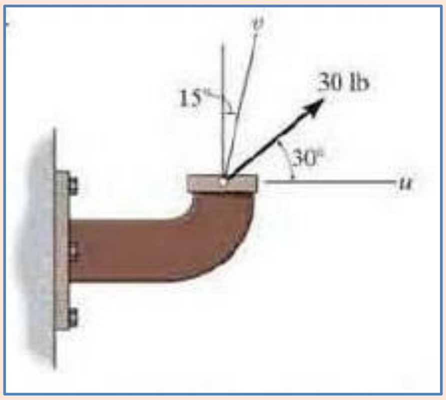


Fig 2-17

Ans: $F_u = 22.0\,\text{lb}$ $F_v = 15.5\,\text{lb}$

Exercise 2.8:

If force $\mathbf{F}^{\rightarrow}$ is to have a component along the *u* axis of F_U=6 kN, determine the magnitude of $\mathbf{F}^{\rightarrow}$ and the magnitude of its component F_v along the *v* axis.

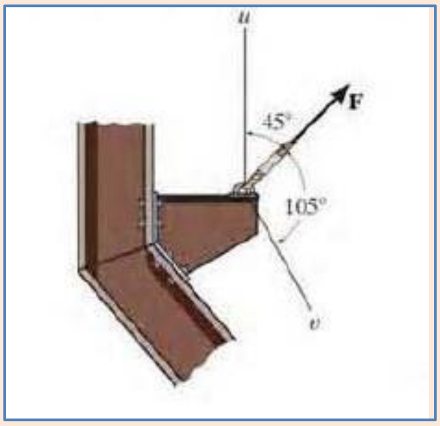


Fig 2-18

Ans: $F = 3.11\,\text{kN}$ $F_v = 4.39\,\text{kN}$

Exercise 2.9:

If $\theta = 60°$ and *T*=5kN, determine the magnitude of the resultant force acting on the eyebolt and its direction measured clockwise from the positive *x* axis.

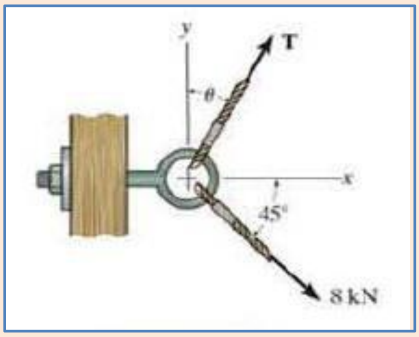


Fig 2-19

Ans: $F_R = 10.47\text{kN}$ $\emptyset = 17.5°$

Exercise 2.10:

Resolve $\mathbf{F_1}^{\rightarrow}$ into components along *u* and *v* axes and determine the magnitudes of these components.

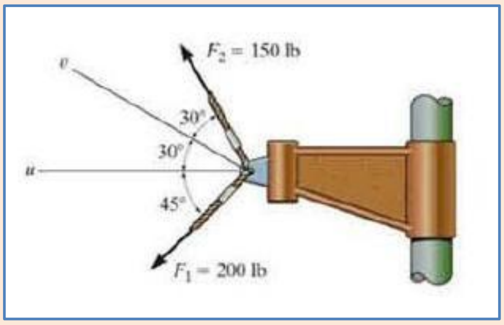


Fig 2-20

Ans: $F_u = 386\,\text{lb}$ $F_v = 283\,\text{lb}$

Exercise 2.11:

Resolve $\vec{F_2}$ into components along *u* and *v* axes, and determine the magnitudes of these components.

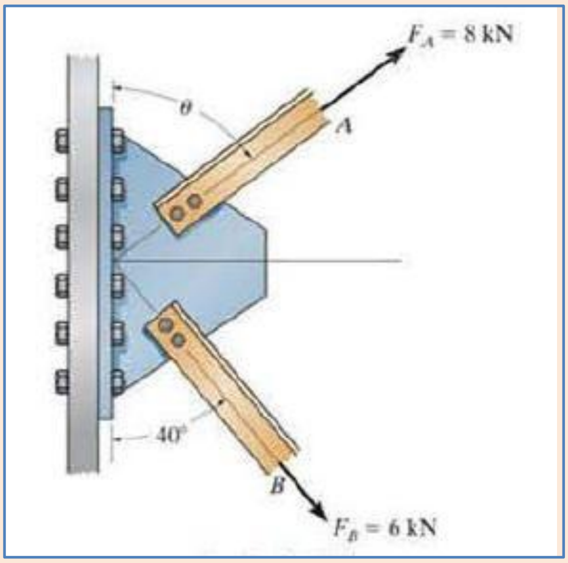
See Fig 2-20

Ans: $F_u = 150\ \text{lb}$ $F_v = 260\ \text{lb}$

Exercise 2.12:

The plate is subjected to the two forces at *A* and *B* as shown. If θ=60° determine the magnitude of the resultant of these two forces and its direction measured clockwise from the horizontal.

Fig 2-21



Ans: $F_R = 10.8\ \text{kN}$ $\emptyset = 3.16°$

Exercise 2.13:

Determine the angle of θ for connecting member *A* to the plate so that the resultant force of $\vec{F_A}$ and $\vec{F_B}$ is directed horizontally to the right. Also what is the magnitude of the resultant force?

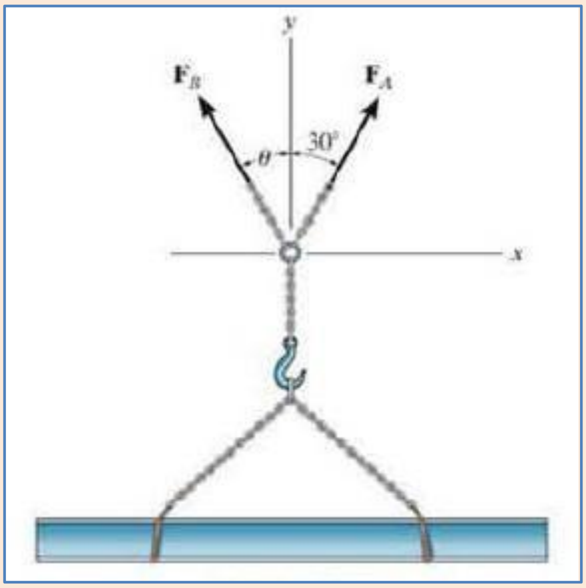
See Fig 2-21

Ans: $\theta = 54.9°$ $F_R = 10.4\ \text{KN}$

Exercise 2.14:

The beam is to be hoisted using two chains. If the resultant force is to be 600 N directed along the positive *y* axis, determine the magnitudes of forces $\vec{F_A}$ and $\vec{F_B}$ acting on each chain and the angle θ of $\vec{F_B}$ so that the magnitude of $\vec{F_B}$ is minimum, $\vec{F_A}$ act at 30° from the *y* axis as shown.

Fig 2-22



Ans: $F_A = 520\ \text{N}$ $F_B = 300\ \text{N}$

2.4 addition of a system of coplanar forces

When a force in resolved into two components along the *x* and *y* axes the components are then called **rectangular components**.

The rectangular components of force F shown in Fig 2.23 are found using the parallelogram law, so that

التمثيل هنا يكون بتحويلها إلى مركبات سينية وصادية على مستوى المحاور الديكارتية

$$\vec{F} = \vec{F_x} + \vec{F_y}$$

$$F_x = F\cos\theta$$

$$F_y = F\sin\theta$$

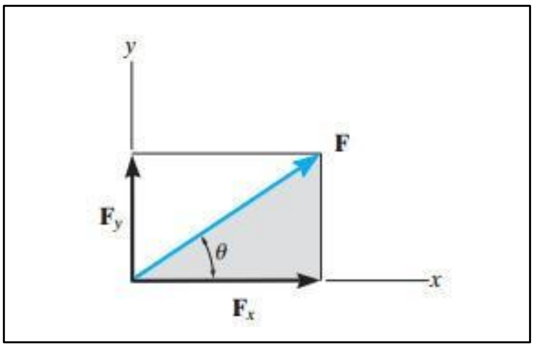


Fig 2-23

instead of using the angle θ, the direction of $\vec{F}$ can also be defined **using a small "slope" triangle**, such as shown in fig 2.24

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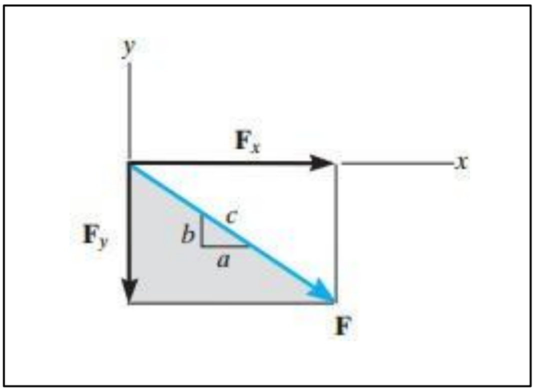


Fig 2-24

$$\frac{F_X}{F} = \frac{a}{c} \Rightarrow F_X = F\left(\frac{a}{c}\right) \qquad \text{And} \qquad \frac{F_y}{F} = \frac{b}{c} \Rightarrow F_y = \left(\frac{b}{c}\right)F$$

It is also possible to represent the *x* and *y* components of a force in terms of **Cartesian unit vectors i** and **j** (Fig 2.25).

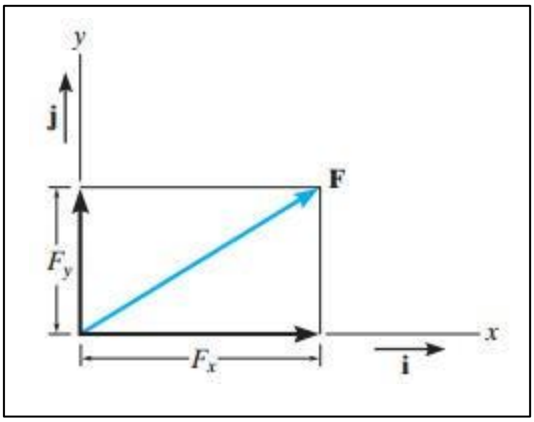


Fig 2-25

We can express $\vec{F}$ as a **Cartesian vector**.

$$\vec{F} = F_x\vec{i} + F_y\vec{j}$$

In **coplanar force** resultant case, **each force is resolved** into its *x* and *y* components, and then the respective components are added using **scalar algebra** since they are collinear. For example, consider the three concurrent forces in Fig 2.26.

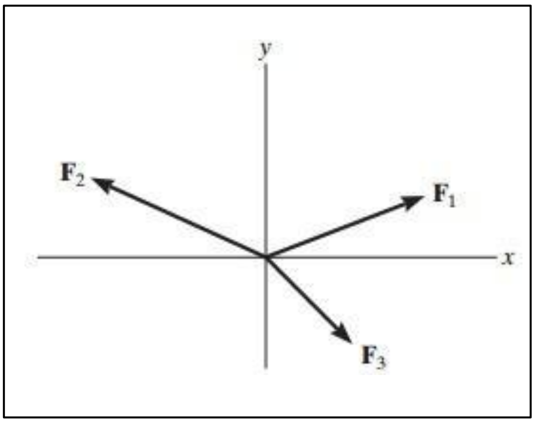


Fig 2-26

Each force is represented as a Cartesian vector.

$$\vec{F_1} = F_{1x}\vec{i} + F_{1y}\vec{j}$$

$$\vec{F_2} = -F_{2x}\vec{i} + F_{2y}\vec{j}$$

$$\vec{F_3} = F_{3x}\vec{i} - F_{3y}\vec{j}$$

The vector resultant is therefore.

$$\vec{F_R} = \vec{F_1} + \vec{F_2} + \vec{F_3} = (F_{1x} + F_{2x} + F_{3x})\vec{i} + (F_{1y} + F_{2y} + F_{3y})\vec{j}$$

$$\vec{F_R} = \vec{F_1} + \vec{F_2} + \vec{F_3} = (F_{Rx})\vec{i} + (F_{Ry})\vec{j}$$

We can **represent** the components of the resultant force of any number of coplanar forces symbolically by the **algebraic sum the x and y** components of all the forces.

$$F_{Rx} = \Sigma F_x$$

$$F_{Ry} = \Sigma F_y$$

Once these components are determined, they may be sketched along the x and y axes with their proper sense of direction, and the resultant force can be determined from vector addition as shown in Fig 2-27.

The magnitude of $\vec{F_R}$ is then found from the by **Pythagorean theorem**: that is

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

$$\theta = \tan^{-1} \left| \frac{F_{Ry}}{F_{Rx}} \right|$$

كمية و زاوية المحصلة يكون باستعمال نظرية بيثاغورث للمثلثات القائمة

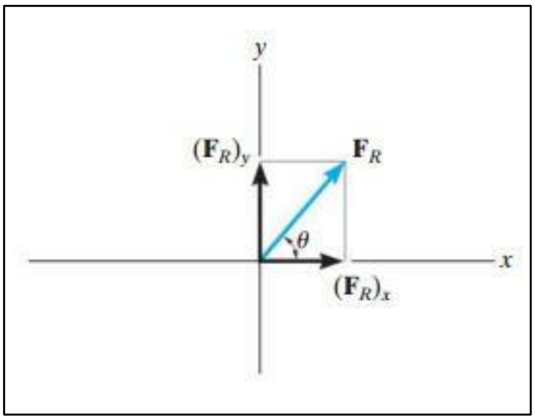
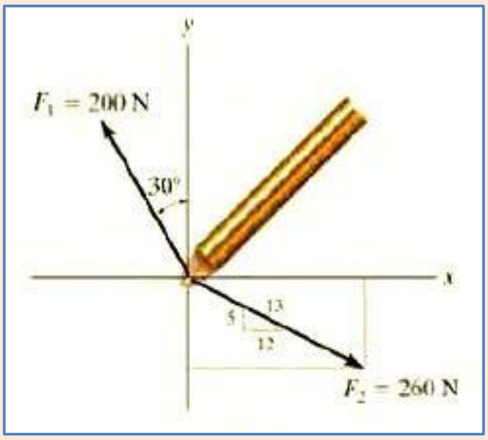


Fig 2-27

Exercise 2.15:

Determine the *x* and *y* components of $\vec{F_1}$ and $\vec{F_2}$ acting on the boom shown in Fig 2.28 express each force as a Cartesian vector.

Fig 2-28



Ans: $F_{1x} = -100$ N $F_{1y} = 173$ N $F_{2x} = 240$ N $F_{2y} = -100$ N

$\vec{F_1} = (-100\vec{i} + 173\vec{j})$ N $\vec{F_2} = (240\vec{i} - 100\vec{j})$ N

Exercise 2.16:

The link in Fig 2.29 is subjected to two forces $\vec{F_1}$ and $\vec{F_2}$. Determine the magnitude and direction of the resultant force.

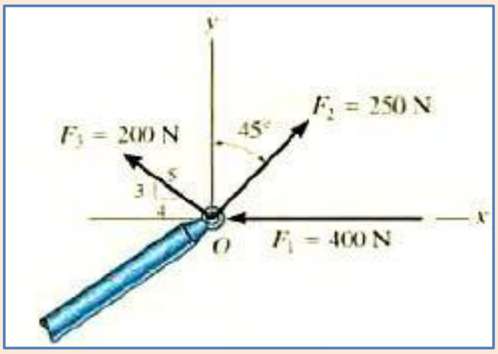
Fig 2-29

Ans: $F_R = 629$ N $\theta = 67.9°$

Exercise 2.17:

The end of boom O in Fig 2.30 is subjected to three concurrent and coplanar forces. Determine the magnitude and direction of the resultant force.

Fig 2-30

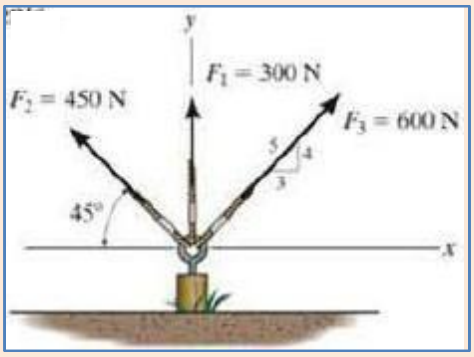


Ans: $F_R = 485$ N $\theta = 37.8°$

Exercise 2.18:

Resolve each force acting on the post into its *x* and *y* components.

Fig 2-31

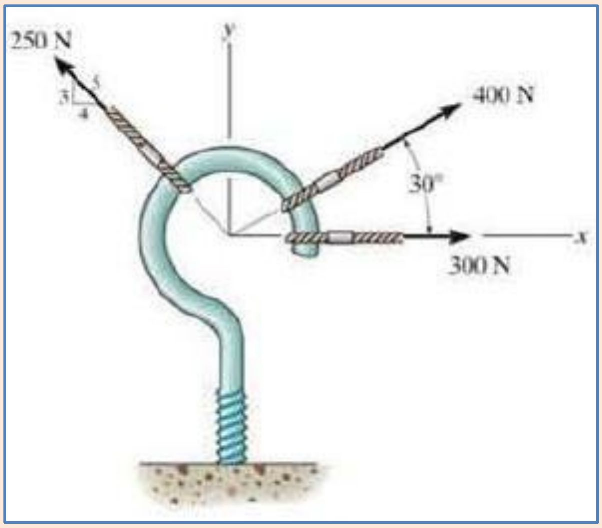


Ans: $F_{1x} = 0$ N $F_{1y} = 300$ N $F_{2x} = -318$ N $F_{2y} = 318$ N $F_{3x} = 360$ N $F_{3y} = 480$ N

Exercise 2.19:

Determine the magnitude and direction of the resultant force.

Fig 2-32

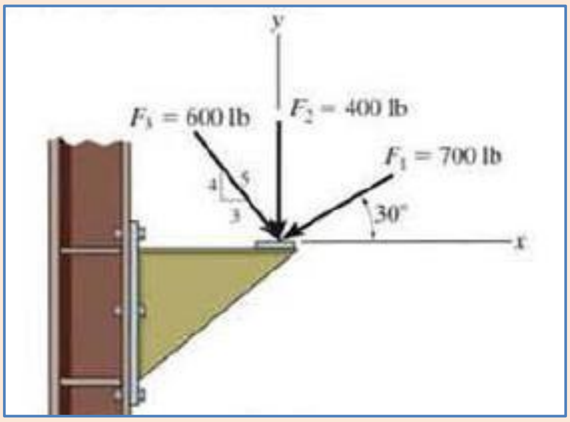


Ans: $F_R = 567\ \mathrm{N}$ $\theta = 38.1°$

Exercise 2.20:

Determine the magnitude of the resultant force acting on the corbel and its direction θ measured counterclockwise from the *x* axis.

Fig 2-33

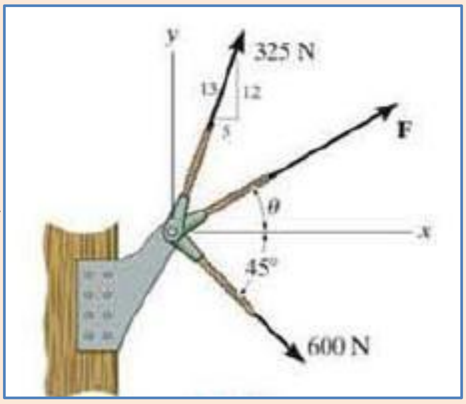


Ans: $F_R = 1254\ \mathrm{lb}$ $\Phi = 78.68°$ $\theta = 180 + \Phi = 259°$

Exercise 2.21:

If the resultant force acting on the bracket is to be 750 N directed along the positive *x* axis, determine the magnitude of $\vec{\mathbf{F}}$ and its direction θ.

Fig 2-34



Ans: $\theta = 31.76°$ $F = 236\ \mathrm{N}$

Exercise 2.22:

Determine the magnitude and direction measured counterclockwise from the positive *x* axis of the resultant force of the three forces acting on the ring A. Take F_1 = 500N and θ =20°.

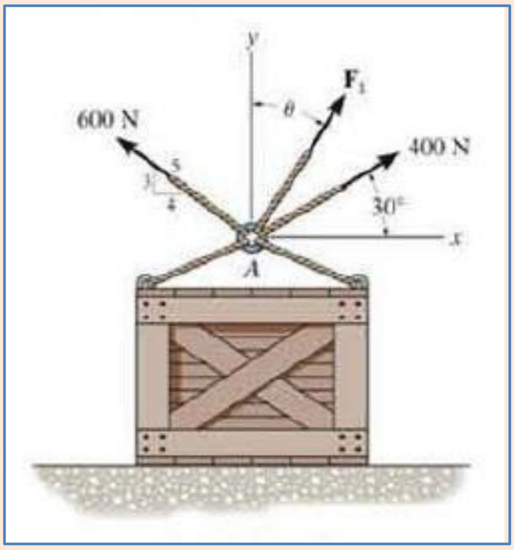


Fig 2-35

Ans: $F_R = 1.03\text{ kN} \quad \theta = 87.9°$

Exercise 2.23:

Determine the magnitude and direction measured counterclockwise from the positive *x* axis of the resultant force acting on the ring at O, if F_A = 750 N and θ = 45°.

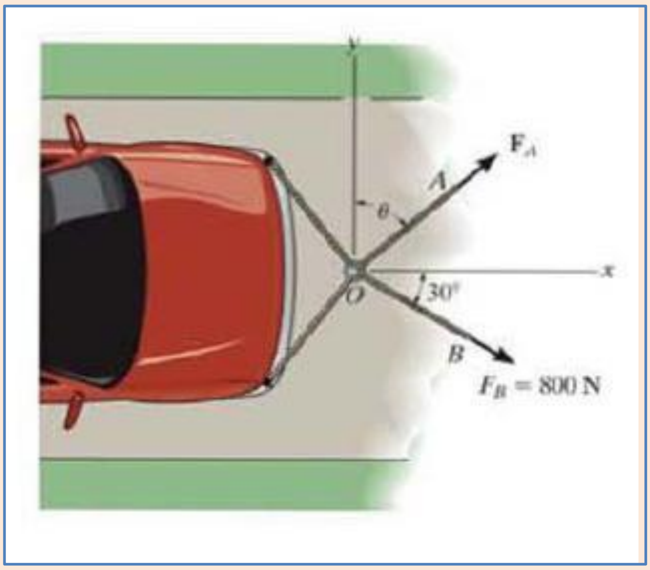


Fig 2-36

Ans: $F_R = 1.23\ \text{kN} \qquad \theta = 6.08°$

Exercise 2.24:

Express each of the three forces acting on the bracket in Cartesian vector form with respect to the *x* and *y* axes. Determine the magnitude and direction θ of $\vec{F_1}$ so that the resultant force is directed along the positive *x′* axis and has a magnitude of F_R = 600 N.

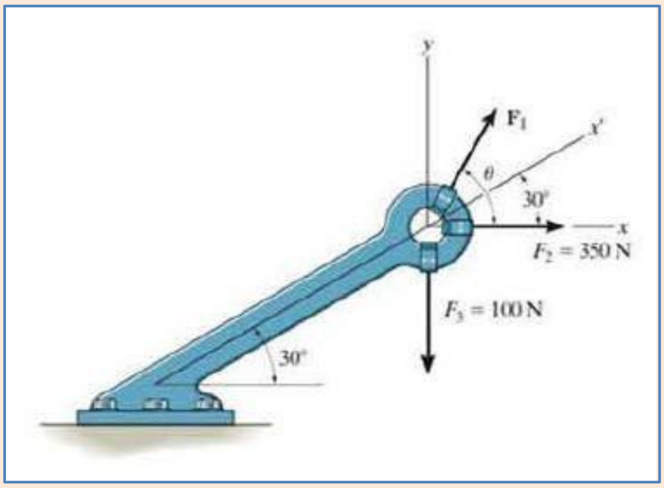


Fig 2-37

Ans: $F_1 = 434.5\text{ N} \qquad \theta = 67°$

2.5 Cartesian vectors

A vector $\vec{A}$ may have three rectangular components along the *x, y, z* coordinate axes and is represented by the vector sum of its **three rectangular components** (Fig 2-38).

القوى المؤثرة في البعد الثالث لها مركبات حسب المحاور **x** و **y** و **z**

$$\vec{A} = \vec{A_x} + \vec{A_y} + \vec{A_z}$$

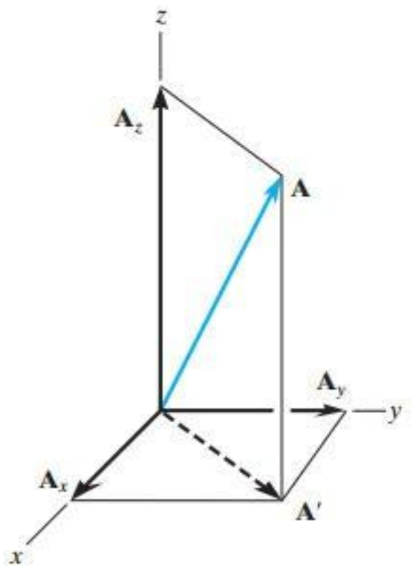


Fig. 2.38

In three dimensions, the set of Cartesian unit $\vec{i}$, $\vec{j}$, $\vec{k}$ is used to designate the directions of the x, y, z axes, respectively. The positive Cartesian unit vectors are shown in Fig 2-39.

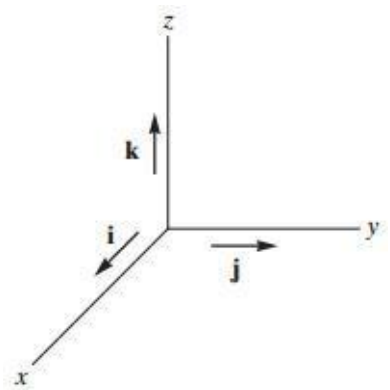


Fig. 2.39

We can write $\vec{A}$ in **Cartesian vector** form as

نستخدم متجهات الوحدة الديكارتية $\vec{i}$ و $\vec{j}$ و $\vec{k}$ لتعريف كمية المتجه $\vec{A}$

$$\vec{A} = A_x\vec{i} + A_y\vec{j} + A_z\vec{k}$$

The **magnitude** of $\vec{A}$ is expressed in Cartesian vector from as

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

The **direction of** $\vec{\mathbf{A}}$ is defined by the **coordinate direction** angles α, β, and γ (Fig 2.40).

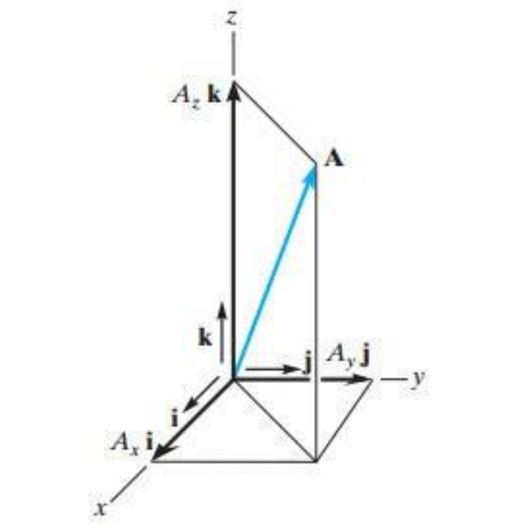


Fig. 2.40

$$\cos \alpha = \frac{A_x}{A} \qquad \cos \beta = \frac{A_y}{A} \qquad \cos \gamma = \frac{A_z}{A}$$

With

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

The addition (or subtraction) of two or more vectors are greatly simplified in terms of their Cartesian components. For example, the resultant $\vec{\mathbf{R}}$ in Fig 2.41 is written as

$$\vec{\mathbf{R}} = (A_x + B_x)\vec{\mathbf{i}} + (A_y + B_y)\vec{\mathbf{j}} + (A_z + B_z)\vec{\mathbf{k}}$$

حالة جمع متجهين في الفضاء

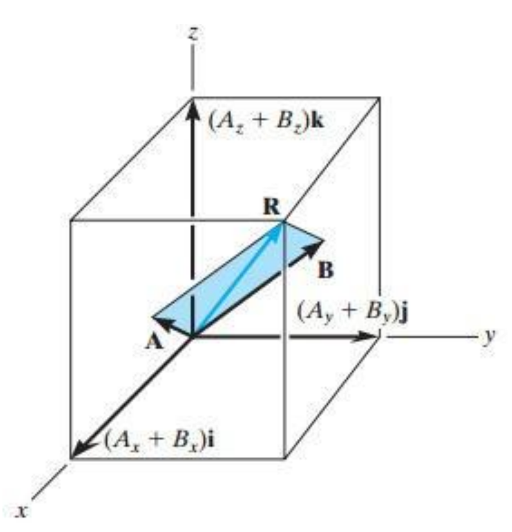


Fig. 2.41

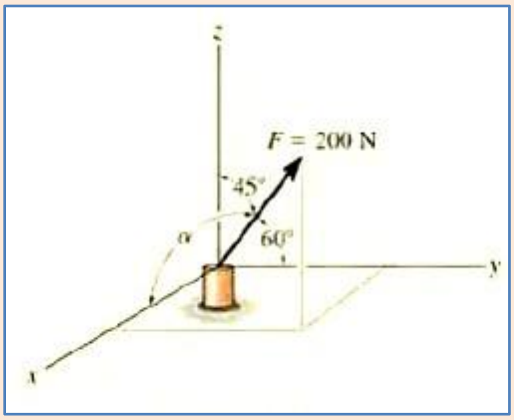
If this is generalized and applied to a system of several concurrent forces, then the force resultant is the vector sum of all the forces in the system and can be written as

$$\vec{\mathbf{F}}_R = \Sigma\vec{\mathbf{F}} = \Sigma F_x\vec{\mathbf{i}} + \Sigma F_y\vec{\mathbf{j}} + \Sigma F_z\vec{\mathbf{k}}$$

Exercise 2.25:

Express the force $\vec{F}$ shown in Fig 2.38 as a cartesian vector.

Fig 2-38

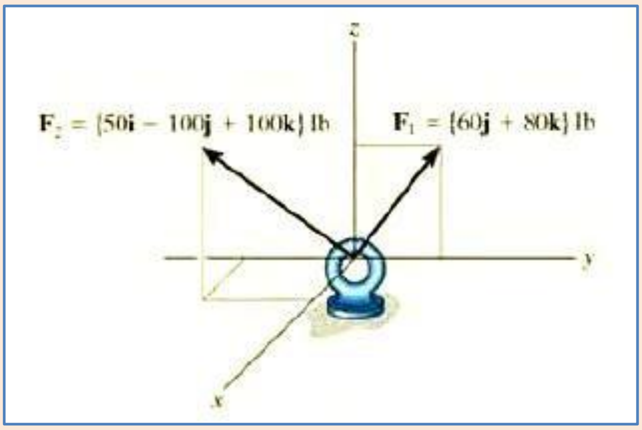


Ans: $\vec{F} = \{100\vec{i} + 100\vec{j} + 141.4\vec{k}\}$ N

Exercise 2.26:

Determine the magnitude and the coordinate direction angles of the resultant force acting on the ring in Fig 2-39

Fig 2-39

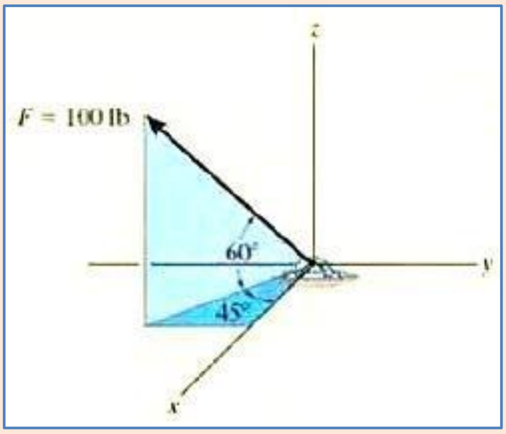


Ans: $F_R = 191$ lb $\cos\alpha = 0.2617$ $\alpha = 74.8°$ $\cos\beta = -0.2094$ $\beta = 102°$ $\cos\gamma = 0.9422$ $\gamma = 19{,}6°$

Exercise 2.27:

Express the force $\vec{F}$ shown in Fig 2.40 as a Cartesian vector, And determine its coordinate direction angles.

Fig 2-40



Ans: $\vec{F} = \{35.4\vec{i} - 35.4\vec{j} + 86.6\vec{k}\}$ lb $\alpha = 69.3°$ $\beta = 111°$ $\gamma = 30°$

Exercise 2.28:
Two forces act on the hook in Fig 2-41, specify the magnitude of $\vec{F_2}$ and its coordinate direction angles of $\vec{F_2}$ that the resultant force $\vec{F_R}$ acts along the positive *y* axis and has magnitude of 800 N.

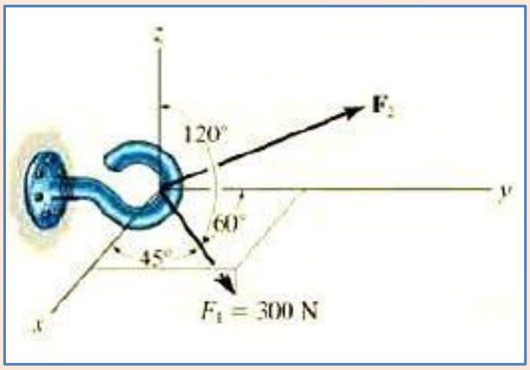


Fig 2-41

Ans: $F_2 = 700$ N $\cos\alpha_2 = \frac{-212.1}{700} \Rightarrow \alpha_2 = 108°$

$\cos\beta_2 = \frac{650}{700} \Rightarrow \beta_2 = 21.8°$

$\cos\gamma_2 = \frac{150}{700} \Rightarrow \gamma_2 = 77.6°$

Exercise 2.29:
Determine its coordinate direction angles of the force.

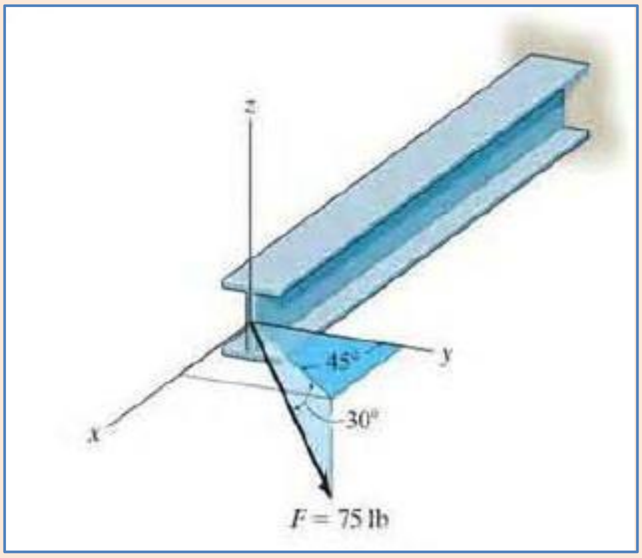


Fig 2-42

Ans: $\alpha = 52.2°$ $\beta = 52.2°$ $\gamma = 120°$

Exercise 2.30:
Express the force as a Cartesian vector.

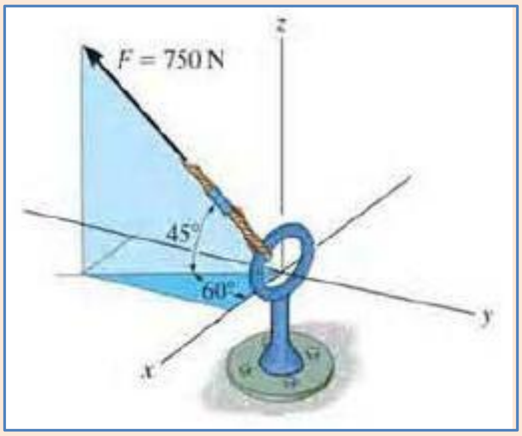


Fig 2-43

Ans: $\vec{F} = \{265\vec{i} - 459\vec{j} + 530\vec{k}\}$ N

Exercise 2.31:
Determine the resultant force acting on the hook.

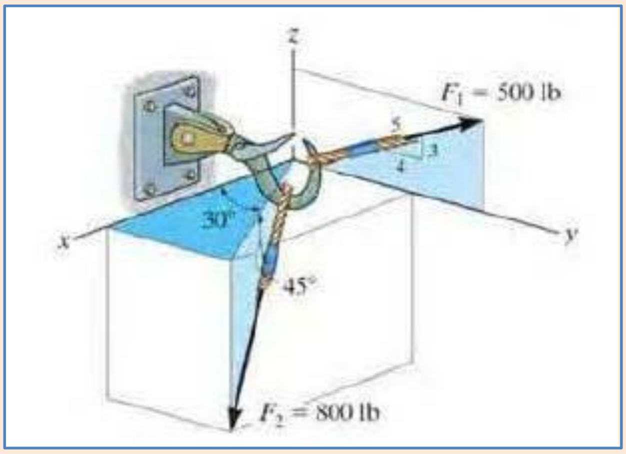


Fig 2-45

Ans: $\vec{F_R} = \vec{F_1} + \vec{F_2} = \{490\,\vec{i} + 683\,\vec{j} - 266\,\vec{k}\}$ lb

Exercise 2.32:
The mast is subject to the three forces shown. Determine the coordinate direction angles $\alpha_1, \beta_1, \gamma_1$ of $\vec{F_1}$ so that the resultant force acting on the mast is $\vec{F_R} = \{350\,\vec{i}\}$ N. Take F_1=500 N.

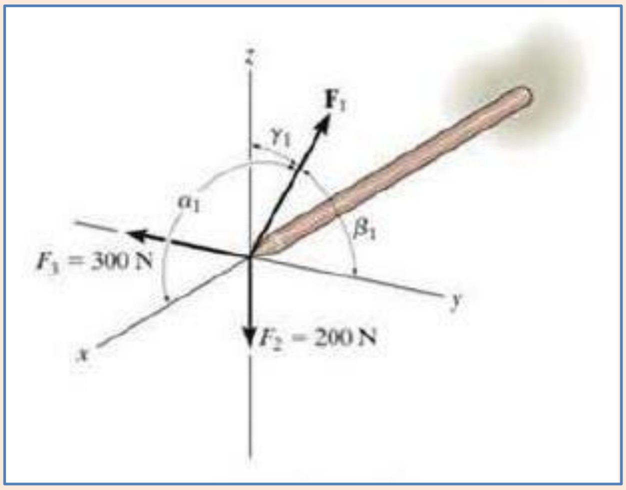


Fig 2-46

Ans: $\alpha_1 = 45.6°$ $\beta_1 = 53.1°$ $\gamma_1 = 66.4°$

Exercise 2.33:
The mast is subject to the three forces shown. Determine the coordinate direction angles $\alpha_1, \beta_1, \gamma_1$ of $\vec{F_1}$ so that the resultant force acting on the mast is zero (see Fig. 2.46).

Ans: $\alpha_1 = 90°$ $\beta_1 = 53.1°$ $\gamma_1 = 66.4°$

Exercise 2.34:
The two forces $\vec{F_1}$ and $\vec{F_2}$ acting at A have a resultant force of $\vec{F_R} = \{-100\,\vec{k}\}$ lb. Determine the magnitude and coordinate direction angles of $\vec{F_2}$.

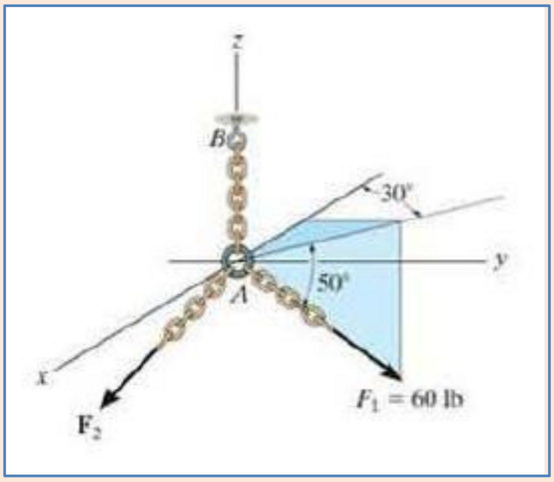


Fig 2-47

Ans: $F_2 = 66.4$ lb $\alpha = 59.8°$ $\beta = 107°$ $\gamma = 144°$

Chapter Three

Equilibrium Of a Particle

أخي الطالب عليك آداب جمة يجب أن تلتزم بها ولا تتخلى عنها وهي أن تكون إرادتك بطلب العلم هي الإخلاص لله ثم لوطنك، وأن تتخلق بمحاسن الأخلاق و تتحلى بمكارم الآداب و تتصف بالصفات الحميدة و تتجمل بالشيم المرضية وأن تتجنب الحسد والرياء والعجب والكبر

Equilibrium of a Particle

3.1 Condition for the equilibrium of a particle.

A particle is said to be in equilibrium if it **remains at rest** if **originally at rest, or has a constant velocity** if **originally in motion**. to maintain equilibrium, it is necessary to satisfy Newton's first law of motion which requires the resultant force acting on a particle to be equal to zero. This condition may be stated mathematically as

$$\sum \vec{F} = 0 \qquad (3.1)$$

Where $\sum \vec{F}$ is the vector sum of all the forces acting on the particle.

شرط توازن جسم هو أن تكون محصلة القوى الخارجية المؤثرة على الجسم تساوي صفر.

3.2 The free body diagram

A **drawing** that **shows** the particle with **all the forces** that act on it is called a **free body diagram** (**FBD**).

مخطط الجسم الحر هو تمثيل تصويري يستخدم لتحليل القوى المؤثرة على الجسم الحر.

We will consider **a springs connections** often encountered in particle equilibrium problems.

Springs: If a linearly elastic spring of undeformed length l_0 is used to support a particle, **the length of the spring will change in direct proportion to the force F acting on it**, Fig 3.1. A **characteristic** that defines the **elasticity of a spring** is the **spring constant** or **stiffness** *k*. The magnitude of force exerted on a linearly elastic spring is stated as

النوابض ذات الملف الحلزوني هي عناصر مرنة تقاوم القوى الضاغطة المطبقة باتجاه محورها، وهي تخضع لقانون هوك للمرونة الذي يعتبر بأن الامتداد الناتج يتناسب مباشرة مع الحمل.

$$F = k\,s$$

Where

$$s = l - l_0$$

s هو الفرق بين موضع الجسم الجديد و موقعه الأصلي. **k** هو ثابت المرونة.

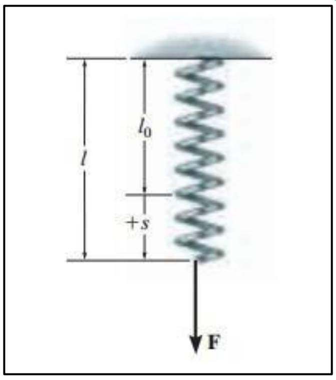
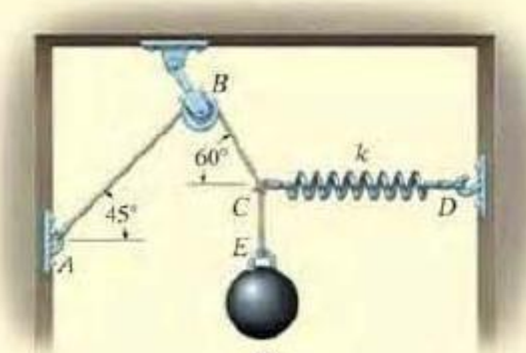


Fig 3.1

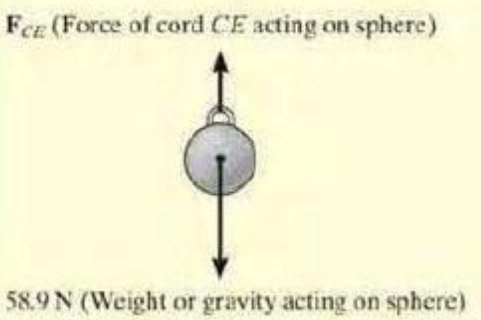
The following example **shows a drawing of the free body diagram** of a sphere.

The sphere has a mass of 6 kg and is supported as shown. Draw a free-body diagram of the sphere, the cord *CE*, and the knot at *C*.



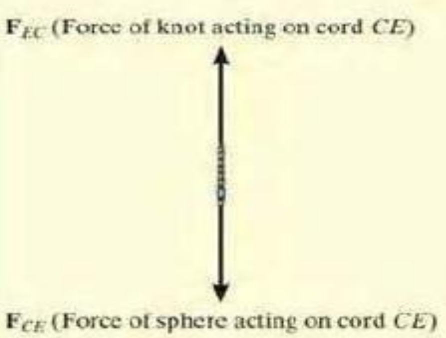
(a)

يظهر مخطط الجسم الحر كل قوى التلامس المؤثرة على الجسم الكروي و الحبل و العقدة **C.**



(b)

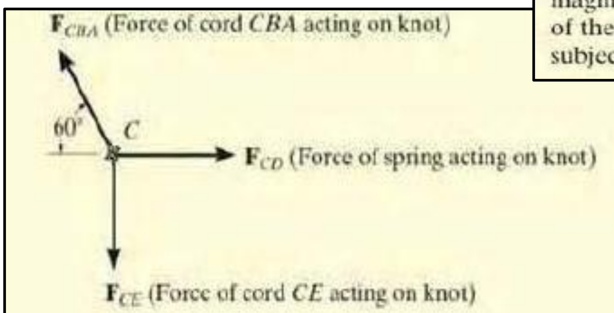
Sphere. By inspection, there are only two forces acting on the sphere, namely, its weight, 6 kg $(9.81\ \text{m/s}^2) = 58.9$ N, and the force of cord *CE*. The free-body diagram is shown in Fig.



(c)

Cord CE. When the cord *CE* is isolated from its surroundings, its free-body diagram shows only two forces acting on it, namely, the force of the sphere and the force of the knot, Fig. 3–3*c*. Notice that $\mathbf{F}_{CE}$ shown here is equal but opposite to that shown i *b*, a consequence of Newton's third law of action–reaction. Also, $\mathbf{F}_{CE}$ and $\mathbf{F}_{EC}$ pull on the cord and keep it in tension so that it doesn't collapse. For equilibrium, $F_{CE} = F_{EC}$.

Knot. The knot at *C* is subjected to three forces. They are caused by the cords *CBA* and *CE* and the spring *CD*. As required, the free-body diagram shows all these forces labeled with their magnitudes and directions. It is important to recognize that the weight of the sphere does not directly act on the knot. Instead, the cord *CE* subjects the knot to this force.



(d)

3.3 Coplanar force systems

If a particle is subjected to a system of coplanar forces as in Fig 3-2, then each force can be resolved into its $\vec{\mathbf{i}}$ an $\vec{\mathbf{j}}$ components. **For equilibrium these forces must sum to produce a zero free resultant**.

$$\sum \vec{\mathbf{F}} = 0$$

$$\sum F_x \vec{\mathbf{i}} + \sum F_y \vec{\mathbf{j}} = 0$$

بعد تحليل القوى إلى مركباتها الأفقية و العمودية يمكن كتابة معادلة الاتزان الجسم في اتجاه كل محور.

Hence

$$\sum F_x = 0$$

مجموع القوى في اتجاه المحور **x** تساوي **0**.

$$\sum F_y = 0$$

مجموع القوى في اتجاه المحور **y** تساوي **0**.

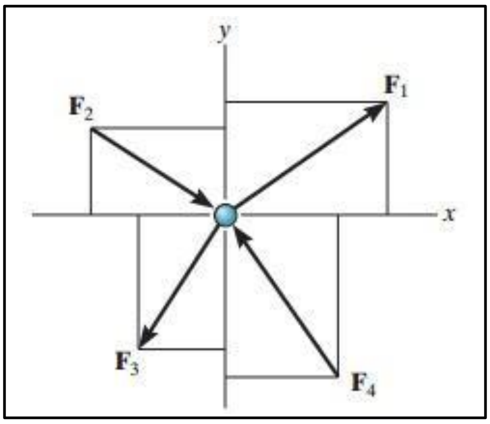


Fig 3-2

Exercise 3.1:

Determine the tension in cables BA and BC necessary to support the 60 kg cylinder in fig 3-3.

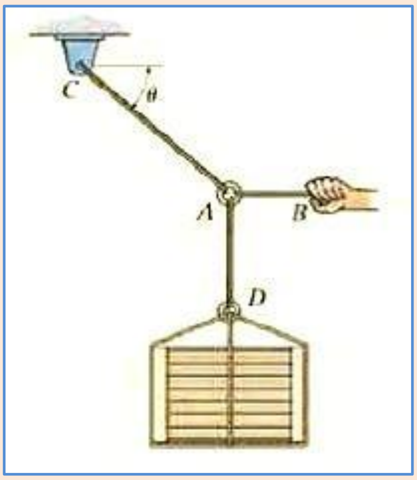


Fig 3-3

Ans: $T_C = 476 \text{ N}$ $T_A = 420 \text{ N}$

Exercise 3.2:

The 200 kg crate in fig 3.4 a is suppended using the ropes AB and AC. Each rope can withstand a maximum forces of 10 kN, before it breaks. If AB always remains horizontally, determine the smallest angle θ to which the crate can be suspended before one of the ropes breaks.

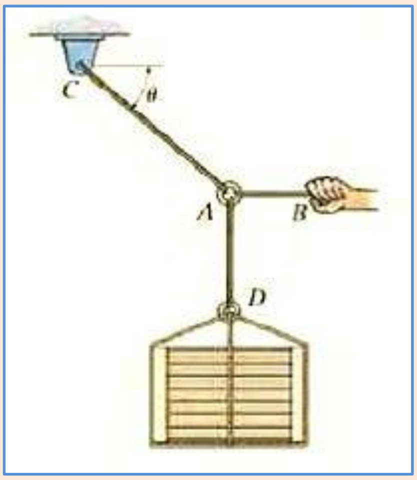


Fig 3-4

Ans: $\theta = 11.31°$ $F_B = 9.81 \text{ N}$

Exercise 3.3:

Determine the required length of AC in fig 3.5 so that the 8 kg lamp can be suspended in the position shown. The undeformed length of spring AB is l'_{AB}=0.4 m, and the spring has a stiffness of k_{AB} =300 N/m.

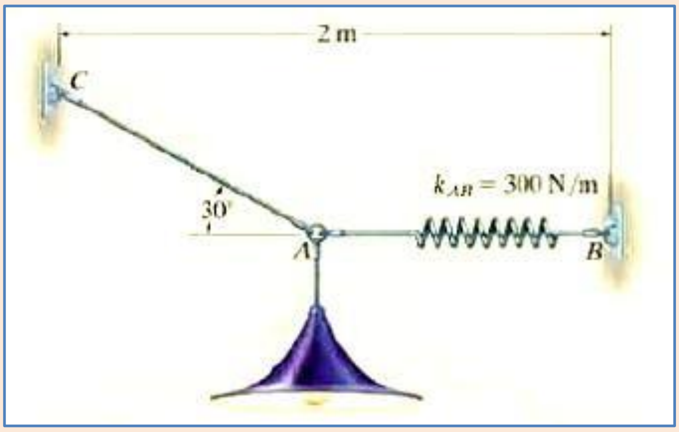


Fig 3-5

Ans: $l_{AC} = 1.32\ m$

Exercise 3.4:

The crate has a weight of 550 lb. Determine the force in each supporting cable.

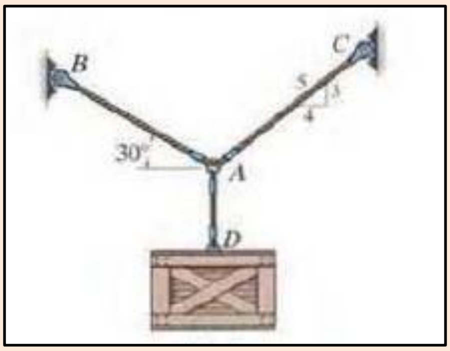


Fig 3-6

Ans: $F_{AB} = 478\ lb$ $F_{AC} = 518\ lb$

Exercise 3.5:

If the mass of cylinder C is 40 kg, determine the mass of cylinder A in order to hold the assembly in the position shown.

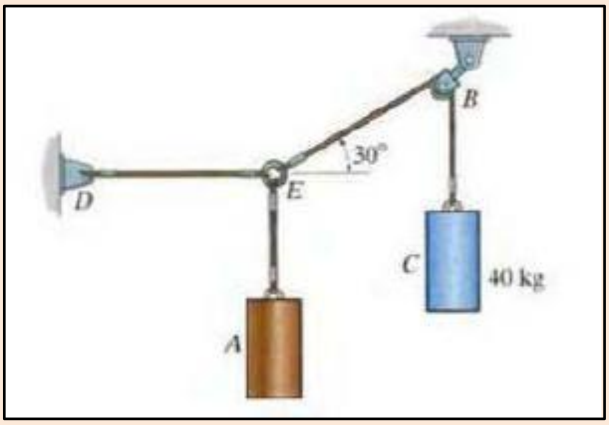


Fig 3-7

Ans: $\sum F_y = 0$ $m_a = 20kg$

Exercise 3.6:

The members of a truss are connected to the gusset plate. If the forces are concurrent at point O, determine the magnitudes of $\vec{F}$ and $\vec{T}$ for equilibrium. Take θ = 30°.

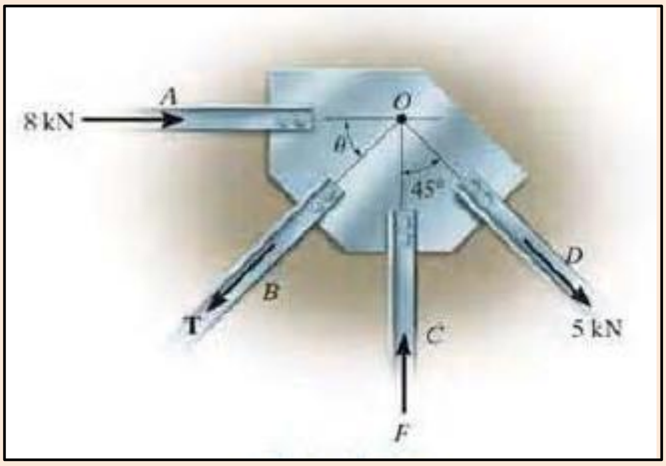


Fig 3-8

Ans: $T = 13.3$ kN $F = 10.2$ kN

Exercise 3.7:

The gusset plate is subjected to the forces of four members. Determine the force in member B and its proper orientation θ for equilibrium. The forces are concurrent at point O. Take *F* = 12 kN.

See Fig 3-8

Ans: $T = 14.3$ kN $\theta = 36.27°$

Exercise 3.8:
The 200 lb uniform tank is suspended by means of a 6 ft long cable which is attached to the sides of the tank and passes over the small pulley located at O. If the cable can be attached at either points A and B or C and D. Determine which attachment produces the least amount of Tension in the cable. What is this tension?

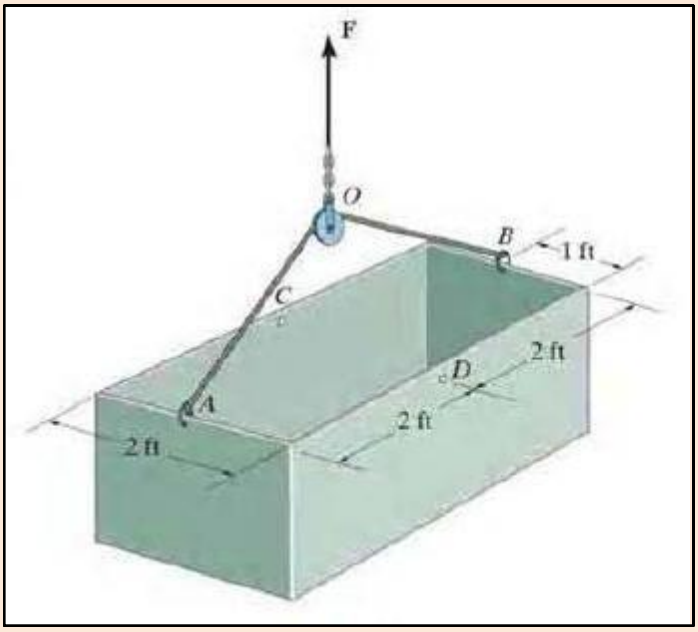


Fig 3-9

Ans: $T = 106\ lb$ *related to CD attac emen*

3.4 Three dimensional force systems

In the case of three dimensional force system, as in fig 3.10, we can resolve the forces into their respective $\vec{i}$, $\vec{j}$, $\vec{k}$ components **For equilibrium**, so that.

تذ نحيوً اقّ إىّ شٍمثاخ ثلاثخً
الأتاد نْ◌ِ◌ِ مراتح ◌ٍادىح الاذضاُ
اىجسُ ف اذجآ مو حٍسّ.

$$\sum F_x \vec{i} + \sum F_y \vec{j} + \sum F_k \vec{k} = 0$$

To satisfy this equation we require

جٍعً اىقّ ف اذجآ اىحَسّ x ذسأً
.0

جٍعً اىقّ ف اذجآ اىحَسّ y ذسأً
.0

جٍعً اىقّ ف اذجآ اىحَسّ
z ذسأً .0

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$

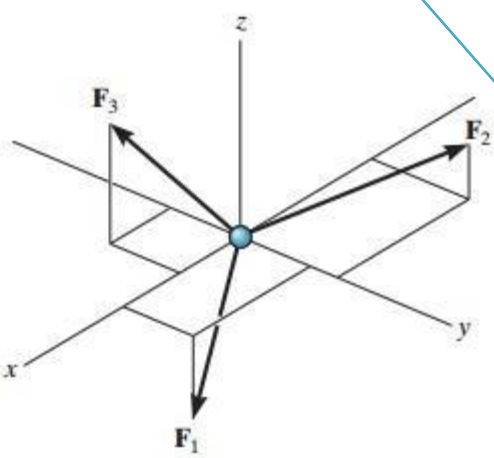
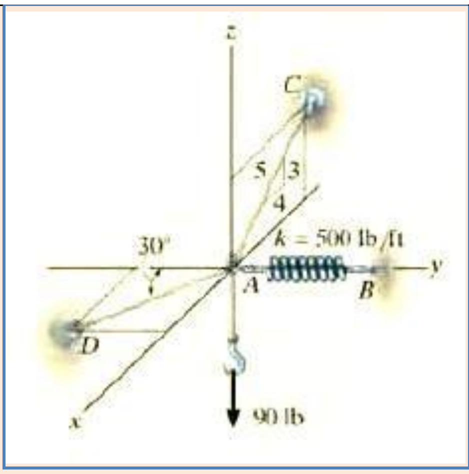


Fig 3-10

Exercise 3.9:

A 90 lb is suspended from the hook shown in fig 3-11. If the load is supported by two cables and a spring having a stiffness *k = 500 lb/ft*, determine the force in cables and the stretch of the spring for equilibrium. Cable AD lies in the *x-y* plane and cable AC lies in *x-z* plane.

Fig 3-11

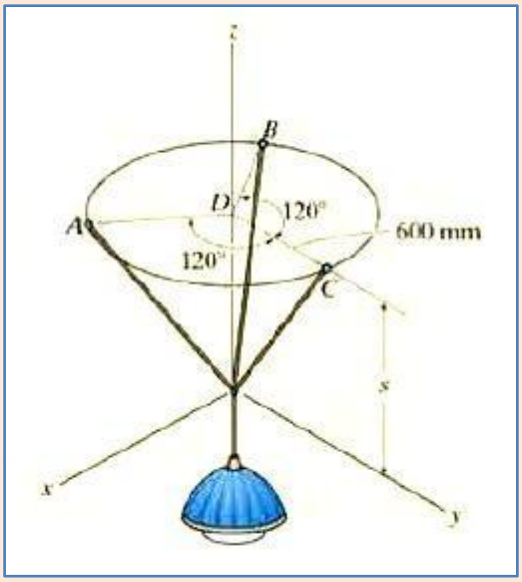


Ans: $F_C = 150\ lb$ $F_D = 240\ lb$ $F_B = 207.8\ lb$ $S_{AB} = 0.416\ ft$

Exercise 3.10:

The 10 kg lamp in fig 3.12 is suspended from the three equal length cords. Determine its smallest vertical distance *s* from the ceiling if the force developed in any cord is not allowed to exceed 50N.

Fig 3-12

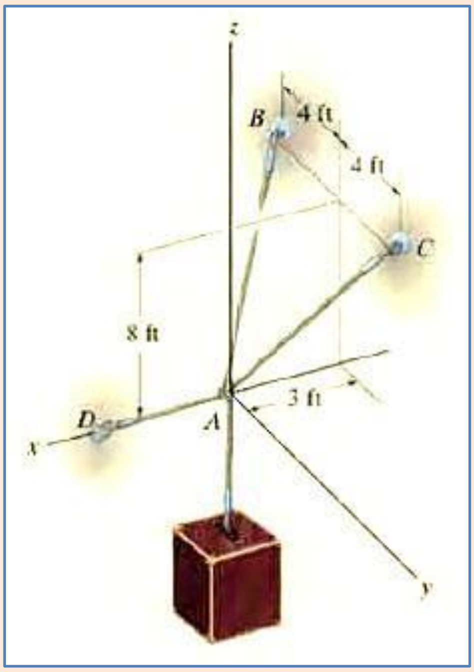


Ans: $S = 519mm$

Exercise 3.11:

Determine the force in each cable used to support the 40 lb crate shown fig 3-13.

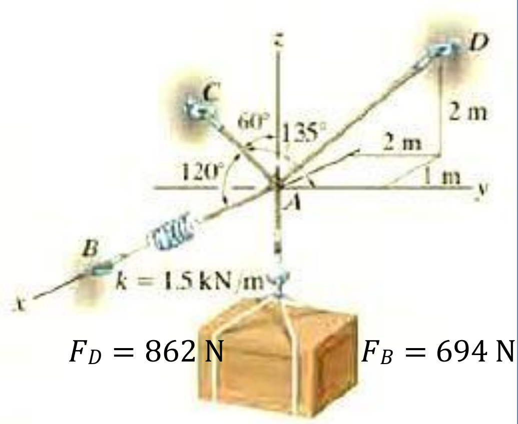
Fig 3-13



Ans: $F_B = F_C = 23.6\ lb$ $F_D = 15\ lb$

Determine the tension in each cord used to support the 100 kg crate shown fig 3-14.

Fig 3-14

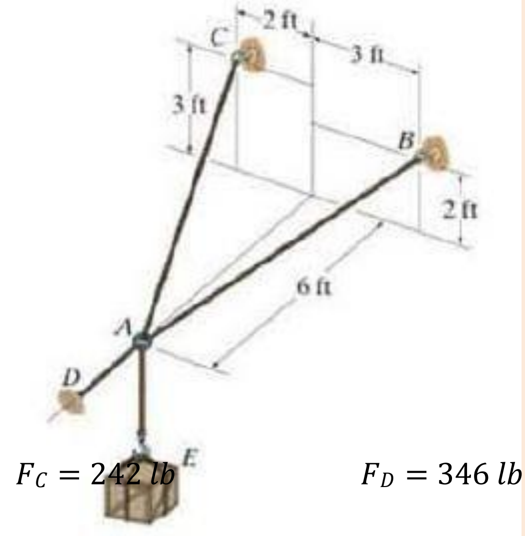


Ans: $F_C = 813$ N $F_D = 862$ N $F_B = 694$ N

Exercise 3.13:

The 150 lb crate is supported by cables AB, AC and AD. Determine the tension in these wires.

Fig 3-15

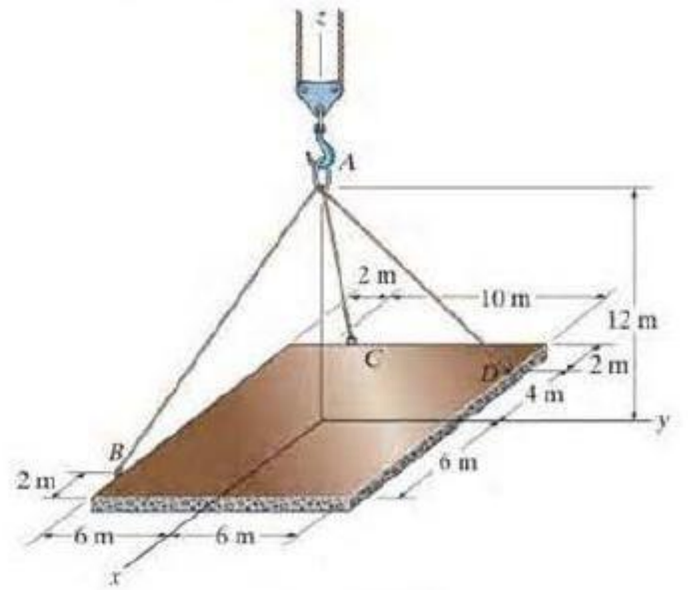


Ans: $F_B = 162\ lb$ $F_C = 242\ lb$ $F_D = 346\ lb$

Exercise 3.14:

The ends of the three cables are attached to a ring at A and to the edge of a uniform 150 kg plate. Determine the tension in each of the cables for equilibrium.

Fig 3-16



It is observed that when the bed of the dump truck is raised to an angle of $\theta = 25°$ the vending machines will begin to slide off the bed, Fig. 8-4,. Determine the static coefficient of friction between a vending machine and the surface of the truck bed.

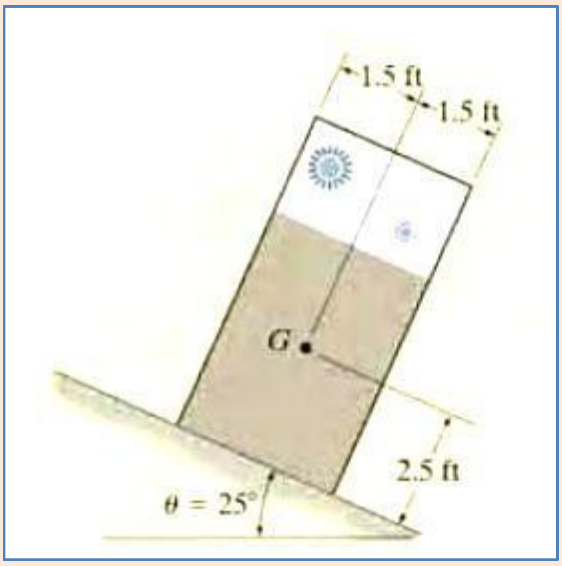


Fig. 8-4

Ans: $\mu_s = \tan 25° = 0.466$

Chapter
Four

Friction

Friction

8.1 Characteristics of Dry Friction:

Friction is a **force** that **resists** the **movement** of **two contacting surfaces** that **slide relative to one another**. This force always acts **tangent** to the surface at the points of contact and is directed so as to oppose the possible or existing motion between the surfaces.

الاحتكاك هي القوة المقاومة التي تحدث عند تحرك سطحين متلاصقين باتجاهين متعاكسين عندما يكون بينهما قوة ضاغطة تعمل على تلاحمهما معا.

In this chapter, we will study, the effects of **dry friction**, which is sometimes called **Coulomb friction** since its characteristics were studied extensively by C. A. Coulomb in 1781. **Dry friction occurs** between the contacting surfaces of bodies **when there is no lubricating fluid**.

الاحتكاك الجاف (لا يوجد تشحيم ولا تزييت) ينشأ نتيجة وجود نتوءات وفجوات بين الأسطح. تصطدم هذه النتوءات عند تطبيق قوة (سحب) على الجسم المتحرك مما ينتج قوة احتكاكية بين السطحين. يمكن تجزئة القوى على مستوى السطحين إلى قوتين، ضاغطة و عمودية و معاكسة لقوة وزن الجسم و قوة احتكاك معاكسة لقوة السحب وهي مماسة للسطحين.

Theory of Dry Friction:

The theory of dry friction can be explained by considering the effects caused by pulling horizontally on a block of uniform weight $\vec{W}$ which is resting on a rough horizontal surface that is **nonrigid or deformable** Fig. 8- 1a. The upper portion of the block, however, can be considered rigid. As shown on the free-body diagram of the block, Fig. 8-1b, the floor exerts an uneven **distribution** of both **normal force** ΔN_a and **frictional force** ΔF_a along the contacting surface. For equilibrium, the normal forces must act upward to balance the block's weight $\vec{W}$, and the frictional forces act to the left to prevent the applied force $\vec{P}$ from moving the block to the right. Close examination of the contacting surfaces between the floor and block reveals how these frictional and normal forces develop, Fig. 8- 1c. It can be seen that many microscopic irregularities exist between the two surfaces and, as a result, reactive forces $\Delta \vec{R}_n$ are developed at each point of contact. As shown, each reactive force contributes both a frictional component $\Delta \vec{F}_n$ and a normal component $\Delta \vec{N}_n$.

Equilibrium:

The effect of the **distributed** normal and frictional loadings is indicated by their **resultants** $\vec{\mathbf{N}}$ and $\vec{\mathbf{F}}$ On the free body diagram, Fig. 8- 1d. Notice that N acts distance *x* to the right of the line of action of $\vec{\mathbf{W}}$. Fig. 8-1d. This location, which coincides with the centroid or geometric center of the normal force distribution in Fig. 8-1b, is necessary in order to balance the "tipping effect" caused by $\vec{\mathbf{P}}$. For example, if $\vec{\mathbf{P}}$ is applied at a height *h* from the surface, Fig. 8-1d, then moment equilibrium about point O is satisfied if $w\,x = p$ or $x = p\ /w$

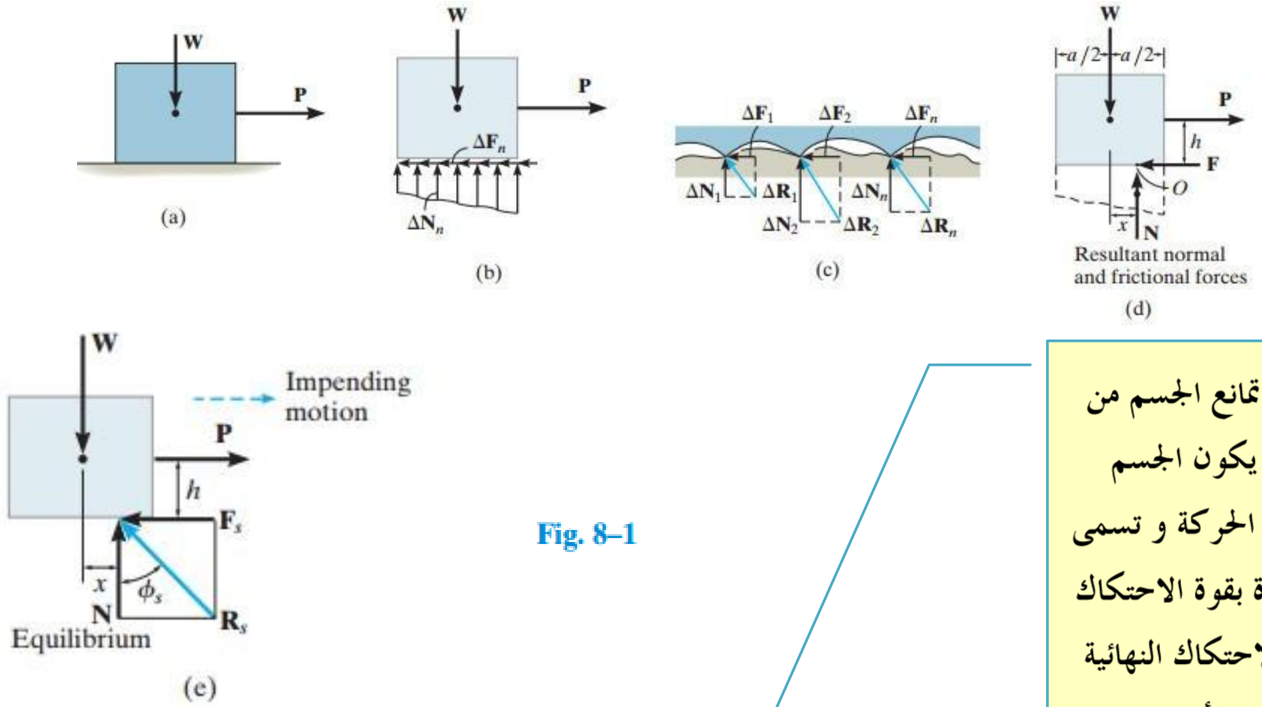


Fig. 8–1

تصل القوة التي تمانع الجسم من الحركة إلى قيمة يكون الجسم فيها على وشك الحركة و تسمى عندئذ هذه القوة بقوة الاحتكاك الحرج أو قوة الاحتكاك النهائية و تعرف في أغلب الأحيان بقوة الاحتكاك الساكن، وهي تتناسب طرديا مع القوة الضاغطة بين الجسمين.

Impending Motion:

In cases where the surfaces of contact are rather "slippery", the frictional force $\vec{\mathbf{F}}$ may **not** be great enough to balance $\vec{\mathbf{p}}$. and consequently the block will lend to slip. In other words, as *P* is slowly increased, F correspondingly increases until it attains a certain **maximum value** F_x called the limiting static frictional force, Fig. 8-1e. When this value is reached, the block is in **unstable equilibrium** since any further increase in *P* will cause the block to move. Experimentally, it has been determined that this **limiting static frictional** force *F*, is **directly proportional** to the resultant normal force *N*. Expressed mathematically.

معامل الاحتكاك μ_s هو كمية عددية تستخدم للتعبير عن النسبة بين قوة الاحتكاك بين جسمين والقوة الضاغطة بينهما، وليس له وحدة قياس، ويعتمد على ماديْ الجسمين.

$$F_s = \mu_s N$$

where the constant of proportionality, μ_s (mu "sub" s), is called the **coefficient of static friction.**

Thus, when the block is on the **verge of sliding**, the normal force $\vec{N}$ and frictional force $\vec{F_s}$ combine to create a resultant $\vec{R_s}$, Fig. 8-1e the angle $\emptyset_s$ (phi "sub" s) that $\vec{R_s}$ makes with $\vec{N}$ is called the **angle of static friction.** From the figure.

$$\emptyset_s = \tan^{-1}\left(\frac{F_s}{N}\right) = \tan^{-1}\left(\frac{\mu_s N}{N}\right) = \tan^{-1}\mu_s$$

Typical values for μ_s are given in Table 8-1. Note that these values can vary since experimental testing was done under variable conditions of roughness and cleanliness of the contacting surfaces. For applications, therefore, it is important that both caution and judgment be exercised when selecting a coefficient of friction for a given set of conditions. When a more accurate calculation of F_x , is required, the coefficient of friction should be determined directly by an experiment that involves the two materials to be used.

تتوقف قيمة معامل الاحتكاك على نوع مادة السطحين المتلامسين فهو يختلف من مادة إلى أخرى كما يتوقف على طبيعة السطحين من حيث الخشونة، النعومة، درجة الحرارة، ووجود شوائب والجدول التالي يعطيك قيم معاملات الاحتكاك لبعض المواد وهي في الواقع قيم تقريبية

Table 8-1
Typical Values for μ_s

Contact Materials	***coefficient of static Friction μ_s***
Metal on ice	0.03 - 0.05
Wood on wood	0.30 - 0.70
Leather on wood	0.20 - 0.50
Leather on metal	0.30 - 0.60
Aluminum on aluminum	1.10 - 1.7

Motion:

If the magnitude of $\vec{P}$ acting on the block is increased so that it becomes slightly greater than F_s, the frictional force at the contacting surface will drop to a smaller value F_k . called the **kinetic frictional force**. The block will begin to slide with increasing speed, Fig. 8-2a. As this Occurs, the block will "ride" on top of these peaks at the points of contact, as shown in Fig. 8-2b. The continued breakdown of the surface is the dominant mechanism creating kinetic friction. Experiments with sliding blocks indicate that the magnitude of the kinetic friction force is directly proportional to the magnitude of the resultant normal force, expressed mathematically as

$$F_k = \mu_k N$$

بمجرد أن يبدأ الجسم في التحرك تقل قوة الاحتكاك عن قيمتها النهائية، وتسمى قوة الاحتكاك هذه، والتي تؤثر في الجسم أثناء حركته بقوة الاحتكاك الحركي. وهي تتناسب طرديا مع القوة الضاغطة بين الجسمين. معامل الاحتكاك الحركي يكون عادة أقل من معامل الاحتكاك الساكن.

Here the constant of proportionality, μ_k, is called the **coefficient of kinetic friction**. Typical values for μ_k are approximately 25 percent **smaller** than those listed in Table 8-1 for μ_s. As shown in Fig. 8-2a, in this case, the resultant force at the surface of contact, $\vec{R}_k$, has a line of action defined by ϕ_k. This angle is referred to as the **angle of kinetic friction**, where

$$\phi_s = \tan^{-1}\left(\frac{F_k}{N}\right) = \tan^{-1}\left(\frac{\mu_k N}{N}\right) = \tan^{-1}\mu_k$$

By comparison, $\phi_s \geq \phi_k$.

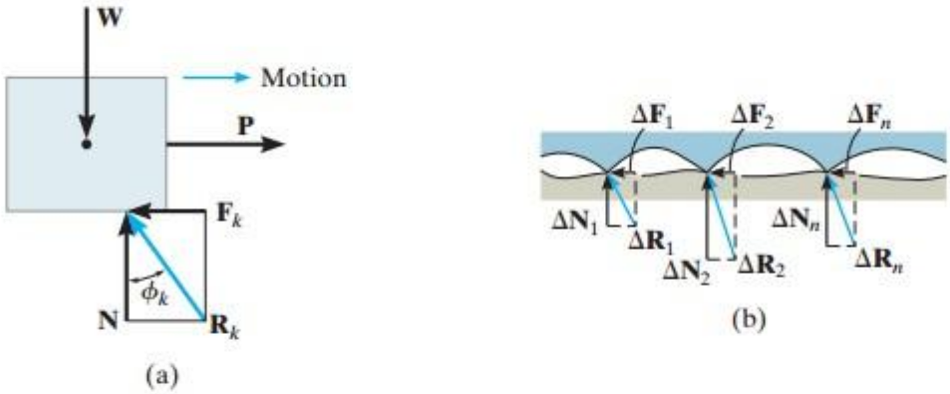


Fig. 8–2

Exercise 8.1:

The uniform crate shown in Fig. 8-3 has a mass of 20 kg. If a force P = 80 N is applied to the crate, determine if it remains in equilibrium. The coefficient of static friction is $\mu_s = 0.3$.

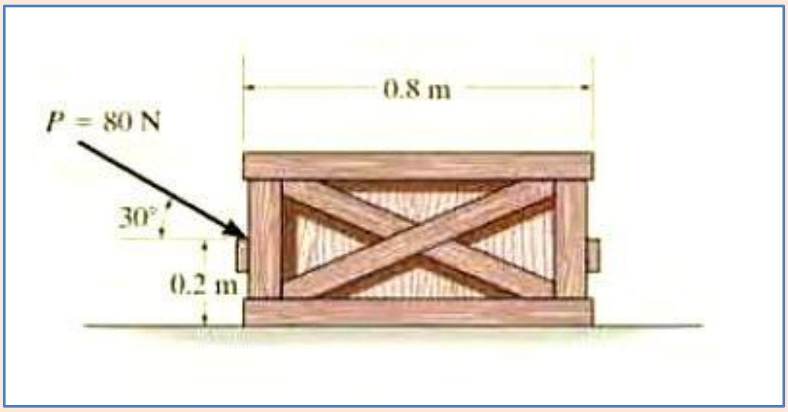
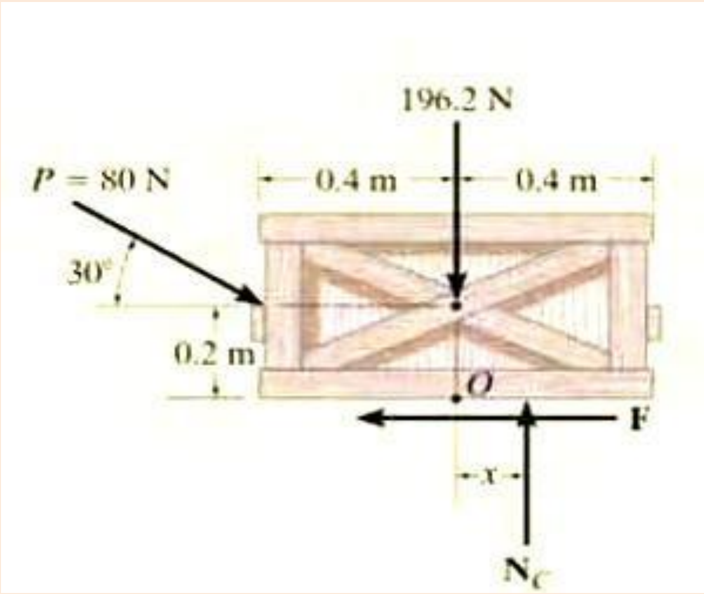


Fig. 8-3

Ans:

Free-Body Diagram:

As shown in Fig. 8-3. the resultant normal force $\vec{\overline{N}}_c$ must act a distance x from the crate's center line in order to counteract the tipping effect caused by $\vec{\overline{P}}$. There are three unknowns F, N_c, and x, which can be determined strictly from the three equations of equilibrium.



equations of equilibrium:

$\overset{+}{\rightarrow} \sum F_x = 0 \qquad 80\ \cos 30°\ N\ (0.2m) + N_c(x) = 0$

$+\uparrow \sum F_y = 0 \qquad -80\ \sin 30°\ N\ + N_c - 196.2\ N = 0$

$G + \sum M_o = 0 \qquad 80\ \sin 30°\ N(0.4m) - 80\ \cos 30° N(0.2m) + N_c(x) = 0$

$$\boldsymbol{F = 69.3N}$$

$$\boldsymbol{N_C = 236\ N}$$

$$\boldsymbol{x = -0.00908\ m = -\ 9.08\ mm}$$

Since x is negative it indicates the resultant normal force acts (slightly) to the left of the crate's center line. No tipping will occur since x < 0.4 m. Also, the maximum frictional force which can be developed at the surface of contact is $F_{max} = \mu_s Nc = 0.3(236N) = 70.8N$. Since F = 69.3 N < 70.8 N, the crate will not slip although it is very close to doing so.

Exercise 8.2:

It is observed that when the bed of the dump truck is raised to an angle of $\theta = 25°$ the vending machines will begin to slide off the bed, Fig. 8-4,. Determine the static coefficient of friction between a vending machine and the surface of the truck bed.

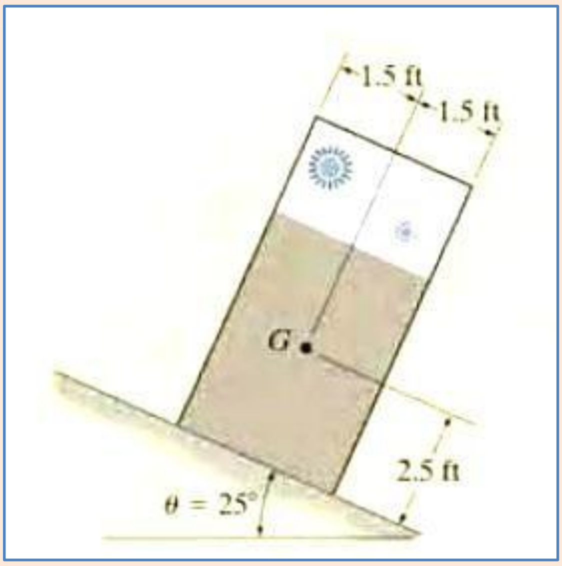


Fig. 8-4

Ans: $\mu_s = \tan 25° = 0.466$

UNIVERSTECHNICA LY

Lecture Notes and Exercises on ***STATICS***

Ashraf Emad

Chapter Four

Force System

Resultantes

العلم صيد والكتابة قيده   قيد صيودك بالحبال الواثقة
فمن الحماقة أن تصيد غزالة   وتتركها بين الخلائق طالقة

Force System Resultants

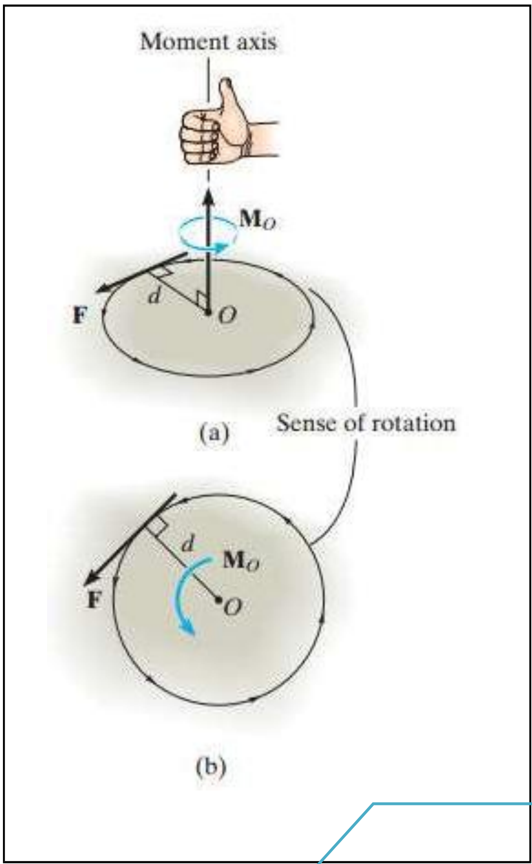
4.1 Moment of a force scalar formulation.

The moment $\overrightarrow{\mathbf{M_o}}$ **about point O**, or about an axis passing through O and perpendicular to the plane, **is a vector quantity** since it has a specified **magnitude and direction** (fig 4-1).

العزم هو الأثر الدوراني الذي تحدثه القوة في الجسم

فهو ممثل في متجه لقياس مدى قدرة قوة على تدوير جسم حول نقطة معينة أو محور ما.

Fig 4-1



تأثير الدوران يتناسب طرديا مع القوة والمسافة حول نقطة معينة ويعطى بحاصل ضرب مقدار القوة في ذراعها

The magnitude of $\mathbf{M_o}$ is

$$M_o = F.d \qquad 4$$

يقصد بالذراع ***d*** المسافة العمودية بين خط عمل القوة و النقطة المطلوب حساب العزم حولها.

Where *d* **is the moment arm** or **perpendicular distance from the axis at point O** to the line of action of the force. **Units** of moment is **N.m** or **lb.ft**.

The **direction** of $\overrightarrow{\mathbf{M_o}}$ is defined by its moment axis which is **perpendicular** to the **plane** that contains the force **F** and its moment arm *d*. The **right-hand rule** is used establish the sense of the direction of $\overrightarrow{\mathbf{M_o}}$.

For two dimensional problems, where all the forces lie within the *x-y* plane, fig 4-2, the resultant moment $\overrightarrow{(\mathbf{M_R})_o}$ **about point O** (the z axis) can be determined by finding the **algebraic sum of the moments** caused by all the forces in the system. As a **convention** we will generally consider **positive moments** as a **counterclockwise** since they are directed along the positive z axis (out of page). **Clockwise** moments will be **negative**. Using the sign convention, the resultant moment in fig 4-3 is therefore

اتفاق:

يعتبر العزم موجب إذا كان اتجاهه عكس اتجاه عقارب الساعة و سالب إذا كان اتجاهه هو نفس اتجاه عقارب الساعة

$$(M_R)_0 = \sum Fd$$

$$(M_R)_0 = F_1 d_1 - F_2 d_2 + F_3 d_3$$

إذا كان لدينا جسم تحت تأثير مجموعة من القوى، فإن قيمة العزم حول نقطة معينة تساوي الجمع الجبري لعزوم القوى منفردة

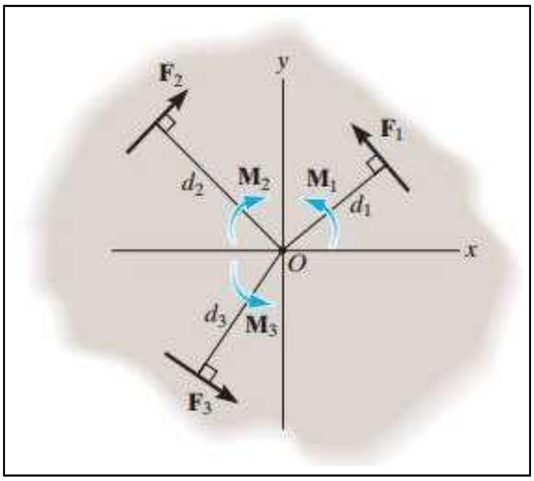
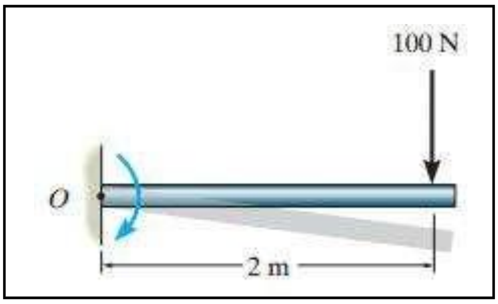


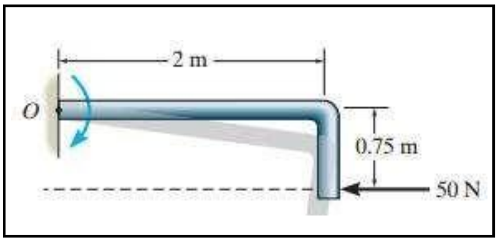
Fig 4-2

Example:

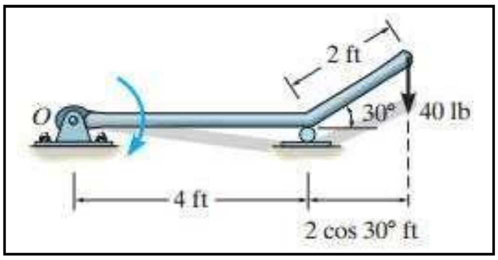
For each case illustrated below, the moments of the forces are:



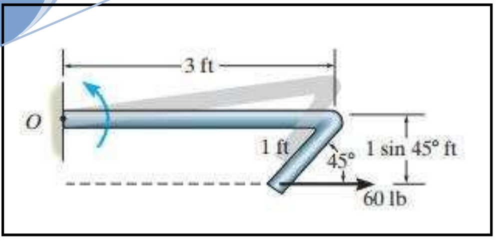
$$M_O = (100\text{ N})(2\text{ m}) = 200\text{ N}\cdot\text{m} \circlearrowright$$



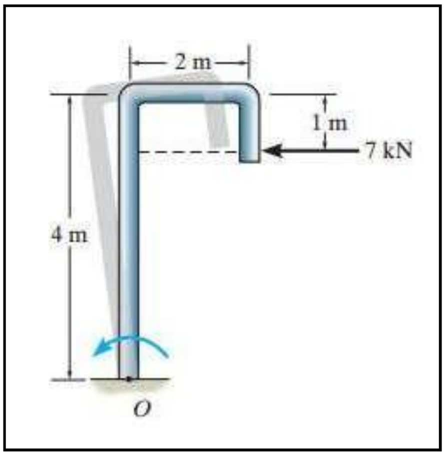
$$M_O = (50\text{ N})(0.75\text{ m}) = 37.5\text{ N}\cdot\text{m} \circlearrowright$$



$$M_O = (40\text{ lb})(4\text{ ft} + 2\cos 30^\circ\text{ ft}) = 229\text{ lb}\cdot\text{ft} \circlearrowright$$



$M_O = (60\text{ lb})(1 \sin 45°\text{ ft}) = 42.4\text{ lb}\cdot\text{ft}$ ↺



$M_O = (7\text{ kN})(4\text{ m} - 1\text{ m}) = 21.0\text{ kN}\cdot\text{m}$ ↺

Exercise 4.1:
Determine the resultant moment of the four Forces acting on the rod shown in fig 4-3 about point O.

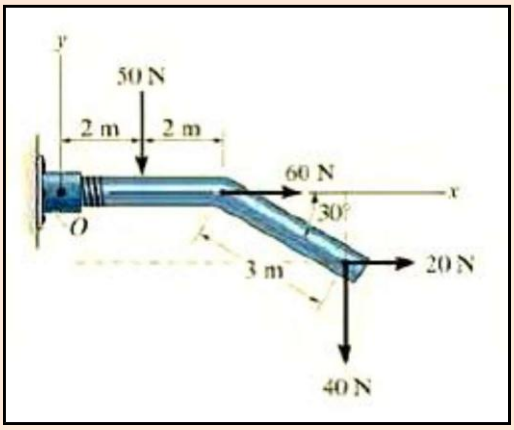


Fig 4-3

Ans: $M_{R_0} = -334\ \mathrm{N.m}$

4.2 Cross product

الضرب الإتجاهي: سمي بهذا الاسم لأن ناتج الضرب عبارة عن كمية اتجاهية (vector)، بمعنى، أن حاصل الضرب الإتجاهي لمتجهين هو متجه ثالث، اتجاهه يكون عموديا على المستوي الذي يحوي المتجهين المضروبين ببعضهما.

مقدار المتجه الثالث يعطى بعلاقة C.

يمكن معرفة اتجاه المتجه الثالث باستخدام قاعدة اليد اليمنى.

The cross product of two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ yields the vector C which is written

$$\vec{\mathbf{C}} = \vec{\mathbf{A}} \times \vec{\mathbf{B}} \qquad 4.2$$

And read $\vec{\mathbf{C}}$ equals $\vec{\mathbf{A}}$ cross $\vec{\mathbf{B}}$.

The magnitude of $\vec{\mathbf{C}}$ is defined as the product of the magnitudes $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ and the sine of the angle θ between their tails $(0° \le \theta \le 180°)$, thus

$$C = A\,B \sin\,\theta$$

$\vec{\mathbf{C}}$ has a direction that is perpendicular to the plane containing $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ such that $\vec{\mathbf{C}}$ is specified by the right-hand rule.

Knowing both the magnitude and direction of $\vec{\mathbf{C}}$, we can write

$$\vec{\mathbf{C}} = \vec{\mathbf{A}} \times \vec{\mathbf{B}} = (A\,B\,\sin\theta)\vec{\mathbf{u_c}} \qquad 4.3$$

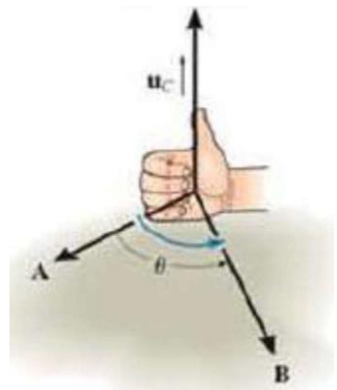


Fig 4-4

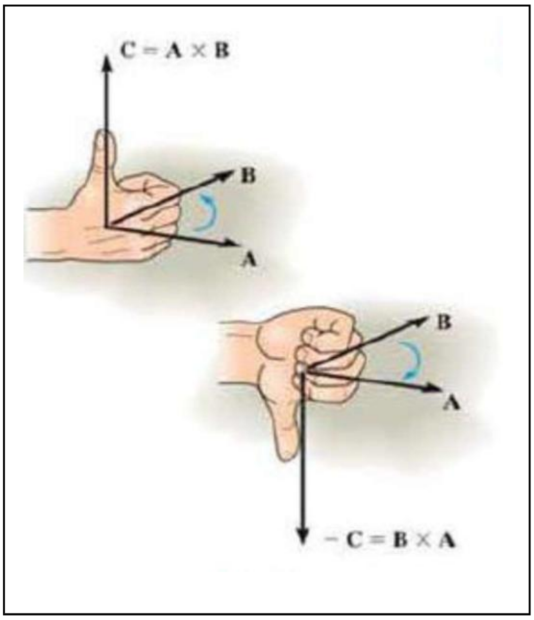
Where the scalar *(A B* sin θ) defines the magnitude of $\vec{\mathbf{C}}$ and the unit vector $\vec{\mathbf{u_c}}$ defines the direction of $\vec{\mathbf{C}}$ (fig 4-4).

Laws of operation:

$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} \neq \vec{\mathbf{B}} \times \vec{\mathbf{A}}$$

$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$ **(commutative law is not valid)**

Fig 4-5



$a(\vec{A} \times \vec{B}) = (a\vec{A}) \times \vec{B} = \vec{A} \times (a\vec{B}) = (\vec{A} \times \vec{B})a$ **(associative law)**

عملية الضرب الإتجاهي تجميعية

$\vec{A} \times (\vec{B} + \vec{D}) = (\vec{A} \times \vec{B}) + (\vec{A} \times \vec{D})$ **(distributive law)**

عملية الضرب الإتجاهي توزيعية

Cartesian vector formulation:

Equation 4.3 may be used to find the cross product of any pair of Cartesian unit vectors. For example, to find $\mathbf{i} \times \mathbf{j}$, the magnitude of the resultant vector is

$$(i)(j)\sin 90° = (1)(1)(1) = 1$$
$$(i)(i)\sin 0° = 0$$

and its direction is determined using the right-hand rule (fig 4-6), the resultant vector points in the $+\vec{k}$ direction. Thus $\vec{i} \times \vec{j} = (1)\vec{k}$.
In similar maner.

$\vec{i} \times \vec{j} = \vec{k}.$ $\quad \vec{i} \times \vec{k} = -\vec{j}$ $\quad \vec{i} \times \vec{i} = 0$
$\vec{j} \times \vec{k} = \vec{i}$ $\quad \vec{j} \times \vec{i} = -\vec{k}$ $\quad \vec{j} \times \vec{j} = 0$
$\vec{k} \times \vec{i} = \vec{j}$ $\quad \vec{k} \times \vec{j} = -\vec{i}$ $\quad \vec{k} \times \vec{k} = 0$

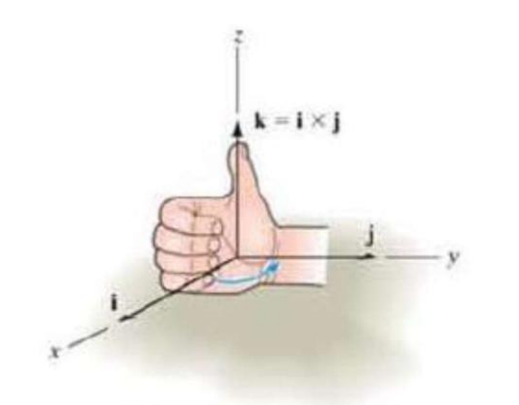


Fig 4-6

للتبسيط نستخدم الدائرة التالية من أجل إيجاد نتائج الضرب الإتجاهي لمتجهات الوحدة.

A simple scheme shown in fig 4-7 is helpful
for obtaining the same results when the need arises.

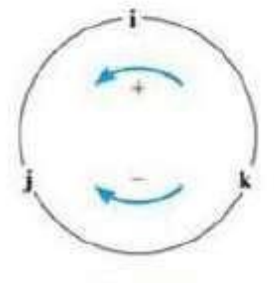


Fig 4-7

Let us now consider the cross product of two general vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$.

$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = \left(A_x\vec{\mathbf{i}} + A_y\vec{\mathbf{j}} + A_z\vec{\mathbf{k}}\right) \times \left(B_x\vec{\mathbf{i}} + B_y\vec{\mathbf{j}} + B_z\vec{\mathbf{k}}\right)$$

$$\begin{aligned}\vec{\mathbf{A}} \times \vec{\mathbf{B}} = &(A_xB_x(\vec{\mathbf{i}} \times \vec{\mathbf{i}}) + A_xB_y(\vec{\mathbf{i}} \times \vec{\mathbf{j}}) + A_xB_z(\vec{\mathbf{i}} \times \vec{\mathbf{k}}) \\ &+A_yB_x(\vec{\mathbf{j}} \times \vec{\mathbf{i}}) + A_yB_y(\vec{\mathbf{j}} \times \vec{\mathbf{j}}) + A_yB_z(\vec{\mathbf{j}} \times \vec{\mathbf{k}}) \\ &+A_zB_x\left(\vec{\mathbf{k}} \times \vec{\mathbf{i}}\right) + A_zB_y\left(\vec{\mathbf{k}} \times \vec{\mathbf{j}}\right) + A_zB_z(\vec{\mathbf{k}} \times \vec{\mathbf{k}})\end{aligned}$$

$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = \left(A_yB_z - A_zB_y\right)\vec{\mathbf{i}} - (A_xB_z - A_zB_x)\vec{\mathbf{j}} + \left(A_xB_y - A_yB_x\right)\vec{\mathbf{k}}$$

This equation may also be written in a more compact determinant form as

$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

الكتابة الديكارتية للضرب الإتجاهي تعطى بالمحددات كما هو مبين في العلاقة التالية.

4.3 Moment of a force – vector formulation

The moment of a force F about a point O (fig 4-8) can be expressed using the vector cross product namely

$$\vec{\mathbf{M}_0} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} \qquad 4.4$$

Here $\vec{\mathbf{r}}$ represents a position vector direct from O to any point on the line of action of $\vec{\mathbf{F}}$

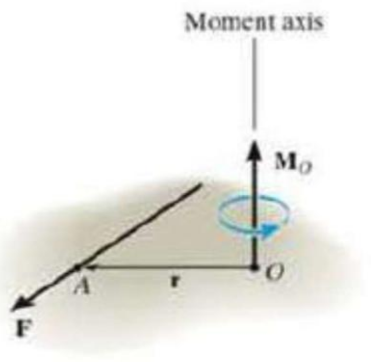


Fig 4-8

الصيغة الاتجاهية للضرب الإتجاهي تعطى بالعلاقة التالية

The magnitude of the cross product is defined from Eq. 4-3 as

$$M_0 = r\ F\ sin\theta$$

were θ is measured between the tails of $\vec{\mathbf{r}}$ and $\vec{\mathbf{F}}$.

The direction and sense of $\overrightarrow{\mathbf{M_O}}$ in Eq. 4-4 are determined by the right-hand rule as it applies to the cross product (fig 4-9).

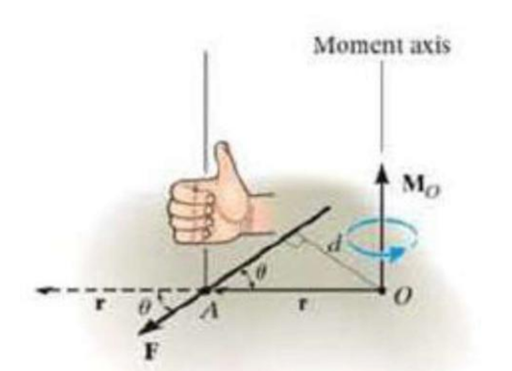


Fig 4-9

Cartesian vector formulation:

If we establish *x, y, z* coordinate axes, then the position vector $\vec{\mathbf{r}}$ and force $\vec{\mathbf{F}}$ can be expressed as Cartesian vectors (fig 4-10)

$$\overrightarrow{\mathbf{M_0}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Where r_x, r_y, r_z represent the *x, y, z* components of the position vector drawn from point O to any point on the line of action of the force.

F_x, F_y, F_z represent the *x, y, z* of the force vector.

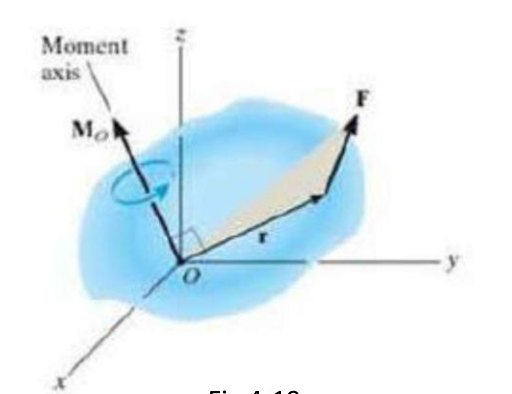


Fig 4-10

Resultant Moment of a system of forces:

If a body is acted upon by a system of forces (fig 4-11), the resultant moment of the forces about point O can be determined by vector addition of the moment of each force. This resultant can be written symbolically as

$$\overrightarrow{\mathbf{M_{R_o}}} = \sum (\vec{\mathbf{r}} \times \vec{\mathbf{F}})$$

إذا كان لدينا جسم تحت تأثير مجموعة من القوى، فإن قيمة العزم حول نقطة معينة تساوي جمع عزوم القوى المنفردة.

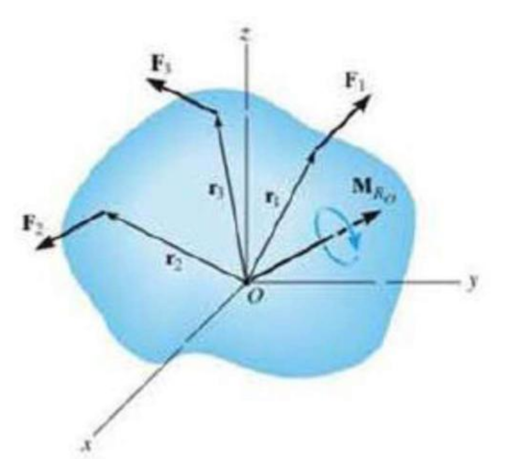


Fig 4-11

Exercise 4.2:
Express the moment produces by the force $\vec{\mathbf{F}}$ in Fig 4-12 about point O, as a Cartesian vector.

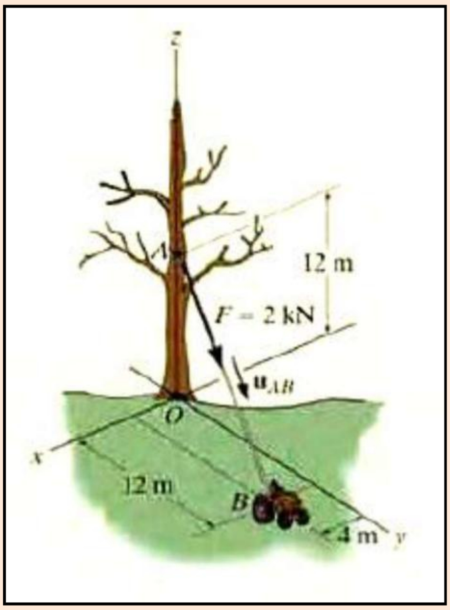


Fig 4-12

Ans: $\overrightarrow{\mathbf{M}_{\mathbf{R_0}}} = \sum(\vec{\mathbf{r}} \times \vec{\mathbf{F}}) = \{-16.5\mathbf{i} - 5.51\mathbf{j}\}\ \text{kN.m}$

Exercise 4.3:
Two forces act on the rod shown in fig 4-13. Determine the resultant moment they create about the flange at O. Express the result as a Cartesian vector.

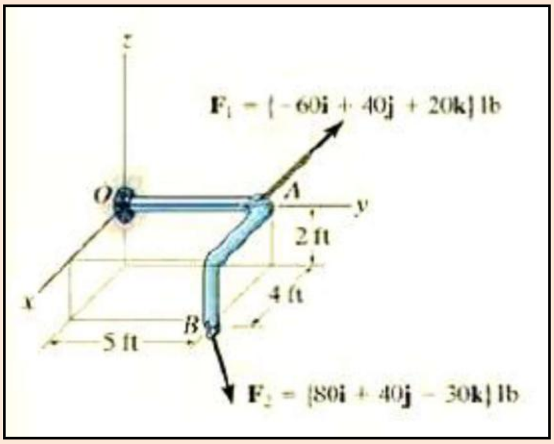


Fig 4-13

Ans: $\overrightarrow{\mathbf{M}_{\mathbf{R_0}}} = \sum(\vec{\mathbf{r}} \times \vec{\mathbf{F}}) = \{30\vec{\mathbf{i}} - 40\vec{\mathbf{j}} + 60\vec{\mathbf{k}}\}\ \text{lb.ft}$

4.4 Principle of moments

تنص نظرية فارنيون أن عزم قوة حول نقطة يساوي مجموع عزوم مركبات هذه القوة حول نفس النقطة.

The principle of moments is referred to the French mathematician Varignon (1654-1722). It states that the moment of a force about a point is equal to the sum of the moments of the components of the force about the point. If we consider the case of fig 4-14, we have.

$$\overrightarrow{\mathbf{M}_\mathbf{o}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} = \vec{\mathbf{r}} \times \left(\overrightarrow{\mathbf{F}_1} + \overrightarrow{\mathbf{F}_2}\right) = \vec{\mathbf{r}} \times \overrightarrow{\mathbf{F}_1} + \vec{\mathbf{r}} \times \overrightarrow{\mathbf{F}_2}$$

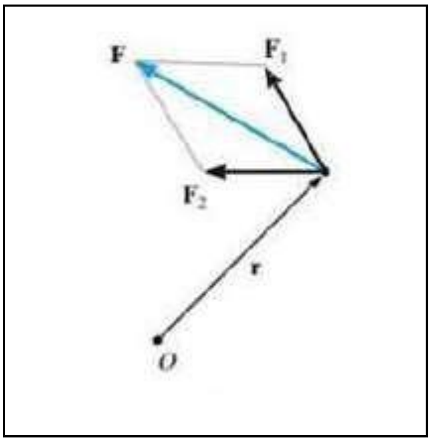
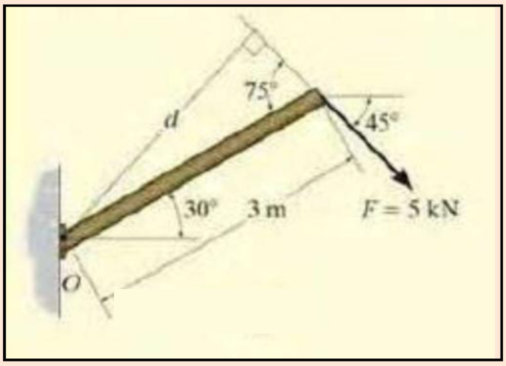


Fig 4-14

Exercise 4.4:

Determine the moment of the force in fig 4-15 about the point O.

Fig 4-15

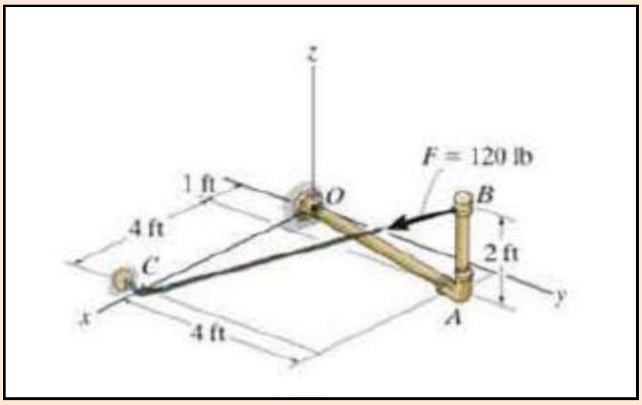


Ans: $M_{R_0} = -14.5 \text{ kN.m}$

Exercise 4.5:

Determine the moment of the force in fig 4-16 about point O. Express the result as a Cartesian vector.

Fig 4-16

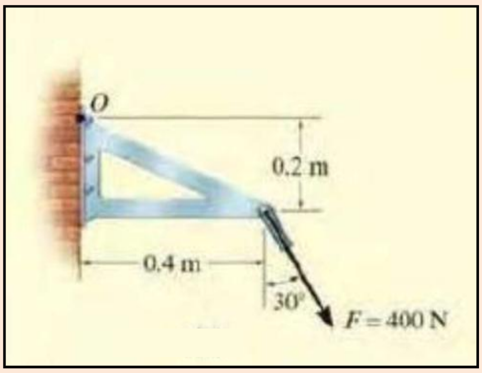


Ans: $\overrightarrow{\mathbf{M_0}} = \{200\vec{\mathbf{j}} - 400\vec{\mathbf{k}}\}\, lb.ft$

Exercise 4.6:

Force $\vec{\mathbf{F}}$ acts at the end of the angle bracket shown in fig 4-17. Determine the moment of the force about point O.

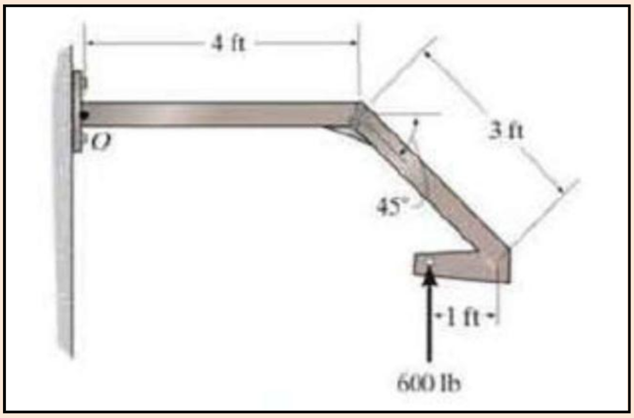
Fig 4-17



Ans: $\overrightarrow{\mathbf{M_0}} = \{-98.6\vec{\mathbf{k}}\} \text{ N.m}$

Exercise 4.7:
Determine the moment of the force about point O.

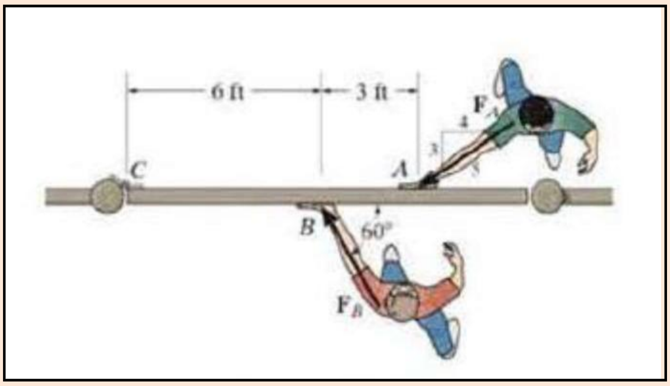
Fig 4-18



Ans: $M_0 = 36.7 \text{ N.m}$

Exercise 4.8:
The two boys push the gate with forces of $F_A = 30$ lb and $F_B = 50$ lb as shown. Determine the moment of each force about C. Which way willthe gate rotate clockwise or counterclockwise
Neglect the thickness of the gate.

Fig 4-19



Ans: $(M_{F_A})_C = -162\ lb.ft$ $\quad (M_{F_B})_C = 260\ lb.ft$

$since\ (M_{F_B})_C > (M_{F_A})_C\ the\ gate\ will\ rotate\ counterclockwise$

Exercise 4.9:
Two boys push on the gate as shown. If the boy at B exerts a force of $F_B = 30$ lb , determine the magnitude of the force F_A the boy at ***A*** must exert in order to prevent the gate from turning. Neglect the thickness of the gate.

See Fig 4-19

Ans: $F_A = 28.9\ lb$

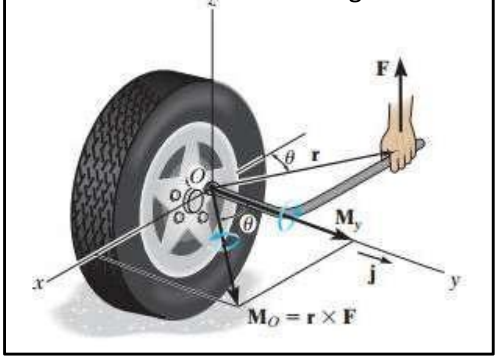
4.5 Moment of a Force about a specified axis

Scalar analysis

In general, for any axis (fig 4-20) the moment is

$$M_a = F\ d_a$$

Fig 4 20



Vector Analysis

If the vectors are written in Cartesian form, we have

$$M_a = \overrightarrow{\mathbf{u_a}}.(\vec{\mathbf{r}} \times \vec{\mathbf{F}}) = \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Where u_{a_x}, u_{a_y}, u_{a_z} represent the *x, y, z* components of unit vector defining the direction of the *a* axis.

r_x, r_y, r_z represent the *x, y, z* components of the position vector extended from any point O on the *a* axis to any point A on the line of action of the force.

F_x, F_y, F_z represent the *x, y, z* components of the force vector.

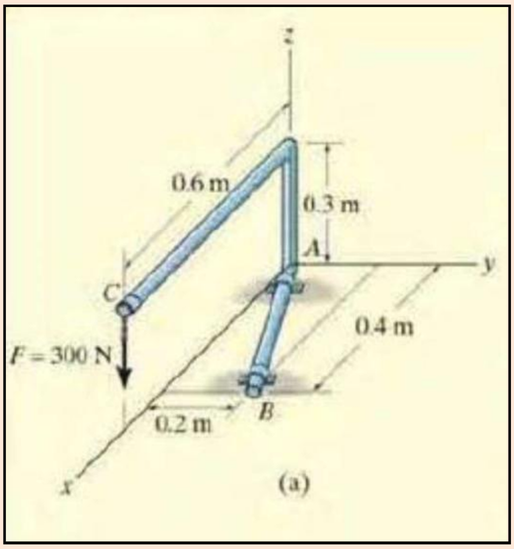
Once M_a is determined, we can then express $\overrightarrow{\mathbf{M_a}}$ as a Cartesian vector namely.

$$\overrightarrow{\mathbf{M_a}} = M_a \overrightarrow{\mathbf{u_a}}$$

Exercise 4.10:

Determine the moment M_{AB} produced by the force $\vec{\mathbf{F}}$ in fig 4-21, which tends to rotate the rod about the AB axis.

Fig 4-21

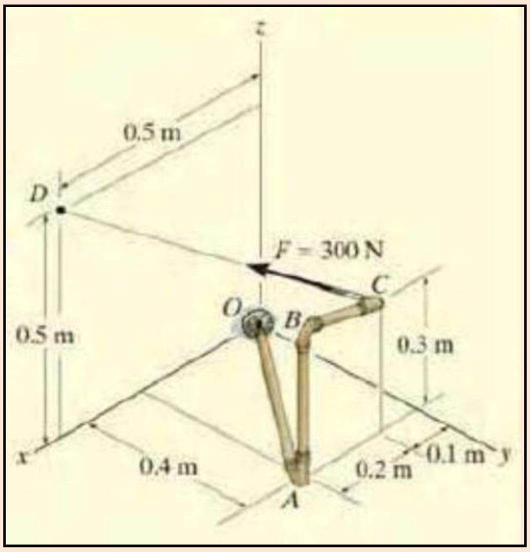


Ans: $\overrightarrow{\mathbf{M_{AB}}} = \{72\vec{\mathbf{i}} + 36\vec{\mathbf{j}}\}$ N.m

Exercise 4.11:

Determine the magnitude of the moment of force $\vec{\mathbf{F}}$ about segment OA of force the pipe assembly in fig 4-24a

Fig 4-22



Ans: $M_{OA} = 100$ N.m

Exercise 4.12:

Determine the magnitude of the moment of the force $\vec{F} = \{300\vec{i} - 200\vec{j} + 150\vec{k}\}$N about the *x* axis. Express the result as a Cartesian vector.

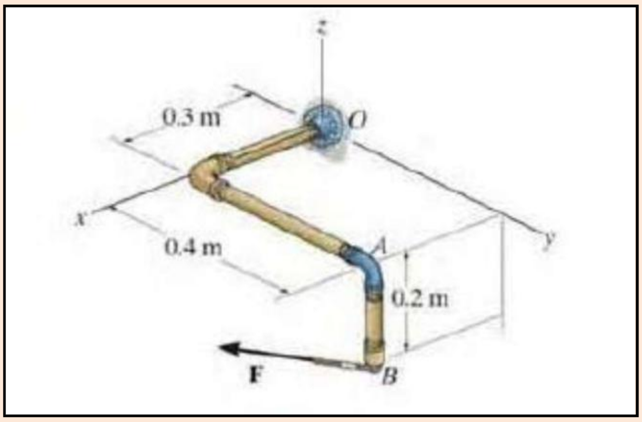


Fig 4-23

Ans: $M_x = 20$ N.m

Exercise 4.13

Determine the magnitude of the moment of the force $\vec{F} = \{300\vec{i} - 200\vec{j} + 150\vec{k}\}$N about the OA axis. Express the result as a Cartesian vector

See Fig 4-23

Ans: $M_{OA} = -72$ N.m

4.6 Moment of a couple

a couple is defined as a two parallel forces that have the same magnitude, but opposite directions, and are separated by a perpendicular distance *d* (fig 4-24). The moment produced by a couple is called a couple moment.

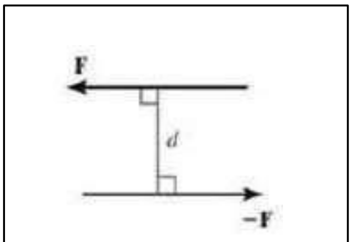


Fig 4-24

Scalar Formulation

The moment of a couple $\vec{M}$ (fig 4-25), is defined as having a magnitude of

$$M = F\,d$$

Where *F* is the magnitude of one of the forces and *d* is the perpendicular distance or moment arm between the forces. The direction and sense of the couple moment are determined by the right hand rule. $\vec{M}$ will act perpendicular to the plane containing these forces.

يعرف عزم الازدواج بأنه تأثير قوتين متساويتين في المقدار و متوازيتين و متضادتين في الاتجاه يأثران في نقطة معينة.

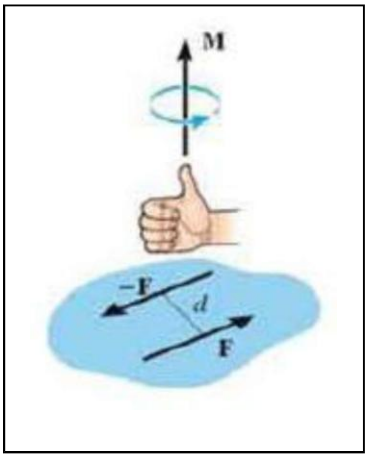


Fig 4-25

Vector Formulation

The moment of a couple can also be expressed by the vector Cross product as

$$\vec{\mathbf{M}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$$

Resultant couple moment

Since couple moments are vectors, their resultant can be determined by vector addition.

If more than two couple moments act on the body, we may generalize this concept and write the vector resultant as

$$\vec{\mathbf{M}_{\mathbf{R}}} = \sum (\vec{\mathbf{r}} \times \vec{\mathbf{F}})$$

Exercise 4.14:

Determine the resultant couple moment of the three couples acting on the plate in fig 4-26.

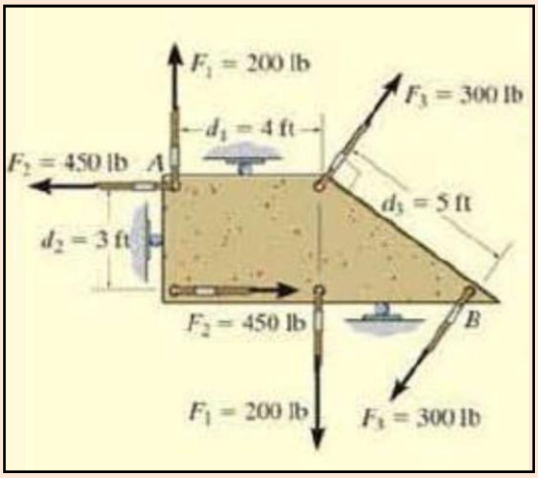


Fig 4-26

Ans: $M_R = -950\ \mathrm{lb.ft}$

Exercise 4.15:

Determine the magnitude and direction of the couple moment acting on the gear in fig 4-27.

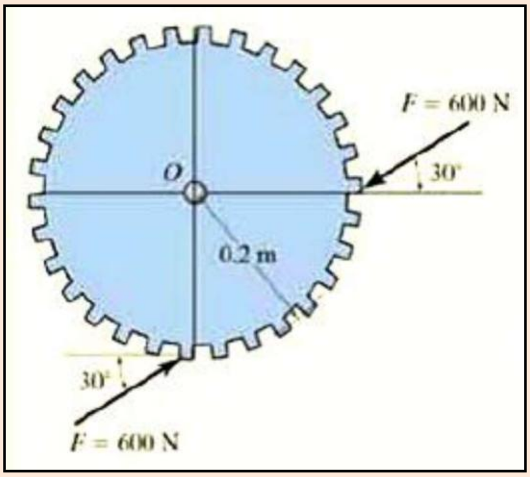


Fig 4-27

Ans: $M = 43.9\ \mathrm{N.m}$

Exercise 4.16:

Determine the couple moment acting on the pipe shown in fig 4-28 Segment AB is directed 30° below the *x-y* plane. Take OA=8 in and AB=6 in.

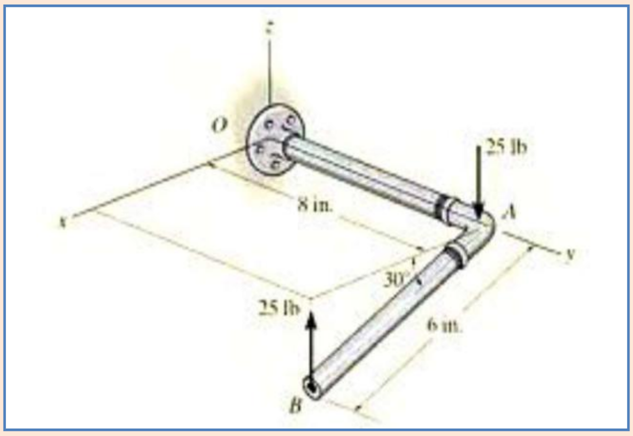


Fig 4-28

Ans: $M = -130 \text{ lb. in}$

Exercise 4.17:

Replace the two couples acting on the pipe Column in fig 4-29 by a resultant couple moment.

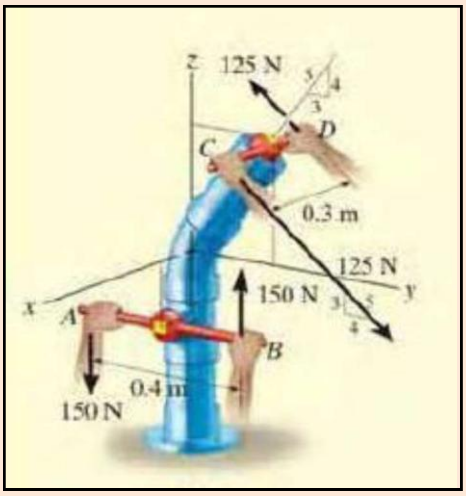


Fig 4-29

Ans: $\overrightarrow{M_R} = \{60\,\vec{i} + 22.5\,\vec{j} + 30\,\vec{k}\} \text{ N.m}$

Exercise 4.18:

Determine the couple moment acting on the pipe assembly and express the result as a Cartesian vector.

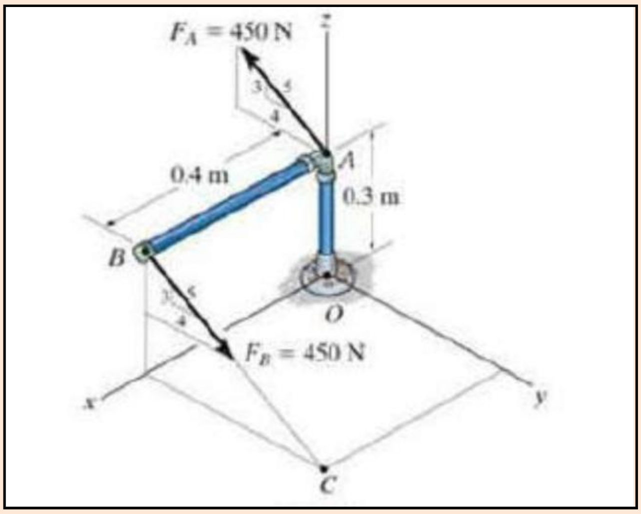


Fig 4-30

Ans: $\overrightarrow{\mathbf{M_C}} = \overrightarrow{\mathbf{r_{AB}}} \times \overrightarrow{\mathbf{F_B}} = \{108\,\vec{\mathbf{j}} + 144\,\vec{\mathbf{k}}\} \text{ N.m}$

Exercise 4.19:

Two couples act on the beam as shown.
Determine the magnitude of $\vec{F}$ so that the resultant couple moment is 300 lb.ft couterclokwise. Where on the beam does the resultant couple act?

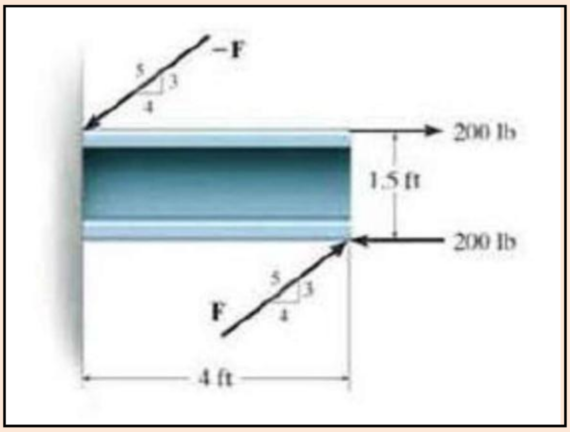


Fig 4-31

Ans: $F = 167$ lb