Chapter One

Physical properties of Fluids

Definition: Fluid mechanics is the study of fluids either in motion (fluid *dynamics*) or at rest (fluid *statics*) and the subsequent effects of the fluid upon the boundaries, which may be either solid surfaces or interfaces with other fluids. Both gases and liquids are classified as fluids.

Some Characteristics of Fluids:

One of the first questions we need to explore is, What is a fluid? Or we might ask, What is the difference between a solid and a fluid?
A solid is "hard" and not easily deformed, whereas a fluid is "soft" and is easily deformed. Although quite descriptive, these casual observations of the differences between solids and fluids are not very satisfactory from a scientific or engineering point of view.

Although the differences between solids and fluids can be explained qualitatively on the basis of molecular structure, a more specific distinction is based on how they deform under the action of an external load. Specifically, *a fluid is defined as a substance that deforms continuously when acted on by a shearing stress of any magnitude*. A shearing stress (force per unit area) is created whenever a tangential force acts on a surface. When common solids such as steel or other metals are acted on by a shearing stress, they will initially deform (usually a very small deformation), but they will not continuously deform (flow). However, common fluids such as water, oil, and air satisfy the definition of a fluid—that is, they will flow when acted on by a shearing stress.

Dimensions, Dimensional Homogeneity, and Units:

A standard unit for length might be a (meter or foot), for time might be (hour or second), and for mass a (slug or kilogram). Such standards are called *units*, and several systems of units are in common use as described in the following section. The qualitative description is conveniently given in terms of certain *primary quantities*, such as length, (L), time, (T), mass, (M), and temperature, (θ). These primary quantities can then be used to provide a qualitative description of any other *secondary quantity*: for example, Area=L^2, Velocity=LT^{-1}, Density=ML^{-3} and so on, where the symbol is used to indicate the *dimensions* of the secondary quantity in terms of the primary quantities. Thus, to describe qualitatively a velocity, V, we would write $V = LT^{-1}$ and say that "the dimensions of a velocity equal length divided by time." The primary quantities are also referred to as *basic dimensions*.

For a wide variety of problems involving fluid mechanics, only the three basic dimensions, (L, T, and M) are required. Alternatively, (L, T, and F) could be used, where F is the basic dimensions of force. Since Newton's law states that force is equal to mass times acceleration, it follows that $F=MLT^{-2}$

Table (1.1)

Quantity	FLT system	MLT system
Acceleration	LT^{-2}	LT^{-2}
Angular acceleration	T^{-2}	T^{-2}
Angular velocity	T^{-1}	T^{-1}
Area	L^2	L^2
Density	$FL^{-4}T^2$	ML^{-3}
Energy	FL	ML^2T^{-2}
Force	F	MLT^{-2}
Heat	FL	ML^2T^{-2}
Length	L	L
Mass	$FL^{-1}T^2$	M
Modulus of elasticity	FL^{-2}	$ML^{-1}T^{-2}$
Moment of force	FL	ML^2T^{-2}
Moment of inertia	L^4	L^4
Momentum	FT	MLT^{-1}
Power	FLT^{-1}	ML^2T^{-3}
Pressure	FL^{-2}	$ML^{-1}T^{-2}$
Specific weight	FL^{-3}	$ML^{-2}T^{-2}$
Strain	1	1
Stress	FL^{-2}	$ML^{-1}T^{-2}$
Surface tension	FL^{-1}	MT^{-2}
Temperature	θ	θ
Torque	FL	ML^2T^{-2}
Velocity	LT^{-1}	LT^{-1}
Dynamic viscosity	$FL^{-2}T$	$ML^{-1}T^{-2}$
Kinematic viscosity	L^2T^{-1}	L^2T^{-1}
Work	FL	ML^2T^{-2}

Ex:

1.1 Verify the dimensions, in both the *FLT* and *MLT* systems, of the following quantities which appear in Table 1.1: **(a)** volume, **(b)** acceleration, **(c)** mass, **(d)** moment of inertia (area), and **(e)** work.

(a) volume $\doteq \underline{\underline{L^3}}$

(b) acceleration = time rate of change of velocity

$$\doteq \frac{LT^{-1}}{T} \doteq \underline{\underline{LT^{-2}}}$$

(c) mass $\doteq \underline{\underline{M}}$

or with $F \doteq MLT^{-2}$

mass $\doteq \underline{\underline{FL^{-1}T^{2}}}$

(d) moment of inertia (area) = second moment of area

$$\doteq (L^2)(L^2) \doteq \underline{\underline{L^4}}$$

(e) work = force × distance

$\doteq \underline{\underline{FL}}$

or with $F \doteq MLT^{-2}$

work $\doteq \underline{\underline{ML^2T^{-2}}}$

Ex

1.3 Verify the dimensions, in both the *FLT* system and the *MLT* system, of the following quantities which appear in Table 1.1: **(a)** acceleration, **(b)** stress, **(c)** moment of a force, **(d)** volume, and **(e)** work.

(a) acceleration $= \dfrac{\text{velocity}}{\text{time}} \doteq \dfrac{L}{T^2} \doteq \underline{\underline{LT^{-2}}}$

(b) stress $= \dfrac{\text{force}}{\text{area}} \doteq \dfrac{F}{L^2} \doteq \underline{\underline{FL^{-2}}}$

Since $F \doteq MLT^{-2}$,

stress $\doteq \dfrac{MLT^{-2}}{L^2} = \underline{\underline{ML^{-1}T^{-2}}}$

(c) moment of a force = force x distance $\doteq \underline{\underline{FL}}$

$\doteq (MLT^{-2})L \doteq \underline{\underline{ML^2T^{-2}}}$

(d) volume $= (\text{length})^3 \doteq \underline{\underline{L^3}}$

(e) work = force x distance $\doteq \underline{\underline{FL}}$

$\doteq (MLT^{-2})L \doteq \underline{\underline{ML^2T^{-2}}}$

1.2 Determine the dimensions, in both the *FLT* system and *MLT* system, for **(a)** the product of force times volume, **(b)** the product of pressure times mass divided by area, and **(c)** moment of a force divided by velocity.

(a) $\text{force} \times \text{volume} \doteq (F)(L^3) \doteq \underline{\underline{FL^3}}$

Since $F \doteq MLT^{-2}$

$\text{force} \times \text{volume} \doteq (MLT^{-2})(L^3) \doteq \underline{\underline{ML^4T^{-2}}}$

(b) $$\frac{\text{pressure} \times \text{mass}}{\text{area}} \doteq \frac{(FL^{-2})(M)}{L^2} \doteq \frac{(FL^{-2})(FL^{-1}T^2)}{L^2}$$

$$\doteq \underline{\underline{F^2L^{-5}T^2}}$$

$$\doteq \frac{(MLT^{-2})(L^{-2})(M)}{L^2}$$

$$\doteq \underline{\underline{M^2L^{-3}T^{-2}}}$$

(c) $$\frac{\text{moment of a force}}{\text{velocity}} \doteq \frac{FL}{LT^{-1}} \doteq \underline{\underline{FT}}$$

$$\doteq (MLT^{-2})(T) \doteq \underline{\underline{MLT^{-1}}}$$

Systems of Units:

British Gravitational (BG) System. In the BG system the unit of length is the *foot* (ft), the time unit is the *second* (s), the force unit is the *pound* (lb), and the temperature unit is the degree *Fahrenheit* (°F)or the absolute temperature unit is the degree *Rankine* (R) Where:

R=F+459.67

The mass unit, called the *slug*, is defined from Newton's second law (force=mass ×acceleration) as: 1 lb = (1 slug) ×(1 ft/s^2)

The weight, W (which is the force due to gravity, *g*) of a mass, *m*, is given by the equation W=m×g and in BG units, W (lb)=m (slug)×g (ft/s^2)

Since the earth's standard gravity is taken as g=32.2 ft/s^2 , it follows that a mass of 1 slug weighs 32.2 lb under standard gravity.

International System (SI). In SI the unit of length is the meter (m), the time unit is the second (s), the mass unit is the kilogram (kg), and the temperature unit is the kelvin (K). The Kelvin temperature scale is an absolute scale and is related to the Celsius (centigrade) scale (°C)through the relationship: K=°C+273.15

The force unit, called the Newton (N), is defined from Newton's second law as (force=mass ×acceleration due to gravity), or 1 N=(1 kg)×(9.81 m/s^2)

Table (1.3)

Quantity	SI. Units	BG.units	Conversion factor
Mass	Kilogram (Kg)	Slug	1 slug=14.6 kg
Length	meter (m)	foot (ft)	1 ft=0.305 m
Time	Second (s)	Second (s)	1 s=1 s
Force	Newton (N)	Pound (1bf)	1bf=4.45 N
Temperature	Kelvin (K)	Rankine (R)	1 K=1.8 R
Area	m^2	ft^2	1 m^2=10.76 ft^2
Velocity	m/s	ft/s	1 ft/s=0.305 m/s
Pressure or stress	Pascal (N/m^2)	1bf/ft^2	1 1bf/ft^2=47.8 N/m^2
Energy or work	Joule (N.m)	1bf.ft	1bf.ft=1.35 N.m
Power	Watt (N.m/s)	1bf.ft/s	1bf.ft/s=1.35 N.m/s
Density	Kg/m^3	Slug/ft^3	Slug/ft^3=515 kg/m^3
Viscosity	Kg/(m.s)	Slug/(ft.s)	Slug/(ft.s)=47.8 kg/(m.s)

Hint: 1mL=1cm^3,
1m^3=1000 liter
1 inch=2.54 cm

1.14 Make use of Table 1.3 to express the following quantities in SI units: **(a)** 10.2 in./min, **(b)** 4.81 slugs, **(c)** 3.02 lb, **(d)** 73.1 ft/s², **(e)** 0.0234 lb·s/ft².

$$(a)\ 10{,}2\ \frac{in.}{min} = \left(10{,}2\ \frac{in.}{min}\right)\left(2.540\times10^{-2}\ \frac{m}{in.}\right)\left(\frac{1\ min}{60\ s}\right)$$

$$= 4.32\times10^{-3}\ \frac{m}{s} = \underline{\underline{4.32\ \frac{mm}{s}}}$$

$$(b)\ 4.81\ slugs = \left(4.81\ slugs\right)\left(1.459\times10\ \frac{kg}{slug}\right) = \underline{\underline{70{,}2\ kg}}$$

$$(c)\ 3.02\ lb = \left(3.02\ lb\right)\left(4.448\ \frac{N}{lb}\right) = \underline{\underline{13.4\ N}}$$

$$(d)\ 73.1\ \frac{ft}{s^2} = \left(73.1\ \frac{ft}{s^2}\right)\left(3.048\times10^{-1}\ \frac{\frac{m}{s^2}}{\frac{ft}{s^2}}\right) = \underline{\underline{22{,}3\ \frac{m}{s^2}}}$$

$$(e)\ 0.0234\ \frac{lb\cdot s}{ft^2} = \left(0.0234\ \frac{lb\cdot s}{ft^2}\right)\left(4.788\times10\ \frac{\frac{N\cdot s}{m^2}}{\frac{lb\cdot s}{ft^2}}\right)$$

$$= \underline{\underline{1.12\ \frac{N\cdot s}{m^2}}}$$

1.15 Make use of Table 1.4 to express the following quantities in BG units: **(a)** 14.2 km, **(b)** 8.14 N/m³, **(c)** 1.61 kg/m³, **(d)** 0.0320 N·m/s, **(e)** 5.67 mm/hr.

(a) $14.2\ \mathrm{km} = (14.2 \times 10^{3}\ \mathrm{m})\left(3.281\ \frac{\mathrm{ft}}{\mathrm{m}}\right) = \underline{\underline{4.66 \times 10^{4}\ \mathrm{ft}}}$

(b) $8.14\ \frac{\mathrm{N}}{\mathrm{m}^3} = \left(8.14\ \frac{\mathrm{N}}{\mathrm{m}^3}\right)\left(6.366 \times 10^{-3}\ \frac{\frac{\mathrm{lb}}{\mathrm{ft}^3}}{\frac{\mathrm{N}}{\mathrm{m}^3}}\right) = \underline{\underline{5.18 \times 10^{-2}\ \frac{\mathrm{lb}}{\mathrm{ft}^3}}}$

(c) $1.61\ \frac{\mathrm{kg}}{\mathrm{m}^3} = \left(1.61\ \frac{\mathrm{kg}}{\mathrm{m}^3}\right)\left(1.940 \times 10^{-3}\ \frac{\frac{\mathrm{slugs}}{\mathrm{ft}^3}}{\frac{\mathrm{kg}}{\mathrm{m}^3}}\right) = \underline{\underline{3.12 \times 10^{-3}\ \frac{\mathrm{slugs}}{\mathrm{ft}^3}}}$

(d) $0.0320\ \frac{\mathrm{N \cdot m}}{\mathrm{s}} = \left(0.0320\ \frac{\mathrm{N \cdot m}}{\mathrm{s}}\right)\left(7.376 \times 10^{-1}\ \frac{\frac{\mathrm{ft \cdot lb}}{\mathrm{s}}}{\frac{\mathrm{N \cdot m}}{\mathrm{s}}}\right)$

$= \underline{\underline{2.36 \times 10^{-2}\ \frac{\mathrm{ft \cdot lb}}{\mathrm{s}}}}$

(e) $5.67\ \frac{\mathrm{mm}}{\mathrm{hr}} = \left(5.67 \times 10^{-3}\ \frac{\mathrm{m}}{\mathrm{hr}}\right)\left(3.281\ \frac{\mathrm{ft}}{\mathrm{m}}\right)\left(\frac{1\ \mathrm{hr}}{3600\ \mathrm{s}}\right)$

$= \underline{\underline{5.17 \times 10^{-6}\ \frac{\mathrm{ft}}{\mathrm{s}}}}$

Physical properties of fluids:

1. Mass Density: The *density* of a fluid, designated by the Greek symbol ρ(rho), is defined as its mass per unit volume. Density is typically used to characterize the mass of a fluid system. In the BG system ρ has units of (slug/ft^3) and in SI the units are (kg/m^3). The density of water is equal to 1000 kg/m^3, or equal to1.94 slug/ft^3 under standard conditions.

The *specific volume,* $\forall$, is the *volume* per unit mass (m^3/kg) and is therefore the reciprocal of the density—that is,

$$\forall = \frac{1}{\rho} \qquad \text{....(1.1)}$$

2. Weight Density or (Specific weight): The *specific weight* of a fluid, designated by the Greek symbol γ (gamma), is defined as its *weight* per unit volume. Thus, specific weight is related to density through the equation:

$$\gamma = \rho \times g \qquad \text{....(1.2)}$$

where *g* is the local acceleration of gravity which it has units of (32.2 ft^2/s) in BG system and (9.81m^2/s) in SI the units. Just as density is used to characterize the mass of a fluid system, the specific weight is used to characterize the weight of the system. In the BG system, γ has units of (1b/ft^3) and in SI the units are (N/m^3). The specific weight of water is equal to (9810 N/m^3), and it equal to (62.4 1b/ft^3).

3. Relative Density or (Specific Gravity): The *specific gravity* of a fluid, designated as *SG,* is defined as the ratio of the density of the fluid to the density of water at some specified temperature. In equation form, specific gravity is expressed as:

$$SG = \frac{\rho_{anyfluid}}{\rho_{water}} \qquad \text{....(1.3)}$$

Since the specific gravity of mercury (SG$_{hg}$) is equal to (13.56).

1.24 The specific gravity of mercury at 80 °C is 13.4. Determine its density and specific weight at this temperature. Express your answer in both BG and SI units.

$$\rho = SG \times \rho_{H_2O@4°C} \qquad \gamma = \rho g$$

In BG units

$$\rho = 13.4\left(1.94\ \frac{slugs}{ft^3}\right) = \underline{\underline{26.0\ \frac{slugs}{ft^3}}}$$

$$\gamma = \left(26.0\ \frac{slugs}{ft^3}\right)\left(32.2\ \frac{ft}{s^2}\right) = \underline{\underline{837\ \frac{lb}{ft^3}}}$$

In SI units:

$$\rho = 13.4\left(1000\ \frac{kg}{m^3}\right) = \underline{\underline{13.4 \times 10^3\ \frac{kg}{m^3}}}$$

$$\gamma = \left(13.4 \times 10^3\ \frac{kg}{m^3}\right)\left(9.81\ \frac{m}{s^2}\right) = \underline{\underline{131\ \frac{kN}{m^3}}}$$

1.25 A *hydrometer* is used to measure the specific gravity of liquids. (See **Video V2.6.**) For a certain liquid a hydrometer reading indicates a specific gravity of 1.15. What is the liquid's density and specific weight? Express your answer in SI units.

$$SG = \frac{\rho}{\rho_{H_2O@4°C}}$$

$$1.15 = \frac{\rho}{1000\ \frac{kg}{m^3}}$$

$$\rho = (1.15)\left(1000\ \frac{kg}{m^3}\right) = \underline{\underline{1150\ \frac{kg}{m^3}}}$$

$$\gamma = \rho g = \left(1150\ \frac{kg}{m^3}\right)\left(9.81\ \frac{m}{s^2}\right) = \underline{\underline{11.3\ \frac{kN}{m^3}}}$$

4. Dynamic Viscosity or (Absolute viscosity): The viscosity can be defined as the fluid resistance to move (flow) under any magnitude of shear stress. When a fluid is flowing, it begins to move at a strain rate proportional to shear stress, and the constant of proportionality is called *coefficient of viscosity* μ. Consider a fluid element sheared in one plane by a single shear stress (τ), as shown in Fig. (1.1).The velocity δu will continuously grow along the normal distance between the fluid layers δy as long as the stress τ is maintained constant. The upper surface is moving at speed δu larger than the lower surface. Such common fluids as water, oil, and air show a linear relation between applied shear stress and resulting strain rate, $\tau \alpha \dfrac{\delta u}{\delta y}$.

Where, $\dfrac{\delta u}{\delta y}$ is called velocity gradient or rate of strain. Then the constant of proportionality is called dynamic viscosity as shown below:

$$\tau = \mu \frac{\delta u}{\delta y} \quad \text{..(1.4)}$$

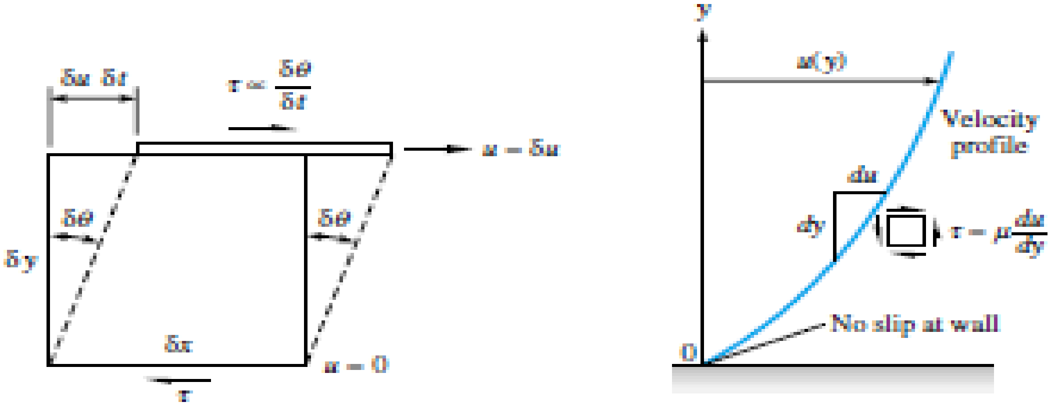


Fig. (1.1)

Equation (1.4) is dimensionally consistent; therefore μ has dimensions of (shear stress ×time) which means $\{F.T/L^2\}$ or $\{M/(L.T)\}$. The BG unit is (slugs /foot×second), and the SI unit is (kilograms /meter×second). The linear fluids which follow Eq. (1.4) are called *Newtonian fluids*, after Sir Isaac Newton, who first postulated this resistance law in 1687.

The second form of viscosity that the ratio of dynamic viscosity to mass density which it has the name of *(kinematic viscosity), υ:*

$$\upsilon = \frac{\mu}{\rho} \quad \text{..(1.5)}$$

It is called kinematic because the mass units cancel, having the units of $\{m^2/s\}$ in SI unit and $\{ft^2/s\}$ in BG unit. It has another units such as (poises, and stokes). Each 1 poise = $(N/m^2)*10$, and Each 1 stoke $(cm^2/s)=10^{-4}m^2/s$.

Ex: The viscosity of a certain fluid is 5×10^{-4} poise. Determine its viscosity in both SI. and BG. units.

From Appendix E $10^{-1}\,\frac{N.s}{m^2} = 1$ poise. Thus,

$$\mu = (5\times10^{-4}\,\text{poise})\left(10^{-1}\,\frac{\frac{N.s}{m^2}}{\text{poise}}\right) = \underline{\underline{5\times10^{-5}\,\frac{N.s}{m^2}}}$$

and From Table 1.4

$$\mu = \left(5\times10^{-5}\,\frac{N.s}{m^2}\right)\left(2.089\times10^{-2}\,\frac{\frac{lb.s}{ft^2}}{\frac{N.s}{m^2}}\right) = \underline{\underline{10.4\times10^{-7}\,\frac{lb.s}{ft^2}}}$$

Flow Between Two Plates:

A classic problem is the flow induced between a fixed lower plate and an upper plate moving steadily at velocity **V**, as shown in Fig. (1.2). The clearance between plates is *h*, and the fluid is Newtonian and does not slip at either plate. For Newtonian fluids equation (1.4) becomes:

$$\frac{du}{dy} = \frac{\tau}{\mu} = \text{constant}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.6)$$

which we can integrate to obtain:

$$u = a + by\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.7)$$

The *velocity distribution is linear*, as shown in Fig. (1.2), and the constants *a* and *b* can be evaluated from the no-slip condition at the upper and lower walls:

$$u = \begin{cases} 0 = a + b(0) & \text{at } y = 0 \\ V = a + b(h) & \text{at } y = h \end{cases}$$

Hence *a* = 0 and *b* = *V*/*h*. Then the velocity profile between the plates is given by:

$$\frac{u}{y} = \frac{V}{h}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.8)$$

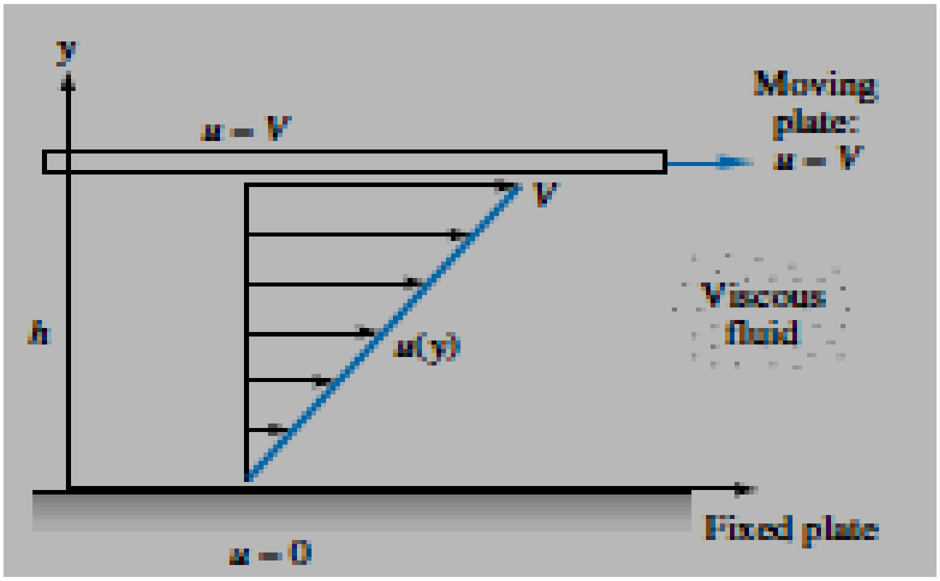


Fig. (1.2)

The actual value of the viscosity depends on the *particular fluid*, and for a particular fluid the viscosity is also highly dependent on *temperature* as illustrated in Fig. (1.3) with the two curves for water. Fluids for which the shearing stress is *linearly* related to the rate of velocity gradient (du/dy) are designated as *Newtonian fluids* **I. Newton** (1642–1727). Fortunately most common fluids, both liquids and gases, are Newtonian.

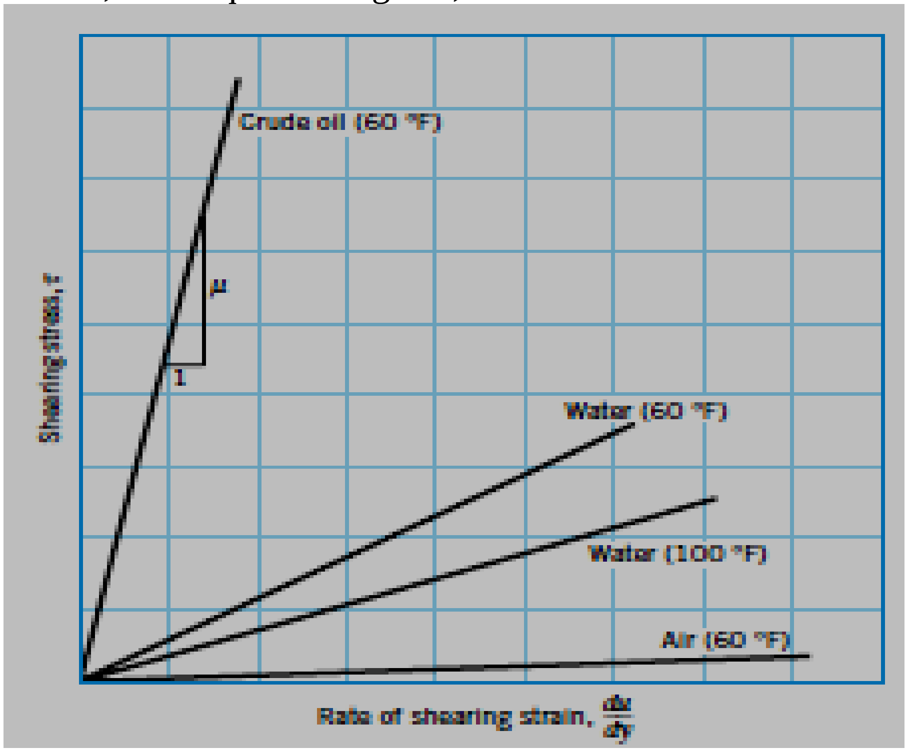


Fig.(1.3)

1.53 For a parallel plate arrangement of the type shown in Fig. 1.3 it is found that when the distance between plates is 2 mm, a shearing stress of 150 Pa develops at the upper plate when it is pulled at a velocity of 1 m/s. Determine the viscosity of the fluid between the plates. Express your answer in SI units.

$$\tau = \mu \frac{du}{dy}$$

$$\frac{du}{dy} = \frac{U}{b}$$

$$\mu = \frac{\tau}{\left(\frac{U}{b}\right)} = \frac{150\ \frac{N}{m^2}}{\left(\frac{1\ \frac{m}{s}}{0.002\ m}\right)} = \underline{\underline{0.300\ \frac{N \cdot s}{m^2}}}$$

The effect of temperature on viscosity can be closely approximated using two empirical formulas. For gases the *Sutherland equation* can be expressed as: $\mu = \frac{CT^{3/2}}{T+S}$ ……………………………………………………………….(1.9)

where *C* and *S* are empirical constants, and *T* is absolute temperature. Thus, if the viscosity is known at two temperatures, *C* and *S* can be determined.

Some Applications of Viscosity Property:

1. Body slides on inclined surface.
2. Flow between two or more plates.
3. Oil between Piston and cylinder.
4. Oil between Bush and shaft.
5. Oil between rotating disc and cylinder.
6. Fluid flow inside a circular or non circular pipe.

1.45 A block of weight W slides down an inclined plane on a thin film of oil, as in Fig. P1.45 at right. The film contact area is A and its thickness h. Assuming a linear velocity distribution in the film, derive an analytic expression for the terminal velocity V of the block.

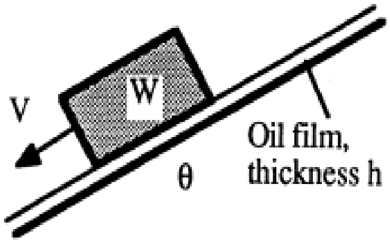


Fig. P1.45

$$\sum F_x = W\sin\theta - \tau A = W\sin\theta - \left(\mu\frac{V}{h}\right)A$$

$$\text{or:}\quad V_{terminal} = \frac{hW\sin\theta}{\mu A}\quad \textit{Ans.}$$

Hints:

1. **if the body moves at constant velocity then acceleration is equal to zero, which means (∑Forces=m×a), but for constant velocity (a=0) as occurred in this example.**
2. **if the body moves at variable velocity, the its moves at acceleration, which means ($a \neq 0$).**

1.47 A shaft 6.00 cm in diameter and 40 cm long is pulled steadily at V = 0.4 m/s through a sleeve 6.02 cm in diameter. The clearance is filled with oil, $\nu = 0.003\ m^2/s$ and SG = 0.88. Estimate the force required to pull the shaft.

Solution: Assuming a linear velocity distribution in the clearance, the force is balanced by resisting shear stress in the oil:

$$F = \tau A_{wall} = \left(\mu\frac{V}{\Delta R}\right)(\pi D_i L) = \frac{\mu V \pi D_i L}{R_o - R_i}$$

For the given oil, $\mu = \rho\nu = (0.88 \times 998\ kg/m^3)(0.003\ m^2/s) \approx 2.63\ N\cdot s/m$ (or $kg/m\cdot s$). Then we substitute the given numerical values to obtain the force:

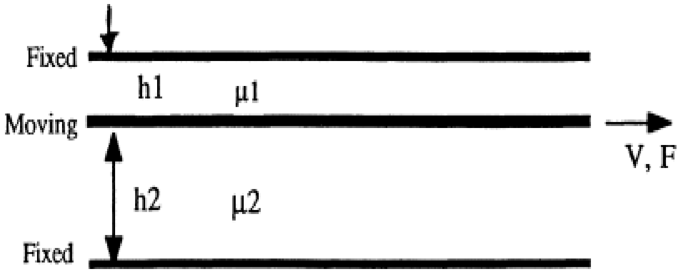
$$F = \frac{\mu V \pi D_i L}{R_o - R_i} = \frac{(2.63\ N\cdot s/m^2)(0.4\ m/s)\pi(0.06\ m)(0.4\ m)}{(0.0301 - 0.0300\ m)} \approx 795\ N \quad \textit{Ans.}$$

Hints:

Lateral area of shaft =π×D_i (diameter of shaft×L (length of shaft)

Thickness of oil layer between shaft and sleeve=(R_o-R_i)

1.48 A thin moving plate is separated from two fixed plates by two fluids of unequal viscosity and unequal spacing, as shown below. The contact area is A. Determine (a) the force required, and (b) is there a necessary relation between the two viscosity values?



Solution: (a) Assuming a linear velocity distribution on each side of the plate, we obtain

$$F = \tau_1 A + \tau_2 A = \left(\frac{\mu_1 V}{h_1} + \frac{\mu_2 V}{h_2}\right) A \quad \textit{Ans.}\ (a)$$

1.52 The belt in Fig. P1.52 moves at steady velocity V and skims the top of a tank of oil of viscosity μ. Assuming a linear velocity profile, develop a simple formula for the belt-drive power P required as a function of (h, L, V, B, μ). Neglect air drag. What power P in watts is required if the belt moves at 2.5 m/s over SAE 30W oil at 20°C, with L = 2 m, b = 60 cm, and h = 3 cm?

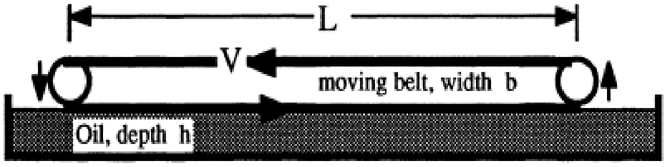


Fig. P1.52

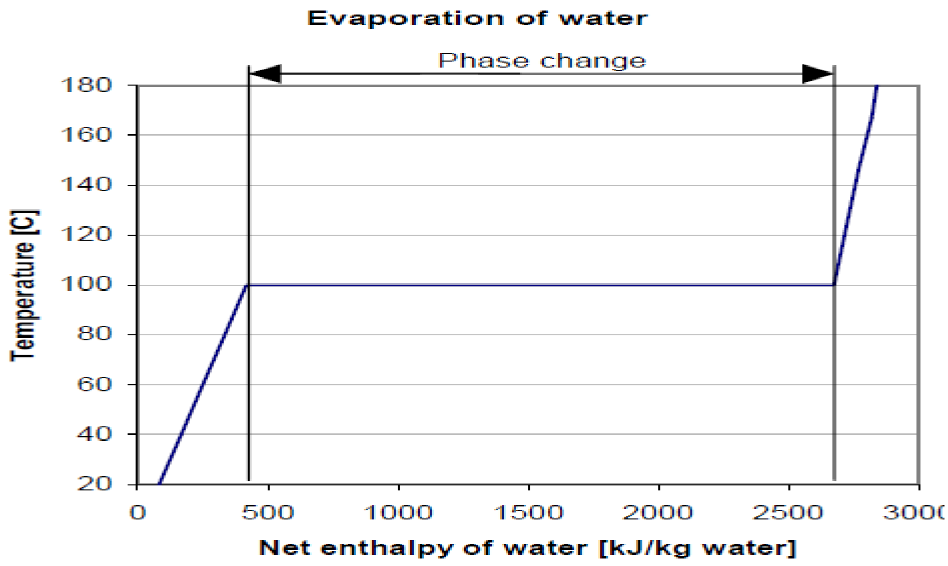
Solution: The power is the viscous resisting force times the belt velocity:

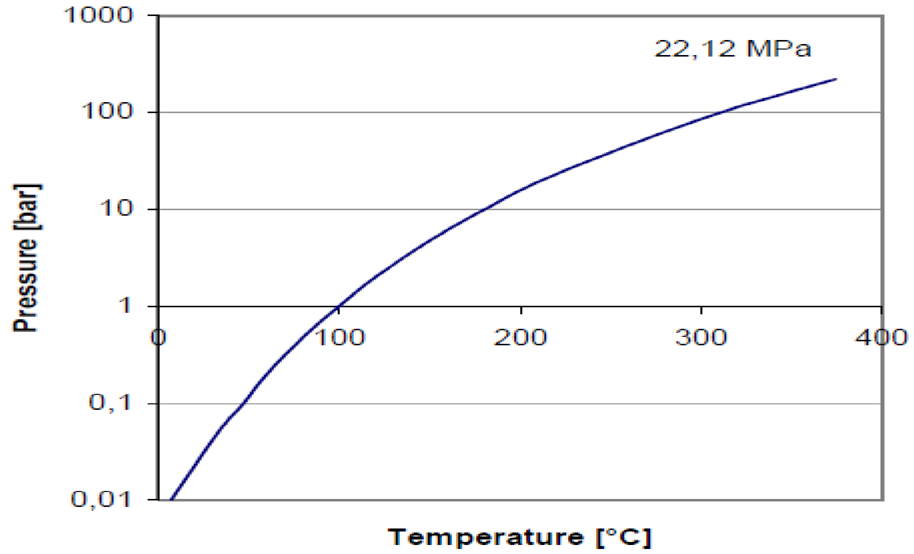
$$P = \tau_{oil}A_{belt}V_{belt} \approx \left(\mu\frac{V}{h}\right)(bL)V = \mu V^2 b\frac{L}{h} \quad \textit{Ans.}$$

(b) For SAE 30W oil, $\mu \approx 0.29\ \mathrm{kg/m\cdot s}$. Then, for the given belt parameters,

$$P = \mu V^2 bL/h = \left(0.29\ \frac{kg}{m\cdot s}\right)\left(2.5\ \frac{m}{s}\right)^2 (0.6\ m)\frac{2.0\ m}{0.03\ m} \approx 73\ \frac{kg\cdot m^2}{s^3} = \mathbf{73\ W} \quad \textit{Ans.}\ (b)$$

5**. Vapor Pressure:** It is a common observation that liquids such as water and gasoline will evaporate if they are simply placed in a container open to the atmosphere. Evaporation takes place because some liquid molecules at the surface have sufficient momentum to overcome the intermolecular cohesive forces and escape into the atmosphere. If the container is closed with a small air space left above the surface, and this space evacuated to form a vacuum, a pressure will develop in the space as a result of the vapor that is formed by the escaping molecules. When an equilibrium condition is reached so that the number of molecules leaving the surface is equal to the number entering, the vapor is said to be saturated and the pressure that the vapor exerts on the liquid surface is termed the *vapor pressure*.





Boiling, which is the formation of vapor bubbles within a fluid mass, it is initiated when the absolute pressure in the fluid reaches the vapor pressure. As commonly observed in the kitchen when we using (pressure cooker), water at standard atmospheric pressure will boil when the temperature reaches(100^{o}C) when the vapor pressure of water is 101.325 kPa. However, if we attempt to boil water at a higher elevation, say 10,000 ft above sea level, where the atmospheric pressure is 10.1 psi . We find that boiling will start when the temperature is about (89.4^{o}C). At this temperature the vapor pressure of water is 10.1 psi. Thus, *boiling* can be induced at a given pressure acting on the fluid by *raising the temperature*, or at a given fluid temperature by *lowering the pressure*. From the above diagram we can see that when we exceed a certain pressure to 22,12 Mpa (the corresponding boiling temperature is 374°C).

An important reason for our interest in vapor pressure and boiling lies in the common observation that in flowing fluids it is possible to develop very low pressure due to the fluid motion, and if the pressure is lowered to the vapor pressure, boiling will occur. For example, this phenomenon may occur in flow through the irregular, narrowed passages of a valve or pump. When vapor bubbles are formed in a flowing fluid they are swept along into regions of higher pressure where they suddenly collapse with sufficient intensity to actually cause structural damage. The formation and subsequent collapse of vapor bubbles in a flowing fluid, called *cavitations*, is an important fluid flow phenomenon to be given further attention in next chapters**.**

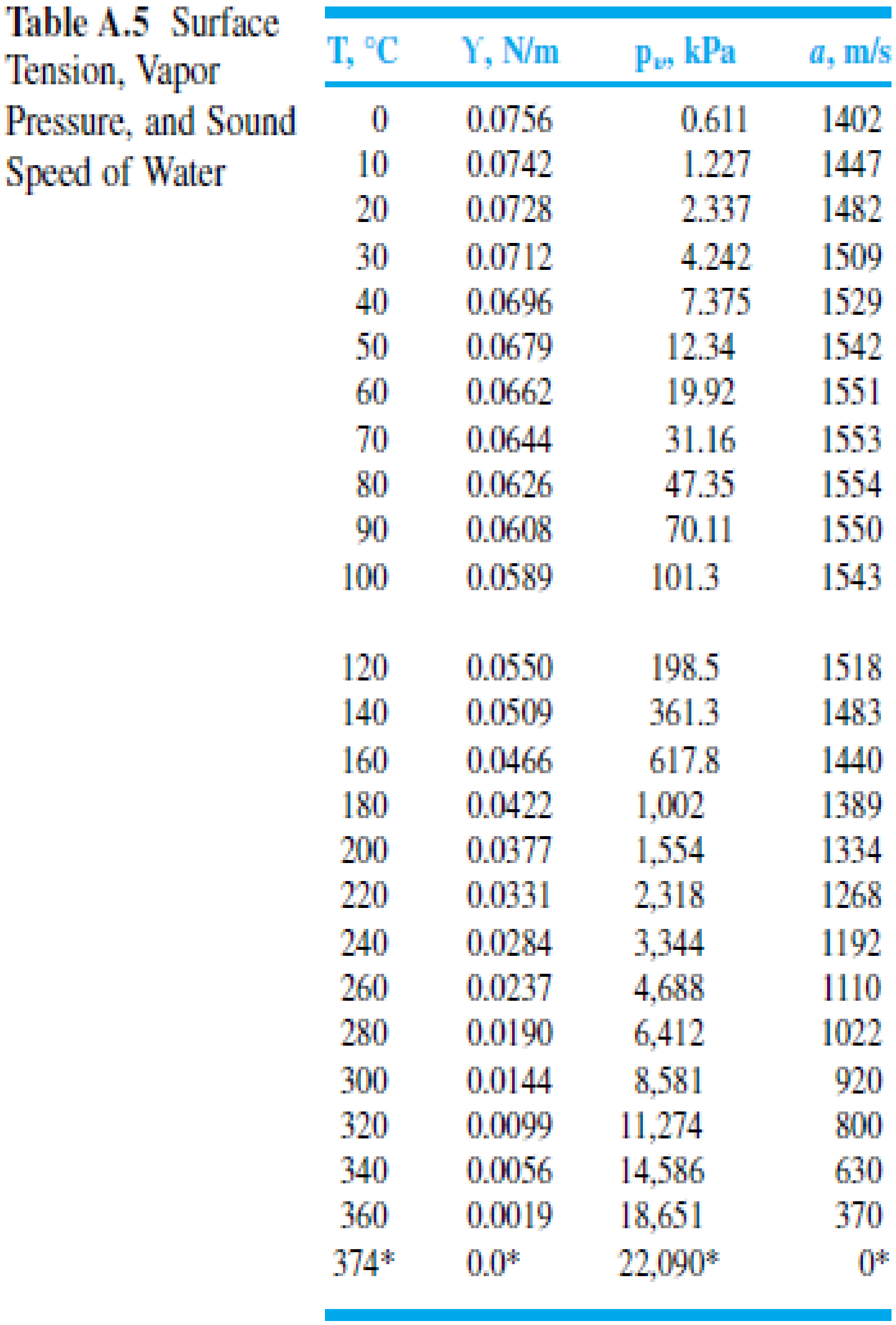


Table A.5 Surface Tension, Vapor Pressure, and Sound Speed of Water

T, °C	Y, N/m	p_v, kPa	a, m/s
0	0.0756	0.611	1402
10	0.0742	1.227	1447
20	0.0728	2.337	1482
30	0.0712	4.242	1509
40	0.0696	7.375	1529
50	0.0679	12.34	1542
60	0.0662	19.92	1551
70	0.0644	31.16	1553
80	0.0626	47.35	1554
90	0.0608	70.11	1550
100	0.0589	101.3	1543
120	0.0550	198.5	1518
140	0.0509	361.3	1483
160	0.0466	617.8	1440
180	0.0422	1,002	1389
200	0.0377	1,554	1334
220	0.0331	2,318	1268
240	0.0284	3,344	1192
260	0.0237	4,688	1110
280	0.0190	6,412	1022
300	0.0144	8,581	920
320	0.0099	11,274	800
340	0.0056	14,586	630
360	0.0019	18,651	370
374*	0.0*	22,090*	0*

The saturated vapor pressure of water may be approximated by the following equation (in order of increasing accuracy)

$$P_{(mmHg)} = \exp\left(20.386 - \frac{5132}{T}\right) \quad \text{...}(1.11)$$

where P is the vapor pressure in mmHg and T is the temperature in kelvin.

6. Surface tension: At the interface between a liquid and a gas, or between two different liquids, forces develop in the liquid surface which causes the surface to behave as a “skin” stretched over the fluid mass. For example, a steel needle will float on water if placed on the surface because the tension developed in the skin supports the needle. Small droplets of mercury will form into spheres when placed on a smooth surface because the *cohesive forces* (قوة تماسك) in the surface tend to hold all the molecules together in a compact shape.

These various types of surface phenomena are due to the unbalanced cohesive forces acting on the liquid molecules at the fluid surface. Molecules in the interior of the fluid mass are surrounded by molecules that are attracted to each other equally. However, molecules along the surface are subjected to a net force toward the interior (قوة تجاذب نحو ألاسفل). The apparent physical consequence of this unbalanced force along the surface is to create the hypothetical skin.

A tensile force may be considered to be acting in the plane of the surface along any line in the surface. The intensity of the molecular attraction per unit length along any line in the surface is called the *surface tension* and is designated by the Greek symbol σ (sigma). The dimensions of surface tension are with BG units of (1b/ft) and SI units of (N/m). The value of the surface tension decreases as the temperature increases.

The pressure inside a drop of fluid can be calculated using the free-body diagram in Fig. (1.7).

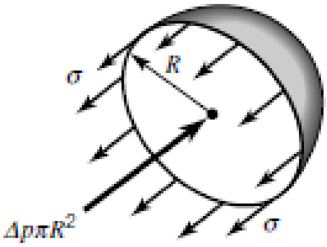


Fig.(1.7)

If the spherical drop is cut in half (as shown) the force developed around the edge due to surface tension ($2\pi R\times\sigma$). This force must be balanced by the pressure difference, (ΔP) between the internal pressure, (p_i) and the external pressure, (p_e) acting over the circular area, (πR^2). Thus,

$$2\pi R\times\sigma=\Delta P\times\pi R^2 \quad \textbf{……………………………………………..…….(1.12)}$$

Or

$$p_i - p_e = \frac{2\sigma}{R} \quad \textbf{..(1.13)}$$

Among common phenomena associated with surface tension is the rise (or fall) of a liquid in a capillary tube. If a small open tube is inserted into water, the water level in the tube will rise above the water level outside the tube as is illustrated in Fig. (1.8*a*). In this situation we have a liquid–gas–solid interface. For the case illustrated there is an attraction (adhesion) (قوة تلاصق) between the wall of the tube and liquid molecules which is strong enough to overcome the attraction (cohesion) of the molecules and pull them up the wall. Hence, the liquid is said to <u>*wet*</u> the solid surface.

The height, *h*, is governed by the value of the surface tension, σ the tube radius, *R*, the specific weight of the liquid, γ and the *angle of contact,ϴ* between the fluid and tube. From the free-body diagram of Fig. (1.8*b*) we see that the vertical force due to the surface tension is equal to $(2\pi R \times \sigma\cos\theta)$ and the weight is $(\gamma\pi R^2 \times h)$ and these two forces must balance for equilibrium. Thus,

$$2\pi R \times \sigma\cos\theta = \gamma\pi R^2 \times h \quad \text{..(1.14)}$$

so that the height is given by the relationship:

$$h = \frac{2\sigma\cos\theta}{\gamma R} \quad \text{..(1.15)}$$

For water in contact with clean glass is equal to (ϴ=0). It is clear from Eq. 1.14 that the height is inversely proportional to the tube radius, and therefore the rise of a liquid in a tube as a result of capillary action becomes increasingly pronounced as the tube radius is decreased.

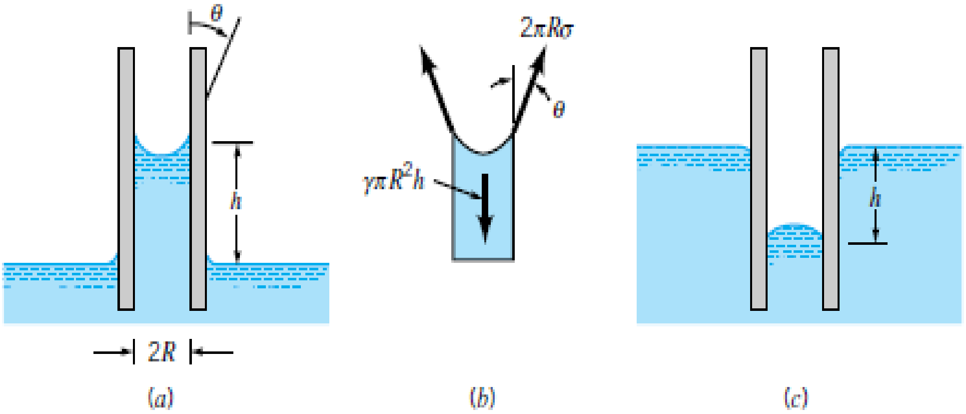
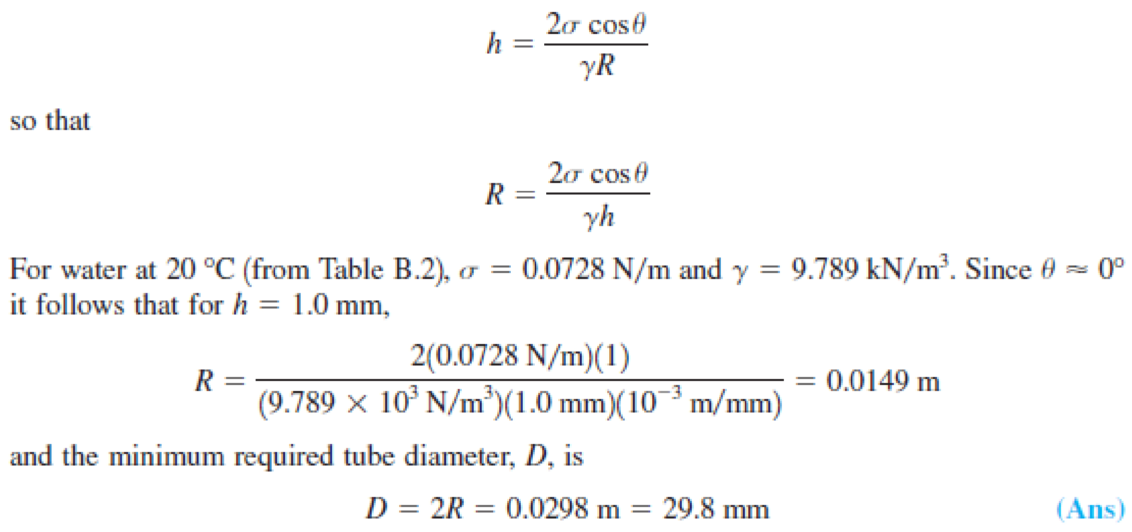


Fig. (1.8) Effect of capillary action in small tubes. a) rise of fluid column, b) free body diagram for calculating column height, c)depression of fluid column.

Ex: What is the diameter of clean glass tube that containing water at 20 ºC which causes to rise the water inside the tube to 1.0 mm due to capillary action ?
Hint: take surface tension σ for water at 20 ºC is equal to 0.0728 N/m, and specific weight γ of water is equal to9.789 kN/m^3. For cleaning water, (Θ=0º).



$$h = \frac{2\sigma\cos\theta}{\gamma R}$$

so that

$$R = \frac{2\sigma\cos\theta}{\gamma h}$$

For water at 20 °C (from Table B.2), $\sigma = 0.0728$ N/m and $\gamma = 9.789$ kN/m^3. Since $\theta \approx 0°$ it follows that for $h = 1.0$ mm,

$$R = \frac{2(0.0728\ \text{N/m})(1)}{(9.789 \times 10^3\ \text{N/m}^3)(1.0\ \text{mm})(10^{-3}\ \text{m/mm})} = 0.0149\ \text{m}$$

and the minimum required tube diameter, D, is

$$D = 2R = 0.0298\ \text{m} = 29.8\ \text{mm} \qquad \textbf{(Ans)}$$

Ex: Derive an expression for the change in height h in a circular tube of a liquid with surface tension σ and contact angle θ, as in Fig. (1.9).

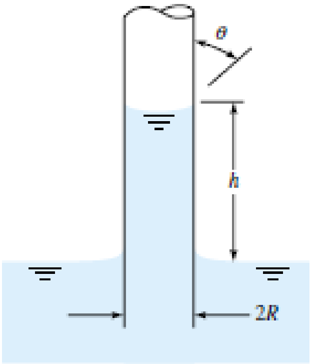


Fig.(1.9)

Thus the capillary height increases inversely with tube radius R and is positive if $\theta \langle 90°$ (wetting liquid) and negative (capillary depression) if $\theta \rangle$ 90°.

Suppose that R=1 mm. Then the capillary rise for a water-air-glass interface, θ=0°, σ=0.073 N/m, and ρ= 1000 kg/m^3 is:

$$h = \frac{2\sigma\cos\theta}{\gamma R},$$

$$h = \frac{2(0.073\text{ N/m})(\cos 0^\circ)}{(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.001\text{ m})} = 0.015\ (\text{N}\cdot\text{s}^2)/\text{kg} = 0.015\text{ m} = 1.5\text{ cm}$$

For a mercury-air-glass interface, with $\theta = 130^\circ$, $\sigma = 0.48$ N/m, and $\rho = 13{,}600$ kg/m^3, the capillary rise is:

$$h = \frac{2(0.48)(\cos 130^\circ)}{13{,}600(9.81)(0.001)} = -0.46\text{ cm}$$

1.62 The hydrogen bubbles in Fig. 1.13 have D ≈ 0.01 mm. Assume an “air-water” interface at 30°C. What is the excess pressure within the bubble?

Solution: At 30°C the surface tension from Table A-1 is 0.0712 N/m. For a droplet or bubble with one spherical surface, from Eq. (1.32),

$$\Delta p = \frac{2Y}{R} = \frac{2(0.0712\text{ N/m})}{(5\text{E–}6\text{ m})} \approx \mathbf{28500\ Pa} \quad \textit{Ans.}$$

Chapter Two

Static Fluids

Pressure at Point:

Figure 2.1 shows a small wedge of fluid at rest of size Δ*x* by Δ*z* by Δ*s* and depth *b* into the paper. There is no shear by definition when fluid at rest), but we suppose that the pressures p_x, p_z, and p_n may be different on each face. The weight of the element also may be important. Summation of forces must equal zero (no acceleration) in both the *x* and *z* directions.

$\sum F_x = 0 = p_x b\Delta z - p_n b\Delta s \sin\theta$,also,

$$\sum F_z = 0 = p_z b\Delta x - p_n b\Delta s \cos\theta - (1/2)\gamma b\Delta x\Delta z \quad \text{....……………………(2.1)}$$

But we know that: $\Delta s \sin\theta = \Delta z$, and $\Delta s \cos\theta = \Delta x$

Then by substituting in Eq. (2.1), and re-arrangement:

$$p_x = p_n \text{, and} \quad \text{………………………………….………...…(2.2a)}$$

$$p_z = p_n + (1/2)\gamma\Delta z \quad \text{…………………………………...………(2.2b)}$$

In the limit as the fluid wedge shrinks to a "point,'' Δz → 0 and Eqs. (2.2) becomes:

$$p_x = p_z = p_n = p \quad \text{………….……………..…………………...…(2.3)}$$

These relations illustrate one important principle of the hydrostatic, or shear-free, condition:

There is no pressure change in the horizontal direction. We conclude that the pressure *p* at a point in a static fluid is independent *of direction as long as there are no shearing stresses present*, This important result is known as *Pascal's law* named in honor of **Blasé Pascal** 11623–16622, a French mathematician who made important contributions in the field of hydrostatics or (*the pressure at point inside static fluid is equal from all sides*).

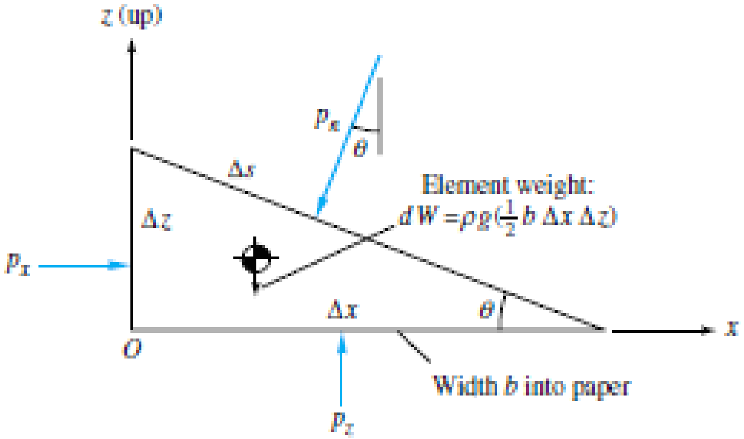


Fig. (2.1)

Pressure Variation in Static Fluids:

For static fluids only the normal stresses are present ($\tau = 0$), then by applying the Newton second law on fluid element that shown in Fig.(2.2) we get: The resultant surface force in the y direction is

$$\sum F_y = 0, \text{ or } (F_{in})_y - (F_{out})_y = 0$$

$$\left(P - \frac{\partial P}{\partial y}\frac{\delta y}{2}\right)\delta x\,\delta z - \left(P + \frac{\partial P}{\partial y}\frac{\delta y}{2}\right)\delta x\,\delta z = 0 \text{ or,}$$

$$\left(-\frac{\partial P}{\partial y}\right)\delta y\,\delta x\,\delta z = 0 \longrightarrow \frac{\partial P}{\partial y} = 0 \quad \text{......(2.4)}$$

Similarly, for the x direction the resultant surface forces are:

$$\sum F_x = 0, \text{ or } (F_{in})_x - (F_{out})_x = 0$$

$$\left(P - \frac{\partial P}{\partial x}\frac{\delta x}{2}\right)\delta y\,\delta z - \left(P + \frac{\partial P}{\partial x}\frac{\delta x}{2}\right)\delta y\,\delta z = 0 \text{ or,}$$

$$\left(-\frac{\partial P}{\partial x}\right)\delta y\,\delta x\,\delta z = 0 \longrightarrow \frac{\partial P}{\partial x} = 0 \quad \text{......(2.5)}$$

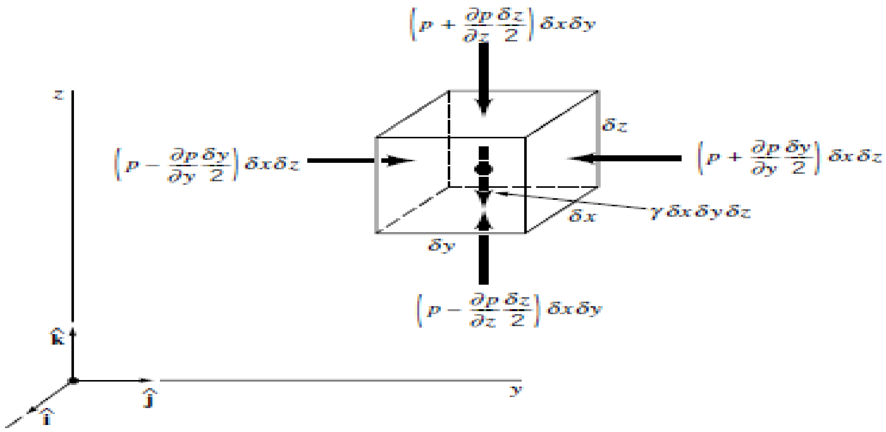


Fig. (2.2)

But, in the Z direction the resultant surface forces are:

$\sum F_z = 0$, or $(F_{in})_z - (F_{out})_z + W = 0$

$$\left(P - \frac{\partial P}{\partial z}\frac{\delta z}{2}\right)\delta x \delta y - \left(P + \frac{\partial P}{\partial z}\frac{\delta z}{2}\right)\delta x \delta y + \gamma \delta x \delta y \delta z = 0 \text{ or,}$$

$\left(-\frac{\partial P}{\partial z}\right)\delta z \delta x \delta y + \gamma \delta z \delta x \delta y = 0$,and by divided by volume ($\delta x \delta y \delta z$) we get:

$$\frac{\partial P}{\partial z} = -\gamma \quad \text{……………………………………………………..(2.6)}$$

These relations illustrate one important principle of the hydrostatic, or shear-free, condition: *The pressure is varied with vertical depth*.

These equations show that the pressure does not depend on *x* or *y (which means pressure don't varied horizontally)*. Since *p* depends only on *z*, the last of Eq. (2.6) can be written as the ordinary differential equation:

$$\frac{dP}{dz} = -\gamma \quad \text{…………………………………………………..…(2.7)}$$

Incompressible Fluid: Since the specific weight is equal to the product of fluid density and acceleration of gravity ($\gamma = \rho.g$) changes in are caused either by a change in ρ or *g*. For most engineering applications the variation in *g* is negligible, so our main concern is with the possible variation in the fluid density (which it called compressible). For liquids the variation in density is usually negligible (which it called incompressible), so that the assumption of constant specific weight when dealing with liquids. For this instance, Eq. (2.7) can be directly integrated:

$\int_{p_1}^{p_2} dp = -\gamma \int_{z_1}^{z_2} dz$ or $(p_2 - p_1) = -\gamma(z_2 - z_1)$ or in final form:

$$(p_1 - p_2) = \gamma(z_2 - z_1) \quad \text{……………………………………………..(2.8)}$$

where p_1 and p_2 are pressures at the vertical elevations as is illustrated in Fig. (2.3). Equation (2.8) can be written in the compact form:

$$(p_1 - p_2) = \gamma h \quad \text{……………………………………………………...(2.9)}$$

Eq. (2.9) shows that in an incompressible fluid at rest the pressure varies linearly with depth and (h *is called pressure head*) which has units of length (m) or (ft). When one works with liquids there is often a free surface, as is illustrated in Fig. (2.3), and it is convenient to use this surface as a reference plane. The reference pressure p_o would correspond to the pressure acting on the free surface (which would frequently be *atmospheric pressure*), and thus if we let $p_2 = p_o$ in Eq. (2.9) it follows that the pressure *p* at any depth *h* below the free surface is given by the equation:

$$p = \gamma h + p_o \quad \text{……………………………………………………….(2.10)}$$

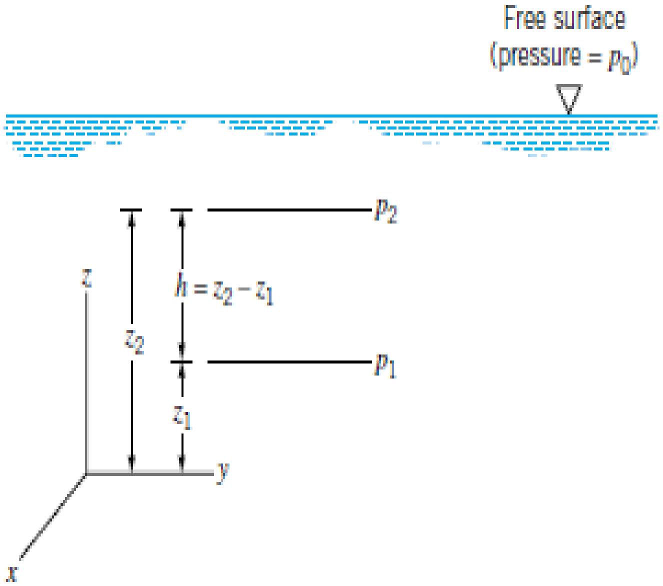


Fig. (2.3)

Ex: A tank is containing Water and Gasoline and open from upper side to atmosphere as shown in Figure below, water has seeped in to the depth shown. If the specific gravity of the gasoline is SG=0.68, determine the pressure at the gasoline-water interface and at the bottom of the tank. Express the pressure in units of lbf/ft^2, as a pressure head in feet of water.

Sol: the pressure variation can be found from the equation:

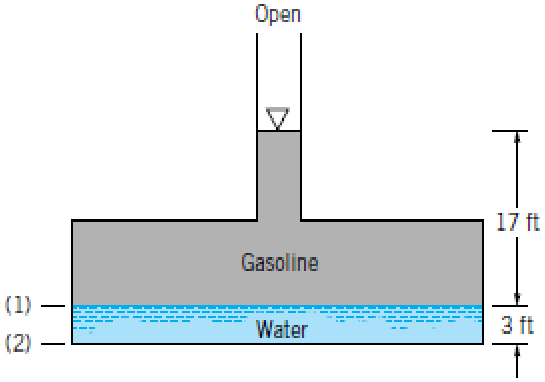
$p_1 = \gamma h + p_o$ or

$$p_1 = \gamma_{Gasoline} \times h + p_o = (0.68)\times(62.4)\times(17) + p_o$$
$$= 721 + p_o \,(\text{1b/ft}^2)$$

If we measure the pressure relative to atmospheric pressure (gage pressure), it follows that ($p_o = 0$) and therefore: $p_1 = 721(\text{1b/ft}^2)$

We can now apply the same relationship to determine the pressure at the tank bottom; that is:

$$p_2 = \gamma_{water} \times h + p_1 = (62.4)(3) + 721$$
$$= 908(\text{1b/ft}^2)$$



Compressible Fluid: We normally think of gases such as air, oxygen, and nitrogen as being (*compressible fluids*) since the density of the gas can change significantly with changes in γ pressure and temperature. Thus, although Eq. (2.7) applies at a point in a gas, it is necessary to consider the possible variation in before the equation can be integrated. The equation of state for an ideal (or perfect) gas is equal to ($P = \rho RT$).This relationship can be combined with Eq. (2.7) to give:

$\frac{dp}{dz} = -\frac{gp}{RT}$ or in separating variables, becomes:

$$\int_{p_1}^{p_2} \frac{dp}{p} = \ln\frac{p_2}{p_1} = -\frac{g}{R}\int_{z_1}^{z_2}\frac{dz}{T} \quad \text{……………………………………………..(2.11)}$$

Or, for isothermal process(T=constant)

$$p_2 = p_1 \exp\left[-\frac{g(z_2 - z_1)}{RT_o}\right] \quad \text{……………………………………………….(2.12)}$$

This equation provides the desired pressure-elevation relationship for an isothermal layer.

Absolute, Gage, Vacuum, and Atmospheric Pressures

Measurement of Pressure: the pressure at a point within a fluid mass will be designated as either an (*absolute* pressure or a *gage* pressure). Absolute pressure is measured relative to a perfect vacuum (absolute zero pressure), whereas gage pressure is measured relative to the local atmospheric pressure. Thus, a gage pressure of zero corresponds to a pressure that is equal to the local atmospheric pressure. Absolute pressures are always positive, but gage pressures can be either positive or negative depending on whether the pressure is above atmospheric pressure (a positive value) or below atmospheric pressure (a negative value). A negative gage pressure is also referred to as a *suction* or *vacuum* pressure as shown in Fig. (2.4).

Hint:

absolute pressure The actual pressure at a given position is called the absolute pressure, and it is measured relative to absolute vacuum (i.e., absolute zero pressure).

gage pressure Gage pressure is the pressure relative to the atmospheric pressure. In other words, how much above or below is the pressure with respect to the atmospheric pressure.

vacuum pressure - Pressures below atmospheric pressure are called vacuum pressures and are measured by vacuum gages that indicate the difference between the atmospheric pressure and the absolute pressure.

atmospheric pressure - The atmospheric pressure is the pressure that an area experiences due to the force exerted by the atmosphere.

Equations:-

$P_{gage} = P_{abs} - P_{atm}$	gage pressure
$P_{vac} = P_{atm} - P_{abs}$	vacuum pressure
$P_{abs} = P_{atm} + P_{gage}$	absolute pressure

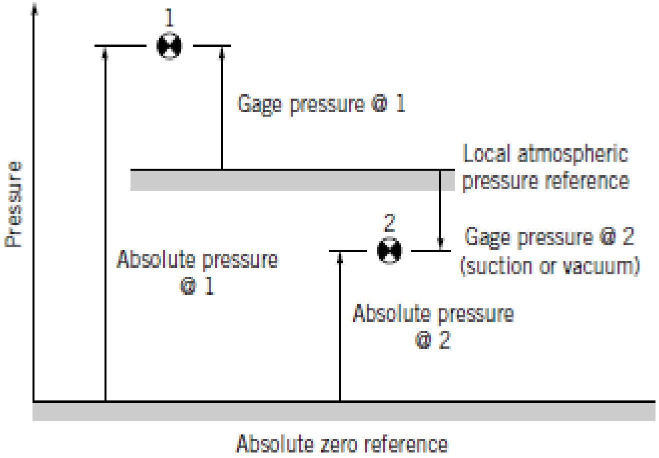


Fig. (2.4)

The measurement of atmospheric pressure is usually accomplished with a mercury *barometer*, which in its simplest form consists of a glass tube closed at one end with the open end immersed in a container of mercury as shown in Fig. (2.5). The tube is initially filled with mercury (inverted with its open end up) and then turned upside down (open end down) with the open end in the container of mercury. The column of mercury will come to an equilibrium position where its weight plus the force due to the vapor pressure (which develops in the space above the column) balances the force due to the atmospheric pressure. Thus, $p_{atm} = \gamma_{Hg} \times h$

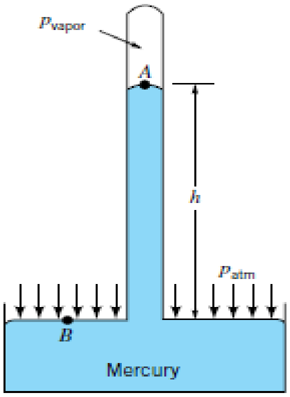


Fig. (2.5)

Ex: A mountain lake has an average temperature of 10 ^oC and a maximum depth of 40 m. For a barometric pressure of 598 mm Hg, determine the absolute pressure (in Pascal) at the deepest part of the lake.
Sol: Take: $\gamma_{water} = 9.804 kN/m^3$, and $\gamma_{Hg} = 133 kN/m^3$

The pressure in the lake at any depth, h, is given by the equation

$$p = \gamma h + p_0$$

where p_0 is the pressure at the surface. Since we want the absolute pressure, p_0 will be the local barometric pressure expressed in a consistent system of units; that is

$$\frac{p_{\text{barometric}}}{\gamma_{\text{Hg}}} = 598 \text{ mm} = 0.598 \text{ m}$$

and for $\gamma_{\text{Hg}} = 133 \text{ kN/m}^3$

$$p_0 = (0.598 \text{ m})(133 \text{ kN/m}^3) = 79.5 \text{ kN/m}^2$$

From Table B.2, $\gamma_{\text{H}_2\text{O}} = 9.804 \text{ kN/m}^3$ at 10 °C and therefore

$$p = (9.804 \text{ kN/m}^3)(40 \text{ m}) + 79.5 \text{ kN/m}^2$$
$$= 392 \text{ kN/m}^2 + 79.5 \text{ kN/m}^2 = 472 \text{ kPa (abs)} \qquad \text{(Ans)}$$

Manometry: A standard technique for measuring pressure involves the use of liquid columns in vertical or inclined tubes. Pressure measuring devices based on this technique are called manometers. The mercury barometer is an example of one type of manometer, but there are many other configurations possible, depending on the particular application. Three common types of manometers include the (piezometer tube, the U-tube manometer, and the inclined-tube manometer).

1. Piezometer: The simplest type of manometer consists of a vertical tube, open at the top, and attached to the container in which the pressure is desired, as illustrated in Fig. (2.6). Since manometers involve columns of fluids at rest, the fundamental equation describing their use is Eq. (2.10)

$$p = \gamma h + p_o$$

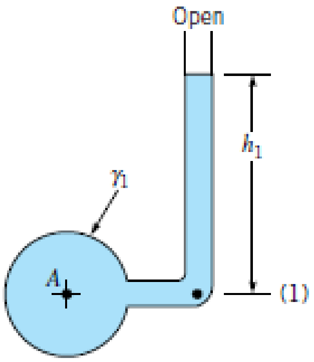


Fig. (2.6)

which gives the pressure at any elevation within a homogeneous fluid in terms of a reference pressure p_o and the vertical distance *h*. Remember that in a fluid at rest pressure will *increase* as we moving *downward* and will decrease as we moving *upward*. Apply this equation to the piezometer tube of Fig. (2.6) indicates that the pressure p_A can be determined by a measurement of h_1 through the relationship: $p_A = \gamma_1 h_1$

where γ_1 is the specific weight of the liquid in the container. Note that since the tube is open at the top, the pressure p_o can be set equal to zero (we are now using gage pressure), with the height h_1 measured from the meniscus at the upper surface to point (1). Since point (1) and point *A* within the container are at the same elevation, then $p_A = p_1$.

2. U-tube manometer: Another type of manometer which is widely used consists of a tube formed into the shape of a U as is shown in Fig. (2.7). The fluid in the manometer is called the *gage fluid*. To find the pressure p_A in terms of the various column heights, we start at one end of the system and work our way around to the other end, simply utilizing Eq. (2.10). Thus, for the U-tube manometer shown in Fig. (2.7), we will start at point *A* and work around to the open end. The pressure at points *A* and (1) are the same, and as we move from point (1) to (2) the pressure will increase by $\gamma_1 h_1$. The pressure at point (2) is equal to the pressure at point (3), since the pressures at equal elevations in a continuous mass of fluid at rest must be the same. Note that we could not simply "jump across" from point (1) to a point at the same elevation in the right-hand tube since these would not be points within the same continuous mass of fluid. With the pressure at point (3) specified we now move to the open end where the pressure is zero. As we move vertically upward the pressure decreases by an amount $\gamma_2 h_2$. In equation form these various steps can be expressed as:

$p_A + \gamma_1 h_1 - \gamma_2 h_2 = 0$, or:

$p_A = \gamma_2 h_2 - \gamma_1 h_1$

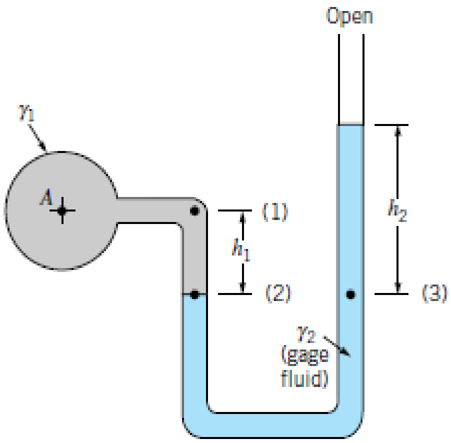


Fig. (2.7)

A major advantage of the U-tube manometer lies in the fact that the gage fluid can be different from the fluid in the container in which the pressure is to be determined. For example, the fluid in *A* in Fig. (2.7) can be either a liquid or a gas. If *A* does contain a gas, the contribution of the gas column, $\gamma_1 h_1$ is almost always negligible so that $p_A = p_2$ and in this instance the above equation becomes: $p_A = \gamma_2 h_2$.

Ex: A closed tank contains compressed air and oil (SG_{oil}=0.9) as is shown in Fig. (2.8). A U-tube manometer using mercury ($SG_{mercury}$=13.6) is connected to the tank as shown. For column heights $h_1 = 36$ in, $h_2 = 6$ in, and $h_3 = 9$ in determine the pressure reading (in 1b/ft^2) of the gage.

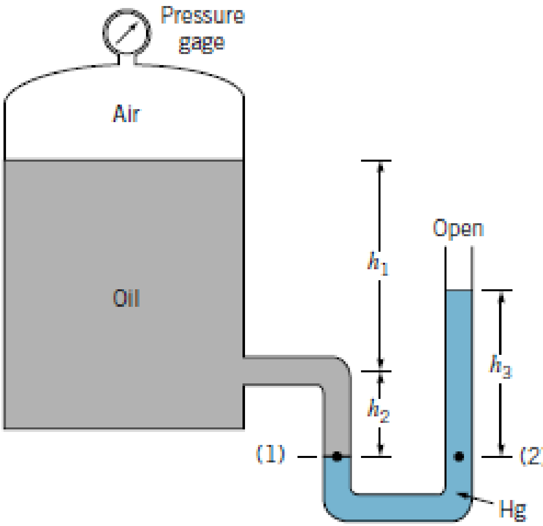


Fig.(2.8)

Following the general procedure of starting at one end of the manometer system and working around to the other, we will start at the air–oil interface in the tank and proceed to the open end where the pressure is zero. The pressure at level (1) is

$$p_1 = p_{\text{air}} + \gamma_{\text{oil}}(h_1 + h_2)$$

This pressure is equal to the pressure at level (2), since these two points are at the same elevation in a homogeneous fluid at rest. As we move from level (2) to the open end, the pressure must decrease by $\gamma_{\text{Hg}}h_3$, and at the open end the pressure is zero. Thus, the manometer equation can be expressed as

$$p_{\text{air}} + \gamma_{\text{oil}}(h_1 + h_2) - \gamma_{\text{Hg}}h_3 = 0$$

or

$$p_{\text{air}} + (SG_{\text{oil}})(\gamma_{\text{H}_2\text{O}})(h_1 + h_2) - (SG_{\text{Hg}})(\gamma_{\text{H}_2\text{O}})h_3 = 0$$

For the values given

$$p_{\text{air}} = -(0.9)(62.4 \text{ lb/ft}^3)\left(\frac{36 + 6}{12} \text{ ft}\right) + (13.6)(62.4 \text{ lb/ft}^3)\left(\frac{9}{12} \text{ ft}\right)$$

so that

$$p_{\text{air}} = 440 \text{ lb/ft}^2$$

3. The Differential Manometer:

The U-tube manometer is also widely used to measure the *difference* in pressure between two containers or two points in a given system. Consider a manometer connected between containers *A* and *B* as is shown in Fig. (2.9). The difference in pressure between *A* and *B* can be found by again starting at one end of the system and working around to the other end. For example, at *A* the pressure is p_A which is equal to p_1 and as we move to point (2) the pressure increases by $\gamma_1 h_1$. The pressure at p_2 is equal to p_3, and as we move upward to point (4) the pressure decreases by $\gamma_2 h_2$.Similarly, as we continue to move upward from point (4) to (5) the pressure decreases by $\gamma_3 h_3$.Finally, $p_5 = p_B$ since they are at equal elevations. Thus,

$$p_A + \gamma_1 h_1 - \gamma_2 h_2 - \gamma_3 h_3 = p_B$$

and the pressure difference is:

$$p_A - p_B = \gamma_2 h_2 + \gamma_3 h_3 - \gamma_1 h_1$$

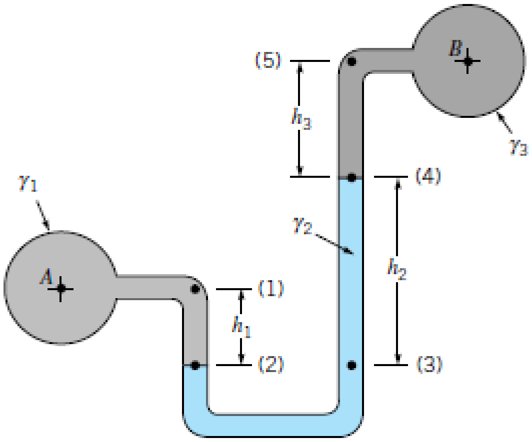


Fig. (2.9)

4. Inclined-Tube Manometer: To measure small pressure changes, a manometer of the type shown in Fig. (2.10) is frequently used. One leg of the manometer is inclined at an angle θ and the differential reading l_2 is measured along the inclined tube. The difference in pressure $p_A - p_B$ can be expressed as:

$p_A + \gamma_1 h_1 - \gamma_2 l_2 \sin\theta - \gamma_3 h_3 = p_B$ or

$p_A - p_B = \gamma_2 l_2 \sin\theta + \gamma_3 h_3 - \gamma_1 h_1$

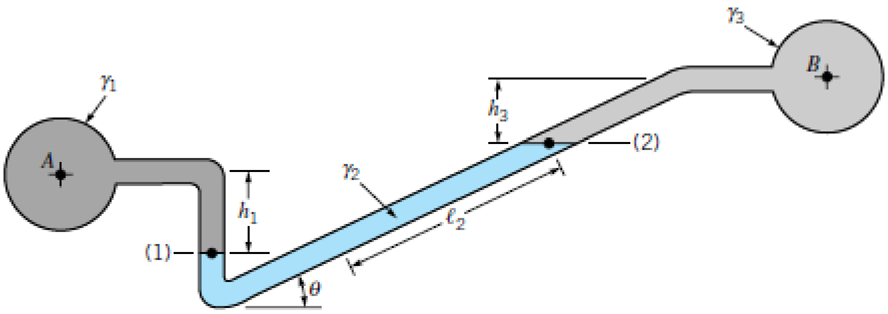
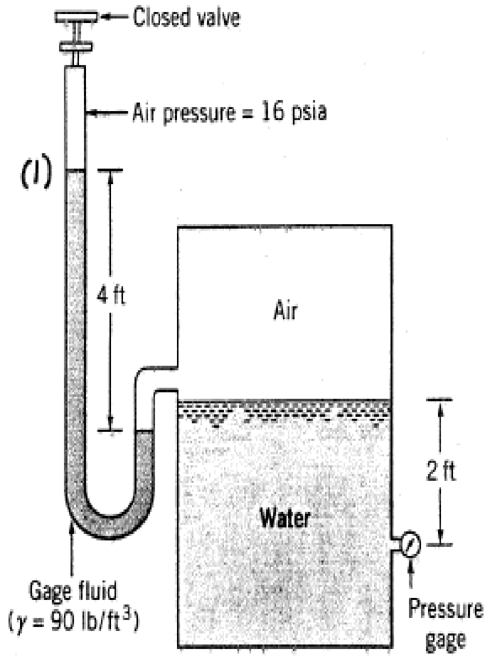


Fig. (2.10)

2.24 A U-tube manometer is connected to a closed tank containing air and water as shown in Fig. P2.24. At the closed end of the manometer the air pressure is 16 psia. Determine the reading on the pressure gage for a differential reading of 4 ft on the manometer. Express your answer in psi (gage). Assume standard atmospheric pressure, and neglect the weight of the air columns in the manometer.



■ FIGURE P2.24

$P_{abs}=P_{atm}+P_{gage}$, or $P_{abs}-P_{atm}=P_{gage}$, P_{atm}=14.7psi at sea level

$$p_1 + \gamma_{gf}\,(4\,ft) + \gamma_{H_2O}\,(2\,ft) = p_{gage}$$

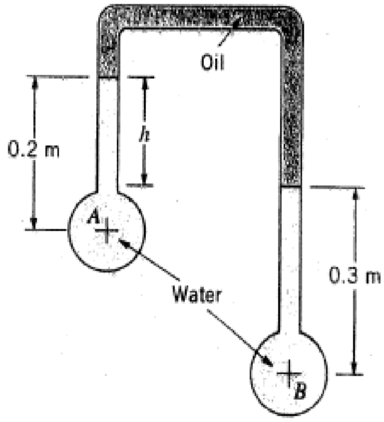
Thus,

$$p_{gage} = \left(16\,\frac{lb}{in.^2} - 14.7\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right) + \left(90\,\frac{lb}{ft^3}\right)(4\,ft) + \left(62.4\,\frac{lb}{ft^3}\right)(2\,ft)$$

$$= 672\,\frac{lb}{ft^2} = \left(672\,\frac{lb}{ft^2}\right)\left(\frac{1\,ft^2}{144\,in.^2}\right) = \underline{\underline{4.67\ psi}}$$

H.W:

2.29 The inverted U-tube manometer of Fig. P2.29 contains oil ($SG = 0.9$) and water as shown. The pressure differential between pipes A and B, $p_A - p_B$, is -5 kPa. Determine the differential reading, h.



■ FIGURE P2.29

$$p_A - \gamma_{H_2O}(0.2 m) + \gamma_{oil}(h) + \gamma_{H_2O}(0.3 m) = p_B$$

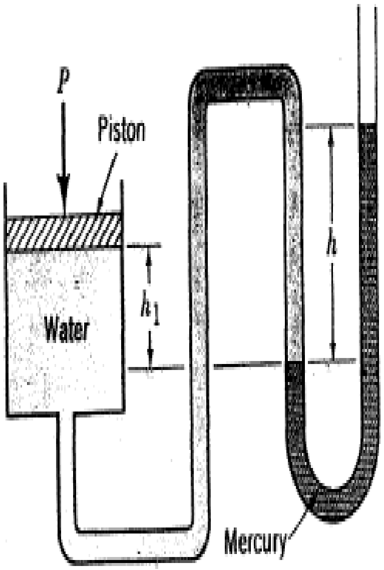
Thus,

$$h = \frac{(p_B - p_A) + \gamma_{H_2O}(0.2 m) - \gamma_{H_2O}(0.3 m)}{\gamma_{oil}}$$

$$= \frac{5\times10^3 \frac{N}{m^2} - (9.80\times10^3 \frac{N}{m^3})(0.1 m)}{8.95\times10^3 \frac{N}{m^3}} = \underline{\underline{0.449 m}}$$

H.W:

2.31 A piston having a cross-sectional area of $0.07 m^2$ is located in a cylinder containing water as shown in Fig. P2.31. An open U-tube manometer is connected to the cylinder as shown. For $h_1 = 60$ mm and $h = 100$ mm, what is the value of the applied force, P, acting on the piston? The weight of the piston is negligible.



2.32 For the inclined-tube manometer of Fig. P2.32 the pressure in pipe A is 0.6 psi. The fluid in both pipes A and B is water, and the gage fluid in the manometer has a specific gravity of 2.6. What is the pressure in pipe B corresponding to the differential reading shown?

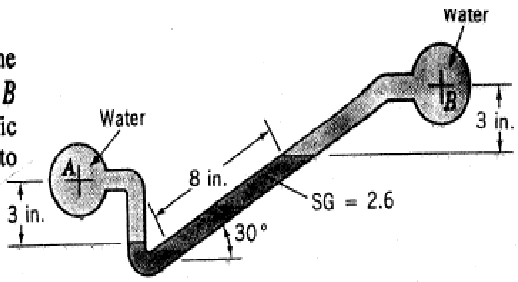


FIGURE P2.32

$$p_A + \gamma_{H_2O}\left(\tfrac{3}{12}\,\text{ft}\right) - \gamma_{gf}\left(\tfrac{8}{12}\,\text{ft}\right)\sin 30° - \gamma_{H_2O}\left(\tfrac{3}{12}\,\text{ft}\right) = p_B$$

(where γ_{gf} is the specific weight of the gage fluid)

Thus,

$$p_B = p_A - \gamma_{gf}\left(\tfrac{8}{12}\,\text{ft}\right)\sin 30°$$

$$= \left(0.6\,\frac{\text{lb}}{\text{in.}^2}\right)\left(144\,\frac{\text{in.}^2}{\text{ft}^2}\right) - (2.6)\left(62.4\,\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{8}{12}\,\text{ft}\right)(0.5) = 32.3\,\frac{\text{lb}}{\text{ft}^2}$$

$$= 32.3\,\text{lb/ft}^2 / 144\,\text{in.}^2/\text{ft}^2 = \underline{\underline{0.224\ \text{psi}}}$$

2.36 Determine the elevation difference, Δh, between the water levels in the two open tanks shown in Fig. P2.36.

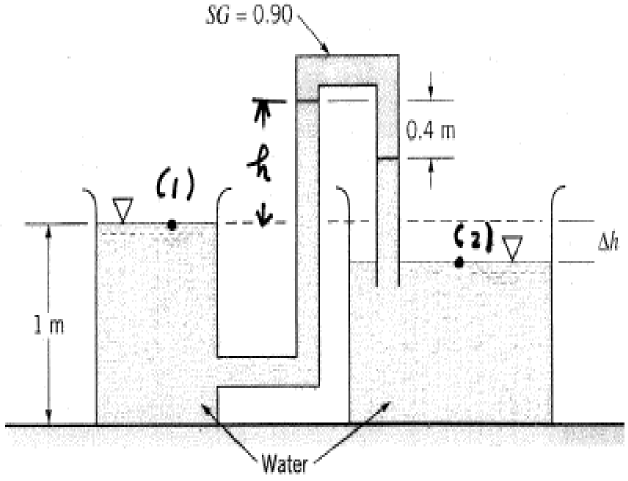


FIGURE P2.36

$$p_1 - \gamma_{H_2O}\,h + (SG)\,\gamma_{H_2O}\,(0.4\,\text{m}) + \gamma_{H_2O}\,(h - 0.4\,\text{m}) + \gamma_{H_2O}(\Delta h) = p_2$$

Since $p_1 = p_2 = 0$

$$\Delta h = 0.4\,\text{m} - (0.9)(0.4\,\text{m}) = \underline{\underline{0.040\,\text{m}}}$$

Mechanical Pressure Measuring Devices:

Although manometers are widely used, they are not well suited for measuring very high pressures, or pressures that are changing rapidly with time. In addition, they require the measurement of one or more column heights, which, although not particularly difficult, can be time consuming. To overcome some of these problems numerous other types of pressure measuring instruments have been developed. Most of these make use of the idea that when a pressure acts on an elastic structure the (*structure will deform*), and this deformation can be related to the magnitude of the pressure. Probably the most familiar device of this kind is the *Bourdon* pressure gage, which is shown in Fig. 2.11*a*. The essential mechanical element in this gage is the hollow elastic curved tube (Bourdon tube) which is connected to the pressure source as shown in Fig. 2.11*b*. As the pressure within the tube increases the tube tends to straighten, and although the deformation is small, it can be translated into the motion of a pointer on a dial as illustrated. Since it is the difference in pressure between the outside of the tube (atmospheric pressure) and the inside of the tube that causes the movement of the tube, the indicated pressure is *gage pressure*. A zero reading on the gage indicates that the measured pressure is equal to the *local atmospheric pressure*.

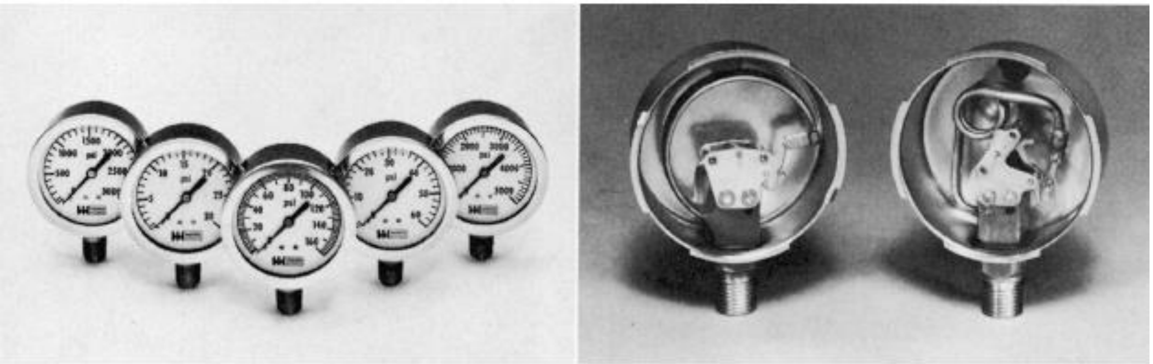


Fig. (2.11)

Hydrostatic Force on a Plane Surface:

When a surface is submerged in a fluid, forces develop on the surface due to the fluid. The determination of these forces is important in the design of storage tanks, ships, dams, and other hydraulic structures. For fluids at rest we know that the force must be *perpendicular* to the surface since there are no shearing stresses present. We also know that the pressure will vary linearly with depth if the fluid is incompressible. For a horizontal surface, such as the bottom of a liquid-filled tank (Fig. 2.12), the magnitude of the resultant force is simply $F_R = pA$ where p is the uniform pressure on the bottom and A is the area of the bottom. For the open tank shown, $p = \gamma h$. Note that if atmospheric pressure acts on both sides of the bottom, as is illustrated, the *resultant force* on the bottom is simply due to the liquid in the tank. Since the pressure is constant and uniformly distributed over the bottom, the resultant force acts through the *centroid of the area* as shown in Fig. (2.12).

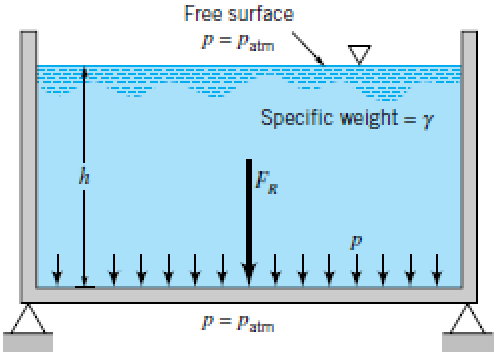


Fig. (2.12)

For the more general case in which a submerged plane surface is inclined, as is illustrated in Fig. (2.13), the determination of the resultant force acting on the surface is more involved. For the present we will assume that the fluid surface is open to the atmosphere. Let the plane in which the surface lies intersect the free surface at 0 and make an angle θ with this surface as in Fig. (2.13). The x–y coordinate system is defined so that 0 is the origin and y is directed along the surface as shown. The area can have an arbitrary shape as shown. We wish to determine the direction, location, and magnitude of the resultant force acting on one side of this area due to the liquid in contact with the area. At any given depth, h, the force acting on dA (the differential area of Fig. 2.13) is $dF = \gamma h dA$ and is perpendicular to the surface. Thus, the magnitude of the resultant force can be found by summing these differential forces over the entire surface. In equation form:

$$F_R = \int_A \gamma h dA = \int_A \gamma y \sin\theta dA, \text{ or,}$$

$$F_R = \gamma \sin\theta \int_A y dA \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.13)$$

The integral appearing in Eq. (2.13) is the *first moment of the area* or centroid with respect to the x axis, so we can write $\int_A y dA = y_c A$,

Where (y_c) is the y coordinate of the centroid measured from the x axis which passes through 0. Equation (2.13) can thus be written as:

$F_R = \gamma A y_c \sin\theta$, ……………………………………………………..(2.14a)

or more simply as: $F_R = \gamma A h_c$ ……………………………………………(2.14b)

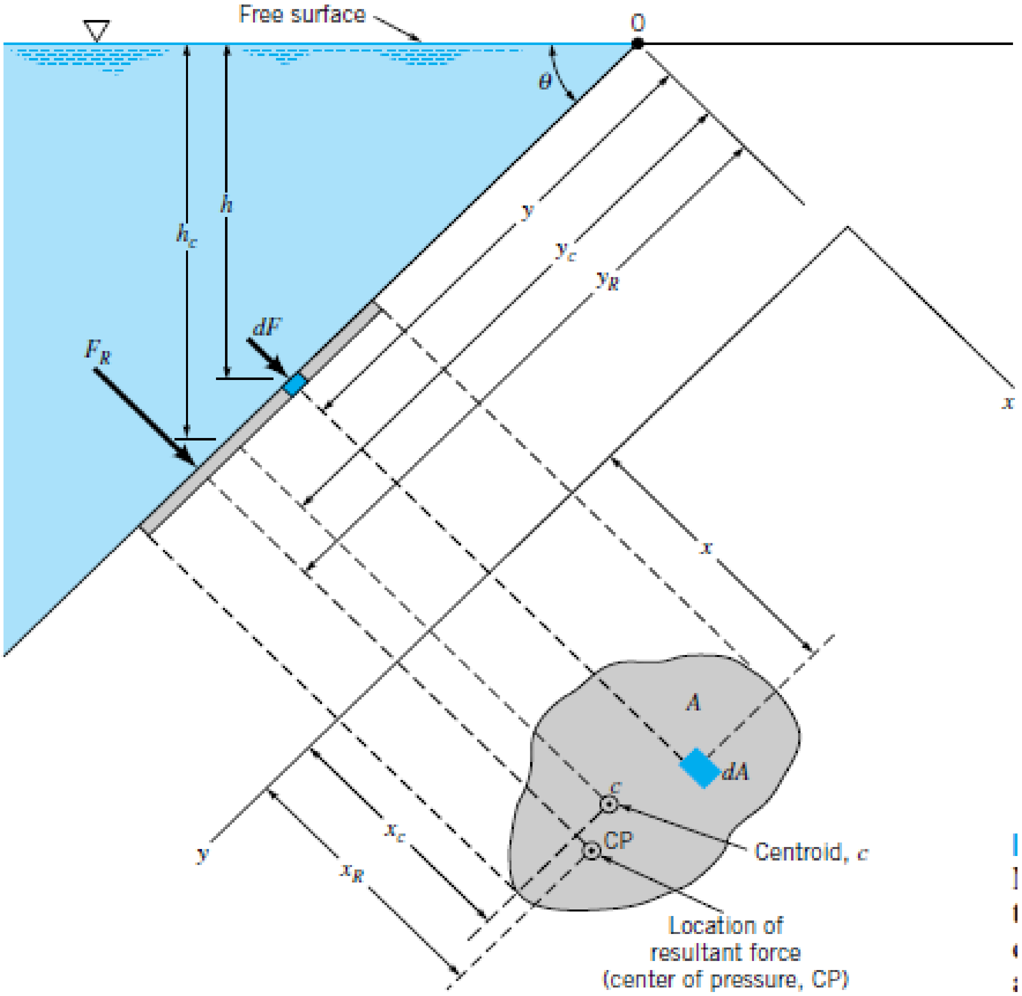


Fig. (2.13)

Where (h_c) is the vertical distance from the fluid surface to the centroid of the area. Note that the magnitude of the force is independent of the angle θ and depends only on the specific weight of the fluid, the total area, and the depth of the centroid of the area below the surface.

In effect, Eq. (2.14) indicates that the magnitude of the resultant force is equal to the pressure at the centroid of the area multiplied by the total area. Since all the differential forces that were summed to obtain(F_R) are perpendicular to the surface, the resultant F_R must also be perpendicular to the surface. Although our intuition might suggest that the resultant force should pass through the centroid of the area, this is not actually in this case.

The *y* coordinate, (y_R) of the resultant force can be determined by summation of moments around the *x* axis. That is, the moment of the resultant force must equal the moment of the distributed pressure force, or:

$F_R y_R = \int_A y\,dF = \int_A \gamma \sin\theta\, y^2 dA$, and therefore, since: $F_R = \gamma A y_c \sin\theta$

Where: $y_R = \dfrac{\int_A y^2 dA}{y_c A}$

The integral in the numerator is the *second moment of the area (moment of inertia),* I_x with respect to an axis formed by the intersection of the plane containing the surface and the free surface (*x* axis). Thus, we can write:

$y_R = \dfrac{I_x}{y_c A}$, Use can now be made of the parallel axis theorem to express I_x as:

$I_x = I_{xc} + A y_c^2$

Where I_{xc} is the second moment of the area with respect to an axis passing through its *centroid* and parallel to the *x* axis. Thus,

$y_R = \dfrac{I_{xc}}{y_c A} + y_c$ …………………………………………..……………..(2.15a)

or

$h_R = \dfrac{I_{xc} \sin\theta}{h_c A} + h_c$ ……………………………………………………….(2.15b)

with respect to vertical distance from center of pressure to surface of water.

The *x* coordinate, (x_R) for the resultant force can be determined in a similar manner by summing moments about the *y* axis. Thus,

$F_R x_R = \int_A \gamma \sin\theta\, xy\,dA$, and therefore, $x_R = \dfrac{\int_A xy\,dA}{y_c A} = \dfrac{I_{xy}}{y_c A}$

where I_{xy} is the product of inertia with respect to the *x* and *y* axes. Again, using the parallel axis theorem, we can write:

$x_R = \dfrac{I_{xyc}}{y_c A} + x_c$ …………………………………………………………..(2.16)

where I_{xyc} is the product of inertia with respect to an orthogonal coordinate system passing through the *centroid* of the area and formed by a translation of the *x*-*y* coordinate system. If the submerged area is symmetrical with respect to an axis passing through the centroid and parallel to either the *x* or *y* axes, the resultant force must lie along the line $x = x_c$ since I_{xyc} is identically zero in this case. The point through which the resultant force acts is called the *center of pressure*.

Centroid coordinates and moments of inertia for some common areas are given in Fig. (2.14).

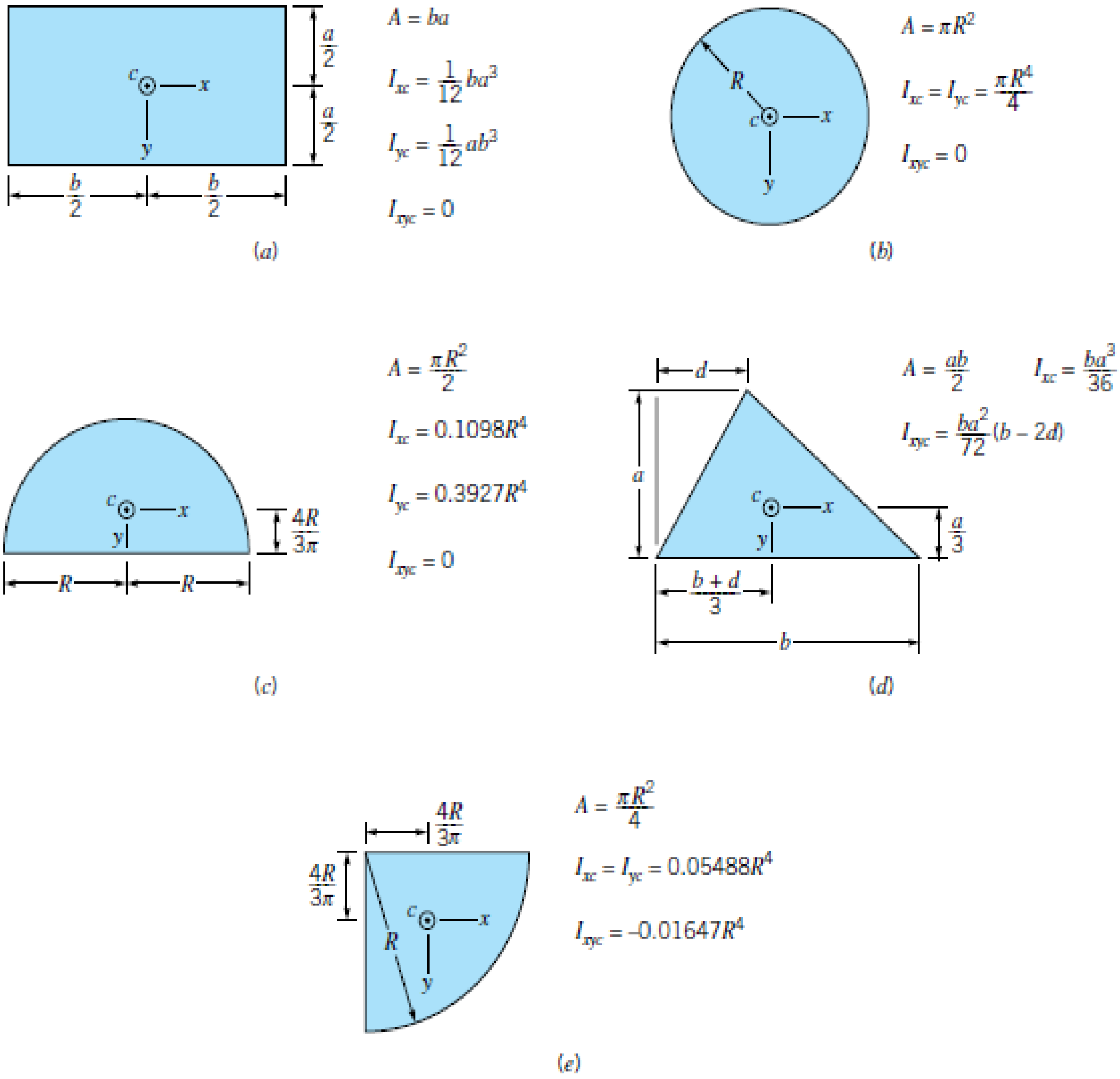
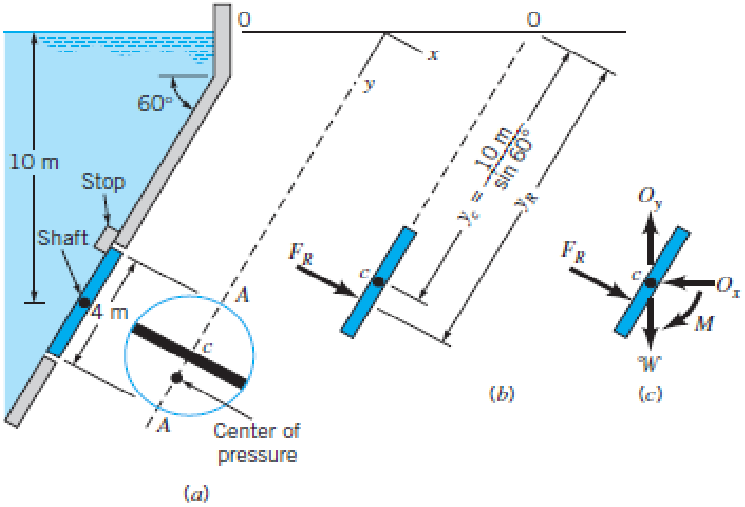


Fig. (2.14)

Ex: The 4-m-diameter circular gate of Fig. (E2.6*a*) is located in the inclined wall of a large reservoir containing water ($\gamma_w = 9.80 kN/m^3$). The gate is mounted on a shaft along its horizontal diameter. For a water depth of 10 m above the shaft determine: (a) the magnitude and location of the resultant force exerted on the gate by the water, and (b) the moment that would have to be applied to the shaft to open the gate.



■ FIGURE E2.6

SOLUTION

(a) To find the magnitude of the force of the water we can apply Eq. 2.18,

$$F_R = \gamma h_c A$$

and since the vertical distance from the fluid surface to the centroid of the area is 10 m it follows that

$$F_R = (9.80 \times 10^3 \text{ N/m}^3)(10 \text{ m})(4\pi \text{ m}^2)$$
$$= 1230 \times 10^3 \text{ N} = 1.23 \text{ MN}$$ **(Ans)**

To locate the point (center of pressure) through which F_R acts, we use Eqs. 2.19 and 2.20,

$$x_R = \frac{I_{xyc}}{y_c A} + x_c \qquad y_R = \frac{I_{xc}}{y_c A} + y_c$$

For the coordinate system shown, $x_R = 0$ since the area is symmetrical, and the center of pressure must lie along the diameter *A-A*. To obtain y_R, we have from Fig. 2.18

$$I_{xc} = \frac{\pi R^4}{4}$$

and y_c is shown in Fig. E2.6*b*. Thus,

$$y_R = \frac{(\pi/4)(2\text{ m})^4}{(10\text{ m}/\sin 60°)(4\pi\text{ m}^2)} + \frac{10\text{ m}}{\sin 60°}$$

$$= 0.0866\text{ m} + 11.55\text{ m} = 11.6\text{ m}$$

and the distance (along the gate) below the shaft to the center of pressure is

$$y_R - y_c = 0.0866\text{ m}$$ **(Ans)**

We can conclude from this analysis that the force on the gate due to the water has a magnitude of 1.23 MN and acts through a point along its diameter *A-A* at a distance of 0.0866 m (along the gate) below the shaft. The force is perpendicular to the gate surface as shown.

b) The moment required to open the gate can be obtained with the aid of the free-body diagram of Fig. E2.6*c*. In this diagram $\mathcal{W}$ is the weight of the gate and O_x and O_y are the horizontal and vertical reactions of the shaft on the gate. We can now sum moments about the shaft

$$\sum M_c = 0$$

and, therefore,

$$M = F_R(y_R - y_c)$$

$$= (1230 \times 10^3\text{ N})(0.0866\text{ m})$$

$$= 1.07 \times 10^5\text{ N}\cdot\text{m}$$ **(Ans)**

Hydrostatic Force on a Curved Surface:

For example, consider the curved section *BC* of the open tank of Fig. (2.15*a*). We wish to find the resultant fluid force acting on this section,which has a unit length perpendicular to the plane of the paper. We first isolate a volume of fluid that is bounded by the surface of interest, in this instance section *BC*, the horizontal plane surface *AB*, and the vertical plane surface *AC*. The free-body diagram for this volume is shown in Fig. (2.15*b*). The magnitude and location of forces F_1 and F_2 can be determined from the relationships for planar surfaces.

The weight, W is simply the specific weight of the fluid times the enclosed volume and acts through the center of gravity (CG) of the mass of fluid contained within the volume. The forces F_H and F_V represent the components of the force that the tank *exerts on the fluid*.

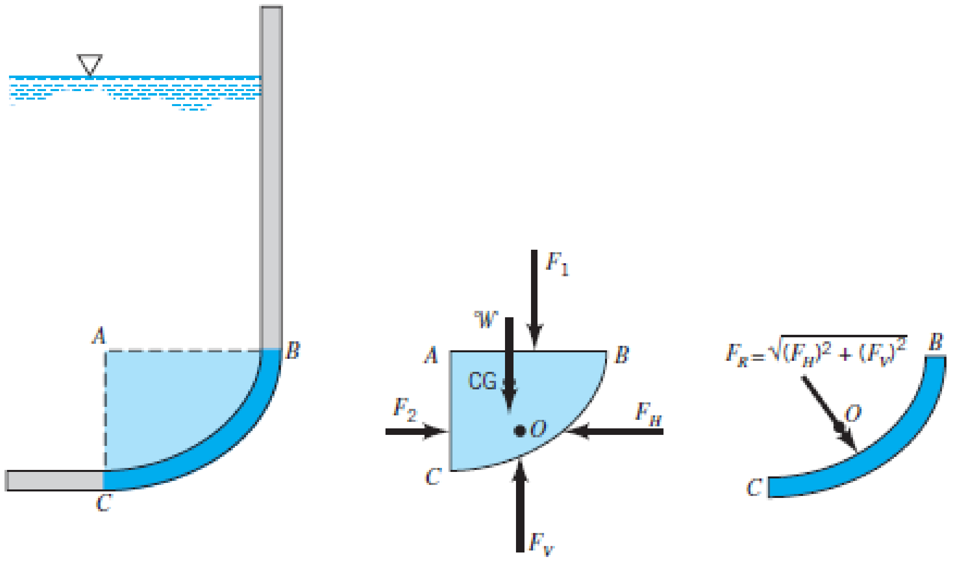
In order for this force system to be in equilibrium, the horizontal component F_H must be equal in magnitude and collinear with F_2 and the vertical component F_V equal in magnitude and collinear with the resultant of the vertical forces F_1 and W.This follows since the three forces acting on the fluid mass (F_2 the resultant of F_1 and W ,and the resultant force that the tank exerts on the mass) must form a *concurrent* force system. Thus,

$$F_H = F_2$$

$$F_V = F_1 + W$$

and the magnitude of the resultant is obtained from the equation”

$$F_R = \sqrt{(F_H)^2 + (F_V)^2}$$

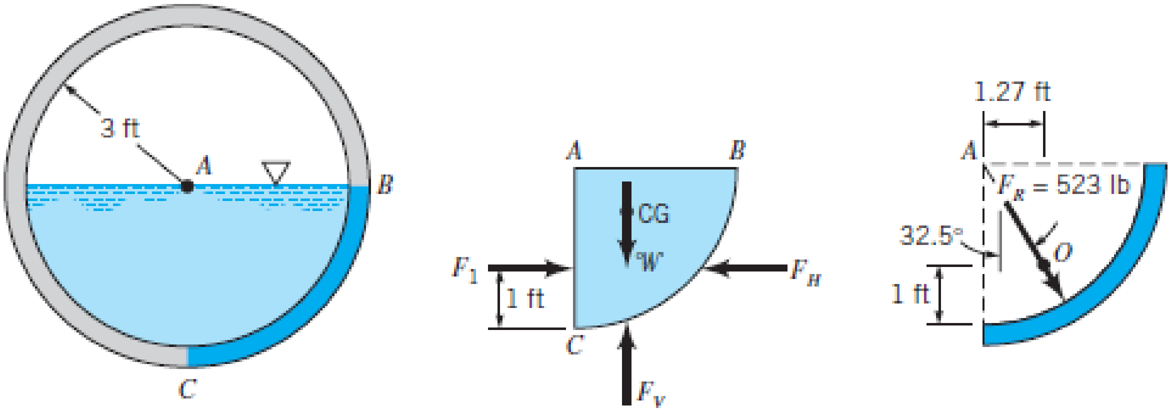


(*a*) (*b*) (*c*)

Fig. (2.15)

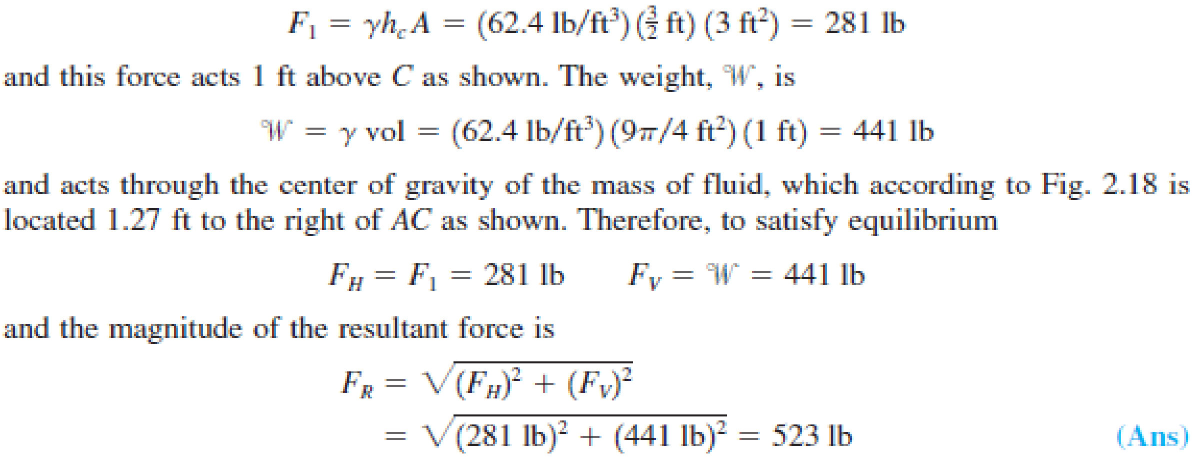
The resultant F_R passes through the point *O*, which can be located by summing moments about an appropriate axis. The resultant force of the fluid acting *on the curved surface BC* is equal and opposite in direction to that obtained from the free-body diagram of Fig. (2.15*b*).The desired fluid force is shown in Fig. (2.15*c*).

Ex: The 6-ft-diameter drainage conduit of Fig. E2.9*a* is half full of water at rest. Determine the magnitude and line of action of the resultant force that the water exerts on a 1-ft length of the curved section *BC* of the conduit wall.



(*a*) (*b*) (*c*)

Sol: We first isolate a volume of fluid bounded by the curved section *BC*, the horizontal surface *AB*, and the vertical surface *AC*, as shown in Fig. E2.9*b*. The volume has a length of 1 ft. The forces acting on the volume are the horizontal force, F_1 which acts on the vertical surface *AC*, the weight, W of the fluid contained within the volume, and the horizontal and vertical components of the force of the conduit wall on the fluid, F_H and F_V respectively. The magnitude of F_1 is found from the equation:



$$F_1 = \gamma h_c A = (62.4\ \text{lb/ft}^3)\left(\tfrac{3}{2}\ \text{ft}\right)(3\ \text{ft}^2) = 281\ \text{lb}$$

and this force acts 1 ft above C as shown. The weight, $\mathcal{W}$, is

$$\mathcal{W} = \gamma\ \text{vol} = (62.4\ \text{lb/ft}^3)(9\pi/4\ \text{ft}^2)(1\ \text{ft}) = 441\ \text{lb}$$

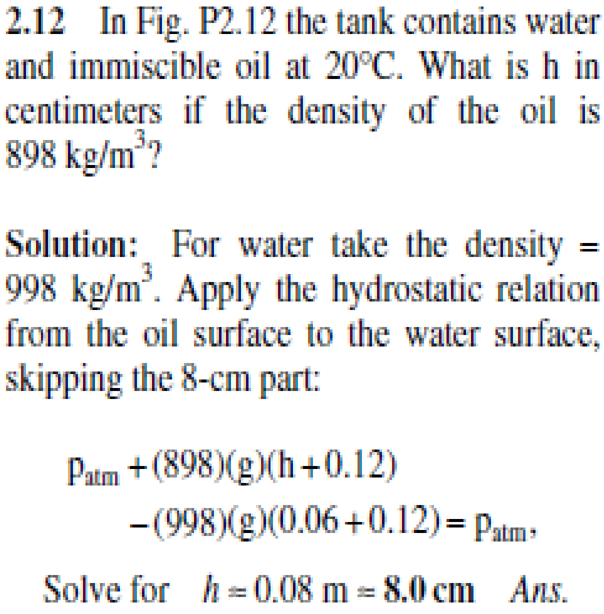
and acts through the center of gravity of the mass of fluid, which according to Fig. 2.18 is located 1.27 ft to the right of AC as shown. Therefore, to satisfy equilibrium

$$F_H = F_1 = 281\ \text{lb} \qquad F_V = \mathcal{W} = 441\ \text{lb}$$

and the magnitude of the resultant force is

$$F_R = \sqrt{(F_H)^2 + (F_V)^2}$$
$$= \sqrt{(281\ \text{lb})^2 + (441\ \text{lb})^2} = 523\ \text{lb} \qquad \textbf{(Ans)}$$

The force the water exerts *on* the conduit wall is equal, but *opposite in direction*, to the forces F_H and F_V shown in Fig. E2.9*b*. Thus, the resultant force *on the conduit wall* is shown in Fig. E2.9*c*.
This force acts through the point *O* at the angle shown.



2.12 In Fig. P2.12 the tank contains water and immiscible oil at 20°C. What is h in centimeters if the density of the oil is 898 kg/m³?

Solution: For water take the density = 998 kg/m³. Apply the hydrostatic relation from the oil surface to the water surface, skipping the 8-cm part:

$$p_{atm} + (898)(g)(h + 0.12) - (998)(g)(0.06 + 0.12) = p_{atm},$$

Solve for $h \approx 0.08\ \text{m} \approx \mathbf{8.0\ cm}$ *Ans.*

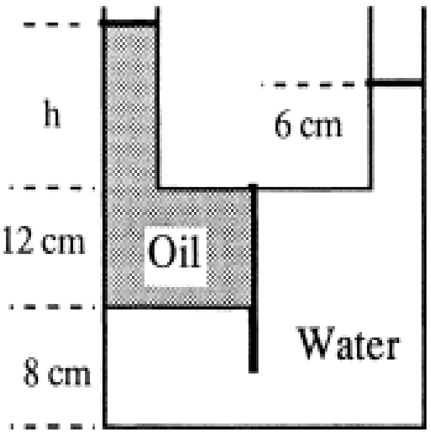


Fig. P2.12

2.13 In Fig. P2.13 the 20°C water and gasoline are open to the atmosphere and are at the same elevation. What is the height h in the third liquid?

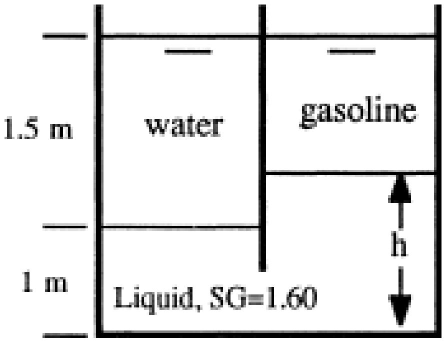


Fig. P2.13

Solution: Take water = 9790 N/m^3 and gasoline = 6670 N/m^3. The bottom pressure must be the same whether we move down through the water or through the gasoline into the third fluid:

$$p_{\text{bottom}} = (9790\ \text{N/m}^3)(1.5\ \text{m}) + 1.60(9790)(1.0) = 1.60(9790)h + 6670(2.5 - h)$$

Solve for $h = \mathbf{1.52\ m}$ *Ans.*

2.23 In Fig. P2.23 both fluids are at 20°C. If surface tension effects are negligible, what is the density of the oil, in kg/m^3?

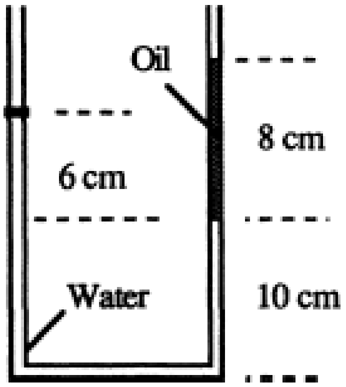


Fig. P2.23

Solution: Move around the U-tube from left atmosphere to right atmosphere:

$$p_a + (9790\ \text{N/m}^3)(0.06\ \text{m}) - \gamma_{\text{oil}}(0.08\ \text{m}) = p_a,$$

$$\text{solve for}\quad \gamma_{\text{oil}} \approx 7343\ \text{N/m}^3,$$

$$\text{or:}\quad \rho_{\text{oil}} = 7343/9.81 \approx \mathbf{748\ kg/m^3}\quad \textit{Ans.}$$

2.32 For the manometer of Fig. P2.32, all fluids are at 20°C. If $p_B - p_A = 97$ kPa, determine the height H in centimeters.

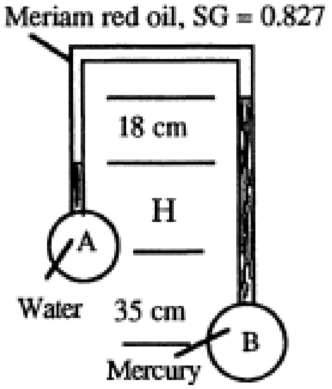


Fig. P2.32

Solution: Gamma = 9790 N/m^3 for water and 133100 N/m^3 for mercury and (0.827)(9790) = 8096 N/m^3 for Meriam red oil. Work your way around from point A to point B:

$$p_A - (9790\ \text{N/m}^3)(\text{H meters}) - 8096(0.18)$$

$$+133100(0.18 + H + 0.35) = p_B = p_A + 97000.$$

$$\text{Solve for}\quad H \approx 0.226\ \text{m} = \mathbf{22.6\ cm}\quad \textit{Ans.}$$

2.51 Gate AB in Fig. P2.51 is 1.2 m long and 0.8 m into the paper. Neglecting atmospheric-pressure effects, compute the force F on the gate and its center of pressure position *X*.

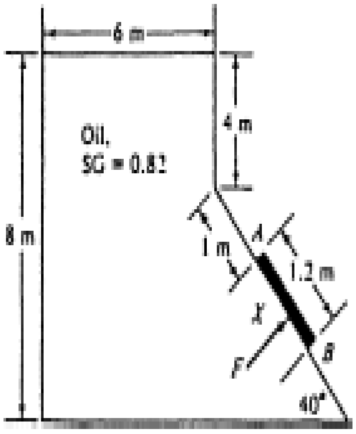


Fig. P2.51

Solution: The centroidal depth of the gate is

$$h_{CG} = 4.0 + (1.0 + 0.6)\sin 40° = 5.028 \text{ m},$$

$$\text{hence} \quad F_{AB} = \gamma_{oil} h_{CG} A_{gate} = (0.82 \times 9790)(5.028)(1.2 \times 0.8) = \mathbf{38750\ N} \quad \textit{Ans.}$$

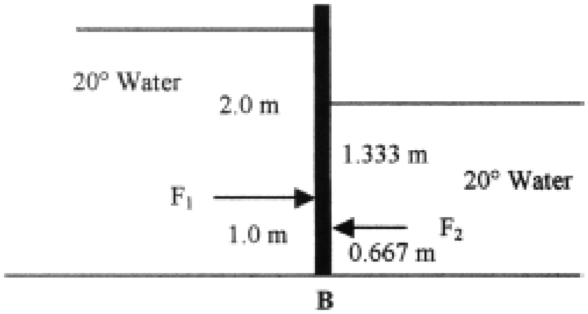
The line of action of F is slightly below the centroid by the amount

$$y_{CP} = -\frac{I_{xx}\sin\theta}{h_{CG}A} = -\frac{(1/12)(0.8)(1.2)^3 \sin 40°}{(5.028)(1.2 \times 0.8)} = -0.0153 \text{ m}$$

Thus the position of the center of pressure is at $X = 0.6 + 0.0153 \approx \mathbf{0.615\ m}$ *Ans.*

2.52 A vertical lock gate is 4 m wide and separates 20°C water levels of 2 m and 3 m, respectively. Find the moment about the bottom required to keep the gate stationary.

Solution: On the side of the gate where the water measures 3 m, F_1 acts and has an h_{CG} of 1.5 m; on the opposite side, F_2 acts with an h_{CG} of 1 m.



$$F_1 = \gamma h_{CG1}A_1 = (9790)(1.5)(3)(4) = 176,220\ \mathrm{N}$$

$$F_2 = \gamma h_{CG2}A_2 = (9790)(1.0)(2)(4) = 78,320\ \mathrm{N}$$

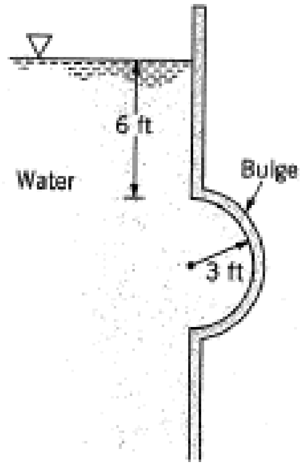
$$y_{CP1} = [-(1/12)(4)(3)^3 \sin 90°]/[(1.5)(4)(3)] = -0.5\ \mathrm{m};\ \text{so } F_1 \text{ acts at } 1.5 - 0.5$$
$$= 1.0\ \mathrm{m\ above\ B}$$

$$y_{CP2} = [-(1/12)(4)(2)^3 \sin 90°]/[(1)(4)(2)] = -0.333\ \mathrm{m};\ F_2 \text{ acts at } 1.0 - 0.33$$
$$= 0.67\ \mathrm{m\ above\ B}$$

Taking moments about points B (see the figure),

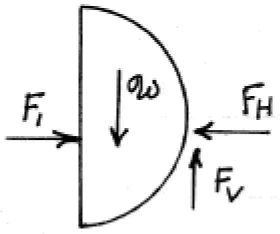
$$\sum M_B = (176,220\ \mathrm{N})(1.0\ \mathrm{m}) - (78,320\ \mathrm{N})(0.667\ \mathrm{m})$$
$$= 124,000\ \mathrm{N\cdot m};\quad \boldsymbol{M_{bottom} = 124\ kN\cdot m.}$$

2.78 An open tank containing water has a bulge in its vertical side that is semicircular in shape as shown in Fig. P2.78. Determine the horizontal and vertical components of the force that the water exerts on the bulge. Base your analysis on a 1-ft length of the bulge.



F_H ~ horizontal force of wall on fluid

F_V ~ vertical force of wall on fluid



$$\mathcal{W} = \gamma_{H_2O} \, \forall ol$$

$$= \left(62.4 \frac{lb}{ft^3}\right)\left(\frac{\pi (3 ft)^2}{2}\right)(1 \; ft)$$

$$= 882 \; lb$$

$$F_1 = \gamma h_c A = \left(62.4 \frac{lb}{ft^3}\right)(6 ft + 3 ft)(6 ft \times 1 ft)$$

$$= 337 \, lb$$

For equilibrium, $F_V = \mathcal{W} = 882 \; lb \uparrow$

and $F_H = F_1 = 337 \; lb \leftarrow$

The force the water exerts <u>on</u> the bulge is equal to, but opposite in direction to F_V and F_H above. Thus,

$$\underline{(F_H)_{wall} = 337 \; lb \longrightarrow}$$

$$\underline{(F_V)_{wall} = 882 \; lb \downarrow}$$

Chapter Three
Fluid Dynamics

Objective of this chapter:

- Introduce the necessary concept to analyze fluids in motion case.
- Identify differences between Steady and unsteady uniform and non-uniform compressible and incompressible flow.
- Demonstrate streamlines and stream tubes.
- Introduce the Continuity principle through conservation of mass and control volumes. This section discusses the analysis of fluid in motion - fluid dynamics. The motion of fluids can be predicted in the same way as the motion of solids are predicted using the fundamental laws of physics together with the physical properties of the fluid.

Classifications of Fluids:

3.1 Uniform Flow, Steady Flow:

It is possible - and useful - to classify the type of flow which is being examined into small number of groups. If we look at a fluid flowing under normal circumstances - a river for example - the conditions at one point will vary from those at another point (e.g. different velocity) we have non-uniform flow. If the conditions at one point vary as time passes then we have unsteady flow. The following terms describe the states which are used to classify fluid flow:

- *uniform flow:* If the flow velocity has is the same magnitude and direction at every point in the fluid it is said to be *uniform*.
- *non-uniform:* If at a given instant, the velocity is **not** the same at every point the flow is *non-uniform*.

(In practice, by this definition, every fluid that flows near a solid boundary will be non-uniform – as the fluid at the boundary must take the speed of the boundary, usually zero. However if the size and shape of the the cross-section of the stream of fluid is constant the flow is considered *uniform*.)

- *steady:* A steady flow is one in which the conditions (velocity, pressure and cross- section) may differ from point to point in direction of flow but DO NOT change with respect to time.
- *unsteady:* If at any point in the fluid, the conditions change with time, the flow is described as *unsteady*. (In practice there is always slight variations in velocity and pressure, but if the average values are constant, the flow is considered *steady*.

Combining the above we can classify any flow in to one of four type:

1. *Steady uniform flow*. Conditions do not change with position in the stream or with time. An example is the flow of water in a pipe of constant diameter at constant velocity.

2. *Steady non-uniform flow*. Conditions change from point to point in the stream but do not change with time. An example is flow in a tapering pipe with constant velocity at the inlet - velocity will change as you move along the length of the pipe toward the exit.

3. *Unsteady uniform flow*. At a given instant in time the conditions at every point are the same, but will change with time. An example is a pipe of constant diameter connected to a pump pumping at a constant rate which is then switched off.

4. *Unsteady non-uniform flow*. Every condition of the flow may change from point to point and with time at every point. For example waves in a channel.

If you imaging the flow in each of the above classes you may imagine that one class is more complex than another. And this is the case - *steady uniform flow* is by far the most simple of the four. You will then be pleased to hear that this course is restricted to only this class of flow. We will not be encountering any non-uniform or unsteady effects in any of the examples (except for one or two quasi-time dependent problems which can be treated at steady).

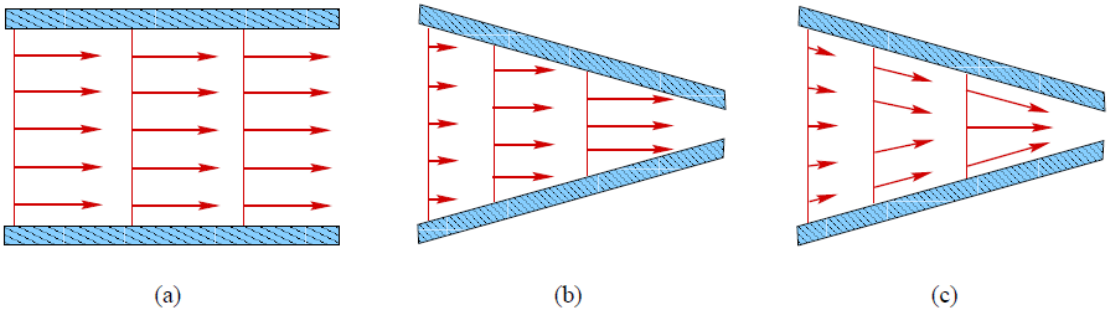


Figure 2.15: Uniform and non-uniform flows; (a) uniform flow, (b) non-uniform, but "locally uniform" flow, (c) non-uniform flow.

3.2 Compressible or Incompressible

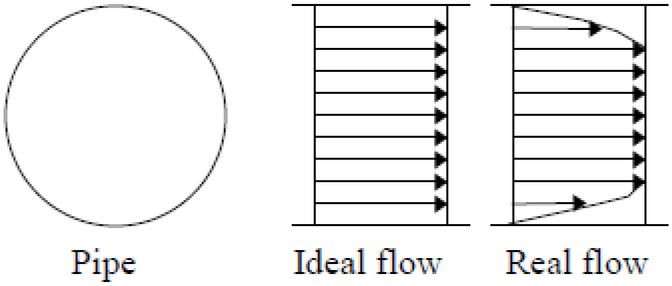
All fluids are compressible - even water - their <u>*density will change as pressure changes*</u>. Under steady conditions, and provided that the changes in pressure are small, it is usually possible to simplify analysis of the flow by assuming it is <u>*incompressible and has constant density*</u>. As you will appreciate, liquids are quite difficult to compress - so under most steady conditions they are treated as incompressible. In some unsteady conditions very high pressure differences can occur and it is necessary to take these into account - even for liquids. Gasses, on the contrary, are very easily compressed, it is essential in most cases to treat these as compressible, taking changes in pressure into account.

3.3 Three-dimensional flow

Although in general all fluids flow three-dimensionally, with pressures and velocities and other flow properties varying in all directions, in many cases the greatest changes only occur in two directions or even only in one. In these cases changes in the other direction can be effectively ignored making analysis much more simple.

Flow is *one dimensional* if the flow parameters (such as velocity, pressure, depth etc.) at a given instant in time only vary in the direction of flow and not across the cross-section. The flow may be unsteady, in this case the parameter vary in time but still not across the cross-section. An example of *one-dimensional flow is the flow in a pipe*. Note that since flow must be zero at the pipe wall - yet non-zero in the centre – there is a difference of parameters across the cross-section. Should this be treated as two-dimensional flow?

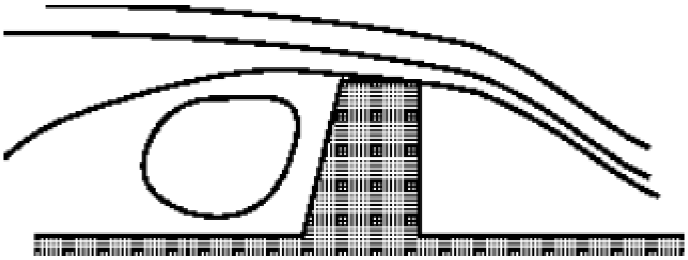
Possibly - but it is only necessary if very high accuracy is required. A correction factor is then usually applied.



One dimensional flow in a pipe.

Flow is *two-dimensional* if it can be assumed that the flow parameters vary in the direction of flow and in one direction at right angles to this direction. Streamlines in two-dimensional flow are curved lines on a plane and are the same on all parallel planes. An example is flow over a weir which typical streamlines can be seen in the figure below. Over the majority of the length of the weir the flow is the same - only at the two ends does it change slightly. Here correction factors may be applied.

In this course we will **only** be considering steady, incompressible one and two-dimensional flow.



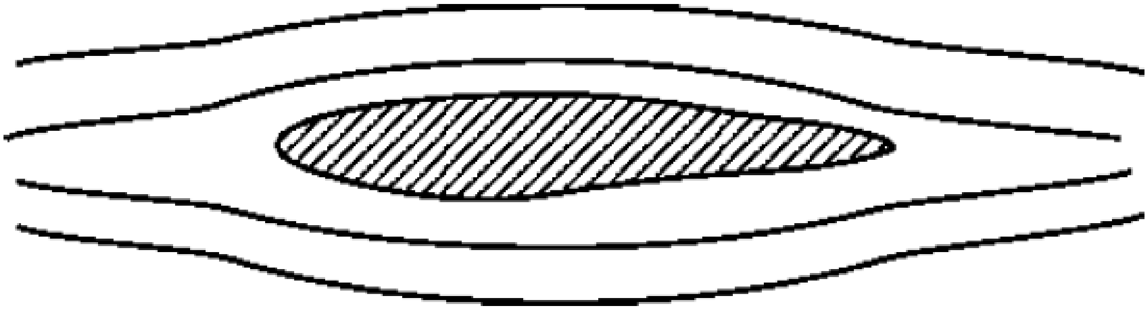
Two-dimensional flow over a weir.

3.4 Viscous or Inviscid flow:

It is a physical fact that all fluids possess the property of viscosity which we have already treated in some detail. But in some flow situations it turns out that the forces on fluid elements that arise from viscosity are small compared with other forces. Hence when $\tau = 0$, or $\mu = 0$ the flow is called inviscid flow.

3.5 Streamlines and streamtubes

In analyzing fluid flow it is useful to visualize the flow pattern. This can be done by drawing lines joining points of equal velocity - velocity contours. These lines are know as *streamlines*. Here is a simple example of the streamlines around a cross-section of an aircraft wing shaped body:



Streamlines around a wing shaped body

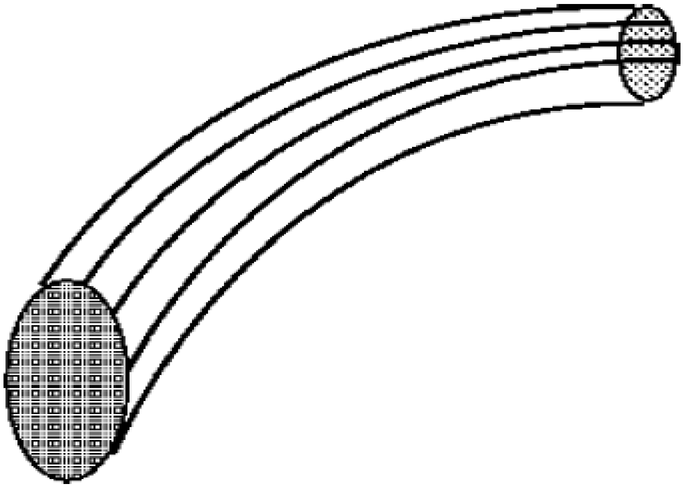
When fluid is flowing past a solid boundary, e.g. the surface of an aerofoil or the wall of a pipe, fluid obviously does not flow into or out of the surface. So very close to a boundary wall the flow direction must be parallel to the boundary.

At all points, the direction of the streamline is the direction of the fluid velocity: this is how they are defined. Close to the wall the velocity is parallel to the wall so the streamline is also parallel to the wall. It is also important to recognize that the position of streamlines can change with time - this is the case in unsteady flow. In steady flow, the position of streamlines does not change.

Some things to know about streamlines:-

- Because the fluid is moving in the same direction as the streamlines, fluid can not cross a streamline.
- Streamlines can not cross each other. If they were to cross this would indicate two different velocities at the same point. This is not physically possible.
- The above point implies that any particles of fluid starting on one streamline will stay on that same streamline throughout the fluid.

A useful technique in fluid flow analysis is to consider only a part of the total fluid in isolation from the rest. This can be done by imagining a tubular surface formed by streamlines along which the fluid flows. This tubular surface is known as a *streamtube*.



A Streamtube in a two-dimensional flow have a streamtube which is flat (in the plane of the paper):

The "walls" of a streamtube are made of streamlines. As we have seen above, fluid cannot flow across a streamline, so fluid cannot cross a streamtube wall. The streamtube can often be viewed as a solid walled pipe. A streamtube is **not** a pipe - it differs in unsteady flow as the walls will move with time. And it differs because the "wall" is moving with the fluid.

3.6 Conservation of Mass:-

3.6.1 Mass flow rate

If we want to measure the rate at which water is flowing along a pipe. A very simple way of doing this is to catch all the water coming out of the pipe in a bucket over a fixed time period. Measuring the weight of the water in the bucket and dividing this by the time taken to collect this water gives a rate of accumulation of mass. This is known as the *mass flow rate*. For example an empty bucket weighs 2.0kg. After 7 seconds of collecting water the bucket weighs 8.0kg, then:

$$\begin{aligned}\text{mass flow rate} = \dot{m} &= \frac{\text{mass of fluid in bucket}}{\text{time taken to collect the fluid}}\\ &= \frac{8.0-2.0}{7}\\ &= 0.857\,kg/s \quad (kg\,s^{-1})\end{aligned}$$

Performing a similar calculation, if we know the mass flow is 1.7kg/s, how long will it take to fill a container with 8kg of fluid?

$$\begin{aligned}\text{time} &= \frac{\text{mass}}{\text{mass flow rate}}\\ &= \frac{8}{1.7}\\ &= 4.7\,s\end{aligned}$$

3.6.2 Volume flow rate - Discharge.

More commonly we need to know the volume flow rate - this is more commonly known as *discharge*. (It is also commonly, but inaccurately, simply called *volume flow rate*). The symbol normally used for discharge is *Q*. The discharge is the volume of fluid flowing per unit time. Multiplying this by the density of the fluid gives us the mass flow rate. Consequently, if the density of the fluid in the above example is 850 kg/m^3 then:

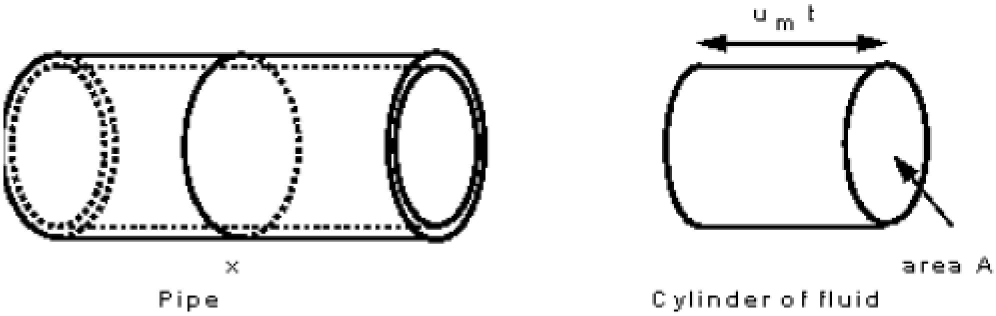
$$\begin{aligned}\text{discharge, } Q &= \frac{\text{volume of fluid}}{\text{time}}\\ &= \frac{\text{mass of fluid}}{\text{density} \times \text{time}}\\ &= \frac{\text{mass flow rate}}{\text{density}}\\ &= \frac{0.857}{850}\\ &= 0.001008\,m^3/s \quad (m^3\,s^{-1})\\ &= 1.008\times 10^{-3}\,m^3/s\\ &= 1.008\,l/s\end{aligned}$$

3.6.3 Discharge and mean velocity.

If we know the size of a pipe, and we know the discharge, we can deduce the mean velocity. If the area of cross section of the pipe at point X is A, and the mean velocity here is u_m . During a time t, a cylinder of fluid will pass point X with a volume $A \times u_m \times t$. The volume per unit time (the discharge) will thus be equal to:

$$Q = \frac{\text{volume}}{\text{time}} = \frac{A \times u_m \times t}{t}$$

$$Q = Au_m$$

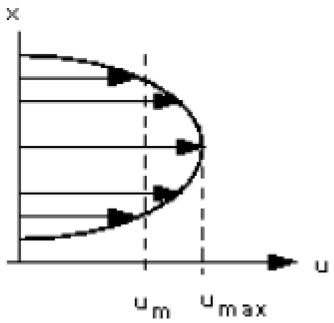


Discharge in a pipe

So if the cross-section area, A= $(1.2 \times 10^{-3})\, m^2$ and the discharge, Q= is 24 $l\,/\,s$, then the mean velocity, u_m , of the fluid is

$$\begin{aligned} u_m &= \frac{Q}{A} \\ &= \frac{2.4 \times 10^{-3}}{1.2 \times 10^{-3}} \\ &= 2.0\, m/s \end{aligned}$$

Note how carefully we have called this the *mean* velocity. This is because the velocity in the pipe is not constant across the cross section. Crossing the centreline of the pipe, the velocity is zero at the walls increasing to a maximum at the centre then decreasing symmetrically to the other wall. This variation across the section is known as the velocity profile or distribution. A typical one is shown in the figure below.

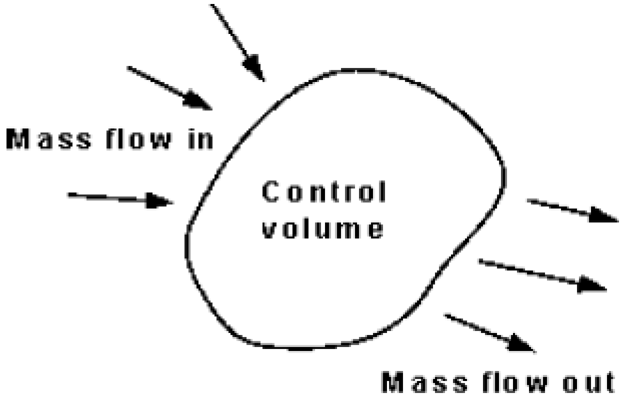


A typical velocity profile across a pipe

3.7 Continuity Equation:-

3.7.1 Continuity equation in integral form

principle of *conservation of mass* law states (Matter cannot be created or destroyed, but it is simply changed in to a different forms of matter). This principle is known as the *conservation of mass* and we use it in the analysis of flowing fluids. The principle is applied to fixed volumes, known as control volumes (or surfaces), like that in the figure below:



An arbitrarily shaped control volume.

For any control volume the principle of *conservation of mass* says:-

[Mass entering per unit time = Mass leaving per unit time + Increase of mass in the control volume per unit time].

Or: $\frac{\partial}{\partial t}\int_{cv}\rho\times d\forall+\int_{cs}\rho\times V\times dA=0$ ……………………………………………..(1)

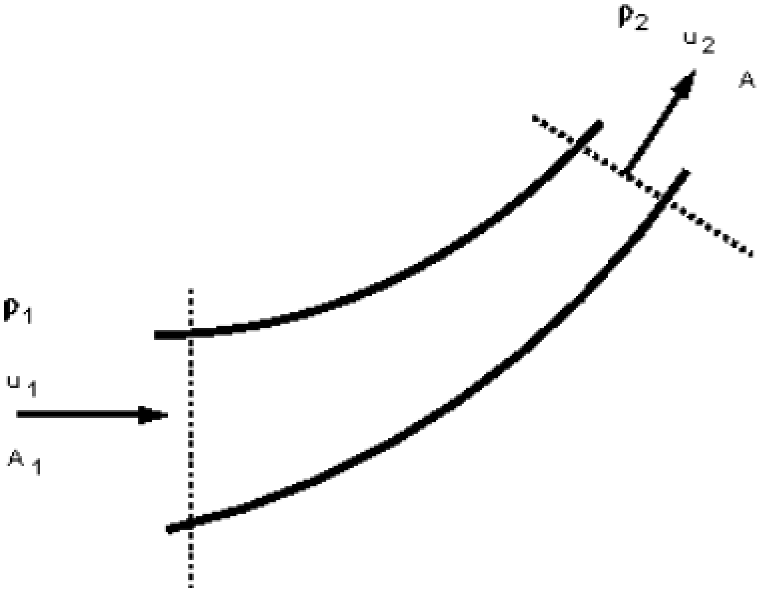
Where the equation (commonly called the *continuity equation*) can be applied to a finite control volume (c.v), which is bounded by a control surface (c.s). The first integral on the left side of Eq. 1 represents the rate at which the mass within the control volume is changing, and the second integral represents the net rate at which mass is flowing out through the control surface (rate of mass outflow- rate of mass inflow).

For **steady** flow there is no increase in the mass within the control volume, so: $\frac{\partial}{\partial t}\int_{cv} \rho \times d\forall = 0$, or $\int_{A_{in}}^{A_{out}} \rho \times V \times dA = 0$

$(\rho \times V \times A)_{in} = (\rho \times V \times A)_{out}$

∑Mass entering per unit time = ∑Mass leaving per unit time

This can be applied to a streamtube such as that shown below. No fluid flows across the boundary made by the streamlines so mass only enters and leaves through the two ends of this streamtube section.



mass entering per unit time at end 1 = mass leaving per unit time at end 2, or

$\sum (\rho_1 \times \delta A_1 \times u_1) = \sum (\rho_2 \times \delta A_2 \times u_2) = \dot{m}$ ……………………………………….(2)

This equation is called equation of continuity.

Where:

ρ : is the fluid density (kg/m^3).

δA: is the pipe or duct cross-sectional area (m^2).

u : is the fluid velocity (m/s).

$\dot{m}$: is the mass flow rate (kg/s).

The flow of fluid through a real pipe (or any other vessel) will vary due to the presence of a wall - in this case we can use the *mean* velocity (u_m) and write:

$\sum (\rho_1 \times A_1 \times u_{m1}) = \sum (\rho_2 \times A_2 \times u_{m2}) = \dot{m}$ ……………………….…………..(3)

Since: $\dot{m} = \rho \times A \times u_m$ which is called mass flow rate.

When the fluid can be considered *incompressible*, i.e. the density does not change, *then* $\rho_1 = \rho_2 =$ constant. Then equation (3) drops to:

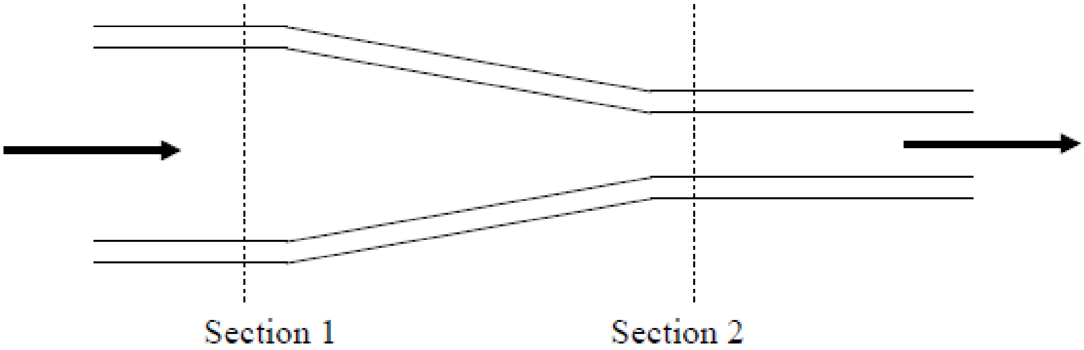
$\sum (A_1 \times u_{m1}) = \sum (A_2 \times u_{m2}) = Q$ ………………………………………..………..(4)

Where Q: is the fluid volume flow rate or discharge (m^3/s), hence:

$Q = A \times u_m$

Some example applications

We can apply the principle of continuity to pipes with cross sections which change along their length. Consider the diagram below of a pipe with a contraction:



A liquid is flowing from left to right and the pipe is narrowing in the same direction. By the continuity principle, the *mass flow rate* must be the same at each section - the mass going into the pipe is equal to the mass going out of the pipe. So we can write:

$\sum (\rho_1 \times A_1 \times u_{m1}) = \sum (\rho_2 \times A_2 \times u_{m2}) = \dot{m}$

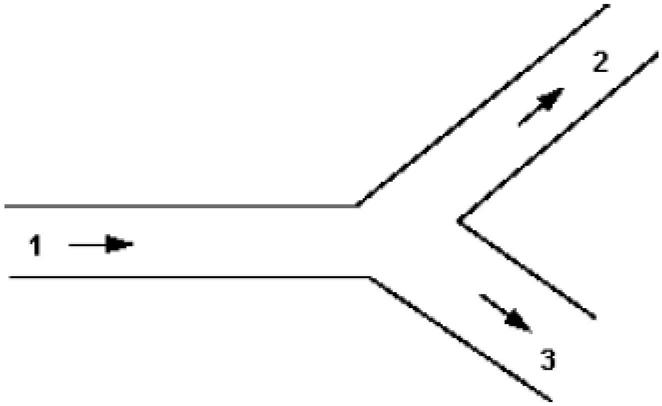
As we are considering a liquid, usually water, which is *not* very compressible, the density changes very little so we can say $\rho_1 = \rho_2 = constant$. This also says that the *volume flow rate* is constant or that:

$\sum Q_1 = \sum Q_2$, or $\sum (A_1 \times u_{m1}) = \sum (A_2 \times u_{m2})$

Ex: If the area $A_1 = (10 \times 10^{-3})$ m^2 and ($A_2 = 3 \times 10^{-3}$) m^2 and the upstream mean velocity, $u_{m1} =$ 2.1 m/s , then the downstream mean velocity can be calculated by:

$$u_{m2} = \frac{A_1 \times u_{m1}}{A_2} = 7m/s$$

Another example of the use of the continuity principle is to determine the velocities in pipes coming from a junction.



Total mass flow into the junction = Total mass flow out of the junction

$\sum \dot{m}_{in} = \sum \dot{m}_{out}$ or

$\dot{m}_1 = \dot{m}_2 + \dot{m}_3$

Then: $\rho_1 Q_1 = \rho_2 Q_2 + \rho_3 Q_3$

When the flow is incompressible (e.g. if it is water): then:

$\rho_1 = \rho_2 = \rho_3 =$ *constant:*

$Q_1 = Q_2 + Q_3$ or

$A_1 \times u_1 = A_2 \times u_2 + A_3 \times u_3$

Ex: If pipe 1 has a diameter of 50mm, mean velocity of 2m/s, pipe 2 has a diameter of 40mm takes 30% of total discharge and pipe 3 has a diameter of 60mm. What are the values of discharge and mean velocity in each pipe?

$$Q_1 = A_1 u_1 = \left(\frac{\pi d^2}{4}\right) u$$

$$= 0.00392\, m^3 / s$$

$$Q_2 = 0.3 Q_1 = 0.001178 m^3 / s$$

$$Q_1 = Q_2 + Q_3$$

$$Q_3 = Q_1 - 0.3 Q_1 = 0.7 Q_1$$

$$= 0.00275 m^3 / s$$

$$Q_2 = A_2 u_2$$

$$u_2 \quad 0\;936 m \quad s$$

$$Q_3 = A_3 u_3$$

$$u_3 \quad 0\;972 m \quad s$$

3.8 Reynold`s Number (Re): It is a dimensional number regards by scientist Reynold that used this number to measure the type of flow in viscous region.

$\mathrm{Re} = \frac{\rho V L}{\mu}$ or,

$\mathrm{Re} = \frac{VL}{\upsilon}$ **for flow over flat plate,**

$\mathrm{Re} = \frac{VD}{\upsilon}$ **for flow inside the pipe.**

Laminar flow: $\mathrm{Re} < 2100$

Transitional flow: $2000 < \mathrm{Re} < 4000$

Turbulent flow: $\mathrm{Re} > 4000$

Where:
ρ is the fluid density.
V is the fluid velocity.
L is the flow path length.
μ is the dynamic viscosity.
υ is the kinematic viscosity.

BOUNDARY LAYER ON FLAT PLATE

(y scale greatly enlarged)