



Lecture 1

STRENGTH OF MATERIALS

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2022-2023

Strength of Materials

Objective of the Course:

- 1) To know different types of the stresses which may subjected to the mechanical elements and their expected effects such as strain.
- 2) To study the shear forces and bending moment diagrams with essential stresses

Syllabus of the First Semester:

Week	Subject
1 st - 2 nd	Simple stress
3 rd - 4 th	Shearing stress, Bearing stress
5 th - 6 th	Thin wall cylinders
7 th - 8 th	Simple strain, stress-strain diagram, Hook's law
9 th - 10 th	Thermal stress
11 th - 12 th	Welded connection
13 th - 14 th	Riveted joints
15 th - 16 th	Torsion

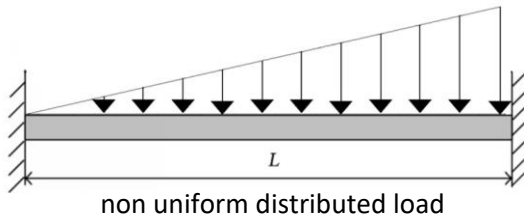
Types of Loads :



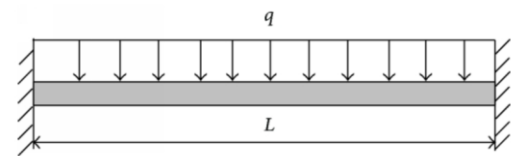
Cantilever



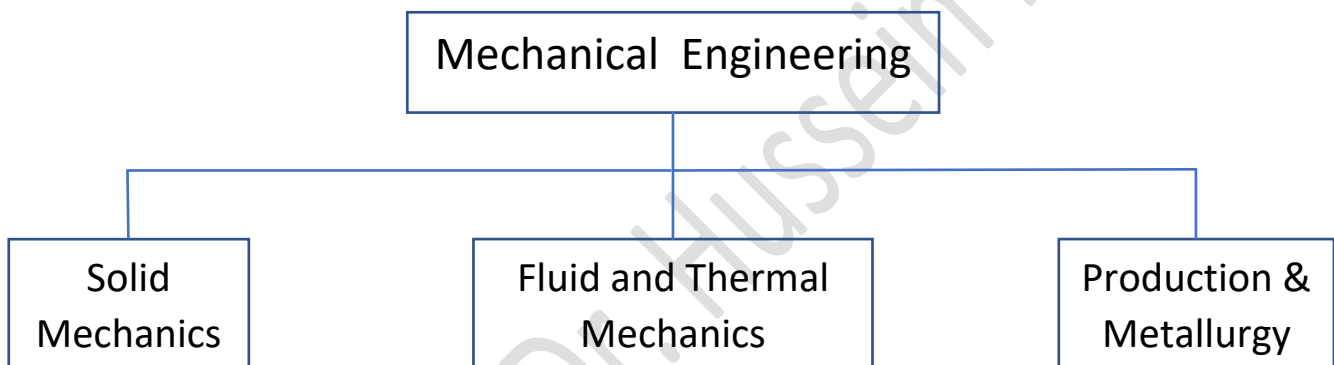
Simple supported beam



non uniform distributed load



uniform distributed load



It is a branch of applied mechanics that deal with behaviors of solid bodies subjected to various loading

- Strength of material or, mechanic of material or, mechanic of solid

❖ In strength of material (S.O.M) it is assuming:

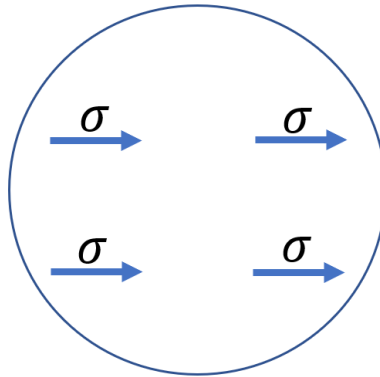
- 1) The body is deformable
- 2) Both the internal and external force are considered

Mechanical properties:

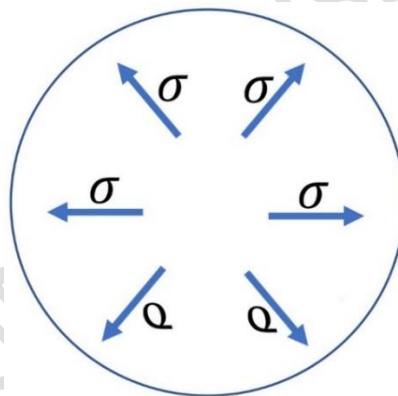
- 1) Strength: it is a material property which refer to the resistance of the material to the failure
- 2) Stiffness: it is a material property refer to the resistance of the material to the deformation

Assumptions:

- 1) The material is continuous (no voids, no cracks) which means there are no defects
- 2) The material is homogenous (it means at any point in one direction, the properties are the same) example: wood, steel, gold



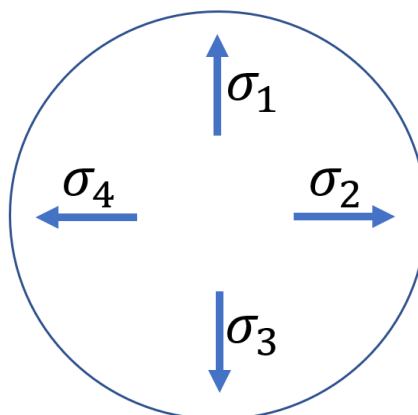
- 3) The material is isotropic (it means one point in any direction; the properties are the same) example: graphite



Note: orthotropic (anisotropic) material (directional properties)

في كل الاتجاهات الخواص الميكانيكية تختلف

Example: composite material



- ❖ Load: it is the external force acting on the body
- ❖ Stress: the force of resistance per unit area offered by a body against deformation

i.e. the load is applied on the body while the stress is induced in material of the body

Define stress and strain

1) **Stress:** it is a measure of : $\frac{\text{Applied force}}{\text{Area over which that force is applied}}$

$$\text{Normal stress } (\sigma) = \frac{F}{A}$$

- ❖ Tensile stress: it the applied force is tensile force

$$\sigma = +ve$$

- ❖ compressive stress: it the applied force is compression force

$$\sigma = -ve$$

$$\text{unit of stress } \frac{N}{m^2}, \frac{Kg}{cm^2}$$

where:

$$\sigma = \text{stress in } \frac{N}{m^2}, \frac{N}{mm^2}, \frac{Kg}{cm^2}$$

F = Applied force

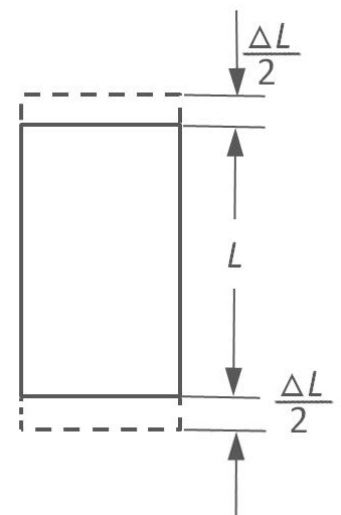
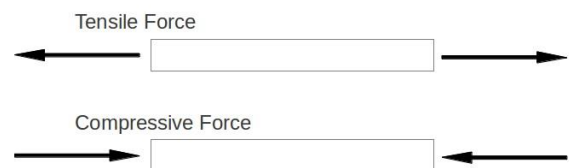
A = the cross-section area normal to the applied force

2) **Strain:** it is a measure of : $\frac{\text{Elongation of material}}{\text{The original length}}$

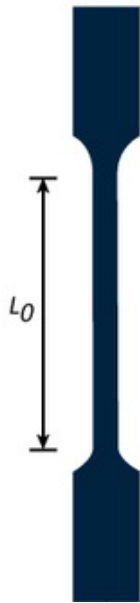
$$\text{Normal strain } (\epsilon) = \frac{\Delta L}{L} = \frac{L_2 - L_1}{L_1} = \frac{\delta l}{l} \times 100\%$$

$$\delta l = \frac{\delta l}{2} + \frac{\delta l}{2} \quad \text{or} \quad \frac{\Delta L}{L_0}$$

Units: dimensionless

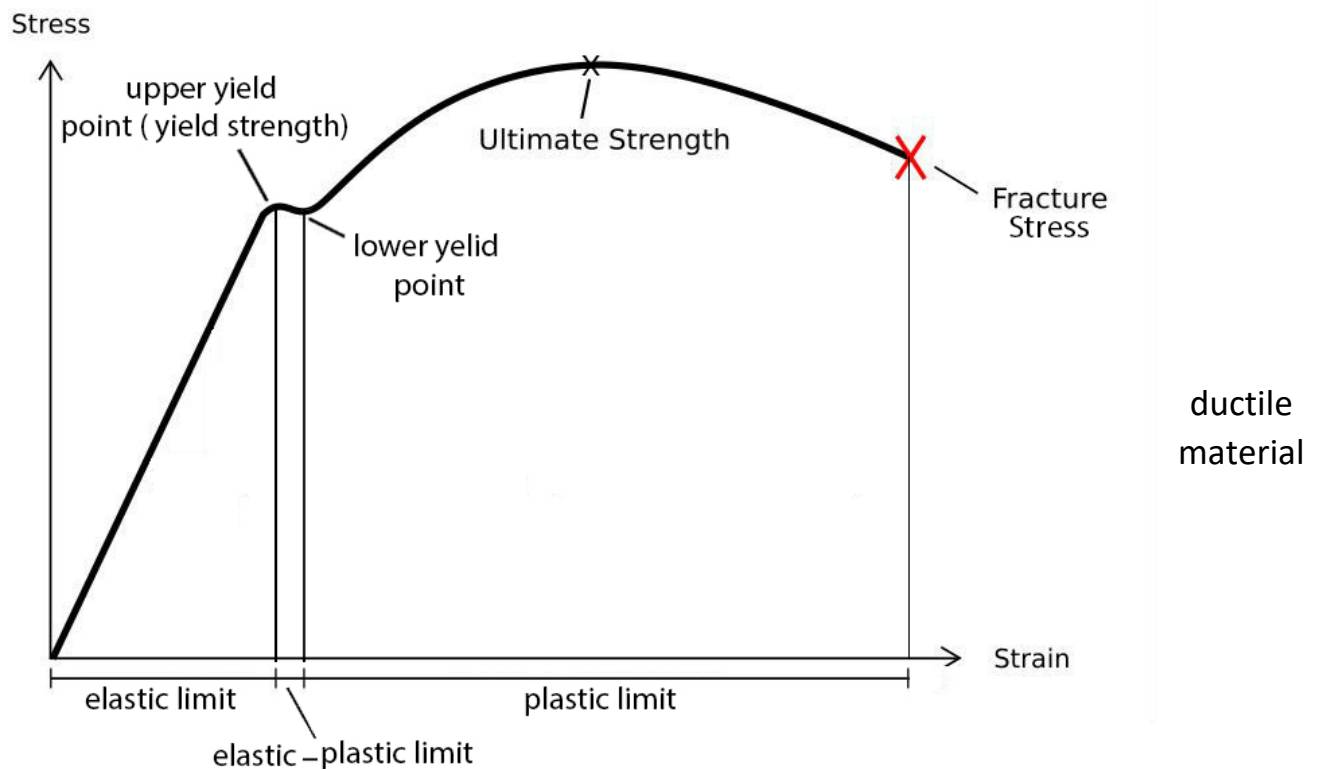


The relationship between stress and strain



Stranded specimen

عينة قياسية يتم تحضيرها لكل مادة لغرض اجراء فحص الشد (Tensile test) باستخدام جهاز فحص الشد



Typical stress-strain curve for steel

- ❖ Hook's law defines the relationship between the stress and strain where:
stress (σ) $\propto$ strain (ε)
- ❖ the stress is Proportional to the strain within the elastic limit

mathematically:

$$\sigma \propto \varepsilon$$

$$\sigma = E \cdot \varepsilon$$

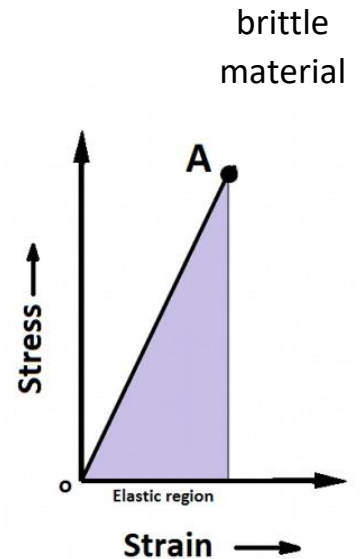
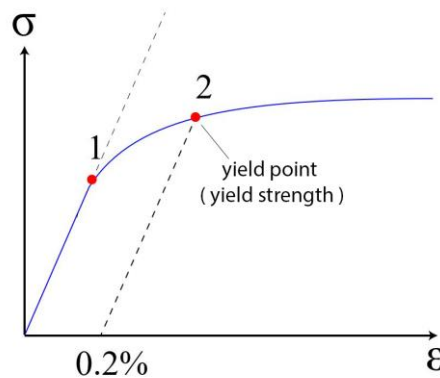
$$\therefore E = \frac{\sigma}{\varepsilon}$$

Where:

σ = Normal stress

ε = Normal strain

E = modulus of elasticity



For the material which has no yield point in the stress-strain curve, offset method is used



Lecture 2

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$$(\sigma) = \frac{F}{A} = \frac{P}{A} \dots\dots\dots (1)$$

$$(\varepsilon) = \frac{\Delta L}{L} = \frac{\delta l}{l} \dots\dots\dots (2)$$

$$E = \frac{\sigma}{\varepsilon} \dots\dots\dots (3)$$

If we substitute equation 1 & 2 in equation 3, we get

$$\delta l = \frac{P.L}{A.E} \dots\dots\dots (4)$$

Ex1: A hollow steel tube with an inside diameter of 100 mm must carry a tensile load of 400 kN. Determine the outside diameter of the tube if the stress is limited to 120 MN/m²

Sol:

$$P = \sigma A$$

where:

$$P = 400 \text{ kN} = 400\,000 \text{ N}$$

$$\sigma = 120 \text{ MPa}$$

$$A = \frac{1}{4}\pi D^2 - \frac{1}{4}\pi(100^2)$$

$$A = \frac{1}{4}\pi(D^2 - 10\,000)$$

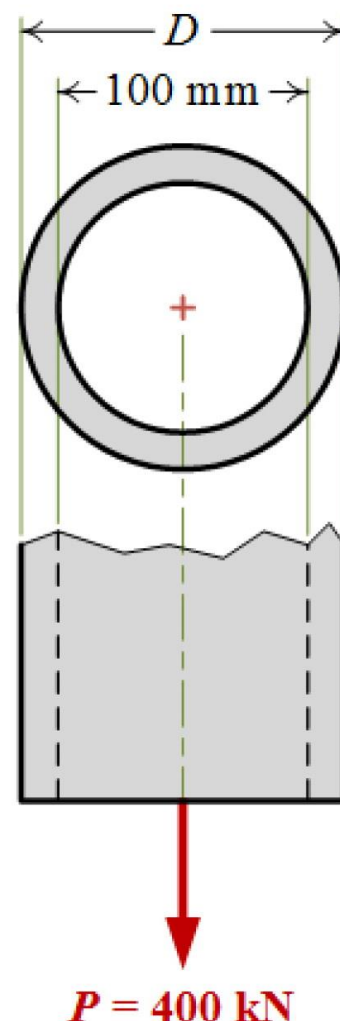
Thus,

$$400\,000 = 120 \left[\frac{1}{4}\pi(D^2 - 10\,000) \right]$$

$$400\,000 = 30\pi D^2 - 300\,000\pi$$

$$D^2 = \frac{400\,000 + 300\,000\pi}{30\pi}$$

$$D = 119.35 \text{ mm} \quad \text{answer}$$



Ex2: A spring 4mm in diameter has original length of 2m. The spring is pulled by force 200 N if the final length of the spring is 2.02 m. Determine

- 1) Stress
- 2) Strain
- 3) Young's modulus or modulus of elasticity

Sol:

Known :

$$\text{Diameter (d)} = 4 \text{ mm} = 0.004 \text{ m}$$

$$\text{Radius (r)} = 2 \text{ mm} = 0.002 \text{ m}$$

$$\text{Area (A)} = \pi r^2 = (3.14)(0.002 \text{ m})^2$$

$$\text{Area (A)} = 0.00001256 \text{ m}^2 = 12.56 \times 10^{-6} \text{ m}^2$$

$$\text{Force (F)} = 200 \text{ N}$$

$$\text{Original length of spring (l}_0\text{)} = 2 \text{ m}$$

$$\text{The change in length } (\Delta l) = 2.02 - 2 = 0.02 \text{ m}$$

Solution :

(a) The stress

$$\text{Stress} = \frac{\text{Force (F)}}{\text{Area (A)}}$$

$$\text{Stress} = \frac{200 \text{ N}}{12.56 \times 10^{-6} \text{ m}^2}$$

$$\text{Stress} = 15.92 \times 10^6 \text{ N/m}^2$$

(b) The strain

$$\text{Strain} = \frac{\text{The change in length } (\Delta l)}{\text{Original length (l}_0\text{)}}$$

$$\text{Strain} = \frac{0.02 \text{ m}}{2 \text{ m}}$$

$$\text{Strain} = 0.01$$

(c) Young's modulus

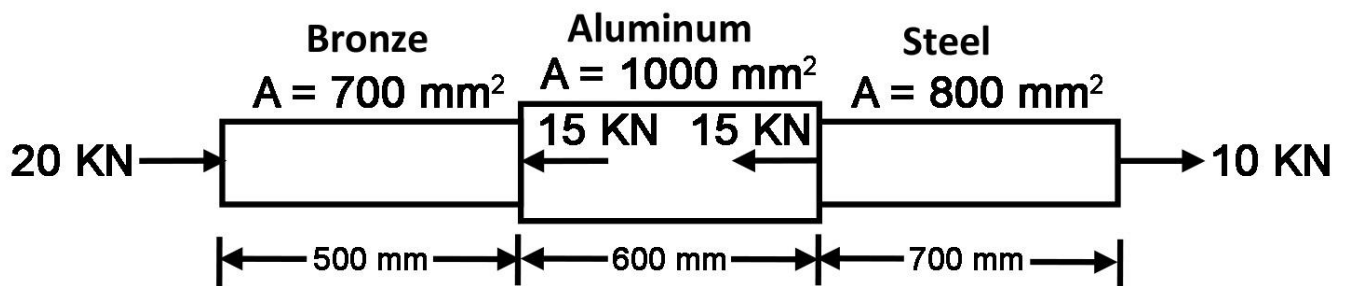
$$\text{Young's modulus} = \frac{\text{Stress}}{\text{Strain}}$$

$$\text{Young's modulus} = \frac{15.92 \times 10^6 \text{ N/m}^2}{0.01}$$

$$\text{Young's modulus} = 1592 \times 10^6 \text{ N/m}^2$$

$$\text{Young's modulus} = 1.6 \times 10^9 \text{ N/m}^2$$

Ex3: An aluminum tube is rigidly fastened between a bronze rod and a steel rod as shown in the fig. below. Axial loads are applied at the positions indicated. Determine the stress in each material.



Sol:

$$(\sigma_B) = \frac{F_B}{A_B}$$

$$(\sigma_A) = \frac{F_A}{A_A}$$

$$(\sigma_S) = \frac{F_S}{A_S}$$

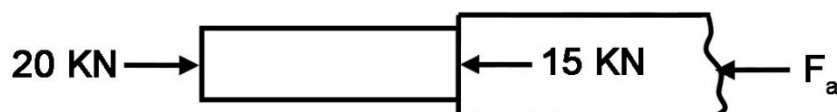
To know the force of applied on each material, we must draw the free body diagram (F.B.D) for each material.

1. F.B.D for Bronze tube



$$F_b = 20 \text{ KN}$$

2. F.B.D for Aluminum tube



$$F_a = 5 \text{ KN}$$

3. F.B.D for Steel tube



$$F_s = 10 \text{ kN}$$

$$\therefore (\sigma_B) = \frac{20 \times 10^3 \text{ N}}{700 \times 10^{-6} \text{ m}^2} = 28.6 \times 10^6 \text{ N/m}^2 = 28.6 \text{ MPa}$$

$$(\sigma_A) = \frac{5 \times 10^3 \text{ N}}{1000 \times 10^{-6} \text{ m}^2} = 5 \times 10^6 \text{ N/m}^2 = 5 \text{ MPa}$$

$$(\sigma_S) = \frac{10 \times 10^3 \text{ N}}{800 \times 10^{-6} \text{ m}^2} = 12.5 \times 10^6 \text{ N/m}^2 = 12.5 \text{ MPa}$$

Factor of safety (F.O.S) or Safety Factor (S.F)

Factor of safety: - is refers to how much the part will stand

$$F.O.S = \frac{\text{yield stress or ultimate stress or maximum stress}}{\text{working stress (allowable stress)}}$$

$$F.O.S = \frac{\sigma_{max}}{\sigma_{working}(\sigma_{allowable})} \dots \dots \dots (5)$$

$$\sigma_{working}(\sigma_{allowable}) = \frac{\sigma_{maximum}}{F.O.S} \dots \dots \dots (*)$$

إذا لم يتطرق في السؤال الى F.O.S نعتبره 1 وبالتالي يعتبر $\sigma_{working} = \sigma_{maximum}$

أما إذا اعطانا في السؤال قيمة F.O.S فسوف نطبق معادلة (*)



Lecture 3

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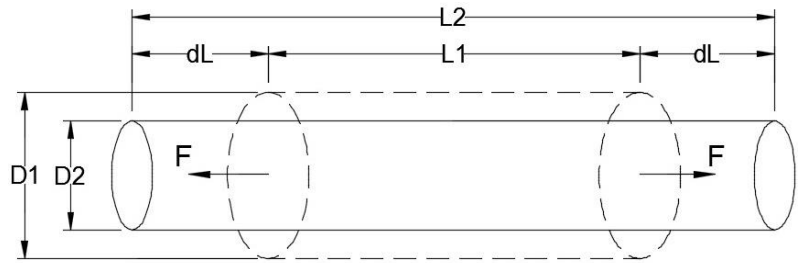
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$$\delta l = \Delta L = L_2 - L_1$$

$$(\epsilon) = \frac{\Delta L}{L} = \frac{\delta l}{l}$$

Elongational strain: -

$$(\epsilon_{lateral \text{ or } transverse}) = \frac{\delta d}{d} = \frac{-(D_2 - D_1)}{D_1}$$



Poisson's Ratio (μ): - within the elastic limit, the ratio of the lateral strain (transverse strain) (ϵ_{lat}) to the longitudinal strain (ϵ_l) will be always constant for the material. This ratio is termed as Poisson's ratio (μ)

$$\text{Poisson's Ratio } (\mu) = \frac{-\epsilon_{lat}}{\epsilon_l}$$

$$\mu = \frac{\frac{\delta d}{d}}{\frac{\delta l}{l}}$$

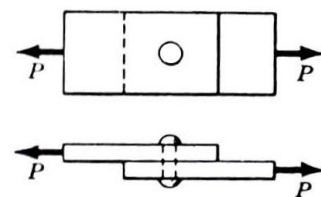
μ its range 0.1 – 0.3

Shear stress (τ)

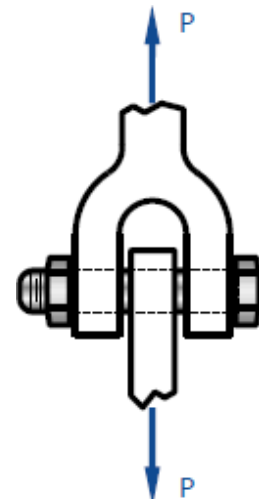
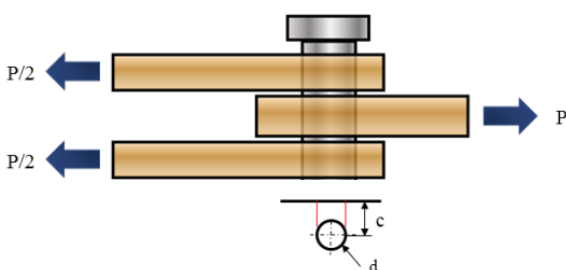
1. Normal stress or simple stress (σ) = $\frac{F}{A}$

2. Shear stress:- its measure of : $\frac{\text{Applied force on a material}}{\text{Area over which that force is applied parallel}}$

$$\tau = \frac{\text{shear force}}{\text{shear Area}} = \frac{V}{A_s} \quad (\text{single shear})$$



$$\tau = \frac{V}{2A_s} = \frac{F}{2\left(\frac{\pi}{4}d^2\right)} \quad (\text{double shear})$$



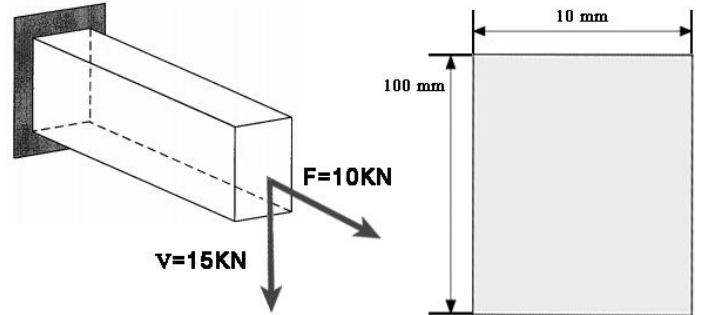
Ex: - for the wooding cantilever shown in the fig. below calculate

- 1) The normal stress due to the 10 KN axial force
- 2) The shear stress due to the 15 KN shear force

Sol:

a)

$$\begin{aligned} \text{Normal stress } (\sigma) &= \frac{F}{A} \\ &= \frac{10 \times 10^3}{100 \times 10} = 10 \text{ N/mm}^2 \end{aligned}$$



b)

$$\text{shear stress } (\tau) = \frac{V}{A_s} = \frac{15 \times 10^3}{100 \times 10} = 15 \text{ N/mm}^2$$

Shear strain (γ)

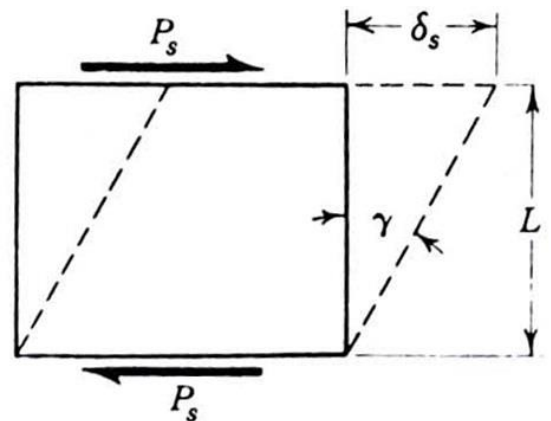
Shearing force cause a shearing deformation

$$\tan \gamma = \frac{\delta_s}{L}$$

Since the angle γ is very small

$\tan \gamma = \gamma$ and we obtain :

$$\gamma = \frac{\delta_s}{L}$$



Where : γ is shear strain which is defined as angular deformation between two perpendicular force of differential element

* عندما تكون الزاوية صغيرة جدا ظلها يساوي الزاوية نفسها

$$\text{Strain } (\varepsilon) = \frac{\delta l}{L}$$

$$\sigma \propto \varepsilon$$

$$\sigma = E \cdot \varepsilon$$

$$\therefore E = \frac{\sigma}{\varepsilon} \text{ within the elastic limit}$$

$$\tau \propto \gamma$$

$$\tau = G \cdot \gamma$$

$$\therefore G = \frac{\tau}{\gamma}$$

Hooke's law is applied in the case of shear stress

Within the elastic limit of the material, the shear stress is proportional to the shear strain

$$\tau \propto \gamma$$

$$\tau = G \cdot \gamma$$

$$\therefore G = \frac{\tau}{\gamma}$$

Where :

$$\tau = \text{shear stress in } \frac{N}{mm^2} \text{ or } \frac{KN}{m^2}$$

$$\gamma = \text{shear strain}$$

$$G = \text{modulus of rigidity}$$



Lecture 4

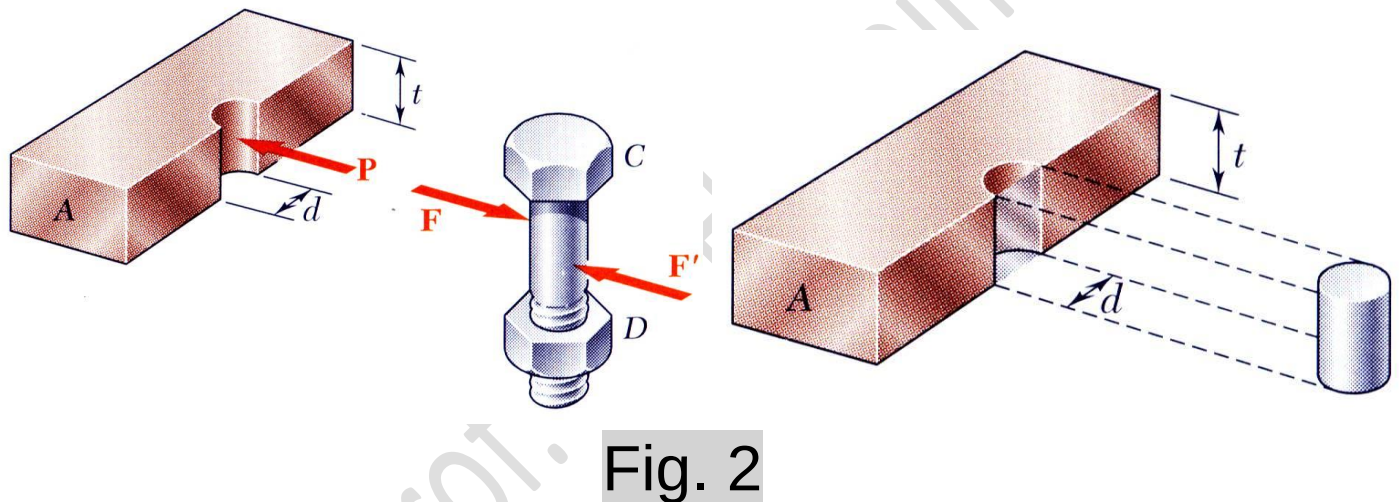
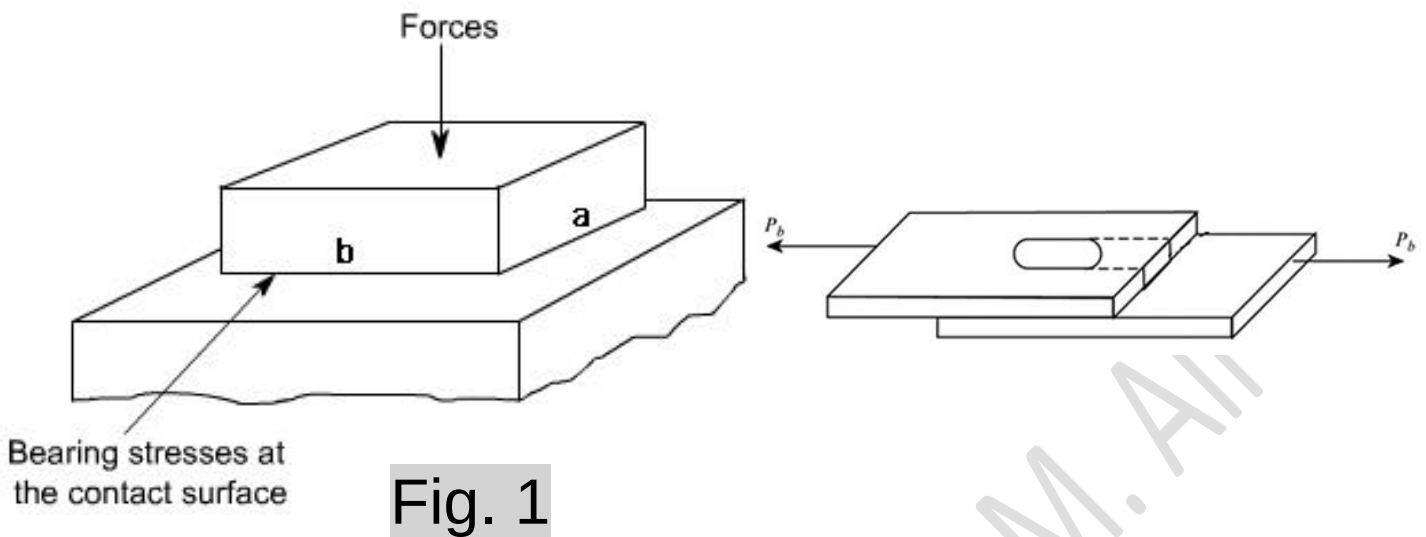
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Bearing stress (σ_b)



It is the contact pressure between the separate bodies. It is different from the compressive stress, as it is an internal stress caused by compressive force.

$$\text{Bearing stress}(\sigma_b) = \frac{F}{A_b}$$

$$A_b = a \times b \quad \text{for fig. 1}$$

$$A_b = d \times t \quad \text{for fig. 2}$$

Where :

a = rivet diameter

b = plate diameter

Thermal stresses (σ_{th})



لا يوجد إجهاد



يوجد إجهاد

when there is some increase or decrease in the temperature of a body, it causes the body expand or contract. If the body is allowed to expand or contract freely with the rise or fall of the temperature, no stresses are induced in the body. But, if the deformation of the body is prevented, some stresses are induced in the body. Such stresses are known as thermal stress

$$\delta l = l \alpha \Delta t$$

Where:

δl = deformation or change length or extension

l = original length of the body

Δt = change in temperature ($t_2 - t_1$)

α = thermal expansion coefficient

$$\delta l = \frac{P \cdot l}{A \cdot E}$$

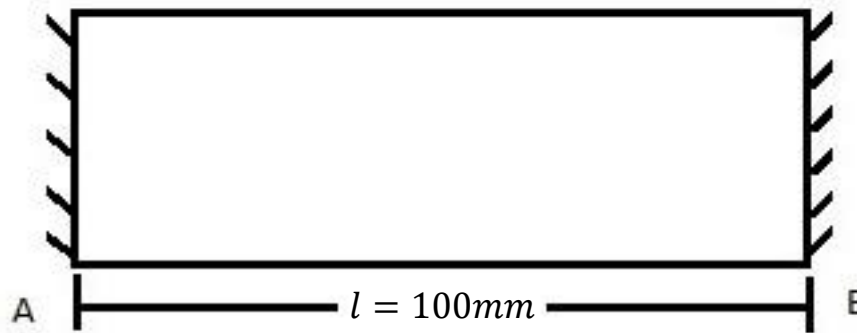
$$\therefore \frac{P \cdot l}{A \cdot E} = l \alpha \Delta t$$

$$P = \alpha \Delta t \cdot A \cdot E$$

$$\sigma_{th} = \frac{P}{A} = \frac{\alpha \Delta t \cdot A \cdot E}{A}$$

$$\therefore \sigma_{th} = \alpha \Delta t \cdot E$$

Ex: - A steel bar is constrained with two end A x B as shown in the figure below. Determine the change in length, thermal stress if the temperature of the bar is raised from 30 °C to 90 °C and the deformation that will happen in the bar. Coefficient of thermal expansion of steel is $11.7\mu\text{m} / \text{m} \cdot ^\circ\text{C}$ and $E = 200 \times 10^9 \text{ N/m}^2$



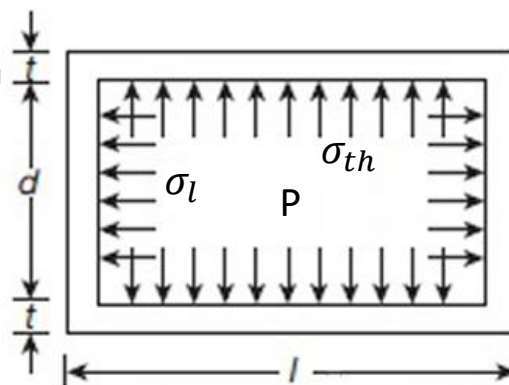
Sol:

$$\delta l = l \alpha \Delta t = 100 \times 10^{-3} \times 11.7 \times 10^{-6} \times (90 - 30) = 7.02 \times 10^{-5} \text{ m}$$

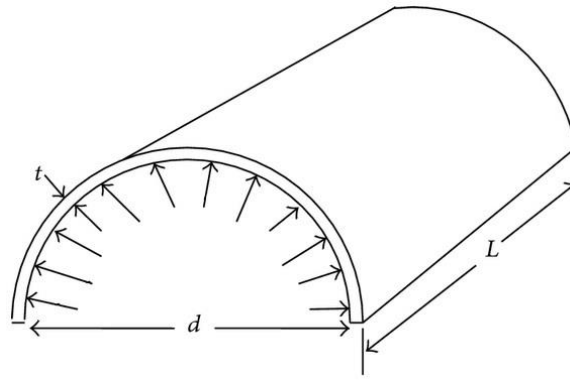
$$\sigma_{th} = \alpha \Delta t \cdot E = 11.7 \times 10^{-6} \times (90 - 30) \times 200 \times 10^9 = 140.4 \times 10^6 \frac{\text{N}}{\text{m}^2}$$

$$\delta l = \text{deformation or change length or extension} = 7.02 \times 10^{-5} \text{ m}$$

Stresses in thin cylindrical shell



A Cylindrical tank or pipe carrying a fluid or gas under pressure is subjected to a tensile force, which resist bursting developed a cross longitudinal and transverse section



1. Longitudinal stress σ_l

$$\sigma_l = \frac{Pd}{4t}$$

2. Hoop or transverse stress

$$\sigma_H = \frac{Pd}{2t}$$

Ex: - A cylindrical steel pressure vessel 400 mm in diameter with a wall thickness of 20 mm is subjected to an internal pressure of 4.5 MN/m²

- 1) calculate the hoop and longitudinal stresses in the steel
- 2) To what value may the internal pressure be increased if the stresses in the steel is limited to 120 MN/m²

Sol:

a)

$$\sigma_l = \frac{Pd}{4t} = \frac{4.5 \times 400}{4 \times 20} = 22.5 \text{ MPa}$$

$$\sigma_H = \frac{Pd}{2t} = \frac{4.5 \times 400}{2 \times 20} = 45 \text{ MPa}$$

b)

$$\sigma_H = \frac{Pd}{2t}$$

$$120 = P \times \frac{400}{2 \times 20}$$

$$\therefore P = 12 \text{ MPa}$$



Lecture 5

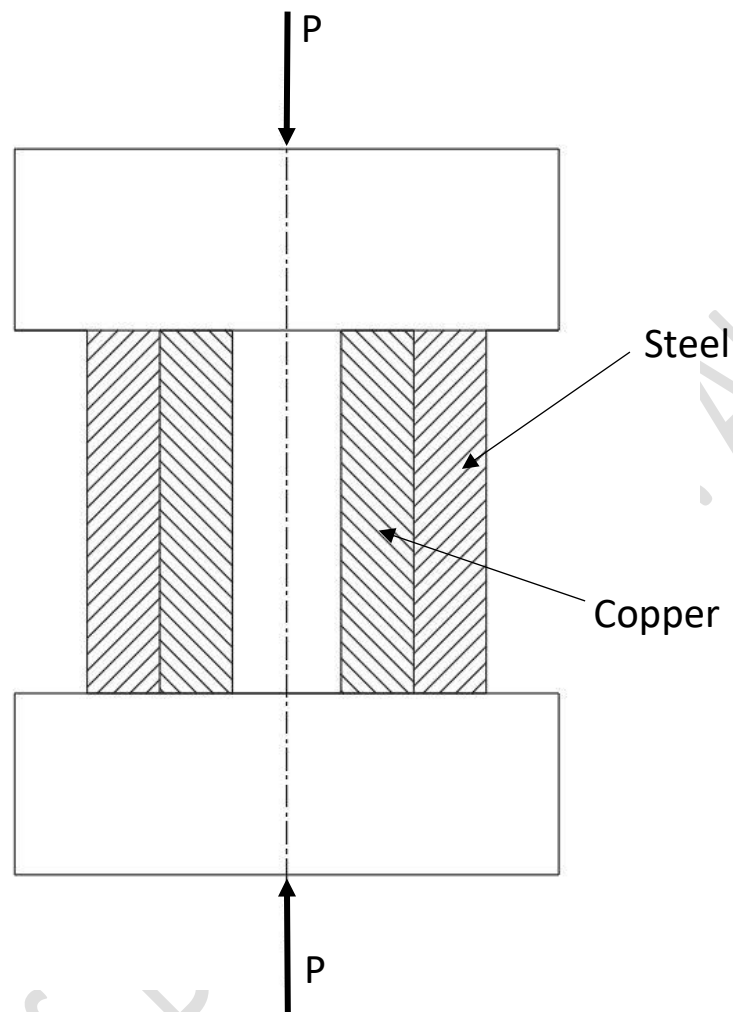
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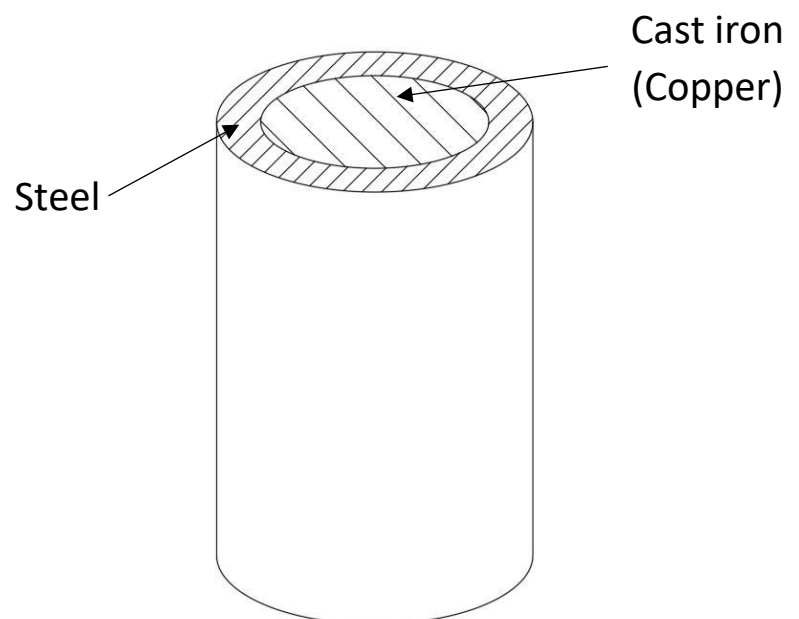
Stress in compound section



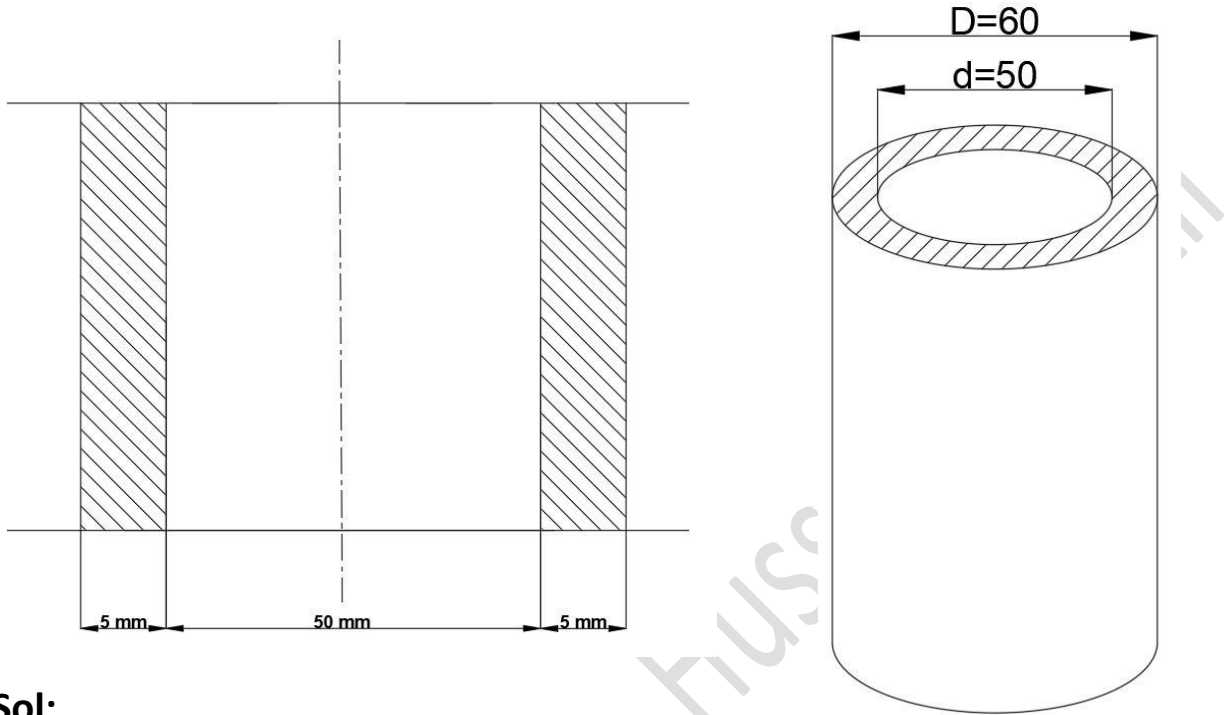
Compound bars are containing two or more materials which can be regarded as one unit

$$\epsilon_{st} = \epsilon_{C.I}$$

$$P = P_{st} + P_{C.I}$$



Ex: - A compound bar of length 2 m is subjected to axial load P. The bar is formed cast iron and steel. The steel diameter is 50 mm and thickness for the cast iron is 5mm. The deflection in both materials is 1 mm. Determine the axial load P if the modules of elasticity for steel $200 \times 10^9 \text{ N/m}^2$ and for cast iron is $100 \times 10^9 \text{ N/m}^2$



Sol:

$$\delta l = \frac{P \cdot L}{A \cdot E}$$

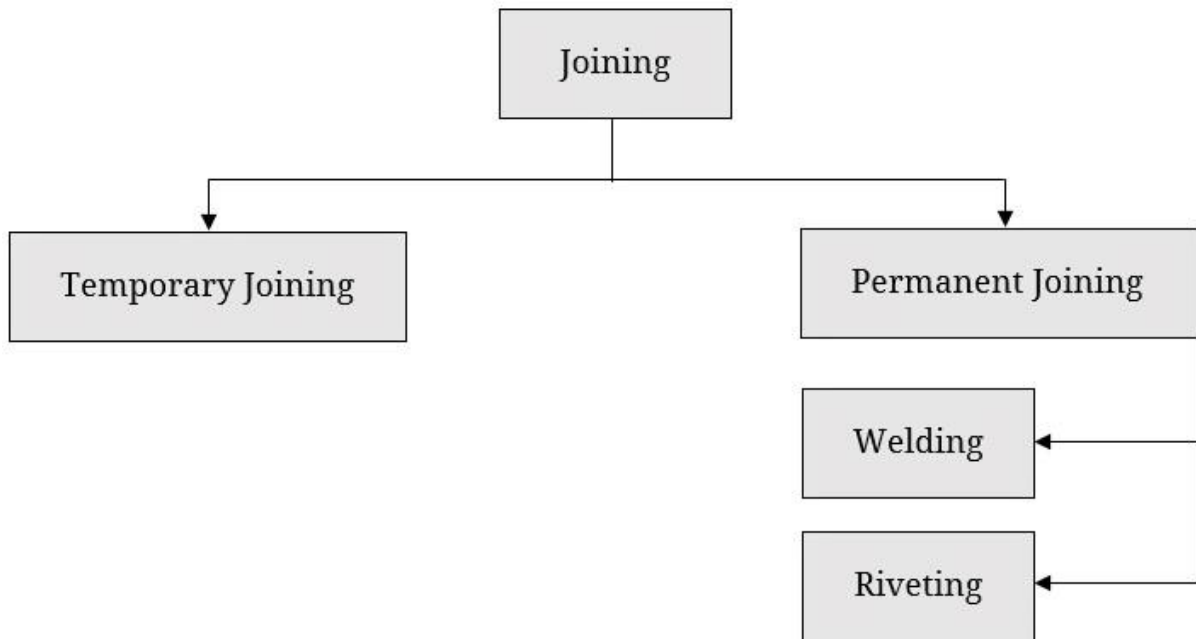
$$\delta l_{st} = \frac{P_{st} \cdot L_{st}}{A_{st} \cdot E_{st}}$$

$$P_{st} = \frac{\delta l_{st} \cdot A_{st} \cdot E_{st}}{L_{st}}$$

$$P_{st} = \frac{1 \times 10^{-3} \times \frac{\pi}{4} \times \left(\frac{50}{1000}\right)^2 \times 200 \times 10^9}{2} = 196.3 \text{ KN}$$

$$P_{C.I} = \frac{\delta l_{C.I} \cdot A_{C.I} \cdot E_{C.I}}{L_{C.I}}$$

$$P_{C.I} = \frac{1 \times 10^{-3} \times \frac{\pi}{4} \times \left(\frac{60 - 50}{1000}\right)^2 \times 100 \times 10^9}{2} = 3.9 \text{ KN}$$



Welding connections

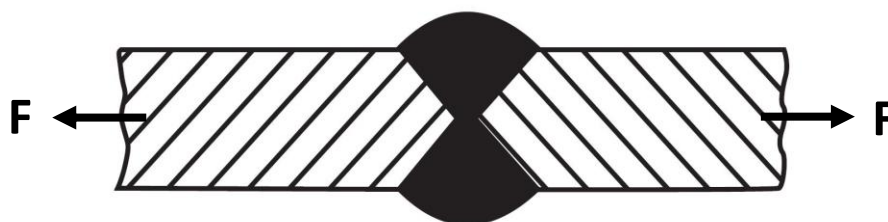
Welding

Welding is a method of joining metals by fusion with heat from either an electric arc or on oxyacetylene touch. The metal at the joint is melted and fuses with additional metal from welding rod. The two principle types of a welds are:

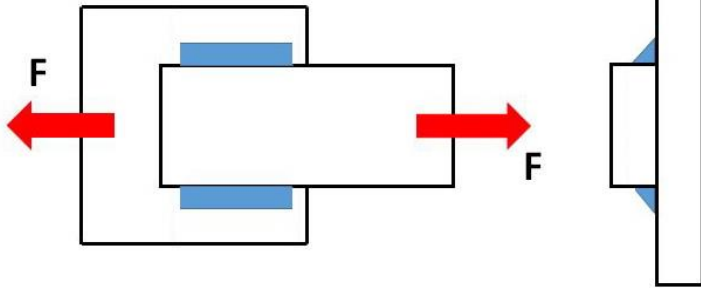
- 1) Butt weld joint
- 2) fillet weld joint



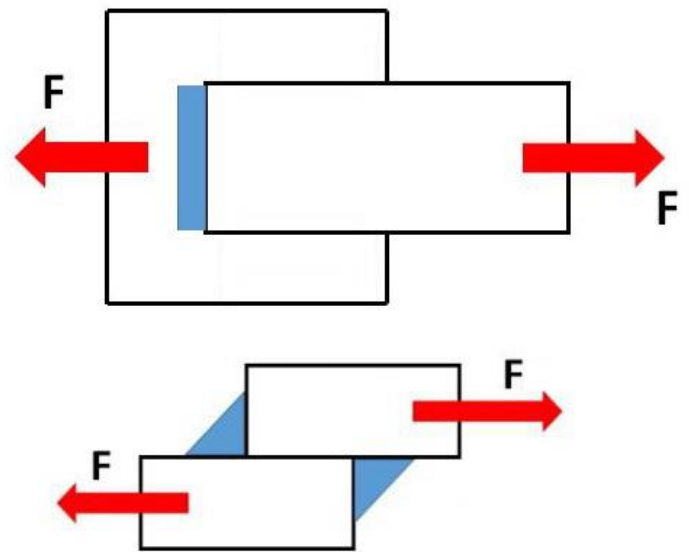
Single V-Butt weld joint



Double V-Butt weld joint

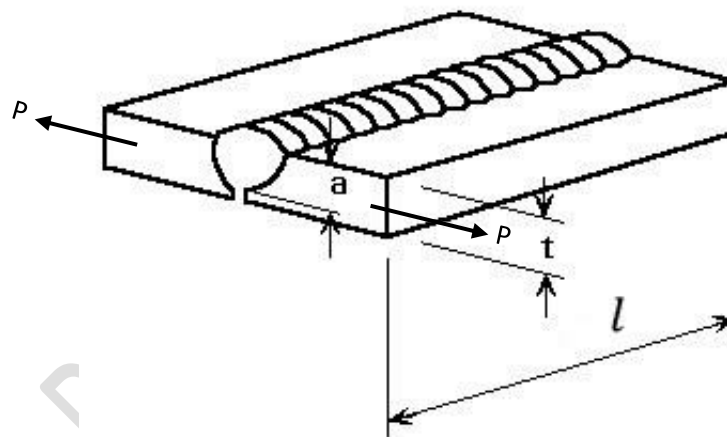


Parallel fillet weld joint



Transverse fillet weld joint

strength of Butt weld joint or load carrying Capacity of Butt weld joint



Let:

t = thickness of the body

σ_t = permissible tensile stress

l = length of the weld

$$\sigma = \frac{P}{A} = \frac{P}{t \times l}$$

$$\therefore P = \sigma \times t \times l$$



Lecture 6

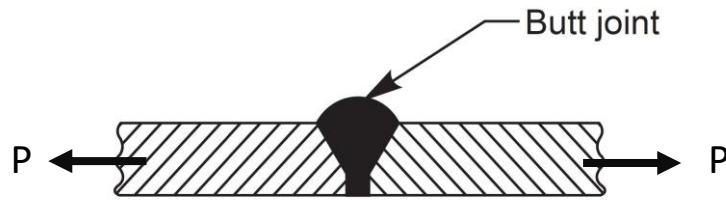
STRENGTH OF MATERIALS

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Dept. of Power Mechanics Techniques Engineering

2022-2023

Butt weld joint



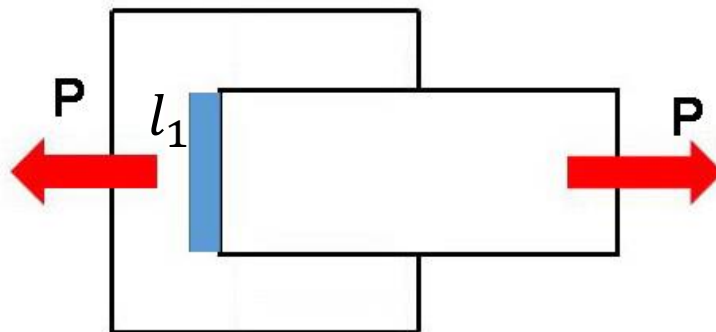
$$P = \sigma_t \times t \times l$$

القوة التي يتحملها اللحام والتي يجب ان لا نسلط أكثر منها على القطع
المربوطة باللحام

P = load carrying capacity or strength of the Butt weld

Fillet weld joint

- Strength of transverse fillet weld joint



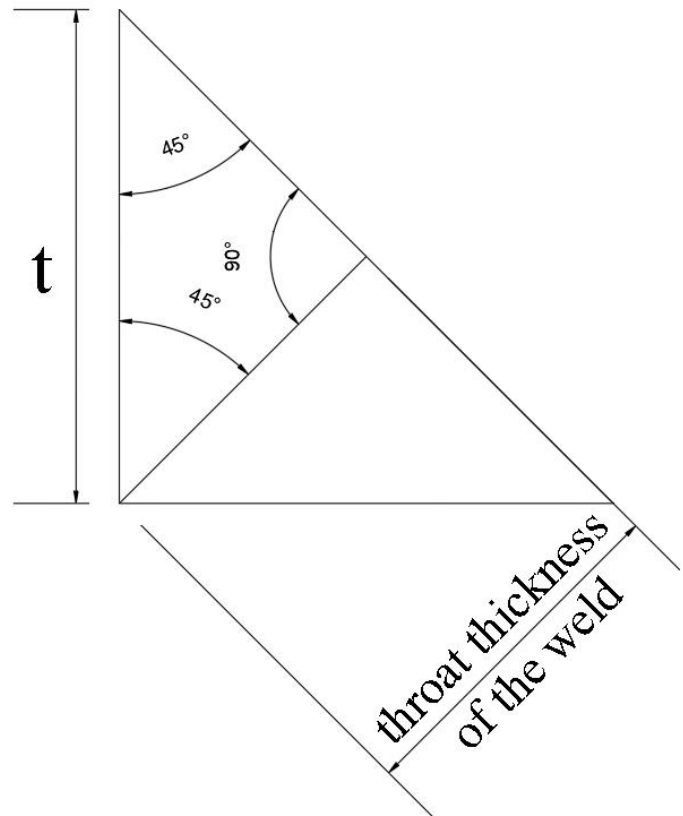
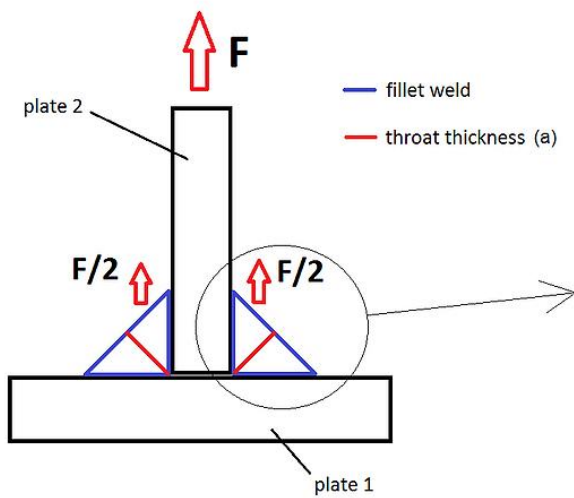
Let :

l_1 = is length of transverse fillet weld joint

σ_t = permissible tensile stress (N/mm²)

$$\sigma = \frac{P}{A}$$

$$P = \sigma \times A$$



$$A = \text{throat thickness} \times \text{length of weld} (l_1)$$

$$\therefore \sin 45 = \frac{\text{throat thickness}}{t}$$

$$\therefore \text{throat thickness} = \sin 45 \times t$$

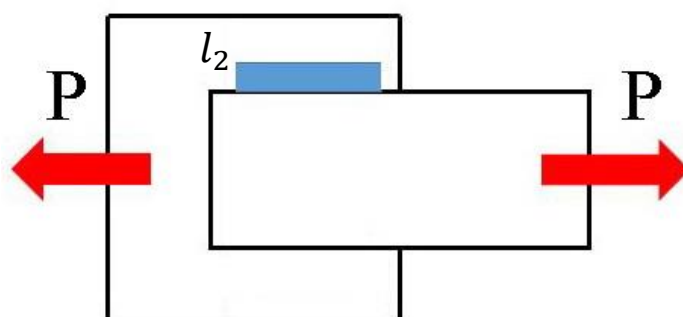
$$\therefore \text{throat thickness} = 0.707 \times t$$

$$\therefore A = 0.707 \times t \times l_1$$

$\therefore$ Strength or load carrying capacity of the transverse weld

$$P = \sigma \times 0.707 \times t \times l_1$$

➤ Strength of a single parallel fillet weld joint



Let :

l_2 = is length of parallel fillet weld joint (mm)

τ = permissible shear stress (N/mm²)

$$\tau = \frac{P}{A}$$

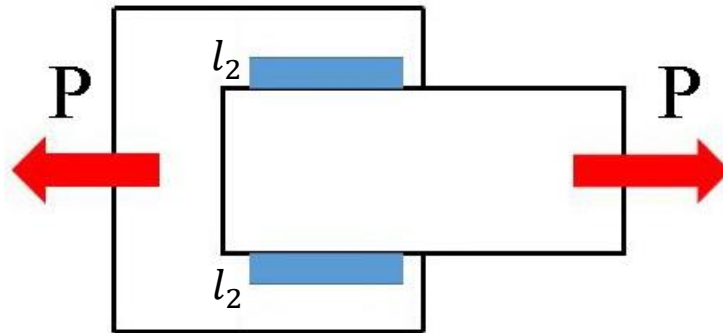
load carrying capacity or strength of the single parallel fillet joint is :

$$P = \tau \times A$$

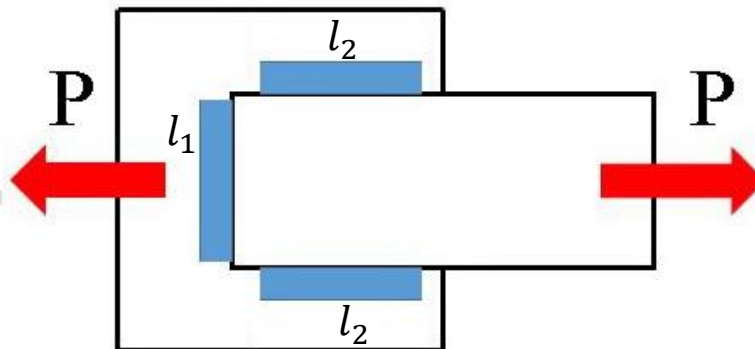
$$P = \tau \times 0.707 \times t \times l_2$$

Similarly, the strength of double fillet weld joint is :

$$P = 2 \times (\tau \times 0.707 \times t \times l_2)$$



Now if we have combined transverse and parallel fillet weld joint



The total load carrying capacity of both transverse and parallel fillet weld joint can be calculate by :

$$P = P_1 + P_2$$

$$P = [\sigma \times 0.707 \times t \times l_1] + [2 \times (\tau \times 0.707 \times t \times l_2)]$$

Example : A plate 75 mm wide and 12.5 mm thick is joined with another plate by a single transverse weld and a double parallel fillet weld as shown in Fig.1.

The maximum tensile and shear stresses are 70 MPa and 56 MPa respectively.

Find the length of each parallel fillet weld.

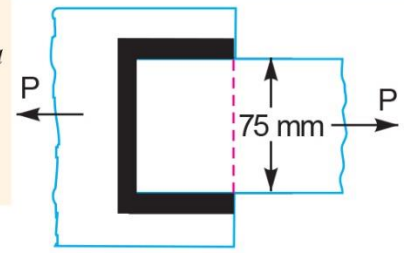


Fig. 1

Solution.

Given : Width = 75 mm ; Thickness = 12.5 mm ;
 $\sigma_t = 70 \text{ MPa} = 70 \text{ N/mm}^2$; $\tau = 56 \text{ MPa} = 56 \text{ N/mm}^2$.

The effective length of weld (l_1) for the transverse weld may be obtained by subtracting 12.5 mm from the width of the plate.

$$\therefore l_1 = 75 - 12.5 = 62.5 \text{ mm}$$

هذا الرقم قياسي ويمثل

start and stop of welding = 12.5 mm

Length of each parallel fillet

Let l_2 = Length of each parallel fillet.

We know that the maximum load which the plate can carry is

$$P = \text{Area} \times \text{Stress} = 75 \times 12.5 \times 70 = 65.625 \text{ N}$$

Load carried by single transverse weld,

$$P_1 = 0.707 t \times l_1 \times \sigma_t = 0.707 \times 12.5 \times 62.5 \times 70 = 38.664 \text{ N}$$

and the load carried by double parallel fillet weld,

$$P_2 = 2 \times 0.707 t \times l_2 \times \tau = 2 \times 0.707 \times 12.5 \times l_2 \times 56 = 990 l_2 \text{ N}$$

$\therefore$ Load carried by the joint (P),

$$65.625 = P_1 + P_2 = 38.664 + 990 l_2 \quad \text{or} \quad l_2 = 27.2 \text{ mm}$$

Adding 12.5 mm for starting and stopping of weld run, we have

$$l_2 = 27.2 + 12.5 = 39.7 \text{ say } 40 \text{ mm} \quad \text{Ans.}$$

عند حساب l نضيف 12.5 mm

ولكن عندما l تكون موجودة نطرح منها 12.5 mm

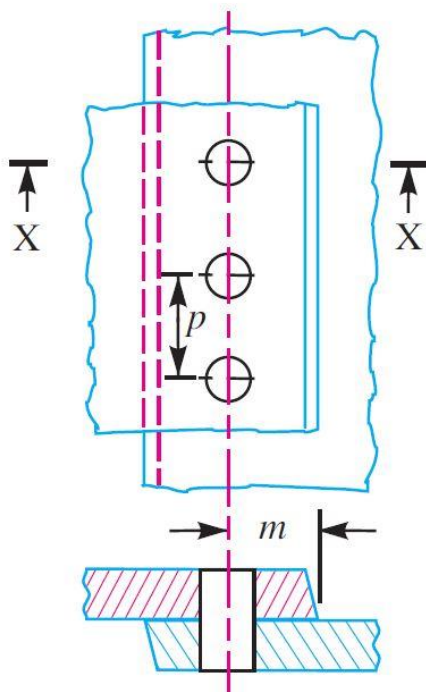
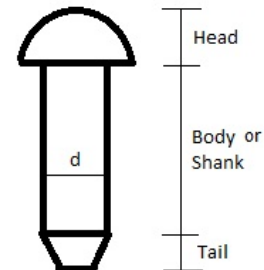
Riveted joint

It is one type of the permanent joining of the parts used mainly for fastening sheet and shaped rolled metals. Riveting may be used in lap, abutment and double- cover plate joints. Riveted joints are replaced by the more economical welded and glued joints.

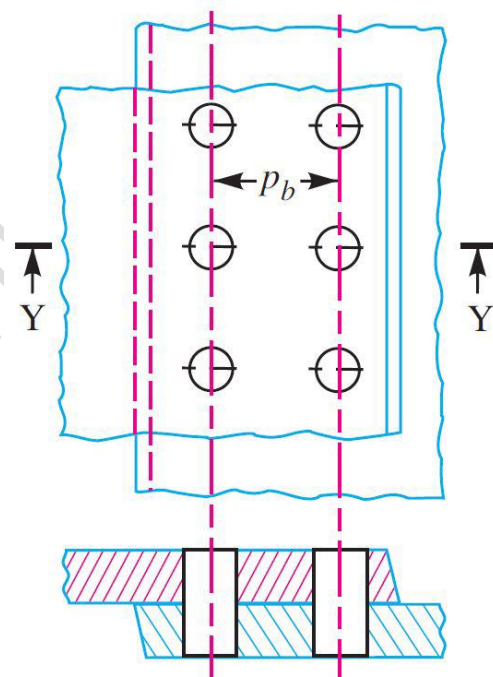
Types of Riveted joint

There are two types of riveted joint

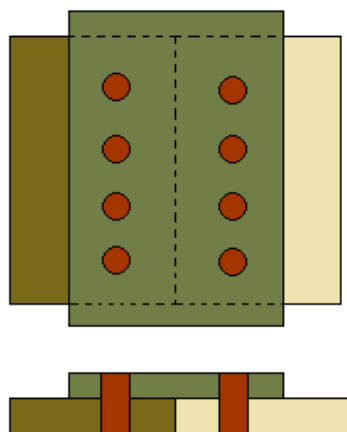
- 1) Lap joints
- 2) Butt joints



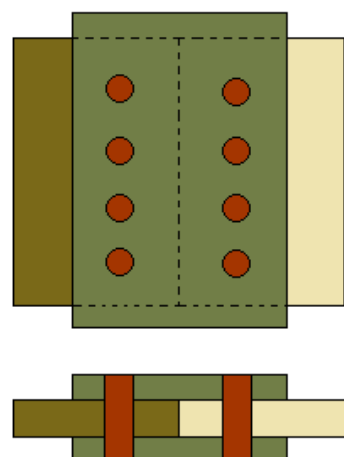
(a) Single riveted lap joint.



(b) Double riveted lap joint (Chain riveting).



Single strap butt joint



Double strap butt joint

Important terms used in riveted join

Pitch (P): it is the distance from the center of one rivet to the center of the next rivet.

Back Pitch (P_b): it is the perpendicular distance between the center lines of the successive rows.

Diagonal Pitch (P_d): it is the distance between the rivets in adjacent rows of zig-zag riveted joint.

Margin(M): it is the distance between the center of riveted hole to the nearest edge of the plate.



Lecture 7

STRENGTH OF MATERIALS

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Failures in riveted joints (strength of riveted joints)

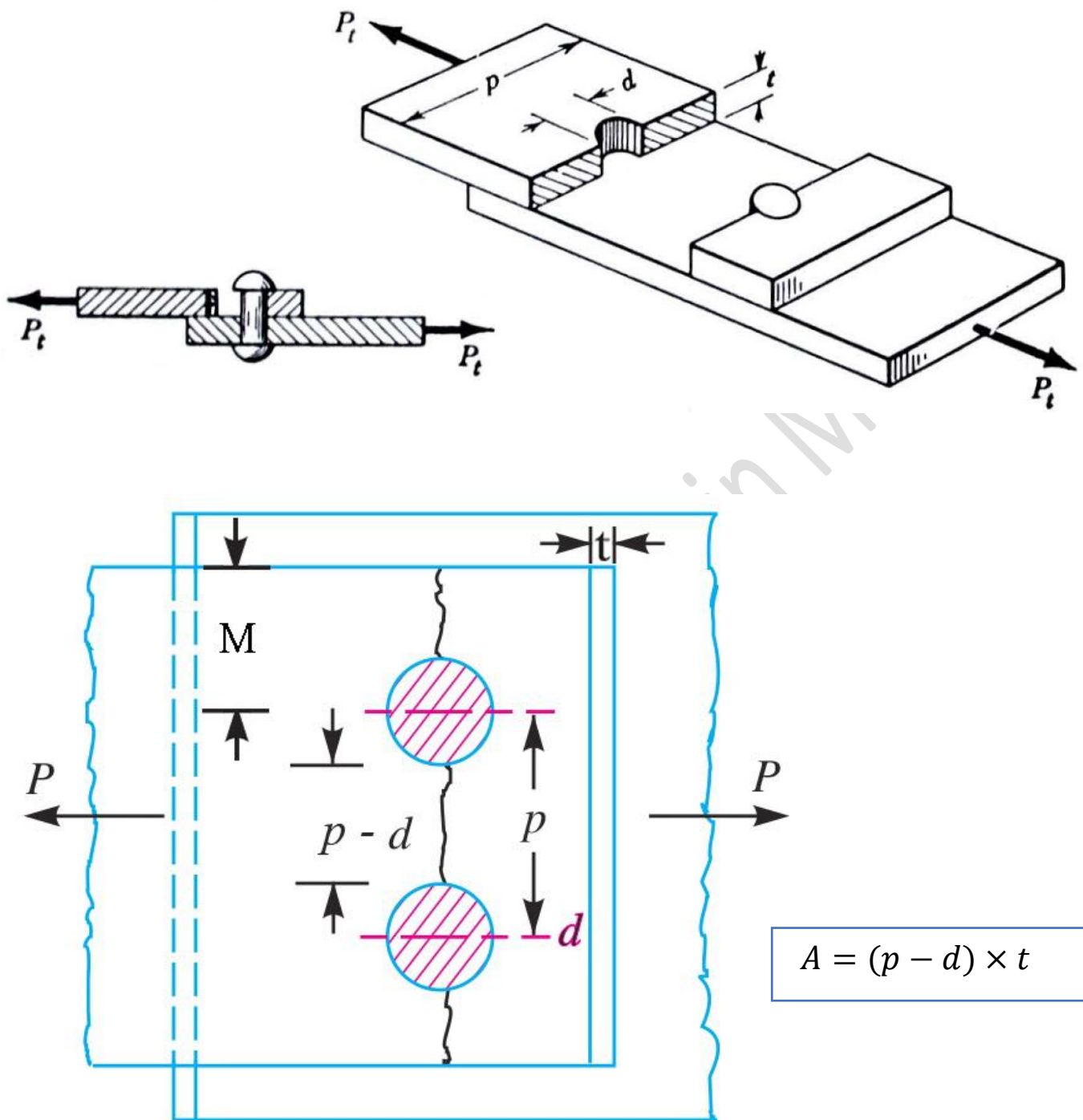


Fig. 9.14. Tearing of the plate across the rows of rivets.

- 1) Tearing
- 2) Shearing
- 3) Crushing

1. Tearing

a) Tearing at the nearest edge of the plate from the center of the rivet. This type of failure can be avoided if $M = 1.5 d$

b) Tearing on the plate across row of rivets (P_t)

$$P_{(t)} = (p - d) \times t \times \sigma_t$$

Where :

$P_{(t)}$ = tearing strength resistance of the plate

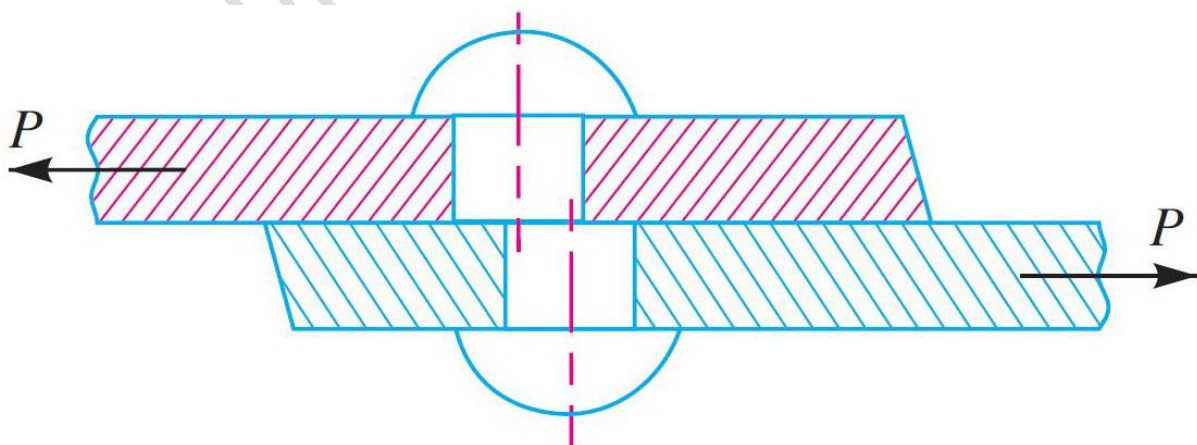
p = pitch

t = thickness of the plate

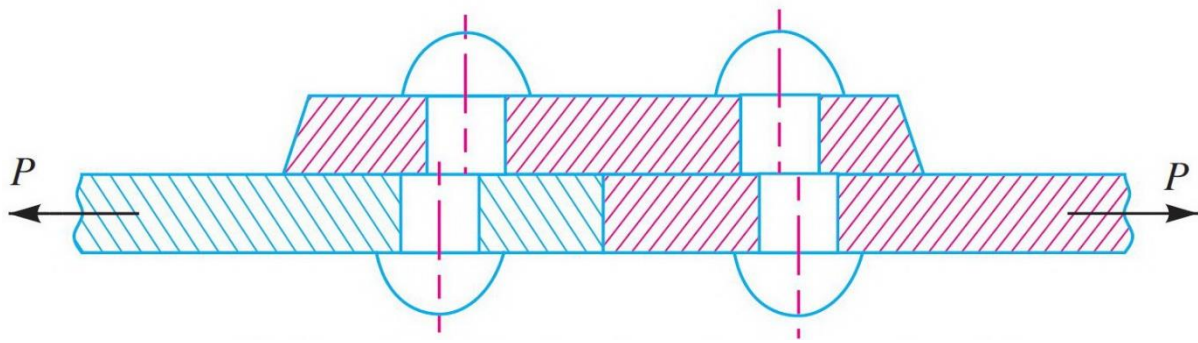
τ = permissible tensile stress

Tearing of type (b) can be avoided if $P_{(t)} > \text{applied force } \{ P_{(t)} > P \}$

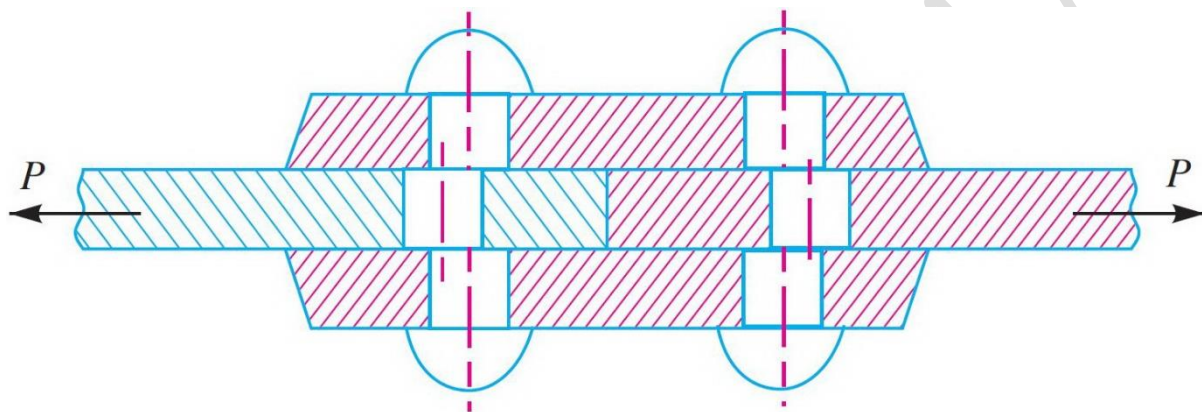
2. Shearing of the rivets $P_{(s)}$



(a) Shearing of a single riveted lap joint



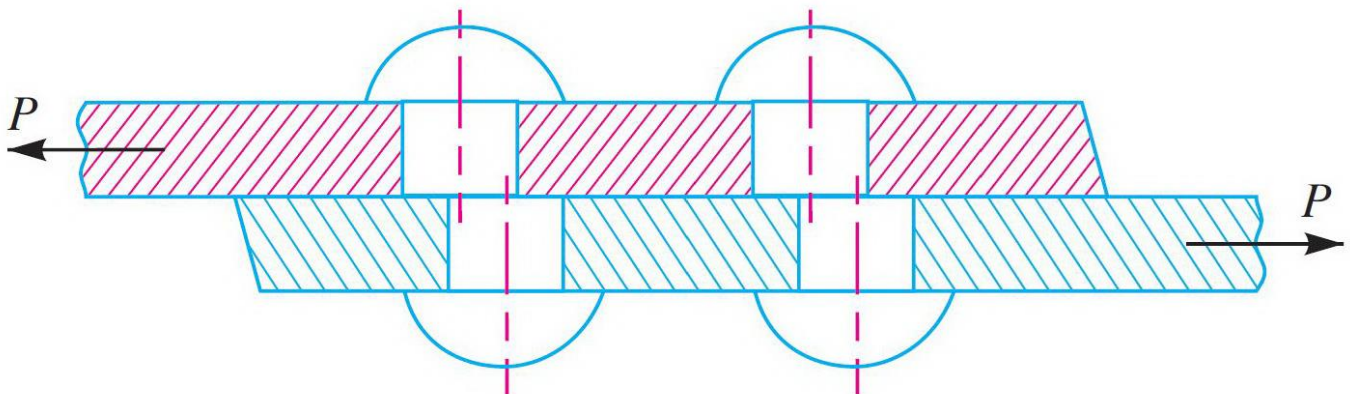
Single rivet single strap Butt joint



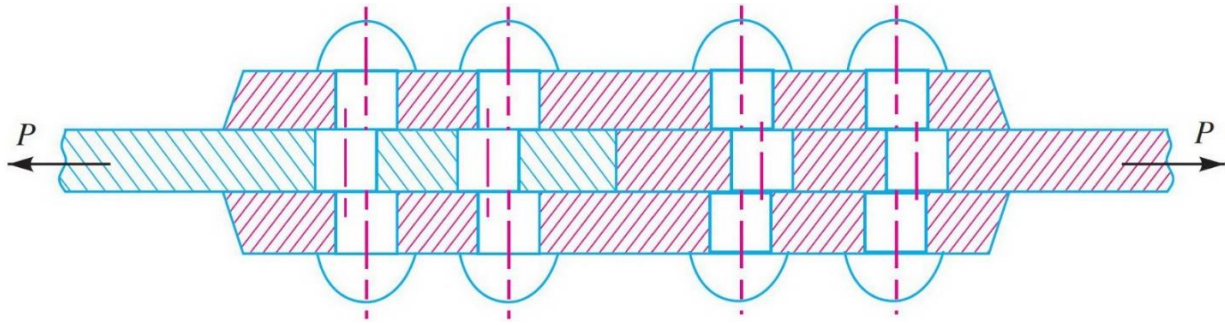
Single rivet double strap Butt joint

$$n \times 2 \times \frac{\pi}{4} \times d^2 = 1 \times 2 \times \frac{\pi}{4} \times d^2$$

n = number of rivets row



double rivet lap joint



double rivet double strap

$$n \times 2 \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times 2 \times \frac{\pi}{4} \times d^2 \times \tau$$

$$P_{(s)} = n \times \frac{\pi}{4} \times d^2 \times \tau \quad \text{for single shear}$$

$P_{(s)}$ = shearing resistance of the rivet material **or** shearing strength of the rivet

n = number of rivets row

$$P_{(s)} = n \times 2 \times \frac{\pi}{4} \times d^2 \times \tau \quad \text{shear strength of double shear}$$

double strap يحصل عندما يكون هنالك double shear

Shearing failures can be avoided if $P_{(s)} > \text{applied force } \{ P_{(t)} > P \}$

3. Crushing of the plate or rivets P_{cr}

مثال على Crushing هو Bearing stress

$$P_{cr} = n \times d \times t \times \sigma_{cr}$$

Where :

P_{cr} = Crushing strength of crushing resistance of the rivets or plate

d = hole diameter

σ_{cr} = permissible crushing stress

If $P_{cr} > P$

Crushing failure will not happen



Lecture 8

STRENGTH OF MATERIALS

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Efficiency of riveted joints

$$\eta_{\text{of riveted joints}} = \frac{\text{least of } P_{(t)}, P_{(s)}, P_{cr}}{\text{strength of unriveted}} = \frac{\text{least of } P_{(t)}, P_{(s)}, P_{cr}}{p \times t \times \sigma_t}$$

Example 9.1. A double riveted lap joint is made between 15 mm thick plates. The rivet diameter and pitch are 25 mm and 75 mm respectively. If the ultimate stresses are 400 MPa in tension, 320 MPa in shear and 640 MPa in crushing, find the minimum force per pitch which will rupture the joint.

If the above joint is subjected to a load such that the factor of safety is 4, find out the actual stresses developed in the plates and the rivets.

Solution. Given : $t = 15 \text{ mm}$; $d = 25 \text{ mm}$; $p = 75 \text{ mm}$; $\sigma_t = 400 \text{ MPa} = 400 \text{ N/mm}^2$; $\tau = 320 \text{ MPa} = 320 \text{ N/mm}^2$; $\sigma_{cr} = 640 \text{ MPa} = 640 \text{ N/mm}^2$

Minimum force per pitch which will rupture the joint

Since the ultimate stresses are given, therefore we shall find the ultimate values of the resistances of the joint. We know that ultimate tearing resistance of the plate per pitch,

$$P_t = (p - d)t \times \sigma_t = (75 - 25)15 \times 400 = 300\,000 \text{ N}$$

Ultimate shearing resistance of the rivets per pitch,

$$P_s = n \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times \frac{\pi}{4} (25)^2 320 = 314\,200 \text{ N} \quad \dots (\because n = 2)$$

and ultimate crushing resistance of the rivets per pitch,

$$P_{cr} = n \times d \times t \times \sigma_{cr} = 2 \times 25 \times 15 \times 640 = 480\,000 \text{ N}$$

From above we see that the minimum force per pitch which will rupture the joint is 300 000 N or 300 kN. **Ans.**

Actual stresses produced in the plates and rivets

Since the factor of safety is 4, therefore safe load per pitch length of the joint
 $= 300\,000 / 4 = 75\,000 \text{ N}$

Let σ_t , τ and σ_{cr} be the actual tearing, shearing and crushing stresses produced with a safe load of 75 000 N in tearing, shearing and crushing.

We know that actual tearing resistance of the plates (P_t),

$$75\,000 = (p - d)t \times \sigma_t = (75 - 25)15 \times \sigma_t = 750 \sigma_t$$

$$\therefore \sigma_t = 75\,000 / 750 = 100 \text{ N/mm}^2 = 100 \text{ MPa} \quad \text{Ans.}$$

Actual shearing resistance of the rivets (P_s),

$$75\,000 = n \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times \frac{\pi}{4} (25)^2 \tau = 982 \tau$$

$$\therefore \tau = 75\,000 / 982 = 76.4 \text{ N/mm}^2 = 76.4 \text{ MPa} \quad \text{Ans.}$$

and actual crushing resistance of the rivets (P_{cr}),

$$75\,000 = n \times d \times t \times \sigma_{cr} = 2 \times 25 \times 15 \times \sigma_{cr} = 750 \sigma_{cr}$$

$$\therefore \sigma_{cr} = 75\,000 / 750 = 100 \text{ N/mm}^2 = 100 \text{ MPa} \quad \text{Ans.}$$

Example 9.2. Find the efficiency of the following riveted joints :

1. Single riveted lap joint of 6 mm plates with 20 mm diameter rivets having a pitch of 50 mm.
2. Double riveted lap joint of 6 mm plates with 20 mm diameter rivets having a pitch of 65 mm.

Assume

Permissible tensile stress in plate = 120 MPa

Permissible shearing stress in rivets = 90 MPa

Permissible crushing stress in rivets = 180 MPa

Solution. Given : $t = 6$ mm ; $d = 20$ mm ; $\sigma_t = 120$ MPa = 120 N/mm² ; $\tau = 90$ MPa = 90 N/mm² ; $\sigma_{cr} = 180$ MPa = 180 N/mm²

1. Efficiency of the first joint

Pitch, $p = 50$ mm ... (Given)

First of all, let us find the tearing resistance of the plate, shearing and crushing resistances of the rivets.

(i) Tearing resistance of the plate

We know that the tearing resistance of the plate per pitch length,

$$P_t = (p - d) t \times \sigma_t = (50 - 20) 6 \times 120 = 21\,600 \text{ N}$$

(ii) Shearing resistance of the rivet

Since the joint is a single riveted lap joint, therefore the strength of one rivet in single shear is taken. We know that shearing resistance of one rivet,

$$P_s = \frac{\pi}{4} \times d^2 \times \tau = \frac{\pi}{4} (20)^2 90 = 28\,278 \text{ N}$$

(iii) Crushing resistance of the rivet

Since the joint is a single riveted, therefore strength of one rivet is taken. We know that crushing resistance of one rivet,

$$P_{cr} = d \times t \times \sigma_{cr} = 20 \times 6 \times 180 = 21\,600 \text{ N}$$

∴ Strength of the joint

$$= \text{Least of } P_t, P_s \text{ and } P_{cr} = 21\,600 \text{ N}$$

We know that strength of the unriveted or solid plate,

$$P = p \times t \times \sigma_t = 50 \times 6 \times 120 = 36\,000 \text{ N}$$

∴ Efficiency of the joint,

$$\eta = \frac{\text{Least of } P_t, P_s \text{ and } P_{cr}}{P} = \frac{21\,600}{36\,000} = 0.60 \text{ or } 60\% \quad \text{Ans.}$$

2. Efficiency of the second joint

Pitch, $p = 65$ mm ... (Given)

(i) Tearing resistance of the plate,

We know that the tearing resistance of the plate per pitch length,

$$P_t = (p - d) t \times \sigma_t = (65 - 20) 6 \times 120 = 32\,400 \text{ N}$$

(ii) Shearing resistance of the rivets

Since the joint is double riveted lap joint, therefore strength of two rivets in single shear is taken. We know that shearing resistance of the rivets,

$$P_s = n \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times \frac{\pi}{4} (20)^2 90 = 56\,556 \text{ N}$$

(iii) Crushing resistance of the rivet

Since the joint is double riveted, therefore strength of two rivets is taken. We know that crushing resistance of rivets,

$$P_{cr} = n \times d \times t \times \sigma_{cr} = 2 \times 20 \times 6 \times 180 = 43\,200 \text{ N}$$

∴ Strength of the joint

$$= \text{Least of } P_t, P_s \text{ and } P_{cr} = 32\,400 \text{ N}$$

We know that the strength of the unriveted or solid plate,

$$P = p \times t \times \sigma_t = 65 \times 6 \times 120 = 46\,800 \text{ N}$$

∴ Efficiency of the joint,

$$\eta = \frac{\text{Least of } P_t, P_s \text{ and } P_{cr}}{P} = \frac{32\,400}{46\,800} = 0.692 \text{ or } 69.2\% \quad \text{Ans.}$$

Example 9.3. A double riveted double cover butt joint in plates 20 mm thick is made with 25 mm diameter rivets at 100 mm pitch. The permissible stresses are :

$$\sigma_t = 120 \text{ MPa}; \quad \tau = 100 \text{ MPa}; \quad \sigma_{cr} = 150 \text{ MPa}$$

Find the efficiency of joint, taking the strength of the rivet in double shear as twice than that of single shear.

Solution. Given : $t = 20 \text{ mm}$; $d = 25 \text{ mm}$; $p = 100 \text{ mm}$; $\sigma_t = 120 \text{ MPa} = 120 \text{ N/mm}^2$; $\tau = 100 \text{ MPa} = 100 \text{ N/mm}^2$; $\sigma_{cr} = 150 \text{ MPa} = 150 \text{ N/mm}^2$

First of all, let us find the tearing resistance of the plate, shearing resistance and crushing resistance of the rivet.

(i) Tearing resistance of the plate

We know that tearing resistance of the plate per pitch length,

$$P_t = (p - d) t \times \sigma_t = (100 - 25) 20 \times 120 = 180\,000 \text{ N}$$

(ii) Shearing resistance of the rivets

Since the joint is double riveted butt joint, therefore the strength of two rivets in double shear is taken. We know that shearing resistance of the rivets,

$$P_s = n \times 2 \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times 2 \times \frac{\pi}{4} (25)^2 100 = 196\,375 \text{ N}$$

(iii) Crushing resistance of the rivets

Since the joint is double riveted, therefore the strength of two rivets is taken. We know that crushing resistance of the rivets,

$$P_{cr} = n \times d \times t \times \sigma_{cr} = 2 \times 25 \times 20 \times 150 = 150\,000 \text{ N}$$

∴ Strength of the joint

$$= \text{Least of } P_t, P_s \text{ and } P_{cr} = 150\,000 \text{ N}$$

Efficiency of the joint

We know that the strength of the unriveted or solid plate,

$$P = p \times t \times \sigma_t = 100 \times 20 \times 120 = 240\,000 \text{ N}$$

$$\therefore \text{Efficiency of the joint} = \frac{\text{Least of } P_t, P_s \text{ and } P_{cr}}{P} = \frac{150\,000}{240\,000} = 0.625 \text{ or } 62.5\% \quad \text{Ans.}$$

Stress concentration

It is the increase in the stress near holes, notches, sharp corner and other changes in section.

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Lecture 9

STRENGTH OF MATERIALS

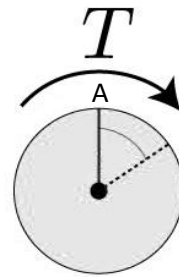
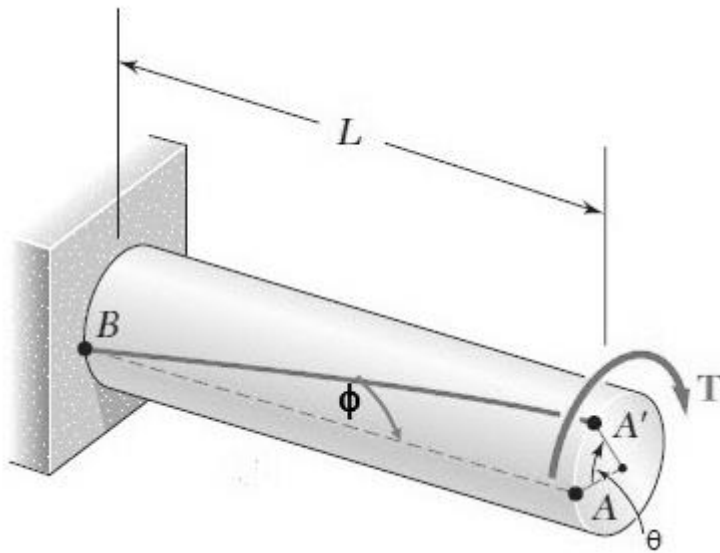
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2022-2023

Torsion of circular shafts

Represent (pure shear stress)



If a twisting load is transmitted through a member (like shaft), then it is subjected to torsion

Torsion of a solid circular shaft: **T** is the twisting moment (Torque)

After application of the torque **T**, BA come to BĀ

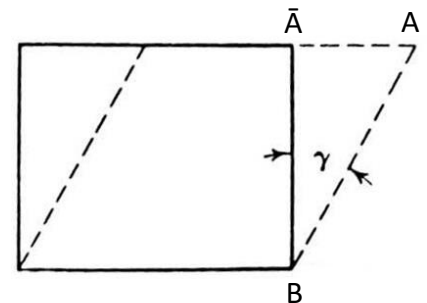
γ : is the shear strain measured in radian

As γ is small

$$\tan \gamma = \gamma = \frac{A\bar{A}}{BA} = \frac{A\bar{A}}{l} \quad \text{لأنه موازي } BA = l$$

$$A\bar{A} = R\theta \quad \tan \theta = \theta \quad \text{لأنها small}$$

$$\therefore \gamma = \frac{R\theta}{l} \quad \text{----- (1)}$$



Where:

θ is the angular of twist

* يجب ان تحول θ الى radian

$$\text{modulus of rigidity} = \frac{\text{shear stress}}{\text{shear strain}}$$

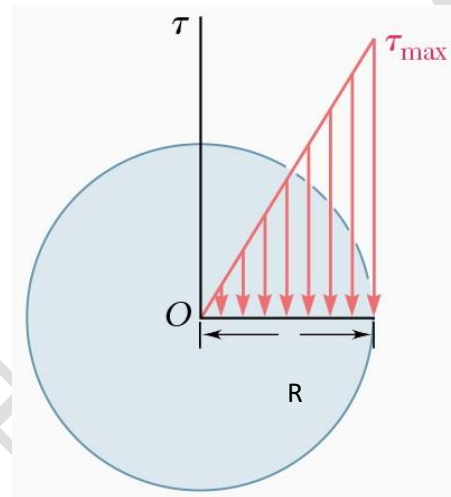
$$G = \frac{\tau}{\gamma}$$

$$\gamma = \frac{\tau}{G} \quad \text{--- (2)}$$

eq (1) = eq (2)

$$\frac{\tau}{G} = \frac{R \theta}{l} \quad \text{or} \quad \boxed{\frac{\tau}{R} = \frac{G \theta}{l}}$$

* عند تغير R تتغير τ فقط حيث تبقى l و θ ثابتة



At $r = R \quad \therefore \tau$ is maximum

This analysis could have been made for any radius r ($r > R$), since θ and l do not depend on r .

$$\tau = \frac{G \theta}{l} \times R = G \gamma$$

$$\frac{q}{r} = \frac{G \theta}{l}$$

$$\therefore q = \frac{G \theta r}{l} = G \bar{\gamma}$$

Where q is the shear stress at radius r .

So, the Shear stress and shear strain vary linearly with radius and have their max value at the outside radius R .

$$\boxed{\therefore \frac{q}{r} = \frac{\tau}{R} = \frac{G \theta}{l}}$$

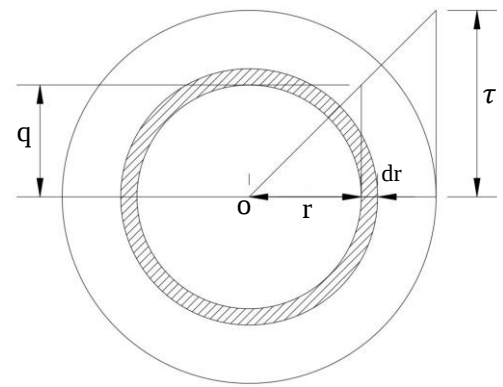
Relation between torsion and shear stress: -

Consider the annular area which is $= 2 \pi r dr = da$

$$\text{Shear stress} = \frac{\text{Shear force}}{\text{Area of shear}}$$

$$q = \frac{\text{Shear force}}{2 \pi r dr}$$

$$\text{Shear force} = 2 \pi r q dr$$



$$\begin{aligned} \text{Moment of this elementary force about the point O} &= 2 \pi r q dr \times r \\ &= 2 \pi r^2 q dr \end{aligned}$$

Summation of such elementary moment area the entire cross section area =
Twisting moment applied (T)

$$\therefore \int_0^R 2 \pi r^2 q dr = \int_0^R q \cdot r da = T$$

$$q = \frac{G \theta r}{l}$$

$$\therefore T = \int_0^R \frac{G \theta}{l} \cdot r \cdot r da$$

$$T = \frac{G \theta}{l} \int_0^R r^2 da$$

J = polar moment of inertia

$$T = \frac{G \theta J}{l}$$

$$\therefore \frac{T}{J} = \frac{G \theta}{l}$$

$$\frac{\tau}{R} = \frac{q}{r} = \frac{T}{J} = \frac{G \theta}{l}$$

$$J = \int_0^R r^2 da = \int_0^R r^2 \cdot 2\pi r dr = \int_0^R 2\pi r^3 dr = 2\pi \int_0^R r^3 dr$$

$$J = 2\pi \left[\frac{r^4}{4} \right]_0^R = \frac{2\pi R^4}{4}$$

$$J = \frac{\pi D^4}{32}$$

$$\frac{\tau}{R} = \frac{T}{J} \longrightarrow \tau = \frac{TR}{J} = \frac{TD}{2\left(\frac{\pi D^4}{32}\right)}$$

$$\tau = \frac{16T}{\pi D^3} \quad \text{max } \tau \text{ for solid shaft} = \frac{16T}{\pi D^3} = \frac{T}{Z} \quad \text{where } Z = \frac{\pi D^3}{16}$$

For a hollow shaft of internal radius $r = \frac{d}{2}$

$$J = 2\pi \int_r^R r^3 dr = \frac{\pi}{2} (R^4 - r^4) \quad \text{or} \quad J = \frac{\pi}{32} (D^4 - d^4)$$

$$\tau_{max} = \frac{16T}{\pi \frac{(D^4 - d^4)}{D}} \quad \text{max } \tau \text{ for hollow shaft}$$

$$\tau_{max} = \frac{T}{Z} \quad \text{where } Z = \frac{\pi (D^4 - d^4)}{16D}$$

T = Torque applied

J = polar moment of inertia

G = modulus of rigidity

θ = angle of twist of the shaft

l = length of shaft

τ = shear stress at the surface = maximum shear

R = radius of the shaft

D = diameter of the shaft

q = shear stress at any radius r

r = the radius at which q is acting

γ = is the shear strain

Ex: Calculate the max shear stress developed in a solid shaft of diameter 10 cm. If the torque transmitted through the shaft be 5 kN

Sol :

$$\tau = \frac{16 T}{\pi D^3} \quad \text{or} \quad \frac{T}{J} = \frac{\tau}{R}$$

$$\frac{T}{J} = \frac{\tau}{R} \longrightarrow \tau = \frac{TR}{J}$$

$$J = \frac{\pi D^4}{32}$$

$$J = \frac{\pi (0.1)^4}{32} = 9.81 \times 10^{-6} \text{ m}^4$$

$$\tau = \frac{5 \times 10^3 \times 0.05}{9.81 \times 10^{-6}} = 25.4 \text{ MN/m}^2$$

Section modulus (Z)

For solid shaft:

$$\frac{T}{J} = \frac{\tau_{max}}{R}$$

$$\tau_{max} = \frac{TR}{J} = \frac{T}{\frac{J}{R}}$$

$$\boxed{\frac{J}{R} = Z} \quad (\text{Section modulus})$$

$$\boxed{\tau_{max} = \frac{T}{Z}}$$

$$\tau_{max} = \frac{16 T}{\pi D^3} = \frac{T}{Z}$$

$$Z = \frac{J}{R} = \frac{\frac{\pi}{32} D^4}{\frac{D}{2}}$$

$$\therefore \boxed{Z = \frac{\pi}{16} D^3} \quad m$$

فقط
عندما

For hollow shaft:

$$\frac{T}{J} = \frac{\tau_{max}}{R}$$

$$\tau_{max} = \frac{TR}{J} = \frac{T}{\frac{J}{R}}$$

$$\frac{J}{R} = Z \quad (\text{Section modulus})$$

$$Z = \frac{\frac{\pi}{32}(D^4 - d^4)}{\frac{D}{2}}$$

$$Z = \frac{\pi}{16} \left(\frac{D^4 - d^4}{D} \right)$$

$$\tau_{max} = \frac{16 T}{\pi \frac{(D^4 - d^4)}{D}} = \frac{T}{Z}$$

$$P = T \times \omega$$

$$= N.m \times \frac{1}{sec} = \frac{N.m}{sec} = \frac{Joule}{sec} = \text{watt or kw}$$

Power transmitted by the shaft: -

$$T = \frac{60 \times P}{2\pi N}$$

Where :

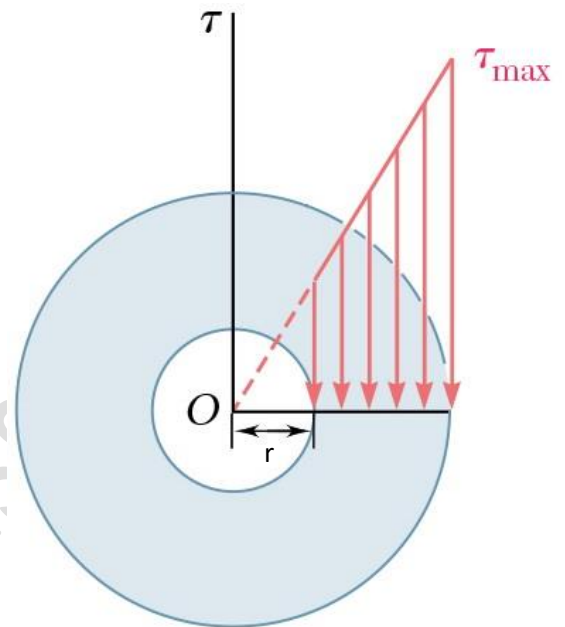
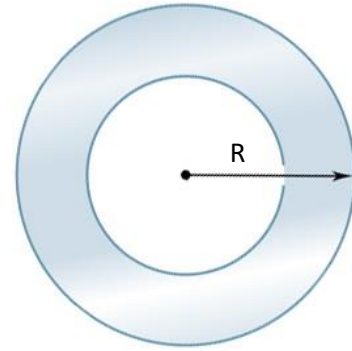
T = Applied Torque (N.m)

N = Frequency of rotation (rev/min)

$2\pi N$ = angular velocity (rad/min)

$$\omega = \frac{2\pi N}{60} \quad \text{angular velocity (rad/sec)}$$

$$P = \frac{T \times 2\pi N}{60} = \frac{N.m}{sec} = \frac{Joule}{sec} = \text{watt}$$





Lecture 10

STRENGTH OF MATERIALS

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2022-2023

Springs

It is an energy absorbing units.

Its function is to store and restore energy. (e.g. spring application in a motor vehicle to acts as buffers between the vehicle and external forces, applied through the wheels uneven road condition) without permanent distortion of the material.

Types of springs:

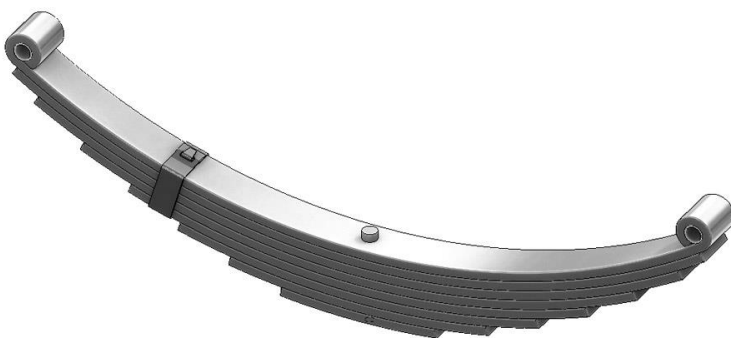
- 1) Close-coiled helical spring.
- 2) Open-coiled helical spring.
- 3) Leaf spring (carriage spring).
- 4) Spiral spring.



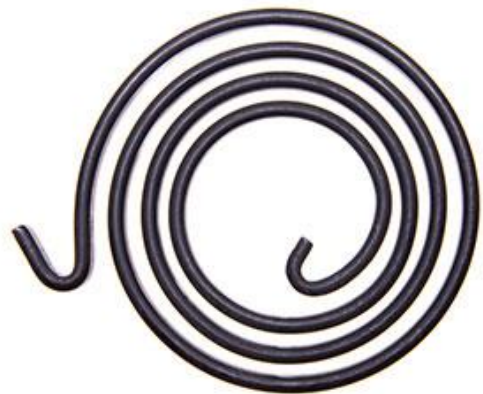
Closed coil helical spring



Open coiled helical spring



Leaf spring



Spiral spring

Close-coiled helical spring: -

Each turn is so close to the adjacent, i.e. each turn may be considered to lie in horizontal plane if the center axis of the spring is vertical.

$$R = \frac{D}{2} \text{ mean coil radius (cm, mm)}$$

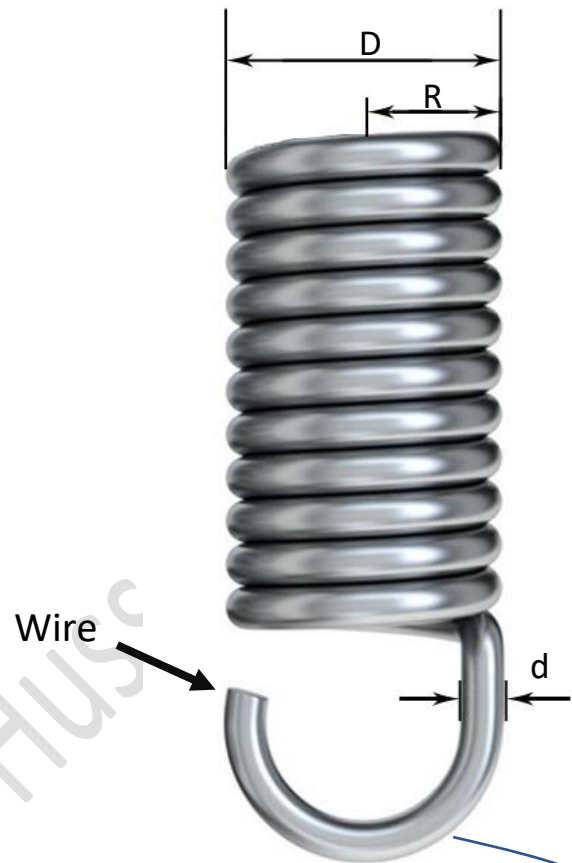
$$r = \frac{d}{2} \text{ wire radius}$$

n = number of turns (number of coils)

l = length of the wire = $\pi D n$

δ = axial deflection

w = axial load (N, KN)



consider one half turn of the spring every cross section will be subjected to torque (**T**) = **W R**, tends to twist the section. To determine the max stress, torsion theory will be applied

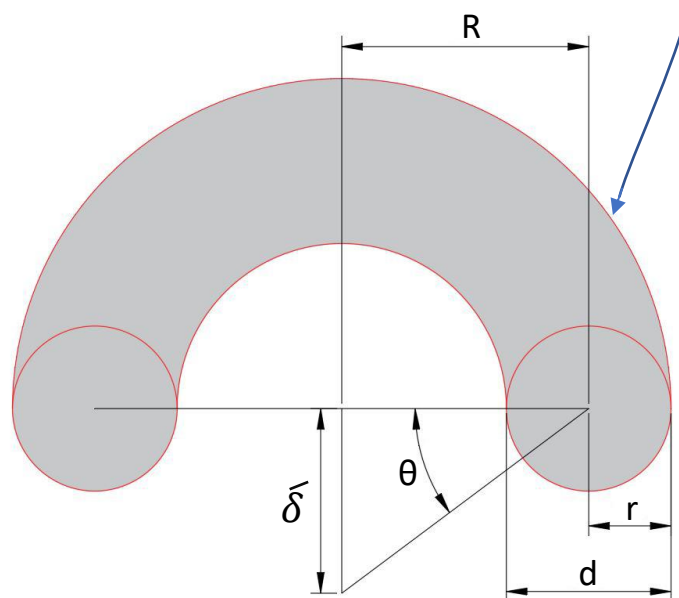
$$\frac{\tau_{max}}{r} = \frac{T}{J}$$

$$T = W R$$

$$J = \frac{\pi}{32} d^4 = \frac{\pi}{2} r^4$$

$$\tau_{max} = \frac{T r}{J} = \frac{W \times R \times r}{\frac{\pi}{2} r^4}$$

$$\boxed{\text{max stress} = \frac{2WR}{\pi r^3}}$$



Also, from half turn: -

$$\therefore \frac{T}{J} = \frac{G \theta}{l}$$

$$\theta = \frac{Tl}{GJ} = \frac{W \times R \times (\pi R)}{G \left(\frac{\pi}{2} r^4 \right)}$$

$$\theta = \frac{2WR^2}{Gr^4} \quad \text{angle of twist of the turn of the spring}$$

$$\text{deflection of half turn} = \delta = R \theta$$

$$\therefore \text{Total deflection} = \delta = 2\delta n = 2 \times R \times \frac{2WR^2}{Gr^4} \times n$$

$$\therefore \delta = \frac{4R^3Wn}{Gr^4} \quad (cm, mm)$$

Where:

δ = deflection of the spring

W = axial load on the spring

n = number of coils

R = mean coil radius

r = wire radius

G = modulus of rigidity

Ex: A close-coiled helical spring of round steel wire 6mm in diameter having 14 complete coils of 6cm mean diameter is subjected to an axial load of 150 N, find the deflection of the spring and the max shearing stress in the material if $G = 80 \text{ GN/m}^2$

Sol:

$$\delta = \frac{4WR^3n}{G r^4}$$

$$\delta = \frac{4 \times 150 \times (3)^3 \times 10^{-6} \times 14}{80 \times 10^9 \times 3^4 \times 10^{-12}} = 0.035 \text{ m} = 3.5 \text{ cm}$$

$$\text{max shear stress} = \frac{2WR}{\pi r^3}$$

$$\therefore \text{max shear stress} = \frac{2 \times 150 \times 3 \times 10^2}{\pi (3)^3 \times 10^{-9}} = 106 \frac{\text{MN}}{\text{m}^2}$$

$$\text{Deflection} = \frac{\text{load } (W) \text{ (N)}}{\text{stiffness } (s) \text{ (N/m)}} = \quad (\text{m})$$

1. Series connection

$$S = \frac{W}{s}$$

$$S = \frac{W}{s} = s_1 + s_2 = \frac{W}{s_1} + \frac{W}{s_2}$$

$$\frac{W}{s} = W \left(\frac{1}{s_1} + \frac{1}{s_2} \right)$$

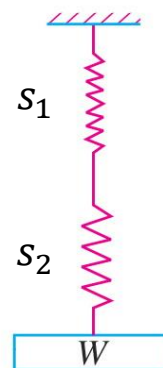
$$\frac{1}{s} = \frac{1}{s_1} + \frac{1}{s_2} = \frac{s_2 + s_1}{s_1 s_2}$$

$$\text{combined stiffness } S = \frac{s_1 s_2}{s_1 + s_2}$$

Where:

s_1 = is the stiffness of spring 1

s_2 = is the stiffness of spring 2



Springs in series.
Two spring carrying a common load (W)

2. Parallel connection

$$\text{Total load} = W = w_1 + w_2$$

$$S = \frac{W}{s} = \frac{w_1}{s_1} = \frac{w_2}{s_2}$$

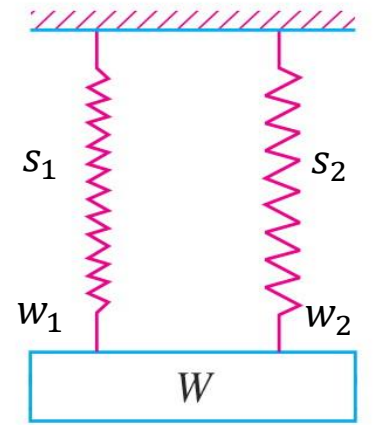
$$w_1 = \frac{W s_1}{s} \quad w_2 = \frac{W s_2}{s}$$

$$W = w_1 + w_2$$

$$W = \frac{W s_1}{s} + \frac{W s_2}{s}$$

$$W = \frac{W}{s} (s_1 + s_2)$$

$$\therefore \text{combined stiffness } S = s_1 + s_2$$



Springs in parallel.

Ex: A close-coiled helical spring is subjected to a load of 100 N, and the max shear stress is limited to 10 KN/cm². Determine the spring diameter for coil and wire if $\frac{D}{d} = 8$

Sol:

$$\tau_{max} = \frac{2R.W}{\pi r^3}$$

$$D = 8d$$

$$\tau_{max} = \frac{2 \times \left(\frac{8}{2} \times d\right) \times 100}{\pi \frac{d^3}{8}}$$

$$\frac{10 \times 10^3}{10^{-4}} = \frac{2 \times \left(\frac{8}{2} \times d\right) \times 100}{\pi \frac{d^3}{8}}$$

$$d = 0.00451 \text{ m}$$

$$\therefore D = 8 \times 0.00451 = 0.03608 \text{ m} = 36.08 \text{ cm}$$



Lecture 11

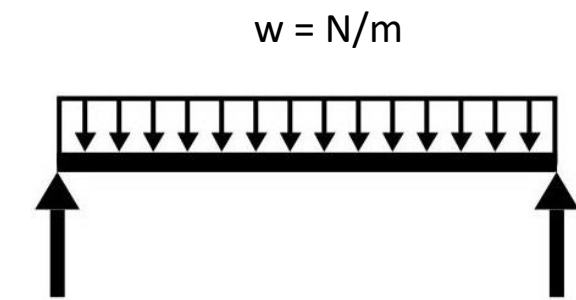
STRENGTH OF MATERIALS

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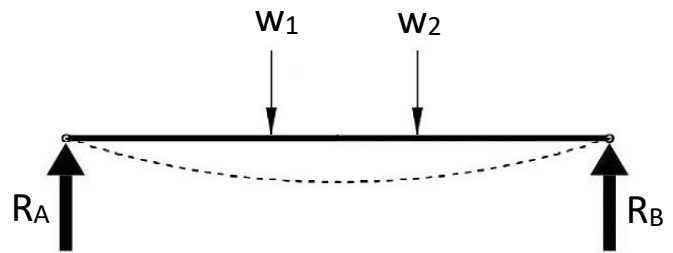
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Shearing Force & Bending Moment Diagram in beams



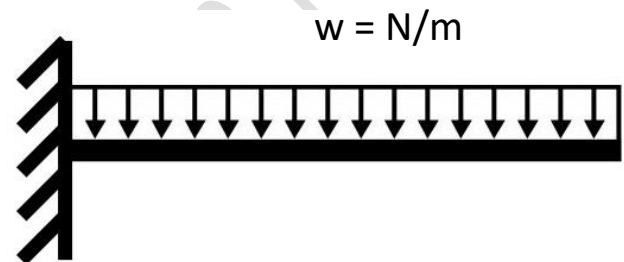
Simply supported beam with uniformly distributed load (U.d.l) w .



Simply supported beam with concentrated load w_1 and w_2 .



Cantilever beam with concentrated load w .



Cantilever beam with uniform distributed load w .

a) Simply supported beam

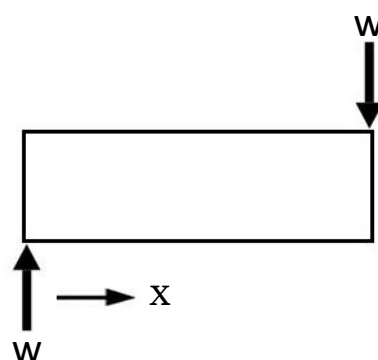
Shearing force (S.F.): - It is defined as the algebraic sum of the forces to the left or to the right of a section of the beam.

Bending Moment (B.M.): - It is defined as the algebraic sum of the moment of the forces acting to the left or to the right of a section of the beam.

Shearing Force and Bending Moment Diagram: - It gives the value and variation of the Shearing Force and Bending Moment at any section respectively

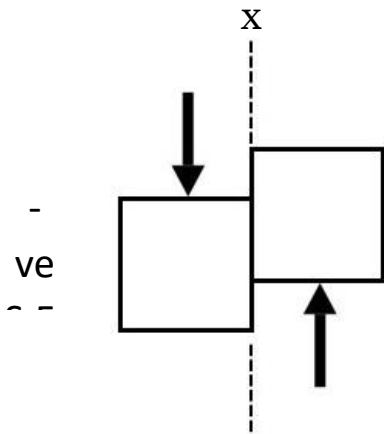
$$S.F. = w$$

$$B.M. = w \times x$$

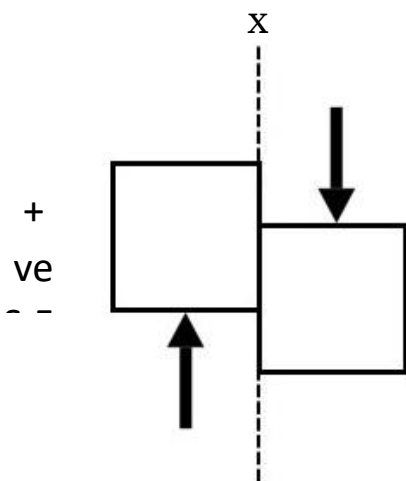


Sign convention: -

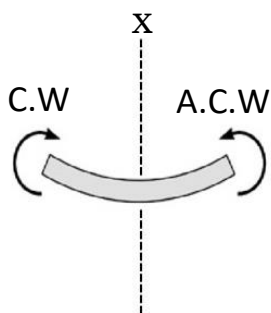
* عندما نحل أسئلة Shear Force & Bending Moment Diagram
(Sign convention) مباشرة نضع فرضيات الإشارات



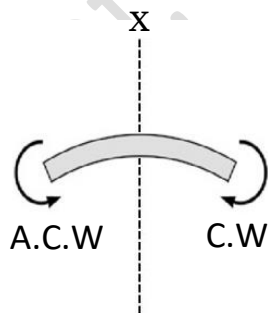
عندما تكون القوة الواقعة على يمين section line (الخط المنقط) الى الأعلى وكانت القوة الواقعة على يسار section line (الخط المنقط) الى الأسفل هذا يعني أن (shear force) تكون في الاتجاه السالب وتكون إشارتها سالبة (- ve)



وعندما تكون القوة الواقعة على يمين section line (الخط المنقط) الى الأسفل وكانت القوة الواقعة على يسار section line (الخط المنقط) الى الأعلى هذا يعني أن (shear force) تكون في الاتجاه الموجب وتكون إشارتها موجبة (+ ve)



Sagging beam

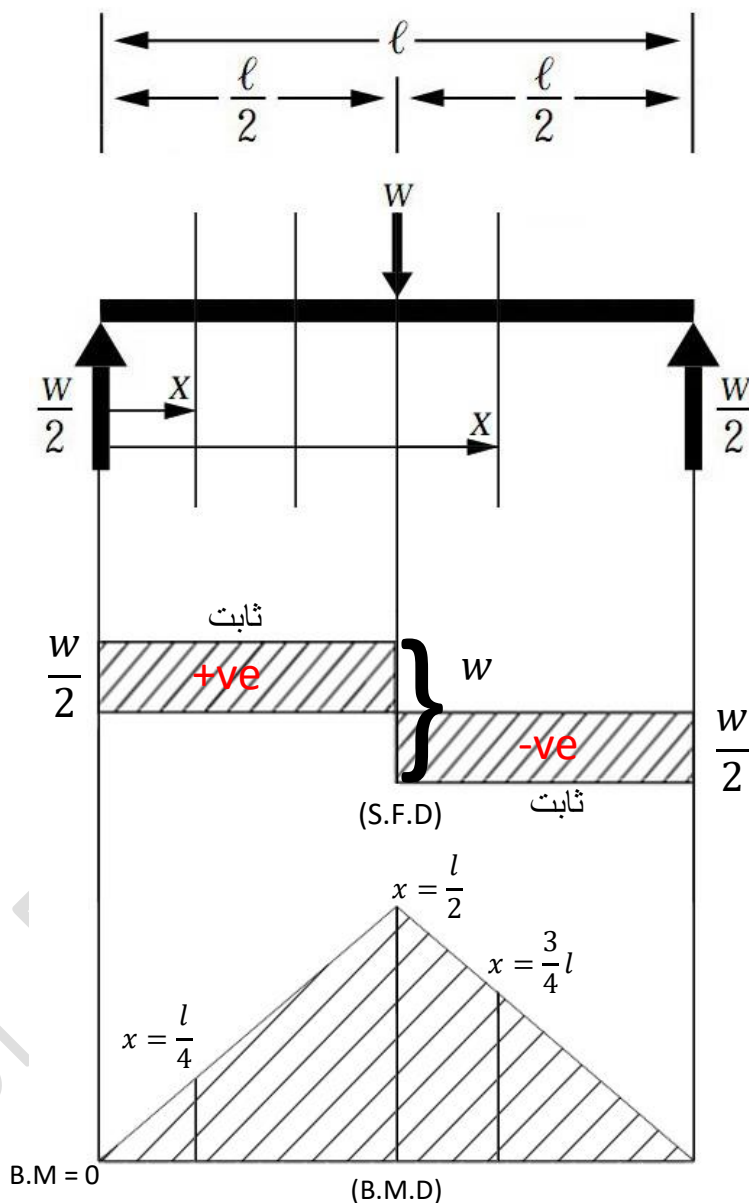


Hogging beam

عندما يكون moment الذي يقع على يمين section line (الخط المنقط) عكس عقارب الساعة وكان moment الواقع على يسار section line (الخط المنقط) مع عقارب الساعة هذا يعني أن إشارة (Bending Moment) تكون موجبة (+ ve)

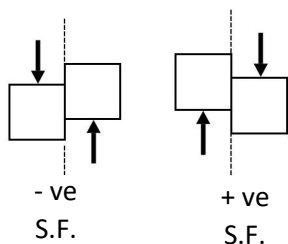
في حين عندما يكون moment الذي يقع على يمين section line (الخط المنقط) مع عقارب الساعة وكان moment الواقع على يسار section line (الخط المنقط) عكس عقارب الساعة هذا يعني أن إشارة (Bending Moment) تكون سالبة (- ve)

Ex: Find the shear force (S.F.) and bending moment (B.M.) diagram for the following beam



Sol:

1-



2-

When x lies between 0 and $\frac{l}{2}$

$$B.M = \frac{w}{2} \cdot x$$

* كل خط افقي في shear force يقابله خط مائل في bending moment

$$B.M = \frac{w}{2} \cdot 0 = 0 \text{ at } x = 0$$

$$B.M = \frac{wl}{4} \text{ at } x = \frac{l}{2}$$

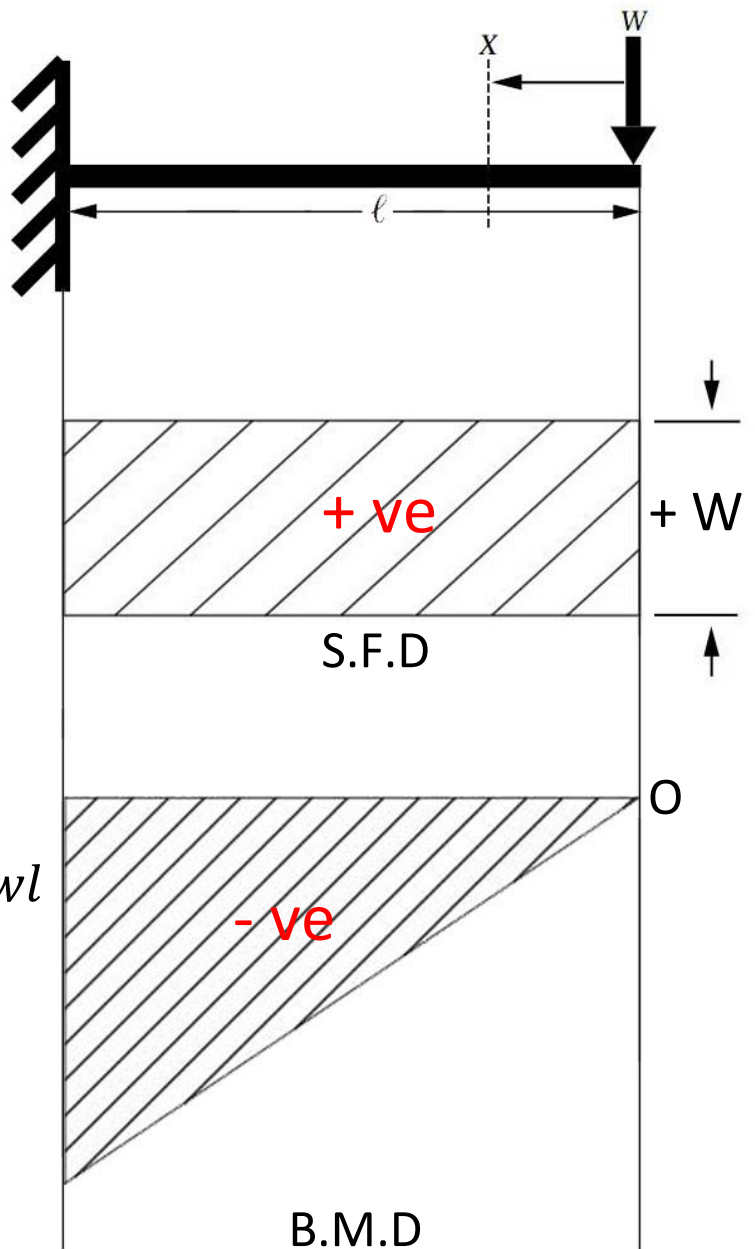
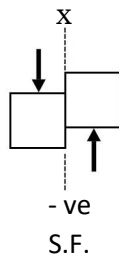
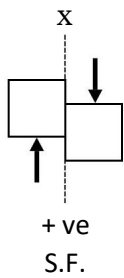
$$B.M = \frac{w}{2} \cdot \frac{l}{4} = \frac{wl}{8} \text{ at } x = \frac{l}{4}$$

$$\text{at } x = \frac{3}{4}l$$

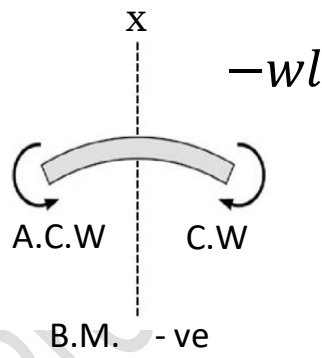
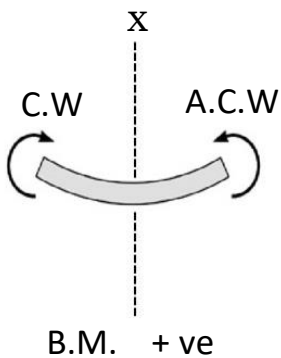
$$B.M = \frac{w}{2} \cdot \frac{3}{4}l - w \left(\frac{3}{4}l - \frac{l}{2} \right) = \frac{3wl}{8} - \frac{wl}{4} = \frac{3wl - 2wl}{8} = \frac{wl}{8}$$

b) Cantilever beam with concentrated load

S.F.



B.M



$$M_x = -wx$$

At $x = 0$

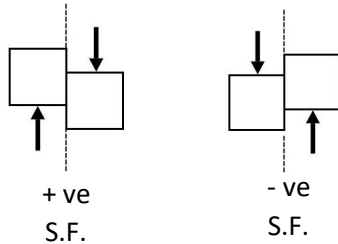
$$B.M. = -w \times 0 = 0$$

At $x = l$

$$B.M. = -w \times l = -wl$$

Ex: Draw the shear force (S.F.) and bending moment (B.M.) diagram for the following beam

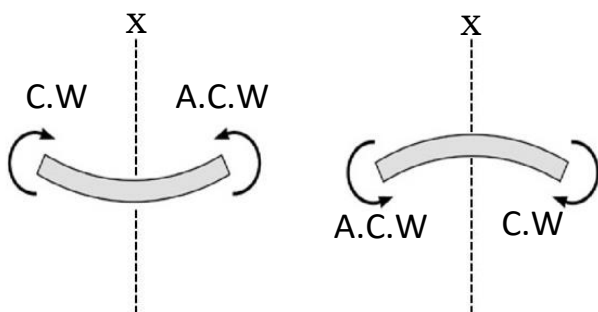
S.F.



S.F. at x between 0 and 1 = 5 kN

At x between 1 and 4

B.M.



$$M_x = -wx$$

At $x = 0$

$$B.M. = -5 \times 0 = 0$$

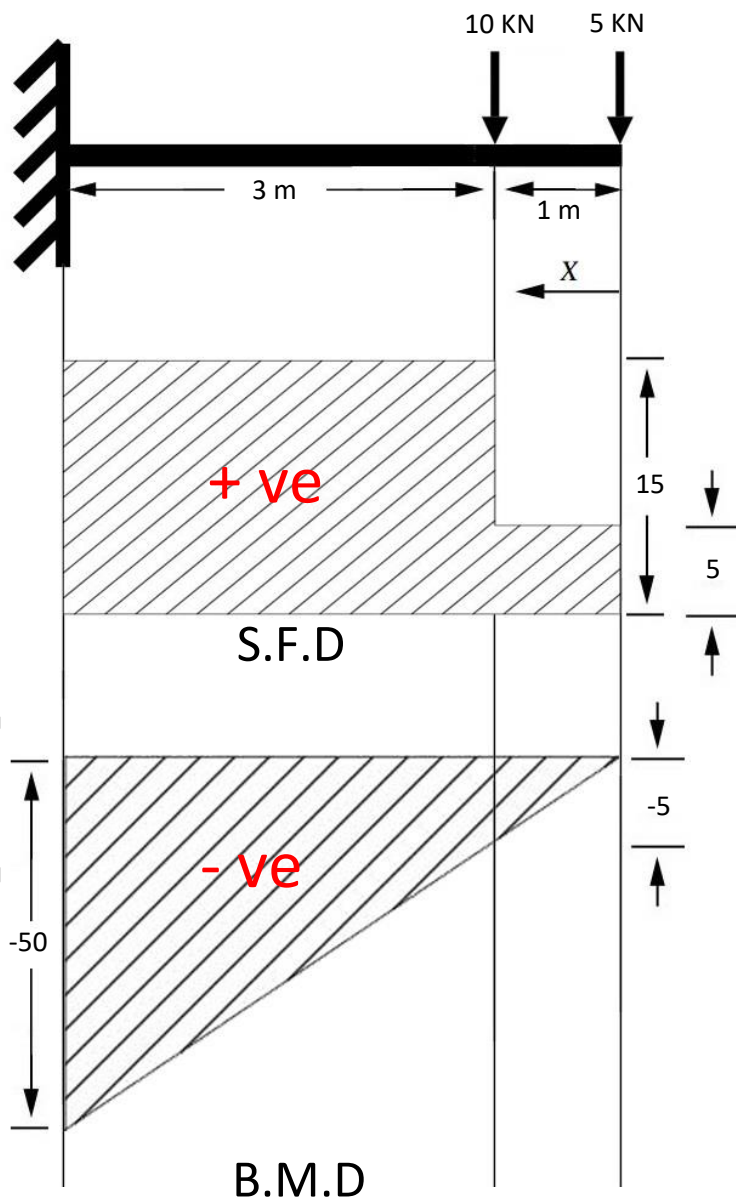
$$B.M. = -5 \times 1 = -5 \quad \text{At } x = 1$$

At x between 1 and 4

$$B.M. = -5 \times x - 10(x - 1)$$

$$B.M. = -5 \times 4 - 10(4 - 1) \quad \text{At } x = 4$$

$$B.M. = -20 - 30 = -50$$





Lecture 12

STRENGTH OF MATERIALS

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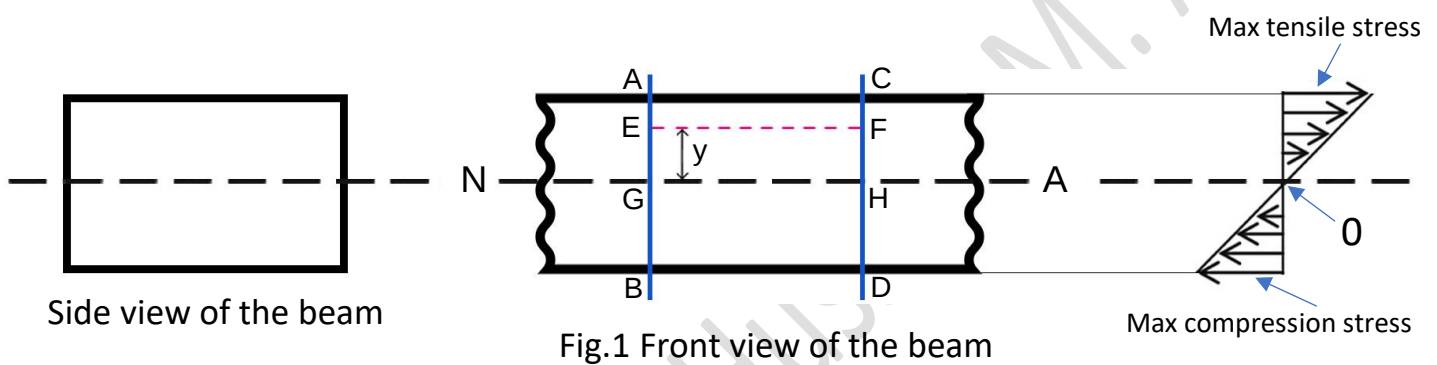
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2022-2023

Bending stress (σ_b)

Assumption: -

- 1) The beam is initially straight and unstressed.
- 2) The material of the beam is homogeneous.
- 3) The elastic limit is nowhere exceeded, i.e. stresses do not exceed the limit of proportionality.
- 4) Plane cross section remain plane before and after bending.
- 5) There is no resultant force perpendicular to any cross section.



Linear stress distribution with max tensile and compression stresses at the external fibers and zero at N.A (neutral axis)

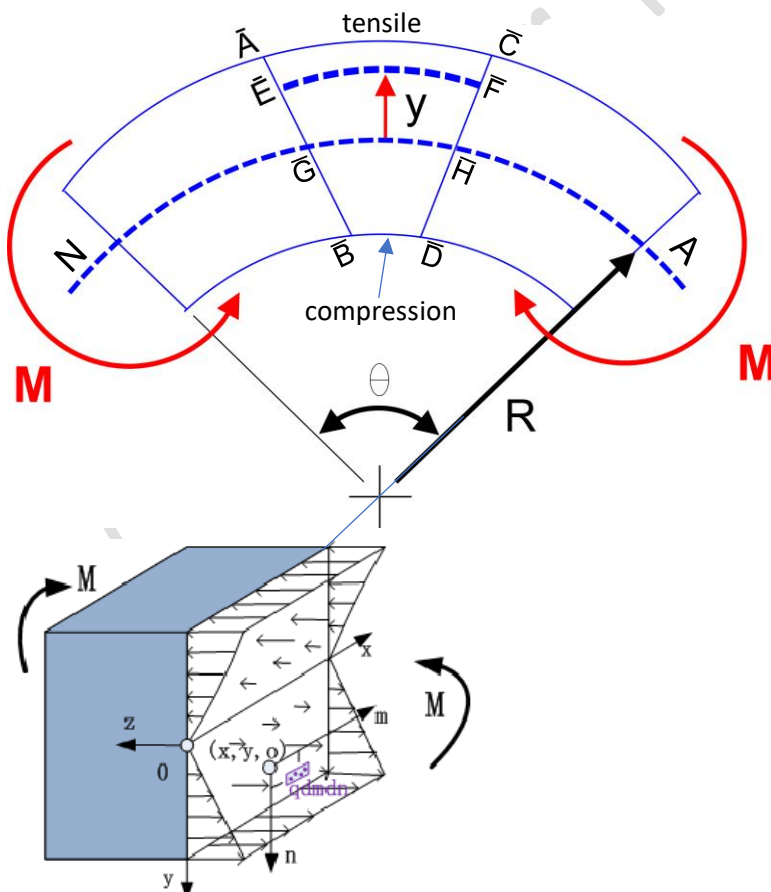


Fig. 2 stress distribution along the beam

Neutral axis (N.A) : هو الخط الذي يمر بالـ centroid للـ beam يعني الذي يمر بـ مركز ثقل الـ beam وعنده الـ stress يكون صفر

N.A represent the locus at which the stress is zero

R is the radius curvature of the N.A

Before bending, AB is parallel to CD

After bending AB and CD will subtend the angle θ

Consider the fiber EF, at a distance y from N.A. After bending, it will be stretched to $\bar{E}\bar{F}$

$$\therefore \text{strain in fiber } \bar{E}\bar{F} = \frac{\bar{E}\bar{F} - EF}{EF}$$

$EF = GH = \bar{G}\bar{H}$ because at the N.A the stress = zero

$$\therefore \text{strain in } EF = \frac{\bar{E}\bar{F} - EF}{EF} = \frac{(R + y)\theta - R\theta}{R\theta} = \frac{R\theta + y\theta - R\theta}{R\theta}$$

$$\therefore \text{strain in } EF = \frac{y}{R} \quad \text{————— (1)}$$

$$\text{strain} = \frac{\text{stress } (\sigma)}{\text{young's modulus } (E)} \quad \text{————— (2)}$$

If we substitute equation 1 in equation 2, we get

$$\frac{\sigma}{E} = \frac{y}{R}$$

$$\therefore \frac{\sigma}{y} = \frac{E}{R} \quad \text{————— (3)}$$

$$\therefore \sigma_b = \frac{E}{R} \cdot y \quad (\text{so } \sigma_b \text{ will vary linearly with } y)$$

لذلك فإن إجهاد الانحناء σ_b سيتغير تغيراً خطياً مع المسافة y

y = هي مقدار المسافة التي تبعد عن N.A

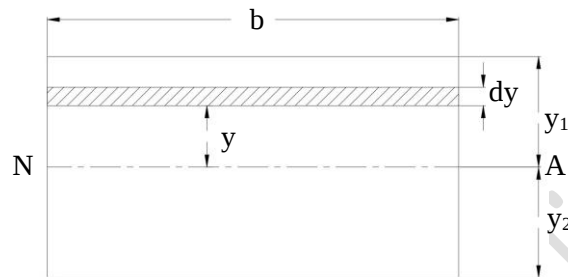
وفي حال لم يعطي في السؤال مقدار المسافة y فستكون قيمتها تساوي المسافة من N.A الى سطح الـ beam

Consider the area dA .

The stress on the strip of area dA , at a distance y from the N.A is: -

$$\sigma_b = \frac{E}{R} \cdot y$$

$$\text{Stress} = \frac{\text{Force}}{\text{Area}}$$



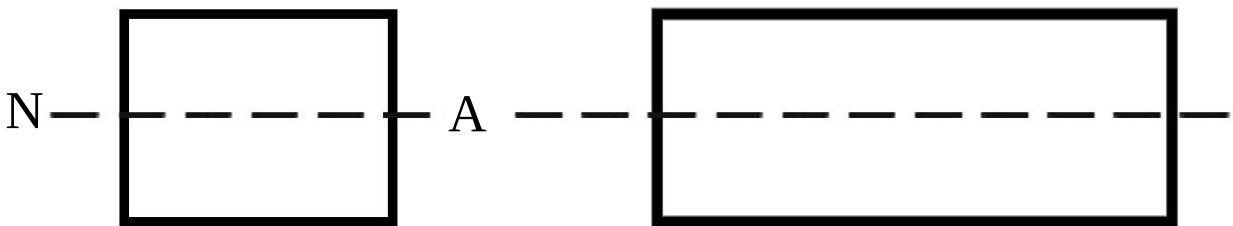
The force on the strip of area $dA = \sigma \times dA = \frac{E}{R} \cdot y \times dA$

Moment of the strip of area dA about N.A = $dM = \frac{E}{R} \cdot y \cdot dA \times y = \frac{E}{R} \cdot y^2 \cdot dA$

$\therefore$ Total moment for the whole cross-section = $\frac{E}{R} \sum y^2 dA$

Where $\sum y^2 \cdot dA$ represent the second moment of area

$$I = \sum y^2 \cdot dA = \int_{y_2}^{y_1} y^2 \cdot b \cdot dy = b \int_{y_2}^{y_1} y^2 \cdot dy$$



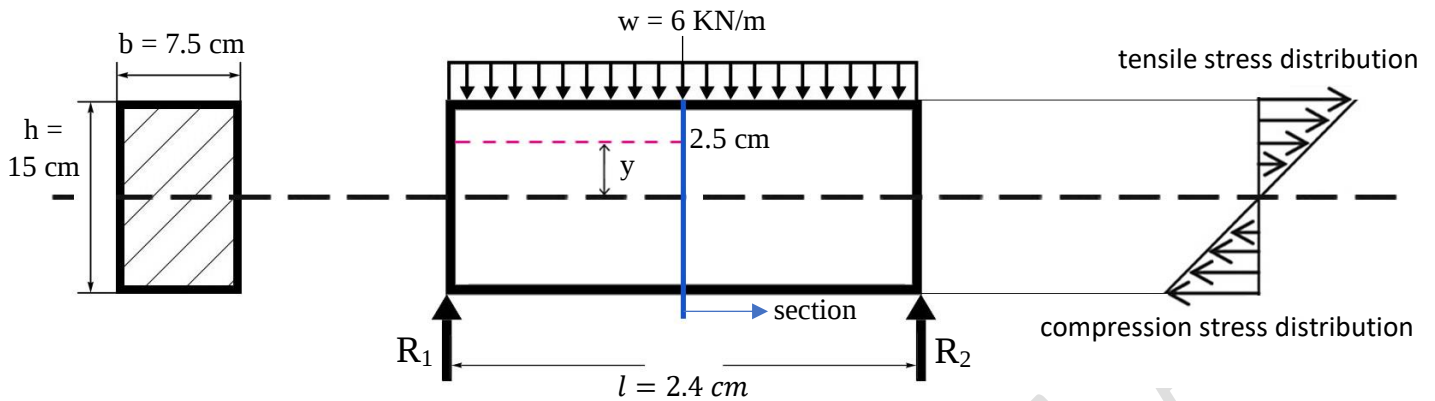
$$M = \frac{E}{R} \cdot I \longrightarrow \frac{M}{I} = \frac{E}{R}$$

$$\boxed{\frac{M}{I} = \frac{E}{R} = \frac{\sigma_b}{y}} \quad \text{————— (4)}$$

$E, I, M = \text{constant}$

R is the radius of curvature, where the beam will bend as a circular arc.

Ex: Determine the bending stress at a section midway between the supports at 2.5 cm below the upper surface at the beam.



Sol:

From equation 4 we get $\sigma_b = \frac{M}{I} \cdot y$

$$R_1 = R_2 = \frac{wl}{2}$$

$$B.M = R \times \frac{l}{2} - \frac{wl}{2} \times \frac{l}{4} = \frac{wl^2}{4} - \frac{wl^2}{8} = \frac{8wl^2 - 4wl^2}{32} = \frac{4wl^2}{32} = \frac{wl^2}{8}$$

$$B.M = \frac{6 \times 10^3 \times 2.4^2}{8} = 4320 \text{ N.m}$$

$$y = \frac{15}{2} - 2.5 = 5 \text{ cm}$$

$$I = \frac{bh^3}{12} = \frac{(7.5 \times 10^{-2}) \times (15 \times 10^{-2})^3}{12} = 2.109 \times 10^{-5}$$

$$\sigma_b = \frac{M}{I} \cdot y = \frac{4320}{2.109 \times 10^{-5}} \times 5 \times 10^{-2}$$

$$\sigma_b = 10.23 \text{ MN/m}^2$$

$$\text{bending stress } (\sigma_b) = \frac{M}{I} \cdot y$$

$$\sigma_{max} = \frac{M}{I} \cdot y_{max} = \frac{M}{\left(\frac{I}{y_{max}}\right)}$$

Where $\frac{I}{y_{max}} = z = \text{modulus of section}$ or **Section modulus**

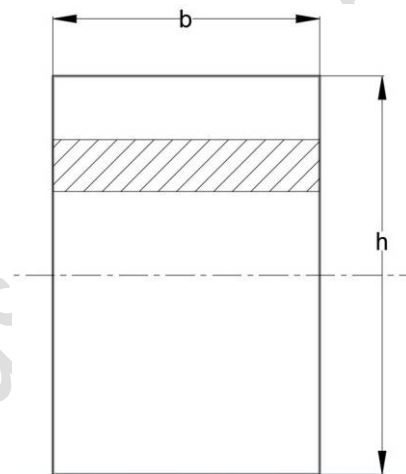
$$\therefore \sigma_{max} = \frac{M}{Z}$$

For rectangular section: -

$$I = \frac{1}{12} b h^3$$

$$y_{max} = \frac{h}{2}$$

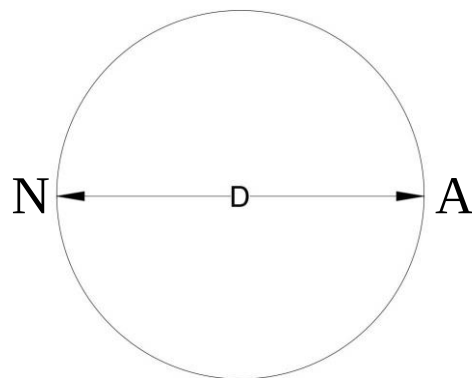
$$\therefore Z = \frac{I}{y_{max}} = \frac{\frac{b h^3}{12}}{\frac{h}{2}} = \frac{b h^2}{6}$$



For circular cross-section: -

$$I = \frac{\pi D^4}{64}, \quad y_{max} = \frac{D}{2}$$

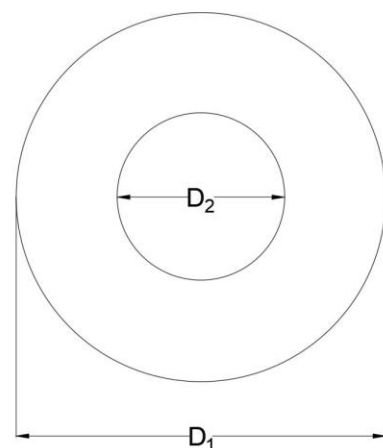
$$\therefore Z = \frac{\frac{\pi D^4}{64}}{\frac{D}{2}} = \frac{\pi D^3}{32}$$



For hollow circular beam: -

$$I = \frac{\pi(D_1^4 - D_2^4)}{64}, \quad y_{max} = \frac{D_1}{2}$$

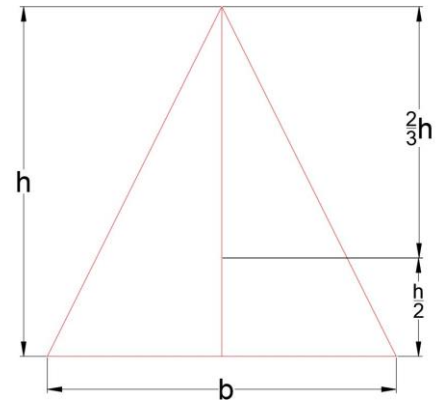
$$\therefore Z = \frac{\frac{\pi(D_1^4 - D_2^4)}{64}}{\frac{D_1}{2}} = \frac{\pi(D_1^4 - D_2^4)}{32 D_1}$$



For triangle

$$I = \frac{bh^3}{32}, \quad y_{max} = \frac{2}{3}h$$

$$\therefore Z = \frac{I}{y_{max}} = \frac{\frac{bh^3}{32}}{\frac{2}{3}h} = \frac{bh^2}{24}$$



Ex: For the beam shown, the allowable working stress in tensile and compression = 144 MN/m^2 , height of the beam $\{h = 2 \text{ width } (b)\}$. find b and h ?

Sol:

$$h = 2b$$

$$\sigma_b = \frac{M}{I} \cdot y$$

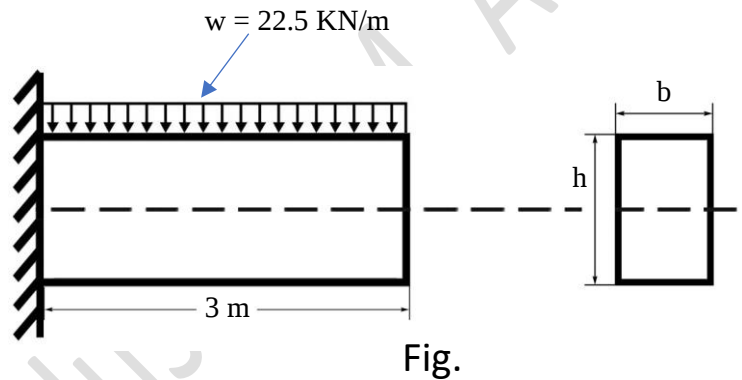
$$144 \times 10^6 = \frac{\frac{wl^2}{2}}{\frac{bh^3}{12}} \times \frac{h}{2}$$

$$\frac{144 \times 10^6}{10^4} = \frac{\left(\frac{22.5 \times 10^3}{2} \times 9\right) \times 10^2}{\frac{h}{2} \frac{h^3}{12}} \times \frac{h}{2}$$

$$h = 20.35 \text{ cm}$$

$$\therefore h = 2b$$

$$\therefore b = \frac{h}{2} = \frac{20.35}{2} = 10.175 \text{ cm}$$





Lecture 13

STRENGTH OF MATERIALS

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Moment of inertia for built up sections (I)

$$I = \frac{BD^3}{12} - 2 \left(\frac{bd^3}{12} \right)$$

Neutral axis (N.A) هذا الشكل متماثل حول

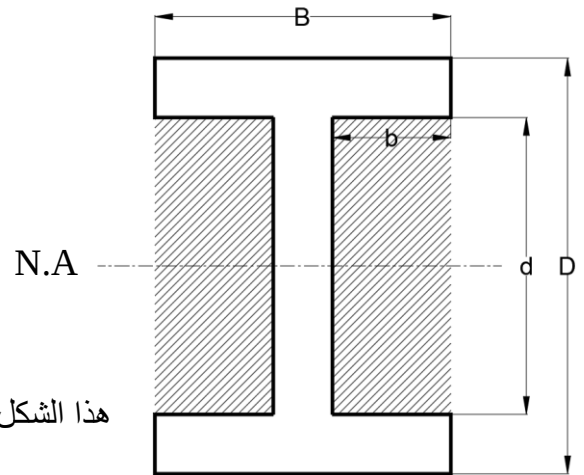


Fig. 1

$$I = I_{ABCD} - I_{\text{Shaded area about DC}} + I_{EFGH \text{ about GH}}$$

Neutral axis (N.A) هذا الشكل غير متماثل حول

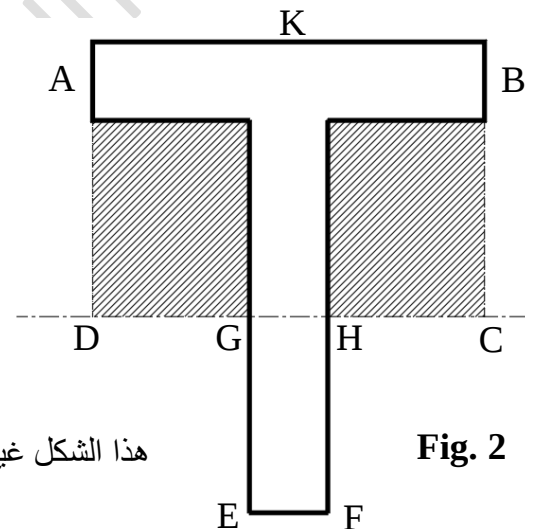


Fig. 2

Centroid of the area A is ($\bar{y}$)

$$A \cdot \bar{y} = \sum A \cdot y = A_1 y_1 + A_2 y_2$$

$$(A) \text{ total area} = A_1 + A_2$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} \longrightarrow \text{parallel axis theorem}$$

$$I_{XO} A = I_{XO} A_1 + I_{XO} A_2$$

$$= (I_{N.A} + A_1 d_1^2) + (I_{N.A} + A_2 d_2^2)$$

$$I_{XO} A = \frac{bd^3}{12} + (bd)d_1^2 + \frac{ca^3}{12} + (Ca)d_2^2$$

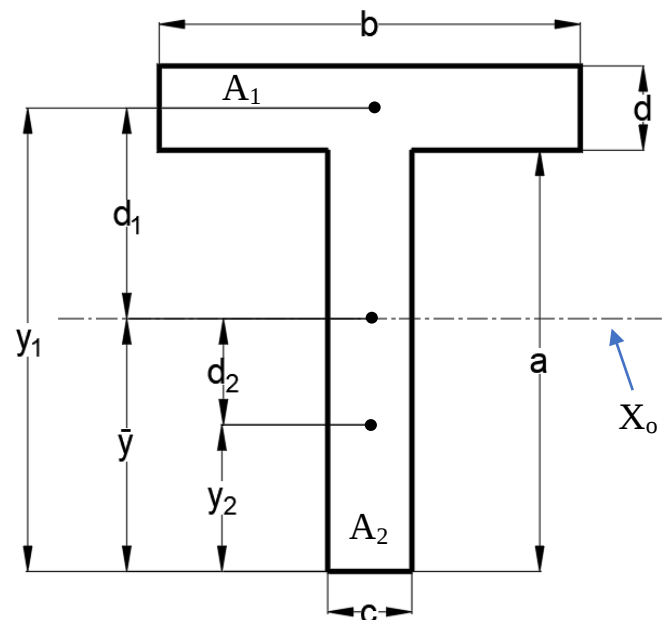


Fig. 3

Neutral axis (N.A) هذا الشكل غير متماثل حول



Lecture 14

STRENGTH OF MATERIALS

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Shaft subjected to bending moment and torque

1) To find stress due to bending effect

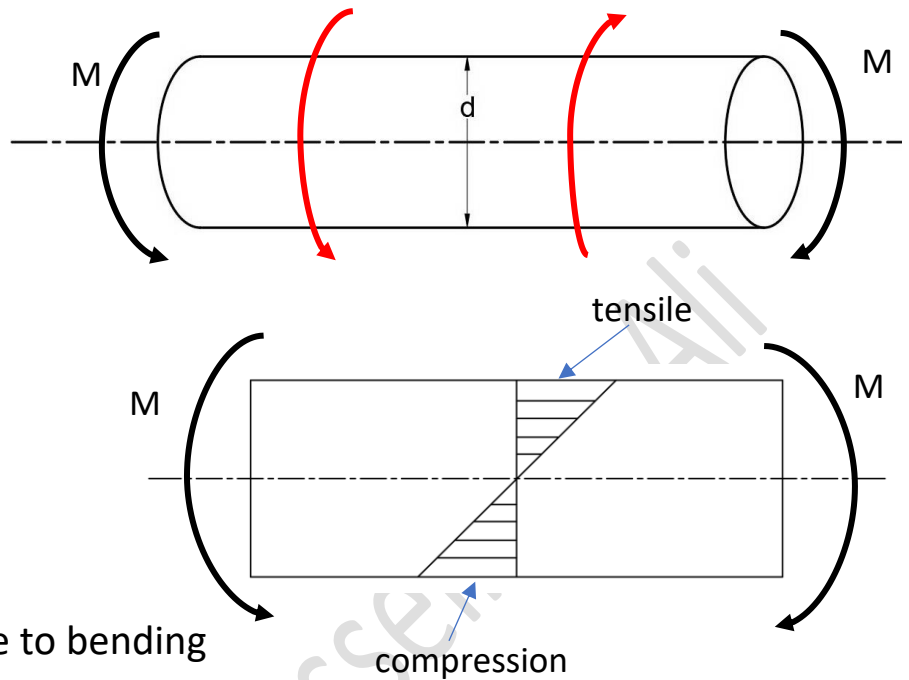
$$\frac{\sigma}{y} = \frac{M}{I}$$

$$I = \frac{\pi d^4}{64}, \quad y = \frac{d}{2}$$

$$\frac{\sigma}{y} = \frac{M}{I}$$

$$\sigma = \frac{My}{I} = \frac{M \cdot \frac{d}{2}}{\frac{\pi d^4}{64}}$$

$$\sigma = \frac{32 M}{\pi d^3} \quad \text{Direct stress due to bending}$$



2) To find stress due to torsion

$$\frac{\tau}{R} = \frac{T}{J}$$

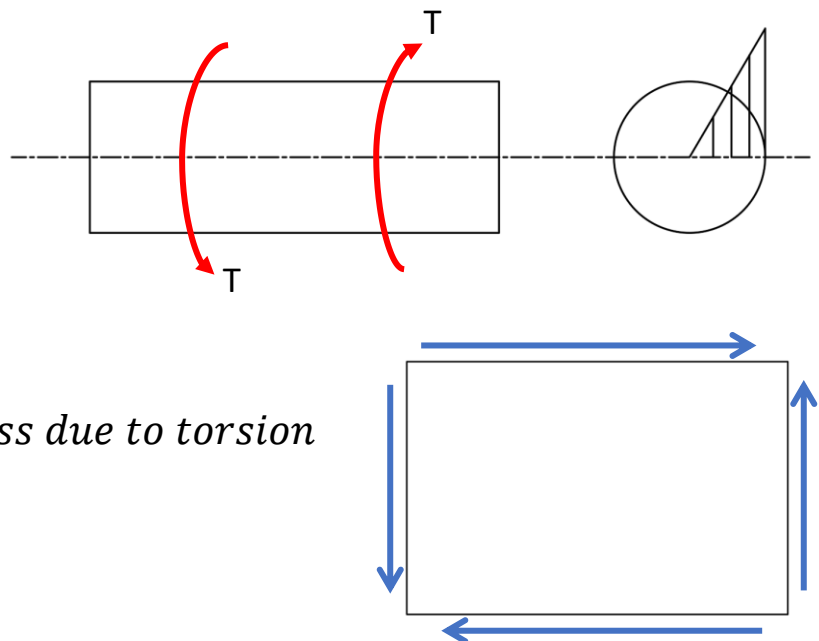
J = polar moment of inertia

$$J = \frac{\pi d^4}{32}$$

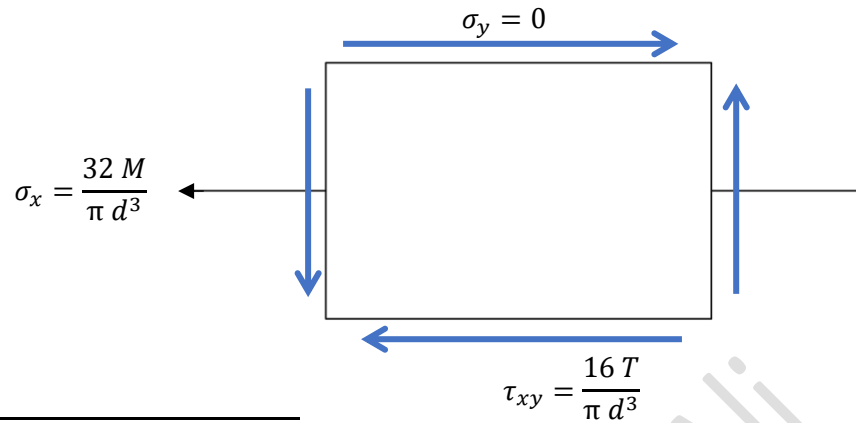
$$R = \frac{d}{2}$$

$$\tau = \frac{TR}{J} = \frac{T \cdot \frac{d}{2}}{\frac{\pi d^4}{32}}$$

$$\therefore \tau = \frac{16 T}{\pi d^3} \quad \text{shear stress due to torsion}$$



∴ An element at the top is loaded as shown: -



$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_y = 0$$

$$\sigma_1 = \frac{\sigma_x}{2} + \frac{1}{2} \sqrt{\sigma_x^2 + 4 \tau_{xy}^2}$$

$$\sigma_1 = \frac{32 M}{\pi d^3} \times \frac{1}{2} + \frac{1}{2} \sqrt{\left(\frac{32 M}{\pi d^3}\right)^2 + 4 \left(\frac{16 T}{\pi d^3}\right)^2}$$

Maximum shear stress on the shaft

$$\sigma_1 = \frac{32 M}{\pi d^3} \times \frac{1}{2} + \frac{1}{2} \times \frac{32}{\pi d^3} \sqrt{M^2 + T^2}$$

in the shaft

$$\sigma_1 = \frac{32}{\pi d^3} \left[\frac{1}{2} M + \frac{1}{2} \sqrt{M^2 + T^2} \right] = \sigma_{max} \quad \text{--- (1)}$$

Maximum bending stress (σ_{max}) on the shaft due to the combined torque and bending moment

$$\tau_{max} = \frac{1}{2} \sqrt{\left(\frac{32 M}{\pi d^3}\right)^2 + 4 \left(\frac{16 T}{\pi d^3}\right)^2}$$

$$\tau_{max} = \frac{1}{2} \times \frac{32}{\pi d^3} \sqrt{M^2 + T^2}$$

$$\therefore \tau_{max} = \frac{16}{\pi d^3} \sqrt{M^2 + T^2}$$

Maximum shear stress on the shaft due to the combined torque and bending moment

1- Equivalent bending moment (Me)

Let (Me) be the equivalent bending moment (that bending moment if acting alone will cause the same direct stress which is caused by the combined effect of (M & T))

$$\frac{Me}{I} = \frac{\sigma_{max}}{y}$$

$$I = \frac{\pi d^4}{64} \quad , \quad y = \frac{d}{2}$$

$$\frac{Me}{\frac{\pi d^4}{64}} = \frac{\sigma_{max}}{\frac{d}{2}}$$

$$Me = \frac{\pi d^4}{64} \times \frac{\sigma_{max}}{\frac{d}{2}}$$

$$\sigma_{max} = \frac{32 Me}{\pi d^3} \quad \text{Put it in equation (1), we get:}$$

$$Me = \frac{1}{2} \left[M + \sqrt{M^2 + T^2} \right]$$

2- Equivalent torque (T_e)

Let (T_e) be the equivalent torque (That torque if acting alone will cause the same maximum shear stress which caused by the combined effect of (M & T))

$$\frac{T_e}{J} = \frac{\tau_{max}}{R}$$

$$J = \frac{\pi d^4}{32}$$

$$R = \frac{d}{2}$$

$$T_e = \frac{\pi d^4}{32} \times \frac{\tau_{max}}{\frac{d}{2}}$$

$$\frac{16 T_e}{\pi d^3} = \tau_{max} \quad \text{————— (2)}$$

$$\text{But } \tau_{max} = \frac{1}{2} \sqrt{\left(\frac{32 M}{\pi d^3}\right)^2 + 4 \left(\frac{16 T}{\pi d^3}\right)^2} = \frac{16}{\pi d^3} \sqrt{M^2 + T^2} \quad \text{————— (3)}$$

From 2 and 3

$$\frac{16 T_e}{\pi d^3} = \frac{16}{\pi d^3} \sqrt{M^2 + T^2}$$

$$\therefore T_e = \sqrt{M^2 + T^2}$$

Ex: A hollow shaft of 200mm outside diameter and 125 mm bore is subjected simultaneously to a B.M of 43 kN.m and a torque of 65 kN.m. calculate the bending stress and the torsional shear stress. Hence find the max. shear stress in the shaft due to the combined torque and bending moment.

Sol:

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

$$I = \frac{\pi(D_o^4 - D_i^4)}{64} = \frac{\pi(0.2^4 - 0.125^4)}{64} = 6.65 \times 10^{-3} \text{ m}^4$$

$$y = \frac{D_o}{2} = \frac{200}{2} = 0.1 \text{ m}$$

$$\sigma_b = \frac{My}{I} = \frac{43 \times 10^3 \times 0.1}{6.65 \times 10^{-3}} = 64.6 \text{ MN/m}^2$$

$$\frac{\tau}{R} = \frac{T}{J}$$

$$J = \text{polar moment of inertia} = \frac{\pi d^4}{32} = 13.3 \times 10^{-5} \text{ m}^4$$

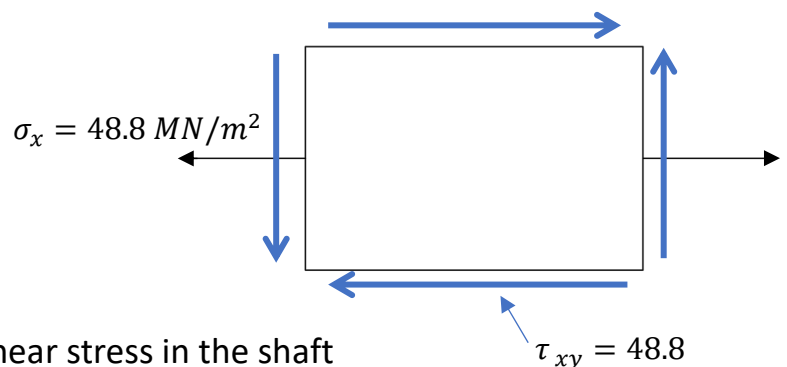
$$R = \frac{D_o}{2} = 0.1 \text{ m}$$

$$\tau = \frac{TR}{J} = \frac{65 \times 10^3 \times 0.1}{13.3 \times 10^{-5}} = 48.8 \text{ MN/m}^2 \text{ shear stress due to torsion}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{max} = \sqrt{\left(\frac{64.6}{2}\right)^2 + (48.8)^2}$$

$$\tau_{max} = 58.52 \text{ MN/m}^2 \text{ max shear stress in the shaft}$$





Lecture 15

STRENGTH OF MATERIALS

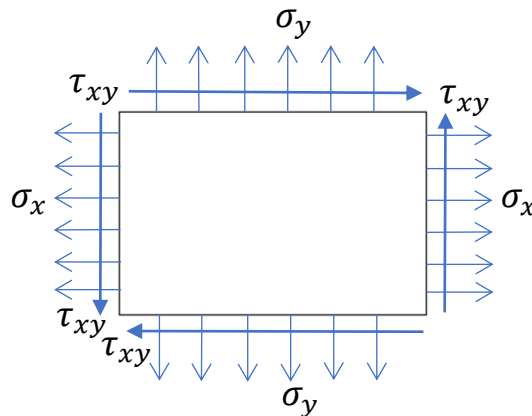
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Material subjected to direct and shear stress

If the material is subjected to the two types of stresses at the same time as shown in fig.



the max. and min. stress will be find as follows:

$$\sigma_{\max, \min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{\max} = \sigma_1 \quad , \quad \sigma_{\min} = \sigma_2$$

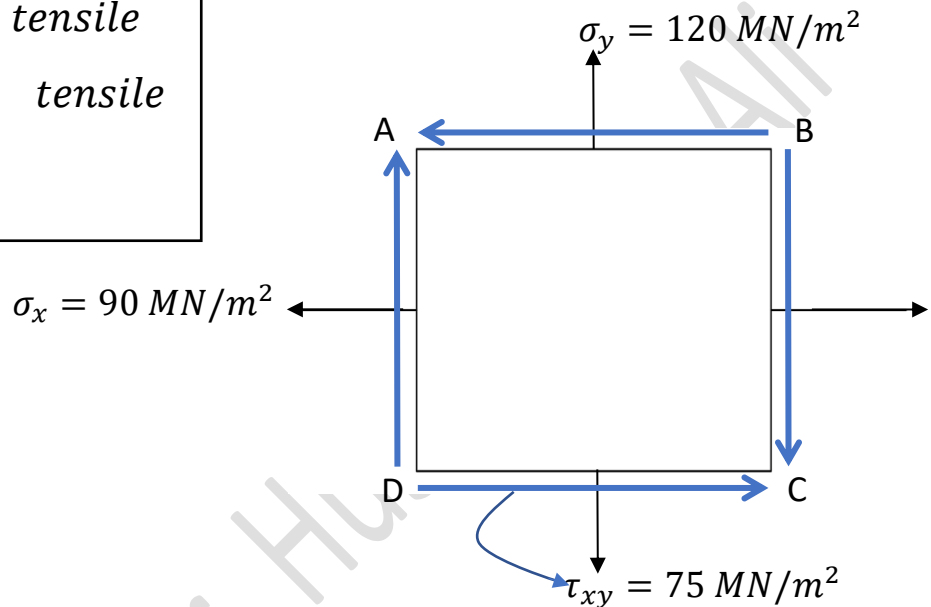
$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

These stresses are termed as the principal stresses of the system acting on the mutually perpendicular planes called principal planes.

Ex: Under certain condition of loading the stresses in the walls of a cylinder are as shown below. Calculate:

- The principal stresses (σ_1, σ_2)
- The max shear stress (τ_{max})
- shear stress and the principal stress if σ_x & σ_y is zero
- max shear stress & direct stress if σ_x & σ_y is zero

$\sigma_x = 90 \text{ MN/m}^2$	<i>tensile</i>
$\sigma_y = 120 \text{ MN/m}^2$	<i>tensile</i>
$\tau_{xy} = 75 \text{ MN/m}^2$	



Sol:

a)

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = \frac{90 + 120}{2} + \sqrt{\left(\frac{90 - 120}{2}\right)^2 + 75^2}$$

$$\sigma_1 = 105 + 76.5 = 181.5 \text{ MN/m}^2 \longrightarrow \textit{tensile}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_2 = 105 - 76.5 = 28.5 \text{ MN/m}^2 \longrightarrow \textit{tensile}$$

b)

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 76.5 \text{ MN/m}^2$$

c)

$$\sigma_x = 0 \quad , \quad \sigma_y = 0$$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = \sqrt{\tau_{xy}^2} = \tau_{xy}$$

$$181.5 = \tau_{xy} \quad \therefore \tau_{xy} = 181.5 \text{ MN/m}^2$$

$$\sigma_2 = 181.5 \text{ MN/m}^2$$

$$\sigma_y = 0$$



d)

$$\sigma_y = 0 \quad , \quad \tau_{xy} = 0$$

$$\sigma_x = ? \quad \tau_{max} = 76.5 \text{ MN/m}^2$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$76.5 = \sqrt{\left(\frac{\sigma_x}{2}\right)^2} = \frac{\sigma_x}{2}$$

$$\sigma_x = 153 \text{ MN/m}^2$$

