Engineering Mechanics

Statics

Assist. Professor Tariq Khalid

Engineering Technical College/ Northern Technical University

General Principles:

Mechanics is a branch of the physical sciences that is concerned with the state of rest or motion of bodies that are subjected to the action of forces. In general, this subject can be subdivided into three branches: *rigid-body mechanics*, *deformable-body mechanics*, and *fluid mechanics*. we will study rigid-body mechanics Rigid-body mechanics is divided into two areas: statics and dynamics. ***Statics*** deals with the equilibrium of bodies, that is, those that are either at rest or move with a constant velocity; whereas ***dynamics*** is concerned with the accelerated motion of bodies. We can consider statics as a special case of dynamics, in which the acceleration is zero;

Fundamental Concepts:

Basic Quantities. The following four quantities are used throughout mechanics.

Length. ***Length*** is used to locate the position of a point in space and thereby describe the size of a physical system.

Time. ***Time*** is conceived as a succession of events. This quantity plays an important role in the study of dynamics.

Mass. ***Mass*** is a measure of a quantity of matter.

Force. In general, ***force*** is considered as a "push" or "pull" exerted by one body on another.

Idealizations. Models or idealizations are used in mechanics in order to simplify application of the theory. Here we will consider three important idealizations.

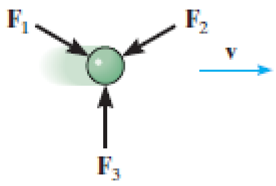
Particle. A ***particle*** has a mass, but a size that can be neglected. For example, the size of the earth is insignificant compared to the size of its orbit.

Rigid Body. A ***rigid body*** can be considered as a combination of a large number of particles in which all the particles remain at a fixed distance from one another, both before and after applying a load.

Concentrated Force. A ***concentrated force*** represents the effect of a loading which is assumed to act at a point on a body. We can represent a load by a concentrated force, provided the area over which the load is applied is very small compared to the overall size of the body.

Newton's Three Laws of Motion

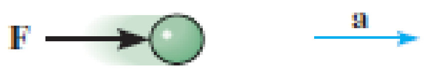
First Law: A particle originally at rest, or moving in a straight line with constant velocity, tends to remain in this state provided the particle is *not* subjected to an unbalanced force.



Equilibrium

Second Law: A particle acted upon by an *unbalanced force* **F** experiences an acceleration **a** that has the same direction as the force and a magnitude that is directly proportional to the force. Fig. 1–1*b*. If **F** is applied to a particle of mass *m*, this law may be expressed mathematically as:

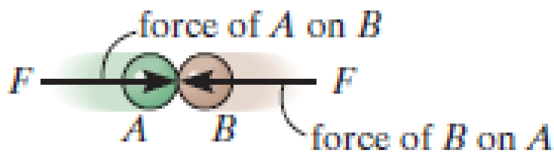
$$\mathbf{F} = m\mathbf{a} \qquad (1\text{–}1)$$



Accelerated motion

(b)

Third Law: For every action, there is an equal, opposite, and collinear reaction.



Action – reaction

Newton's Law of Gravitational Attraction

$$F = G\frac{m_1 m_2}{r^2}$$

where

F = force of gravitation between the two particles

G = universal constant of gravitation; according to experimental evidence,

$G = 66.73(10^{-12})\ m^3/(kg \cdot s^2)$

$m1$, $m2$ = mass of each of the two particles

r = distance between the two particles

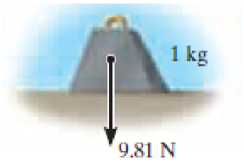
Units of Measurement

The four basic quantities—length, time, mass, and force

SI Units: The International System of units, abbreviated SI after the French "Système International d'Unités,". The SI system defines length in meters (m), time in seconds (s), and mass in kilograms (kg). The unit of force, called a ***newton*** (N), is *derived* from **F** = *m***a**. Thus, 1 newton is equal to a force required to give1 kilogram of mass an acceleration of 1 m/s^2 ($N = kg.\ m/s^2$).

$$W = mg \qquad (g = 9.81 \text{ m/s}^2)$$

Therefore, a body of mass 1 kg has a weight of 9.81 N, a 2-kg body weighs 19.62 N, and so on, Fig. 1–2*a*.



(a)

U.S. Customary: In the U.S. Customary system of units (FPS) length is measured in feet (ft), time in seconds (s), and force in pounds (lb), Table 1–1. The unit of mass, called a ***slug***, is *derived* from **F** = *m***a**. Hence, 1 slug is equal to the amount of matter accelerated at 1 ft/s^2 when acted upon by a force of 1 lb. (slug = lb. s^2/ft). Where $\boldsymbol{g = 32.2}$ **ft/s^2**,
And so a body weighing 32.2 lb has a mass of 1 slug, a 64.4-lb body has a mass of 2 slugs, and so on, Fig. 1–2*b*.

(b)

Fig. 1–2

TABLE 1–1 Systems of Units

Name	Length	Time	Mass	Force
International System of Units SI	meter m	second s	kilogram kg	$\boxed{\text{newton*}}$ N $\left(\frac{\text{kg}\cdot\text{m}}{\text{s}^2}\right)$
U.S. Customary FPS	foot ft	second s	$\boxed{\text{slug*}}$ $\left(\frac{\text{lb}\cdot\text{s}^2}{\text{ft}}\right)$	pound lb

Conversion Tables

Multiply	by	To Obtain / Multiply	by	To Obtain
centimetres (cm)	0.3937	inches (in)	2.54	centimetres (cm)
metres (m)	3.2808	feet (ft)	0.3048	metres (m)
metres (m)	39.37	inches (in)	0.0254	metres (m)
square metres (m^2)	10.76	square feet (sq. ft.)	0.0929	square metres (m^2)
cubic metres (m^3)	35.314	cubic feet (c.f.)	0.0283	cubic metres (m^3)
kilogram (kg)	2.205	pound (lb)	0.4536	kilogram (kg)

Prefixes: When a numerical quantity is either very large or very small, the units used to define its size may be modified by using a prefix.

TABLE 1–3 Prefixes

	Exponential Form	Prefix	SI Symbol
Multiple			
1 000 000 000	10^9	giga	G
1 000 000	10^6	mega	M
1 000	10^3	kilo	k
Submultiple			
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	μ
0.000 000 001	10^{-9}	nano	n

EXAMPLE 1.1

Convert 2 km/h to m/s How many ft/s is this?

SOLUTION

Since 1 km = 1000 m and 1 h = 3600 s, the factors of conversion are arranged in the following order, so that a cancellation of the units can be applied:

$$2\ \text{km/h} = \frac{2\ \cancel{\text{km}}}{\cancel{\text{h}}}\left(\frac{1000\ \text{m}}{\cancel{\text{km}}}\right)\left(\frac{1\ \cancel{\text{h}}}{3600\ \text{s}}\right)$$
$$= \frac{2000\ \text{m}}{3600\ \text{s}} = 0.556\ \text{m/s} \qquad \textit{Ans.}$$

From Table 1–2, 1 ft = 0.3048 m. Thus,

$$0.556\ \text{m/s} = \left(\frac{0.556\ \cancel{\text{m}}}{\text{s}}\right)\left(\frac{1\ \text{ft}}{0.3048\ \cancel{\text{m}}}\right)$$
$$= 1.82\ \text{ft/s} \qquad \textit{Ans.}$$

NOTE: Remember to round off the final answer to three significant figures.

Force Vectors

Scalar: A ***scalar*** is any positive or negative physical quantity that can be completely specified by its *magnitude*. Examples of scalar quantities include length, mass, and time.

Vector: A ***vector*** is any physical quantity that requires both a *magnitude* and a *direction* for its complete description. Examples of vectors encountered in statics are force, position, and moment. A vector is shown graphically by an arrow. The length of the arrow represents the *magnitude* of the vector, and the angle θ between the vector and a fixed axis defines the *direction of its line of action*. The head or tip of the arrow indicates the *sense of direction* of the vector, Fig. 2–1.
In print, vector quantities are represented by boldface letters such as
A, and the magnitude of a vector is italicized, *A*. For handwritten work, it is often convenient to denote a vector quantity by simply drawing an arrow above it,

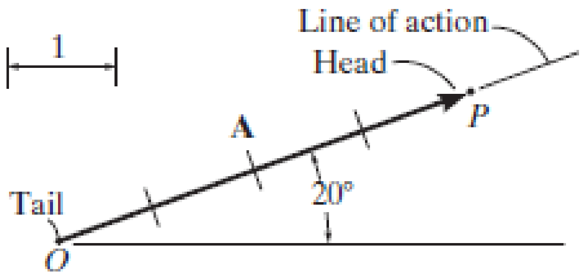
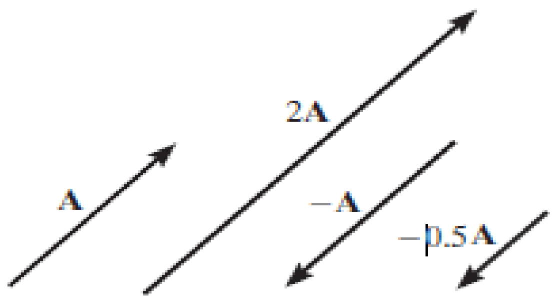


Fig. 2–1



Scalar multiplication and division

Fig. 2–2

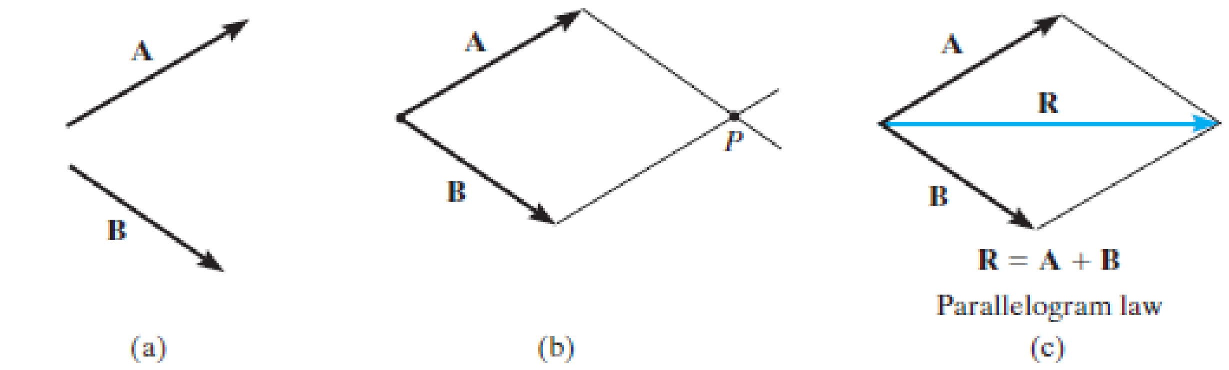


Fig. 2–3

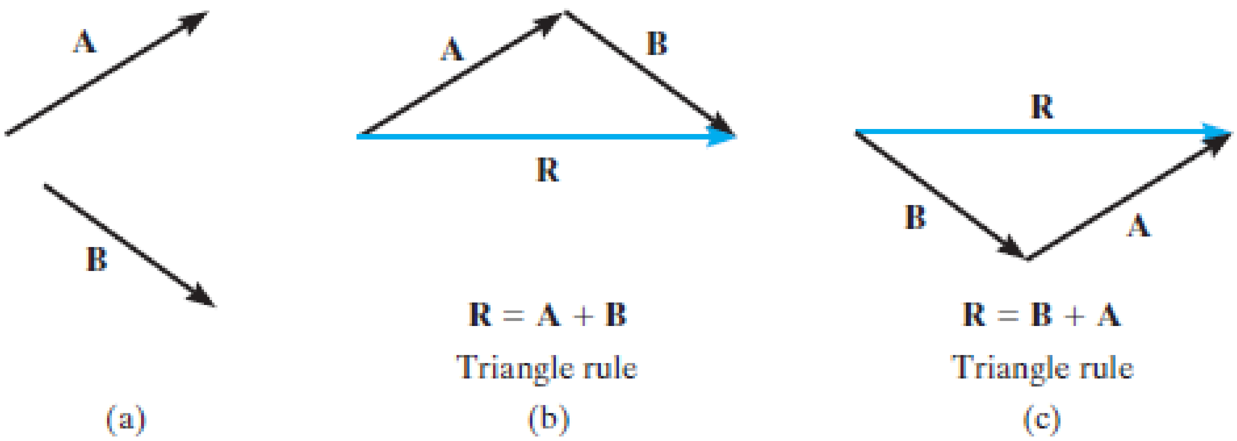


Fig. 2–4

As a special case, if the two vectors **A** and **B** are ***collinear***, i.e., both have the same line of action, the parallelogram law reduces to an *algebraic* or *scalar addition* $R = A + B$, as shown in Fig. 2–5.

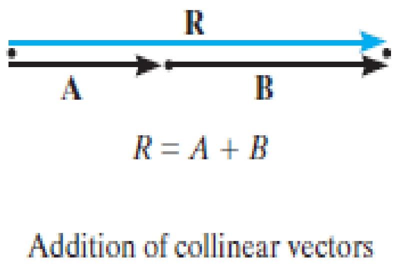
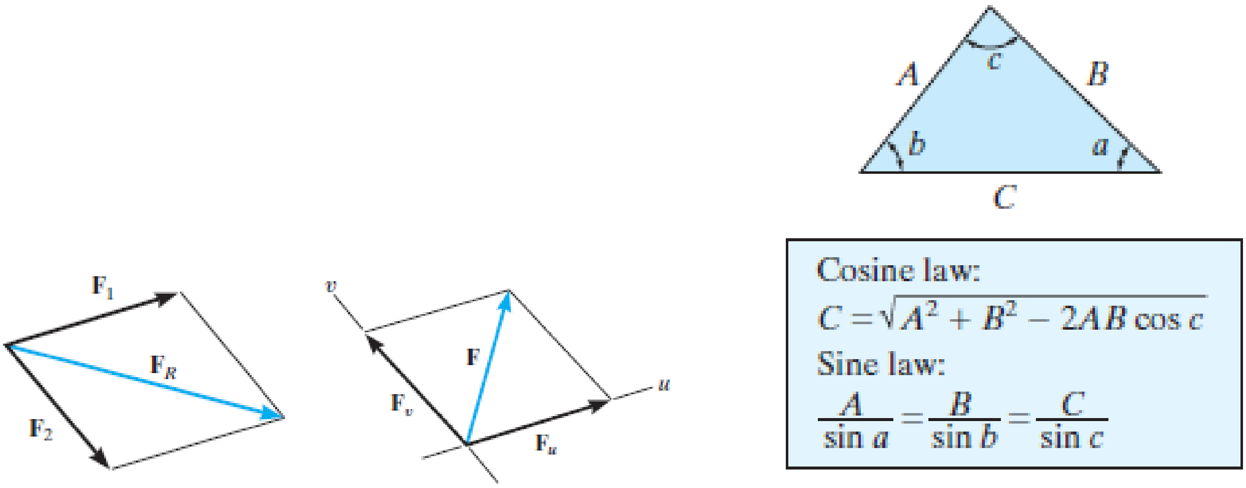
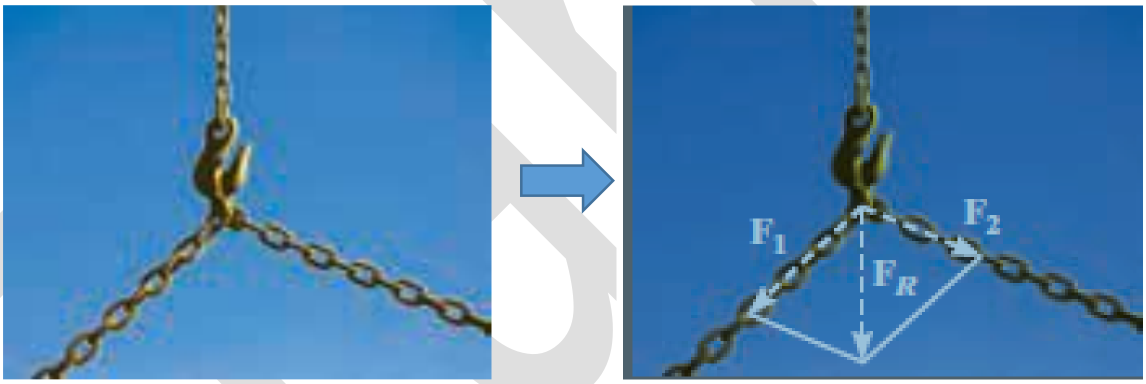


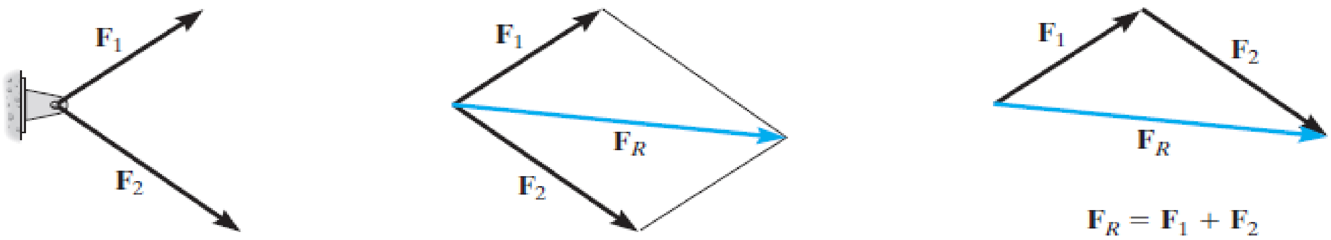
Fig. 2–5



(a) (b) (c)

Finding a Resultant Force





(a) (b) (c)

Fig. 2–7

Resultant force

forces Parallelogram & forces triangle

For triangle (fig.1)

$\alpha + \beta + \theta = 180^\circ$

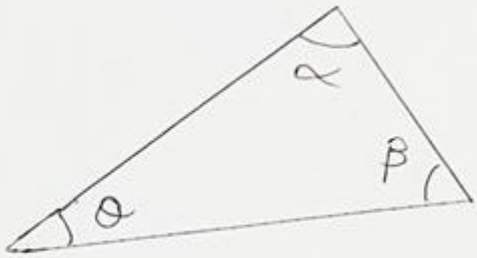


fig. 1

for rectangle (fig. 2)

$\alpha + \beta + \theta + \phi = 360^\circ$

& $\alpha = \beta = \theta = \phi = 90$

fig. 2

for parallelogram (fig. 3)

$\alpha + \beta + \theta + \phi = 360^\circ$

& $\alpha = \theta$, $\beta = \phi$

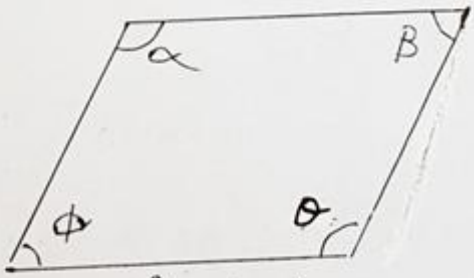


fig. 3

for (fig. 4)

$\alpha = \theta$ & $\phi = \beta$

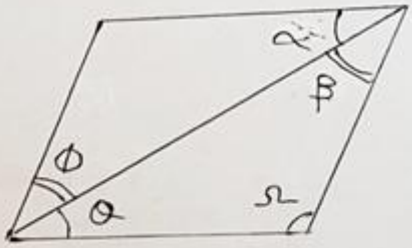


fig. 4

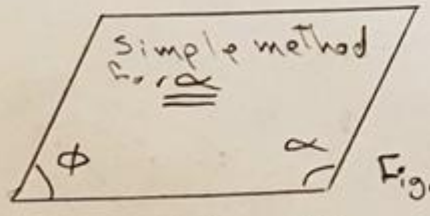
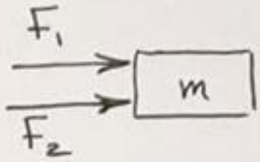


Fig. 2.1

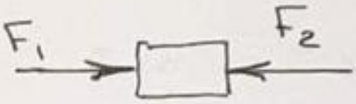
from Fig 2 & Fig. 2.1 $\phi + \alpha = 180$

To find the resultant force for two forces

1- Case A



$F_R = F_1 + F_2$



2- Case B

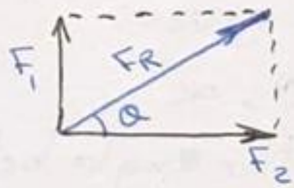
$F_R = F_1 - F_2$

if the result is negative, it means the direction of F_R is with F_2

3- Case C

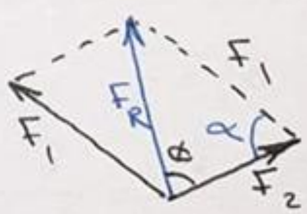
Apply Pythagorean Theom

$F_R = \sqrt{F_1^2 + F_2^2}$



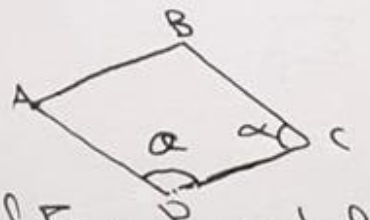
4- Case D

1- Draw the two forces
2- Find the angle betwen the two forces θ



3- refer to the parallelogram ABCD

$\alpha = 180 - \theta$

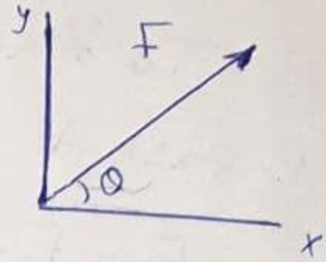


4- $F_R = \sqrt{F_1^2 + F_2^2 - 2F_1F_2\cos\alpha}$

5- To find the direction of F_R you must find angle ϕ and the direction angle $= \beta + \phi$

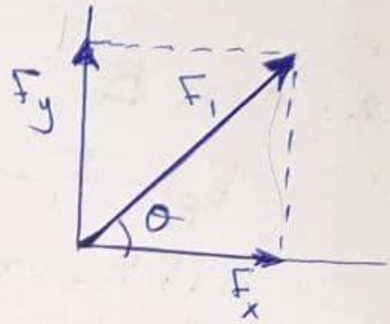
$$\frac{\sin\alpha}{F_R} = \frac{\sin\phi}{F_1}$$

To resolve an inclined Force



$F_x = F\cos\theta$

$F_y = F\sin\theta$

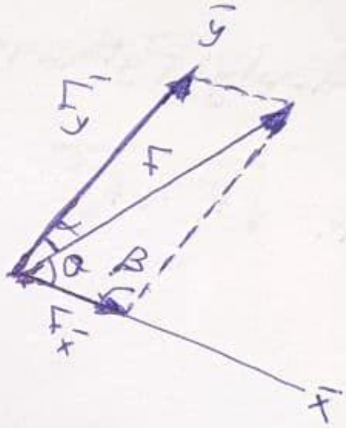


To resolve a force into two non-perpendicular force

1- Complete the parallelogram

data given

F, θ, α

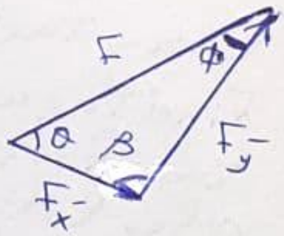


2- Find the angle between the two axes $= \theta + \alpha$

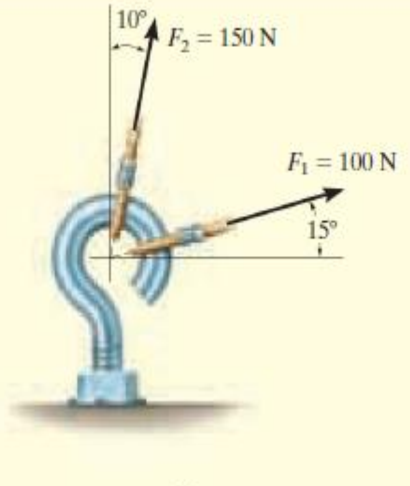
3- Find the angle infront F i.e (β), then find ϕ

4- Use the Sine law to Find $F_{\bar{x}}$ & $F_{\bar{y}}$

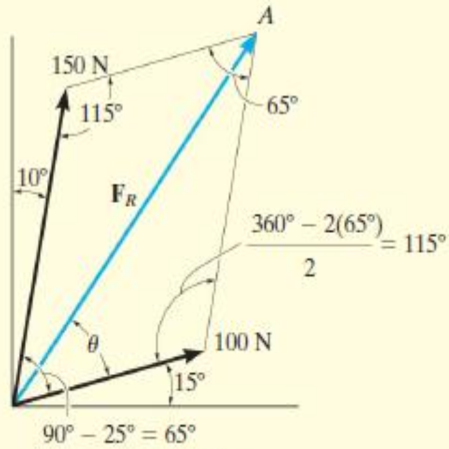
$$\frac{F_{\bar{x}}}{\sin\phi} = \frac{F_{\bar{y}}}{\sin\theta} = \frac{F}{\sin\beta}$$



Ex. 2-1. The screw eye in Fig. below is subjected to two forces, **F**1 and **F**2. Determine the magnitude and direction of the resultant force.



(a)



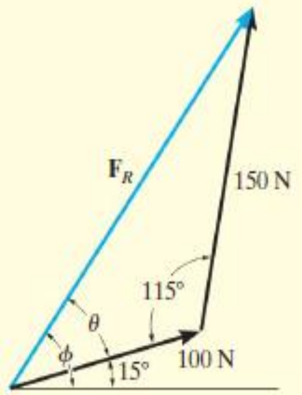
(b)

SOLUTION

Parallelogram Law. The parallelogram is formed by drawing a line from the head of $\mathbf{F}_1$ that is parallel to $\mathbf{F}_2$, and another line from the head of $\mathbf{F}_2$ that is parallel to $\mathbf{F}_1$. The resultant force $\mathbf{F}_R$ extends to where these lines intersect at point A, Fig. 2–11*b*. The two unknowns are the magnitude of $\mathbf{F}_R$ and the angle θ (theta).

Trigonometry. From the parallelogram, the vector triangle is constructed, Fig. 2–11*c*. Using the law of cosines

$$F_R = \sqrt{(100\text{ N})^2 + (150\text{ N})^2 - 2(100\text{ N})(150\text{ N})\cos 115^\circ}$$
$$= \sqrt{10\,000 + 22\,500 - 30\,000(-0.4226)} = 212.6\text{ N}$$
$$= 213\text{ N} \qquad \textit{Ans.}$$



(c)

Applying the law of sines to determine θ,

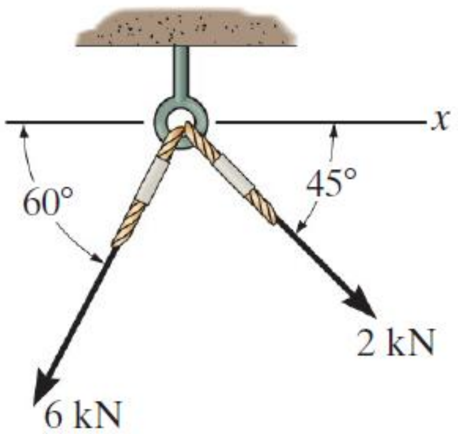
$$\frac{150\text{ N}}{\sin\theta} = \frac{212.6\text{ N}}{\sin 115^\circ} \qquad \sin\theta = \frac{150\text{ N}}{212.6\text{ N}}(\sin 115^\circ)$$
$$\theta = 39.8^\circ$$

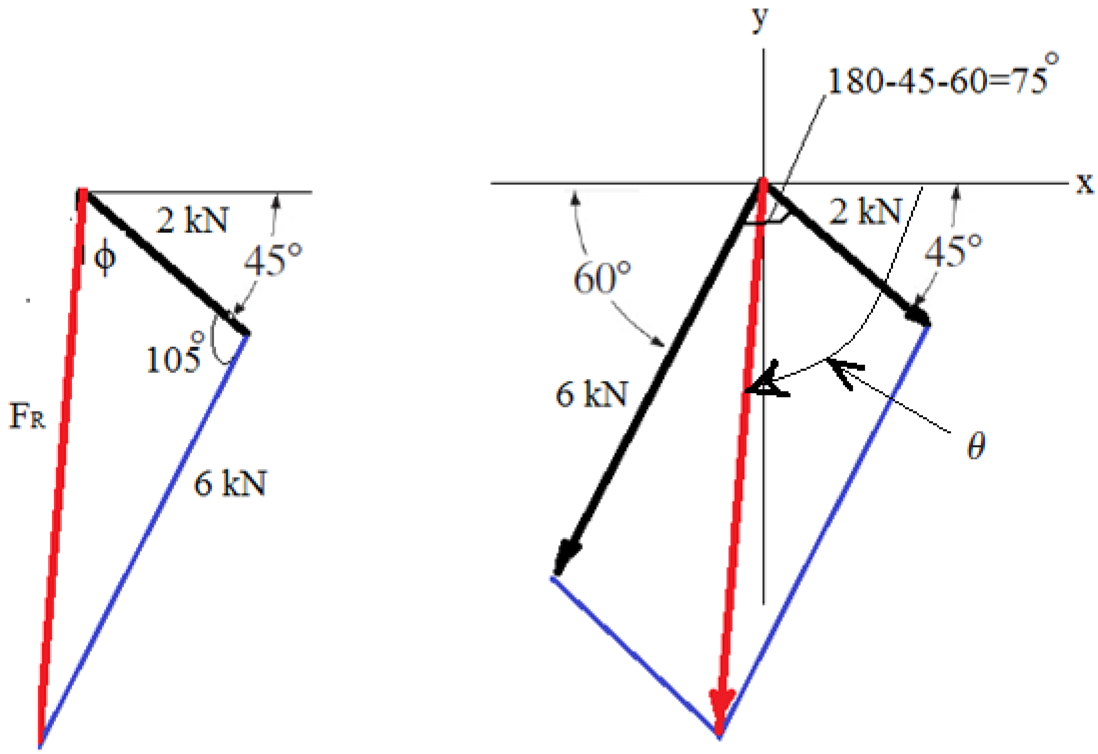
Thus, the direction ϕ (phi) of $\mathbf{F}_R$, measured from the horizontal, is

$$\phi = 39.8^\circ + 15.0^\circ = 54.8^\circ \qquad \textit{Ans.}$$

NOTE: The results seem reasonable, since Fig. 2–11*b* shows $\mathbf{F}_R$ to have a magnitude larger than its components and a direction that is between them.

F2–1. Determine the magnitude of the resultant force acting on the screw eye and its direction measured clockwise from the *x* axis.





$$F_R = \sqrt{(2)^2 + (6)^2 - 2(2)(6)\cos 105^\circ}$$
$$= 6.798 \text{ kN} = 6.80 \text{ kN}$$
$$\frac{\sin\phi}{6} = \frac{\sin 105^\circ}{6.798} \qquad \phi = 58.49^\circ$$
$$\theta = 45^\circ + \phi = 45^\circ + 58.49^\circ = 103^\circ$$

EXAMPLE 2.2

Resolve the horizontal 600-lb force in Fig. 2–12*a* into components acting along the *u* and *v* axes and determine the magnitudes of these components.

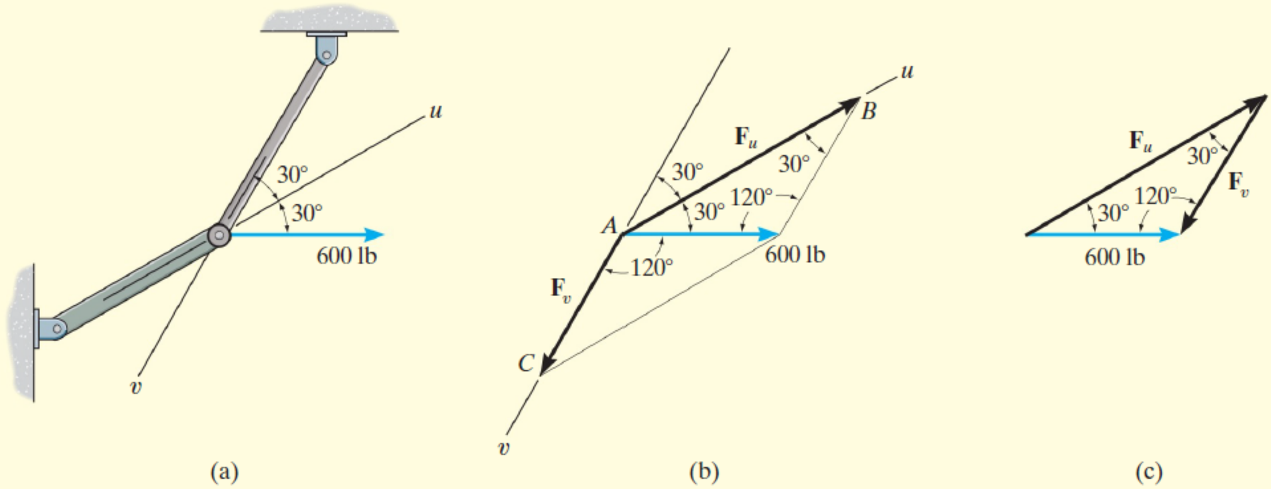


Fig. 2–12

SOLUTION

The parallelogram is constructed by extending a line from the *head* of the 600-lb force parallel to the *v* axis until it intersects the *u* axis at point *B*, Fig. 2–12*b*. The arrow from *A* to *B* represents $\mathbf{F}_u$. Similarly, the line extended from the head of the 600-lb force drawn parallel to the *u* axis intersects the *v* axis at point *C*, which gives $\mathbf{F}_v$.

The vector addition using the triangle rule is shown in Fig. 2–12*c*. The two unknowns are the magnitudes of $\mathbf{F}_u$ and $\mathbf{F}_v$. Applying the law of sines,

$$\frac{F_u}{\sin 120^\circ} = \frac{600 \text{ lb}}{\sin 30^\circ}$$

$$F_u = 1039 \text{ lb} \qquad \textit{Ans.}$$

$$\frac{F_v}{\sin 30^\circ} = \frac{600 \text{ lb}}{\sin 30^\circ}$$

$$F_v = 600 \text{ lb} \qquad \textit{Ans.}$$

NOTE: The result for F_u shows that sometimes a component can have a greater magnitude than the resultant.

EXAMPLE 2.3

Determine the magnitude of the component force **F** in Fig. 2–13*a* and the magnitude of the resultant force $\mathbf{F}_R$ if $\mathbf{F}_R$ is directed along the positive *y* axis.

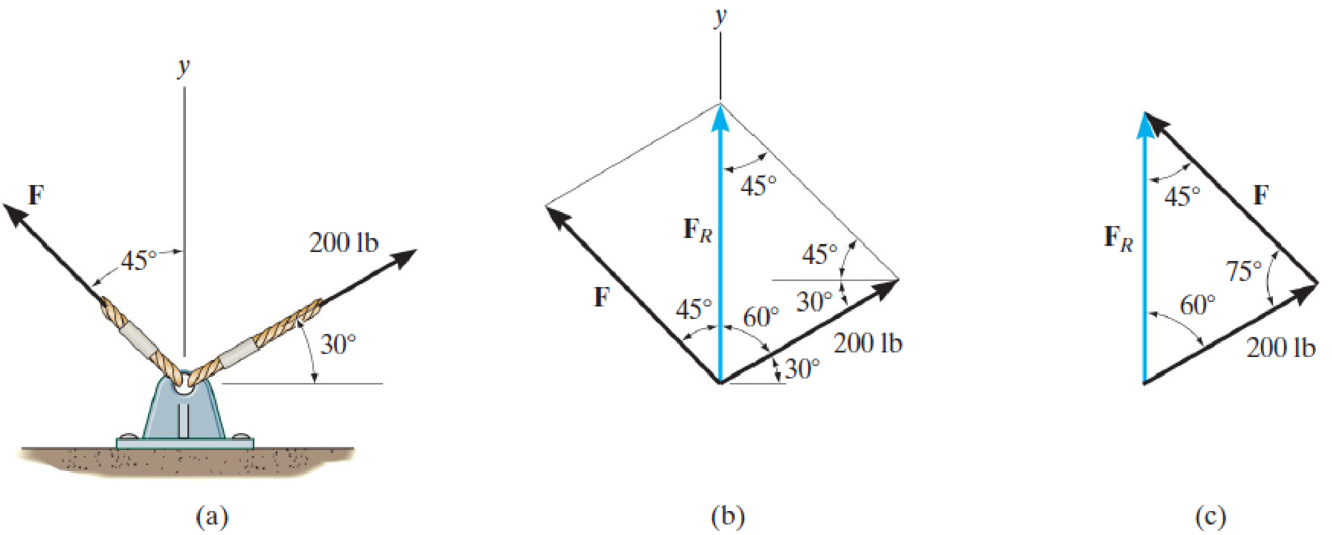


Fig. 2–13

SOLUTION

The parallelogram law of addition is shown in Fig. 2–13*b*, and the triangle rule is shown in Fig. 2–13*c*. The magnitudes of $\mathbf{F}_R$ and $\mathbf{F}$ are the two unknowns. They can be determined by applying the law of sines.

$$\frac{F}{\sin 60^\circ} = \frac{200 \text{ lb}}{\sin 45^\circ}$$

$$F = 245 \text{ lb} \qquad \textit{Ans.}$$

$$\frac{F_R}{\sin 75^\circ} = \frac{200 \text{ lb}}{\sin 45^\circ}$$

$$F_R = 273 \text{ lb} \qquad \textit{Ans.}$$

It is required that the resultant force acting on the eyebolt in Fig. 2–14*a* be directed along the positive *x* axis and that $\mathbf{F}_2$ have a *minimum* magnitude. Determine this magnitude, the angle θ, and the corresponding resultant force.

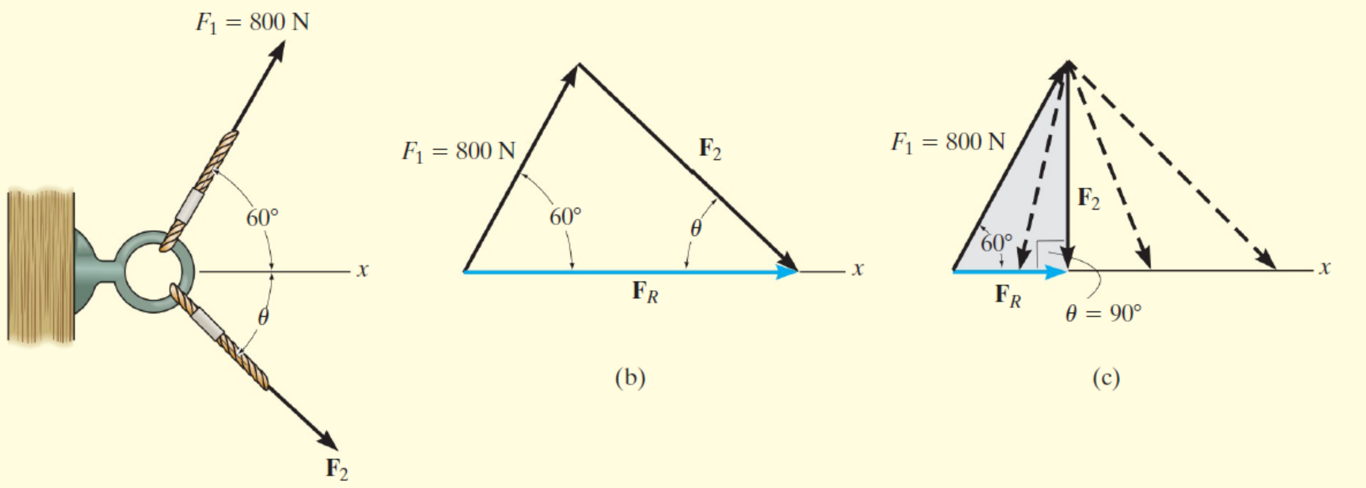


Fig. 2–14

SOLUTION

The triangle rule for $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ is shown in Fig. 2–14*b*. Since the magnitudes (lengths) of $\mathbf{F}_R$ and $\mathbf{F}_2$ are not specified, then $\mathbf{F}_2$ can actually be any vector that has its head touching the line of action of $\mathbf{F}_R$, Fig. 2–14*c*. However, as shown, the magnitude of $\mathbf{F}_2$ is a *minimum* or the shortest length when its line of action is *perpendicular* to the line of action of $\mathbf{F}_R$, that is, when

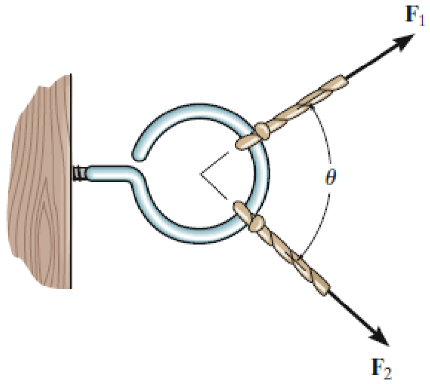
$$\theta = 90° \qquad \textit{Ans.}$$

Since the vector addition now forms the shaded right triangle, the two unknown magnitudes can be obtained by trigonometry.

$$F_R = (800 \text{ N})\cos 60° = 400 \text{ N} \qquad \textit{Ans.}$$

$$F_2 = (800 \text{ N})\sin 60° = 693 \text{ N} \qquad \textit{Ans.}$$

Two forces act on the screw eye. If $F_1 = 400$ N and $F_2 = 600$ N, determine the angle $\theta(0° \le \theta \le 180°)$ between them, so that the resultant force has a magnitude of $F_R = 800$ N.



SOLUTION

The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b*, respectively. Applying law of cosines to Fig. *b*,

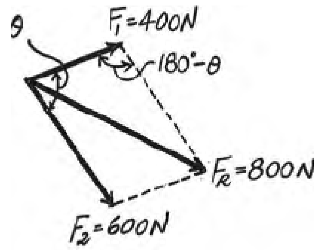
$$800 = \sqrt{400^2 + 600^2 - 2(400)(600)\cos(180° - \theta°)}$$

$$800^2 = 400^2 + 600^2 - 480000\cos(180° - \theta)$$

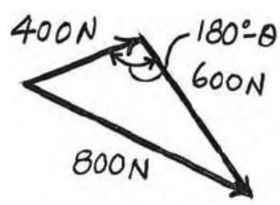
$$\cos(180° - \theta) = -0.25$$

$$180° - \theta = 104.48$$

$$\theta = 75.52° = 75.5° \qquad \textbf{Ans.}$$

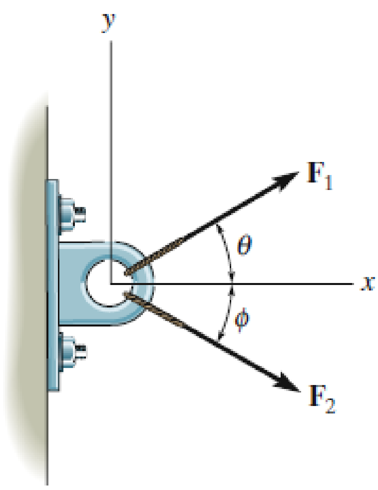


(a)



(b)

If $F_1 = 30$ lb and $F_2 = 40$ lb, determine the angles θ and ϕ so that the resultant force is directed along the positive x axis and has a magnitude of $F_R = 60$ lb.



SOLUTION

Parallelogram Law. The parallelogram law of addition is shown in Fig. *a*,

Trigonometry. Applying the law of cosine by referring to Fig. *b*,

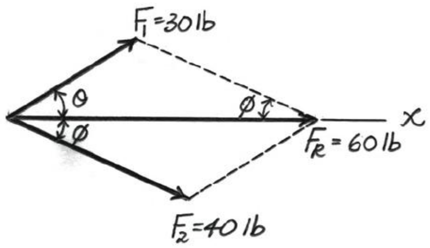
$$40^2 = 30^2 + 60^2 - 2(30)(60)\cos\theta$$

$$\theta = 36.34° = 36.3° \qquad \textbf{Ans.}$$

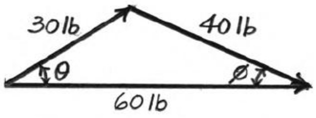
And

$$30^2 = 40^2 + 60^2 - 2(40)(60)\cos\phi$$

$$\phi = 26.38° = 26.4° \qquad \textbf{Ans.}$$



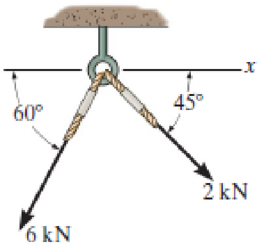
(a)



(b)

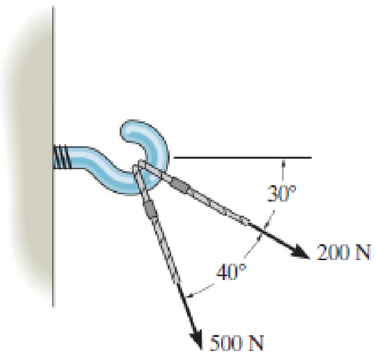
Problems:

F2–1. Determine the magnitude of the resultant force acting on the screw eye and its direction measured clockwise from the x axis.



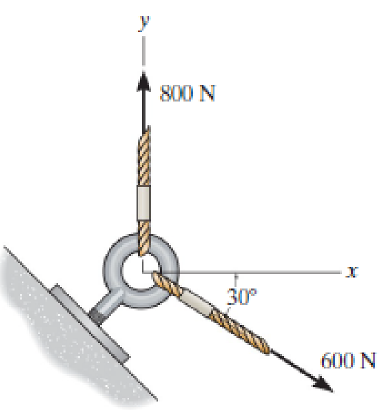
Prob. F2–1

F2–2. Two forces act on the hook. Determine the magnitude of the resultant force.



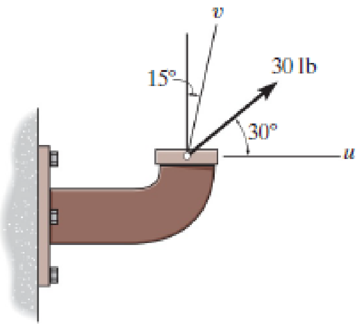
Prob. F2–2

F2–3. Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.



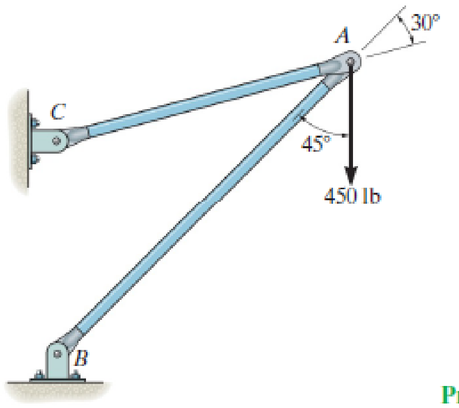
Prob. F2–3

F2–4. Resolve the 30-lb force into components along the u and v axes, and determine the magnitude of each of these components.



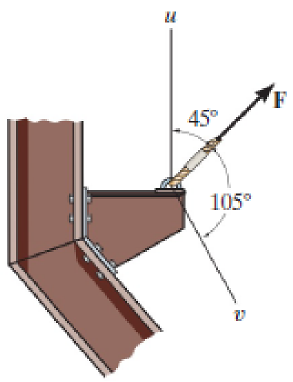
Prob. F2–4

F2–5. The force $F = 450$ lb acts on the frame. Resolve this force into components acting along members AB and AC, and determine the magnitude of each component.



Prob. F2–5

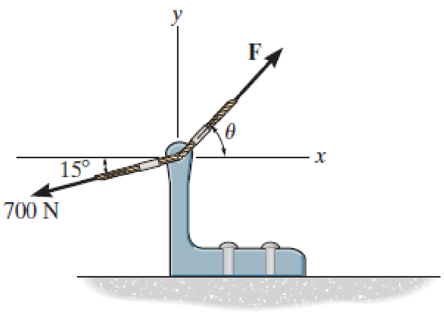
F2–6. If force **F** is to have a component along the u axis of $F_u = 6$ kN, determine the magnitude of **F** and the magnitude of its component $\mathbf{F}_v$ along the v axis.



Prob. F2–6

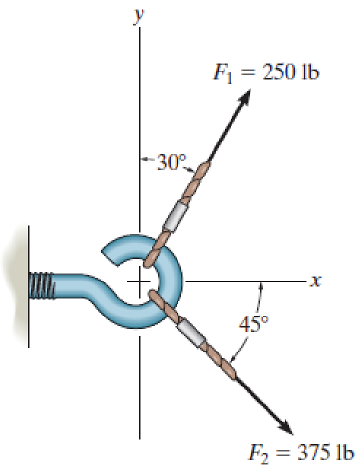
2–1. If $\theta = 60°$ and $F = 450$ N, determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.

2–2. If the magnitude of the resultant force is to be 500 N, directed along the positive y axis, determine the magnitude of force $\mathbf{F}$ and its direction θ.



Probs. 2–1/2

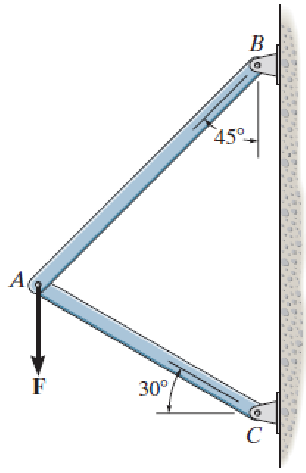
2–3. Determine the magnitude of the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and its direction, measured counterclockwise from the positive x axis.



Prob. 2–3

***2–4.** The vertical force $\mathbf{F}$ acts downward at A on the two-membered frame. Determine the magnitudes of the two components of $\mathbf{F}$ directed along the axes of AB and AC. Set $F = 500$ N.

2–5. Solve Prob. 2–4 with $F = 350$ lb.

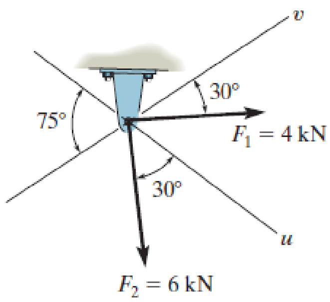


Probs. 2–4/5

2–6. Determine the magnitude of the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and its direction, measured clockwise from the positive u axis.

2–7. Resolve the force $\mathbf{F}_1$ into components acting along the u and v axes and determine the magnitudes of the components.

***2–8.** Resolve the force $\mathbf{F}_2$ into components acting along the u and v axes and determine the magnitudes of the components.



Probs. 2–6/7/8

Addition of a System of Coplanar Forces

When a force is resolved into two components along the *x* and *y* axes, the components are then called ***rectangular components***. For analytical work we can represent these components in one of two ways, using either scalar or Cartesian vector notation.

Scalar Notation

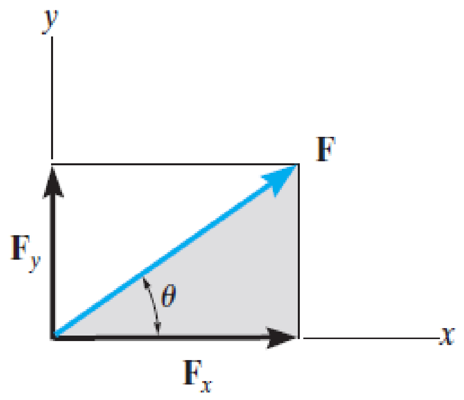
$F_x = F\cos\theta$ and $F_y = F\sin\theta$

$$\frac{F_x}{F} = \frac{a}{c}$$

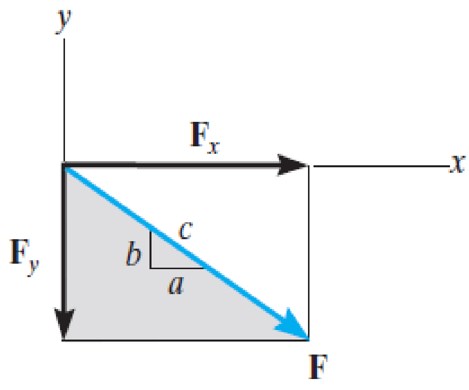
$$F_x = F\left(\frac{a}{c}\right)$$

$$\frac{F_y}{F} = \frac{b}{c}$$

$$F_y = -F\left(\frac{b}{c}\right)$$



(a)

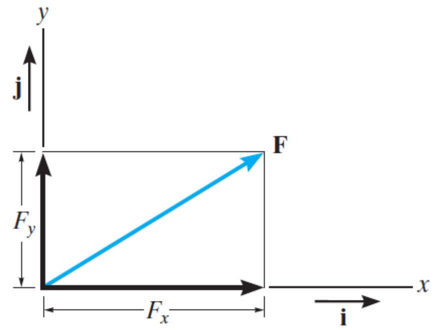


(b)

Cartesian Vector Notation

It is also possible to represent the *x* and *y* components of a force in terms of Cartesian unit vectors **i** and **j**.

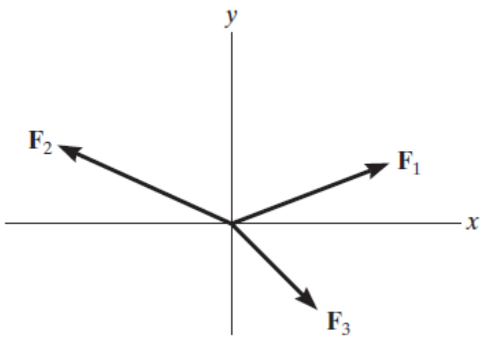
$$\mathbf{F} = F_x\,\mathbf{i} + F_y\,\mathbf{j}$$



Coplanar Force Resultants

We can use either of the two methods just described to determine the resultant of several ***coplanar forces***,

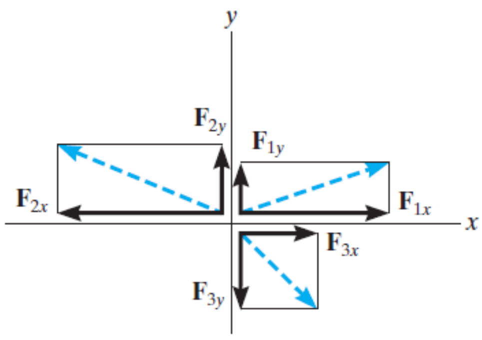
Using **Cartesian vector notation**, each force is first represented as a Cartesian vector, i.e.



(a)

$$\mathbf{F}_1 = F_{1x}\,\mathbf{i} + F_{1y}\,\mathbf{j}$$
$$\mathbf{F}_2 = -F_{2x}\,\mathbf{i} + F_{2y}\,\mathbf{j}$$
$$\mathbf{F}_3 = F_{3x}\,\mathbf{i} - F_{3y}\,\mathbf{j}$$

The vector resultant is therefore



(b)

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3\\ &= F_{1x}\,\mathbf{i} + F_{1y}\,\mathbf{j} - F_{2x}\,\mathbf{i} + F_{2y}\,\mathbf{j} + F_{3x}\,\mathbf{i} - F_{3y}\,\mathbf{j}\\ &= (F_{1x} - F_{2x} + F_{3x})\mathbf{i} + (F_{1y} + F_{2y} - F_{3y})\mathbf{j}\\ &= (F_{Rx})\mathbf{i} + (F_{Ry})\mathbf{j}\end{aligned}$$

If ***scalar notation*** is used, then indicating the positive directions of components along the *x* and *y* axes with symbolic arrows, we have

$$\xrightarrow{+} \qquad (F_R)_x = F_{1x} - F_{2x} + F_{3x}$$
$$+\uparrow \qquad (F_R)_y = F_{1y} + F_{2y} - F_{3y}$$

These are the *same* results as the **i** and **j** components of **F***R* determined above.

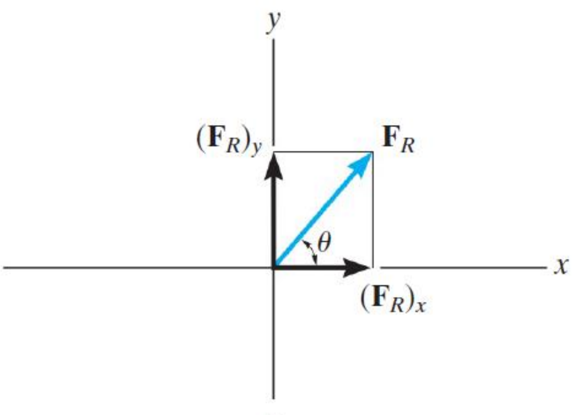
*For handwritten work, unit vectors are usually indicated using a circumflex, e.g., $\hat{\imath}$ and $\hat{\jmath}$.

We can represent the components of the resultant force of any number of coplanar forces symbolically by the algebraic sum of the *x* and *y* components of all the forces, i.e.

$$\boxed{\begin{aligned}(F_R)_x &= \Sigma F_x \\ (F_R)_y &= \Sigma F_y\end{aligned}} \qquad (2\text{–}1)$$

The magnitude of **F***R* is then found from the Pythagorean Theorem; that is,

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2}$$

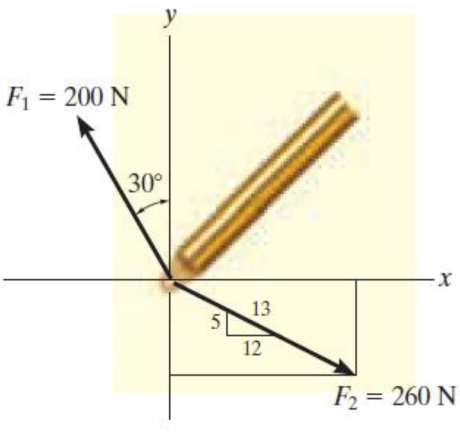


(c)

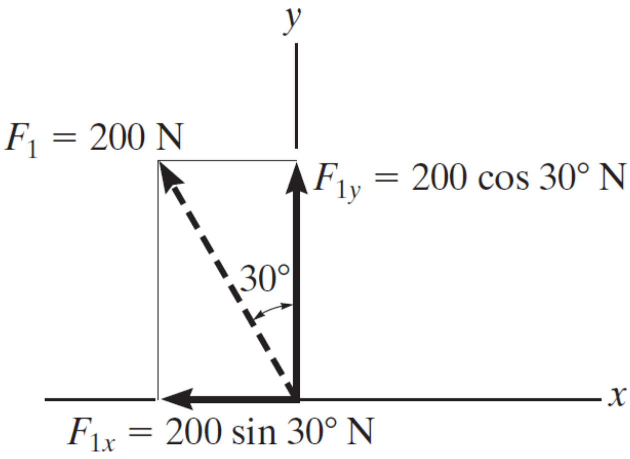
Also, the angle θ, which specifies the direction of the resultant force, is determined from trigonometry:

$$\theta = \tan^{-1}\left|\frac{(F_R)_y}{(F_R)_x}\right|$$

Determine the x and y components of $\mathbf{F}_1$ and $\mathbf{F}_2$ acting on the boom shown in Fig. 2–18a. Express each force as a Cartesian vector.



(a)



(b)

Solution

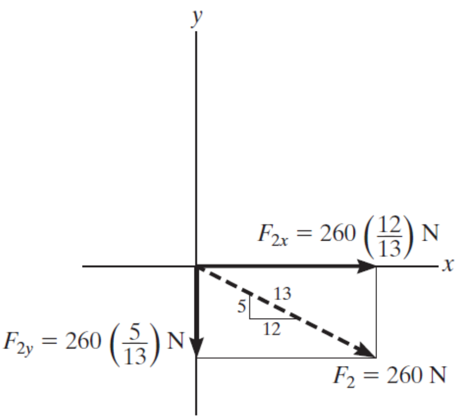
Scalar Notation

F_{1x} = -200 sin 30°
= -100 N = 100 N ← *Ans.*

F_{1y} = 200 cos 30°
= 173 N = 173 N ↑ *Ans.*

$$F_{2x} = 260\text{ N}\left(\frac{12}{13}\right) = 240\text{ N}$$

$$F_{2y} = 260\text{ N}\left(\frac{5}{13}\right) = 100\text{ N}$$



(c)

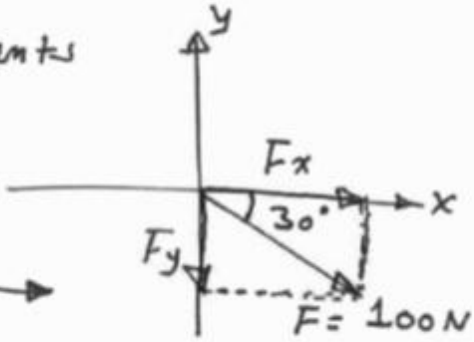
Cartesian Vector Notation. Having determined the magnitudes and directions of the components of each force, we can express each force as a Cartesian vector.

$$\mathbf{F}_1 = \{-100\mathbf{i} + 173\mathbf{j}\}\text{ N} \qquad \textit{Ans.}$$

$$\mathbf{F}_2 = \{240\mathbf{i} - 100\mathbf{j}\}\text{ N} \qquad \textit{Ans.}$$

Examples :

Ex.1 : Determine the x & y components for the 100N-force.



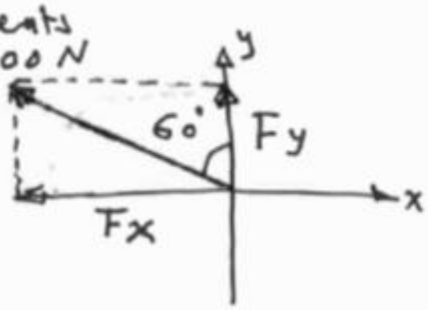
sol.

$\xrightarrow{+} F_x = F\cos 30$
$= 100\cos 30 = 86.60\ N \rightarrow$

$+\uparrow\ F_y = F\sin 30$
$= -100\sin 30 = -50 = 50N \downarrow$

the negative sign of the Fy indicates that Fy is directed in the opposite direction of +ve y.

Ex.2 : Determine the x & y components for the 200N-force.

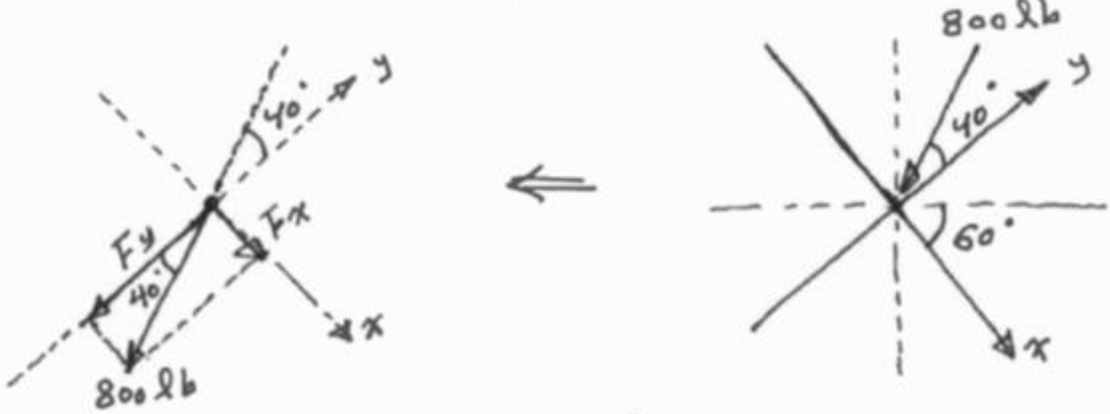


sol.

$\xrightarrow{+} F_x = -200\sin 60$
$= -173.2 = 173.2\ N \leftarrow$

$+\uparrow\ F_y = 200\cos 60 = 100N \uparrow$

Ex.3 : Determine the x & y components for the 800lb-force.

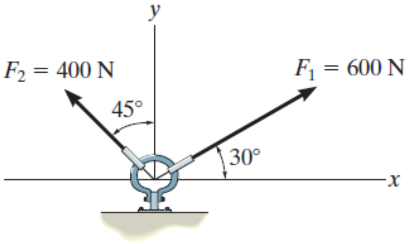


$+\searrow\ F_x = 800\sin 40 = 514.23\ lb \searrow$

$+\nearrow\ F_y = -800\cos 40 = -612.83 = 612.83\ lb \swarrow$

EXAMPLE 2.6

The link in Fig. 2–19*a* is subjected to two forces $\mathbf{F}_1$ and $\mathbf{F}_2$. Determine the magnitude and direction of the resultant force.



SOLUTION I

Scalar Notation. First we resolve each force into its x and y components, Fig. 2–19*b*, then we sum these components algebraically.

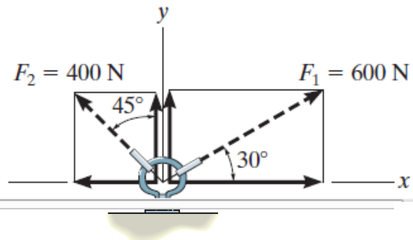
$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = 600 \cos 30° \text{ N} - 400 \sin 45° \text{ N} = 236.8 \text{ N} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 600 \sin 30° \text{ N} + 400 \cos 45° \text{ N} = 582.8 \text{ N}\uparrow$$

The resultant force, shown in Fig. 2–19*c*, has a *magnitude* of

$$F_R = \sqrt{(236.8 \text{ N})^2 + (582.8 \text{ N})^2} = 629 \text{ N} \qquad \textit{Ans.}$$

From the vector addition,



$$\theta = \tan^{-1}\left(\frac{582.8 \text{ N}}{236.8 \text{ N}}\right) = 67.9° \qquad \textit{Ans.}$$

SOLUTION II

Cartesian Vector Notation. From Fig. 2–19*b*, each force is first expressed as a Cartesian vector.

$$\mathbf{F}_1 = \{600 \cos 30° \mathbf{i} + 600 \sin 30° \mathbf{j}\} \text{ N}$$
$$\mathbf{F}_2 = \{-400 \sin 45° \mathbf{i} + 400 \cos 45° \mathbf{j}\} \text{ N}$$

Then,

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 = (600 \cos 30° \text{ N} - 400 \sin 45° \text{ N})\mathbf{i} + (600 \sin 30° \text{ N} + 400 \cos 45° \text{ N})\mathbf{j} = \{236.8\mathbf{i} + 582.8\mathbf{j}\} \text{ N}$$

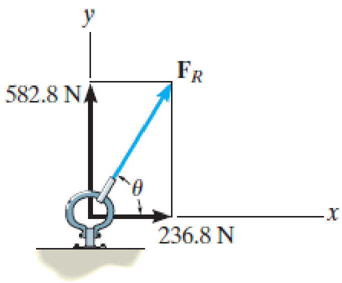


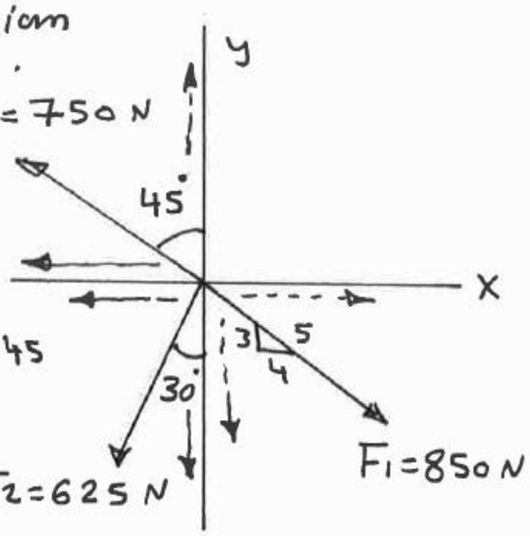
Fig. 2–19

The magnitude and direction of $\mathbf{F}_R$ are determined in the same manner as before.

NOTE: Comparing the two methods of solution, notice that the use of scalar notation is more efficient since the components can be found *directly*, without first having to express each force as a Cartesian vector before adding the components. Later, however, we will show that Cartesian vector analysis is very beneficial for solving three-dimensional problems.

Ex.4: Determine the magnitude and the direction of the resultant force. Use the scalar notation and the Cartesian vector notation methods.

Sol.



① Scalar Notation:

$\xrightarrow{+}$ $F_{Rx} = \Sigma F_x = F_{x1} + F_{x2} + F_{x3}$

$= 850\left(\frac{4}{5}\right) - 625 \sin 30 - 750 \sin 45$

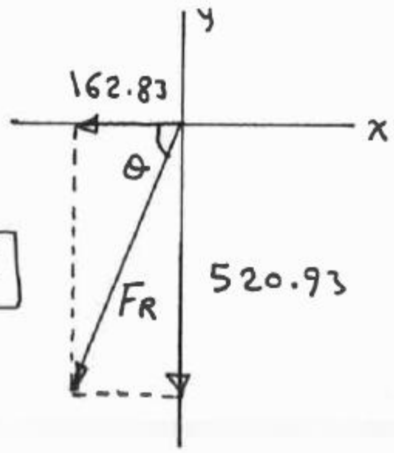
$= -162.83$

$= 162.83\ \text{N} \leftarrow$

$+\uparrow$ $F_{Ry} = \Sigma F_y = F_{y1} + F_{y2} + F_{y3}$

$= -850\left(\frac{3}{5}\right) - 625 \cos 30 + 750 \cos 45 = -520.93$

$= 520.93\ \text{N} \downarrow$



$F_R = \sqrt{(F_{Rx})^2 + (F_{Ry})^2}$

$= \sqrt{(162.83)^2 + (520.93)^2} = \boxed{545.78\ \text{N}}$

$\theta = \tan^{-1} \frac{520.93}{162.83} = \boxed{72.64^\circ}$

② Cartesian Vector Notation:

$\vec{F_R} = \vec{F_1} + \vec{F_2} + \vec{F_3}$

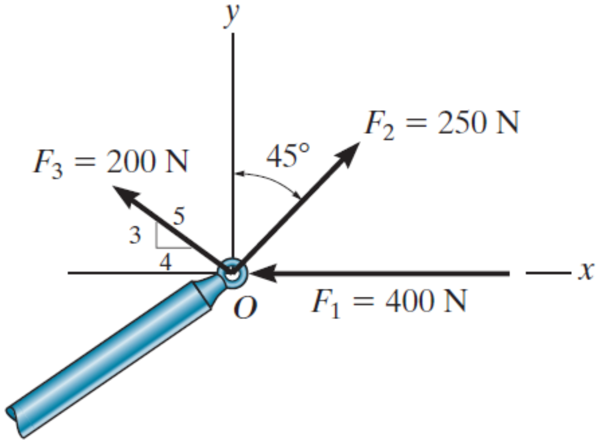
$= \left(850\left(\frac{4}{5}\right) i - 850\left(\frac{3}{5}\right) j\right) + (-625 \sin 30\, i - 625 \cos 30\, j)$

$+ (-750 \sin 45\, i + 750 \cos 45\, j)$

$\Rightarrow \vec{F_R} = (-162.83\, i - 520.93\, j)$ N

The magnitude and the direction of F_R are determined in the same manner as in the scalar notation.

Ex: The end of the boom O in Fig. 2–20a is subjected to three concurrent and coplanar forces. Determine the magnitude and direction of the resultant force.



(a)

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = -400 \text{ N} + 250 \sin 45^\circ \text{ N} - 200\left(\tfrac{4}{5}\right)$$

$$= -383.2 \text{ N} = 383.2 \text{ N} \leftarrow$$

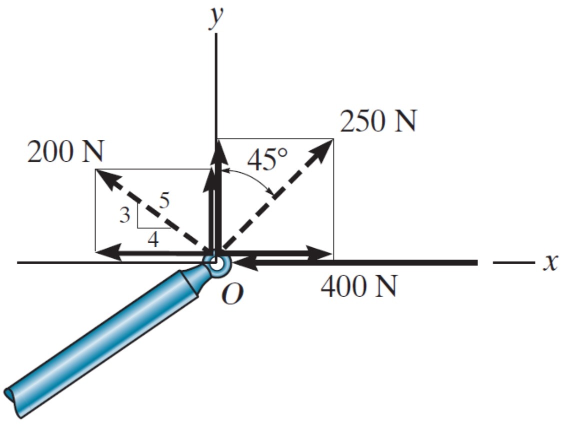
$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 250 \cos 45^\circ \text{ N} + 200\left(\tfrac{3}{5}\right) \text{ N}$$

$$= 296.8 \text{ N} \uparrow$$

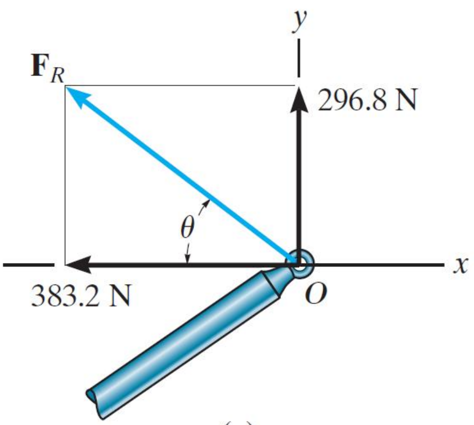
$$F_R = \sqrt{(-383.2 \text{ N})^2 + (296.8 \text{ N})^2}$$

$$= 485 \text{ N}$$

$$\theta = \tan^{-1}\left(\frac{296.8}{383.2}\right) = 37.8^\circ$$

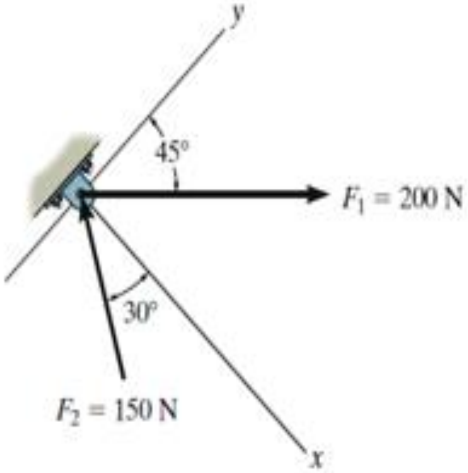


(b)



(c)

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

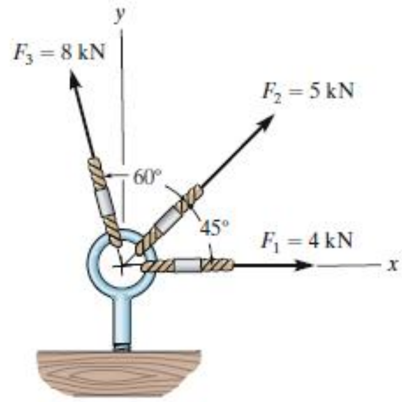
$$+\searrow F_{Rx} = \Sigma F_{x'}; \qquad F_{Rx} = -150\cos 30^\circ + 200\sin 45^\circ = 11.518\text{ N}$$

$$\nearrow + F_{Ry} = \Sigma F_{y'}; \qquad F_{Ry} = 150\sin 30^\circ + 200\cos 45^\circ = 216.421\text{ N}$$

$$F_R = \sqrt{(11.518)^2 + (216.421)^2} = 217\text{ N}$$ **Ans.**

$$\theta = \tan^{-1}\left(\frac{216.421}{11.518}\right) = 87.0^\circ$$ **Ans.**

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

Scalar Notation. Summing the force components along x and y axes algebraically by referring to Fig. a,

$$\xrightarrow{+} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = 4 + 5\cos 45^\circ - 8\sin 15^\circ = 5.465\text{ kN} \rightarrow$$

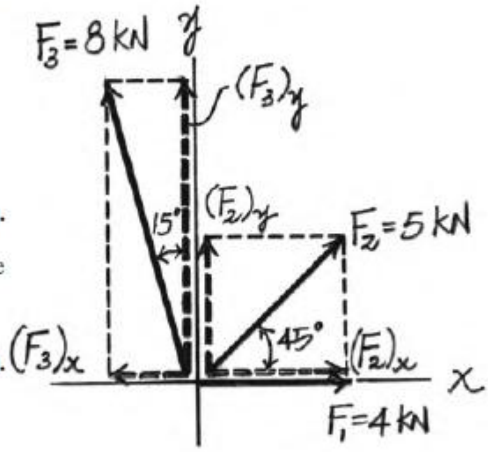
$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 5\sin 45^\circ + 8\cos 15^\circ = 11.263\text{ kN} \uparrow$$

By referring to Fig. b, the magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{5.465^2 + 11.263^2} = 12.52\text{ kN} = 12.5\text{ kN}$$ **Ans.**

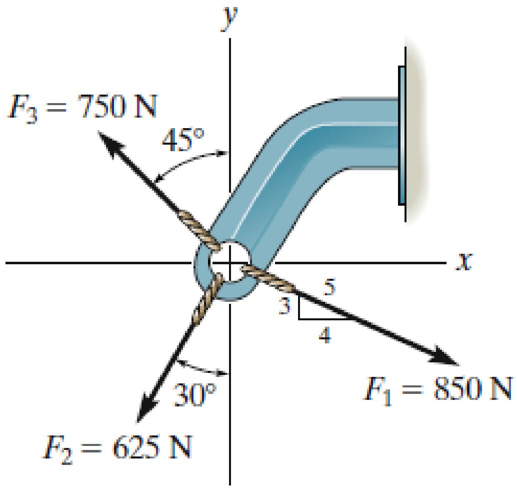
And the directional angle θ of $\mathbf{F}_R$ measured counterclockwise from the positive x axis is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{11.263}{5.465}\right) = 64.12^\circ = 64.1^\circ$$ **Ans.**



(a)

Express $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ as Cartesian vectors.



SOLUTION

$$\mathbf{F}_1 = \frac{4}{5}(850)\,\mathbf{i} - \frac{3}{5}(850)\,\mathbf{j}$$

$$= \{680\,\mathbf{i} - 510\,\mathbf{j}\}\text{ N} \qquad \textbf{Ans.}$$

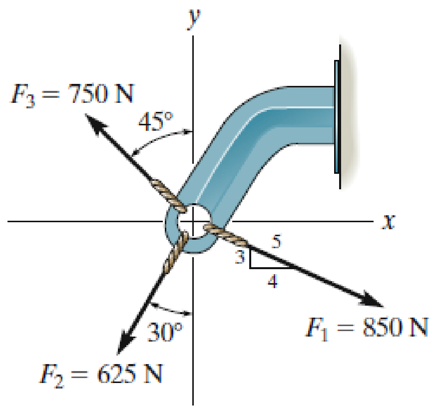
$$\mathbf{F}_2 = -625 \sin 30^\circ\,\mathbf{i} - 625 \cos 30^\circ\,\mathbf{j}$$

$$= \{-312\,\mathbf{i} - 541\,\mathbf{j}\}\text{ N} \qquad \textbf{Ans.}$$

$$\mathbf{F}_3 = -750 \sin 45^\circ\,\mathbf{i} + 750 \cos 45^\circ\,\mathbf{j}$$

$$= \{-530\,\mathbf{i} + 530\,\mathbf{j}\}\text{ N} \qquad \textbf{Ans.}$$

Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.



SOLUTION

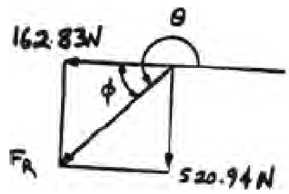
$$\xrightarrow{+} F_{Rx} = \Sigma F_x; \qquad F_{Rx} = \frac{4}{5}(850) - 625 \sin 30^\circ - 750 \sin 45^\circ = -162.83\text{ N}$$

$$+\uparrow F_{Ry} = \Sigma F_y; \qquad F_{Ry} = -\frac{3}{5}(850) - 625 \cos 30^\circ + 750 \cos 45^\circ = -520.94\text{ N}$$

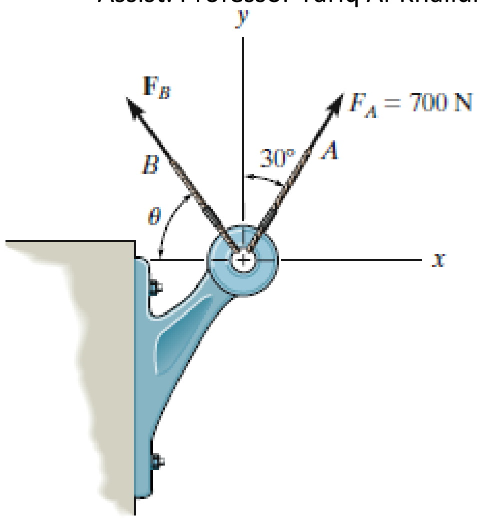
$$F_R = \sqrt{(-162.83)^2 + (-520.94)^2} = 546\text{ N} \qquad \textbf{Ans.}$$

$$\phi = \tan^{-1}\left(\frac{520.94}{162.83}\right) = 72.64^\circ$$

$$\theta = 180^\circ + 72.64^\circ = 253^\circ \qquad \textbf{Ans.}$$



Determine the magnitude and orientation θ of $\mathbf{F}_B$ so that the resultant force is directed along the positive y axis and has a magnitude of 1500 N.



SOLUTION

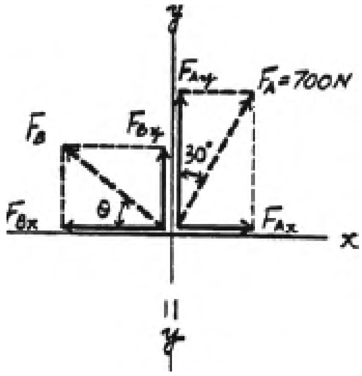
Scalar Notation: Suming the force components algebraically, we have

$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \qquad 0 = 700 \sin 30^\circ - F_B \cos\theta$$

$$F_B \cos\theta = 350 \qquad \textbf{(1)}$$

$$+\uparrow F_{R_y} = \Sigma F_y; \qquad 1500 = 700 \cos 30^\circ + F_B \sin\theta$$

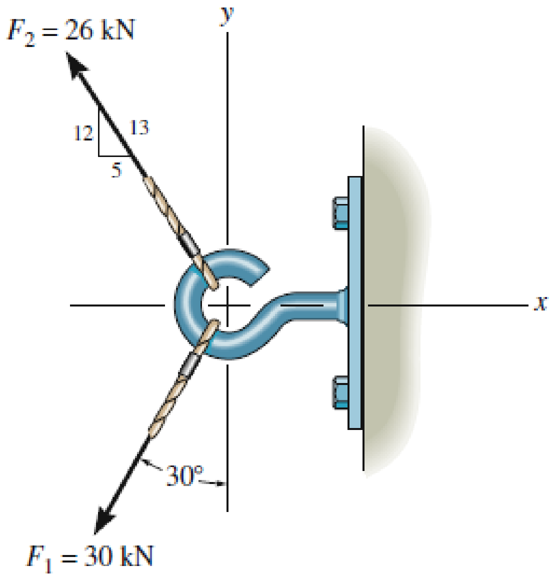
$$F_B \sin\theta = 893.8 \qquad \textbf{(2)}$$



Solving Eq. (1) and (2) yields

$$\theta = 68.6^\circ \qquad F_B = 960 \text{ N} \qquad \textbf{Ans.}$$

Express $\mathbf{F}_1$ and $\mathbf{F}_2$ as Cartesian vectors.



SOLUTION

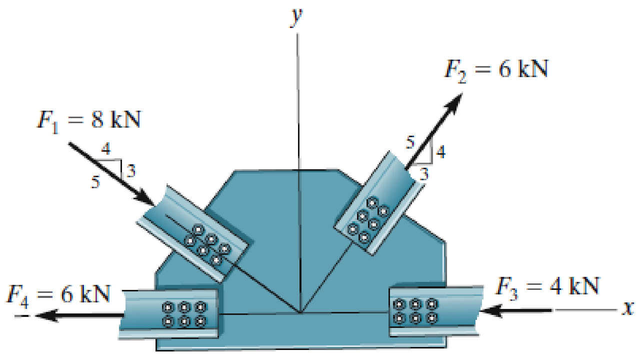
$$\mathbf{F}_1 = -30 \sin 30^\circ\, \mathbf{i} - 30 \cos 30^\circ\, \mathbf{j}$$

$$= \{-15.0\,\mathbf{i} - 26.0\,\mathbf{j}\} \text{ kN} \qquad \textbf{Ans.}$$

$$\mathbf{F}_2 = -\frac{5}{13}(26)\,\mathbf{i} + \frac{12}{13}(26)\,\mathbf{j}$$

$$= \{-10.0\,\mathbf{i} + 24.0\,\mathbf{j}\} \text{ kN} \qquad \textbf{Ans.}$$

Determine the x and y components of each force acting on the *gusset plate* of a bridge truss. Show that the resultant force is zero.



SOLUTION

Scalar Notation. Referring to Fig. *a*, the x and y components of each forces are

$$(F_1)_x = 8\left(\frac{4}{5}\right) = 6.40 \text{ kN} \rightarrow$$ **Ans.**

$$(F_1)_y = 8\left(\frac{3}{5}\right) = 4.80 \text{ kN} \downarrow$$ **Ans.**

$$(F_2)_x = 6\left(\frac{3}{5}\right) = 3.60 \text{ kN} \rightarrow$$ **Ans.**

$$(F_2)_y = 6\left(\frac{4}{5}\right) = 4.80 \text{ kN} \uparrow$$ **Ans.**

$$(F_3)_x = 4 \text{ kN} \leftarrow$$ **Ans.**

$$(F_3)_y = 0$$ **Ans.**

$$(F_4)_x = 6 \text{ kN} \leftarrow$$ **Ans.**

$$(F_4)_y = 0$$ **Ans.**

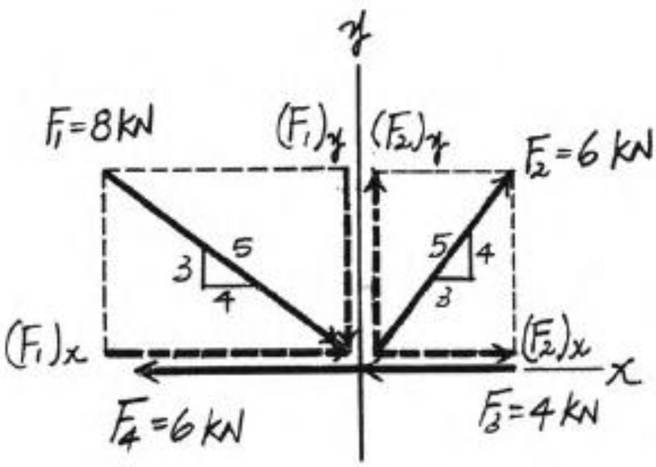
Summing these force components along x and y axes algebraically,

$$\xrightarrow{+} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = 6.40 + 3.60 - 4 - 6 = 0$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 4.80 - 4.80 = 0$$

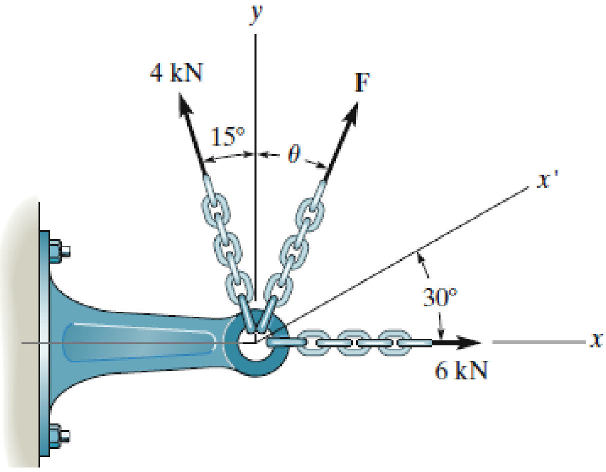
Thus,

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{O^2 + O^2} = O \qquad \textbf{(Q.E.D)}$$



(a)

Three forces act on the bracket. Determine the magnitude and direction θ of $\mathbf{F}$ so that the resultant force is directed along the positive x' axis and has a magnitude of 8 kN.



SOLUTION

Scalar Notation. Equating the force components along the x and y axes algebraically by referring to Fig. a,

$$\xrightarrow{+} (F_R)_x = \Sigma F_x; \qquad 8\cos 30^\circ = F\sin\theta + 6 - 4\sin 15^\circ$$

$$F\sin\theta = 1.9635 \qquad (1)$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad 8\sin 30^\circ = F\cos\theta + 4\cos 15^\circ$$

$$F\cos\theta = 0.1363 \qquad (2)$$

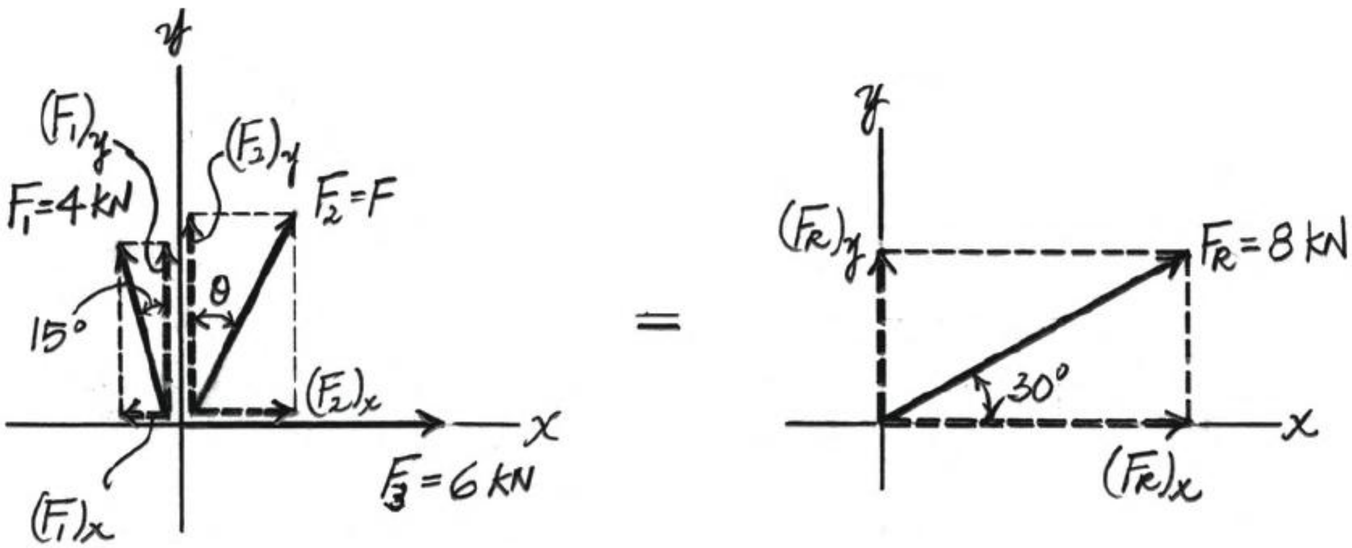
Divide Eq (1) by (2)

$$\tan\theta = 14.406 \qquad \theta = 86.03^\circ = 86.0^\circ \qquad \textbf{Ans.}$$

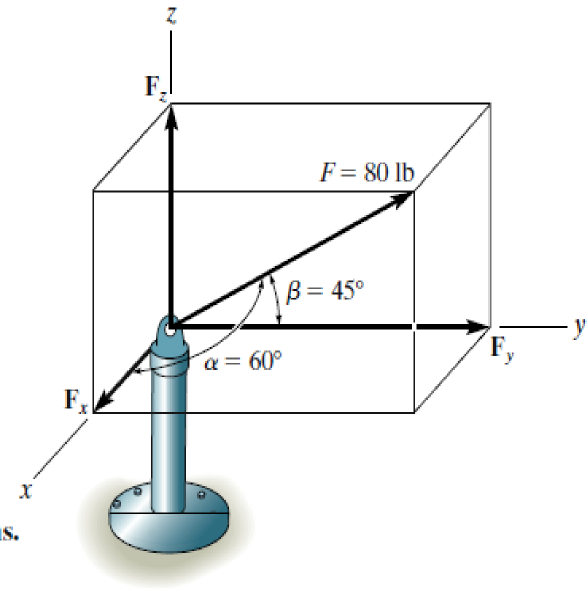
Substitute this result into Eq (1)

$$F\sin 86.03^\circ = 1.9635$$

$$F = 1.968 \text{ kN} = 1.97 \text{ kN} \qquad \textbf{Ans.}$$



The force **F** has a magnitude of 80 lb and acts within the octant shown. Determine the magnitudes of the x, y, z components of **F**.



SOLUTION

$1 = \cos^2 60° + \cos^2 45° + \cos^2 \gamma$

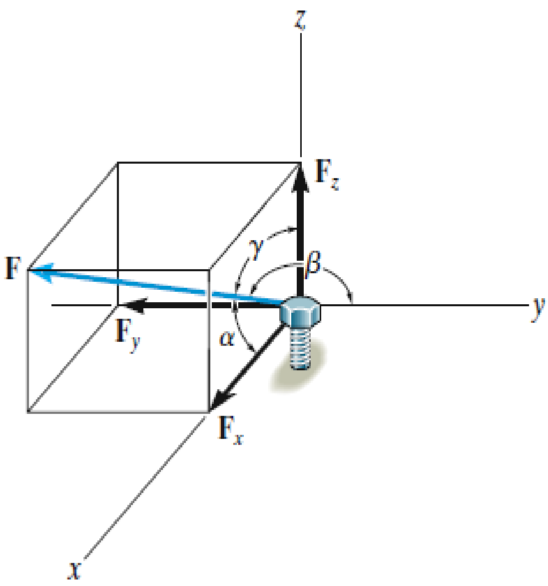
Solving for the positive root, $\gamma = 60°$

$F_x = 80 \cos 60° = 40.0 \text{ lb}$ **Ans.**

$F_y = 80 \cos 45° = 56.6 \text{ lb}$ **Ans.**

$F_z = 80 \cos 60° = 40.0 \text{ lb}$ **Ans.**

The bolt is subjected to the force **F**, which has components acting along the x, y, z axes as shown. If the magnitude of **F** is 80 N, and $\alpha = 60°$ and $\gamma = 45°$, determine the magnitudes of its components.



SOLUTION

$$\cos\beta = \sqrt{1 - \cos^2\alpha - \cos^2\gamma}$$

$$= \sqrt{1 - \cos^2 60° - \cos^2 45°}$$

$$\beta = 120°$$

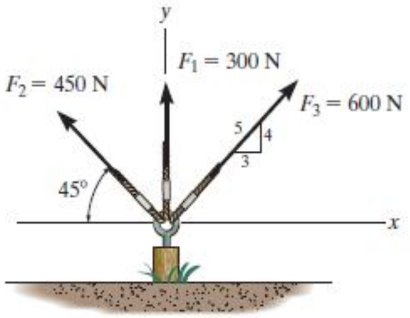
$F_x = |80 \cos 60°| = 40 \text{ N}$ **Ans.**

$F_y = |80 \cos 120°| = 40 \text{ N}$ **Ans.**

$F_z = |80 \cos 45°| = 56.6 \text{ N}$ **Ans.**

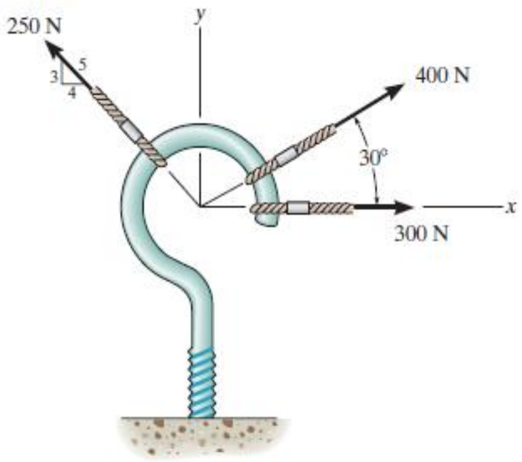
Problems:

F2–7. Resolve each force acting on the post into its x and y components.



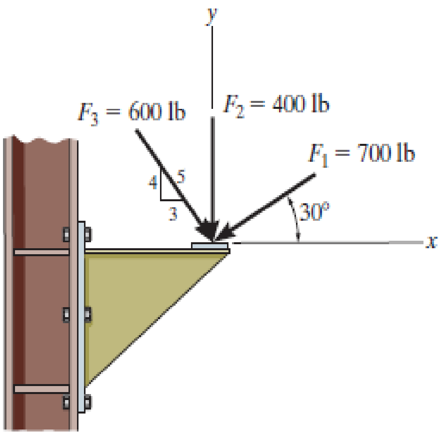
Prob. F2–7

F2–8. Determine the magnitude and direction of the resultant force.



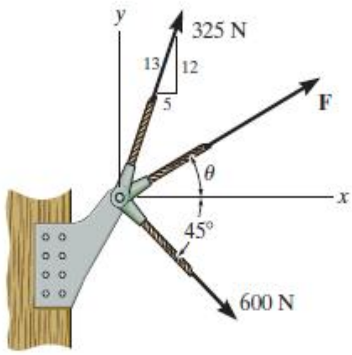
Prob. F2–8

F2–9. Determine the magnitude of the resultant force acting on the corbel and its direction θ measured counterclockwise from the x axis.



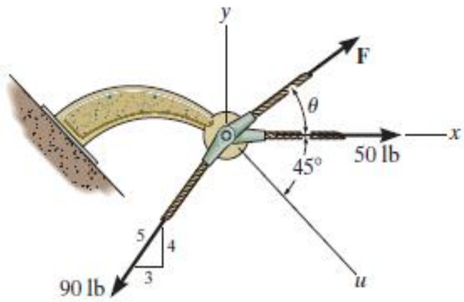
Prob. F2–9

F2–10. If the resultant force acting on the bracket is to be 750 N directed along the positive x axis, determine the magnitude of **F** and its direction θ.



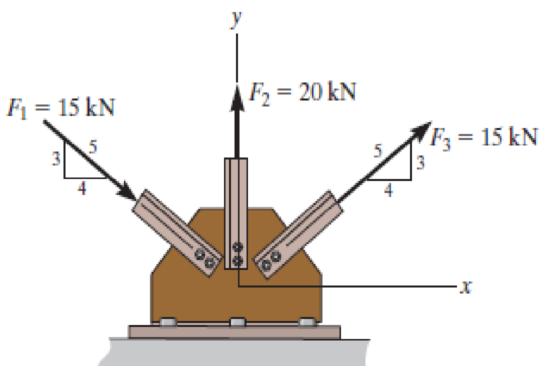
Prob. F2–10

F2–11. If the magnitude of the resultant force acting on the bracket is to be 80 lb directed along the u axis, determine the magnitude of **F** and its direction θ.



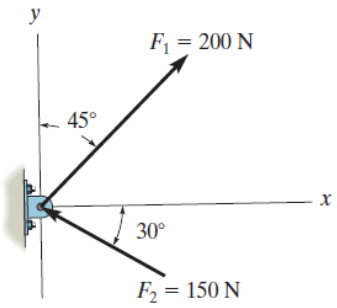
Prob. F2–11

F2–12. Determine the magnitude of the resultant force and its direction θ measured counterclockwise from the positive x axis.



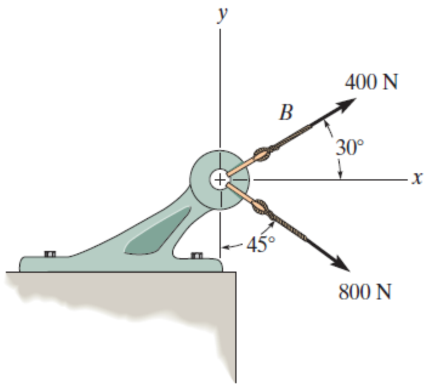
Prob. F2–12

***2–32.** Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



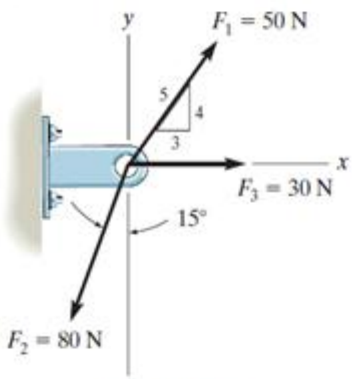
Prob. 2–32

2–33. Determine the magnitude of the resultant force and its direction, measured clockwise from the positive x axis.



Prob. 2–33

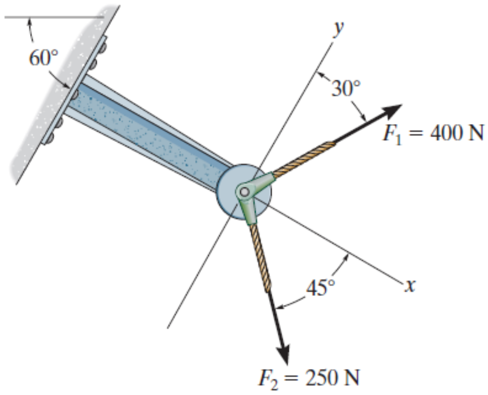
2–38. Express each of the three forces acting on the support in Cartesian vector form and determine the magnitude of the resultant force and its direction, measured clockwise from positive x axis.



Prob. 2–38

2–34. Resolve $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x and y components.

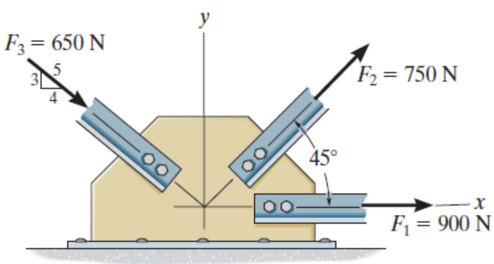
2–35. Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.



Probs. 2–34/35

***2–36.** Resolve each force acting on the *gusset plate* into its x and y components, and express each force as a Cartesian vector.

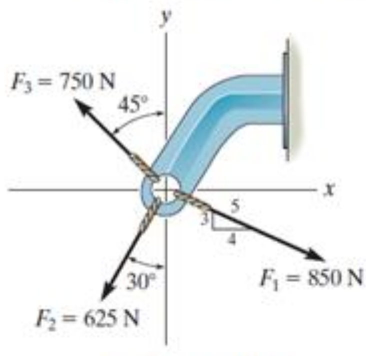
2–37. Determine the magnitude of the resultant force acting on the plate and its direction, measured counterclockwise from the positive x axis.



Probs. 2–36/37

2–42. Express $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ as Cartesian vectors.

2–43. Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



Probs. 2–42/43