Fluid Mechanics

Lecture 4

PRESSURE MEASUREMENT

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1.Pressure Measurement

1.1Pressure of a Liquid

When a fluid is contained in a vessel, it exerts force at all points on the sides and bottom and top of the container. The *force per unit area is called* **pressure.**

If, P = The force, and

A = Area on which the force acts; then

intensity of pressure, $p = \dfrac{P}{A}$ ...(2.1)

The pressure of a fluid on a surface will *always act normal to the surface.*

1.2 Pressure head of a liquid

A liquid is subjected to pressure due to its own weight, this pressure increases as the depth of the liquid increases.

Consider a vessel containing liquid, as shown in Fig. 2.1. The liquid will exert pressure on all sides and bottom of the vessel. Now, let cylinder be made to stand in the liquid, as shown in the figure.

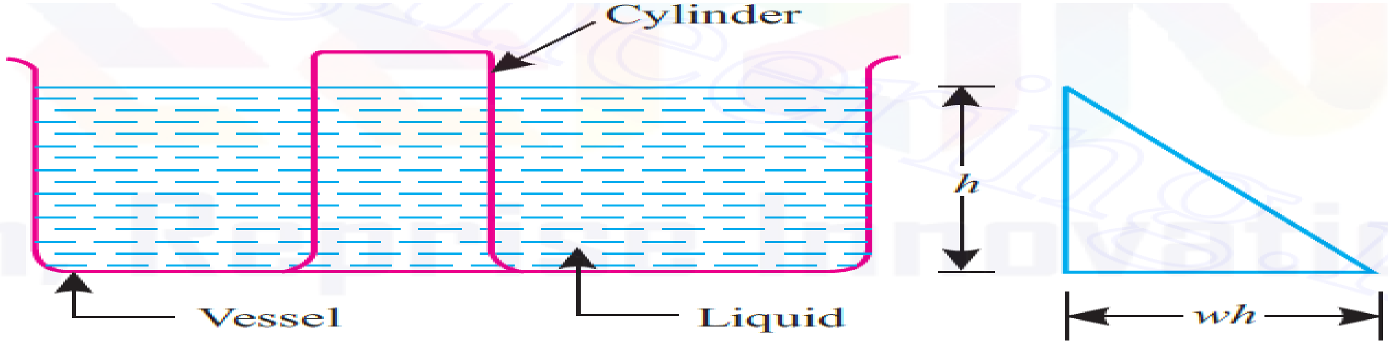


Fig. 2.1. Pressure head.

Let, h = Height of liquid in the cylinder,

A = Area of the cylinder base,

w = Specific weight of the liquid,

and, p = Intensity of pressure.

Now, Total pressure on the base of the cylinder = Weight of liquid in the cylinder

i.e., $p.\,A. = wAh$

$$p = \frac{wAh}{A} = wh \qquad \text{i.e., } p = wh \qquad \ldots(2.2)$$

As $p = wh$, the intensity of pressure in a liquid due to its depth will vary directly with depth.

As the pressure at any point in a liquid depends on height of the free surface above that point, it is sometimes convenient to express a liquid pressure by the *height of the free surface* which would cause the pressure, *i.e.,*

$$h = \frac{p}{w} \qquad \text{[from eqn. (2.2)]}$$

The height of the free surface above any point is known as the *static head* at that point. In this case, static head is h.

Hence, the intensity of pressure of a liquid may be expressed in the following two ways:

1. As a force per unit area (*i.e.*, N/mm^2, N/m^2), and
2. As an equivalent static head (*i.e.*, metres, mm or cm of liquid).

Alternatively:

Pressure variation in fluid at rest:

In order to determine the pressure at any point in a fluid at rest "*hydrostatic law*" is used; the law states as follows:

"The rate of increase of pressure in a vertically downward direction must be equal to the specific weight of the fluid at that point."

The proof of the law is as follows.

Refer to Fig. 2.2

Let, p = Intensity of pressure on face LM,

ΔA = Cross-sectional area of the element,

Z = Distance of the fluid element from free surface, and

ΔZ = Height of the fluid element.

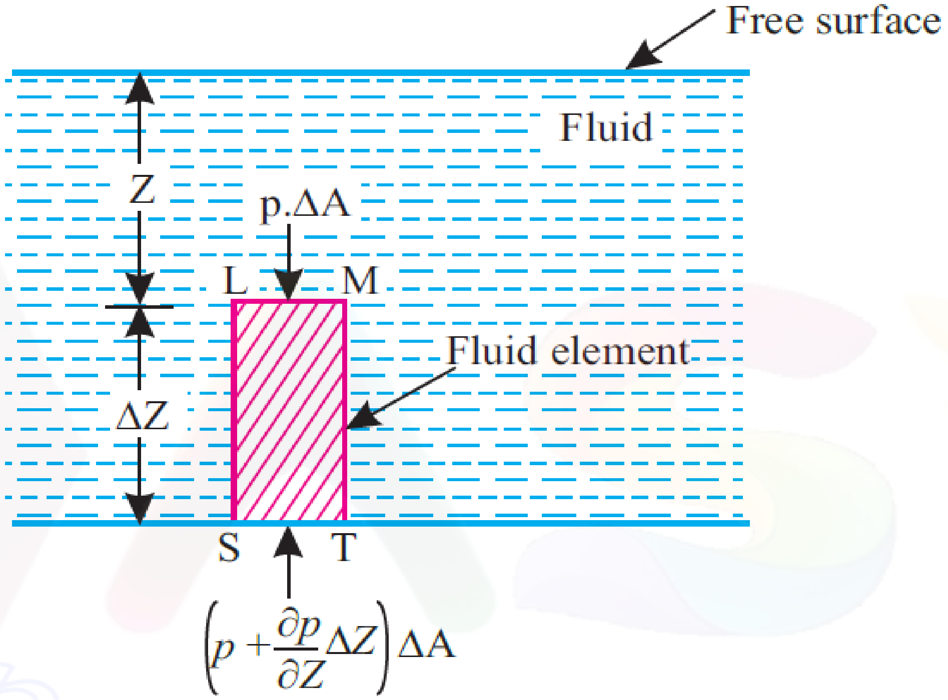


Fig. 2.2. Forces acting on a fluid element.

The forces acting on the element are:

(*i*) Pressure force on the face

$LM = p \times \Delta A$ ...(acting downward)

(*ii*) Pressure force on the face $ST = \left(p + \frac{\partial p}{\partial Z} \times \Delta Z \right) \times \Delta A$ ... (acting upward)

(*iii*) Weight of the fluid element = Weight density × volume

$= w \times (\Delta A \times \Delta Z)$

(*iv*) Pressure forces on surfaces MT and LS ..... are equal and opposite.

For equilibrium of the fluid element, we have:

$$p \times \Delta A - \left[p + \frac{\partial p}{\partial Z} \times \Delta Z\right] \times \Delta A + w \times (\Delta A \times \Delta Z) = 0$$

or, $$p \times \Delta A - p \times \Delta A - \frac{\partial p}{\partial Z} \times \Delta Z \times \Delta A + w \times \Delta A \times \Delta Z = 0$$

or, $$\frac{\partial p}{\partial Z} \Delta Z \times \Delta A + w \times \Delta A \times \Delta Z = 0$$

or, $$\frac{\partial p}{\partial Z} = w \text{ (cancelling } \Delta Z \times \Delta A \text{ from both the sides)}$$

or, $$\frac{\partial p}{\partial Z} = \rho \times g \qquad (\because w = \rho \times g) \qquad \text{...(2.3)}$$

Eqn. (2.3.) states *that rate of increase of pressure in a vertical direction is equal to weight density of the fluid at that point*. This is "**hydrostatic law**".

On integrating the eqn. (2.3), we get:

$$\int dp = \int \rho g . dZ$$

or, $$p = \rho g. \ Z (= wZ) \qquad \text{...(2.4)}$$

where, p is the pressure above atmospheric pressure.

From eqn. (2.4), we have:

$$Z = \frac{p}{\rho . g}\left(= \frac{p}{w}\right) \qquad \text{... (2.5)}$$

Here Z is known as *pressure head*.

Example 2.1. *Find the pressure at a depth of 15 m below the free surface of water in a reservoir.*

Solution. Depth of water, $h = 15$ m

Specific weight of water, $w = 9.81$ kN/m^3

Pressure *p*:

We know that, $p = wh = 9.81 \times 15 = 147.15$ kN/m^2

i.e., $p = 147.15$ kN/m^2 = **147.15 kPa (Ans.)**

Example 2.2. *Find the height of water column corresponding to a pressure of 54 kN/m^2.*

Solution. Intensity of pressure, $p = 54$ kN/m^2

Specific weight of water, $w = 9.81$ kN/m^3

Height of water column, *h*:

Using the relation: $p = wh;\ h = \dfrac{p}{w} = \dfrac{54}{9.81} = \mathbf{5.5\ m}$ **(Ans.)**

1.3 Pascal Law

The **Pascal's law** states as follows :

"The intensity of pressure at any point in a liquid at rest, is the same in all directions".

Proof. Let us consider a very small wedge shaped element *LMN* of a liquid, as shown in Fig. 2.3.

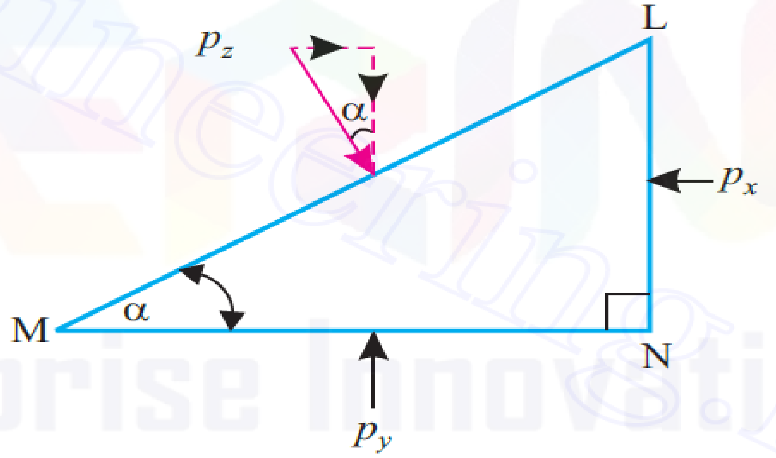


Fig. 2.3. Pressure on a fluid element at rest.

Let, p_x = Intensity of horizontal pressure on the element of liquid,

p_y = Intensity of vertical pressure on the element of liquid,

p_z = Intensity of pressure on the diagonal of the right angled triangular element,

α = Angle of the element of the liquid,

P_x = Total pressure on the vertical side *LN* of the liquid,

P_y = Total pressure on the horizontal side *MN* of the liquid, and

P_z = Total pressure on the diagonal *LM* of the liquid.

Now, $$P_x = p_x \times LN \quad \ldots(i)$$

and, $$P_y = p_y \times MN \quad \ldots(ii)$$

and, $$P_z = p_z \times LM \quad \ldots(iii)$$

As the element of the liquid is at rest, therefore the *sum of horizontal and vertical components of the liquid pressures must be equal to zero.*

Resolving the forces *horizontally*:

$$P_z \sin \alpha = P_x$$

$$p_z \,.\, LM.\sin\alpha = p_x \,.\, LN \qquad (\because P_z = p_z.\, LM)$$

But, $\quad LM \,.\, \sin\alpha = LN$ ... From Fig 2.3

$\therefore \quad p_z = p_x$ ...(*iv*)

Resolving the forces *vertically*:

$$P_z\,.\cos\alpha = P_y - W$$

(where, W = weight of the liquid element)

Since the element is very small, neglecting its weight, we have:

$$P_z \cos\alpha = P_y \quad \text{or} \quad p_z \,.\, LM\cos\alpha = p_y\,.MN$$

But, $\quad LM\cos\alpha = MN$ ...From Fig 2.3

$\therefore \quad p_z = p_y$ ...(*v*)

From (*iv*) and (*v*), we get: $p_x = p_y = p_z$,

which is independent of α.

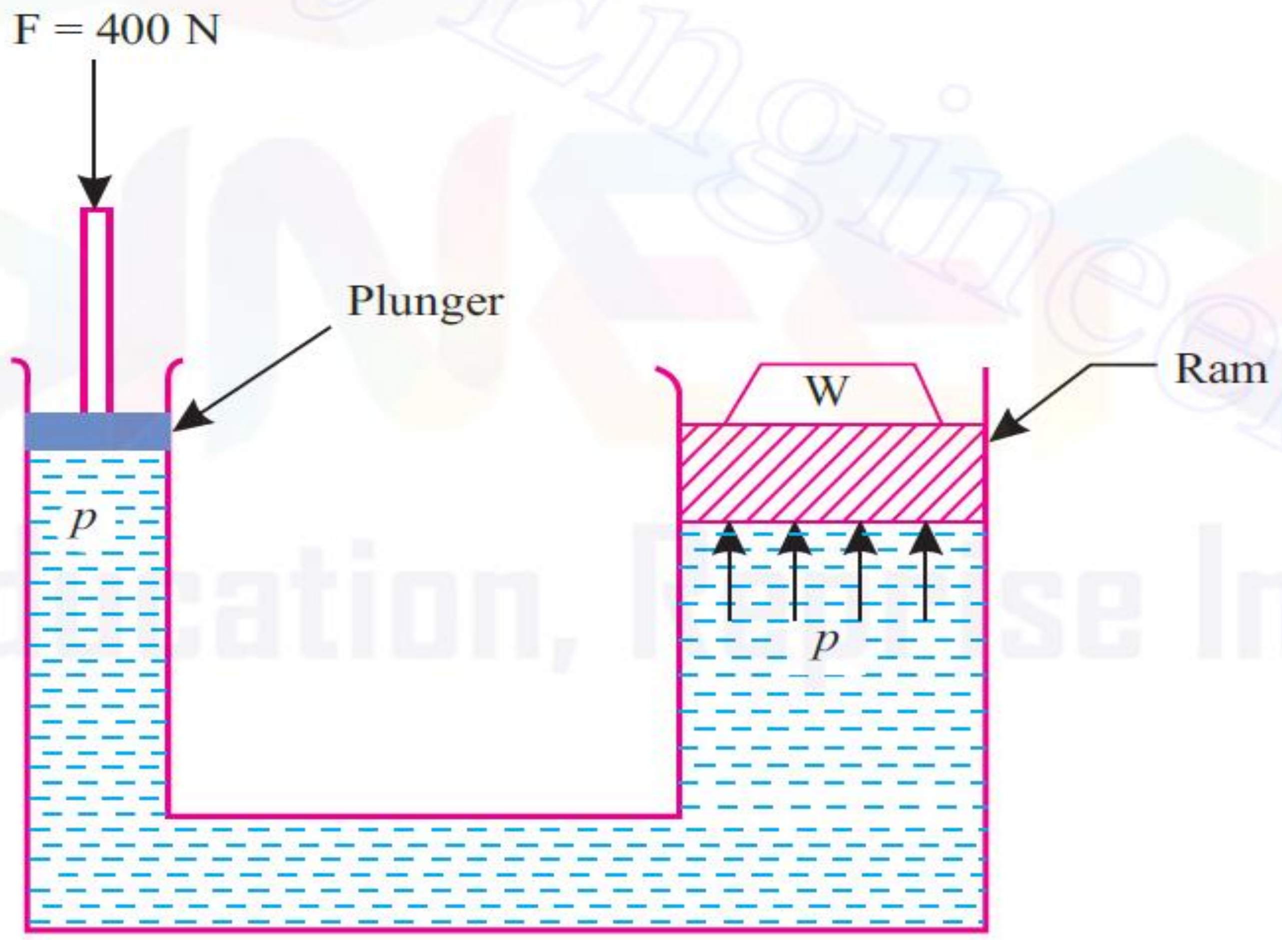
Hence, at *any point in a fluid at rest the intensity of pressure is exerted equally in all directions, which is called* **Pascal's law.**

Example 2.3. *The diameters of ram and plunger of an hydraulic press are 200 mm and 30 mm respectively. Find the weight lifted by the hydraulic press when the force applied at the plunger is 400 N.*

Solution. Diameter of the ram, $D = 200 \text{ mm} = 0.2 \text{ m}$

Diameter of the plunger, $d = 30 \text{ mm} = 0.03 \text{ m}$

Force on the plunger, $F = 400 \text{ N}$



Hydraulic press

Load lifted, W:

$$\text{Area of ram, } A = \frac{\pi}{4}D^2 = \frac{\pi}{4} \times 0.2^2 = 0.0314 \text{ m}^2$$

$$\text{Area of plunger, } a = \frac{\pi}{4}d^2 = \frac{\pi}{4} \times 0.03^2 = 7.068 \times 10^{-4} \text{ m}^2$$

Intensity of pressure due to plunger,

$$p = \frac{F}{a} = \frac{400}{7.068 \times 10^{-4}} = 5.66 \times 10^5 \text{ N/m}^2$$

Since the intensity of pressure will be equally transmitted (due to Pascal's law), therefore the intensity of pressure at the ram is also

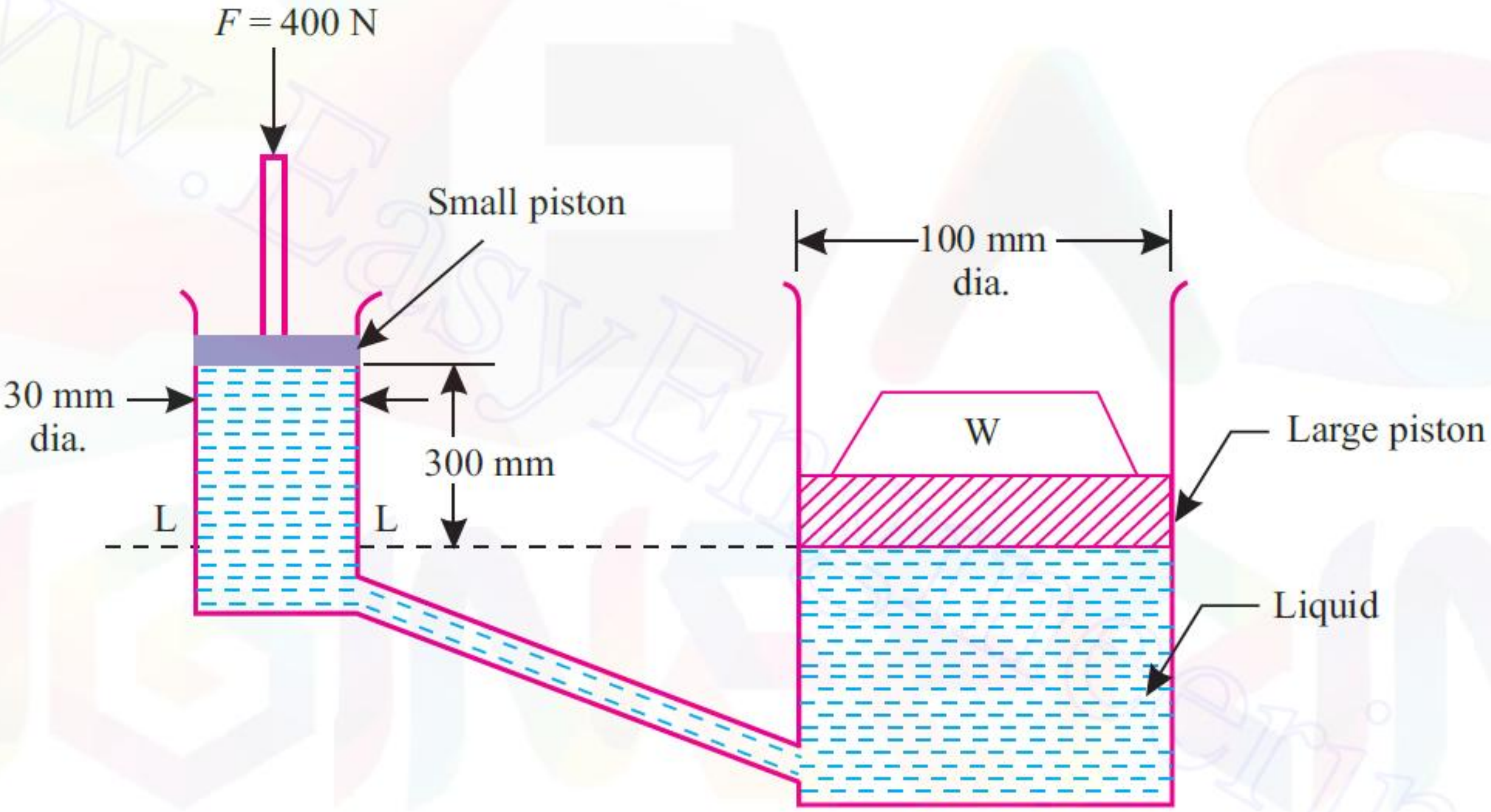
$$= p = 5.66 \times 10^5 \text{ N/m}^2$$

$$\text{But intensity of pressure at the ram} = \frac{\text{Weight}}{\text{Area of ram}} = \frac{W}{A} = \frac{W}{0.0314} \text{ N/m}^2$$

$\therefore \dfrac{W}{0.0314} = 5.66 \times 10^5$ or $W = 0.0314 \times 5.66 \times 10^5 \text{ N} = 17.77 \times 10^3 \text{ N}$ or **17.77 kN** **(Ans.)**

Example 2.4. *For the hydraulic jack shown in Fig. 2.5 find the load lifted by the large piston when a force of 400 N is applied on the small piston. Assume the specific weight of the liquid in the jack is 9810 N/m^3.*

Solution. Diameter of small piston, $d = 30$ mm $= 0.03$ m



$$\text{Area of small piston, } a = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 0.03^2 = 7.068 \times 10^{-4} \text{ m}^2$$

$$\text{Diameter of the large piston, } D = 100 \text{ mm} = 0.1 \text{ m}$$

$$\text{Area of large piston, } A = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times 0.1^2 = 7.854 \times 10^{-3} \text{ m}^2$$

$$\text{Force on small piston, } F = 400 \text{ N}$$

Load lifted, *W*:

$$\text{Pressure intensity on small piston, } p = \frac{F}{a} = \frac{400}{7.068 \times 10^{-4}} = 5.66 \times 105 \text{ N/m}^2$$

Pressure intensity at section LL,

$$p_{LL} = \frac{F}{a} + \text{Pressure intensity due to height of 300 mm of liquid}$$

$$= \frac{F}{a} + wh = 5.66 \times 10^5 + 9810 \times \frac{300}{1000}$$

$$= 5.66 \times 10^5 + 2943 = 5.689 \times 10^5 \text{ N/m}^2$$

Pressure intensity transmitted to the large piston = 5.689×10^5 N/m^2

$$\text{Force on the large piston} = \text{Pressure intensity} \times \text{area of large piston}$$

$$= 5.689 \times 10^5 \times 7.854 \times 10^{-3} = 4468 \text{ N}$$

Hence, *load lifted by the large piston* = **4468 N (Ans.)**

1.4 ABSOLUTE AND GAUGE PRESSURES

Atmospheric pressure:

The atmospheric air exerts a normal pressure upon all surfaces with which it is in contact, and it is known as *atmospheric pressure*. The atmospheric pressure is also known as *'Barometric pressure'*.

The atmospheric pressure at sea level (above absolute zero) is called *'Standard atmospheric pressure'*.

Note. The local atmospheric pressure may be a little lower than these values if the place under question is higher than sea level, and higher values if the place is lower than sea level, due to the corresponding decrease or increase of the column of air standing, respectively.

Gauge pressure:

It is the pressure, measured with the help of pressure measuring instrument, in which the atmospheric pressure is taken as datum. The *atmospheric pressure* on the scale is marked as *zero*.

Gauges record pressure above or below the local atmospheric pressure, since they measure the difference in pressure of the liquid to which they are connected and that of surrounding air. If the pressure of the liquid is *below* the local atmospheric pressure, then the gauge is designated as '*vacuum gauge*' and the recorded value indicates the amount by which the pressure of the liquid is *below* local atmospheric pressure, i.e. *negative pressure*.

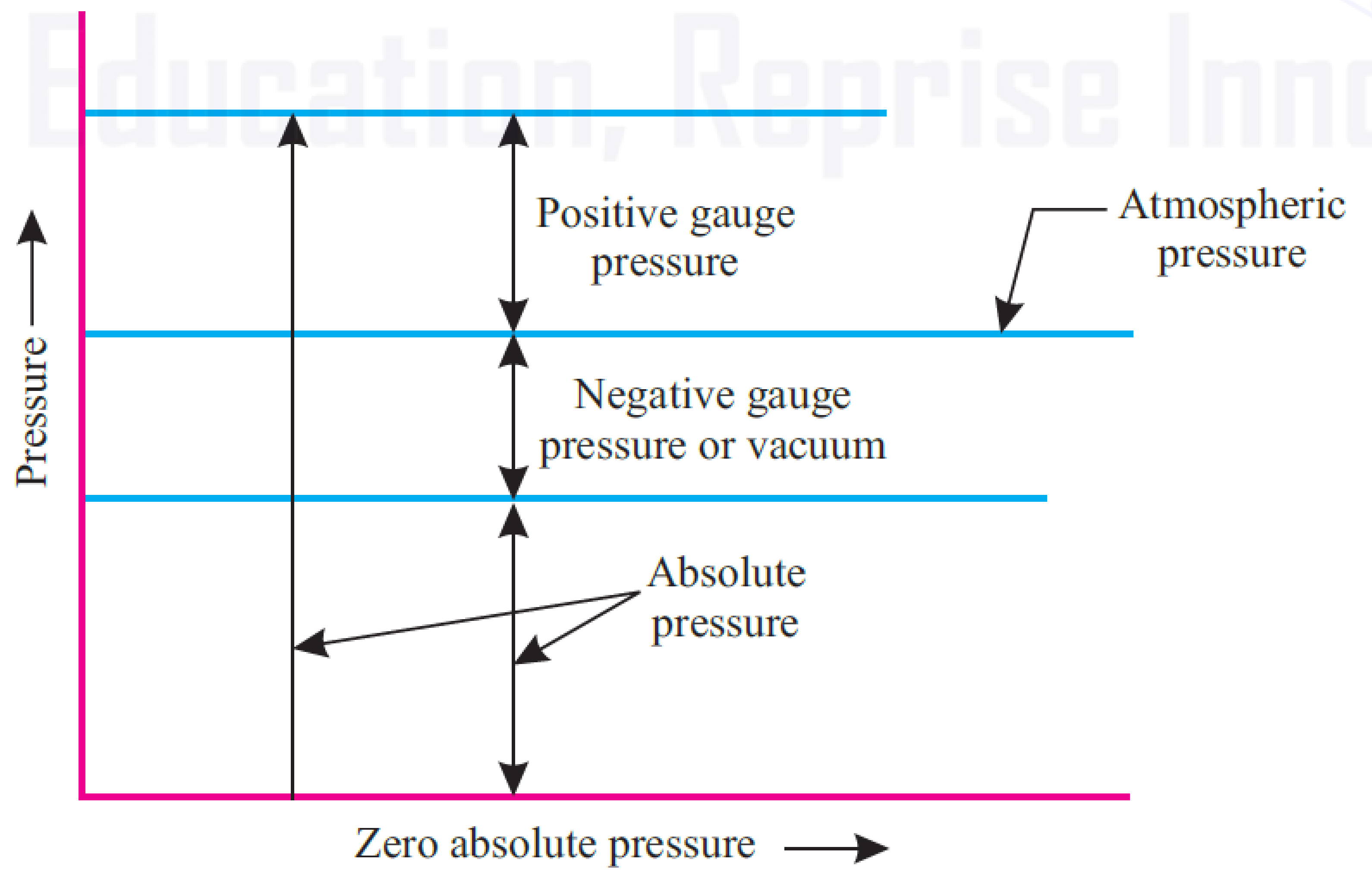
(*Vacuum pressure* is defined as the pressure *below the atmospheric pressure*).

Absolute pressure:

It is necessary to establish an absolute pressure scale which is independent of the changes in atmospheric pressure. A pressure of absolute zero can exist only in complete vacuum.

Any pressure measured above the absolute zero of pressure is termed as an '**absolute pressure**'.

A schematic diagram showing the gauge pressure, vacuum pressure and the absolute pressure is given in Fig. 2.6.


Positive gauge pressure
Atmospheric pressure
Negative gauge pressure or vacuum
Absolute pressure
Pressure
Zero absolute pressure

Mathematically:

1. *Absolute pressure* = Atmospheric pressure + gauge pressure

   i.e., $$p_{abs} = p_{atm} + p_{gauge}$$

2. *Vacuum pressure* = Atmospheric pressure – absolute pressure

Units for pressure:

The fundamental S.I. unit of pressure is newton per square metre (N/m^2). This is also known as *Pascal*.

Low pressures are often expressed in terms of mm of water or mm of mercury. This is an abbreviated way of saying that the pressure is such that will support a liquid column of stated height.

Note. When the local atmospheric pressure is not given in a problem, it is taken as 100 kN/m^2 or 10 m of water for simplicity of calculations.

Standard atmospheric pressure has the following equivalent values:

101.3 kN/m^2 or 101.3 kPa; 10.3 m of water; 760 mm of mercury; 1013 mb (millibar) $\simeq$ 1 bar $\simeq$100 kPa = 10^5 N/m^2.

Example 2.5. *Given that:*
Barometer reading = 740 mm of mercury;
Specific gravity of mercury = 13.6; Intensity of pressure = 40 kPa.
Express the intensity of pressure in S.I. units, both gauge and absolute.

Solution. Intensity of pressure, $p = 40$ kPa

Gauge pressure:

(*i*) $p = 40 \text{ kPa} = 40 \text{ kN/m}^2 = 0.4 \times 10^5 \text{ N/m}^2 =$ **0.4 bar (Ans.)**
$(1 \text{ bar} = 10^5 \text{ N/m}^2)$

(*ii*) $h = \dfrac{p}{w} = \dfrac{0.4 \times 10^5}{9.81 \times 10^3}$ **= 4.077 m of water (Ans.)**

(*iii*) $h = \dfrac{p}{w} = \dfrac{0.4 \times 10^5}{9.81 \times 10^3 \times 13.6}$ **= 0.299 m of mercury (Ans.)**

$$\left[\begin{array}{l}\text{Where, } w = \text{specific weight;}\\ \text{For water : } w = 9.81 \text{ kN/m}^3\\ \text{For mercury : } w = 9.81 \times 13.6 \text{ kN/m}^3\end{array}\right]$$

Absolute pressure:

Barometer reading (atmospheric pressure)

$$= 740 \text{ mm of mercury} = 740 \times 13.6 \text{ mm of water}$$

$$= \frac{740 \times 13.6}{1000} = 10.6 \text{ m of water}$$

Absolute pressure $(p_{abs.})$ = Atmospheric pressure $(p_{atm.})$ + gauge pressure (p_{gauge}).

$\therefore$ $p_{abs} = 10.06 + 4.077 =$ **14.137 m of water (Ans.)**

$= 14.137 \times (9.81 \times 10^3) =$ **1.38×10^5 N/m^2 (Ans.)** $(p = wh)$

$=$ **1.38 bar (Ans.)** $(1 \text{ bar} = 10^5 \text{ N/m}^2)$

$= \dfrac{14.137}{13.6}$ **= 1.039 m of mercury. (Ans.)**

Example 2.7. *On the suction side of a pump a gauge shows a negative pressure of 0.35 bar. Express this pressure in terms of:*

(*i*) *Intensity of pressure, kPa,*

(*ii*) *N/m^2 absolute,*

(*iii*) *Metres of water gauge,*

(*iv*) *Metres of oil (specific gravity 0.82) absolute, and*

(*v*) *Centimetres of mercury gauge,*

Take atmospheric pressure as 76 cm of Hg and relative density of mercury as 13.6.

Solution. *Given*: Reading of the *vacuum* gauge = 0.35 bar

(*i*) Intensity of pressure, kPa:

$$\text{Gauge reading} = 0.35 \text{ bar} = 0.35 \times 10^5 \text{ N/m}^2$$
$$= 0.35 \times 10^5 \text{ Pa} = \mathbf{35 \text{ kPa}} \text{ (Ans.)}$$

(*ii*) N/m² absolute:

$$\text{Atmospheric pressure, } p_{atm.} = 76 \text{ cm of Hg}$$
$$= (13.6 \times 9810) \times \frac{76}{100} = 101396 \text{ N/m}^2$$
$$\text{Absolute pressure} = \text{Atmospheric pressure} - \textit{Vacuum} \text{ pressure}$$
$$p_{abs.} = p_{atm} - p_{vac.}$$
$$= 101396 - 35000 = \mathbf{66396 \text{ N/m}^2 \text{ absolute}} \text{ (Ans.)}$$

(*iii*) Metres of water gauge:

$$p = \rho g h = wh$$
$$\therefore \quad h_{water} \text{ (gauge)} = \frac{p}{w} = \frac{0.35 \times 10^5}{9810} = \mathbf{3.567 \text{ m}} \text{ (}\textit{gauge}\text{) (Ans.)}$$

(*iv*) Metres of oil (sp. gr. = 0.82) absolute:

$$h_{oil} \text{ (absolute)} = \frac{66396}{0.82 \times 9810} = \mathbf{8.254 \text{ m of water}} \text{ (}\textit{absolute}\text{) (Ans.)}$$

(*v*) Centimetres of mercury gauge:

$$h_{mercury} \text{(gauge)} = \frac{0.35 \times 10^5}{13.6 \times 9810} = 0.2623 \text{ m of mercury}$$
$$= \mathbf{26.236 \text{ cm of mercury}} \text{ (Ans.)}$$

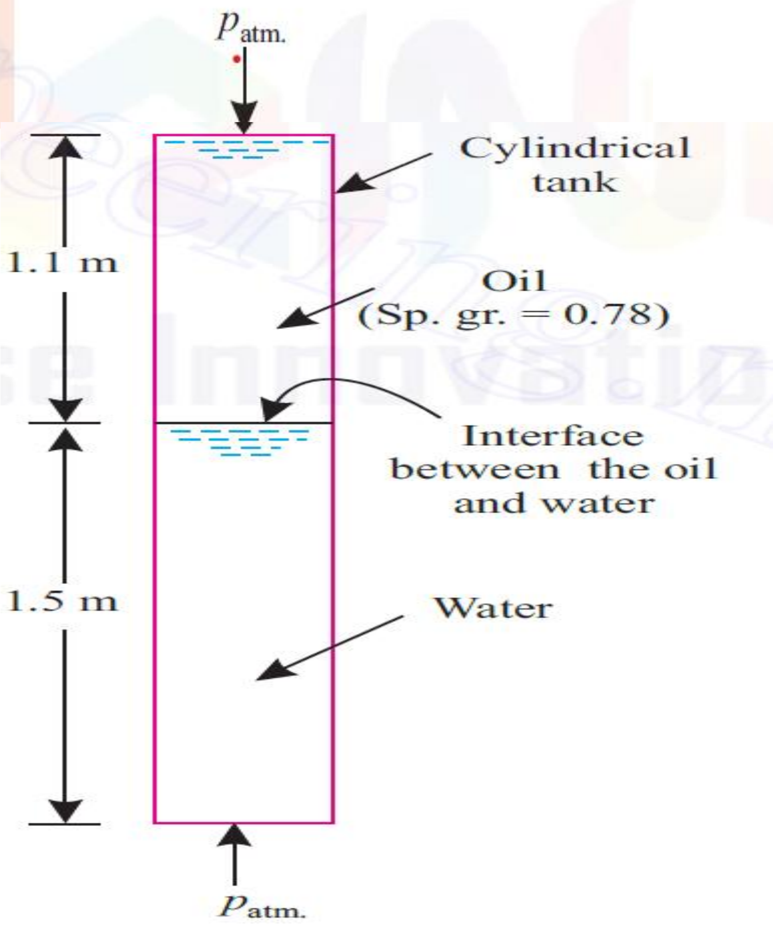
Example 2.9. *A cylindrical tank of cross-sectional area 600 mm² and 2.6 m height is filled with water upto a height of 1.5 m and remaining with oil of specific gravity 0.78. The vessel is open to atmospheric pressure. Calculate:*

(*i*) *Intensity of pressure at the interface.*

(*ii*) *Absolute and gauge pressures on the base of the tank in terms of water head, oil head and N/m².*

(*iii*) *The net force experienced by the base of the tank.*

Assume atmospheric pressure as 1.0132 bar.



Solution. *Given:* Area of cross-section of the tank, $A = 600$ mm² $= 600 \times 10^{-6}$; Sp.gr. of oil $= 0.78$; $p_{atm.} = 1.0132$ bar.

(*i*) Intensity of pressure at the interface:

The pressure intensity at the interface between the oil and water is due to 1.1 m of oil and is given by:

$$p_{interface} = wh$$
$$= (0.78 \times 9810) \times 1.1$$
$$= \mathbf{8417\ N/m^2}\ \textbf{(Ans.)}$$

(*ii*) Absolute and gauge pressures on the base of the tank:

Pressure at the base of the tank

= Pressure at the interface (due to 1.1 m of oil) + pressure due to 1.5 m of water,

i.e., $p_{base\ (gauge)} = 8417 + (9810 \times 1.5)$

$= \mathbf{23132\ N/m^2}$ **(gauge) (Ans.)**

$= \dfrac{23132}{9810}$ **= 2.358 m of water (gauge) (Ans.)**

$= \dfrac{23132}{0.78 \times 9810}$ **= 3.023 m of oil (gauge) (Ans.)**

$$\text{Atmospheric pressure, } p_{atm.} = 1.0132 \text{ bar}$$

$$= 1.0132 \times 10^5 \text{ N/m}^2$$

$$= \frac{1.0132 \times 10^5}{9810} = 10.328 \text{ m of water}$$

$$= \frac{1.0132 \times 10^5}{0.78 \times 9810} = 13.241 \text{ m of oil}$$

$$\text{Absolute pressure} = \text{Atmospheric pressure} + \text{gauge pressure}$$

$$p_{base} \text{ (absolute)} = 10.328 + 2.358 = \mathbf{12.686 \text{ m of water}} \text{ (Ans.)}$$

$$= 13.241 + 3.023 = \mathbf{16.264 \text{ m of oil}} \text{ (Ans.)}$$

$$= 101320 + 23132 = \mathbf{124452 \text{ N/m}^2} \text{ (Ans.)}$$

(*iii*) The net force experienced by the base of the tank:

$$F (= P) = p_{base}\text{(gauge)} \times \text{cross-sectional area}$$

$$= 23132 \times 600 \times 10^{-6} = \mathbf{13.879 \text{ N}} \text{ (Ans.)}$$

Fluid Mechanics AM201
Second level

Northern Technical University

Engineering Technical College/Mosul

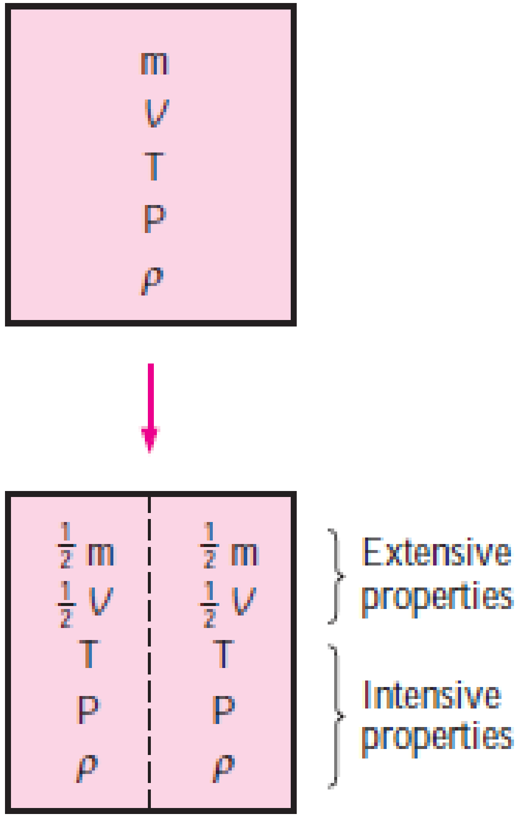
Applied Mechanics Engineering Techniques Dpt.

2nd lesson

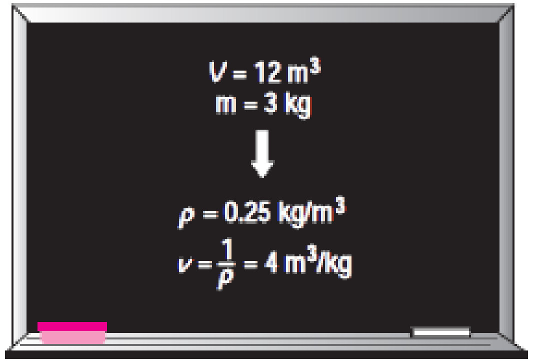
Physical Properties of Fluid

Introduction

- Any characteristic of a system is called a **property.** Some familiar properties are pressure *P*, temperature *T*, volume *V*, and mass *m*. The list can be extended to include less familiar ones such as viscosity, thermal conductivity, modulus of elasticity, thermal expansion coefficient, electric resistivity, and even velocity and elevation.
- Properties are considered to be either intensive or extensive. Intensive properties are those that are independent of the mass of a system, such as temperature, pressure, and density. Extensive properties are those whose values depend on the size—or extent—of the system. Total mass, total volume V, and total momentum are some examples of extensive properties. An easy way to determine whether a property is intensive or extensive is to divide the system into two equal parts with an imaginary partition, as shown in Fig.1. Each part will have the same value of intensive properties as the original system, but half the value of the extensive properties.



Introduction



- Extensive properties per unit mass are called specific properties. Some examples of specific properties are specific volume (v = V/m) and specific total energy (e = E/m).

1- **Density and specific gravity**

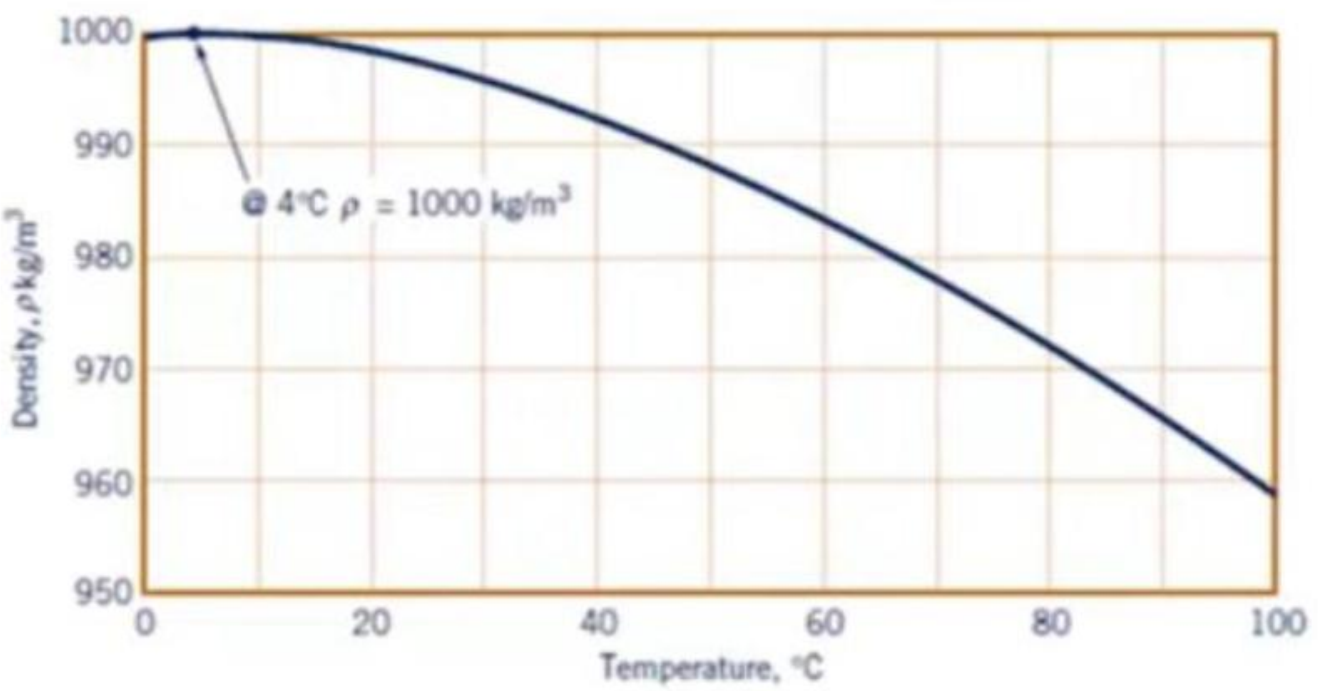
- Density is defined as mass per unit volume $\rho = m/V$ (kg/m^3)
- The reciprocal of density is the specific volume v, which is defined as volume per unit mass. That is, v = V/m = 1/ ρ.
- The density of a substance, in general, depends on temperature and pressure.
- The density of most gases is proportional to pressure and inversely proportional to temperature.
- Sometimes the density of a substance is given relative to the density of a well-known substance. Then it is called specific gravity, or relative density, and is defined as the ratio of the density of a substance to the density of some standard substance at a specified temperature (usually water at 4°C, for which $\rho\ H_2O$ = 1000 kg/m3). That is,

$$SG = \rho \,/\, \rho H_2O$$

TABLE 2–1

Specific gravities of some substances at 0°C

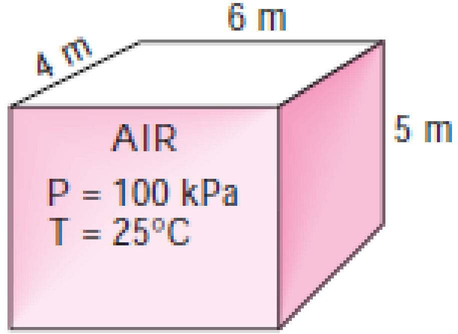
Substance	SG
Water	1.0
Blood	1.05
Seawater	1.025
Gasoline	0.7
Ethyl alcohol	0.79
Mercury	13.6
Wood	0.3–0.9
Gold	19.2
Bones	1.7–2.0
Ice	0.92
Air (at 1 atm)	0.0013



Density of water as a function of temperature.

Example 1

Determine the density, specific gravity, and mass of the air in a room whose dimensions are 4 m × 5 m × 6 m at 100 kPa and 25°C.



Solution The density, specific gravity, and mass of the air in a room are to be determined.

Assumptions At specified conditions, air can be treated as an ideal gas.

Properties The gas constant of air is $R = 0.287\ \text{kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K}$.

Analysis The density of air is determined from the ideal-gas relation $P = \rho RT$ to be

$$\rho = \frac{P}{RT} = \frac{100\ \text{kPa}}{(0.287\ \text{kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(25 + 273)\ \text{K}} = \mathbf{1.17\ kg/m^3}$$

Then the specific gravity of air becomes

$$SG = \frac{\rho}{\rho_{H_2O}} = \frac{1.17\ \text{kg/m}^3}{1000\ \text{kg/m}^3} = \mathbf{0.00117}$$

Finally, the volume and the mass of air in the room are

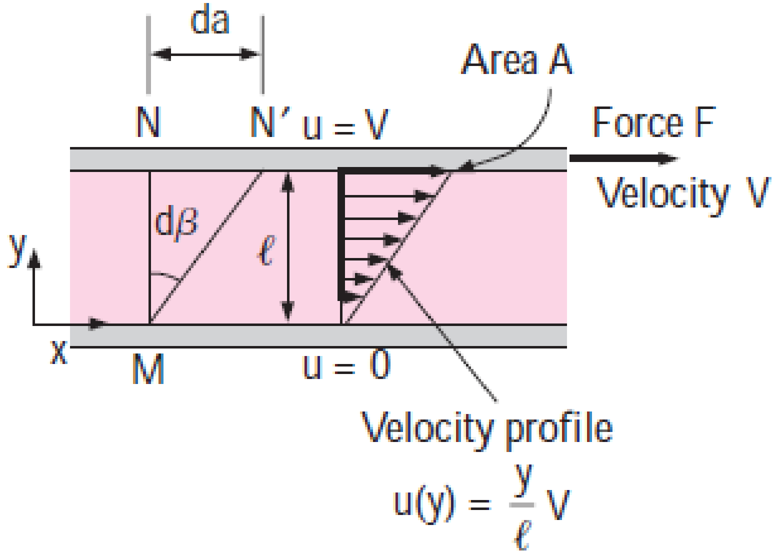
$$V = (4\ \text{m})(5\ \text{m})(6\ \text{m}) = 120\ \text{m}^3$$

$$m = \rho V = (1.17\ \text{kg/m}^3)(120\ \text{m}^3) = \mathbf{140\ kg}$$

2- Viscosity

- When two solid bodies in contact move relative to each other, a friction force develops at the contact surface in the direction opposite to motion. To move a table on the floor, for example, we have to apply a force to the table in the horizontal direction large enough to overcome the friction force. The magnitude of the force needed to move the table depends on the friction coefficient between the table and the floor.

- The situation is similar when a fluid moves relative to a solid or when two fluids move relative to each other. We move with relative ease in air, but not so in water. Moving in oil would be even more difficult, as can be observed by the slower downward motion of a glass ball dropped in a tube filled with oil. It appears that there is a property that represents the internal resistance of a fluid to motion or the "fluidity," and that property is the viscosity



- $\tau = F/A$

$$u(y) = \frac{y}{\ell} V \qquad \text{and} \qquad \frac{du}{dy} = \frac{V}{\ell}$$

$$d\beta \approx \tan\beta = \frac{da}{\ell} = \frac{V\,dt}{\ell} = \frac{du}{dy}dt$$

$$\frac{d\beta}{dt} = \frac{du}{dy}$$

$$\tau \propto \frac{d\beta}{dt} \qquad \text{or} \qquad \tau \propto \frac{du}{dy}$$

$$\tau = \mu \frac{du}{dy} \qquad (N/m^2)$$

FLUIDS AND THEIR PROPERTIES

Factors Effecting Viscosity (μ)

Temperature

- The viscosity of *liquids* ($\mu_{liquids}$) *decreases* with *increase in temperature* (**T**). But, the viscosity of *gases* (μ_{gases}) increases with *increase in temperature* (**T**).

This is due to the reason that in *liquids* the shear stress is due to the inter-molecular cohesion which *decreases* with increase of temperature.

As **T** ⇧ **Cohesive force** ⇩ then $\mu_{liquids}$ ⇩

- In gases the inter-molecular cohesion is negligible and the shear stress is due to exchange of momentum of the molecules. The molecular activity increases with rise in temperature and so does the viscosity of gas.

- As **T** ⇧ **Cohesive force** (**Negligible**), **Exchange of momentum of the molecules** ⇧

then μ_{gases} ⇧

Pressure

- The viscosity under ordinary conditions is not noticeably affected by the changes in pressure. however, the viscosity of some oils has been found to increase with increase in pressure.

- μ *is Dynamic viscosity*

$$\mu = \tau \frac{dy}{du} = \frac{N}{\text{m}^2} \frac{m}{\frac{m}{sec}} = \text{pa . sec} \quad \text{(SI units)}$$

1 pa.sec (SI units) = 10 Poise (CGS units)

10^{-1} pa.sec = 1 poise

$\mu_{\text{water}} = 8.90 \times 10^{-4}$ pa·s at about 25 °c

EXAMPLE 1.3. *A plate 0.05 mm distant from a fixed plate moves at 1.2 m/s and requires a force of 2.2 N/m^2 to maintain this speed. Find the viscosity of the fluid between the plates.*

Solution: Velocity of the moving plate, $u = 1.2$ m/s

Distance between the plates, $dy = 0.05$ mm $= 0.05 \times 10^{-3}$ m

Force on the moving plate, $F = 2.2\ N/m^2$

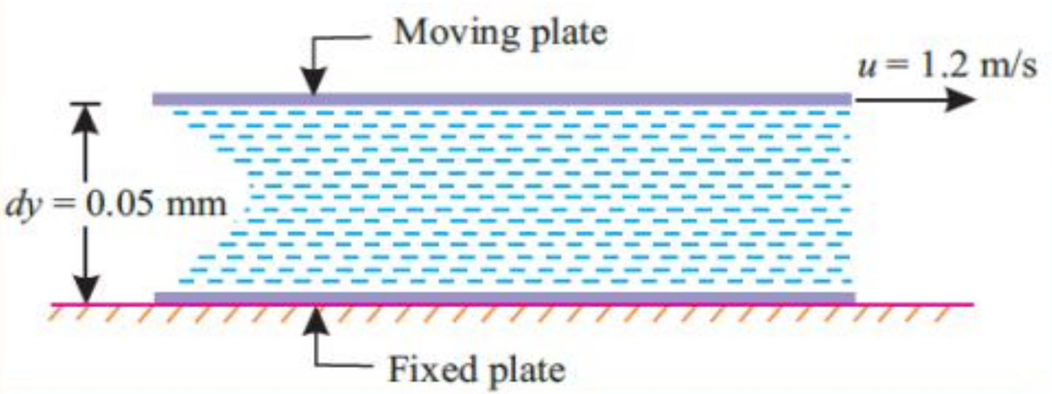


Fig. 1.3

Viscosity of the fluid, μ:

We know, $\tau = \mu . \frac{du}{dy}$

where τ = shear stress or force per unit area $= 2.2\ N/m^2$,

du = change of velocity $= u - 0 = 1.2$ m/s and

dy = change of distance $= 0.05 \times 10^{-3}$ m.

$$\therefore \quad 2.2 = \mu \times \frac{1.2}{0.05 \times 10^{-3}}$$

$$\mu = \frac{2.2 \times 0.05 \times 10^{-3}}{1.2} = 9.16 \times 10^{-5} N.s/m^2$$

Example 1.4. *A plate having an area of 0.6 m² is sliding down the inclined plane at 30° to the horizontal with a velocity of 0.36 m/s. There is a cushion of fluid 1.8 mm thick between the plane and the plate. Find the viscosity of the fluid if the weight of the plate is 280 N.*

Solution: Area of plate, $A = 0.6 \text{ m}^2$

Weight of plate, $W = 280$ N

Velocity of plate, $u = 0.36$ m/s

Thickness of film, $t = dy = 1.8 \text{ mm} = 1.8 \times 10^{-3}$ m

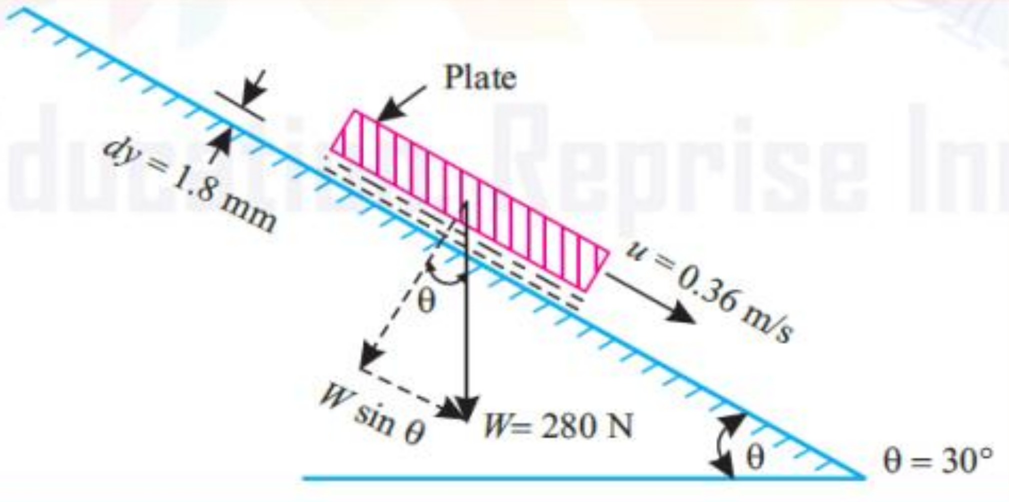


Fig. 1.4

Viscosity of the fluid, μ:

Component of W along the plate $= W \sin\theta = 280 \sin 30° = 140$ N

$$\tau = \frac{F}{A} = \frac{140}{0.6} = 233.33 \text{N/m}^2$$

We know,
$$\tau = \mu \cdot \frac{du}{dy}$$

Where,
$$du = \text{change of velocity} = u - 0 = 0.36 \text{ m/s}$$
$$dy = t = 1.8 \times 10^{-3} \text{ m}$$
$$233.33 = \mu \times \frac{0.36}{1.8 \times 10^{-3}}$$
$$\mu = \frac{233.33 \times 1.8 \times 10^{-3}}{0.36} = 1.166 \text{N.s/m}^2$$

Example 1.16. *Two large fixed parallel planes are 12 mm apart. The space between the surfaces is filled with oil of viscosity 0.972 N.s/m². A flat thin plate 0.25 m² area moves through the oil at a velocity of 0.3 m/s. Calculate the drag force:*

(*i*) *When the plate is equidistant from both the planes, and*

(*ii*) *When the thin plate is at a distance of 4 mm from one of the plane surfaces.*

Solution. Given: Distance between the fixed parallel planes = 12 mm = 0.012 m

Area of *thin* plate, $A = 0.25\ m^2$

Velocity of plate, $u = 0.3$ m/s

Viscosity of oil $= 0.972\ N.s/m^2$

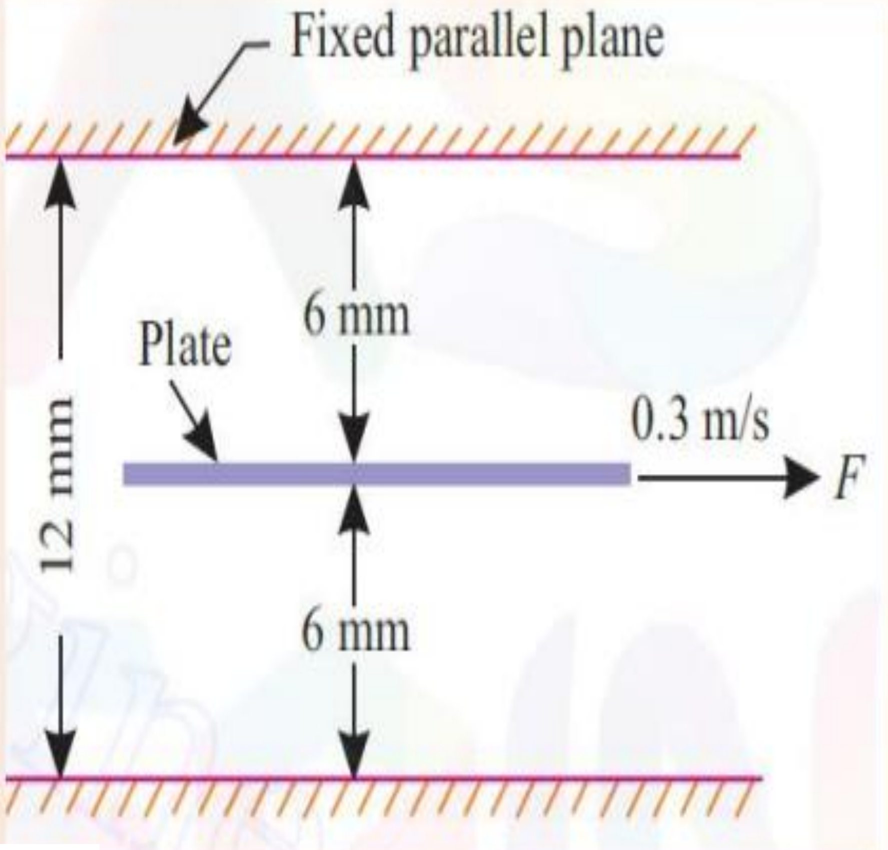


Fig. 1.12

Drag force, F:

(*i*) *When the plate is equidistant from both the planes:*

Let, F_1 = Shear force on the upper side of the thin plate,

F_2 = Shear force on the lower side of the thin plate,

F = Total force required to drag the plate $(= F_1 + F_2)$.

$$\tau_1 = \mu \cdot \left(\frac{du}{dy}\right)_1$$

where, $du = 0.3$ m/s (relative velocity between upper fixed plane and the plate), and $dy =$ 6 mm = 0.006 m (distance between the upper fixed plane and the plate)

(Thickness of the plate neglected).

$$\therefore \qquad \tau_1 = 0.972 \times \frac{0.3}{0.006} = 48.6 \text{ N/m}^2$$

$$\therefore \qquad \text{Shear force, } F_1 = \tau_1 \cdot A = 48.6 \times 0.25 = 12.15 \text{ N}$$

Similarly shear stress (τ_2) on the lower side of the thin plate is given by

$$\tau_2 = u \cdot \left(\frac{du}{dy}\right)_2 = 0.972 \times \frac{0.3}{0.006} = 48.6 \text{ N/m}^2$$

and $$F_2 = \tau_2 \cdot A = 48.6 \times 0.25 = 12.15 \text{ N}$$

$$\therefore \qquad F = F_1 + F_2 = 12.15 + 12.15 = \mathbf{24.30\ N}$$

(*ii*) *When the thin plate is at a distance of 4 mm from one of the plane surfaces*: Refer to Fig. 1.13.

The shear force on the upper side of the thin plate,

$$F_1 = \tau_1 \, . \, A = \mu \, . \left(\frac{du}{dy}\right)_1 \times A$$

$$= 0.972 \times \frac{0.3}{0.008} \times 0.25 = 9.11 \text{N}$$

The shear force on the lower side of the thin plate,

$$F_2 = \tau_2 \times A = \mu \, . \left(\frac{du}{dy}\right)_2 \times A$$

$$= 0.972 \times \left(\frac{0.3}{0.004}\right) \times 0.25 = 18.22 \text{N}$$

$\therefore$ Total force $F = F_1 + F_2 = 9.11 + 18.22 =$ **27.33 N**

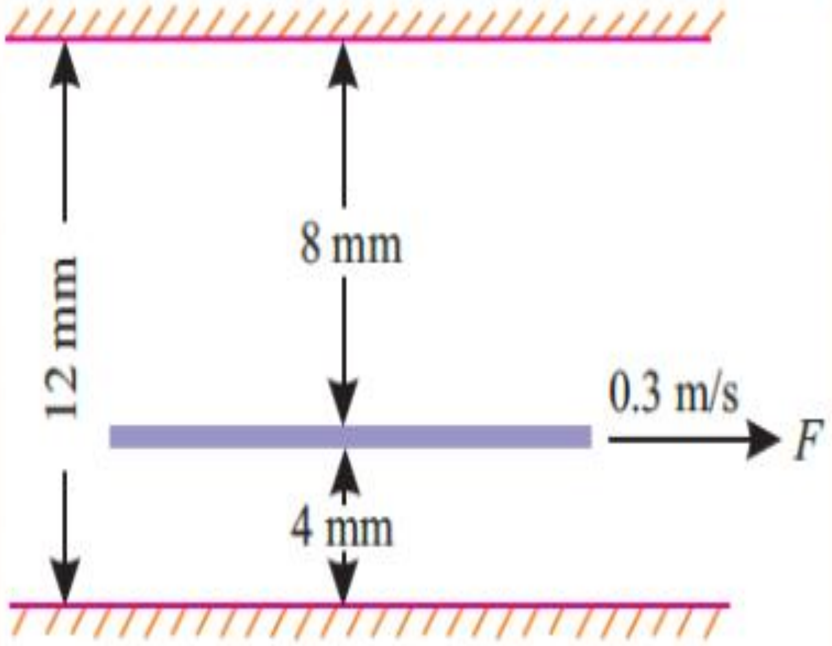


Fig. 1.13

Two concentric cylinders _ Linear movement

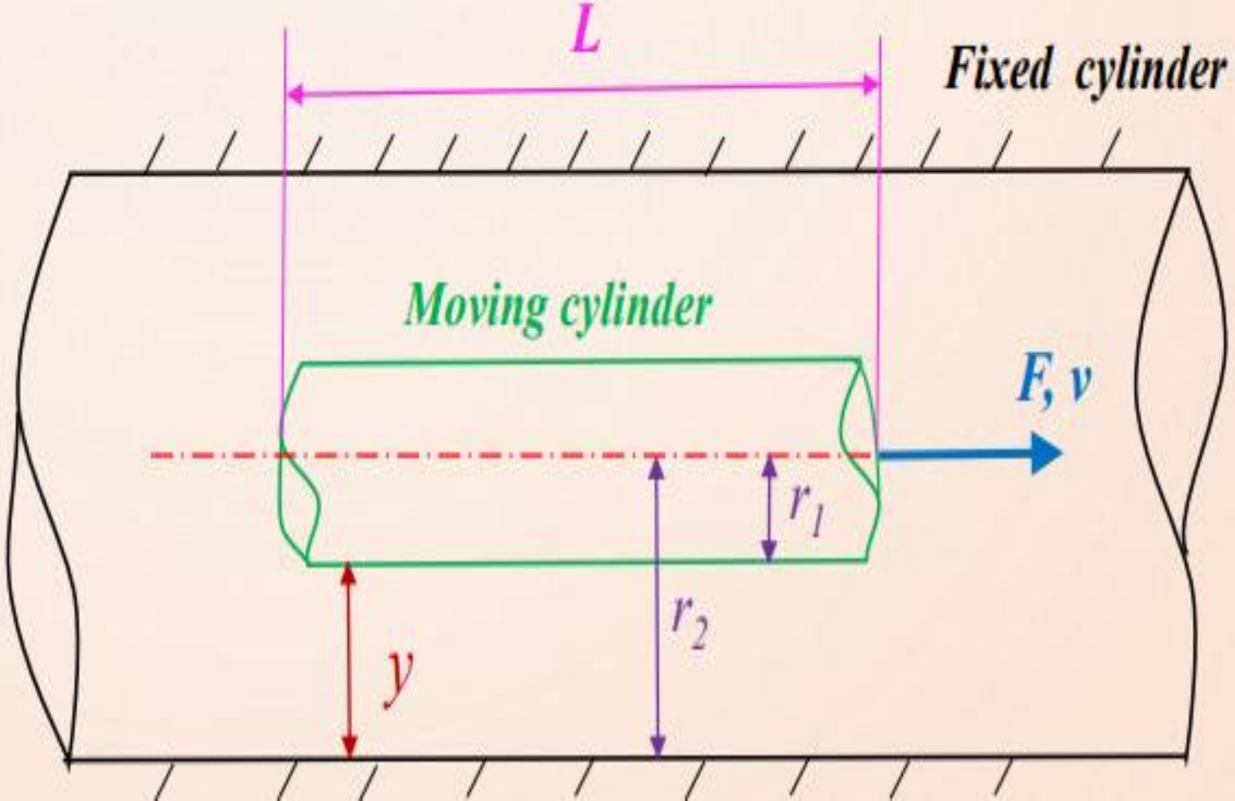
1. Inner cylinder moving with uniform linear velocity

$$F = \mu \frac{v}{y} A$$

$$A = 2 \pi r_1 L$$

$$y = r_2 - r_1$$

$$F = \mu \frac{v}{r_2 - r_1} 2 \pi r_1 L$$



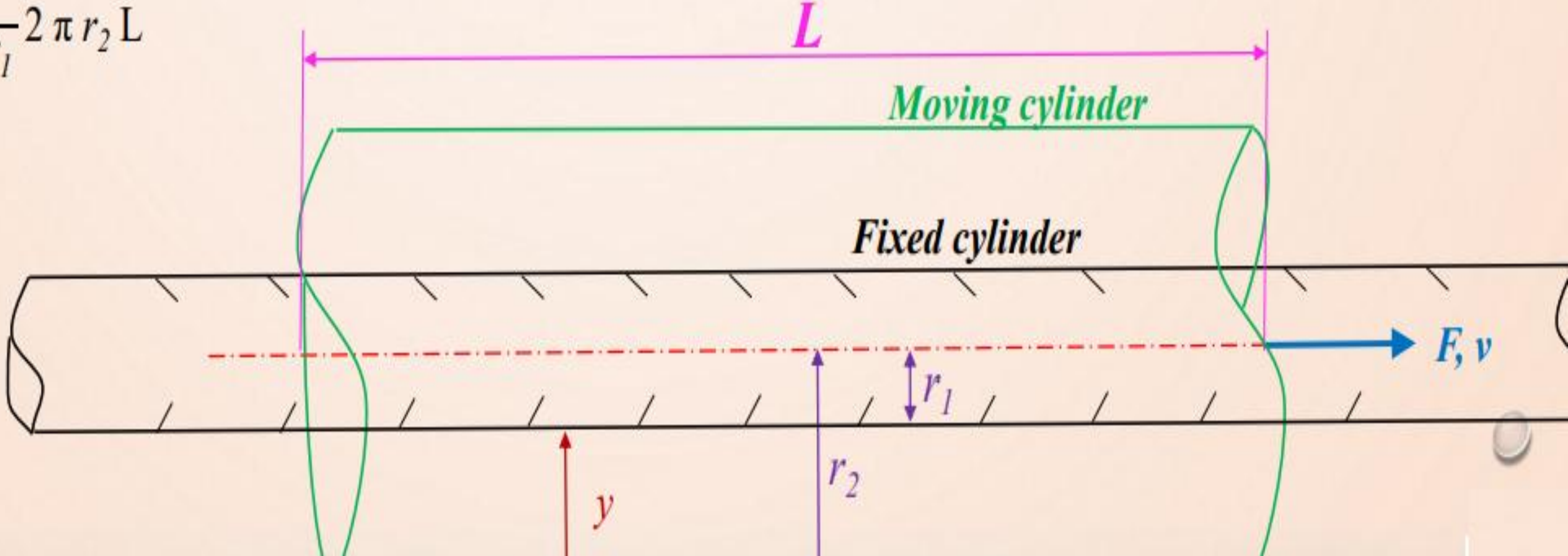
2. Outer cylinder moving with uniform linear velocity while inner cylinder fixed

$$F = \mu \frac{V}{y} A$$

$$A = 2 \pi r_2 L$$

$$y = r_2 - r_1$$

$$F = \mu \frac{V}{r_2 - r_1} 2 \pi r_2 L$$



Two concentric cylinders _ Rotational movement

Example 1.12. *A vertical cylinder of diameter 180 mm rotates concentrically inside another cylinder of diameter 181.2 mm. Both the cylinders are 300 mm high. The space between the cylinders is filled with a liquid whose viscosity is unknown. Determine the viscosity of the fluid if a torque of 20 Nm is required to rotate the inner cylinder at 120 r.p.m.*

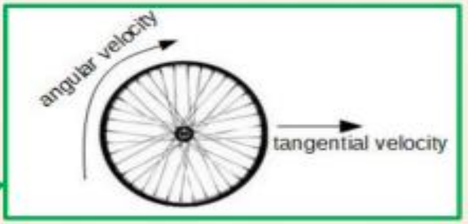
Solution. *Given*: Diameter of inner cylinder, $d = 180 \text{ mm} = 0.18 \text{ m}$

Diameter of outer cylinder, $D = 181.2 \text{ mm} = 0.1812 \text{ m}$

Length of each cylinder, $l = 300 \text{ mm} = 0.3 \text{ m}$

Speed of the inner cylinder, $N = 120 \text{ r.p.m.}$

Torque, $T = 20 \text{ Nm.}$



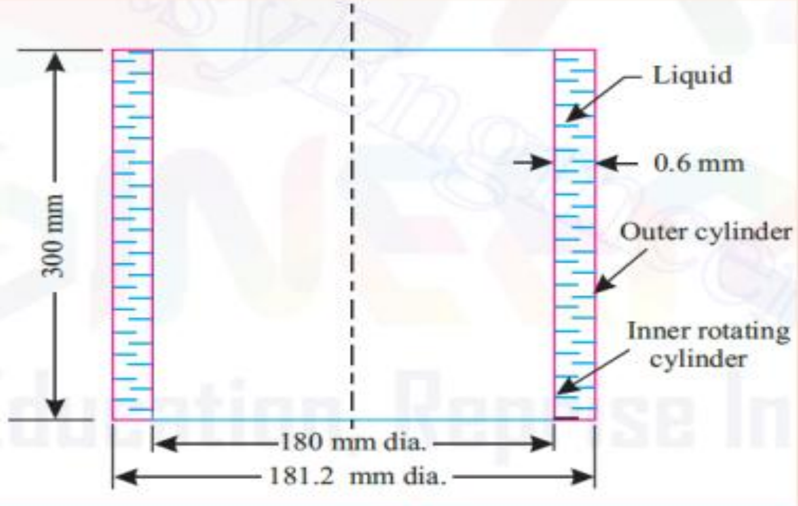


Fig. 1.8

Fig. 1.8

Viscosity of the liquid, μ:

Tangential velocity of the inner cylinder

$$u = \boxed{\frac{\pi d N}{60}} = \frac{\pi \times 0.18 \times 120}{60} = 1.13 \text{ m/s}$$

Surface area of the inner cylinder,

$$A = \pi d l = \pi \times 0.18 \times 0.3$$
$$= 0.1696 \text{ m}^2$$

Using the relation:

$$\tau = \mu . \frac{du}{dy}$$

Hint

$\omega = 2\pi N/60$ (Where N is rpm)

v *or* $u = r\,\omega$ (Where **r** is radius)

or can write it as the following:

$u = \frac{d}{2}\omega($ (Where **d** is diameter) where, $du = u - 0 = 1.13 - 0$

$= 1.13$ m/s

and $dy = \dfrac{0.1812 - 0.180}{2} = 0.0006$ m

$$\tau = \mu \times \frac{1.13}{0.0006} = 1883.33\mu$$

$\therefore$ Shear force, $F = \tau \times A = 1883.33\ \mu \times 0.1696$ N

Torque, $T = \tau \times A \times \dfrac{d}{2}$

$$= 1883.33\ \mu \times 0.1696 \times \frac{0.18}{2}$$

$$20 = 1883.33\ \mu \times 0.1696 \times 0.09$$

$$\mu = \frac{20}{1883.33 \times 0.1696 \times 0.09} = 0.696\ \text{Ns/m}^2$$

μ = **6.96 poise (Ans.)**

Example 1.6. *The velocity distribution for flow over a plate is gives by $u = 2y - y^2$ where u is the velocity in m/s at a distance y metres above the plate. Determine the velocity gradient and shear stress at the boundary and 1.5 m from it.*

Take dynamic viscosity of fluid as 0.9 N.s/m².

Soluton. $u = 2y - y^2$ ...(given) $\quad \therefore \quad \frac{du}{dy} = 2 - 2y$

(*i*) Velocity gradient, $\frac{du}{dy}$:

At the boundary : At $y = 0$, $\left(\frac{du}{dy}\right)_{y=0} = \mathbf{2\ s^{-1}}$ **(Ans.)**

At 0.15 m from the boundary:

$$\text{At } y = 0.15 \text{ m}, \left(\frac{du}{dy}\right)_{y=0.15} = 2 - 2 \times 0.15 = \mathbf{1.7\ s^{-1}}$$ **(Ans.)**

(*ii*) Shear stress, τ:

$$(\tau)_{y=0} = \mu \cdot \left(\frac{du}{dy}\right)_{y=0} = 0.9 \times 2 = \mathbf{1.8\ N/m^2}$$ **(Ans.)**

and,

$$(\tau)_{y=0.15} = \mu \left(\frac{du}{dy}\right)_{y=0.15} = 0.9 \times 1.7 = \mathbf{1.53\ N/m^2}$$ **(Ans.)**

[Where $\mu = 0.9$ N.s/m² ... (given)]

Example 1.7. *A lubricating oil of viscosity μ undergoes steady shear between a fixed lower plate and an upper plate moving at speed V. The clearance between the plates is t. Show that a linear velocity profile results if the fluid does not slip at either plate.*

Solution. For the given geometry and motion, the shear stress τ is constant throughout. From Newton's law of viscosity, we have

$$\frac{du}{dy} = \frac{\tau}{\mu} = \text{constant}$$

or $u = ly + m$

The constantS l and m are evaluated from the no slip conditions at the upper and lower plates.

At $y = 0, \mu = 0 \quad \therefore m = 0$

At $y = t, u = V$

$\therefore \quad V = lt + 0$ or $l = \dfrac{V}{t}$

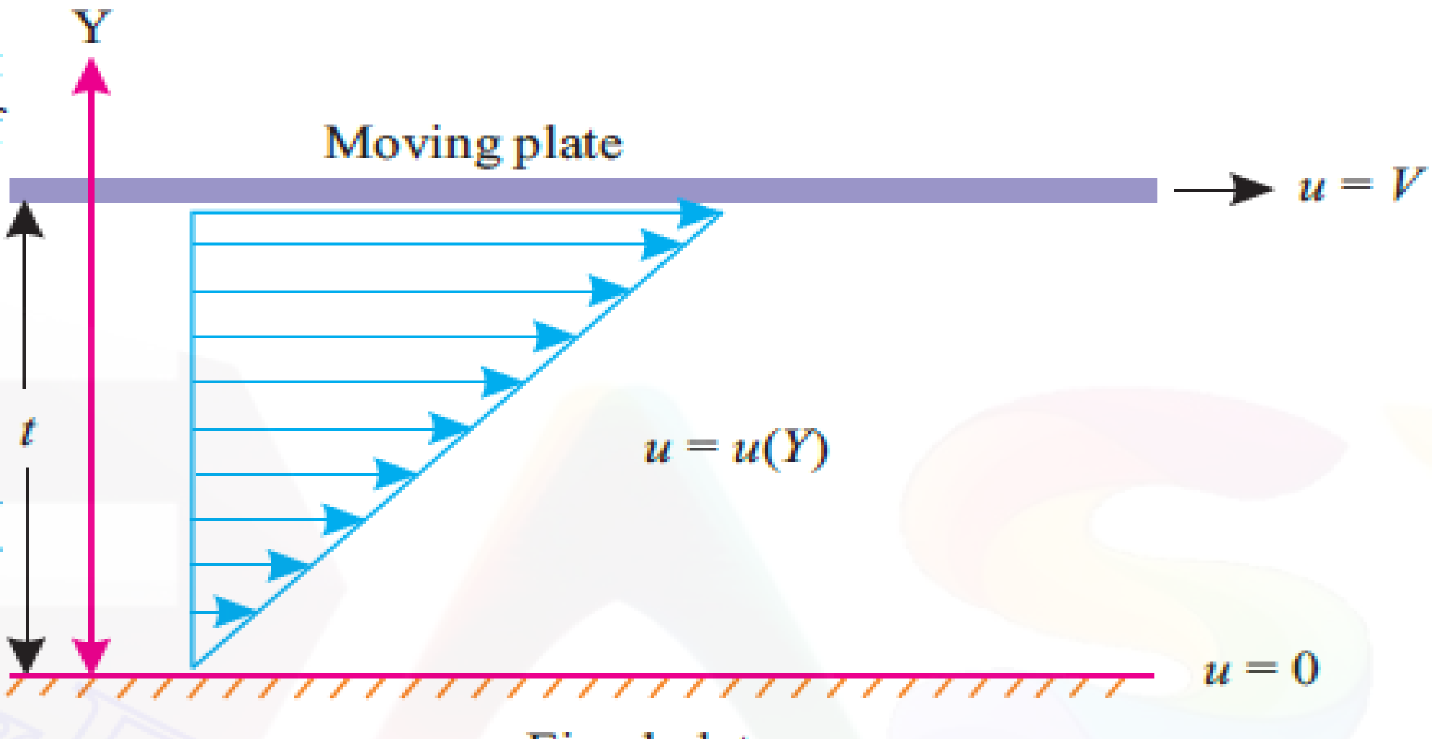


Fig. 1.5

∴ The velocity profile between plates is then given by:

$u = \dfrac{Vy}{t}$ and is *linear* as indicated in Fig 1.5 **(Ans.)**

Example 1.8. *The velocity distribution of flow over a plate is parabolic with vertex 30 cm from the plate, where the velocity is 180 cm/s. If the viscosity of the fluid is 0.9 N.s/m^2 find the velocity gradients and shear stresses at distances of 0, 15 cm and 30 cm from the plate.*

Solution. Distance of the vertex from the plate = 30 cm.

Velocity at vertex, $u = 180$ cm/s

Viscosity of the fluid = 0.9 N.s/m^2

The equation of velocity profile, which is parabolic, is given by

$$u = ly^2 + my + n \qquad ...(1)$$

where l, m and n are constants. The values of these constants are found from the following boundary conditions:

(*i*) At $y = 0$, $u = 0$,

(*ii*) At $y = 30$ cm,

$u = 180$ cm/s and

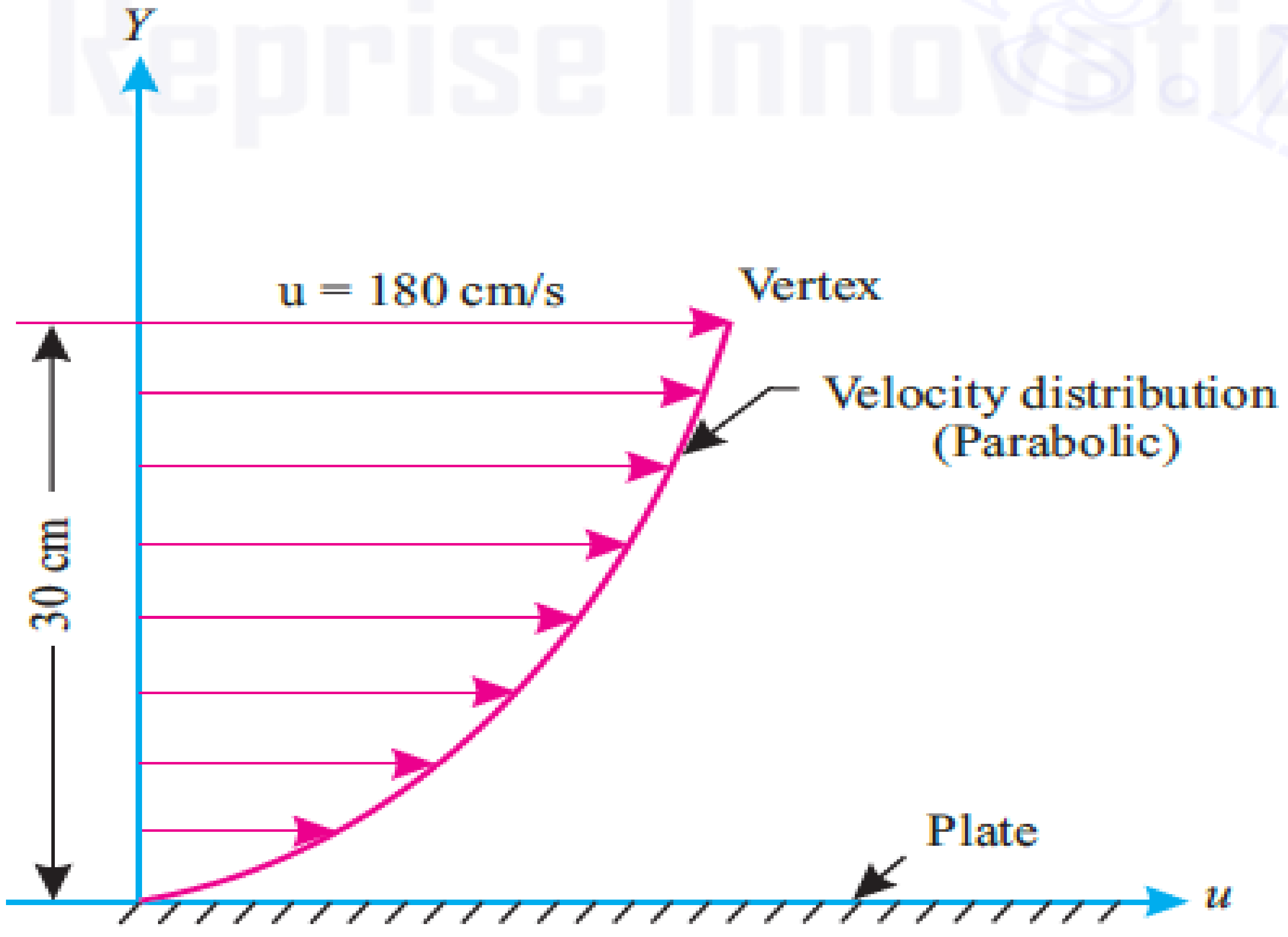


Fig. 1.6

(*iii*) At $y = 30$ cm, $\frac{du}{dy} = 0$.

Substituting boundary conditions (*i*) in eqn. (1), we get

$$0 = 0 + 0 + n \quad \therefore n = 0$$

Substituting boundary conditions (*ii*) in eqn. (1), we get

$$180 = l \times (30)^2 + m \times 30 \quad \text{or} \quad 180 = 900\, l + 30\, m \quad \text{...(2}$$

Substituting boundary conditions (*iii*) in eqn. (1), we get

$$\frac{du}{dy} = 2ly + m \quad \therefore \quad 0 = 2l \times 30 + m \quad \text{or} \quad 0 = 60l + m \quad \text{...(3}$$

Solving eqns. (2) and (3), we have $l = -0.2$ and $m = 12$.

Substituting the values of l, m and n in eqn. (1), we get $u = -0.2\, y^2 + 12y$

Velocity gradients, $\frac{du}{dy}$:

$$\frac{du}{dy} = -0.2 \times 2y + 12 = -0.4y + 12$$

At $y = 0$, $\left(\frac{du}{dy}\right)_{y=0} = \mathbf{12/s}$ **(Ans.)**

At $y = 15$ cm, $\left(\frac{du}{dy}\right)_{y=15} = -0.4 \times 15 + 12 = \mathbf{6/s}$ **(Ans.)**

At $y = 30$ cm, $\left(\frac{du}{dy}\right)_{y=30} = -0.4 \times 30 + 12 = \mathbf{0}$ **(Ans.)**

Shear stresses, τ:

We know, $\tau = \mu \frac{du}{dy}$

At $y = 0$, $(\tau)_{y=0} = \mu \cdot \left(\frac{du}{dy}\right)_{y=0} = 0.9 \times 12 = \mathbf{10.8\ N/m^2}$ **(Ans.)**

At $y = 15$, $(\tau)_{y=15} = \mu \cdot \left(\frac{du}{dy}\right)_{y=15} = 0.9 \times 6 = \mathbf{5.4\ N/m^2}$ **(Ans.)**

At $y = 30$, $(\tau)_{y=30} = \mu \cdot \left(\frac{du}{dy}\right)_{y=30} = 0.9 \times 0 = \mathbf{0}$ **(Ans.)**

- **Surface tension** is caused by the force of cohesion at the free surface.

  At liquid–air interfaces, surface tension results from the greater attraction of liquid molecules to each other (due to cohesion) than to the molecules in the air (due to adhesion).

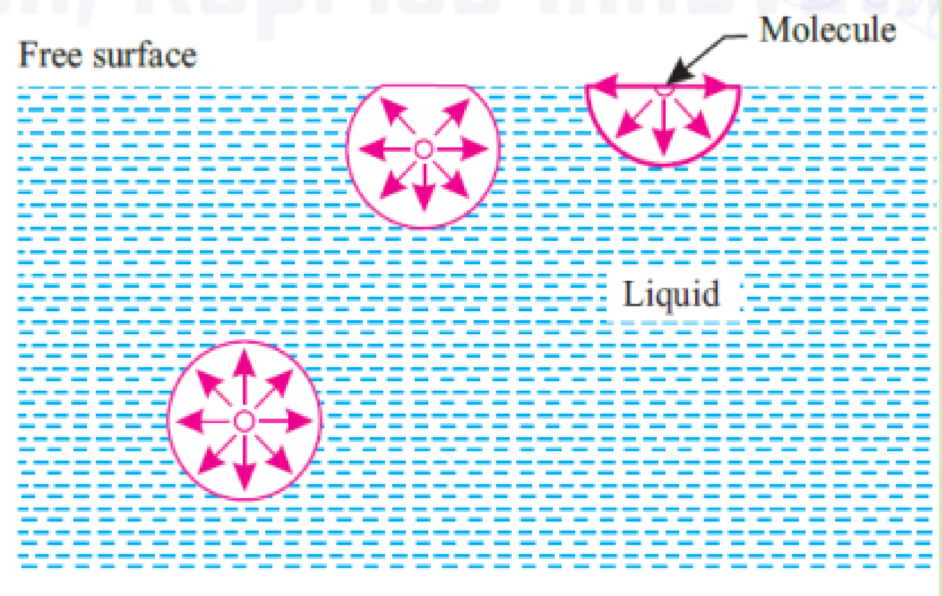
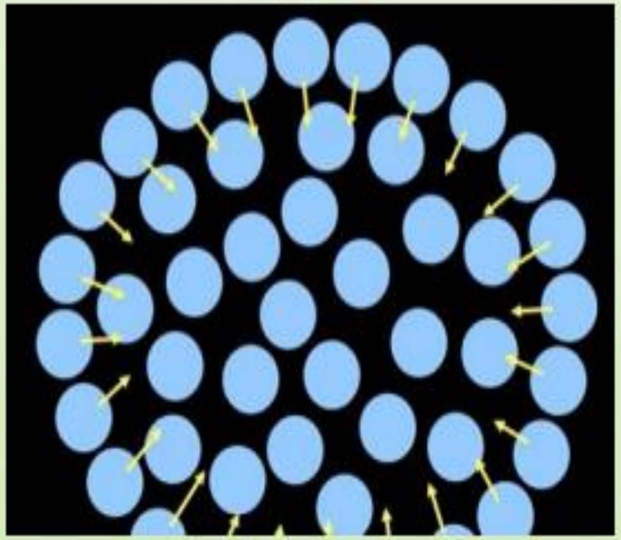
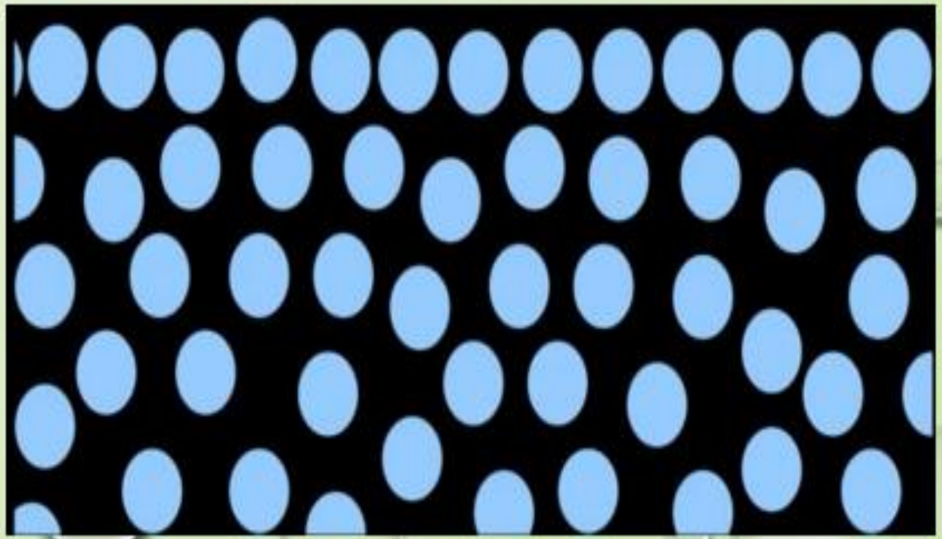


Fig. 1.18





Pressure Inside a Water Droplet, Soap Bubble and a Liquid Jet

Case I. Water droplet:

Let, p = Pressure inside the droplet above outside pressure (*i.e.*, $\Delta p = p - 0 = p$ above atmospheric pressure)

d = Diameter of the droplet and

σ = Surface tension of the liquid. $\frac{F}{L}$

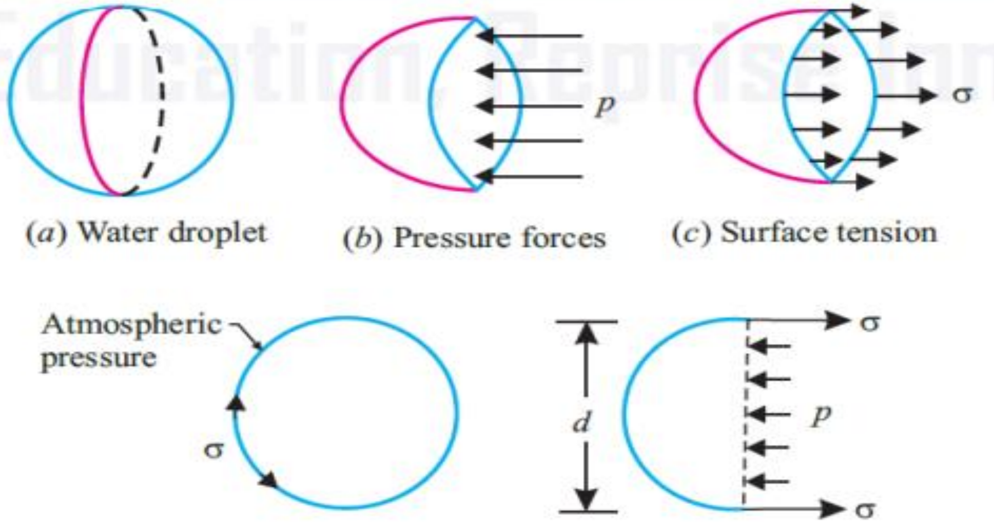
From free body diagram (Fig. 1.19 *d*), we have:

(*i*) Pressure force $= p \times \frac{\pi}{4} d^2$, and

(*ii*) Surface tension force acting around the circumference $= \sigma \times \pi d$.

Under equilibrium conditions these two forces will be equal and opposite,

i.e., $$p \times \frac{\pi}{4} d^2 = \sigma \times \pi d$$



(*a*) Water droplet (*b*) Pressure forces (*c*) Surface tension

$$\therefore \qquad \boxed{p} = \frac{\sigma \times \pi d}{\frac{\pi}{4} d^2} = \boxed{\frac{4\sigma}{d}}$$

The equation above shows that, $\mathbf{P \propto \frac{1}{d}}$

Case II. Soap (or hollow) bubble:

Soap bubbles have two surfaces on which surface tension σ acts.

From the free body diagram (Fig. 1.20), we have

$$p \times \frac{\overleftarrow{\pi}}{4} d^2 = 2 \times (\vec{\sigma} \times \pi d)$$

$$\therefore \qquad \boxed{p} = \frac{2\sigma \times \pi d}{\frac{\pi}{4} d^2} = \boxed{\frac{8\sigma}{d}} \qquad ...(1.18)$$

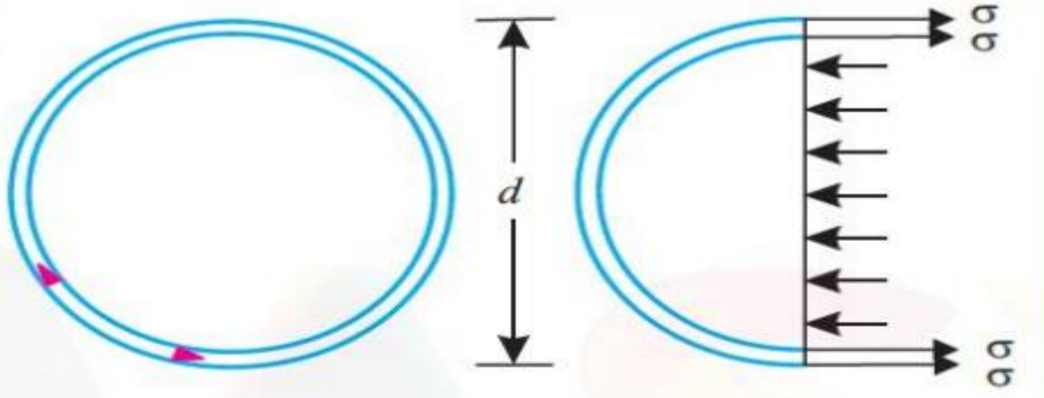
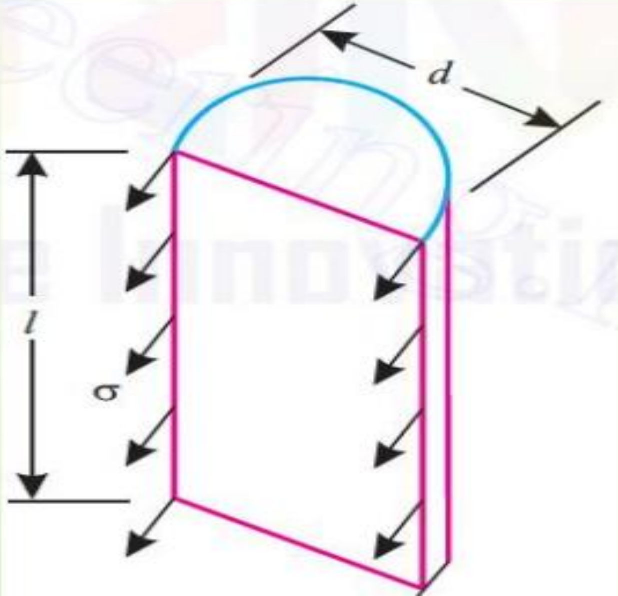


Fig. 1.20. Pressure inside a soap bubble.

Case III. A Liquid jet:

Let us consider a cylindrical liquid jet of diameter d and length l.

Fig. 1.21 shows a semi-jet.



Pressure force $= p \times l \times d$

Surface tension force $= \sigma \times 2l$

Equating the two forces, we have:

$$p \times l \times d = \sigma \times 2l$$

$$\boxed{p} = \frac{\sigma \times 2l}{l \times d} = \boxed{\frac{2\sigma}{d}}$$

Example 1.25. *A soap bubble 62.5 mm diameter has an internal pressure in excess of the outside pressure of 20 N/m². What is tension in the soap film?*

Solution. *Given*: Diameter of the bubble, $d = 62.5 \text{ mm} = 62.5 \times 10^{-3}$ m;

Internal pressure in excess of the outside pressure, $p = 20 \text{ N/m}^2$.

Surface tension, σ:

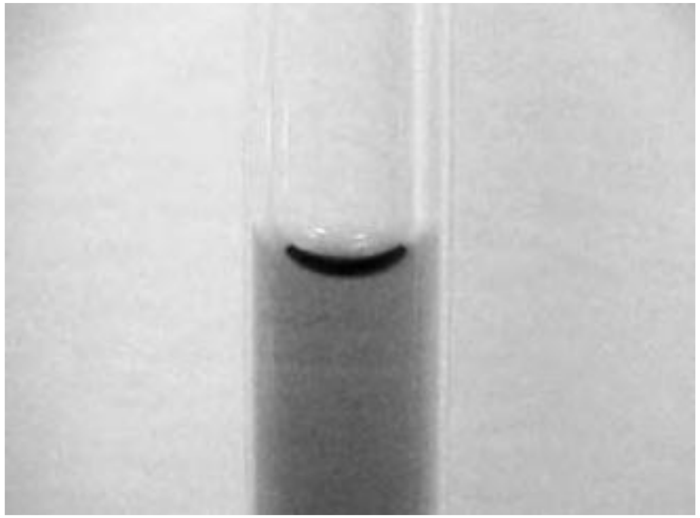
Using the relation, $$p = \frac{8\sigma}{d}$$

i.e., $$20 = \frac{8\sigma}{62.5 \times 10^{-3}}$$

$$\therefore \quad \sigma = 20 \times \frac{62.5 \times 10^{-3}}{8} = \mathbf{0.156 \text{ N/m}}$$

4- Capillary effect

The rise or fall of a liquid in a small-diameter tube inserted into the liquid called capillary effect. Such narrow tubes or confined flow channels are called capillaries



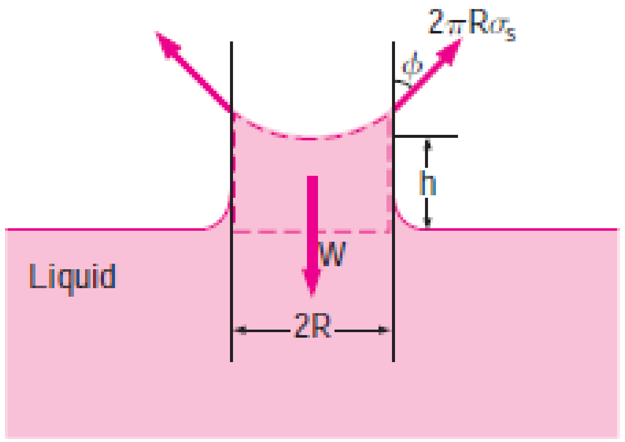
The phenomenon of capillary effect can be explained microscopically by considering cohesive forces (the forces between like molecules, such as water and water) and adhesive forces (the forces between unlike molecules, such as water and glass).

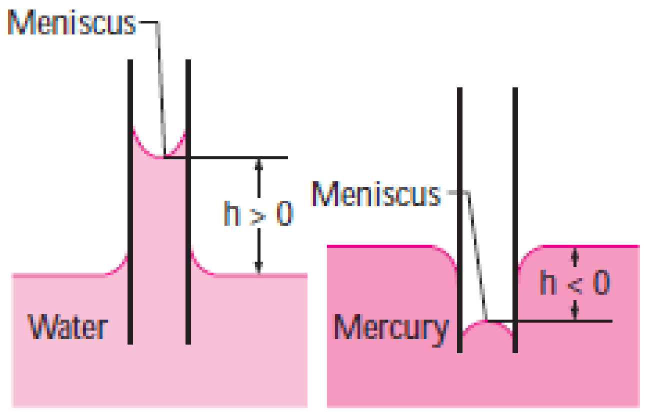
The water molecules are more strongly attracted to the glass molecules than they are to other water molecules, and thus water tends to rise along the glass surface. The opposite occurs for mercury, which causes the liquid surface near the glass wall to be suppressed

W = mg = ρ Vg = ρ g(πR^2h)

Equating the vertical component of the surface tension force to the weight gives

- $W = Fsurface \longrightarrow \rho g(\pi R^2 h) = 2\ \pi R\, \sigma_s\ \cos\varphi,\ h = \dfrac{2\sigma_s}{\rho g R}\cos\varphi,$





Example:

A 0.6-mm-diameter glass tube is inserted into water at 20°C in a cup. Determine the capillary rise of water in the tube

SOLUTION The rise of water in a slender tube as a result of the capillary effect is to be determined.

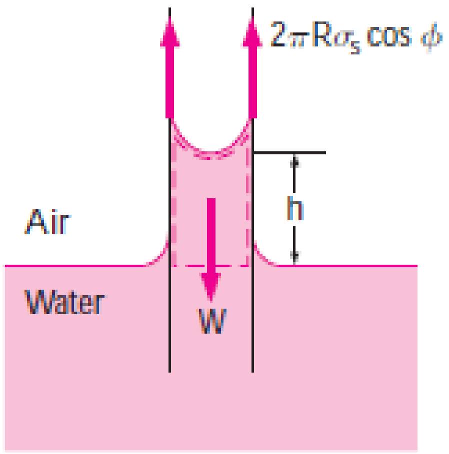
Assumptions 1 There are no impurities in the water and no contamination on the surfaces of the glass tube. 2 The experiment is conducted in atmospheric air.

Properties The surface tension of water at 20°C is 0.073 N/m (Table 2–3). The contact angle of water with glass is 0° (from preceding text). We take the density of liquid water to be 1000 kg/m³.

Analysis The capillary rise is determined directly from Eq. 2–15 by substituting the given values, yielding

$$h = \frac{2\sigma_s}{\rho g R}\cos\phi = \frac{2(0.073\ \text{N/m})}{(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.3\times 10^{-3}\text{m})}(\cos 0°)\left(\frac{1\text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right)$$

$$= 0.050\ \text{m} = 5.0\ \text{cm}$$



H.W

1-A tank contains 500 kg of a liquid whose specific gravity is 2. Determine the volume of the liquid in the tank.

2-Determine the mass of air in a 2 m^3 tank if the air is at room temperature, 20 °C, and the absolute pressure within the tank is 200 kP.

Submission due date: Next lecture

3- In Figure below if the fluid is glycerin at 20°C and the width between plates is 6 mm, what shear stress (in Pa) is required to move the upper plate at V = 5.5 m/s? What is the flow Reynolds number if "L" is taken to be the distance between plates?

Hint: Viscosity=1.5 Pa.s,density=1264kg/m^3

Ans. Shear stress=1.38K Pa

Reynolds's number=28