An overview of steam

Introduction:

The world energy consumption has doubled in the last thirty years and it keeps on increasing with about 1.5 % per year. While the earth's oil and gas reserves are expected to deplete after roughly one hundred years, the coal reserves will last for almost five hundred years into the future. In Finland, 50 % of the electrical power produced, is produced in steam power plants. But there are more reasons to why electricity generation based on steam power plant will continue to grow and why there still will be a demand for steam boilers in the future:

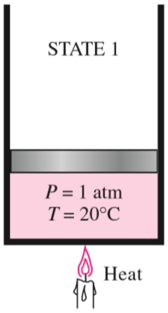
- The cost of the produced electricity is low
- The technology has been used for many decades and is reliable and available
- Wind and solar power are still expensive compared to steam power

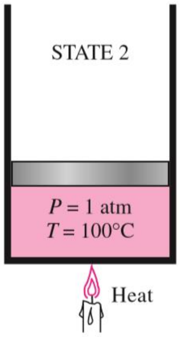
PURE SUBSTANCE:

A substance that has a fixed chemical composition throughout is called a pure substance. Water, nitrogen, helium, and carbon dioxide, for example, are all pure substances. A pure substance does have to be of a single chemical element or compound, however. A mixture of various chemical elements or compounds also qualifies as a pure substance as long as the mixture is homogeneous. Air, for example, is a mixture of several gases, but it is often considered to be a pure substance because it has a uniform chemical composition. However, a mixture of oil and water is not a pure substance. Since oil is not soluble in water, it will collect on top of the water, forming two chemically dissimilar regions. A mixture of two or more phases of a pure substance is still a pure substance as long as the chemical composition of all phases is the same .A mixture of ice and liquid water, for example, is a pure substance because both phases have the same chemical composition. A mixture of liquid air and gaseous air, however, is not a pure substance since the composition of liquid air is different from the composition of gaseous air, and thus the mixture is no longer chemically homogeneous. This is due to different components in air condensing at different temperatures at a specified pressure.

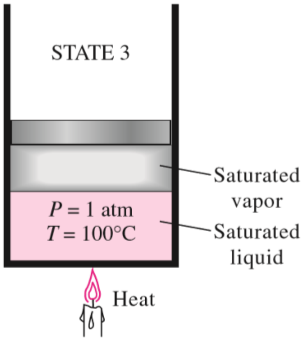
PHASE-CHANGE PROCESSES OF PURE SUBSTANCES:

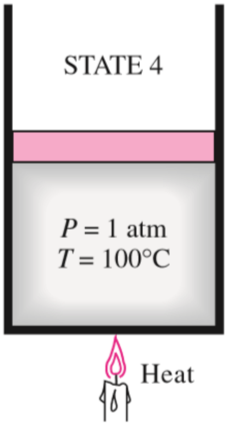
There are many practical situations where two phases of a pure substance coexist in equilibrium. Water exists as a mixture of liquid and vapor in the boiler and the condenser of a steam power plant. The refrigerant turns from liquid to vapor in the freezer of a refrigerator. Even though many home owners consider the freezing of water in underground pipes as the most important phase-change process, attention in this section is focused on the liquid and vapor phases and their mixture. As a familiar substance, water is used to demonstrate the basic principles involved. Remember, however, that all pure substances exhibit the same general behavior. Consider a piston–cylinder device containing liquid water at 20°C and 1 atm pressure (state 1). Under these conditions, water exists in the liquid phase, and it is called a compressed liquid, or a subcooled liquid, meaning that it is not about to vaporize. Heat is now transferred to the water until its temperature rises to, say, 40°C. As the temperature rises, the liquid water expands slightly, and so its specific volume increases. To accommodate this expansion, the piston moves up slightly. The pressure in the cylinder remains constant at 1 atm during this process since it depends on the outside barometric pressure and the weight of the piston, both of which are constant. Water is still a compressed liquid at this state since it has not started to vaporize. As more heat is transferred, the temperature keeps rising until it reaches 100°C (state 2). At this point water is still a liquid, but any heat addition will cause some of the liquid to vaporize. That is, a phase-change process from liquid to vapor is about to take place. A liquid that is about to vaporize is called a saturated liquid. Therefore, state 2 is a saturated liquid state. Once boiling starts, the temperature stops rising until the liquid is completely vaporized. That is, the temperature will remain constant during the entire phase-change process if the pressure is held constant. This can easily be verified by placing a thermometer into boiling pure water on top of a stove. At sea level (P = 1 atm), the thermometer will always read 100°C if the pan is uncovered or covered with a light lid. During a boiling process, the only change we will observe is a large increase in the volume and a steady decline in the liquid level as a result of more



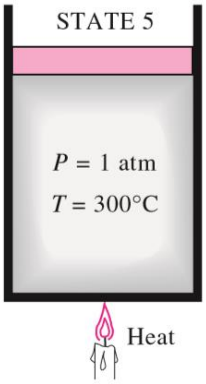


liquid turning to vapor. Midway about the vaporization line (state 3), the cylinder contains equal amounts of liquid and vapor. As we continue transferring heat, the vaporization process continues until the last drop of liquid is vaporized (state 4). At this point, the entire cylinder is filled with vapor that is on the borderline of the liquid phase. Any heat loss from this vapor will cause some of the vapor to condense (phase change from vapor to liquid). A vapor that is about to condense is called a saturated vapor. Therefore, state 4 is a saturated vapor state. A substance at states between 2 and 4 is referred to as a saturated liquid–vapor mixture since the liquid and vapor phases coexist in equilibrium at these states.





Once the phase-change process is completed, we are back to a single phase region again (this time vapor), and further transfer of heat results in an increase in both the temperature and the specific volume. At state 5, the temperature of the vapor is, let us say, 300°C; and if we transfer some heat from the vapor, the temperature may drop somewhat but no condensation will take place as long as the temperature remains above 100°C (for $P = 1\ atm$). A vapor that is not about to condense (i.e., not a saturated vapor) is called a superheated vapor. Therefore, water at state 5 is a superheated vapor. This constant-pressure phase-change process is illustrated on a T-v diagram in Fig.1. If the entire process described here is reversed by cooling the water while maintaining the pressure at the same value,



the water will go back to state 1, retracing the same path, and in so doing, the amount of heat released will exactly match the amount of heat added during the heating process. In our daily life, water implies liquid water and steam implies water vapor. In thermodynamics, however, both water and steam usually mean only one thing: H_2O.

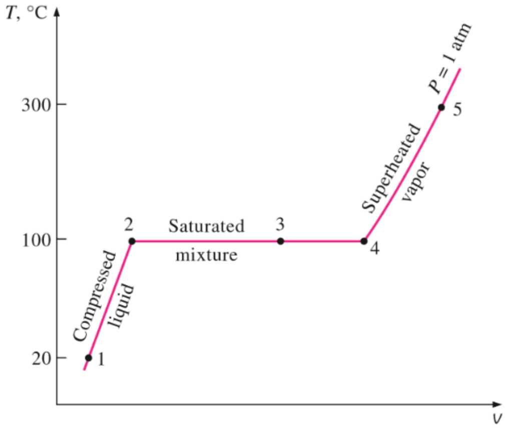


FIGURE |
T-v diagram for the heating process of water at constant pressure.

<u>Saturation Temperature and Saturation Pressure</u> It probably came as no surprise to you that water started to boil at 100°C. Strictly speaking, the statement "water boils at 100°C" is incorrect. The correct statement is "water boils at 100°C at 1 atm pressure." The only reason water started boiling at 100°C was because we held the pressure constant at 1 atm (101.325 kPa). If the pressure inside the cylinder were raised to 500 kPa by adding weights on top of the piston, water would start boiling at 151.8°C. That is, the temperature at which water starts boiling depends on the pressure; therefore, if the pressure is fixed, so is the boiling temperature. At a given pressure, the temperature at which a pure substance changes phase is called the saturation temperature T_{sat}. Likewise, at a given temperature, the pressure at which a pure substance changes phase is called the saturation pressure P_{sat}. At a pressure of 101.325 kPa, T_{sat} is 99.97°C. Conversely, at a temperature of 99.97°C, P_{sat} is 101.325 kPa. (At 100.00°C, P_{sat} is 101.42 kPa)Saturation tables that list the saturation pressure against the temperature (or the saturation temperature against the pressure) are available for practically all substances.

Effect of pressure on evaporation temperature: It is well known that water boils and evaporates at 100°C under atmospheric pressure. By higher pressure, water evaporates at higher temperature - e.g. a pressure of 10 bar equals an evaporation temperature of 184°C. The pressure and the corresponding temperature when a phase change occurs are called the saturation temperature and saturation pressure. During the evaporation process, pressure and temperature are constant, but if the vaporization occurs in a closed vessel, the expansion that occurs due to the phase change of water into steam causes the pressure to rise and thus the boiling temperature rises.

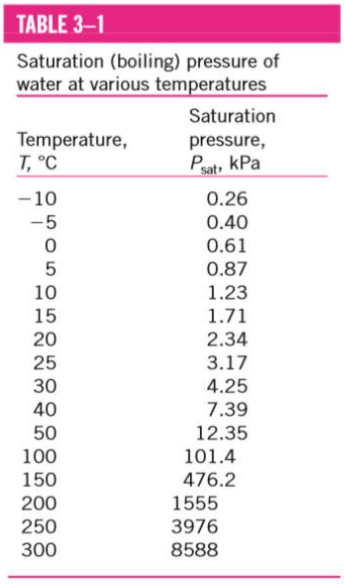


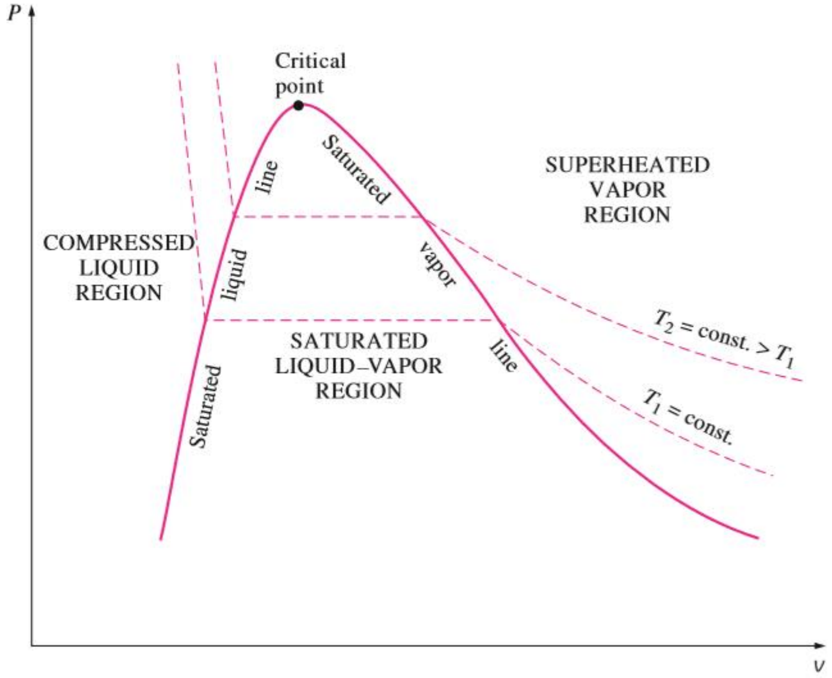
TABLE 3–1

Saturation (boiling) pressure of water at various temperatures

Temperature, T, °C	Saturation pressure, P_{sat}, kPa
−10	0.26
−5	0.40
0	0.61
5	0.87
10	1.23
15	1.71
20	2.34
25	3.17
30	4.25
40	7.39
50	12.35
100	101.4
150	476.2
200	1555
250	3976
300	8588

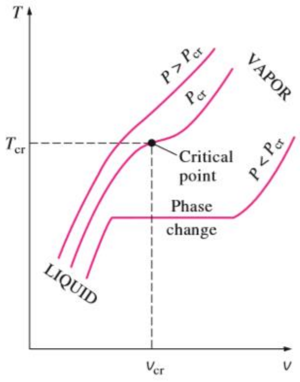
PROPERTY DIAGRAMS FOR PHASE-CHANGE PROCESSES

The variations of properties during phase-change processes are best studied and understood with the help of property diagrams. Next, we develop and discuss the T-v, P-v, and P-T diagrams for pure substances.



Critical point: it is defined as the point at which the saturated liquid and saturated vapor states are identical.

The temperature, pressure, and specific volume of a substance at the critical point are called, respectively, the critical temperature T_{cr}, critical pressure P_{cr}, and critical specific volume vcr. The critical-point properties of water are P_{cr} = 22.06 MPa, T_{cr} = 373.95°C, and v_{cr}= 0.003106 m^3/kg. For helium, they are 0.23 MPa, 267.85°C, and 0.01444 m^3/kg. At pressures above the critical pressure, there is not a distinct phase change process (Fig. 3). Instead, the specific volume of the substance continually increases, and at all times there is only one phase present. Eventually, it resembles a vapor, but we can never tell when the change has occurred. Above the



FIGURE

At supercritical pressures ($P > P_{cr}$), there is no distinct phase-change (boiling) process.

critical state, there is no line that separates the compressed liquid region and the superheated vapor region. However, it is customary to refer to the substance as superheated vapor at temperatures above the critical temperature and as compressed liquid at temperatures below the critical temperature.

From the diagram (Figure 2) we can see that when we exceed a certain pressure, 22.12 Mpa (the corresponding temperature is 374°C), the line stops. The reason is that the border between gas phase and liquid phase is blurred out at that pressure. That point, where the different phases cease to exist, is called the critical point of water.

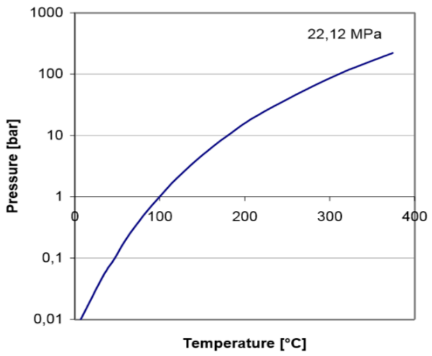
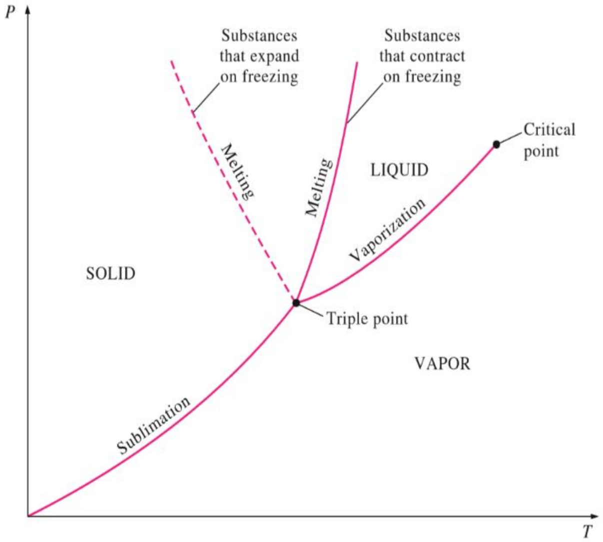


Figure : Evaporation pressure as a function of evaporation temperature.

The P-T Diagram: Figure 4 shows the P-T diagram of a pure substance. This diagram is often called the phase diagram since all three phases are separated from each other by three lines. The sublimation line separates the solid and vapor regions, the vaporization line separates the liquid and vapor regions, and the melting (or fusion) line separates the solid and liquid regions. These three lines meet at



the triple point, where all three phases coexist in equilibrium. The vaporization line ends at the critical point because no distinction can be made between liquid and vapor phases above the critical point. Substances that expand and contract on freezing differ only in the melting line on the P-T diagram.

IMPORTANT TERMS RELATING STEAM FORMATION

1. Latent heat or hidden heat (hfg). It is the amount of heat required to convert water at a given temperature and pressure into steam at the same temperature and pressure. It is expressed by the symbol hfg and its value is available from steam tables. The value of latent heat is not constant and varies according to pressure variation.

2. Dryness fraction (x). The term dryness fraction is related with wet steam. It is defined as the ratio of the mass of actual dry steam to the mass of steam containing it. It is usually expressed by the symbol ‘x’ or ‘q’.

If ms = Mass of dry steam contained in steam considered, and

m_w = Weight of water particles in suspension in the steam considered, Then,

$$x = \frac{m_s}{m_s + m_w}$$

Thus if in 1 kg of wet steam 0.9 kg is the dry steam and 0.1 kg water particles then x = 0.9.

3. Total heat or enthalpy of wet steam (h). It is defined as the quantity of heat required to convert 1 kg of water at 0°C into wet steam at constant pressure. It is the sum of total heat of water and the latent heat and this sum is also called enthalpy.

$$h = hf + xhfg$$

If steam is dry and saturated, then $x = 1$ and $hg = hf + hfg$.

4. Volume of wet and dry steam. If the steam has dryness fraction of x, then 1 kg of this steam will contain x kg of dry steam and (1 – x) kg of water. If vf is the volume of 1 kg of water and vg is the volume of 1 kg of perfect dry steam (also known as specific volume), then volume of 1 kg of wet steam = volume of dry steam + volume of water.

$$= xvg + (1 - x)vf$$

Note. The volume of vf at low pressures is very small and is generally neglected. Thus is general, the volume of 1 kg of wet steam is given by, xvg and density $1/ xvg$

THERMODYNAMIC PROPERTIES OF STEAM AND STEAM TABLES In engineering problem, for any fluid which is used as working fluid, the six basic thermodynamic properties required are : p (pressure), T (temperature), v (volume), u (internal energy), h (enthalpy) and s (entropy). These properties must be known at different pressure for analysing the thermodynamic cycles used for work producing devices. The values of these properties are determined theoretically or experimentally and are tabulated in the form of tables which are known as 'Steam Tables'. The properties of wet steam are then computed from such tabulated data. Tabulated values are also available for superheated steam. It may be noted that steam has only one saturation temperature at each pressure. Following are the thermodynamic properties of steam which are tabulated in the form of table :

p = Absolute pressure (bar or kPa) ;

ts = Saturation temperature (°C) ;

hf = Enthalpy of saturated liquid (kJ/kg) ;

hfg = Enthalpy or latent heat of vaporization (kJ/kg) ;

hg = Enthalpy of saturated vapour (steam) (kJ/kg) ;

sf = Entropy of saturated liquid (kJ/kg K) ;

sfg = Entropy of vaporization (kJ/kg K) ;

sg = Entropy of saturated vapour (steam) (kJ/kg K) ;

vf = Specific volume of saturated liquid (m3/kg) ;

vg = Specific volume of saturated vapour (steam) (m3/kg).

Also, hfg = hg – hf ...... Change of enthalpy during evaporation

sfg = sg – sf ...... Change of entropy during evaporation

The above mentioned properties at different pressures are tabulated in the form of tables as under : The internal energy of steam (u = h – pv) is also tabulated in some steam tables.

COMBUSTION

It is defined as a rapid chemical reaction between fuel and air (oxygen). It is usually a combined by a flame and heat is released during this reaction. Fuel may be chemical or nuclear. Here we shall consider briefly chemical fuels only. A chemical fuel is a substance which releases heat energy on combustion. The principal combustible elements of each fuel are carbon and hydrogen.

In a chemical reaction the terms, reactants and the products are frequently used. 'Reactants' comprise of initial constituents which start the reaction while 'products' comprise of final constituents which are formed by the chemical reaction.

COMBUSTION EQUATIONS:

In a combustion chamber proportionate masses of air and fuel enter where the chemical reaction takes place, and then the combustion products pass to the exhaust. By the conservation of mass, the mass flow remains constant (i.e., total mass of products = total mass of reactants), but the reactants are chemically different from the products, and the products leave at a higher temperature. The total number of atoms of each element concerned in the combustion remains constant, but the atoms are rearranged into groups having different chemical properties. This information is expressed in the chemical equation which shows

(i) The reactants and the products of combustion.

(ii) The relative quantities of the reactants and products. The two sides of the equation must be consistent, each having the same number of atoms of each element involved.

The oxygen supplied for combustion is usually provided by atmospheric air , and it is necessary to use accurate and consistent analysis of air by mass and by volume. It is usual in combustion calculations to take air as 23.3% O_2, 76.7% N_2 by mass, and 21% O_2, 79% N_2 by volume. The small traces of other gases in dry air are included in nitrogen, which is sometimes called 'atmospheric nitrogen'.

Types of Combustion:

1- Stoichiometric: mixture of air and fuel is one that contains just sufficient oxygen for complete combustion of the fuel.

2- Incomplete combustion: occurs when there is an insufficient supply of oxygen to burn the carbon completely to carbon dioxide.

3- Complete Combustion: occurs when there is a more sufficient supply of oxygen to burn the carbon completely to carbon dioxide.

THEORETICAL AIR AND EXCESS AIR:

The minimum amount of air that supplies sufficient oxygen for the complete combustion of all the carbon, hydrogen, and any other elements in the fuel that may oxidize is called the "theoretical air". When complete combustion is achieved with theoretical air, the products contain no oxygen. In practice, it is found that complete combustion is not likely to be achieved unless the amount of air supplied is somewhat greater than the theoretical amount. Thus 150 per cent theoretical air means that air actually supplied is 1.5 times the theoretical air. The complete combustion of methane with minimum amount of theoretical air and 150 per cent theoretical air respectively is written as:

$$CH_4 + 2(O_2 + 3.76N_2) \longrightarrow CO_2 + 2H_2O + (3.76 * 2)N_2$$

$$CH_4 + 2 * 1.5(O_2 + 3.76N_2) \longrightarrow CO_2 + 2H_2O + O_2 + (3.76 * 1.5 * 2)N_2$$

The amount of air actually supplied may also be expressed in terms of per cent excess air. The excess air is the amount of air supplied over and above the theoretical air.

Note. For complete combustion of fuel we need air. As per theoretical basis there is a minimum amount of air which is required by the fuel to burn completely, but always, air in excess is used because whole of air supplied for combustion purposes does not come in contact with the fuel completely and as such portion of fuel may be left unburnt. But if a large quantity of excess air is used it exercises a cooling effect on combustion process which however can be avoided by preheating the air. The amount of excess air supplied varies with the type of fuel and the firing conditions.

STOICHIOMETRIC AIR FUEL (A/F) RATIO:

$$\frac{A}{F} = \frac{mass\ of\ air}{mass\ of\ fuel} = \frac{m_{air}}{m_{fuel}}$$

$$CH_4 + 2(O_2 + 3.76N_2) \longrightarrow CO_2 + 2H_2O + (3.76 * 2)N_2$$

A/F= (2*32+2*3.76*28)/ (1*12+4*1)=17.16

Equivalence Ratio (Mixture Strength) φ:-

$$\varphi = M.S. = \frac{\left({}^{A}/_{F}\right)_{stoich.}}{\left({}^{A}/_{F}\right)_{actual}}$$

φ >1 Incomplete Combustion {CO_2, CO, H_2o, N_2}

φ<1 Complete Combustion {CO_2, O_2, H_2O, N_2}

φ=1 Stoichiometric Combustion {CO_2, H_2O, N_2}

Examples:

1- Isooctane (C_8H_{18}) is burned with (130%) theoretical of air. Find: Φ
2- A hydrocarbon fuel (C_4H_{10}) is burned with the following cases :
    a- 125% theoretical of air
    b- 80% theoretical of air. C- φ= 0.85

3- A propane(C_3H_8) is burned with 90% theoretical of air determine:

a- The actual air-fuel ratio.

b- Mixture strength.

c- The volumetric and gravimetric analysis of products.

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POSITIVE DISPALCEMENT COMPRESSOR

Reciprocating Compressor:

Reciprocating compressors have been the most widely used for industrial plant air systems. The two major types are single acting and double acting, both of which are available as one or two stage compressors. The Single acting cylinder performs compression on one side of the piston during one direction of the power stroke. Two stage compressions reach the final output pressure in two separate compression cycles, or stages, in series.

The double acting compressor is configured to provide a compression stroke as the piston moves in either direction. This is accomplished by mounting a cross head on the crank arm which is then connected to a double acting piston by a piston rod. Distance pieces connect the cylinder to the crankcase. They are sealed to prevent mixing of crank shaft lubricant with the air, but vented so as to prevent pressure built up.

Single Acting Compressor

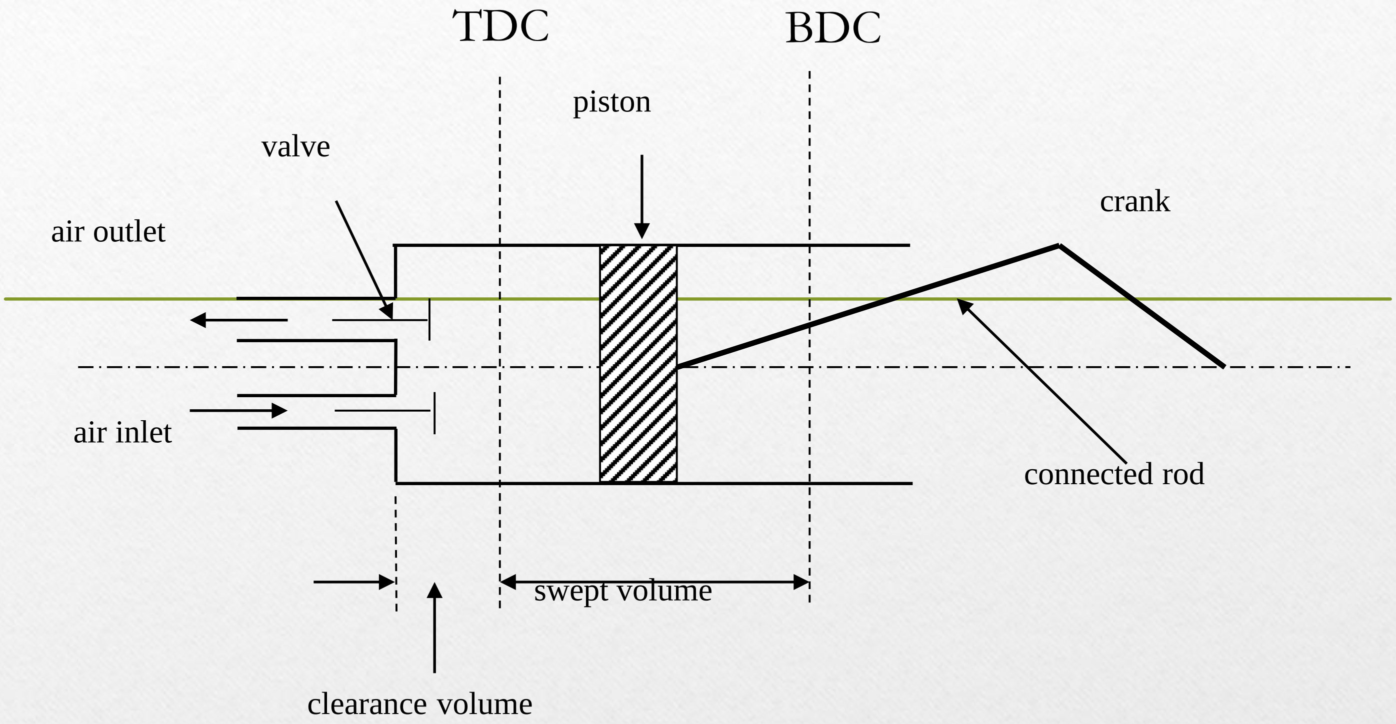
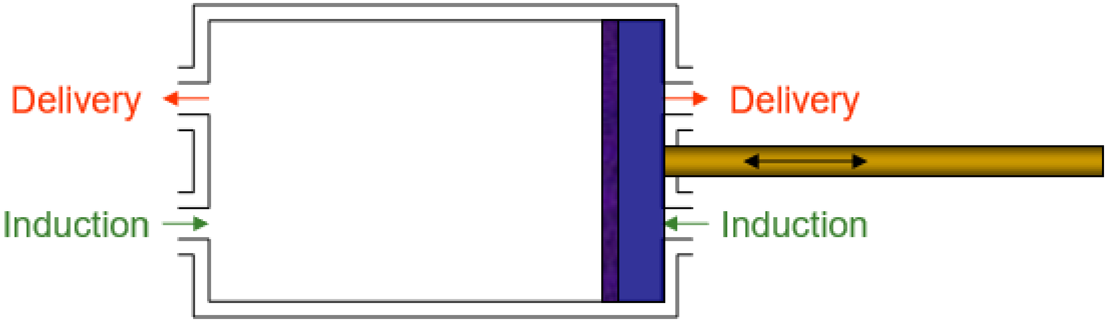


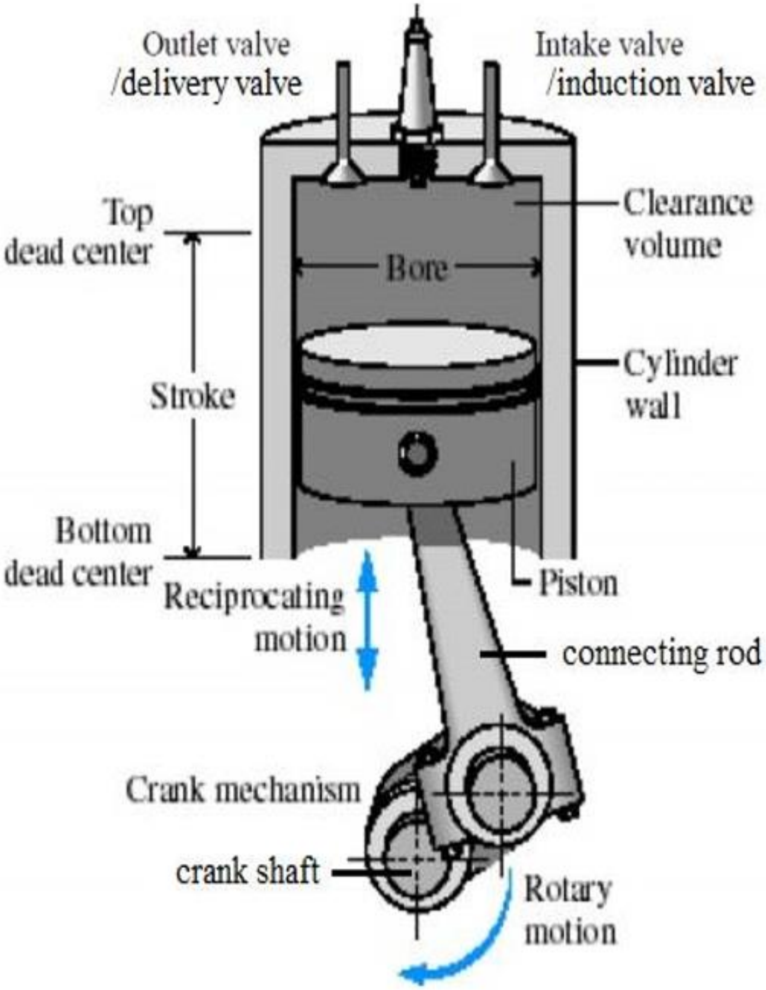
Figure (1): Reciprocating compressor.

Double Acting Compressors

- A **single-acting** compressor completes one compression cycle with one revolution of the crank
- A **double-acting** compressor completes two compression cycles with one revolution of the crank
- So the mass induce per revolution is twice than a single acting where



Basic components of reciprocating compressor



Compression cylinder – provide confinement for the process gas during compression.

Piston – moves through the cylinder in a reciprocating action to compress the gas.

Intake valve – to let the air to drawn into the cylinder.

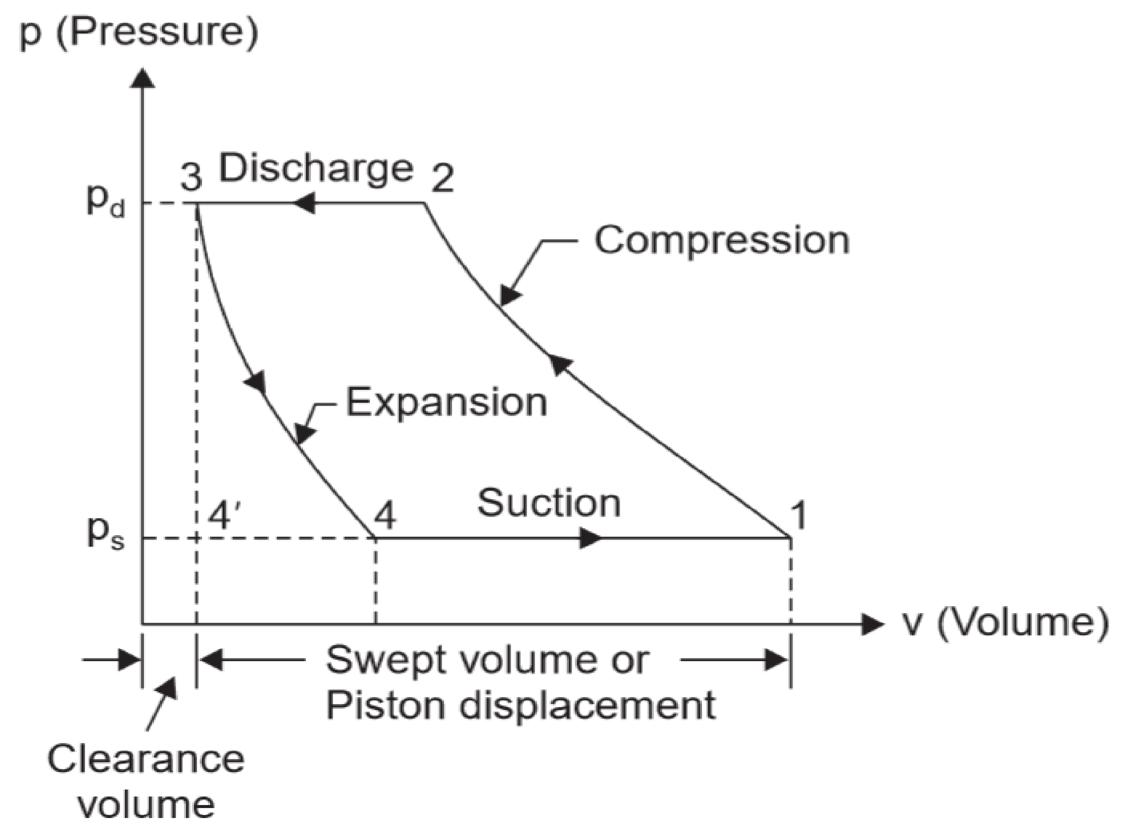
Outlet valve – to let air out of the cylinder.

Crank shaft – connect to the motor to provide the rotary motion.

Connecting rod – connect the crank and the piston – provide the reciprocating ups and down movement.

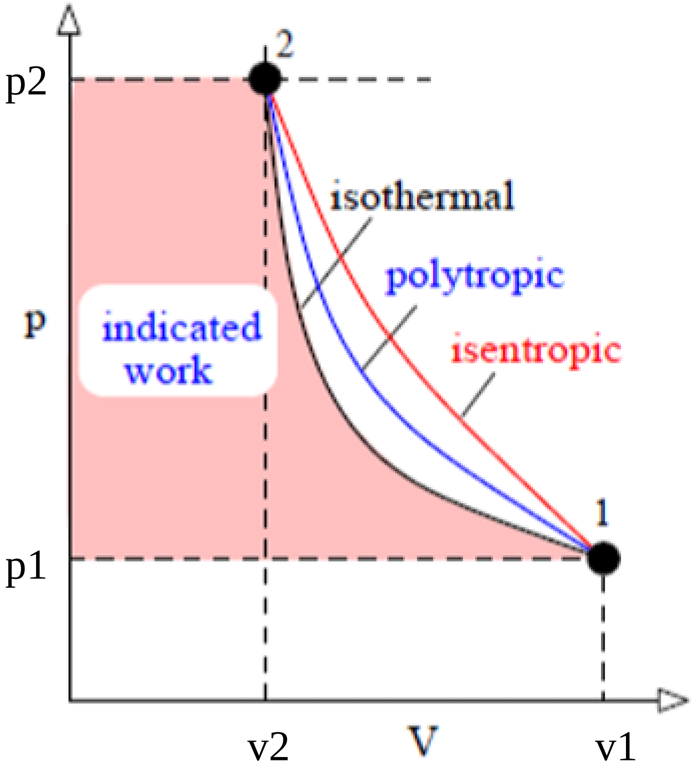
- **Inlet stroke: -**suction or inlet stroke begins with piston at top dead center (a position providing a minimum or clearance volume). During the downward stroke, piston motion reduces the pressure inside the cylinder below the atmospheric pressure. The inlet valve then opens against the pressures of its spring and allows air to flow into the cylinder. The air is drawn into the cylinder until the piston reaches to a maximum volume position (bottom dead center).The discharge valve remains closed during this stroke.

- **Outlet stroke:** During compression stroke piston moves in the opposite direction(Bottom dead center to top dead center), decreasing the volume of the air. As the piston starts moving upwards, the inlet valve is closed and pressure starts to increase continuously until the pressure inside the cylinder is above the pressure of the delivery side which is connected to the receiver. Then the outlet valve opens and air is delivered during the remaining upward motion of the piston to the receiver.



Figure(2): p - V diagram of reciprocating compressor

Conditions for Minimum Work

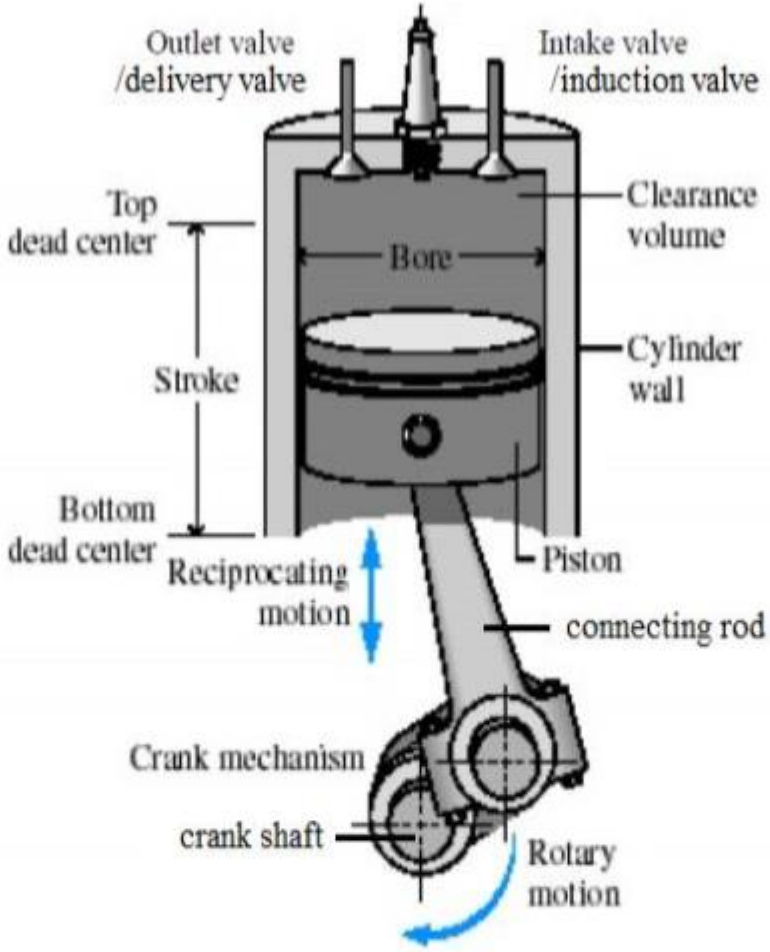


- pV =constant (isothermal)
- pV^{γ} =constant (isentropic)
- pV^{n} =constant (polytropic)

- From here it can be seen that the **isothermal** process **is the best** because it requires minimum work.

- So it is best that the gas temperature is constant throughout the compression cycle.

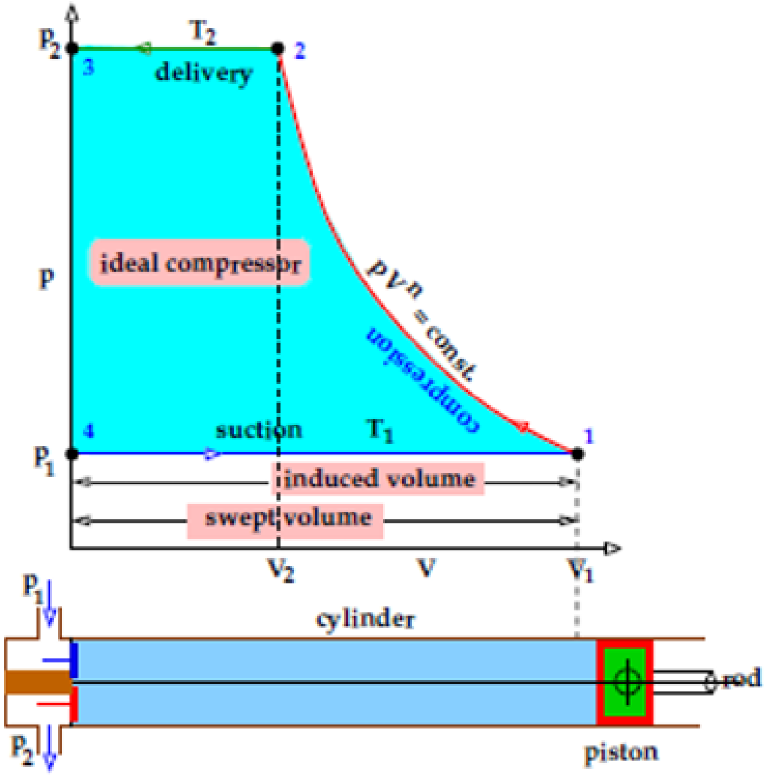
Clearance Volume, V_c



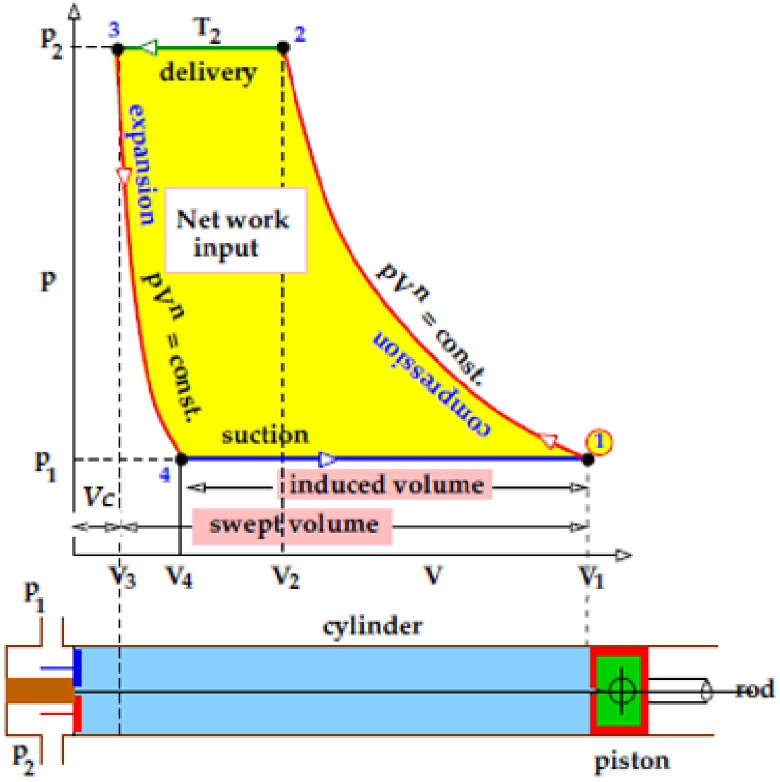
- In **actual** compressors, piston does not reach the top of wall of the cylinder head.
- Instead, it reaches maximum stroke (TDC) at a certain distance from the cylinder head.
- The remaining volume of the cylinder where piston **does not** travel through

  is call the clearance volume V_C.
- The volume where the piston **does** travel through is called the swept volume, V_S.
- Purpose of Vc : to give freedom for working parts and space for valve operations

Cycle different between with and without clearance volume

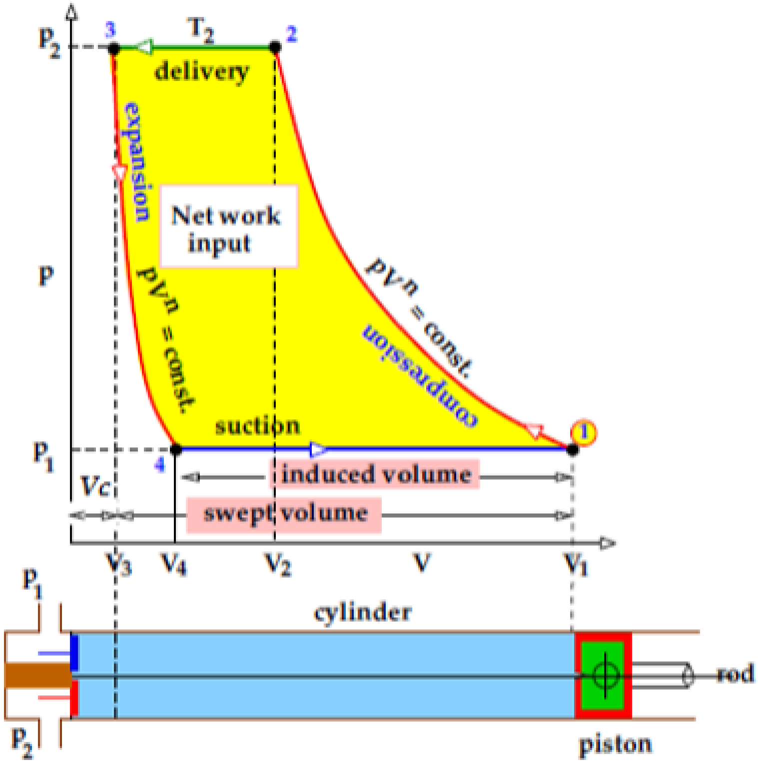


(i) without clearance volume



(ii) with clearance volume

The Effect of Clearance Volume, V_c



- Clearance volume: $V_c = V_3$
- Swept volume: $V_s = V_1 - V_3$

$$= V_1 - V_c$$

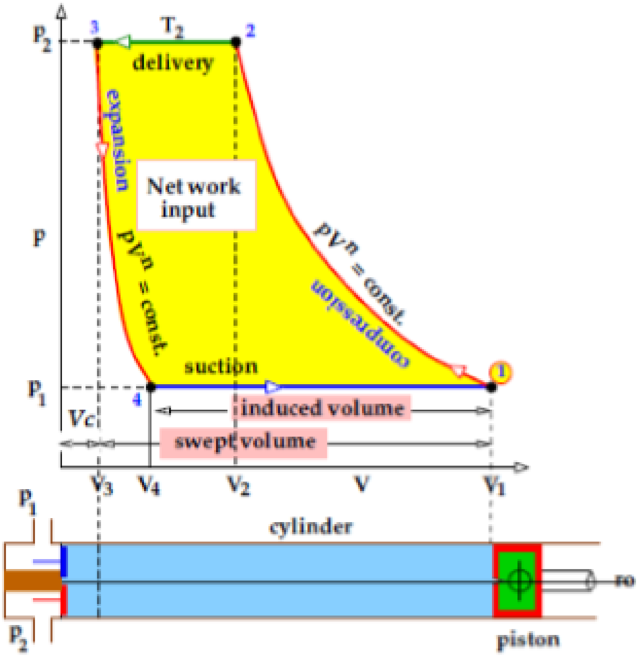
- Because of the expansion of gas remaining in the V_C, *induced volume* is reduced from swept volume V_S to $(V_1 – V_4)$ which is the ***effective volume***
  - Induced volume: $V_{\text{in}} = V_1 - V_4$
- Mass of air per unit time

$$\dot{m}_1 = \dot{m}_2 \; and \; \dot{m}_3 = \dot{m}_4$$

- Mass delivered per unit time = mass induced per unit time

$$\dot{m} = \dot{m}_2 - \dot{m}_3 = \dot{m}_1 - \dot{m}_4$$

Indicated Power for Compressor with Clearance Volume



$$\text{Indicated power}\ \dot{W} = \frac{n}{n-1}(\dot{m}_1 - \dot{m}_4)R(T_2 - T_1)$$

with $\dot{m}_1 - \dot{m}_4 = \dot{m}$ (mass induced per unit time)

and $\left(\dfrac{T_2}{T_1}\right) = \left(\dfrac{p_2}{p_1}\right)^{\frac{n-1}{n}}$, $\dfrac{V_1}{V_2} = \left(\dfrac{p_2}{p_1}\right)^{\frac{1}{n}}$

Therefore

$$\dot{W} = \frac{n}{n-1}\dot{m}R(T_2 - T_1)$$

$$= \frac{n}{n-1}\dot{m}RT_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

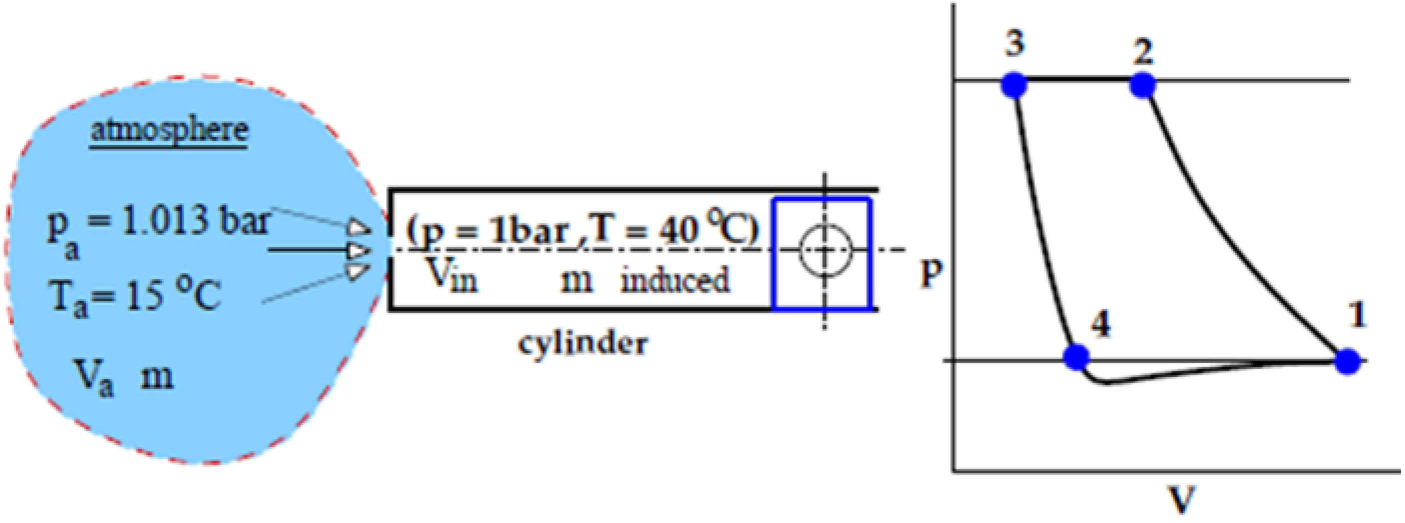
*F*or compressor with N cycles per unit time (N rpm)

$$\dot{m} = m \times N\left(\frac{kg}{\text{min}}\right)$$

or $\dot{m} = (m_1 - m_4) \times N\left(\dfrac{kg}{\text{min}}\right)$

Free Air Delivery (FAD)

- The Free Air Delivery (FAD) is the volume of air drawn into a compressor from the atmosphere.
- It is a standardized measure of the capacity of an air compressor.
- Normally, the pressure inside the compressor during the induction process is slightly lower than the atmospheric pressure outside of the compressor.
- For e.g:



Free Air Delivery (FAD)

Term V_a or $\dot{V}_a$ is also called as Free Air Delivery (FAD) with index "a" refers to the atmospheric conditions.

$$V_a = \frac{m_a R T_a}{p_a}$$

If the suction conditions are given, the FAD can be determined through this conditions.

$$\dot{m} = \frac{p_a V_a}{R T_a} = \frac{p_1 V_{\text{in}}}{R T_1}$$

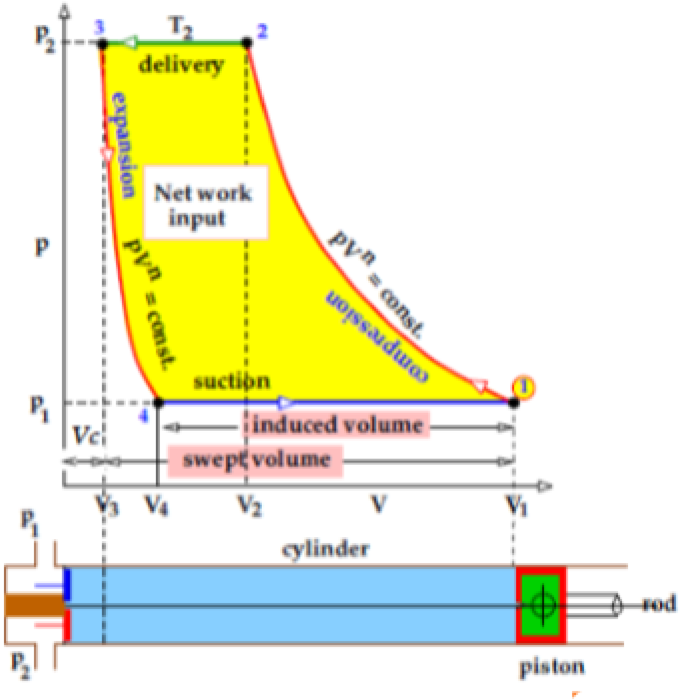
$$\text{FAD} = V_a = V_{\text{in}} \frac{T_a p_1}{T_1 p_a}$$

$$\text{where } V_{\text{in}} = V_1 - V_4$$

Volumetric Efficiency, η_v

- Volumetric efficiency is another definition to measure the performance of a compressor.
- It can be defined as the ratio of the actual delivered gas volume to the swept volume of the cylinder or,

$$\eta_{vol} = \frac{\text{Induced Volume}}{\text{Swept Volume}}$$



- Where,
  - Induced volume: $V_{\text{in}} = V_1 - V_4$
    $= V_s + V_c - V_4$
  - Swept volume: $V_s = V_1 - V_3$
    $= V_1 - V_c$
- If the volumetric efficiency is 100%, it means the compressor has no clearance volume.

Volumetric Efficiency, η_v

K=Vc/Vs

- The volumetric efficiency also can be written as:

$$\eta_v = 1 - \frac{V_3}{V_s}\left(\left[\frac{p_2}{p_1}\right]^{1/n} - 1\right)$$

rp=pressure ratio

- or

$$\eta_v = 1 - \frac{V_c}{V_s}\left(\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}} - 1\right)$$

or

$$\eta_v = 1 + k - krp^{\frac{1}{n}}$$

- From the definition, the volumetric efficiency decreases as the pressure ratio and clearance ratio increase.
- This efficiency is made worse if leaks occur past the valves or piston

Isothermal Efficiency

Isothermal efficiency: the ratio of isothermal work done to the actual work done is called isothermal efficiency.

$$\eta_{iso} = \frac{m\cdot RT \ln \frac{P_2}{P_1}}{\frac{n}{n-1} P_1 V_{in} \left(rp^{\frac{n-1}{n}} - 1 \right)}$$

Multistage Compressors

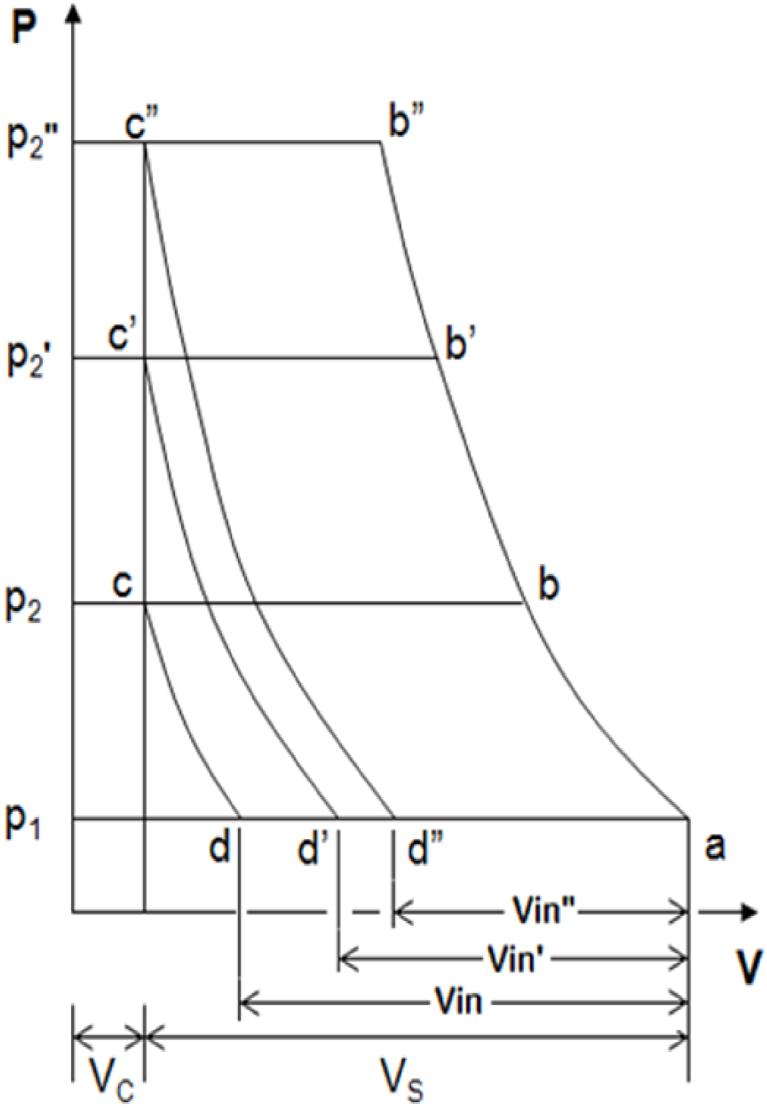
When delivery pressure is **increased to a higher value**, several weaknesses will occur:

i. induced volume will be less

ii. increase in delivery temperature

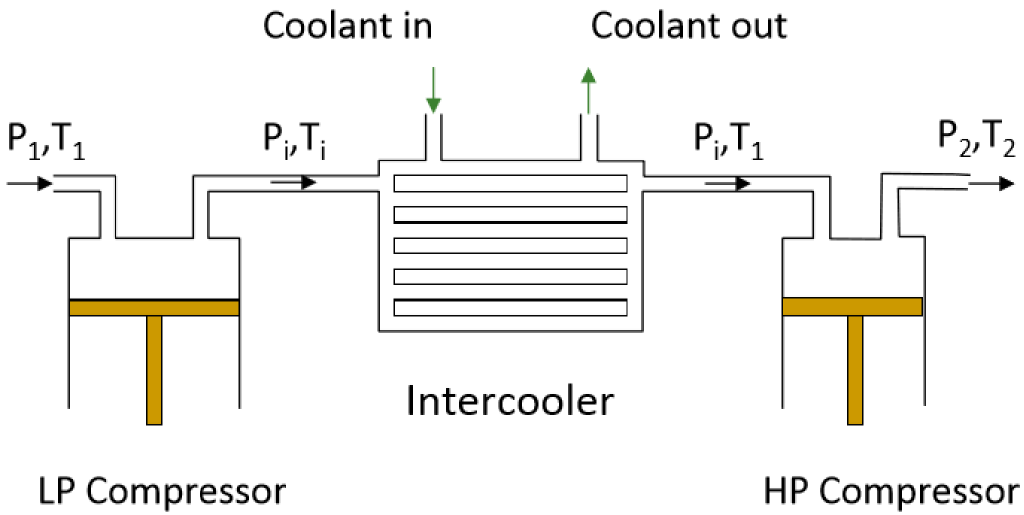
iii. decrease in volumetric efficiency (Vin become less were else no change in Vs)

To overcome those matter, **multi-staging compressor** is introduced.



Multistage Compressors

Perfect intercooler



- It consist of more than one compressor where the air passes through an intercooler before entering the next compressor.
- The size of the next compressor is smaller to compromise Vs.
- In the intercooler, heat is transferred to the surrounding and temperature will decreased. It will be brought back to its inlet temperature (before induction process).
- It is assumed that all compressors will have the same polytropic index.

Multistage Compressors

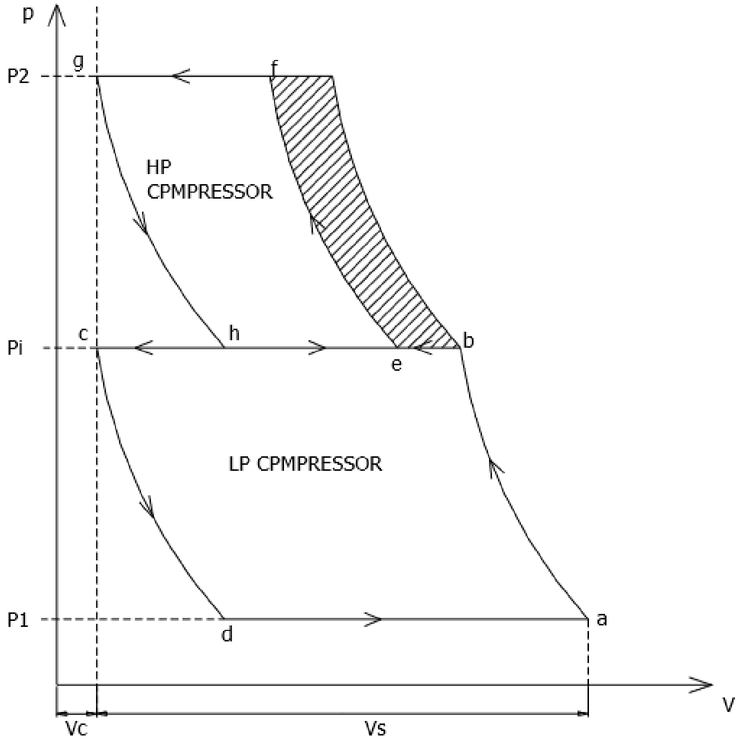
a-b : $PV^n=C$ compression

b-e : Q from air to surrounding
Temperature drops from T_b to T_e.
Ideally $T_e=T_a$

e-f : $PV^n=C$ compression

Advantages:

i. Slight increase in temperature
ii. Increase in volumetric efficiency
iii. Saving in work (shaded area)



*****NOTES:**

- Since no mass is allow to escape during its travel, $\boldsymbol{m_{LPC} = m_{HPC}}$
- If pressure ratio and the ratio of V_c/V_s is the same, volumetric efficiency for both compressor is the same.

The Ideal Intermediate Pressure

Total power input for 2-stage compression:

$$\begin{aligned}\dot{W}_c &= W_{HP} + W_{LP},\\ &= \frac{n}{n-1}\dot{m}RT_1\left(\left[\frac{p_i}{p_1}\right]^{(n-1)/n} - 1\right) + \frac{n}{n-1}\dot{m}RT_1\left(\left[\frac{p_2}{p_i}\right]^{(n-1)/n} - 1\right)\end{aligned}$$

For fixed value of p_1, T_1 and $p_2, \dot{W}$ becomes minimum if

$$\frac{p_i}{p_1} = \frac{p_2}{p_i} \qquad \text{or} \qquad p_i^2 = p_1 p_2 \qquad \text{Or} \qquad p_i = \sqrt{p_1 p_2}$$

So, the total minumum power:

$$\dot{W}_{\mathrm{c,min}} = 2 * \frac{n}{n-1}\dot{m}RT_1\left(\left[\frac{p_i}{p_1}\right]^{(n-1)/n} - 1\right)$$


Thank You!

Dryness fraction measurements

The dryness fraction of steam can be measured by using the following calorimeters:

1. Tank or bucket calorimeter
2. Throttling calorimeter
3. Separating and throttling calorimeter.

Tank or Bucket Calorimeter

The dryness fraction of steam can be found with the help of tank calorimeter as follows: A known mass of steam is passed through a known mass of water and steam is completely condensed. The heat lost by steam is equated to heat gained by the water. Fig. 1 shows the arrangement of this calorimeter. The steam is passed through the sampling tube into the bucket calorimeter containing a known mass of water. The weights of calorimeter with water before mixing with steam and after mixing the steam are obtained by weighing. The temperature of water before and after mixing the steam are measured by mercury thermometer. The pressure of steam passed through the sampling tube is measured with the help of pressure gauge.

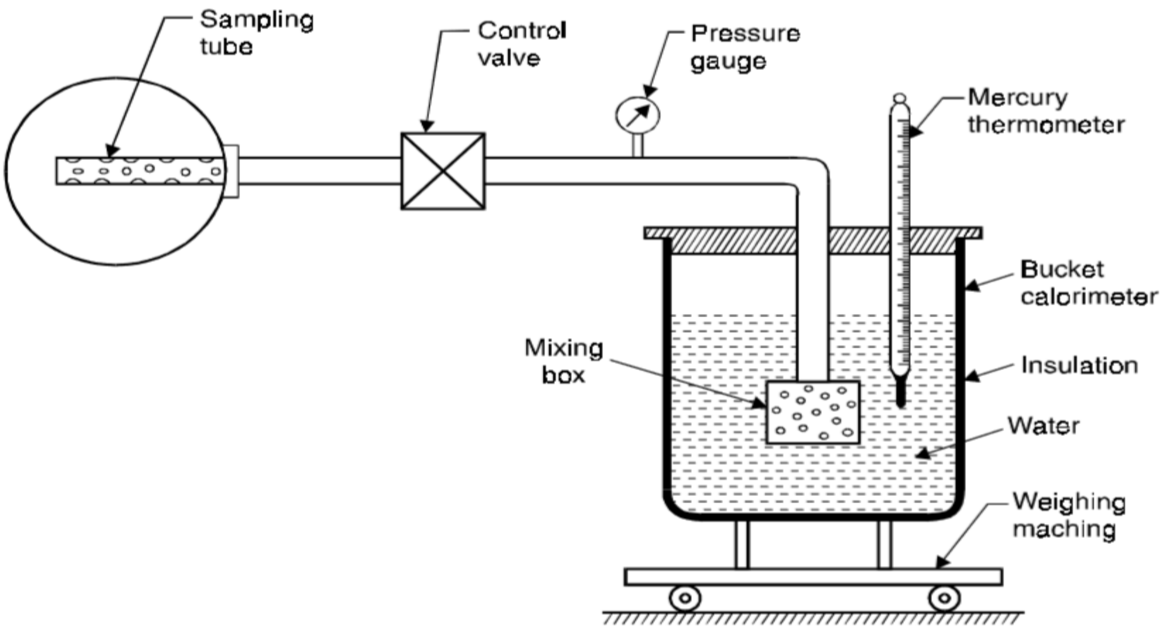


Fig. Tank or bucket calorimeter.

Throttling Calorimeter

The dryness fraction of wet steam can be determined by using a throttling calorimeter which is illustrated diagrammatically in Fig. 2.

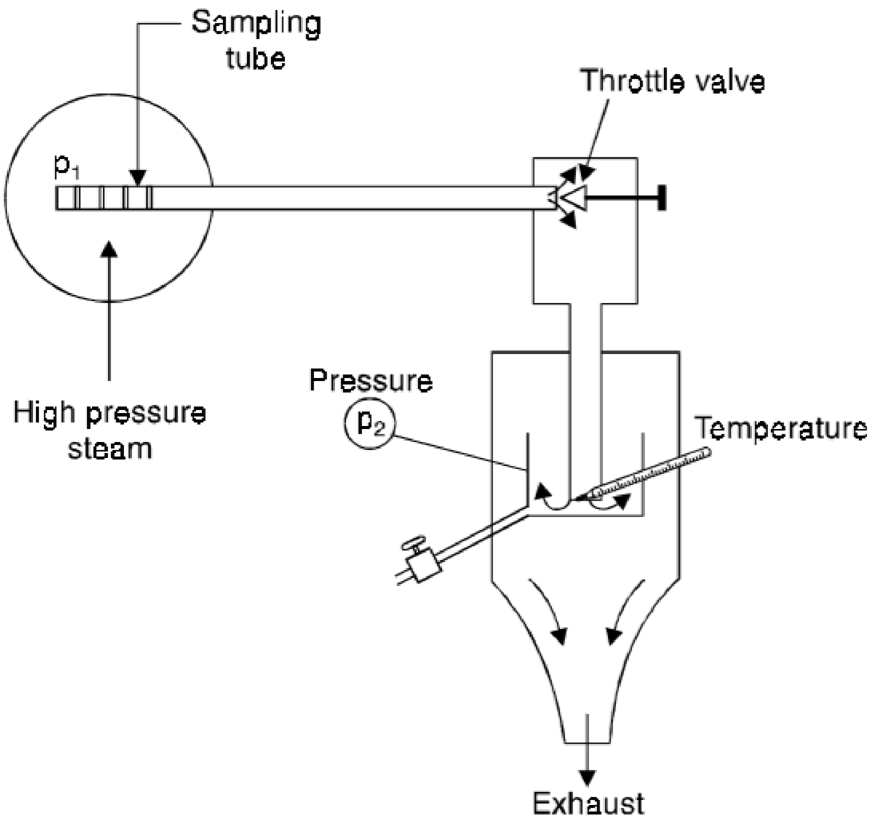


Fig Throttling calorimeter.

The steam to be sampled is taken from the pipe by means of suitable positioned and dimensioned sampling tube. It passes into an insulated container and is throttled through an orifice to atmospheric pressure. Here the temperature is taken and the steam ideally should have about 5.5 K of superheat.

The throttling process is shown on h-s diagram in Fig. 3by the line 1-2. If steam initially wet is throttled through a sufficiently large pressure drop, then the steam at state 2 will become superheated. State 2 can then be defined by the measured pressure and temperature. The enthalpy, h_2 can then be found and hence

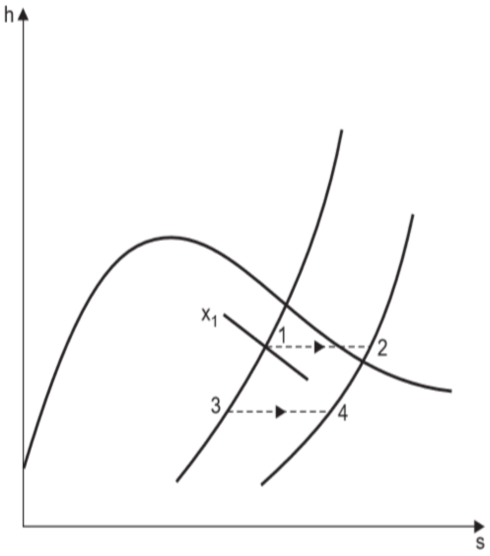


Fig. Throttling process.

$$h_2 = h_1 = (h_{f_1} + x_1 h_{fg_1}) \text{ at } p_1$$

[where $h_2 = h_{f_2} + h_{fg_2} + c_{ps}(T_{sup_2} - T_{s_2})$]

$$\therefore \quad x_1 = \frac{h_2 - h_{f_1}}{h_{fg_1}}$$

Example 1/ A throttling calorimeter is used to measure the dryness fraction of the steam in the steam main which has steam flowing at a pressure of 8 bar. The steam after passing through the calorimeter is at 1 bar pressure and 115°C. Calculate the dryness fraction of the steam in the main. Take cps = 2.1 kJ/kg K.

Separating and Throttling Calorimeter

If the steam whose dryness fraction is to be determined is very wet then throttling to atmospheric pressure may not be sufficient to ensure superheated steam at exit. In this case it is necessary to dry the steam partially, before throttling. This is done by passing the steam sample from the main through a separating calorimeter as shown in Fig. 4. The steam is made to change direction suddenly, and the water, being denser than the dry steam is separated out. The quantity of water which is separated out (m_w) is measured at the separator, the steam remaining, which now has a higher dryness fraction, is passed through the throttling calorimeter. With the combined separating and throttling calorimeter it is necessary to condense the steam after throttling and measure the amount of condensate (m_s). If a throttling calorimeter only is sufficient, there is no need to measure condensate, the pressure and temperature measurements at exit being sufficient.

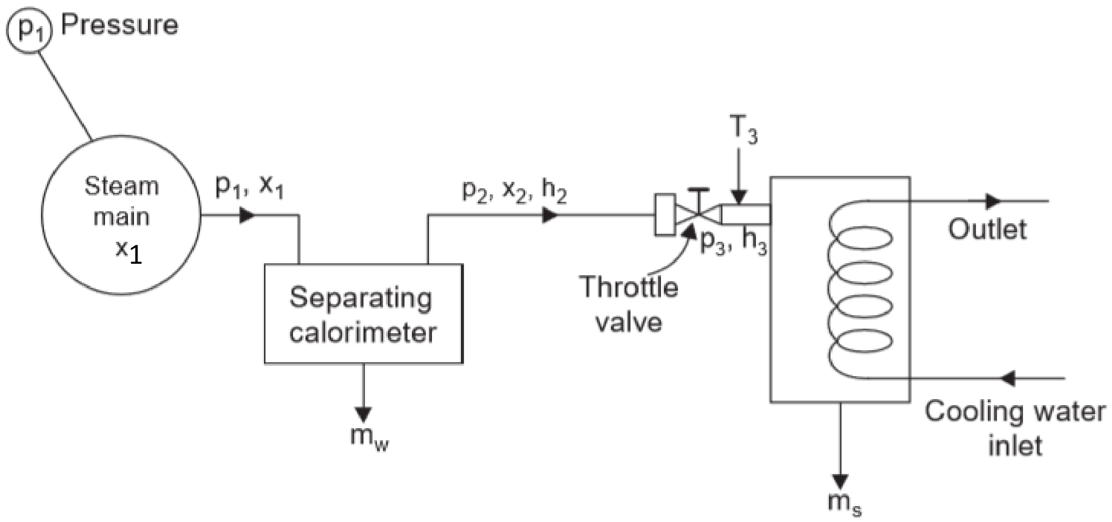


Fig. Separating and throttling calorimeter.

Dryness fraction at 2 is x_2, therefore, the mass of dry steam leaving the separating calorimeter is equal to x_2m_s and this must be the mass of dry vapour in the sample drawn from the main at state 1.

Hence fraction in main, $x_1 = \frac{\text{Mass of dry vapour}}{\text{Total mass}} = \frac{x_2\, m_s}{m_w + m_s}$.

The dryness fraction, x_2, can be determined as follows :

$$^*h_3 = h_2 = h_{f_2} + x_2 h_{fg_2} \text{ at } p_2 \; [^*h_3 = h_{f_3} + h_{fg_3} + c_{ps}\,(T_{sup_3} - T_{s_3}) \text{ at pressure } p_3]$$

$$x_2 = \frac{h_3 - h_{f_2}}{h_{fg_2}}$$

The values of h_{f2} and h_{fg2} are read from steam tables at pressure p_2. The pressure in the separator is small so that p_1 is approximately equal to p_2.

Example 2. The following observations were taken with a separating and a throttling calorimeter arranged in series: Water separated = 2 kg, steam discharged from the throttling calorimeter = 20.5 kg, temperature of steam after throttling = 110°C, initial pressure = 12 bar, barometer = 760 mm of Hg, final pressure = 5 mm of Hg. Estimate the quality of steam supplied.

H.W. The following data were obtained in a test on a combined separating and throttling calorimeter: Pressure of steam sample = 15 bar, pressure of steam at exit = 1 bar, temperature of steam at the exit = 150°C, discharge from separating calorimeter = 0.5 kg/min, discharge from throttling calorimeter = 10 kg/min. Determine the dryness fraction of the sample steam.

GAS MIXTURE

To determine the properties of a mixture, we need to know the composition of the mixture as well as the properties of the individual components. There are two ways to describe the composition of a mixture: either by specifying the number of moles of each component, called ***molar analysis (volumetric analysis)***, or by specifying the mass of each component, called ***gravimetric analysis***.

Consider a gas mixture composed of ***k*** components. The mass of the mixture $\boldsymbol{m_m}$: is the sum of the masses of the individual components, and the mole number of the mixture $\boldsymbol{N_m}$: is the sum of the mole numbers of the individual components (Figs.1 and 2). That is,

$$m_m = \sum_{i=1}^{k} m_i \quad \text{and} \quad N_m = \sum_{i=1}^{k} N_i \qquad \ldots 1$$

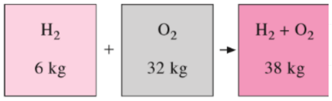


FIGURE 1
The mass of a mixture is equal to the sum of the masses of its components.

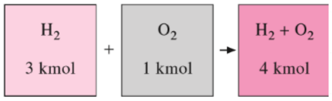


FIGURE 2
The number of moles of a nonreacting mixture is equal to the sum of the number of moles of its components.

The ratio of the mass of a component to the mass of the mixture is called the ***mass fraction*** m_f, and the ratio of the mole number of a component to the mole number of the mixture is called the ***mole fraction*** y:

$$\mathrm{mf}_i = \frac{m_i}{m_m} \quad \text{and} \quad y_i = \frac{N_i}{N_m} \qquad \ldots 2$$

we can easily show that the sum of the mass fractions or mole fractions for a mixture is equal to 1 (Fig. 3):

$$\sum_{i=1}^{k} \mathrm{mf}_i = 1 \quad \text{and} \quad \sum_{i=1}^{k} y_i = 1 \qquad \ldots 3$$

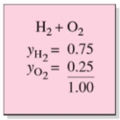


FIGURE 3
The sum of the mole fractions of a mixture is equal to 1.

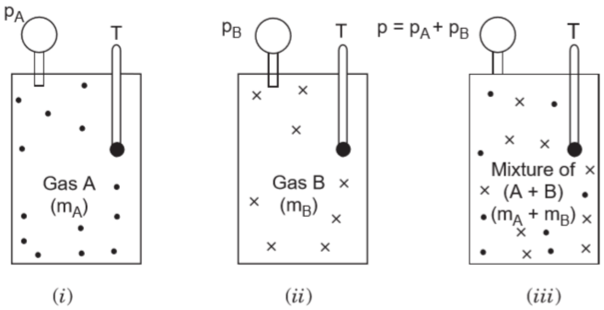
DALTON'S LAW AND GIBBS-DALTON LAW

Dalton's law: Let us consider a closed vessel of volume V at temperature T, which contains a mixture of perfect gases at a known pressure. If some of the mixture were removed, then the pressure would be less than the initial value. If the gas removed were the full amount of one of the constituents then the reduction in pressure would be equal to the contribution of that constituent to the initial total pressure. Each constituent contributes to the total pressure by an amount which is known as the partial pressure of the constituent.

The relationship between the partial pressures of the constituents is expressed by Dalton's law, as follows:

1- The pressure of a mixture of gases(total pressure) is equal to the sum of the partial pressures of the constituents.

2-The partial pressure of each constituent is that pressure which the gas would exert if it occupied alone that volume occupied by the mixtures at the same temperature.



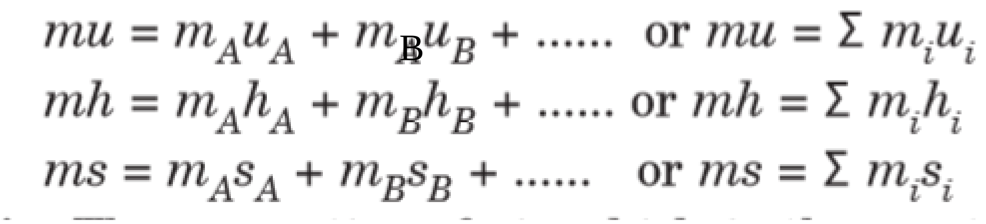
$$p = p_A + p_B + p_c + \ldots\ldots or\ p = \Sigma p_k$$

where p_k = The partial pressure of a constituent.

Gibbs-Dalton law Dalton’s law was re-formulated by Gibbs to include a second statement on the properties of mixtures. The combined statement is known as the Gibbs-Dalton law, and is as follows:

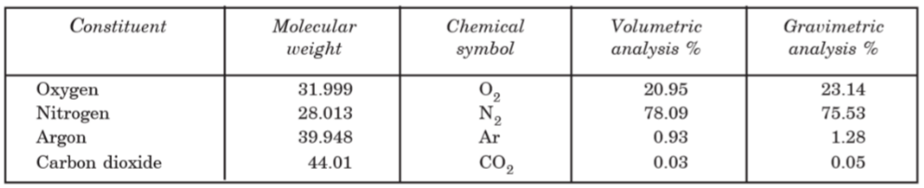
1-The internal energy(u), enthalpy(h), and entropy (s)of a gaseous mixture are respectively equal to the sums of the internal energies, enthalpies, and entropies, of the constituents.

2- Each constituent has that internal energy, enthalpy and entropy, which it could have if it occupied alone that volume occupied by the mixture at the temperature of the mixture. This statement leads to the following equations:



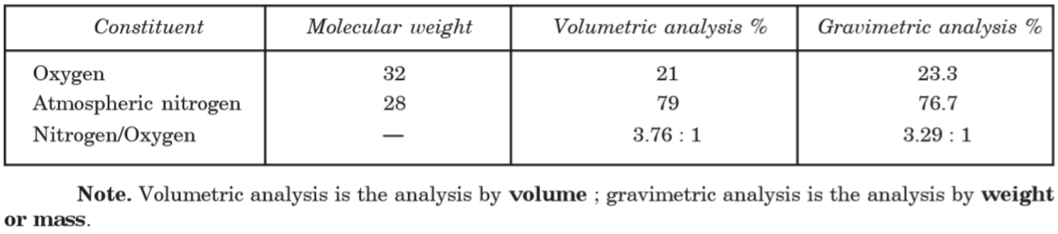
$$mu = m_A u_A + m_B u_B + \ldots\ldots \quad \text{or } mu = \Sigma\, m_i u_i$$
$$mh = m_A h_A + m_B h_B + \ldots\ldots \quad \text{or } mh = \Sigma\, m_i h_i$$
$$ms = m_A s_A + m_B s_B + \ldots\ldots \quad \text{or } ms = \Sigma\, m_i s_i$$

Properties of air: The properties of air which is the most common mixture are given below:



Constituent	*Molecular weight*	*Chemical symbol*	*Volumetric analysis %*	*Gravimetric analysis %*
Oxygen	31.999	O_2	20.95	23.14
Nitrogen	28.013	N_2	78.09	75.53
Argon	39.948	Ar	0.93	1.28
Carbon dioxide	44.01	CO_2	0.03	0.05

For approximate calculations the air is said to be composed of oxygen and ‘atmospheric nitrogen’.

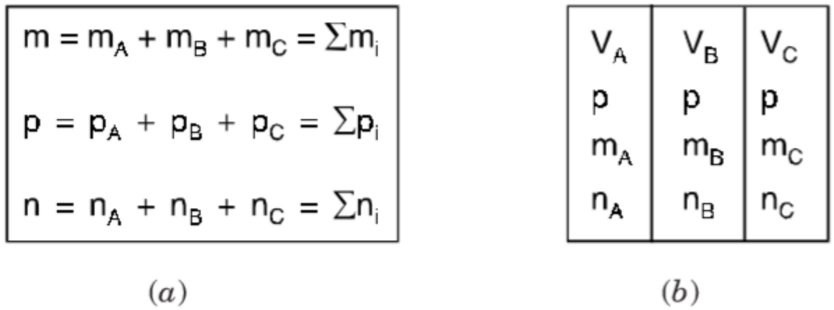


Constituent	*Molecular weight*	*Volumetric analysis %*	*Gravimetric analysis %*
Oxygen	32	21	23.3
Atmospheric nitrogen	28	79	76.7
Nitrogen/Oxygen	—	3.76 : 1	3.29 : 1

Note. Volumetric analysis is the analysis by **volume** ; gravimetric analysis is the analysis by **weight or mass**.

VOLUMETRIC ANALYSIS OF A GAS MIXTURE

Let us consider a volume V of a gaseous mixture at a temperature T, consisting of three constituents A, B and C .Let us further assume that each of the constituents is compressed to a pressure p equal to the total pressure of the mixture, and let the temperature remain constant. The partial volumes then occupied by the constituents will be V_A, V_B and V_C.



$$m = m_A + m_B + m_C = \Sigma m_i$$

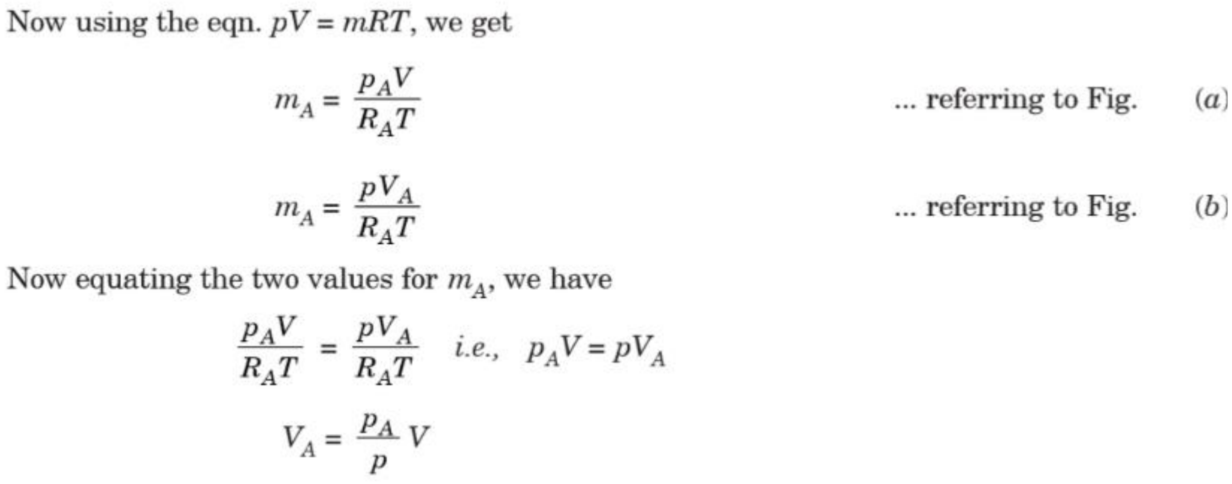
$$p = p_A + p_B + p_C = \Sigma p_i$$

$$n = n_A + n_B + n_C = \Sigma n_i$$

(*a*)

V_A	V_B	V_C
p	p	p
m_A	m_B	m_C
n_A	n_B	n_C

(*b*)



Now using the eqn. $pV = mRT$, we get

$$m_A = \frac{p_A V}{R_A T} \qquad \text{... referring to Fig. } (a)$$

$$m_A = \frac{p V_A}{R_A T} \qquad \text{... referring to Fig. } (b)$$

Now equating the two values for m_A, we have

$$\frac{p_A V}{R_A T} = \frac{p V_A}{R_A T} \quad \text{i.e.,} \quad p_A V = p V_A$$

$$V_A = \frac{p_A}{p} V$$

In general therefore,

$$V_i = \frac{p_i}{p} V \qquad ...(9.8)$$

i.e.,

$$\Sigma V_i = \sum \frac{p_i V}{p} = \frac{V}{p} \Sigma p_i$$

Now from eqn. (9.4), $p = \Sigma p_i$, therefore,

$$\Sigma V_i = V \qquad ...(9.9)$$

Thus, the *volume of a mixture of gases is equal to the sum of the volumes of the individual constituents when each exists alone at the pressure and temperature of the mixture.*

This is the statement of another empirical law, *the law of partial volumes*, sometimes called *Amagat's law* or *Leduc's law*.

— The analysis of mixtures, oftenly, is simplified if it is carried out in moles. The mole is given by the equation

$$n = \frac{m}{M}$$

where, n = Number of moles,
m = Mass of gas, and
M = Molecular weight.

According to *Avogadro's law*, the *number of moles of any gas is proportional to the volume of the gas at a given pressure and temperature.* Referring to Fig. 9.2 (*a*), the volume V contains n moles of the mixture at p and T. In Fig. 9.2 (*b*), the gas A occupies a volume V_A at p and T, and this volume contains n_A moles. Similarly there are n_B moles of gas B in volume V_B and n_C moles of gas C in volume V_C.

From eqn. (9.9), $\Sigma V_i = V$

or $V_A + V_B + V_C = V$

$\therefore$ The *total number of moles in the vessel must equal the sum of the moles of the individual constituents,*

$$n = n_A + n_B + n_C = \Sigma n_i \qquad ...(9.10)$$

9.4. THE APPARENT MOLECULAR WEIGHT AND GAS CONSTANT

The Apparent Molecular Weight

In a gas mixture if a gas occupies a total volume of V at a temperature T, then from the definition of partial pressure and equation $pV = nR_0T$, we have

$$p_iV = n_iR_0T \qquad ...(9.11)$$

(where R_0 is the *universal gas constant*)

$\therefore$ $\Sigma p_iV = \Sigma n_iR_0T$

i.e., $V \Sigma p_i = R_0T \Sigma n_i$

Also $p = \Sigma p_i$ [from eqn. (9.4)]

$\therefore$ $pV = R_0T \Sigma n_i$

Also $n = \Sigma n_i$ [from eqn. (9.10)]

$\therefore$ $pV = nR_0T$

The *mixture therefore acts as a perfect gas,* and *this is the characteristic equation for mixture.*

An apparent molecular weight is defined by the equation

$$M = \frac{m}{n}$$

where, m = Mass of the mixture, and
n = Number of moles of mixture.

The Gas Constant

The *apparent gas constant* (similarly as above) is defined by the equation

$$R = \frac{R_0}{M}$$

It can be assumed that a mixture of perfect gases obeys all the perfect gas laws.

In order to determine the gas constant for the mixture in terms of the gas constants of the constituents let us consider the equation $pV = mRT$ both for the mixture and for a constituent as follows

$$pV = mRT$$

and $$p_iV = m_iR_iT$$

Then $$\Sigma\, p_iV = \Sigma\, m_iR_iT$$

$\therefore$ $$V\,\Sigma\, p_i = T\,\Sigma\, m_iR_i$$

Also $$p = \Sigma\, p_i$$

$\therefore$ $$pV = T\,\Sigma\, m_iR_i$$

or $$pV = mRT = T\,\Sigma\, m_iR_i$$

i.e., $$mR = \Sigma\, m_iR_i$$

or $$R = \Sigma\, \frac{m_i}{m}\, R_i \qquad ...(9.12)$$

where $\frac{m_i}{m}$ = mass fraction of a constituent.

— From equation (9.11), $p_iV = n_iR_0T$, and combining this with eqn. (9.8) $\left(V_i = \frac{p_i}{p}V\right)$ applied to the mixture (*i.e.*, $pV = nR_0T$), we have

$$\frac{p_iV}{pV} = \frac{n_iR_0T}{nR_0T}$$

or $$\frac{p_i}{p} = \frac{n_i}{n} \qquad ...(9.13)$$

On combining this with eqn. (9.8), we get

$$\frac{p_i}{p} = \frac{n_i}{n} = \frac{V_i}{V} \qquad ...(9.14)$$

This means that the *molar analysis is identical with the volumetric analysis*, and *both are equal to the ratio of the partial pressure to the total pressure.*

— The *apparent molecular weight* can be also be determined by the following method.

Let us apply characteristic equation to each constituent and to mixture, we have

$$m_i = \frac{p_iV}{R_iT}$$

$$m = \frac{pV}{RT}$$

Also $m = \Sigma\, m_i$ [from eqn. (9.3)]

$\therefore$ $$\frac{pV}{RT} = \sum \frac{p_i V}{R_i T}$$

$\therefore$ $$\frac{p}{R} = \sum \frac{p_i}{R_i}$$

Using the relation $R = \frac{R_0}{M}$, and substituting, we have

$$\frac{pM}{R_0} = \sum \frac{p_i M_i}{R_0}$$

or $$pM = \sum p_i M_i$$

i.e., $$M = \sum \frac{p_i}{p} M_i \quad \text{...(9.15)}$$

Now using eqn. (9.14), we have

$$M = \sum \frac{V_i}{V} M_i \quad \text{...(9.16)}$$

and $$M = \sum \frac{n_i}{n} M_i \quad \text{...(9.17)}$$

Alternately

$$p = \Sigma\, p_i = p_A + p_B + \text{......}\, p_i$$

Also $$pV = mRT$$

Similarly $$p_A V = m_A R_A T$$
$$p_B V = m_B R_B T$$

$\therefore$ $$pV = p_A V + p_B V + \text{......}$$

or $$mRT = m_A R_A T + m_B R_B T + \text{......}$$

$\therefore$ $$mR = m_A R_A + m_B R_B + \text{......}$$

But $$R = \frac{R_0}{M},\ R_A = \frac{R_0}{M_A},\ R_2 = \frac{R_0}{M_B}$$

Substituting this in the above equation, we get

$$m\,\frac{R_0}{M} = m_A\,\frac{R_0}{M_A} + m_B\,\frac{R_0}{M_B} + \text{......}$$

$\therefore$ $$\frac{1}{M} = \frac{m_A}{m}\cdot\frac{1}{M_A} + \frac{m_B}{m}\cdot\frac{1}{M_B} + \text{......}$$
$$= \frac{m_{fA}}{M_A} + \frac{m_{fB}}{M_B} + \text{......}$$

where m_{fA}, m_{fB} etc. are the *mass fractions* of the constituents.

$\therefore$ $$\frac{1}{M} = \sum \frac{m_{fi}}{M_i}$$

$\therefore$ $$M = \frac{1}{\sum \frac{m_{fi}}{M_i}} \quad \text{...(9.18)}$$

9.5. SPECIFIC HEATS OF A GAS MIXTURE

— As per Gibbs-Dalton law, the internal energy of a mixture of gases is given by

$$mu = \Sigma\, m_i u_i \quad ...(i) \quad \text{[from eqn. (9.5)]}$$

Also

$$u = c_v T \quad ...(ii) \quad \text{[from perfect gas equation]}$$

Now from (*i*) and (*ii*), we have

$$mc_v T = \Sigma\, m_i c_{vi} T$$

$$\therefore \quad mc_v = \Sigma\, m_i c_{vi}$$

or

$$c_v = \sum \frac{m_i}{m}\, c_{vi} \quad ...(9.19)$$

Similarly from equations, $mh = \Sigma\, m_i h_i$

and $h = c_p T$, we get

$$mc_p T = \Sigma\, m_i c_{pi} T$$

$$\therefore \quad m_{cp} = \Sigma\, m_i c_{pi}$$

or

$$c_p = \sum \frac{m_i}{m}\, c_{pi} \quad ...(9.20)$$

From eqns. (9.18) and (9.19),

$$c_p - c_v = \sum \frac{m_i}{m}\, c_{pi} - \sum \frac{m_i}{m}\, c_{vi} = \sum \frac{m_i}{m}\, (c_{pi} - c_{vi})$$

Also $c_{pi} - c_{vi} = R_i$, therefore,

$$c_p - c_v = \sum \frac{m_i}{m}\, R_i$$

Also from eqn. (9.12), $R = \sum \frac{m_i}{m} R_i$, therefore for the mixture

$$c_p - c_v = R$$

The following equations can be applied to a mixture of gases

$$\gamma = \frac{c_p}{c_v}; \quad c_v = \frac{R}{\gamma - 1}; \quad c_p = \frac{\gamma R}{\gamma - 1}$$

It should be borne in mind that γ *must be determined from the eqn.* $\gamma = \frac{c_p}{c_v}$ *; there is no weighted mean expression as there is for* R, c_v *and* c_p.

— In problems on mixtures it is often convenient to work in moles and the specific heats can be expressed in terms of the mole. These are known as *molar heats*, and are denoted by C_p and C_v.

Molar heats are defined as follows :

$$C_p = Mc_p \quad \text{and} \quad C_v = Mc_v \quad ...(9.21)$$

But

$$c_p - c_v = R$$

$$\therefore \quad C_p - C_v = Mc_p - Mc_v = M(c_p - c_v) = MR$$

But

$$MR = R_0$$

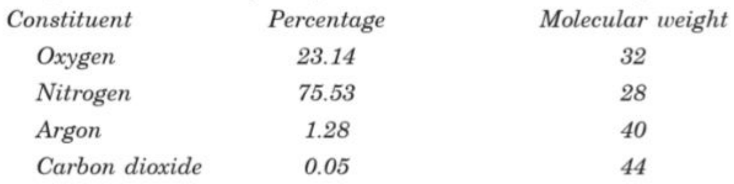
$$\therefore \quad C_p - C_v = R_0 \quad ...(9.22)$$

Also

$$U = mc_v T = \frac{mMc_v T}{M}$$

__Example1__: A vessel of 0.35 m^3 capacity contains 0.4 kg of carbon monoxide (molecular weight = 28) and 1 kg of air at 20°C. Calculate : (i) The partial pressure of each constituent, (ii) The total pressure in the vessel, and The gravimetric analysis of air is to be taken as 23.3% oxygen (molecular weight = 32) and 76.7% nitrogen (molecular weight = 28).

__Example 2__: The gravimetric analysis of air and other data are as follows:



Constituent	*Percentage*	*Molecular weight*
Oxygen	*23.14*	*32*
Nitrogen	*75.53*	*28*
Argon	*1.28*	*40*
Carbon dioxide	*0.05*	*44*

Calculate: (i) Gas constant for air; (ii) Apparent molecular weight.

__Example 3__. Following is the gravimetric analysis of air:

Constituent	Percentage
Oxygen	23.14
Nitrogen	75.53
Argon	1.28
Carbon dioxide	0.05

Calculate the analysis by volume and the partial pressure of each constituent when the total pressure is 1 bar.

Gas Turbine Cycles

The simple constant pressure cycle and the open- and closed-cycle gas turbine units have been considered briefly in Chapter 5. In this chapter the various parts of the cycle will be considered in more detail and the practical limitations and modifications to the ideal cycle will be discussed.

The main use for the gas turbine at the present day is in the aircraft field, although gas turbine units for electric power generation are being used increasingly, usually using natural gas as fuel. Gas turbines are used in marine propulsion, but the oil engine and steam turbine are more frequently used, particularly for larger ships. The gas turbine is also used in conjunction with the oil engine, and as part of total energy schemes in combination with steam plant; this is discussed more fully in Chapter 17.

The inefficiencies in the compression and expansion processes become greater for smaller stand-alone gas turbine units and a heat exchanger is frequently used in order to improve the cycle efficiency. A compact effective heat exchanger is necessary before the small gas turbine can compete for economy with the small oil engine or petrol engine.

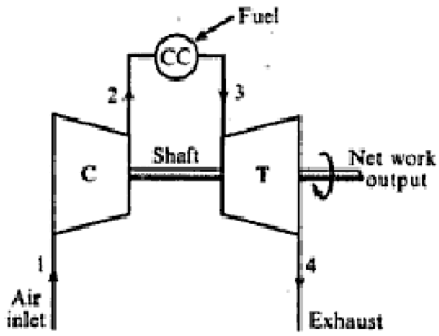
The use of constant pressure combustion with a rotary compressor driven by a rotary turbine, mounted on a common shaft, gives a combination which is ideal for conditions of steady mass flow over a wide operating range.

9.1 The practical gas turbine cycle

The most basic gas turbine unit is one operating on the open cycle in which a rotary compressor and a turbine are mounted on a common shaft, as shown diagrammatically in Fig. 9.1. Air is drawn into the compressor, C, and after compression passes to a combustion chamber, CC. Energy is supplied in the combustion chamber by spraying fuel into the airstream, and the resulting hot gases expand through the turbine, T, to the atmosphere. In order to achieve net work output from the unit, the turbine must develop more gross work output than is required to drive the compressor and to overcome mechanical losses in the drive.

The compressor used is either a centrifugal or an axial flow compressor and

Fig. 9.1 Open-cycle gas turbine unit

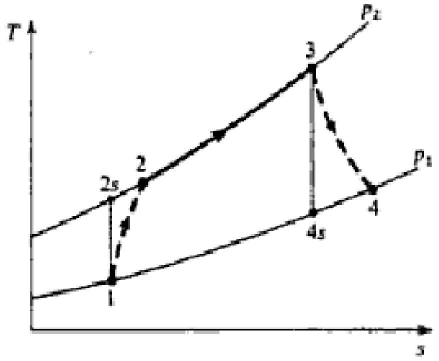


the compression process is therefore irreversible but approximately adiabatic. Similarly the expansion process in the turbine is irreversible but adiabatic. Due to these irreversibilities, more work is required in the compression processes for a given pressure ratio, and less work is developed in the expansion process. It is possible that the compressor and turbine may be so inefficient that the unit is not self-sustaining, and in fact it was the difficulties in improving the compressor and turbine design to cut down irreversibilities that retarded the development of the gas turbine unit.

As stated in section 5.5 the open-cycle gas turbine cannot be compared directly with the ideal constant pressure cycle. The actual cycle involves a chemical reaction in the combustion chamber which results in high-temperature products which are chemically different from the reactants (see section 7.8). During combustion there is no energy exchange with the surroundings, the effect being a gradual decrease in chemical energy with a corresponding increase in enthalpy of the working fluid. The combustion reaction will not be considered in detail here, and a simplification will be made by assuming that the chemical energy released on a combustion is equivalent to a transfer of heat at constant pressure to a working fluid of constant mean specific heat. This simplified approach allows the actual process to be compared with the ideal and to be represented on a T–s diagram.

Neglecting the pressure loss in the combustion chamber the cycle may be drawn on a T–s diagram as shown in Fig. 9.2. Line 1–2 represents irreversible adiabatic compression; line 2–3 represents constant pressure heat supply in the

Fig. 9.2 Gas turbine cycle on a T–s diagram



combustion chamber; line 3–4 represents irreversible adiabatic expansion. The process 1–2*s* represents the ideal isentropic process between the same pressures p_1 and p_2. Similarly the process 3–4*s* represents the ideal isentropic expansion process between the pressures p_2 and p_1. For the moment it will be assumed that the change in kinetic energy between the various points in the cycle is negligibly small compared with the enthalpy changes. Then applying the flow equation to each part of the cycle, we have the following for unit mass. For the compressor:

$$\text{Work input} = c_p(T_2 - T_1)$$

For the combustion chamber:

$$\text{Heat supplied} = c_p(T_3 - T_2)$$

For the turbine:

$$\text{Work output} = c_p(T_3 - T_4)$$

Then $\quad$ $\text{Net work output} = c_p(T_3 - T_4) - c_p(T_2 - T_1)$

and

$$\text{Thermal efficiency} = \frac{\text{net work output}}{\text{heat supplied}}$$

$$= \frac{c_p(T_3 - T_4) - c_p(T_2 - T_1)}{c_p(T_3 - T_2)}$$

The value of the specific heat capacity of a real gas varies with temperature; also, in the open cycle, the specific heat capacity of the gases in the combustion chamber and in the turbine is different from that in the compressor because fuel has been added and a chemical change has taken place. Curves showing the variation of c_p with temperature and air–fuel ratio can be used, and a suitable mean value of c_p and hence γ can be found. It is usual in the gas turbine practice to assume fixed mean values of c_p and γ for the expansion process, and fixed mean values of c_p and γ for the compression process. For the combustion process, curves as shown in Fig. 9.18 (p.282) are used; for simple calculations a mean value of c_p can be assumed. In an open-cycle gas turbine unit the mass flow of gases in the turbine is greater than that in the compressor due to the mass of fuel burned, but it is possible to neglect the mass of fuel, since the air–fuel ratios used are large. Also, in many cases, air is bled from the compressor for cooling purposes, or in the case of aircraft at high altitude, bleed air is used for de-icing and cabin air-conditioning. This amount of air bleed is approximately the same as the mass of fuel injected.

The isentropic efficiency of the compressor is defined as the ratio of the work input required in isentropic compression between p_1 and p_2 to the actual work required.

Neglecting changes in kinetic energy, we have

$$\text{Compressor isentropic efficiency, } \eta_C = \frac{c_p(T_{2s} - T_1)}{c_p(T_2 - T_1)}$$

$$= \frac{T_{2s} - T_1}{T_2 - T_1} \tag{9.1}$$

Similarly the isentropic efficiency of the turbine is defined as the ratio of the actual work output to the isentropic work output between the same pressures.

Neglecting kinetic energy changes

$$\text{Turbine isentropic efficiency, } \eta_T = \frac{c_p(T_3 - T_4)}{c_p(T_3 - T_{4s})}$$

$$= \frac{T_3 - T_4}{T_3 - T_{4s}} \qquad (9.2)$$

Example 9.1 A gas turbine unit has a pressure ratio of 10/1 and a maximum cycle temperature of 700 °C. The isentropic efficiencies of the compressor and turbine are 0.82 and 0.85 respectively. Calculate the power output of an electric generator geared to the turbine when the air enters the compressor at 15 °C at the rate of 15 kg/s. Take $c_p = 1.005$ kJ/kg K and $\gamma = 1.4$ for the compression process, and take $c_p = 1.11$ kJ/kg K and $\gamma = 1.333$ for the expansion process.

Solution A line diagram of the unit is shown in Fig. 9.3(a), and the cycle is shown on a *T–s* diagram in Fig. 9.3(b). In order to evaluate the net work output it is necessary to calculate the temperatures T_2 and T_4. To calculate T_2 we must first calculate T_{2s} and then use the isentropic efficiency.

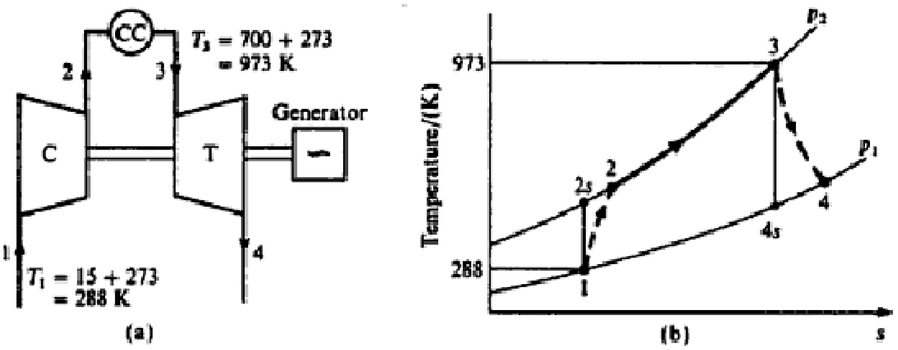


Fig. 9.3 Gas turbine unit (a) and *T–s* diagram (b) for Example 9.1

From equation (3.21) for an isentropic process

$$\frac{T_{2s}}{T_1} = \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma}$$

therefore

$$T_{2s} = 288 \times (10)^{0.4/1.4} = 288 \times 1.931 = 556\text{ K}$$

Then using equation (9.1)

$$\eta_C = \frac{T_{2s} - T_1}{T_2 - T_1} = \frac{556 - 288}{T_2 - 288} = 0.82$$

i.e. $$(T_2 - 288) = \frac{268}{0.82} = 326.8\text{ K}$$

therefore

$$T_2 = 288 + 326.8 = 614.8 \text{ K}$$

Similarly for the turbine

$$\frac{T_3}{T_{4s}} = \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma}$$

therefore

$$T_{4s} = \frac{973}{(10)^{0.333/1.333}} = \frac{973}{1.778} = 547.4 \text{ K}$$

Then from equation (9.2)

$$\eta_T = \frac{T_3 - T_4}{T_3 - T_{4s}} = \frac{973 - T_4}{973 - 547.4} = 0.85$$

i.e. $(973 - T_4) = 425.6 \times 0.85 = 361.8 \text{ K}$

therefore

$$T_4 = 973 - 361.8 = 611.2 \text{ K}$$

Hence Compressor work input $= c_p(T_2 - T_1) = 1.005 \times 326.8$

$= 328.4 \text{ kJ/kg}$

Turbine work output $= c_p(T_3 - T_4) = 1.11 \times 361.8$

$= 401.6 \text{ kJ/kg}$

therefore

Net work output $= (401.6 - 328.4) = 73.2 \text{ kJ/kg}$

i.e. Power output $= 73.2 \times 15 = 1098 \text{ kW}$

Example 9.2 Calculate the cycle efficiency and the work ratio of the plant in Example 9.1, assuming that c_p for the combustion process is 1.11 kJ/kg K

Solution

Heat supplied $= c_p(T_3 - T_2)$

$= 1.11(973 - 614.8) = 1.11 \times 358.2 \text{ kJ/kg}$

i.e. Heat supplied $= 397.6 \text{ kJ/kg}$

Therefore

$$\text{Cycle efficiency} = \frac{\text{net work output}}{\text{heat supplied}} = \frac{73.2}{397.6}$$

i.e. Cycle efficiency = 0.184 or 18.4%

From the definition of work ratio given in section 5.3, we have

$$\text{Work ratio} = \frac{\text{net work output}}{\text{gross work output}} = \frac{73.2}{401.6} = 0.182$$

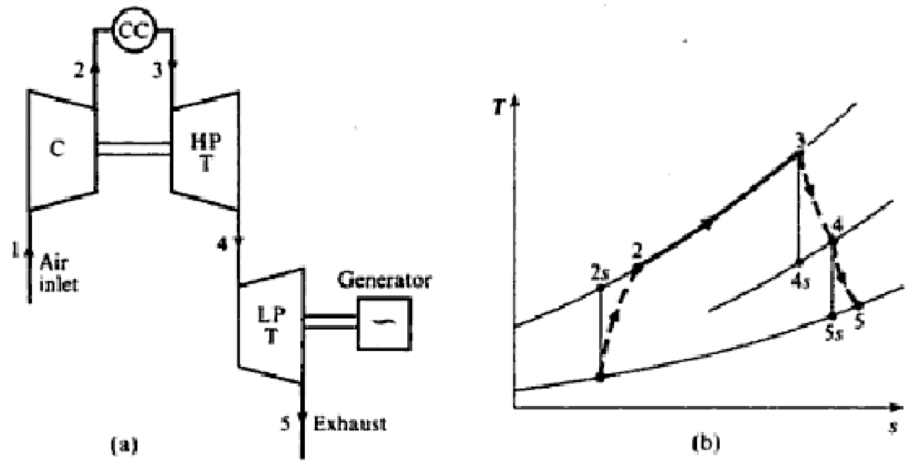
Use of a power turbine

In Examples 9.1 and 9.2 the turbine is arranged to drive the compressor and to develop net work. It is sometimes more convenient to have two separate turbines, one of which drives the compressor while the other provides the power output. The first, or high-pressure (HP) turbine, is then known as the compressor turbine, and the second, or low-pressure (LP) turbine, is called the power turbine. The arrangement is shown in Fig. 9.4(a). Assuming that each turbine has its own isentropic efficiency, the cycle is as shown on a *T–s* diagram in Fig. 9.4(b). The numbers on Fig. 9.4(b) correspond to those of Fig. 9.4(a). Neglecting kinetic energy changes, we have

work from HP turbine = work input to compressor

i.e. $c_{p_g}(T_3 - T_4) = c_{p_a}(T_2 - T_1)$

Fig. 9.4 Gas turbine unit with separate power turbine (a) and the cycle on a *T–s* diagram (b)



where c_{p_g} and c_{p_a} are the specific heat capacities at constant pressure of the gases in the turbine and the air in the compressor respectively. The net work output is then given by the LP turbine,

i.e. Net work output $= c_{p_g}(T_4 - T_5)$

Example 9.3 A gas turbine unit takes in air at 17 °C and 1.01 bar and the pressure ratio is 8/1. The compressor is driven by the HP turbine and the LP turbine drives a separate power shaft. The isentropic efficiencies of the compressor, and the HP and LP turbines are 0.8, 0.85, and 0.83 respectively. Calculate the pressure and temperature of the gases entering the power turbine, the net power developed by the unit per kg/s mass flow rate, the work ratio and the cycle efficiency of the unit. The maximum cycle temperature is 650 °C. For the compression process take $c_p = 1.005$ kJ/kg K and $\gamma = 1.4$; for the combustion process, and for the expansion process take $c_p = 1.15$ kJ/kg K and $\gamma = 1.333$. Neglect the mass of fuel.

Solution The unit is as shown in Figs 9.4(a) and 9.4(b).

From equation (3.21), for an isentropic process,

$$\frac{T_{2s}}{T_1} = \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma}$$

i.e. $$T_{2s} = 290 \times 8^{0.4/1.4} = 290 \times 1.811 = 525\ \text{K}$$

Then, using equation (9.1)

$$\eta_C = \frac{T_{2s} - T_1}{T_2 - T_1} = \frac{525 - 290}{T_2 - 290} = 0.8$$

therefore

$$T_2 - 290 = \frac{235}{0.8}$$

i.e. $$T_2 = 290 + 294 = 584\ \text{K}$$

Then Work input to the compressor $= c_{p_a}(T_2 - T_1)$

$$= 1.005 \times 294 = 295.5\ \text{kJ/kg}$$

Now the work output from the HP turbine must be sufficient to drive the compressor,

i.e. Work output from HP turbine $= c_{p_g}(T_3 - T_4) = 295.5$ kJ/kg

therefore

$$T_3 - T_4 = \frac{295.5}{1.15} = 257\ \text{K}$$

therefore

$$T_4 = T_3 - 257 = 923 - 257 = 666\ \text{K}$$

Then, using equation (9.2),

$$\eta_T \text{ for HP turbine} = \frac{T_3 - T_4}{T_3 - T_{4s}} = \frac{923 - 666}{923 - T_{4s}} = 0.85$$

i.e. $$923 - T_{4s} = \frac{257}{0.85} = 302.5\ \text{K}$$

therefore

$$T_{4s} = 923 - 302.5 = 620.5\ \text{K}$$

Then from equation (3.21) for an isentropic process,

$$\frac{T_3}{T_{4s}} = \left(\frac{p_3}{p_4}\right)^{(\gamma-1)/\gamma}$$

or $$\frac{p_3}{p_4} = \left(\frac{T_3}{T_{4s}}\right)^{\gamma/(\gamma-1)} = \left(\frac{923}{620.5}\right)^{1.333/0.333} = 4.9$$

i.e. $$p_4 = \frac{p_3}{4.9} = \frac{8 \times 1.01}{4.9} = 1.65 \text{ bar}$$

Hence the pressure and temperature at entry to the LP turbine are 1.65 bar and 393 °C, where $t_4 = 666 - 273 = 393$ °C.

To find the power output it is now necessary to evaluate T_5. The pressure ratio, p_4/p_5, is given by $(p_4/p_3) \times (p_3/p_5)$.

i.e. $$\frac{p_4}{p_5} = \frac{p_4}{p_3} \times \frac{p_2}{p_1} \quad (\text{since } p_2 = p_3 \text{ and } p_5 = p_1)$$

therefore

$$\frac{p_4}{p_5} = \frac{8}{4.9} = 1.63$$

Then $$\frac{T_4}{T_{5s}} = \left(\frac{p_4}{p_5}\right)^{(\gamma-1)/\gamma} = 1.63^{0.333/1.333} = 1.131$$

therefore

$$T_{5s} = \frac{666}{1.131} = 588 \text{ K}$$

Then, using equation (9.2)

$$\eta_T \text{ for the LP turbine} = \frac{T_4 - T_5}{T_4 - T_{5s}}$$

i.e. $$T_4 - T_5 = 0.83(666 - 588) = 0.83 \times 78 = 64.8 \text{ K}$$

Then $$\text{Work output from LP turbine} = c_{p_g}(T_4 - T_5)$$
$$= 1.15 \times 64.8 = 74.5 \text{ kJ/kg}$$

i.e. $$\text{Net power output} = 74.5 \times 1 = 74.5 \text{ kW}$$

$$\text{Work ratio} = \frac{\text{net work output}}{\text{gross work output}} = \frac{74.5}{74.5 + 295.5} = \frac{74.5}{370} = 0.201$$

$$\text{Heat supplied} = c_{p_g}(T_3 - T_2) = 1.15(923 - 584)$$

i.e. $$\text{Heat supplied} = 1.15 \times 339 = 390 \text{ kJ/kg}$$

Then $$\text{Cycle efficiency} = \frac{\text{net work output}}{\text{heat supplied}} = \frac{74.5}{390}$$
$$= 0.191 \text{ or } 19.1\%$$

Aircraft engines

In a *jet engine* the propulsion nozzle takes the place of the LP stage turbine, as shown diagrammatically in Fig. 9.5(a). The cycle is shown on a T–s diagram in Fig. 9.5(b), and it can be seen to be identical with Fig. 9.4(b). The aircraft is powered by the reactive thrust of the jet of gases leaving the nozzle, and this high-velocity jet is obtained at the expense of the enthalpy drop from 4 to 5. The turbine develops just enough work to drive the compressor and overcome mechanical losses.

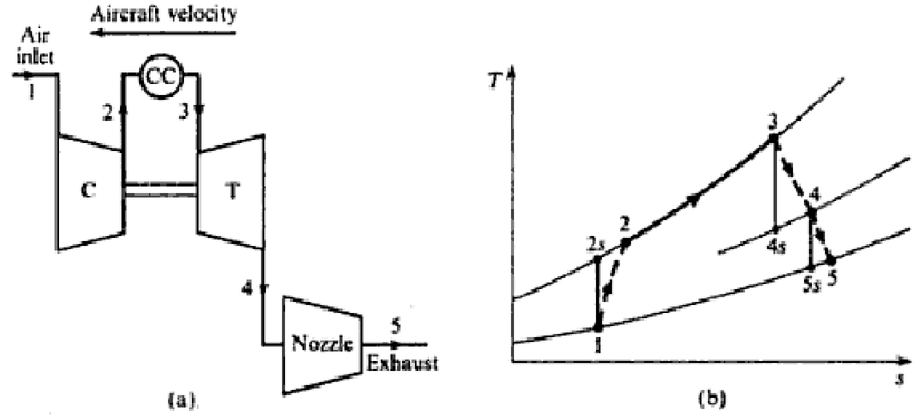


Fig. 9.5 Simple jet engine (a) with the cycle on a T–s diagram (b)

In a *turbo-prop* engine the turbine drives the compressor and also the airscrew, or propeller, as shown in Figs 9.6(a) and 9.6(b). The net work output available to drive the propeller is given by

$$\text{Net work output} = c_{p_g}(T_3 - T_4) - c_{p_a}(T_2 - T_1)$$

(neglecting mechanical losses).

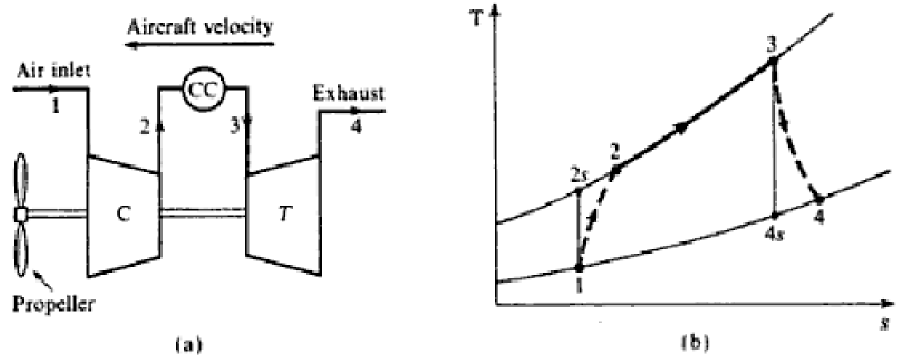


Fig. 9.6 Turboprop engine (a) with the cycle on a T–s diagram (b)

In practice there is also a small jet thrust developed in a turbo-prop aircraft. Jet engines and turbo-prop engines are considered again in Section 10.9.

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Thermodynamics-2

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Maxwell Relations

(1) The combined first and second law

From the first law: $dU = dq + dW$

From the second law: $dS \geq \frac{dq}{T}$

Where, $dS > \frac{dq}{T}$ for irreversible system

and, $dS = \frac{dq}{T}$ for reversible system

For a closed system in which only reversible pV work is involved

$dW = -pdV$ and $dS = \frac{dq}{T}$

$\therefore dU = TdS - pdV$ **Fundamental equation**

The internal energy is a function of S and V

Where U, T, S, P, and V are state functions

$$\therefore dU = TdS - pdV$$

The differential of U

$$\therefore dU = \left(\frac{dU}{dS}\right)_V dS + \left(\frac{dU}{dV}\right)_S dV$$

Thus, we can calculate T and p as

$$\therefore T = \left(\frac{dU}{dS}\right)_V \quad \text{and} \quad -\mathrm{p} = \left(\frac{\mathrm{dU}}{\mathrm{dV}}\right)_{\mathrm{S}}$$ **Equation I**

S and V are natural variables of U represented as U(S,V)

Also since U is a state function we can apply the state function condition

$$\left[\frac{\partial}{\partial \mathrm{V}}\left(\frac{\partial \mathrm{U}}{\partial \mathrm{S}}\right)_{\mathrm{V}}\right]_{\mathrm{S}} = \left[\frac{\partial}{\partial \mathrm{S}}\left(\frac{\partial \mathrm{U}}{\partial \mathrm{V}}\right)_{\mathrm{S}}\right]_{\mathrm{V}} \longrightarrow \left(\frac{\partial \mathrm{T}}{\partial \mathrm{V}}\right)_{\mathrm{S}} = -\left(\frac{\partial \mathrm{p}}{\partial \mathrm{S}}\right)_{\mathrm{V}}$$

This is a Maxwell Relation

Using the fundamental equation we can obtain new equations for dH, dA and dG and use the same procedures as above to obtain three more equations like [I] as well as three other Maxwell Relations. These turn out to be particularly useful in manipulating partials as will be shown shortly.

$H = U + pV$ and $G = H - TS$

For a constant T and p process $dT = 0$ and $dp = 0$ and thus

$G = U - TS + pV$ **$\therefore dG = dU - TdS - SdT + pdV + Vdp$**

Substitute the fundamental equation $dU = TdS - pdV$ into the equation for dG to get: **$dG = Vdp - SdT$**

This equation for dG suggests that we take p and T as the variables for the Gibbs energy.

$$dG = \left(\frac{\partial G}{\partial p}\right)_T dp + \left(\frac{\partial G}{\partial T}\right)_p dT$$

Comparing coefficients of dp and dT for the two equations gives

$$\left(\frac{\partial G}{\partial p}\right)_T = V \quad \text{and} \quad \left(\frac{\partial G}{\partial T}\right)_p = -S$$

Applying the state function condition for G

$$\left(\frac{\partial V}{\partial T}\right)_p = -\left(\frac{\partial S}{\partial p}\right)_T$$

This is a Maxwell Relation

The four Maxwell relations are

$$\left(\frac{\partial p}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T \qquad \left(\frac{\partial V}{\partial T}\right)_p = - \left(\frac{\partial S}{\partial p}\right)_T$$

$$\left(\frac{\partial T}{\partial p}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P \qquad \left(\frac{\partial T}{\partial V}\right)_S = - \left(\frac{\partial p}{\partial S}\right)_V$$

Maxwell relations allow us to develop different equations

Start with the fundamental equation $dU = TdS - pdV$.

Proof that $\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_V - p$

To get the LHS partial of the above equation we divide dU by dV and hold T constant

$$\left.\frac{dU}{dV}\right|_T = \left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial S}{\partial V}\right)_T - p$$

Substitute the Maxwell relation $\left(\frac{\partial p}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$ to obtain the equation

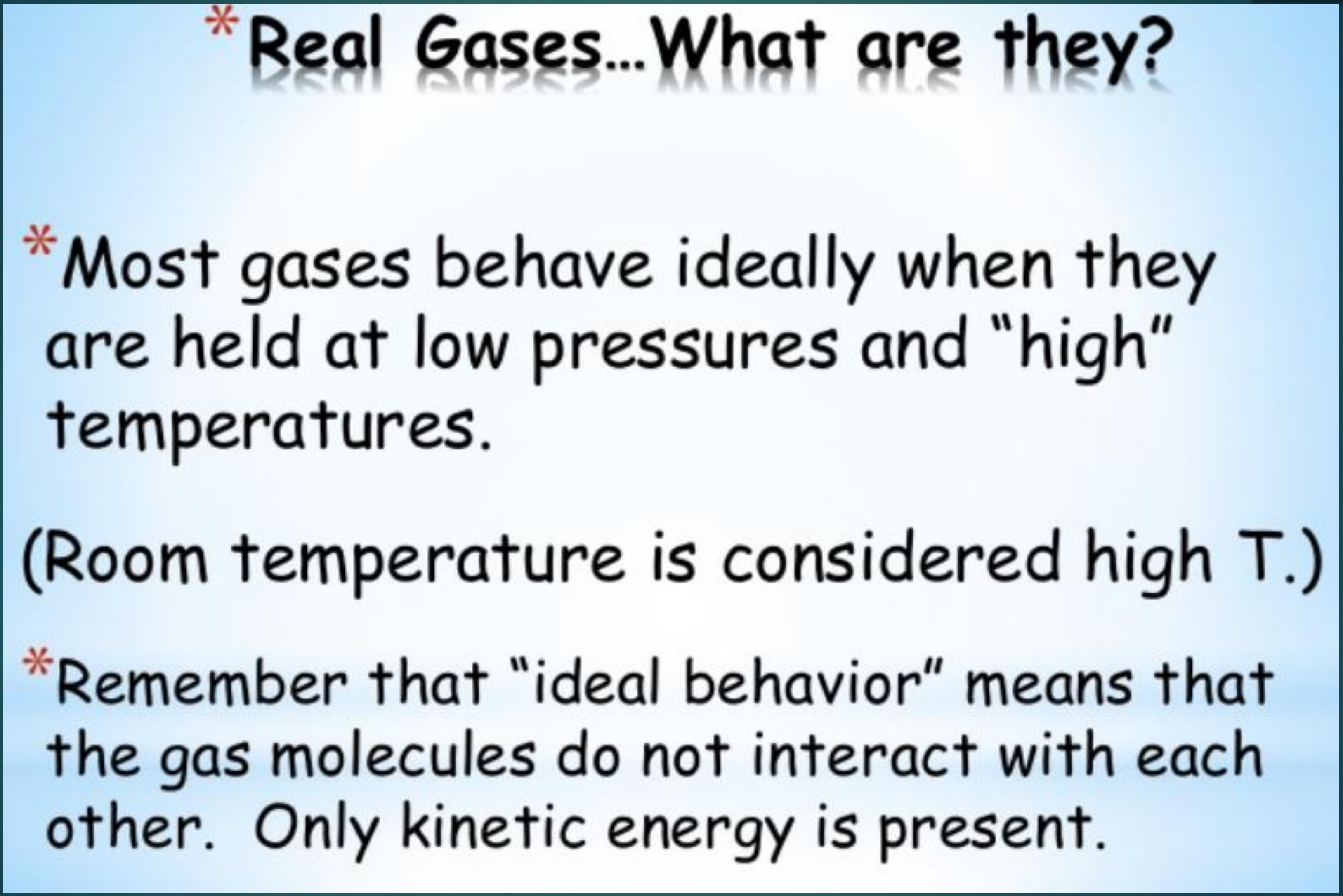
$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_V - p$$

The physical significance of the LHS. It is the change in the energy when the volume of a system, say a gas, is changed. i.e, when the distance between gas molecules is increased or decreased. Thus it is a measure of the change in the potential energy of molecules.

REAL GASES

Ideal gas assumption

- *The ideal gas equation pv = mRT can be derived analytically using the kinetic theory of gases by making the following assumptions :*
- *A finite volume of gas contains large number of molecules.*
- *The collision of molecules with one another and with the walls of the container are perfectly elastic.*

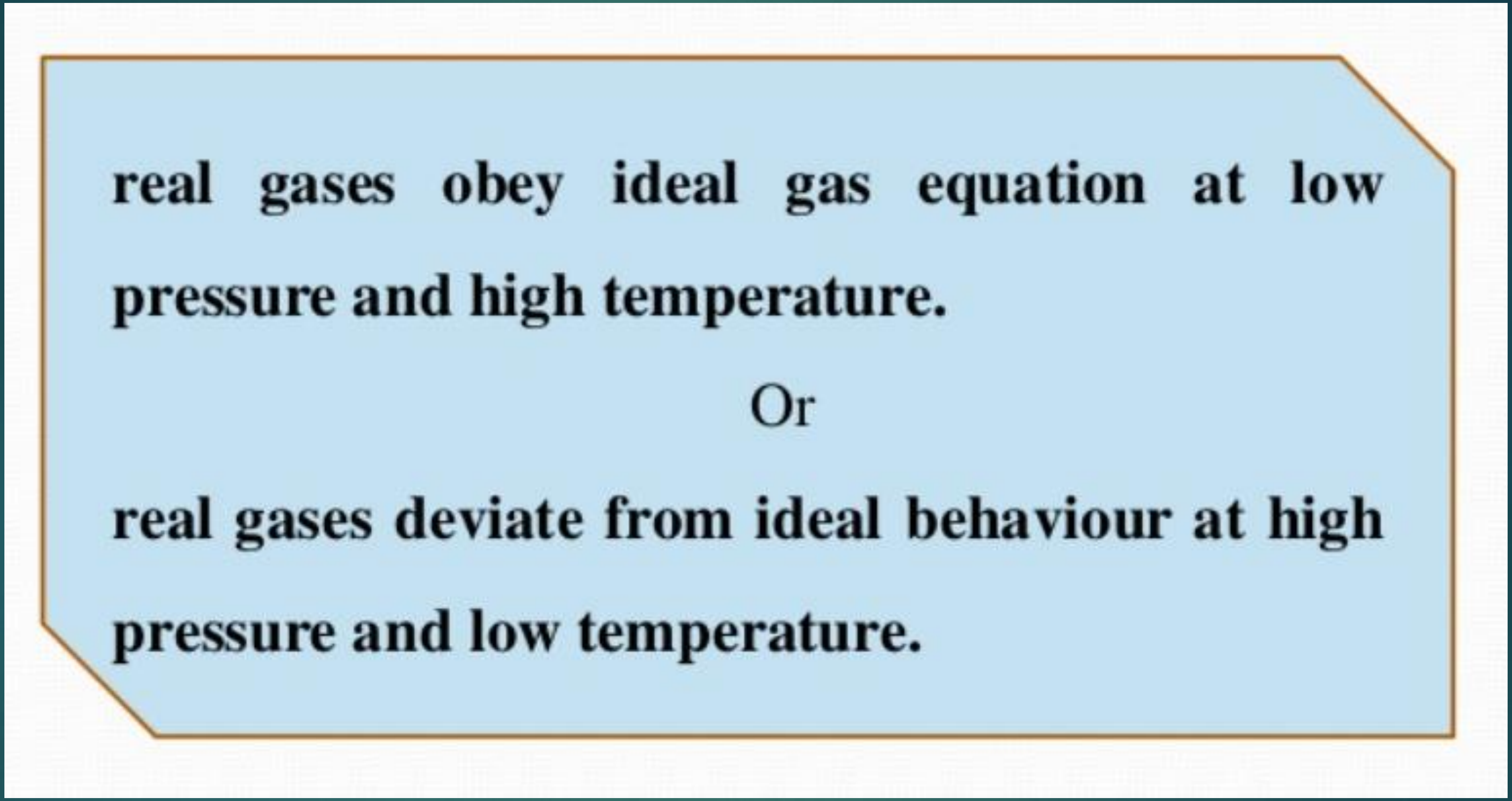


Real Gases…What are they?

- Most gases behave ideally when they are held at low pressures and "high" temperatures.

(Room temperature is considered high T.)

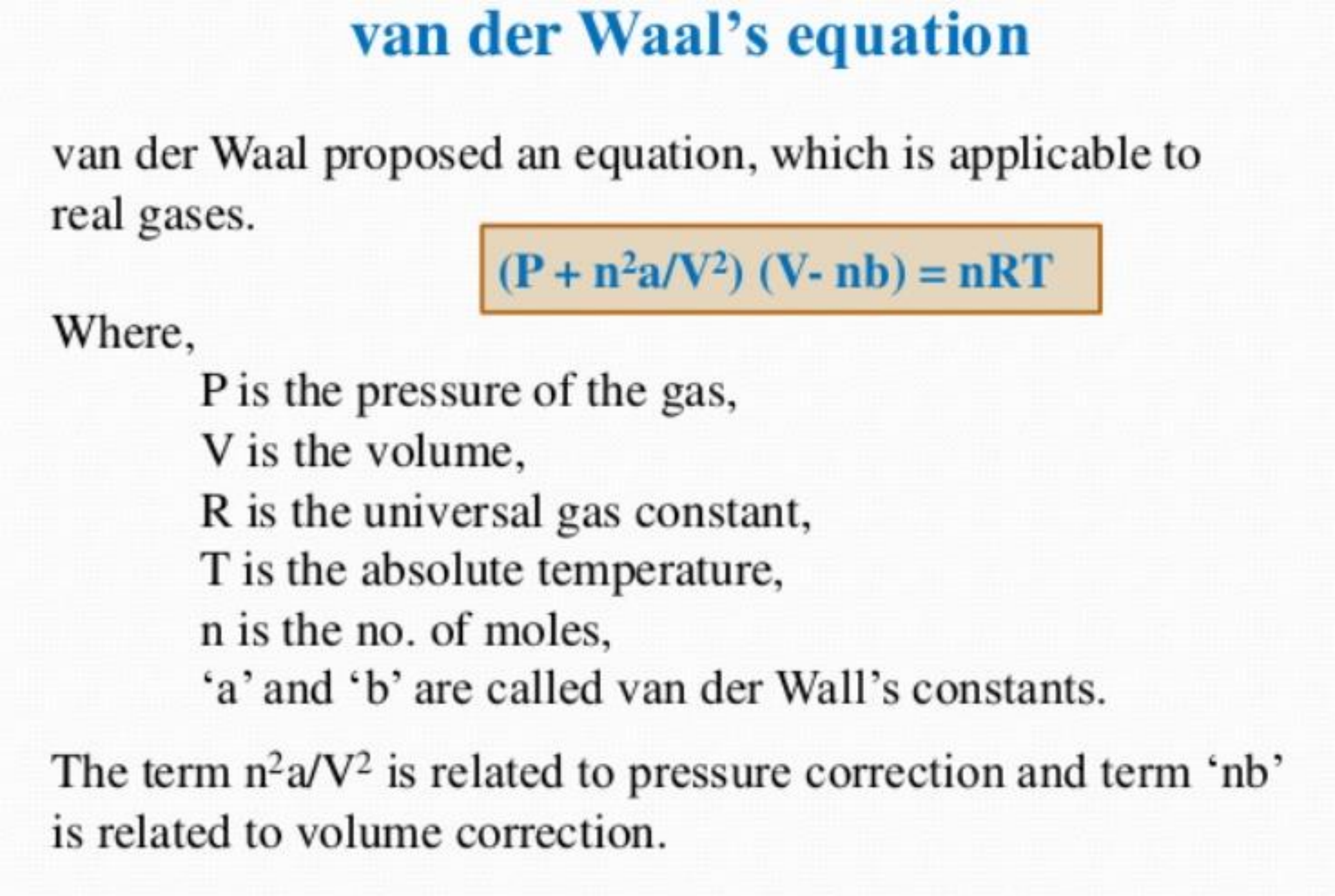
- Remember that "ideal behavior" means that the gas molecules do not interact with each other. Only kinetic energy is present.



real gases obey ideal gas equation at low pressure and high temperature.

Or

real gases deviate from ideal behaviour at high pressure and low temperature.



van der Waal's equation

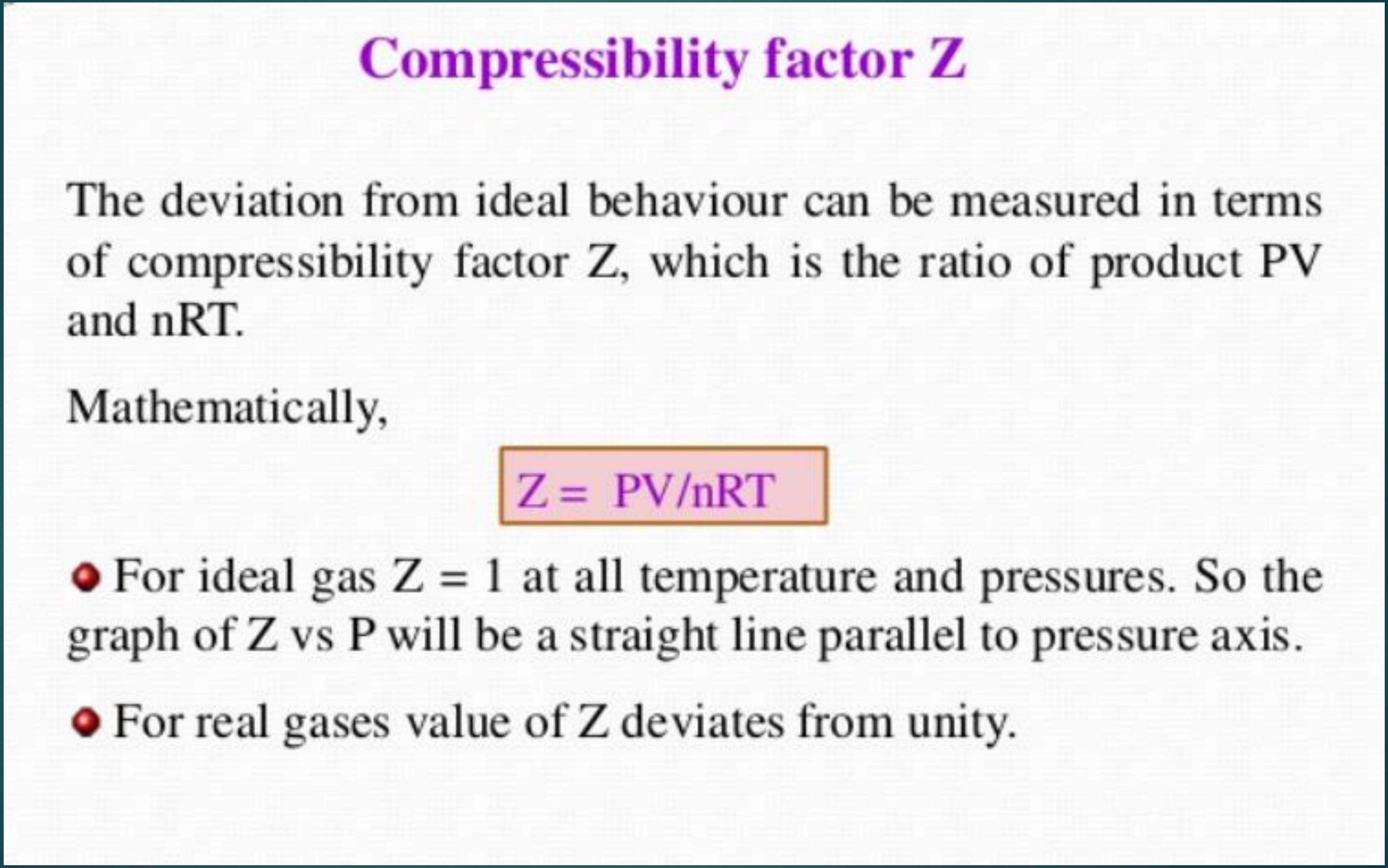
van der Waal proposed an equation, which is applicable to real gases.

$$(P + n^2a/V^2)\ (V - nb) = nRT$$

Where,

P is the pressure of the gas,
V is the volume,
R is the universal gas constant,
T is the absolute temperature,
n is the no. of moles,
'a' and 'b' are called van der Wall's constants.

The term n^2a/V^2 is related to pressure correction and term 'nb' is related to volume correction.



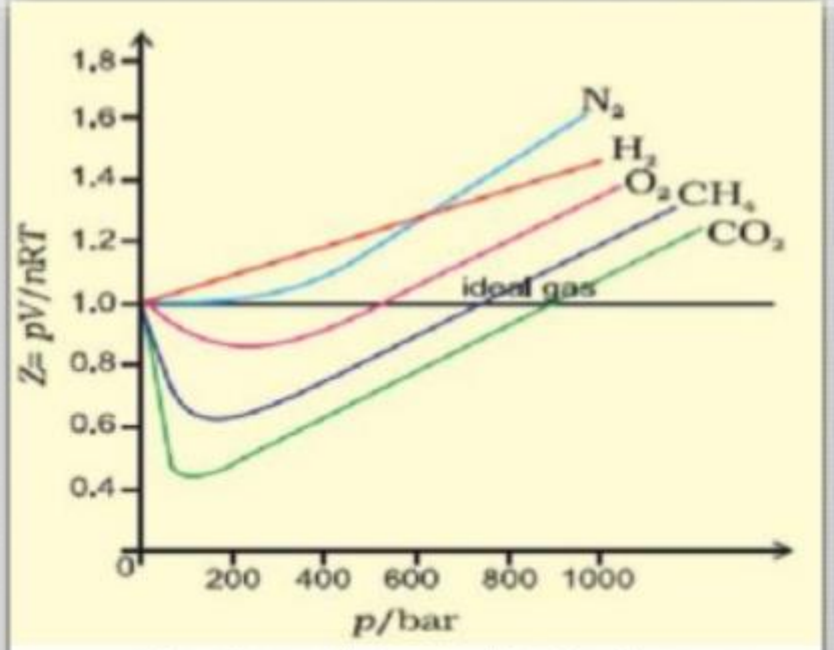
Compressibility factor Z

The deviation from ideal behaviour can be measured in terms of compressibility factor Z, which is the ratio of product PV and nRT.

Mathematically,

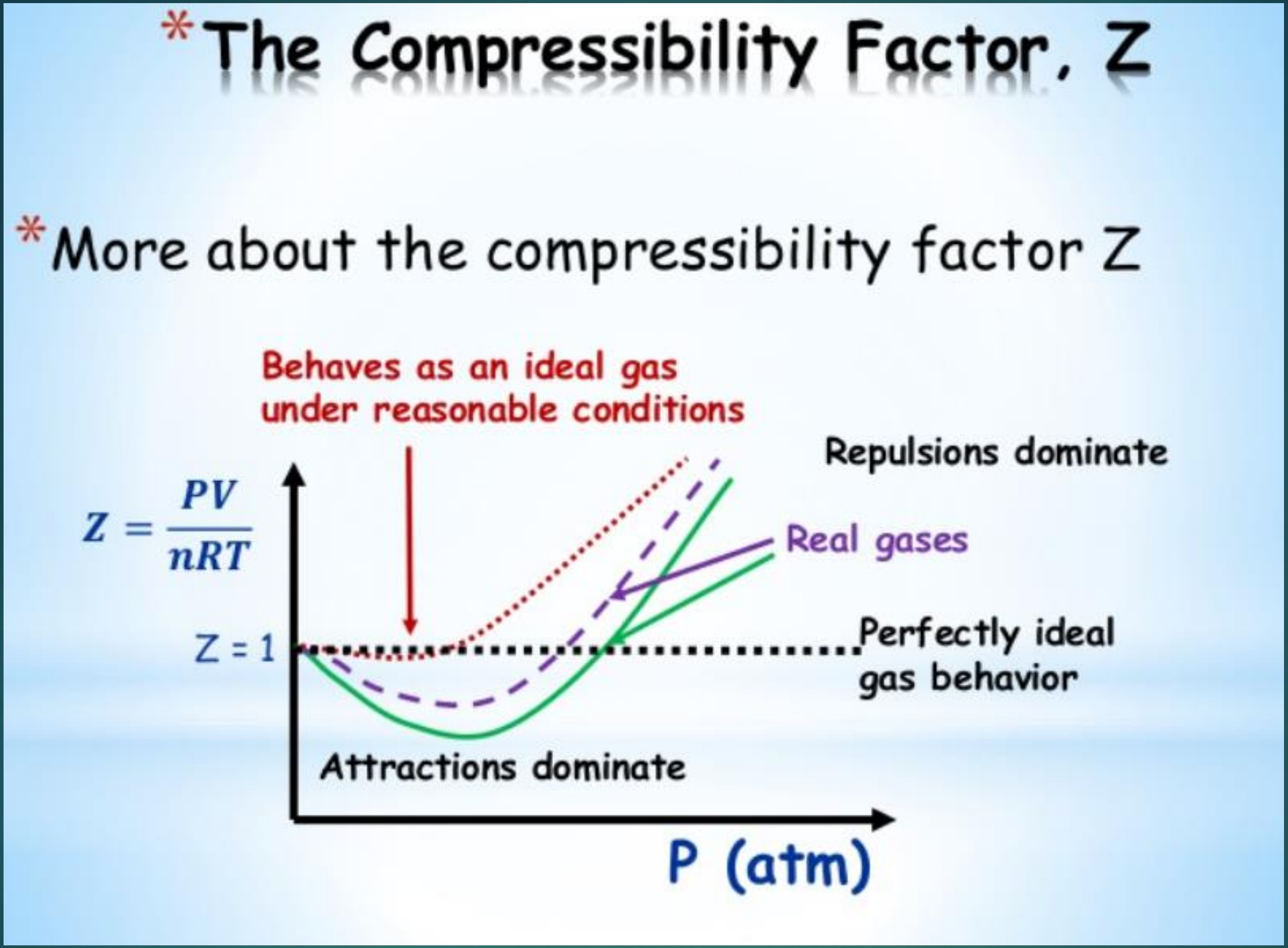
$$\boxed{Z = PV/nRT}$$

- For ideal gas Z = 1 at all temperature and pressures. So the graph of Z vs P will be a straight line parallel to pressure axis.
- For real gases value of Z deviates from unity.



Z vs P graph for real & ideal gases

- For ideal gas Z = 1 at all temperature and pressures. So the graph of Z vs P will be a straight line parallel to pressure axis.
- For real gases value of Z deviates from unity.



*The Compressibility Factor, Z

*More about the compressibility factor Z

- At Low Temperatures the attractive regime is pronounced
  - higher Temperature ->faster motion -> less interaction

Note: $Z = 1$ for an ideal gas.

As Z approaches 1 for a gas at given conditions, the behavior of the gas approaches ideal gas behavior. Although, different gases have very different specific properties at various conditions; *all gases behave in a similar manner relative to their critical pressure, Pcr and critical temparature, Tcr.*

Hence, the gas pressures and temperatures are normalized by the critical values to obtain the *reduced pressure, P_R and temparature, T_R.*
defined as

$P_R=P/Pcr$;

$T_R=T/Tcr$

The reduced values can then be used to determine Z using the Generalized Compressibility Charts

These charts show the conditions for which $Z = 1$ and the gas behaves as an ideal gas:

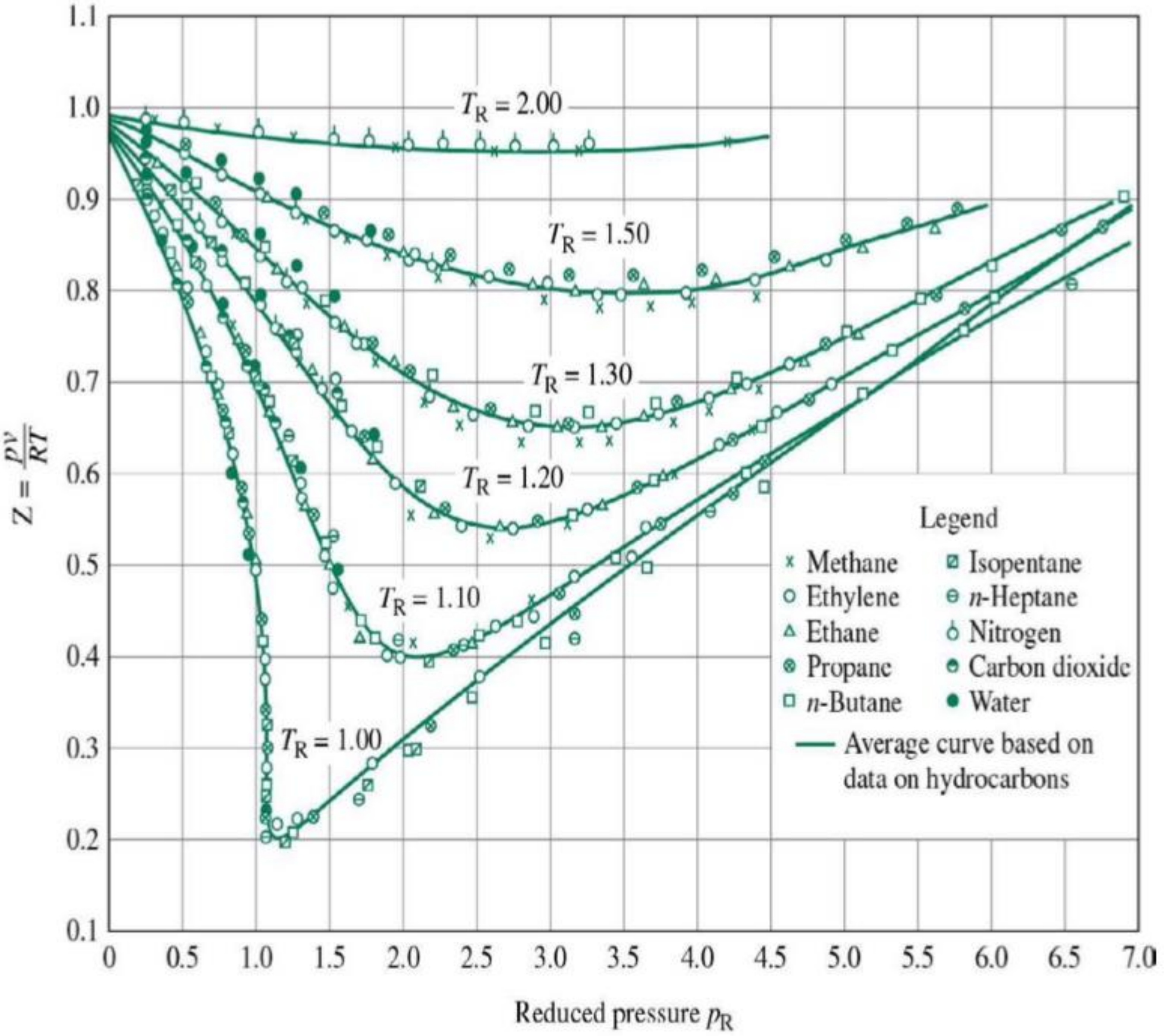
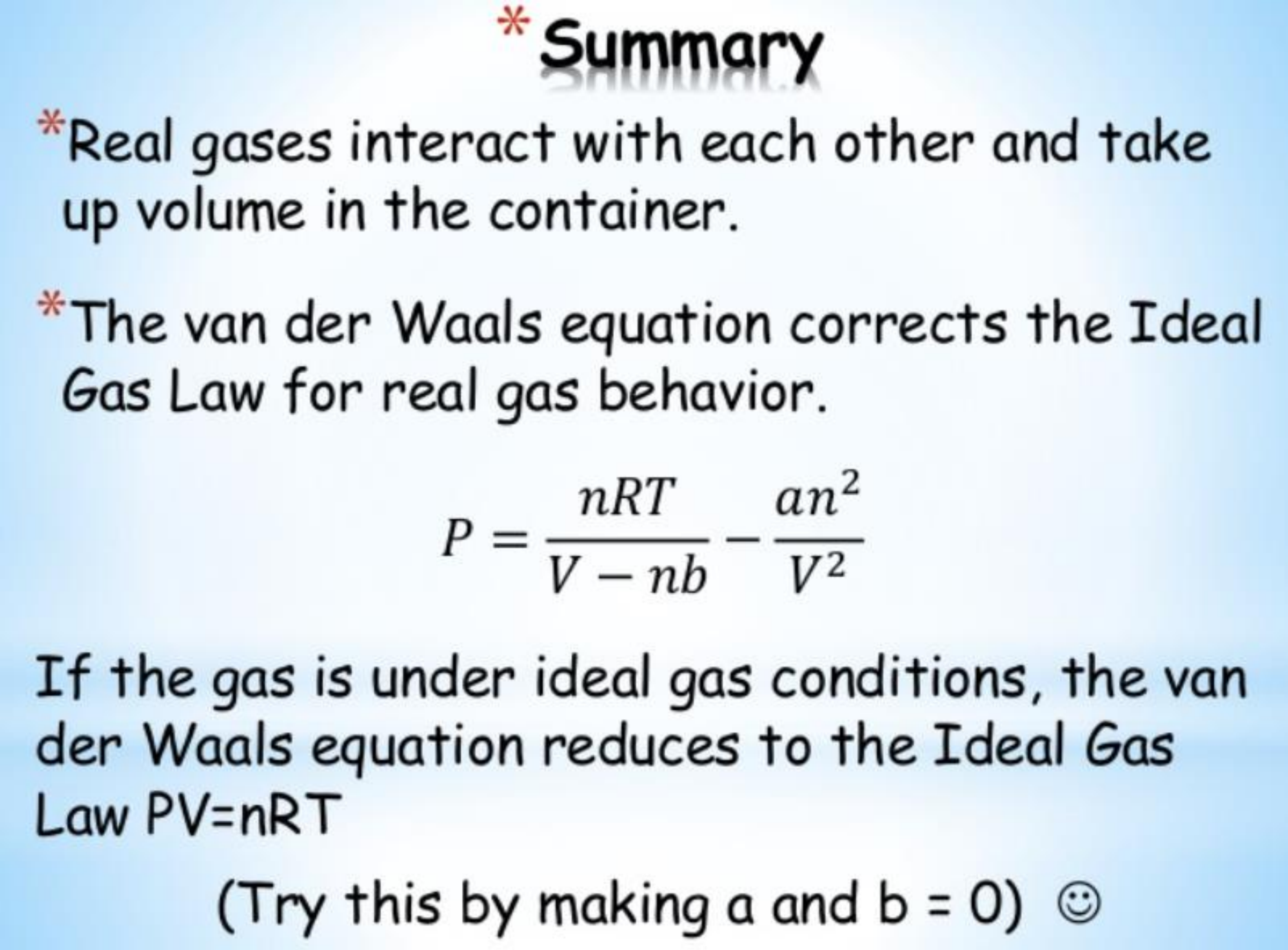


Figure 1 : Compressibilty Chart



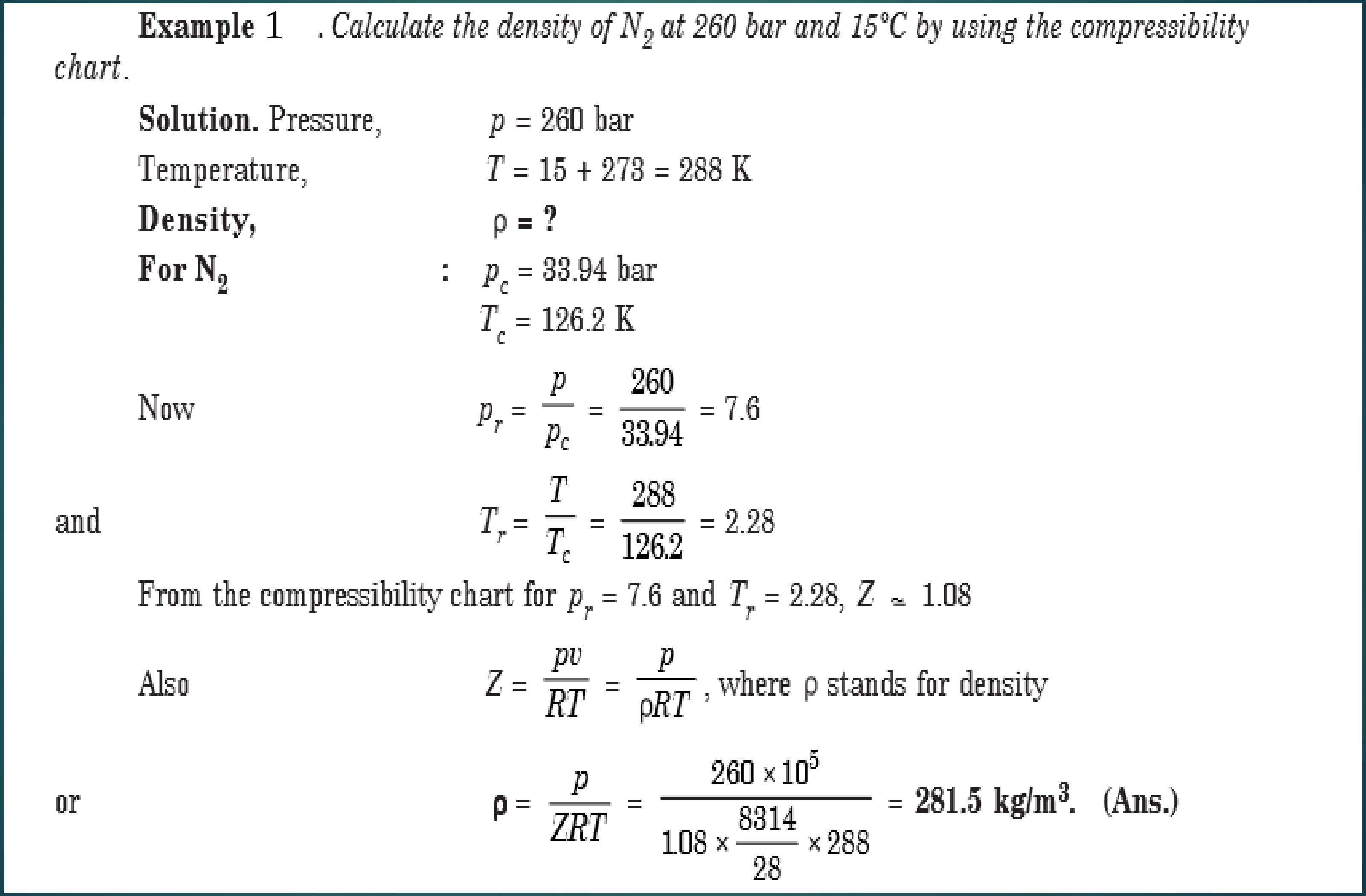
Summary

- Real gases interact with each other and take up volume in the container.
- The van der Waals equation corrects the Ideal Gas Law for real gas behavior.

$$P = \frac{nRT}{V - nb} - \frac{an^2}{V^2}$$

If the gas is under ideal gas conditions, the van der Waals equation reduces to the Ideal Gas Law PV=nRT

(Try this by making a and b = 0) ☺



Example 1 . *Calculate the density of N_2 at 260 bar and 15°C by using the compressibility chart.*

Solution. Pressure, $p = 260$ bar

Temperature, $T = 15 + 273 = 288$ K

Density, $\rho = ?$

For N_2 : $p_c = 33.94$ bar

$T_c = 126.2$ K

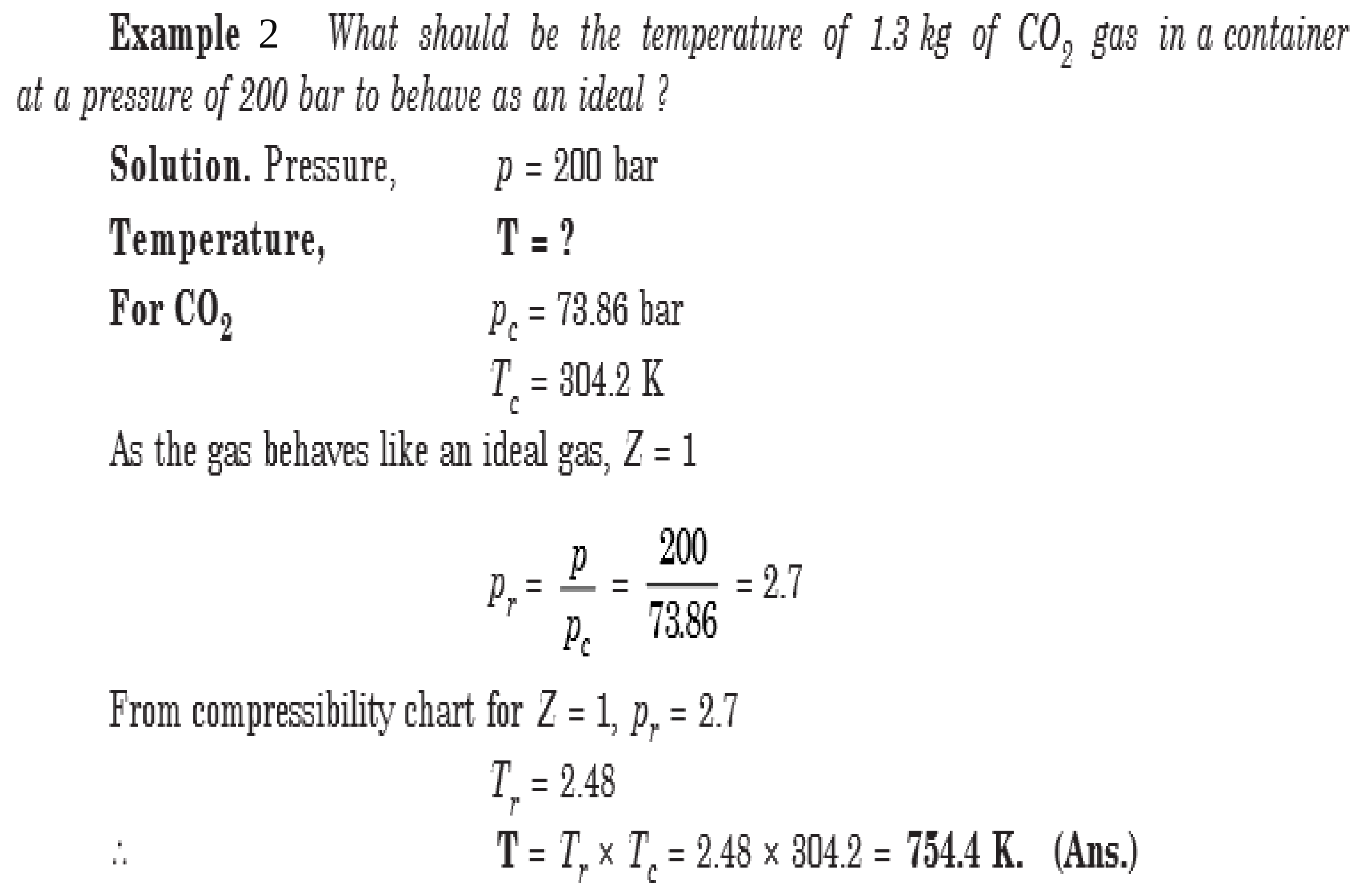
Now
$$p_r = \frac{p}{p_c} = \frac{260}{33.94} = 7.6$$

and
$$T_r = \frac{T}{T_c} = \frac{288}{126.2} = 2.28$$

From the compressibility chart for $p_r = 7.6$ and $T_r = 2.28$, $Z \simeq 1.08$

Also
$$Z = \frac{pv}{RT} = \frac{p}{\rho RT}\text{, where } \rho \text{ stands for density}$$

or
$$\rho = \frac{p}{ZRT} = \frac{260 \times 10^5}{1.08 \times \frac{8314}{28} \times 288} = \mathbf{281.5\ kg/m^3}.\quad \textbf{(Ans.)}$$



Example 2 *What should be the temperature of 1.3 kg of* CO_2 *gas in a container at a pressure of 200 bar to behave as an ideal ?*

Solution. Pressure, $p = 200$ bar

Temperature, $\mathbf{T = ?}$

For $\mathbf{CO_2}$ $p_c = 73.86$ bar

$T_c = 304.2$ K

As the gas behaves like an ideal gas, $Z = 1$

$$p_r = \frac{p}{p_c} = \frac{200}{73.86} = 2.7$$

From compressibility chart for $Z = 1$, $p_r = 2.7$

$T_r = 2.48$

$\therefore$ $\mathrm{T} = T_r \times T_c = 2.48 \times 304.2 =$ **754.4 K.** (Ans.)

THANK YOU

THE IDEAL REGENERATIVE RANKINE CYCLE:

A careful examination of the T-s diagram of the Rankine cycle redrawn in Fig.1 reveals that heat is transferred to the working fluid during process 2-2′ at a relatively low temperature. This lowers the average heat addition temperature and thus the cycle efficiency. To remedy this shortcoming, we look for ways to raise the temperature of the liquid leaving the pump (called the feed water) before it enters the boiler. One such possibility is to transfer heat to the feed water from the expanding steam in a counter flow heat exchanger built into the turbine, that is, to use regeneration. This solution is also impractical because it is difficult to design such a heat exchanger and because it would increase the moisture content of the steam at the final stages of the turbine. A practical regeneration process in steam power plants is accomplished by extracting, or "bleeding," steam from the turbine at various points. This steam, which could have produced more work by expanding further in the turbine, is used to heat the feed water instead. The device where the feed water is heated by regeneration is called a regenerator, or a feed water heater (FWH). Regeneration not only improves cycle efficiency, but also provides a convenient means of deaerating the feed water (removing the air that leaks in at the condenser) to prevent corrosion in the boiler. It also helps control the large volume flow rate of the steam at the final stages of the turbine (due to the large specific volumes at low pressures). Therefore, regeneration has been used in all modern steam power plants since its introduction in the early 1920s. A feed water heater is basically a heat exchanger where heat is transferred from the steam to the feed water either by mixing the two fluid streams (open feed water heaters) or without mixing them (closed feed

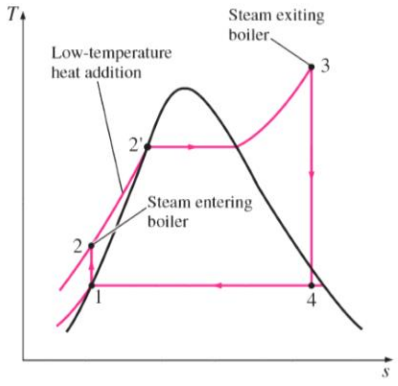


FIGURE 1
The first part of the heat-addition process in the boiler takes place at relatively low temperatures.

water heaters). Regeneration with both types of feed water heaters is discussed below.

Open Feed water Heaters:- An open (or direct-contact) feed water heater is basically a mixing chamber, where the steam extracted from the turbine mixes with the feed water exiting the pump. Ideally, the mixture leaves the heater as a saturated liquid at the heater pressure. The schematic of a steam power plant with one open feed water heater (also called single-stage regenerative cycle) and the T-s diagram of the cycle are shown in Fig. 10–15. In an ideal regenerative Rankine cycle, steam enters the turbine at the boiler pressure (state 5) and expands isentropically to an intermediate pressure (state 6). Some steam is extracted at this state and routed to the feedwater heater, while the remaining steam continues to expand isentropically to the condenser pressure (state 7). This steam leaves the condenser as a saturated liquid at the condenser pressure (state 1). The condensed water, which is also called the feed water, then enters an isentropic pump, where it is compressed to the feedwater heater pressure (state 2) and is routed to the feed water heater, where it mixes with the steam extracted from the turbine. The fraction of the steam extracted is such that the mixture leaves the heater as a saturated liquid at the heater pressure (state 3). A second pump raises the pressure of the water to the boiler pressure (state 4). The cycle is completed by heating the water in the boiler to the turbine inlet state (state 5). In the analysis of steam power plants, it is more convenient to work with quantities expressed per unit mass of the steam flowing through the boiler. For each 1 kg of steam leaving the boiler, y kg expands partially in the turbine and is extracted at state 6. The remaining (1- y) kg expands completely to the condenser pressure. Therefore, the mass flow rates are different in different components. If the mass flow rate through the boiler is $\dot{m}$, for example, it is $(1 - y)\,\dot{m}$. Through the condenser. This aspect of the regenerative Rankine cycle should be considered in the analysis of the cycle as well as in the interpretation of the areas on the T-s diagram. In light of Fig. 2, the heat and work interactions of a regenerative Rankine cycle with one feed water heater can be expressed per unit mass of steam flowing through the boiler as follows:

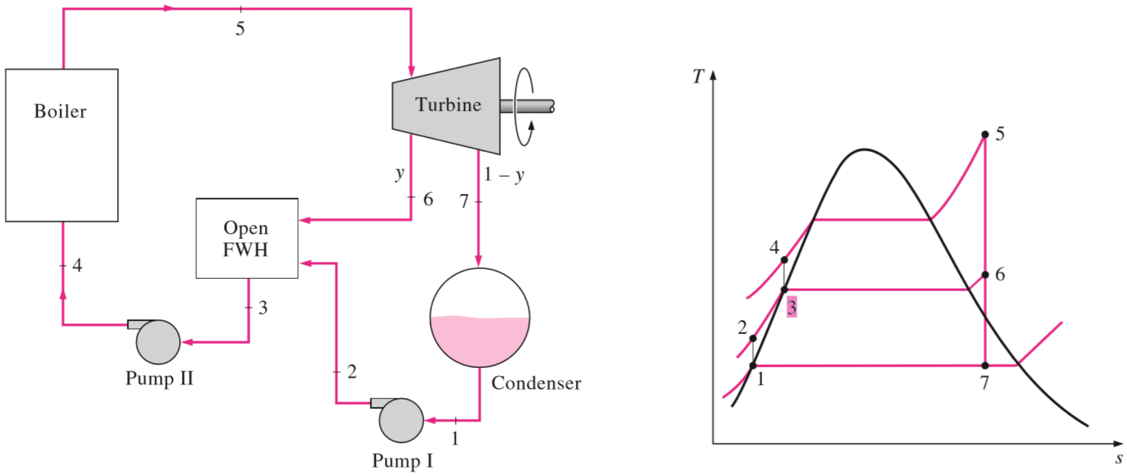


FIGURE 2

The ideal regenerative Rankine cycle with an open feedwater heater.

$$\begin{aligned} q_{\text{in}} &= h_5 - h_4 \\ q_{\text{out}} &= (1-y)(h_7 - h_1) \\ w_{\text{turb,out}} &= (h_5 - h_6) + (1-y)(h_6 - h_7) \\ w_{\text{pump,in}} &= (1-y)w_{\text{pump I,in}} + w_{\text{pump II,in}} \end{aligned}$$

$$y = \dot{m}_6/\dot{m}_5 \qquad \text{(fraction of steam extracted)}$$

$$\begin{aligned} w_{\text{pump I,in}} &= v_1(P_2 - P_1) \\ w_{\text{pump II,in}} &= v_3(P_4 - P_3) \end{aligned}$$

The thermal efficiency of the Rankine cycle increases as a result of regeneration. This is because regeneration raises the average temperature at which heat is transferred to the steam in the boiler by raising the temperature of the water before it enters the boiler. The cycle efficiency increases further as the number of feed water heaters is increased. Many large plants in operation today use as many as eight feed water heaters. The optimum number of feed water heaters is determined from economic considerations. The use of an additional feed water heater cannot be justified unless it saves more from the fuel costs than its own cost.

Closed Feed water Heaters:- Another type of feed water heater frequently used in steam power plants is the closed feed water heater, in which heat is transferred from the extracted steam to the feed water without any mixing taking place. The two streams now can be at different pressures, since they do not mix. The schematic of a steam power plant with one closed feed water heater and the T-s diagram of the cycle are shown in Fig. 3. In an ideal closed feed water heater, the feed water is heated to the exit temperature of the extracted steam, which ideally leaves the heater as a saturated liquid at the extraction pressure. In actual power plants, the feed water leaves the heater below the exit temperature of the extracted steam because a temperature difference of at least a few degrees is required for any effective heat transfer to take place.

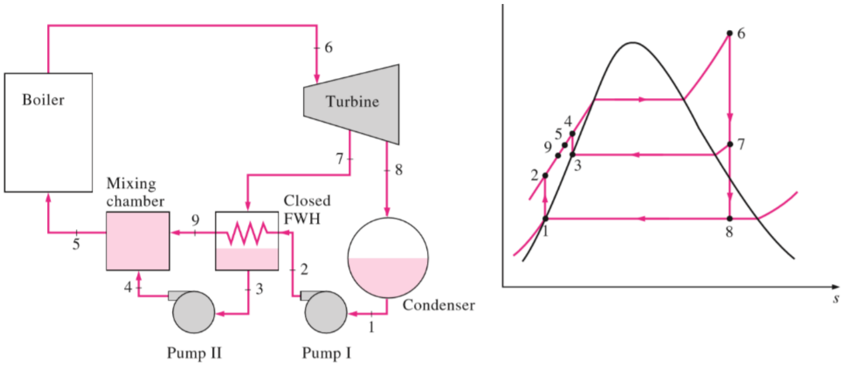


FIGURE 3
The ideal regenerative Rankine cycle with a closed feedwater heater.

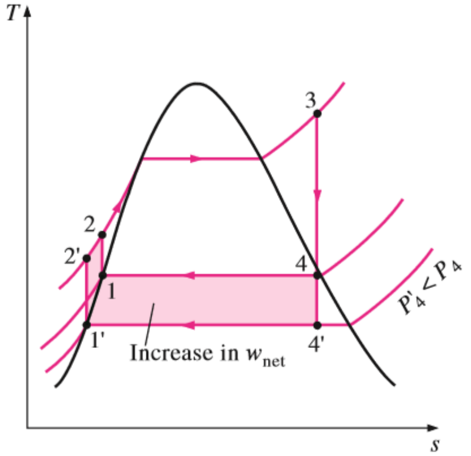
The open and closed feed water heaters can be compared as follows. Open feed water heaters are simple and inexpensive and have good heat transfer characteristics. They also bring the feed water to the saturation state. For each heater, however, a pump is required to handle the feed water. The closed feed water heaters are more complex because of the internal tubing network, and thus they are more expensive. Heat transfer in closed feed water heaters is also less effective since the two streams are not allowed to be in direct contact. However, closed feed water heaters do not require a separate pump for each heater since the extracted steam and the feedwater can be at different pressures.

Example:

Consider a steam power plant operating on the ideal regenerative Rankine cycle with one open feed water heater. Steam enters the turbine at 15 MPa and 600°C and is condensed in the condenser at a pressure of 10 kPa. Some steam leaves the turbine at a pressure of 1.2 MPa and enters the open feed water heater. Determine the fraction of steam extracted from the turbine and the thermal efficiency of the cycle.

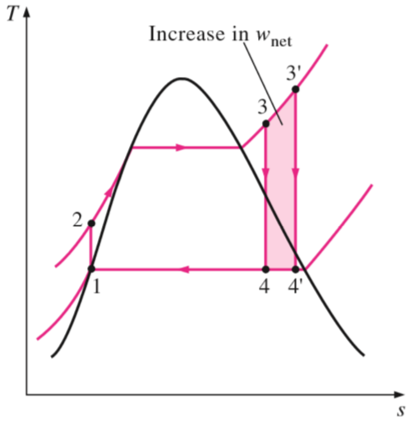
HOW CAN WE INCREASE THE EFFICIENCY OF THE RANKINE CYCLE?

1- Lowering the Condenser Pressure:



Steam exists as a saturated mixture in the condenser at the saturation temperature corresponding to the pressure inside the condenser. Therefore, lowering the operating pressure of the condenser automatically lowers the temperature of the steam, and thus the temperature at which heat is rejected. The effect of lowering the condenser pressure on the Rankine cycle efficiency is illustrated on a T-s diagram. For comparison purposes, the turbine inlet state is maintained the same. The colored area on this diagram represents the increase in net work output as a result of lowering the condenser pressure from P_4 to $P_{4'}$. The heat input requirements also increase (represented by the area under curve $2'$-2), but this increase is very small. Thus the overall effect of lowering the condenser pressure is an increase in the thermal efficiency of the cycle.

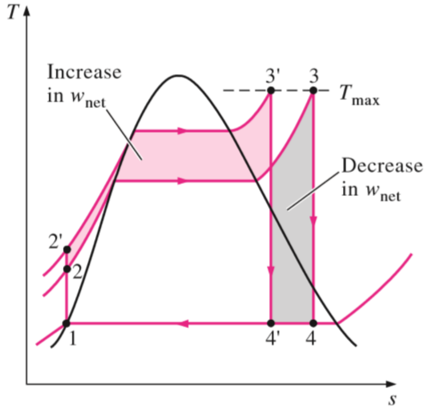
2- Superheating the Steam to High Temperatures:



The average temperature at which heat is transferred to steam can be increased without increasing the boiler pressure by superheating the steam to high temperatures. The effect of superheating on the performance of vapor power cycles is illustrated on a T-s diagram. The colored area on this diagram represents the increase in the network. The total area under the process curve 3-$3'$ represents the increase in the heat input. Thus both the network and heat input increase as a result of superheating the steam to a higher temperature. The overall effect is an increase in thermal efficiency, however, since the average temperature at which heat is added increases. Superheating the steam to higher temperatures has another very desirable effect: It decreases the moisture content of the steam at the turbine exit, as can be seen from the T-s diagram (the quality at state $4'$ is higher than that at state 4). The temperature to which steam can be superheated is limited, however, by metallurgical considerations. Presently the highest steam temperature allowed at the turbine inlet is about 620°C. Any increase in this value depends on improving the present materials or finding new ones that can withstand higher temperatures. Ceramics are very promising in this regard.

3- **Increasing the Boiler Pressure**:

Another way of increasing the average temperature during the heat-addition process is to increase the operating pressure of the boiler, which automatically raises the temperature at which boiling takes place. This, in turn, raises the average temperature at which heat is transferred to the steam and thus raises the thermal efficiency of the cycle. The effect of increasing the boiler pressure on the performance of vapor power cycles is illustrated on a T-s diagram in next Fig. Notice that for a fixed turbine inlet temperature, the cycle shifts to the left and the moisture content of steam at the turbine exit increases. This undesirable side effect can be corrected, however, by reheating the steam.



Example1: Consider a steam power plant operating on the ideal Rankine cycle. Steam enters the turbine at 3 MPa and 350°C and is condensed in the condenser at a pressure of 10 kPa. Determine (a) the thermal efficiency of this power plant, (b) the

thermal efficiency if steam is superheated to 600°C instead of 350°C, and (c) the thermal efficiency if the boiler pressure is raised to 15 MPa while the turbine inlet temperature is maintained at 600°C.

THE IDEAL REHEAT RANKINE CYCLE:

We noted in the last section that increasing the boiler pressure increases the thermal efficiency of the Rankine cycle, but it also increases the moisture content of the steam to unacceptable levels. Then it is natural to ask the following question:

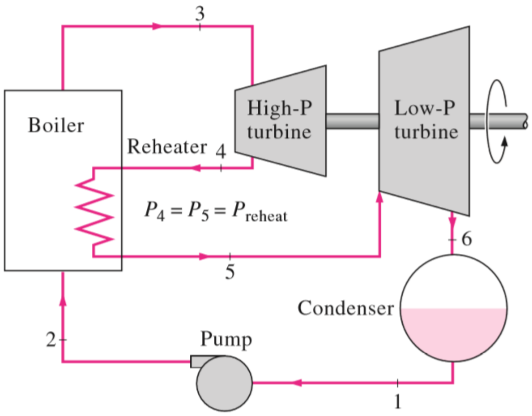
How can we take advantage of the increased efficiencies at higher boiler pressures without facing the problem of excessive moisture at the final stages of the turbine?

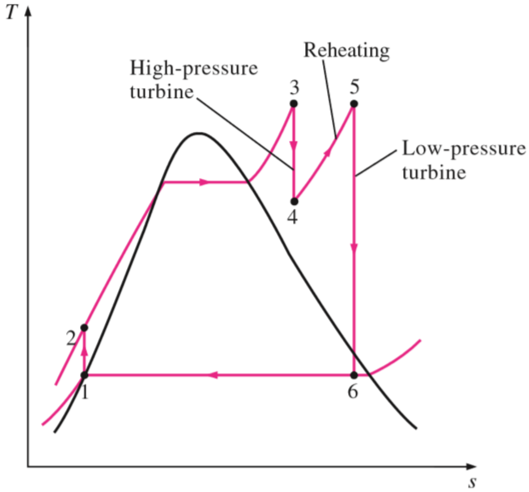
Two possibilities come to mind:

1. Superheat the steam to very high temperatures before it enters the turbine. This would be the desirable solution since the average temperature at which heat is added would also increase, thus increasing the cycle efficiency. This is not a viable solution, however, since it requires raising the steam temperature to metallurgical unsafe levels.

2. Expand the steam in the turbine in two stages, and reheat it in between. In other words, modify the simple ideal Rankine cycle with a reheat process. Reheating is a practical solution to the excessive moisture problem in turbines, and it is commonly used in modern steam power plants.

The T-s diagram of the ideal reheat Rankine cycle and the schematic of the power plant operating on this cycle are shown in Fig.





$$W_{net} = (w_{h.p.t} + w_{l.p.t}) - w_{pump}$$

$$W_T = w_{T1} + w_{T2} = (h_3 - h_4) + (h_5 - h_6)$$

$$W_{Pump} = (h_2 - h_1)$$

$$W_{net} = (h_3 - h_4) + (h_5 - h_6) - (h_2 - h_1)$$

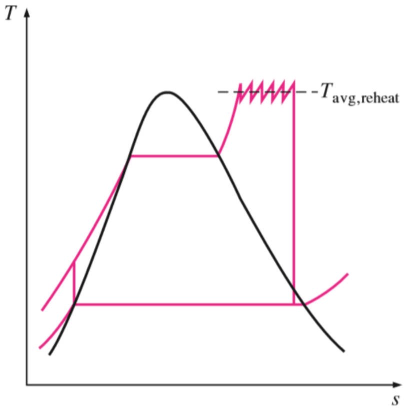
$$Q_{add} = (h_3 - h_2) + (h_5 - h_4)$$

$$Q_{rej} = (h_6 - h_1)$$

$$\eta_R = \frac{w_{net}}{Q_{add}} = \frac{(h_3 - h_4) + (h_5 - h_6) - (h_2 - h_1)}{(h_3 - h_2) + (h_{45} - h_4)}$$

$$W.R. = \frac{w_{net}}{(w_{T1} + w_{T2})}$$

$$S.S.C. = \frac{3600}{w_{net}} \quad \text{Kg/kW.hr}$$



The average temperature at which heat is transferred during reheating increases as the number of reheat stages is increased.

The reheat cycle was introduced in the mid-1920s, but it was abandoned in the 1930s because of the operational difficulties. The steady increase in boiler pressures over the years made it necessary to reintroduce single reheat in the late 1940s and double reheat in the early 1950s. The reheat temperatures are very close or equal to the turbine inlet temperature. The optimum reheat pressure is about one-fourth of the maximum cycle pressure. For example, the optimum reheat pressure for a cycle with a boiler pressure of 12 MPa is about 3 MPa. Remember that the sole purpose of the reheat cycle is to reduce the moisture content of the steam. At the final stages of the expansion process. If we had materials that could withstand sufficiently high temperatures, there would be no need for the reheat cycle.

<u>Ex.2</u>: Find the ideal cycle efficiency and specific steam consumption of a re heat Rankine cycle operation between pressure of 30 and 0.04 bar, with a super heat temperature of 450 ^{o}C. Assume that the first expansion is carried out to the point where the steam is dry saturated and that the steam is re heated to the original super heater temperature.

<u>H.W.</u> Consider a steam power plant operating on the ideal reheat Rankine cycle. Steam enters the high-pressure turbine at 15 MPa and 600°C and is condensed in the condenser at a pressure of 10 kPa. If the moisture content of the steam at the exit of the low-pressure turbine is not to exceed 10.4 percent, determine (a) the pressure at which the steam should be reheated and (b) the thermal efficiency of the cycle. Assume the steam is reheated to the inlet temperature of the high-pressure turbine.

STEAM POWER PLANT

The main objective of a steam power station is to generate electrical power. In a steam power station, the electrical energy is produced according to the principle of “external combustion,” where the “heat of combustion” of the fuel is transferred to a prime mover by a “working medium.” In the steam generator, low-temperature water is the working medium that receives the heat of combustion of fuel and becomes high-energy steam. The heat of steam is converted to mechanical energy in the steam turbine and then to electrical energy in the generator. The sequence of these activities is shown in Figure 2.1. The chemical energy available in fossil fuel (i.e., coal, fuel oil, gas) is converted to heat energy by combustion in a steam generator. The heat thus liberated is absorbed by continuously feeding water in a combination of heat-transfer surfaces, resulting in a continuous generation of steam. The water fed into a steam generator is called feed water. Steam and feed water together is called working fluid. The name “steam generator” is also still called “boiler,”. The fuel-firing equipment of a steam generator should completely burn the fuel used in the furnace to release as much energy as possible. Air is fed into the furnace for combustion of fuel forming products of combustion or flue gas. The heat released by burning fuel is absorbed in different heat-transfer surfaces to the maximum level possible practically and economically to keep the loss of heat to a minimum. In the heating surfaces, the flue gas transfers its heat to the working fluid. Thus, the feed water is pre-heated to the saturation temperature and vaporized. The saturated steam thus formed is further superheated. After passing over the heating surfaces at various zones the flue gas is cooled and discharged to the atmosphere through a stack. The primary function of a steam generator is to generate steam under pressure, but modern steam generators are also required to do the following:

- Ensure generation of exceptionally high - purity steam by eliminating all impurities from saturated steam.
- Raise the degree of superheat of supplied steam as specified and maintain the same temperature over a defined range of load.
- In large power stations after partial expansion in the turbine steam is returned to the steam generator for further superheating and then transmitted to the turbine for complete expansion. This new degree of superheat is called “reheat” and should also be maintained constant over a defined range of load.
- While executing the above duties, a steam generator must utilize the heat of combustion of fuel as efficiently as possible.

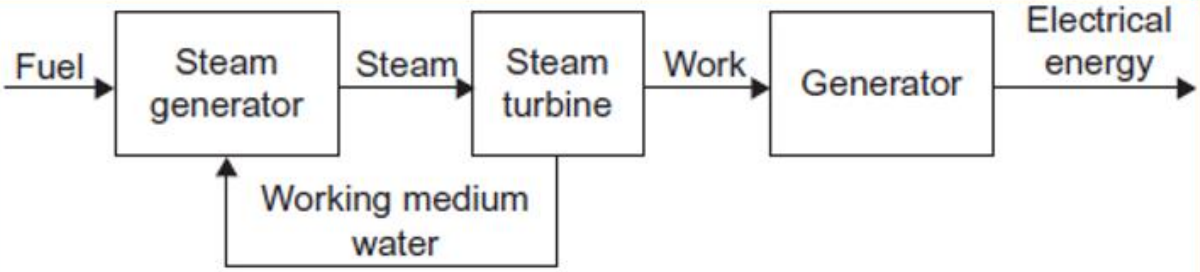


Figure 2.1 Schematic arrangement of a steam power station.

RANKINE CYCLE:-

The ideal cycle for a simple steam power plant is the Rankine cycle, shown in Fig. (2.2)

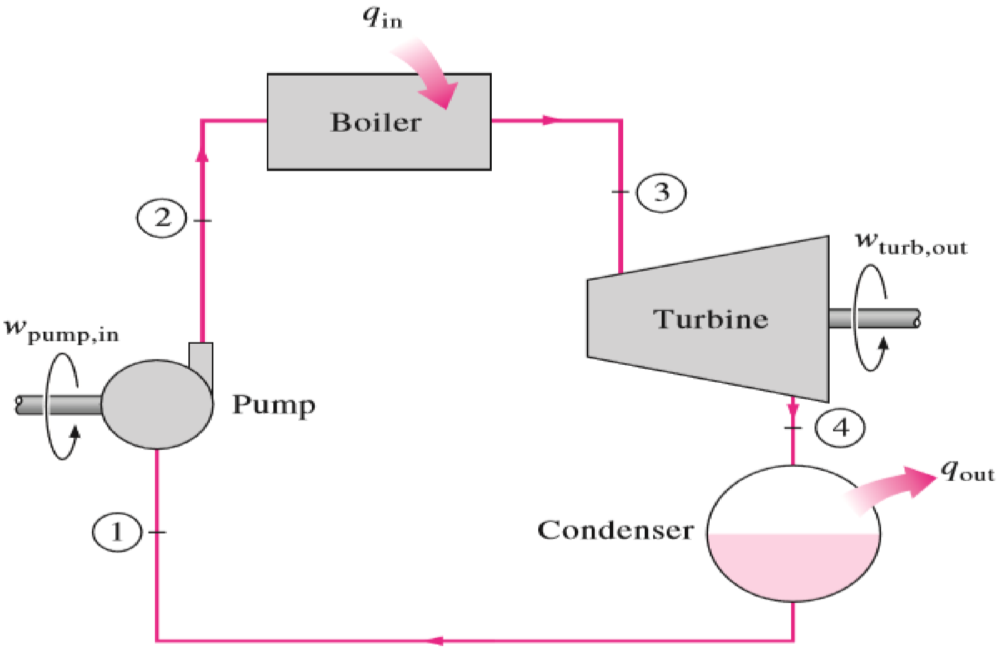


Fig. (2.2 a): Simple steam power plant which operates on the Rankine cycle.

The ideal Rankine cycle does not involve any internal irreversibility and consists of the following four processes (Fig. 2.2 b):

1-2: Isentropic compression in a pump

2-3: Constant pressure heat addition in a boiler

3-4: Isentropic expansion in a turbine

4-1: Constant pressure heat rejection in a condenser

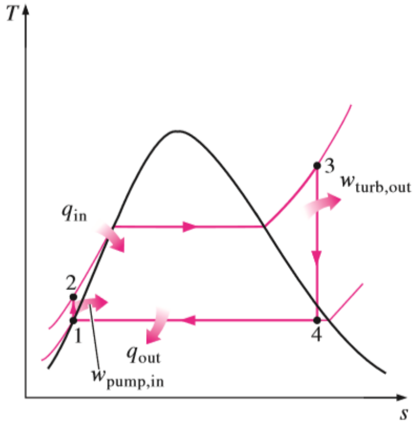


Fig. (2.2 b): T-S diagram of ideal Rankine cycle.

Energy Analysis of the Ideal Rankine Cycle:-

All four components associated with the Rankine cycle (the pump, boiler, turbine, and condenser) are steady-flow devices, and thus all four processes that make up the Rankine cycle can be analyzed as steady-flow processes. The kinetic and potential energy changes of the steam are usually small relative to the work and heat transfer terms and are therefore usually neglected. Then the steady-flow energy equation per unit mass of steam reduces to

$$(q_{in} - q_{out}) + (w_{in} - w_{out}) = h_e - h_i \qquad \dots (1)$$

The boiler and the condenser do not involve any work, and the pump and the turbine are assumed to be isentropic. Then the conservation of energy relation for each device can be expressed as follows:

Pump $(q = 0)$ $w_{pump,in} = h_2 - h_1$ … (2)

Or, $w_{pump,in} = v(P_2 - P_1)$ … (3)

Where $h_1 = h_f @ P_1$ and $v = v_f @ P_1$ … (4)

Boiler $(w = 0)$ $q_{in} = h_3 - h_2$ … (5)

Turbine $(q = 0)$ $w_{turb,out} = h_3 - h_4$ … (6)

Condenser $(w = 0)$ $q_{out} = h_4 - h_1$ … (7)

The thermal efficiency of the Rankine cycle is determined from

$$\eta = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}} \qquad \dots (8)$$

Where $w_{net} = w_{turb,out} - w_{pump,in}$ … (9)

Specific steam consumption:

It is the amount of steam per unit time to produce one unit power. (S.S.C.) is also a comparison criterion in steam power plants. The lower the value the best the plant for the same output power. It gives a direct indication to the relative size of the plant components.

$$S.S.C.=\frac{\dot{m}_s}{P_{out}} \qquad \frac{Kg/h}{kW} \qquad \dots (10)$$

$\dot{m}_s$ is steam mass flow rate in kilogram per hour required. However, steam mass flow rate is usually measured in kilogram per second so it should be multiply by (3600). p_{out} is output power in kilowatt.

$$s.s.c.=\frac{\dot{m}_s * 3600}{\dot{m}_s\, w_{net}}=\frac{3600}{w_{net}} \qquad {}^{Kg}/_{kW.h} \qquad \dots (11)$$

Work ratio (W.R.): It is another comparison criterion of performance in steam power plant. It is define as the ratio of net output work to the total produced work. The higher the value the best the plant for the same output power.

$$W.R.=\frac{w_{net}}{w_{out}}=\frac{w_{turb}-w_{pump}}{w_{turb}} \qquad \dots (12)$$

Example 1: Consider an ideal Rankine cycle that uses water as the working fluid. Saturated vapor leaves the boiler at (175°C) and the condenser pressure is (10 Kpa). Determine: the thermal efficiency of the cycle, S.S.C. and W.R.

Example 2: A steam turbine operating on the Rankine cycle receives steam from boiler at (3.5 MN/m^2) and (350°C), then exhaust to the condenser at (10KN/m^2). Calculate:

1. The energy supplied in the boiler.
2. The dryness fraction of the steam entering the condenser.
3. The efficiency of the cycle.