Northern Technical University
Technical College of Mosul
Building & Construction
Technology Engineering Dept.

THEORY OF STRUCTURES

THIRD CLASS

Lecturer:
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2015 - 2016

Theory of Structures

Sign Convention. Before presenting a method for finding the internal normal force, shear force, and bending moment, we will need to establish a sign convention to define their "positive" and "negative" values.

Although the choice is arbitrary, the sign convention to be adopted here has been widely accepted in structural engineering practice, and is illustrated in Fig. below. On the *left-hand face* of the cut member the normal force **N** acts to the right, the internal shear force **V** acts downward, and the moment **M** acts counterclockwise. In accordance with Newton's third law, an equal but opposite normal force, shear force, and bending moment must act on the right-hand face of the member at the section.

Perhaps an easy way to remember this sign convention is to isolate a small segment of the member and note that *positive normal force tends to elongate the segment, positive shear tends to rotate the segment clockwise,* Fig. 4–1*c*; and *positive bending moment tends to bend the segment concave upward,* so as to "hold water," .

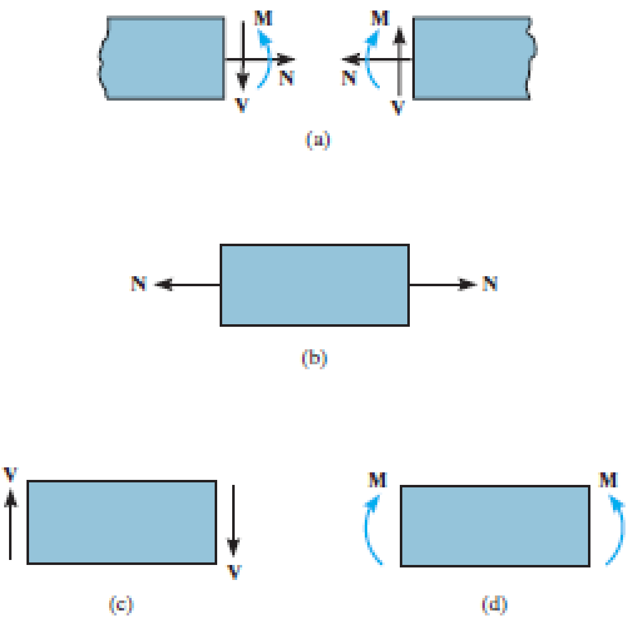


Fig. 4–1

Procedure for Analysis

The following procedure provides a means for applying the method of sections to determine the internal normal force, shear force, and bending moment at a specific location in a structural member.

Support Reactions

- Before the member is "cut" or sectioned, it may be necessary to determine the member's support reactions so that the equilibrium equations are used only to solve for the internal loadings when the member is sectioned.
- If the member is part of a pin-connected structure, the pin reactions can be determined using the methods of Sec. 2–5.

Free-Body Diagram

- Keep all distributed loadings, couple moments, and forces acting on the member in their *exact location*, then pass an imaginary section through the member, perpendicular to its axis at the point where the internal loading is to be determined.
- After the section is made, draw a free-body diagram of the segment that has the least number of loads on it. At the section indicate the unknown resultants **N**, **V**, and **M** acting in their *positive* directions (Fig. 4–1*a*).

Equations of Equilibrium

- Moments should be summed at the section about axes that pass through the *centroid* of the member's cross-sectional area, in order to eliminate the unknowns **N** and **V** and thereby obtain a direct solution for **M**.
- If the solution of the equilibrium equations yields a quantity having a negative magnitude, the assumed directional sense of the quantity is opposite to that shown on the free-body diagram.

Procedure for Analysis

The following procedure provides a method for determining the variation of shear and moment in a beam as a function of position x.

Support Reactions

- Determine the support reactions on the beam and resolve all the external forces into components acting perpendicular and parallel to the beam's axis.

Shear and Moment Functions

- Specify separate coordinates x and associated origins, extending into regions of the beam between concentrated forces and/or couple moments, or where there is a discontinuity of distributed loading.
- Section the beam perpendicular to its axis at each distance x, and from the free-body diagram of one of the segments determine the unknowns V and M at the cut section as functions of x. On the free-body diagram, V and M should be shown acting in their *positive directions*, in accordance with the sign convention given in Fig. 4–1.
- V is obtained from $\Sigma F_y = 0$ and M is obtained by summing moments about the point S located at the cut section, $\Sigma M_S = 0$.
- The results can be checked by noting that $dM/dx = V$ and $dV/dx = w$, where w is positive when it acts upward, away from the beam. These relationships are developed in Sec. 4–3.

Dividing by Δx and taking the limit as $\Delta x \rightarrow 0$, these equations become

$$\frac{dV}{dx} = w(x) \qquad (4\text{–}1)$$

$$\left.\begin{array}{r}\text{Slope of}\\ \text{Shear Diagram}\end{array}\right\} = \left\{\begin{array}{l}\text{Intensity of}\\ \text{Distributed Load}\end{array}\right.$$

$$\frac{dM}{dx} = V \qquad (4\text{–}2)$$

$$\left.\begin{array}{r}\text{Slope of}\\ \text{Moment Diagram}\end{array}\right\} = \{\text{Shear}$$

As noted, Eq. 4–1 states that *the slope of the shear diagram at a point (dV/dx) is equal to the intensity of the distributed load $w(x)$ at the point.* Likewise, Eq. 4–2 states that *the slope of the moment diagram (dM/dx) is equal to the intensity of the shear at the point.*

Equations 4–1 and 4–2 can be "integrated" from one point to another between concentrated forces or couples (such as from B to C in Fig. 4–9a), in which case

$$\Delta V = \int w(x)\,dx \qquad (4\text{–}3)$$

$$\left.\begin{array}{r}\text{Change in}\\ \text{Shear}\end{array}\right\} = \left\{\begin{array}{l}\text{Area under}\\ \text{Distributed Loading}\\ \text{Diagram}\end{array}\right.$$

and

$$\Delta M = \int V(x)\,dx \qquad (4\text{–}4)$$

$$\left.\begin{array}{r}\text{Change in}\\ \text{Moment}\end{array}\right\} = \left\{\begin{array}{l}\text{Area under}\\ \text{Shear Diagram}\end{array}\right.$$

Procedure for Analysis

The following procedure provides a method for constructing the shear and moment diagrams for a beam using Eqs. 4–1 through 4–6.

Support Reactions

- Determine the support reactions and resolve the forces acting on the beam into components which are perpendicular and parallel to the beam's axis.

Shear Diagram

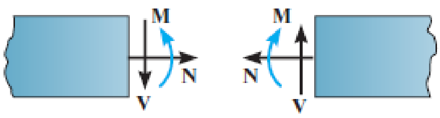
- Establish the V and x axes and plot the values of the shear at the two *ends* of the beam.
- Since $dV/dx = w$, the *slope* of the *shear diagram* at any point is equal to the intensity of the *distributed loading* at the point. (Note that w is positive when it acts upward.)
- If a numerical value of the shear is to be determined at the point, one can find this value either by using the method of sections as discussed in Sec. 4–1 or by using Eq. 4–3, which states that the *change in the shear force* is equal to the *area under the distributed loading diagram*.
- Since $w(x)$ is *integrated* to obtain V, if $w(x)$ is a curve of degree n, then $V(x)$ will be a curve of degree $n + 1$. For example, if $w(x)$ is uniform, $V(x)$ will be linear.

Moment Diagram

- Establish the M and x axes and plot the values of the moment at the ends of the beam.
- Since $dM/dx = V$, the *slope* of the *moment diagram* at any point is equal to the intensity of the *shear* at the point.
- At the point where the shear is zero, $dM/dx = 0$, and therefore this may be a point of maximum or minimum moment.
- If the numerical value of the moment is to be determined at a point, one can find this value either by using the method of sections as discussed in Sec. 4–1 or by using Eq. 4–4, which states that the *change in the moment* is equal to the *area under the shear diagram*.
- Since $V(x)$ is *integrated* to obtain M, if $V(x)$ is a curve of degree n, then $M(x)$ will be a curve of degree $n + 1$. For example, if $V(x)$ is linear, $M(x)$ will be parabolic.

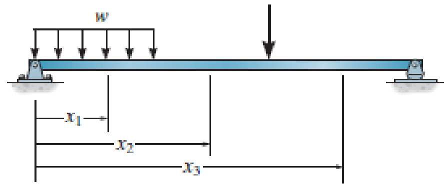
CHAPTER REVIEW

Structural members subjected to planar loads support an internal normal force **N**, shear force **V**, and bending moment **M**. To find these values at a specific point in a member, the method of sections must be used. This requires drawing a free-body diagram of a segment of the member, and then applying the three equations of equilibrium. Always show the three internal loadings on the section in their positive directions.



positive sign convention

The internal shear and moment can be expressed as a function of x along the member by establishing the origin at a fixed point (normally at the left end of the member, and then using the method of sections, where the section is made a distance x from the origin). For members subjected to several loads, different x coordinates must extend between the loads



Shear and moment diagrams for structural members can be drawn by plotting the shear and moment functions. They also can be plotted using the two graphical relationships.

$$\frac{dV}{dx} = w(x)$$

$$\left.\begin{array}{r}\text{Slope of}\\ \text{Shear Diagram}\end{array}\right\} = \left\{\begin{array}{l}\text{Intensity of}\\ \text{Distributed Load}\end{array}\right.$$

$$\frac{dM}{dx} = V$$

$$\left.\begin{array}{r}\text{Slope of}\\ \text{Moment Diagram}\end{array}\right\} = \{\text{Shear}$$

Note that a point of zero shear locates the point of maximum moment since $V = dM/dx = 0$.

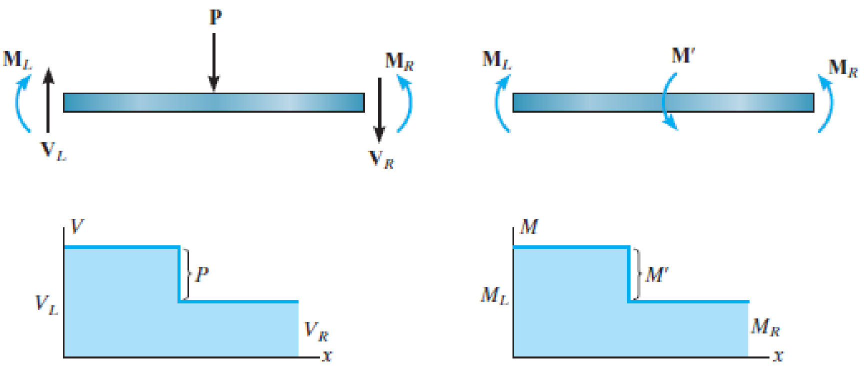
$$\Delta V = \int w(x)\,dx$$

$$\left.\begin{array}{r}\text{Change in}\\ \text{Shear}\end{array}\right\} = \left\{\begin{array}{l}\text{Area under}\\ \text{Distributed Loading}\\ \text{Diagram}\end{array}\right.$$

$$\Delta M = \int V(x)\,dx$$

$$\left.\begin{array}{r}\text{Change in}\\ \text{Moment}\end{array}\right\} = \left\{\begin{array}{l}\text{Area under}\\ \text{Shear Diagram}\end{array}\right.$$

A force acting downward on the beam will cause the shear diagram to jump downwards, and a counterclockwise couple moment will cause the moment diagram to jump downwards.



SHEAR FORCE AND BENDING MOMENT

At every section in a beam carrying transverse loads there will be resultant forces on either side of the section which, for equilibrium must be equal and opposite, and whose combined action tends to shear the section in one of the two ways show in figure 1(a) and figure 1(b).

The shearing force (S.F.) at the section is defined therefore as the algebraic sum of the forces taken on one side of the section. Which side is chosen is purely a matter of convenience but in order that the value obtained on both sides shall have the same magnitude and sign, convenient sign convention has to be adopted.

In addition to the shear, every section of the beam will be subjected to bending (Figure 2(a) and Figure 2(b)) i.e. to a resultant B.M. which is the net effect of the moment of each of the individual loads.

Again for equilibrium, the values on either side of the section must have equal values. The bending moment (B.M.) is defined as the algebraic sum of moments of the force about the section taken either side of the section. As for S.F., a convenient sign convention must be adopted.

Clockwise moments to the left and counter clockwise to the right are positive. Thus figure 2(a) shows a positive bending moment system resulting in sagging of the beam at X – X and figure 2(b) illustrate a negative B.M. system with its associated hogging beam.

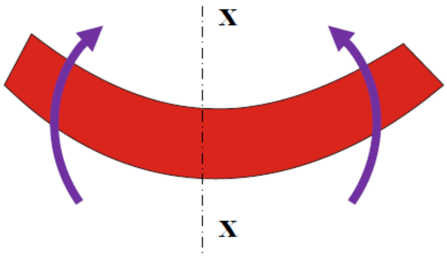


Figure 2 (a) Positive B.M.

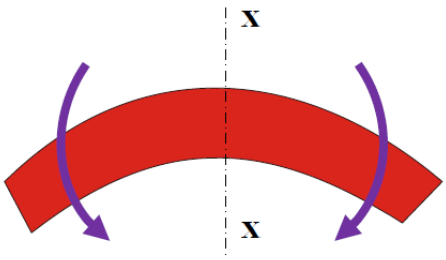


Figure 2 (a) Negative B.M.

RELATIONSHIP BETWEEN LOAD, SHEAR FORCE AND BENDING MOMENT

The beam shown in figure 7(a) carries distributed loading which varies in an arbitrary manner f (w). Consider the free body (figure 7(b)); of a small slice of length dx for which the loading may be regarded as uniform, w.

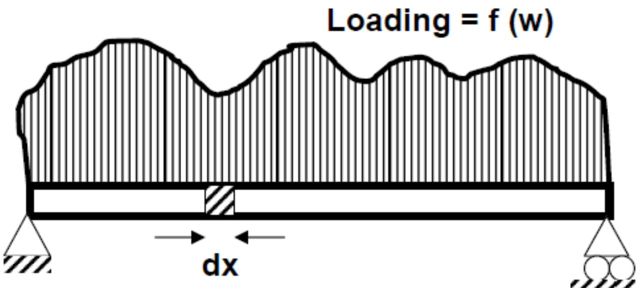


Figure 7(a)

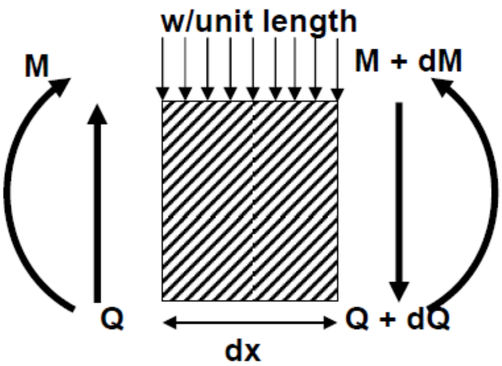


Figure 7(b)

VERTICAL EQUILIBRIUM

$-Q + wdx + (Q + dQ) = 0$

$wdx + dQ = 0$

$$w = -\frac{dQ}{dx} \quad \text{---} \quad (1)$$

MOMENT EQUILIBRIUM

Taking moments about the right-hand edge

$$-M - Qdx + wdx\frac{dx}{2} + (M + dM) = 0$$

$$-Qdx + \frac{w(dx)^2}{2} + dm = 0$$

Neglecting $(dx)^2$

$$-Qdx + dm = 0$$

$$Q = \frac{dM}{dx} \quad \text{---} \quad (2)$$

Substituting equation (2) in equation (1) we have,

$$w = -\frac{d}{dx}\left(\frac{dM}{dx}\right)$$

$$w = \frac{d^2M}{dx^2} \quad \text{---} \quad (3)$$

From equation (1) we have,

$$dQ = -wdx$$

$$\int_1^2 dQ = \int_1^2 -\,wdx$$

$$[Q]_1^2 = \int_1^2 -\,wdx \; + A$$

$$Q_2 - Q_1 = \int -wdx + A \quad \text{---} \quad (4)$$

Thus the change in Shear Force between any two cross-sections may be obtained from the
area under the load distribution curve between those sections.

From equation (2) we have,

$$dm = Qdx$$

$$\int_1^2 dM = \int_1^2 Qdx$$

$$[M]_1^2 = \int_1^2 Qdx + B$$

$$M_2 - M_1 = \int Qdx + B \text{ ------------------------------ } (5)$$

Thus the change in bending moment between any two sections is found from the area under the shear force diagram between those sections.

Example 1

Use the relationships developed between load, shear force and bending moments to find the position of zero shear force and the value of the maximum bending moment for the simply supported beam shown in figure EX. 1. It carries a distributed load which varies linearly in intensity from zero at the left to w newtons per metre at the right-hand end.

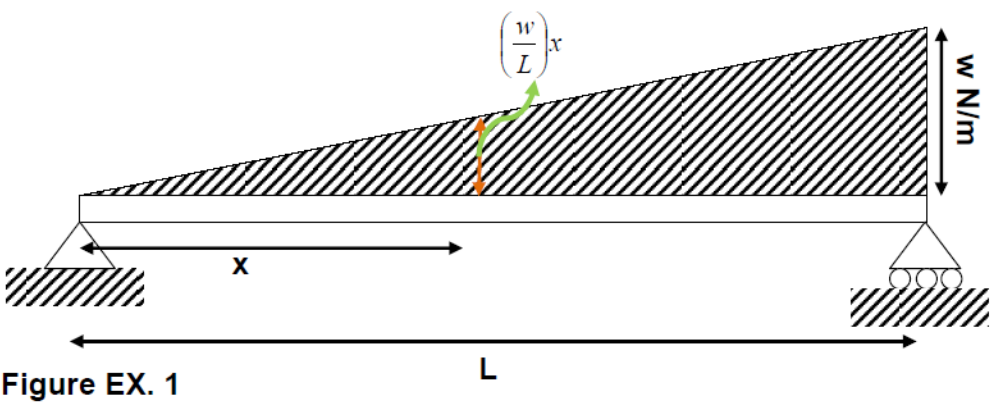


Figure EX. 1

Solution:

By similar triangle:

$$\frac{y}{w} = \frac{\mathrm{x}}{\mathrm{L}}$$

$$y = \left(\frac{\mathrm{w}}{\mathrm{L}}\right)x$$

From equation (4)

$$\text{Shear Force Q} = -\int \frac{wx}{L}\,dx + \mathrm{A}$$

$$\mathrm{Q} = -\frac{wx^2}{2L} + \mathrm{A} \text{ -- } (6)$$

And from equation (5)

$$\mathrm{M} = -\int\left(\frac{wx^2}{2L} + \mathrm{A}\right)dx + \mathrm{A}$$

$$M = \frac{-\,\mathrm{wx}^3}{6L} + \mathrm{Ax} + \mathrm{B} \text{ --------------------------------------- } (7)$$

BOUNDARY CONDITIONS

At x = 0, M = 0 and

At x = L, M = 0

Substituting the boundary values in equation (7) we have,

B = 0 and

$$-\frac{wL^2}{6} + AL = 0$$

$$\therefore \quad A = \frac{wL}{6}$$

$$\text{Shear Force } Q = \frac{-\,wx^2}{2L} + \frac{wL}{6} \text{ and}$$

$$M = \frac{-wx^3}{6L} + \frac{wLx}{6}$$

Position of Zero Shear Force

$$Q = \frac{-\,wx^2}{2L} + \frac{wL}{6}$$

$$\frac{wx^2}{2L} = \frac{wL}{6}$$

$$x^2 = \frac{wL \text{ x } 2L}{6w} = \frac{L^2}{3}$$

$$x = \frac{L}{\sqrt{3}}$$

BEAM SUPPORTING UNIFORMLY DISTRIBUTED LOAD (w) ONLY

Maximum Bending Moment

$$M = \frac{-\,wx^3}{6L} \quad \frac{wLx}{6}$$

$$\frac{dM}{dx} = \frac{-\,wx^2}{2L} + \frac{wL}{6} = Q$$

$\therefore$ M is maximum when Q = 0 that is at point $x = \frac{L}{\sqrt{3}}$

$$M_{MAX} = \frac{-w}{6L} \text{ x } \left(\frac{L}{\sqrt{3}} \right)^3 + \frac{wL}{6} \text{ x } \frac{L}{\sqrt{3}}$$

$$= \frac{-wL^3}{18\sqrt{3}\,L} + \frac{wL^2}{6\sqrt{3}}$$

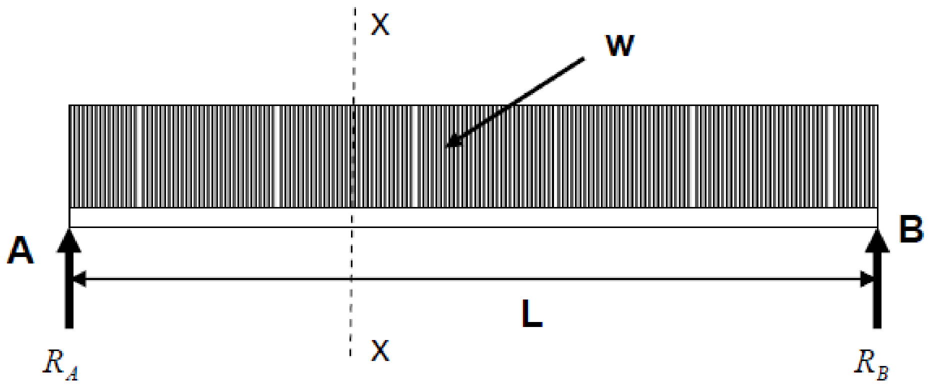
$$= \frac{-\,wL^2}{6\sqrt{3}} \left(-\frac{1}{3} + 1 \right)$$

$$= \frac{wL^2}{9\sqrt{3}}$$

$$M_{MAX} = \frac{wL^2}{9\sqrt{3}}$$

CALCULATION OF BENDING MOMENT AND SHEAR FORCE

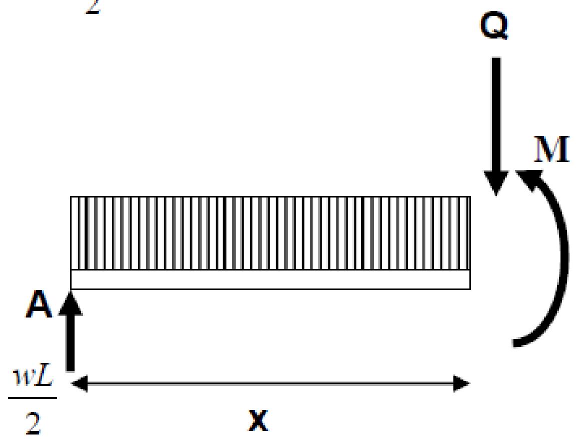
SIMPLY SUPPORTED BEAM



$\Sigma M_B = 0$

$$R_A \text{ x } L - \frac{wL^2}{2} = 0$$

$$R_A \frac{wL}{2}$$



Shear Force

$$Q - \frac{wL}{2} + wx = 0$$

$$Q = \frac{wL}{2} - wx = 0$$

At $x = 0$

$$Q = \frac{wL}{2}$$

At $x = L$

$$Q = -\frac{wL}{2}$$

Bending Moment

$$M + \frac{wx^2}{2} - \frac{\mathbf{wLx}}{\mathbf{2}} = 0$$

$$M = \frac{\mathbf{wLx}}{\mathbf{2}} - \frac{wx^2}{2}$$

At x = 0

M = 0 and

At x = L

M = 0

$$\frac{dM}{dx} = \frac{\mathbf{wL}}{\mathbf{2}} - \mathbf{wx}$$

When $\frac{dM}{dx} = 0$

$$\frac{\mathbf{wL}}{\mathbf{2}} - \mathbf{wx} = 0$$

Therefore, $x = \frac{L}{2}$

$$M_{MAX} = \frac{\mathbf{wL}}{\mathbf{2}} \,\mathbf{x}\, \frac{\mathbf{L}}{\mathbf{2}} - \frac{\mathbf{w}}{\mathbf{2}} \,\mathbf{x}\, \left(\frac{\mathbf{L}}{\mathbf{2}}\right)^2$$

$$M_{MAX} = \frac{\mathbf{wL}^2}{\mathbf{4}} - \frac{\mathbf{wL}^2}{\mathbf{8}}$$

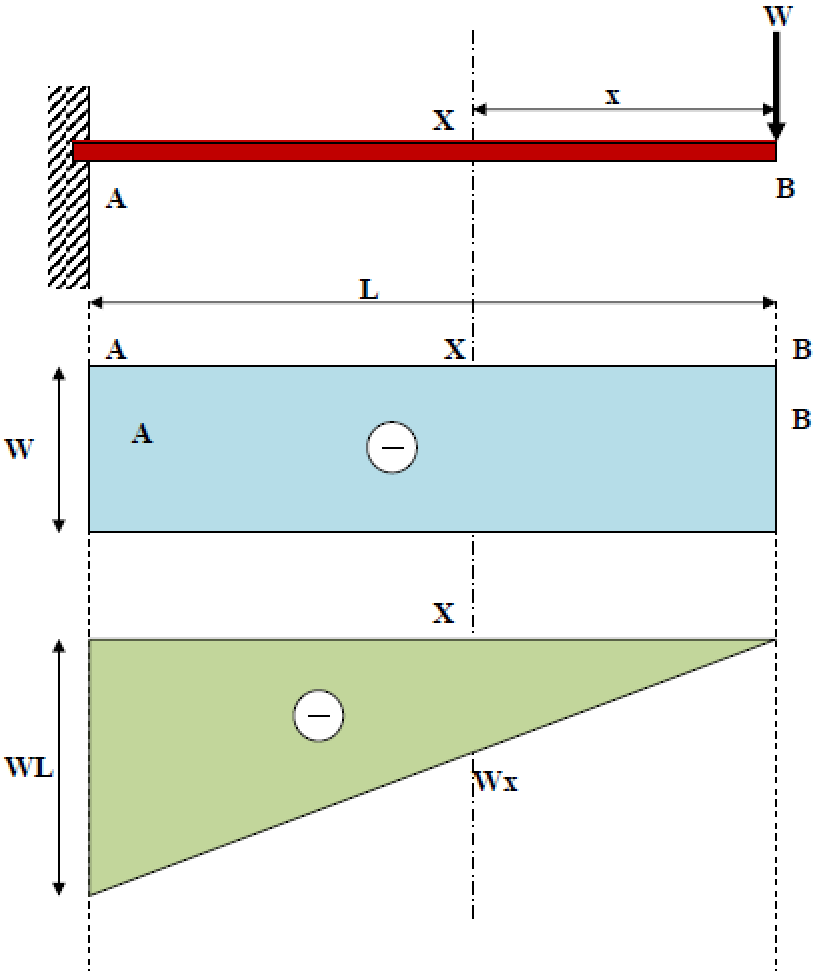
$$M_{MAX} = \frac{\mathbf{wL}^2}{\mathbf{8}}$$

CANTILEVER BEAM

Cantilever Beam with a Point Load at its Free End

Consider a cantilever AB of length L and carrying a point load W at its free end B as shown

in the figure below.



We know that shear force at any section X, at a distance x from the free end, is equal to the total unbalanced vertical force. i.e.

FX = -W (Minus sign due to right downward) And bending moment at this section,

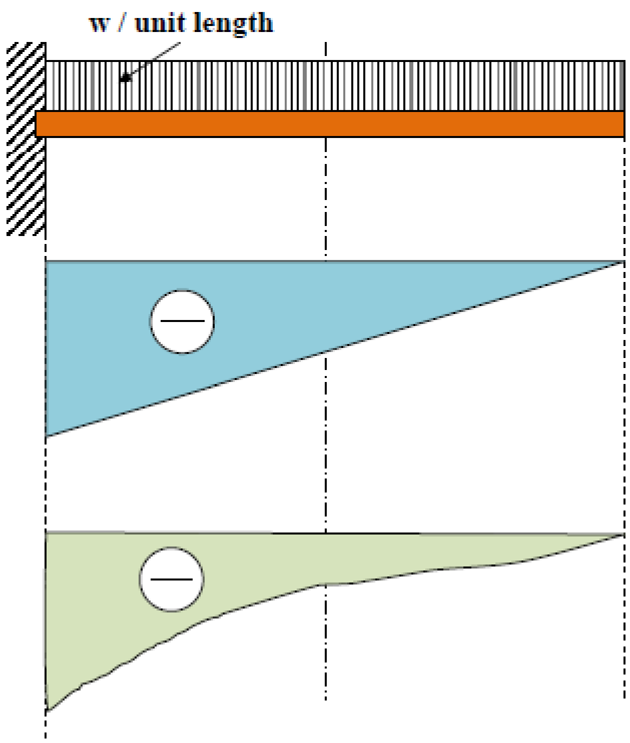
MX = - Wx (Minus sign due to hogging)

Thus from the equation of shear force, we see that the shear force is constant and is equal to – W at all sections between B and A. And from the bending moment equation, we see that the bending moment is zero at B (where x = 0) and increases by a straight line law to –WL; at x = L. Now draw the shear force and bending diagrams, the way graphs are plotted. The curves

obtained give the shear force diagram and the bending moment diagram as shown in the figure.

Cantilever Beam with a Uniformly Distributed Load

Consider a cantilever beam AB of length and carrying a uniformly distributed load of w per unit length, over the entire length of the cantilever as shown in the figure below.



We know that shear force at any section X, at a distance x from,

FX = -wx (Minus sign due to right downwards)

Thus we see that shear force is zero at B (where x = 0) and increases by a straight line law to –wL at A as shown in the figure.

w / unit length 24

We also know that the bending moment at X, MX = -wx . x/2 = -wx2/2

Thus we also see that the bending moment is zero at B (where x = 0) and increases in the form of a parabolic curve to –wL2/2 at B (where x = L) as shown in the figure

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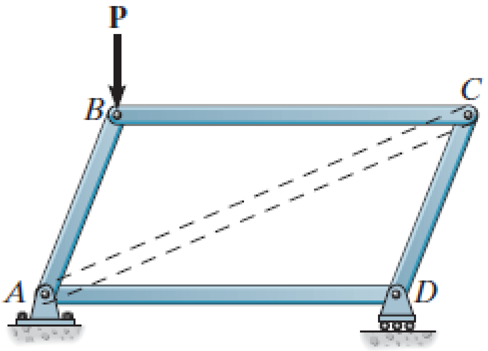
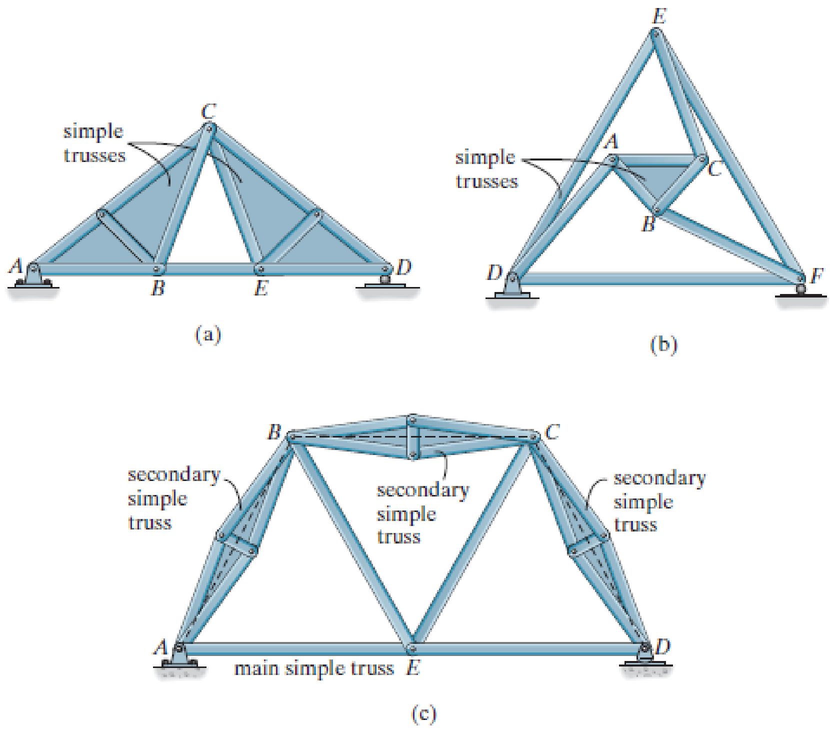
Theory of Structures

Simple Truss. To prevent collapse, the framework of a truss must be rigid. Obviously, the four-bar frame *ABCD* in Fig. 3–7 will collapse unless a diagonal, such as *AC*, is added for support.The simplest framework that is rigid or stable is a *triangle*. Consequently, a *simple truss* is constructed by starting with a basic triangular element, such as *ABC* in Fig. 3–8, and connecting two members (*AD* and *BD*) to form an additional element. Thus it is seen that as each additional element of two members is placed on the truss, the number of joints is increased by one.

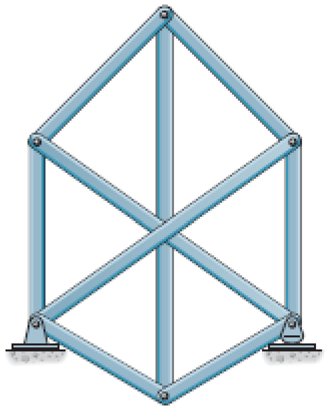


Compound Truss. A *compound truss* is formed by connecting two or more simple trusses together. Quite often this type of truss is used to support loads acting over a *large span,* since it is cheaper to construct a somewhat lighter compound truss than to use a heavier single simple truss.



Various types of compound trusses

Complex Truss. A *complex truss* is one that cannot be classified as being either simple or compound.The truss in Fig. 3–12 is an example.



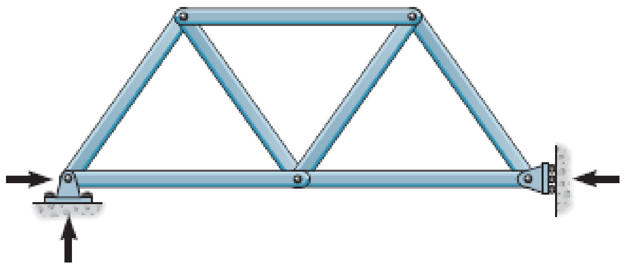
Complex truss

Determinacy. For any problem in truss analysis, it should be realized that the total number of unknowns includes the forces in b number of 0bars of the truss and the total number of external support reactions r.
Since the truss members are all straight axial force members lying in the same plane, the force system acting at each joint is coplanar and concurrent. Consequently, rotational or moment equilibrium is automatically satisfied at the joint (or pin), and it is only necessary to satisfy $\Sigma F_x = 0$ and $\Sigma F_x = 0$ to ensure translational or force equilibrium.Therefore, only two equations of equilibrium can be written for each joint, and if there are j number of joints, the total number of equations available for solution is 2j. By simply comparing the total number of unknowns (b+r)with the total number of available equilibrium equations, it is therefore possible to specify the determinacy for either a simple, compound, or complex truss. We have

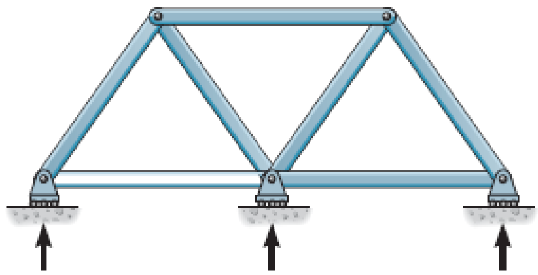
$b + r = 2j$ statically determinate

$b + r > 2j$ statically indeterminate

Stability. If **$b + r < 2j$** ,a truss will be *unstable*, that is, it will collapse, since there will be an insufficient number of bars or reactions to constrain all the joints. Also, a truss can be unstable if it is statically determinate or statically indeterminate. In this case the stability will have to be determined either by inspection or by a force analysis.
External Stability. As stated in Sec. 2–4, a structure (or truss) is externally unstable if all of its reactions are concurrent or parallel. For example, the two trusses in Fig. 3–13 are externally unstable since the support reactions have lines of action that are either concurrent or parallel.

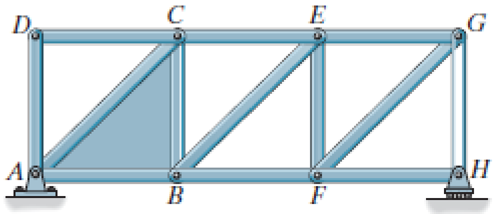


unstable concurrent reactions



unstable parallel reactions

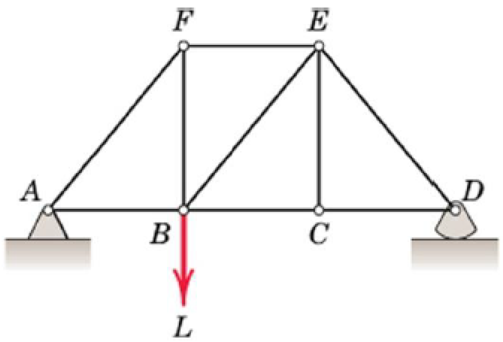
Internal Stability. The internal stability of a truss can often be checked by careful inspection of the arrangement of its members. If it can be determined that each joint is held fixed so that it cannot move in a "rigid body" sense with respect to the other joints, then the truss will be stable. Notice that *a simple truss will always be internally stable*, since by the nature of its construction it requires starting from a basic triangular element and adding successive "rigid elements," each containing two additional members and a joint. The truss in Fig. 3–14 exemplifies this construction, where, starting with the shaded triangle element *ABC*, the successive joints *D*, *E*, *F*, *G*, *H* have been added.
If a truss is constructed so that it does not hold its joints in a fixed position, it will be unstable or have a "critical form."An obvious example of this is shown in Fig. 3–15, where it can be seen that no restraint or
fixity is provided between joints *C* and *F* or *B* and *E*, and so the truss **Fig. 3–14** will collapse under load.



To summarize, if the truss has *b* bars, *r* external reactions, and *j* joints, then if

$b + r < 2j$	unstable
$b + r \geq 2j$	unstable if truss support reactions are concurrent or parallel or if some of the components of the truss form a collapsible mechanism

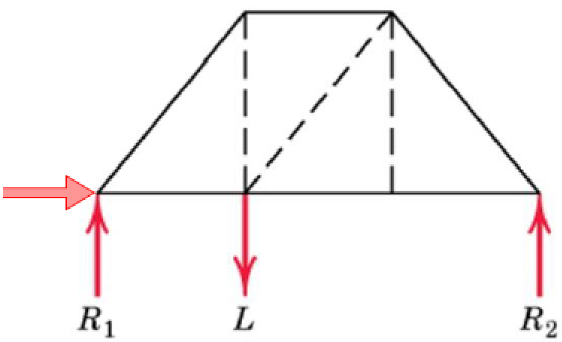
Plane Truss :: Determinacy



No. of unknown reactions = **3**

No. of equilibrium equations = **3**

: **Statically Determinate *(External)***



No. of members (m) = 9

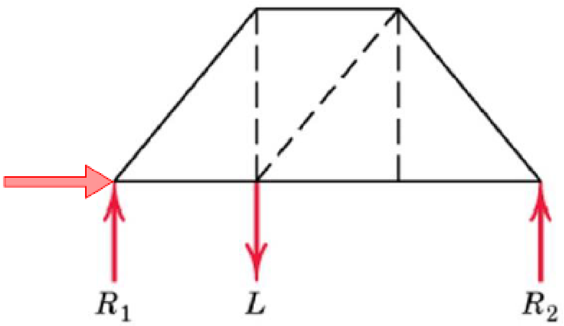
No. of joints (j) = 6

No. of unknown reactions (R) = 3

∴ **m + R = 2j**

: **Statically Determinate *(Internal)***

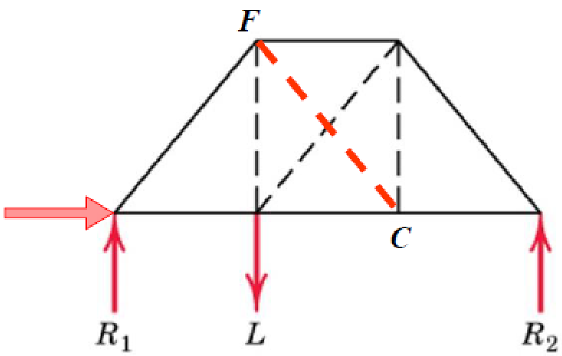
Plane Truss :: Determinacy



Presence of internal members

: **Additional sharing for forces**

: **Additional Stability**



Further addition of internal members

: **Strengthening** of Joints C and F

: **Additional Stability and force sharing**

: **m + R > 2j**

: **Statically Indeterminate *(Internal)***

Plane Truss :: Determinacy

Internal Redundancy or
Degree of Internal Static Indeterminacy
Extra Members than required → Internal Redundancy

Equilibrium of each joint can be specified by two scalar force equations → 2j equations for a truss with "j" number of joints
→ Known Quantities

For a truss with "m" number of two force members, and maximum 3 unknown support reactions → Total Unknowns = m + 3
("m" member forces and 3 reactions for externally determinate truss)

m + 3 = 2j → Statically Determinate Internally
m + 3 > 2j → Statically Indeterminate Internally
m + 3 < 2j → Unstable Truss

Plane Truss :: Determinacy

When more number of members/supports are present than are needed to prevent collapse/stability

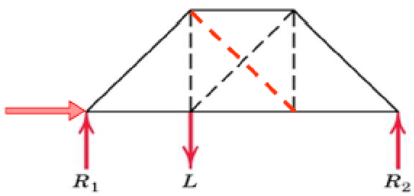
→ Statically Indeterminate Truss

- cannot be analysed using equations of equilibrium alone!
- additional members or supports which are not necessary for maintaining the equilibrium configuration → Redundant

External and Internal Redundancy

Extra Supports than required → **External Redundancy**
– Degree of indeterminacy from available equilibrium equations

Extra Members than required → **Internal Redundancy**



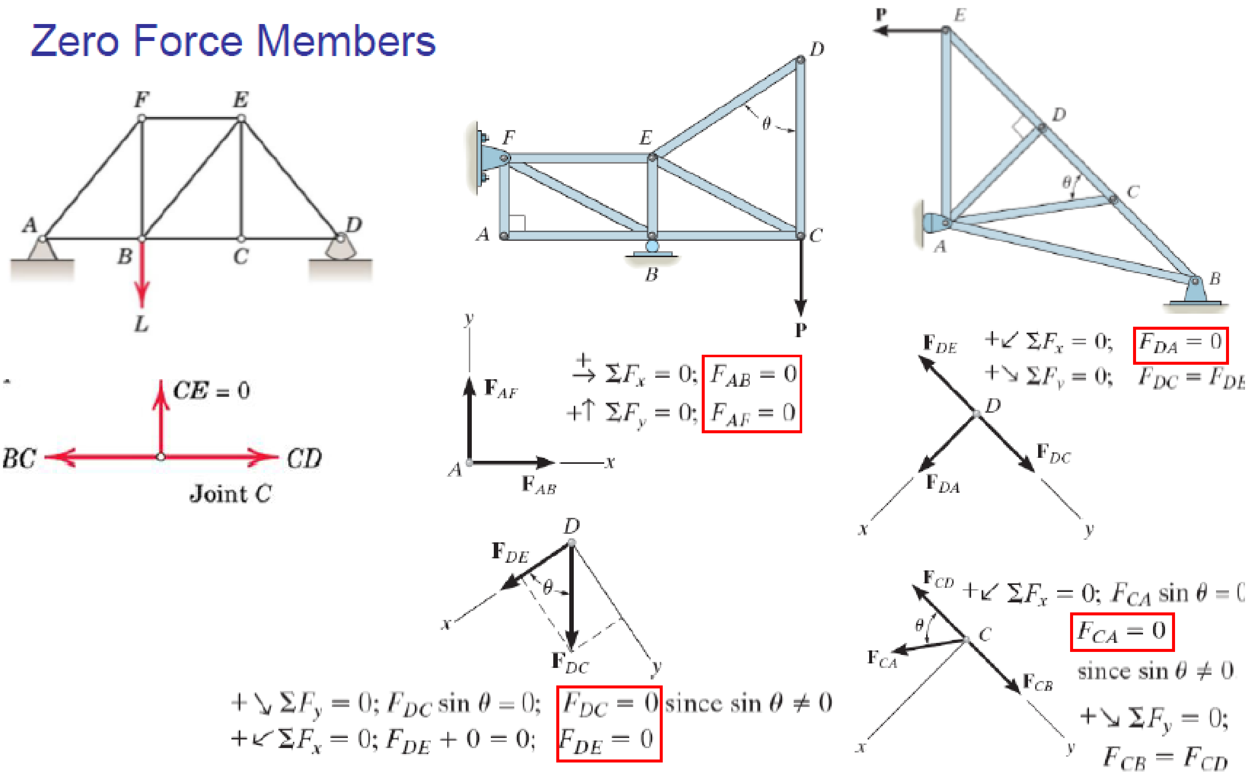
Plane Truss :: Analysis Methods

Why to Provide Redundant Members?

- To maintain alignment of two members during construction
- To increase stability during construction
- To maintain stability during loading (Ex: to prevent buckling of compression members)
- To provide support if the applied loading is changed
- To act as backup members in case some members fail or require strengthening
- Analysis is difficult but possible

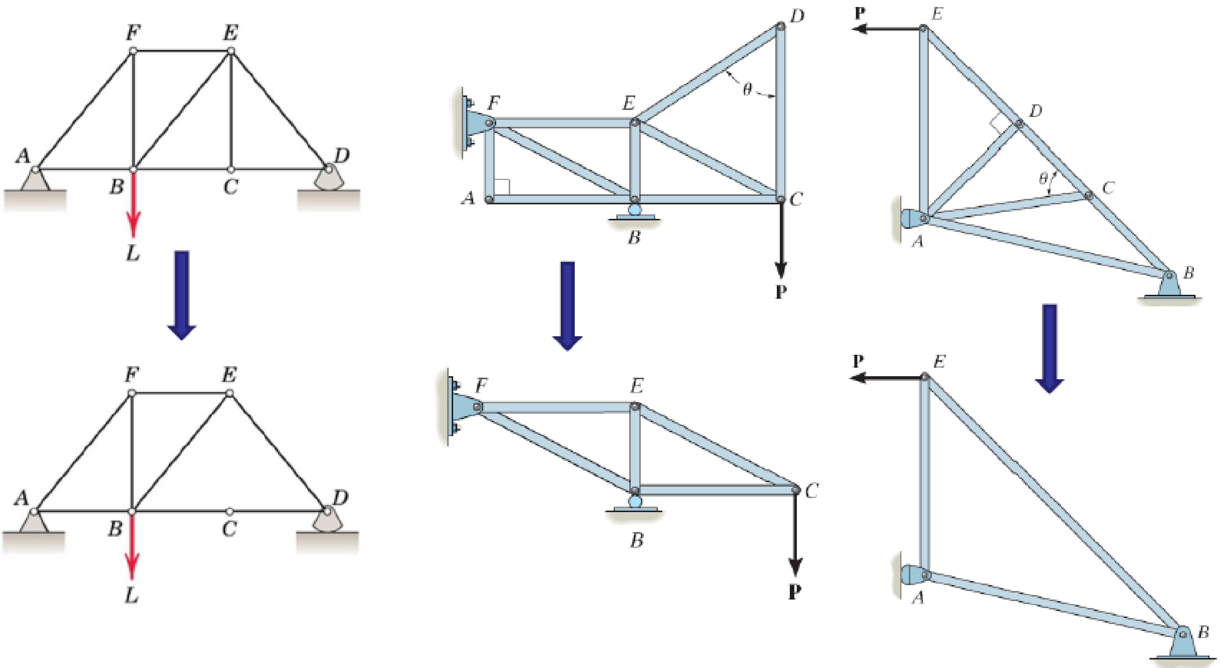
Plane Truss :: Analysis Methods

Zero Force Members



Plane Truss :: Analysis Methods

Zero Force Members: Simplified Structures

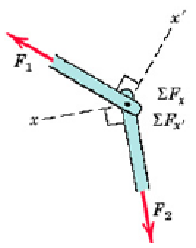


Plane Truss :: Analysis Methods

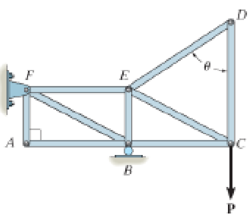
Zero Force Members: Conditions

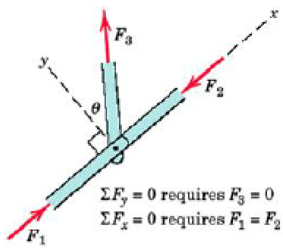
• if only two noncollinear members form a truss joint and no external load or support reaction is applied to the joint, the two members must be zero force members

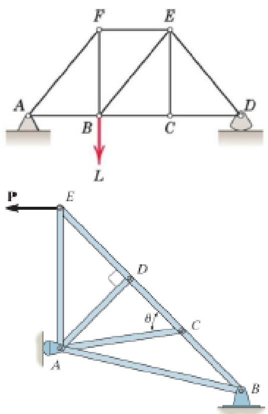
•if three members form a truss joint for which two of the members are collinear, the third member is a zero-force member provided no external force or support reaction is applied to the joint



$\Sigma F_x = 0$ requires $F_1 = 0$
$\Sigma F_{x'} = 0$ requires $F_2 = 0$



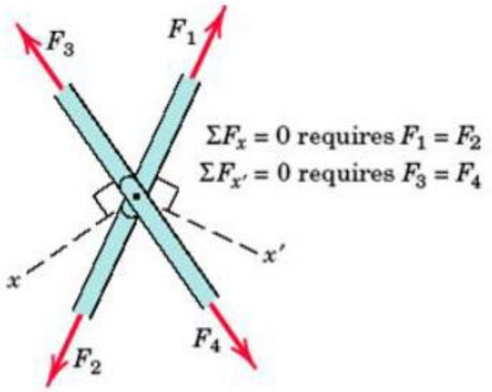




Structural Analysis: Plane Truss

Special Condition

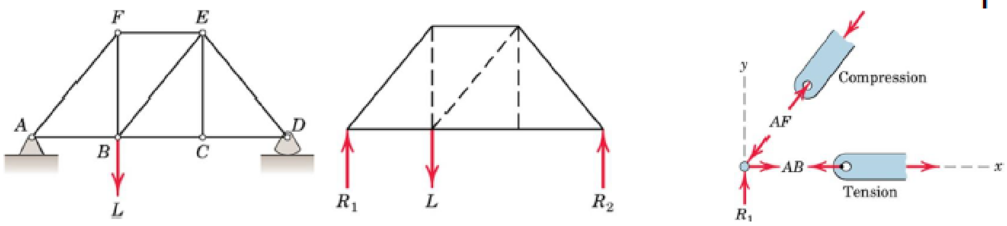
- When two pairs of collinear members are joined as shown in figure, the forces in each pair must be equal and opposite.



Plane Truss :: Analysis Methods

Method of Joints

- Start with any joint where at least one known load exists and where not more than two unknown forces are present.



FBD of Joint A and members AB and AF: Magnitude of forces denoted as *AB* & *AF*

- Tension indicated by an arrow away from the pin
- Compression indicated by an arrow toward the pin

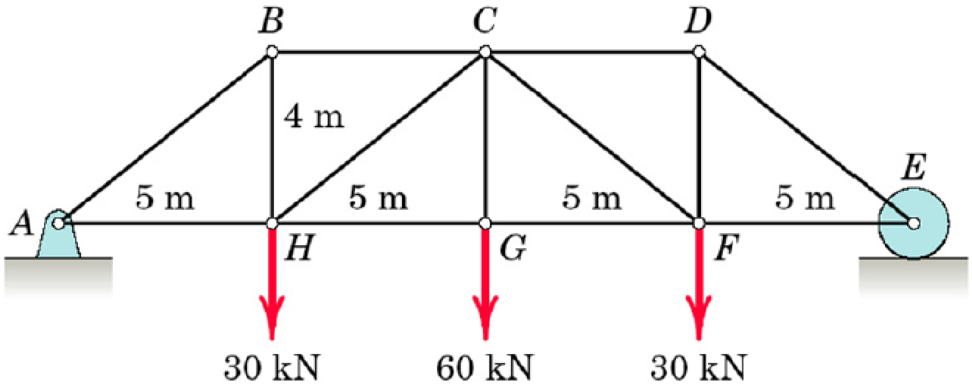
Magnitude of AF from $\Sigma F_y = 0$

Magnitude of AB from $\Sigma F_x = 0$

Analyze joints F, B, C, E, & D in that order to complete the analysis

Method of Joints: Example

Determine the force in each member of the loaded truss by Method of Joints.

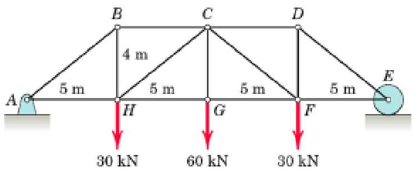


Is the truss statically determinant externally? Yes

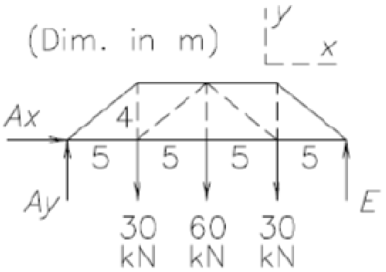
Is the truss statically determinant internally? Yes

Are there any Zero Force Members in the truss? No

Method of Joints: Example



Solution

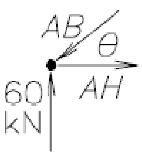


As a whole: $\Sigma F_x = 0 \Rightarrow A_x = 0$

$A_y = E = 60$ kN by

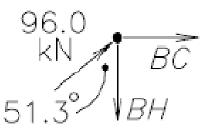
$\Sigma F_y = 0$ and symmetry.

Joint A: $\left(\theta = \tan^{-1}(4/5) = 38.7°\right)$



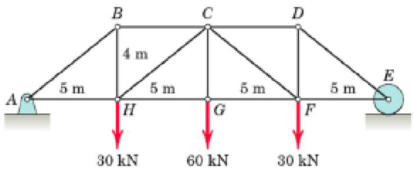
$$\begin{cases} \Sigma F_y = 0 : 60 - AB\sin\theta = 0, \ \underline{AB = 96.0 \text{ kN } C} \\ \Sigma F_x = 0 : AH - 96.0\cos\theta, \ \underline{AH = 75 \text{ kN } T} \end{cases}$$

Joint B:

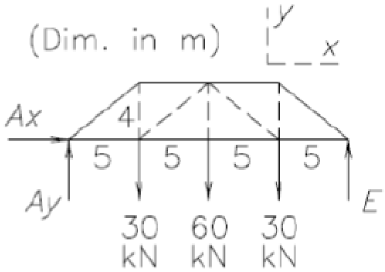


$$\begin{cases} \Sigma F_x = 0 : BC + 96.0\sin 51.3° = 0, \ \underline{BC = -75 \text{ kN } (C)} \\ \Sigma F_y = 0 : -BH + 96.0\cos 51.3° = 0, \ \underline{BH = 60 \text{ kN } T} \end{cases}$$

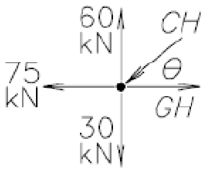
Method of Joints: Example



Solution

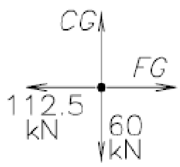


Joint H:



$$\begin{cases} \Sigma F_y = 0: -CH\sin\theta + 30 = 0, \; \underline{CH = 48.0 \text{ kN } C} \\ \Sigma F_x = 0: 48.0\cos\theta + GH - 75 = 0, \; \underline{GH = 112.5 \text{ kN } T} \end{cases}$$

Joint G:



$$\Sigma F_y = 0 \Rightarrow CG = 60 \text{ kN } T$$

By symmetry:

$FG = 112.5$ kN T, $CF = 48.0$ kN C

$CD = 75$ kN C, $DF = 60$ kN T

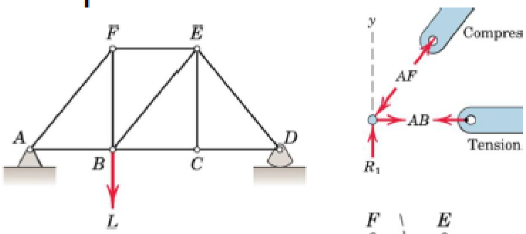
$EF = 75$ kN T, $DE = 96.0$ kN C

Structural Analysis: Plane Truss

Method of Joints: only two of three equilibrium equations were applied at each joint because the procedures involve concurrent forces at each joint

→Calculations from joint to joint

→More time and effort required

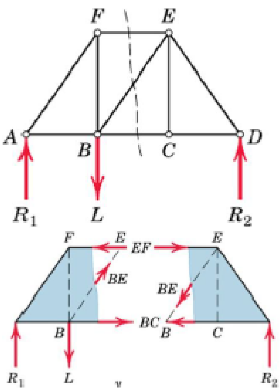


Method of Sections

Take advantage of the 3rd or moment equation of equilibrium by selecting an entire section of truss

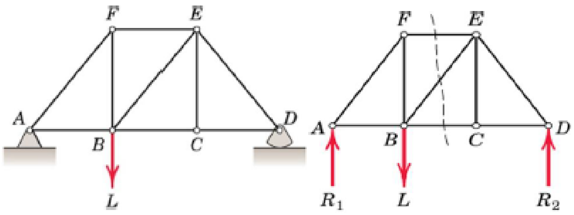
→Equilibrium under non-concurrent force system

→Not more than 3 members whose forces are unknown should be cut in a single section since we have only 3 independent equilibrium equations

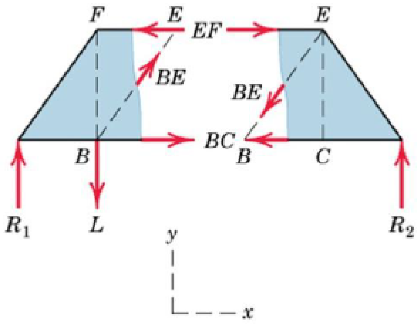


Structural Analysis: Plane Truss

Method of Sections



- Find out the reactions from equilibrium of whole truss
- To find force in member BE:
- Cut an imaginary section (dotted line)
- Each side of the truss section should remain in equilibrium
  - Apply to each cut member the force exerted on it by the member cut away
  - The left hand section is in equilibrium under *L*, R_1, *BC*, *BE* and *EF*
  - Draw the forces with proper senses (else assume)
    - Moment @ B → *EF*
    - $L > R_1$; → $\sum F_y=0$ → *BE*
    - Moment @ E and observation of whole truss → *BC*
  - Forces acting towards cut section → Compressive
  - Forces acting away from the cut section → Tensile



- Find *EF* from $\sum M_B=0$; Find *BE* from $\sum F_y=0$
- Find *BC* from $\sum M_E=0$

→ Each unknown has been determined independently of the other two

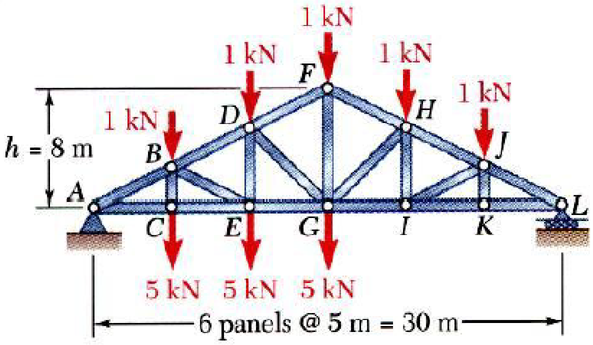
Structural Analysis: Plane Truss

Method of Sections

- Principle: If a body is in equilibrium, then any part of the body is also in equilibrium.
- Forces in few particular member can be directly found out quickly without solving each joint of the truss sequentially
- Method of Sections and Method of Joints can be conveniently combined
- A section need not be straight.
- More than one section can be used to solve a given problem

Structural Analysis: Plane Truss

Method of Sections: Example



Find out the internal forces in members FH, GH, and GI

Find out the reactions

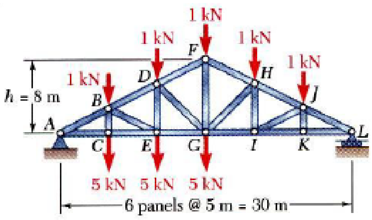
$$\sum M_A = 0 = -(5\text{ m})(6\text{ kN}) - (10\text{ m})(6\text{ kN}) - (15\text{ m})(6\text{ kN}) - (20\text{ m})(1\text{ kN}) - (25\text{ m})(1\text{ kN}) + (30\text{ m})L$$

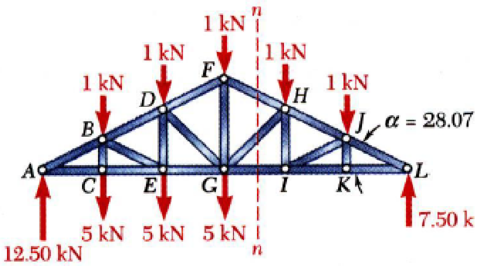
$$L = 7.5\text{ kN} \uparrow$$

$$\sum F_y = 0 = -20\text{ kN} + L + A$$

$$A = 12.5\text{ kN} \uparrow$$

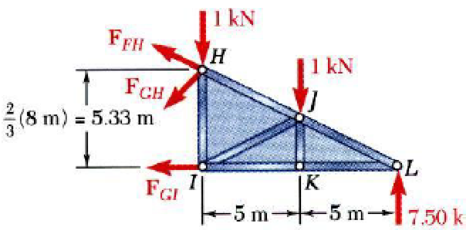
Method of Sections: Example Solution





- Pass a section through members FH, GH, and GI and take the right-hand section as a free body.

$$\tan\alpha = \frac{FG}{GL} = \frac{8\text{ m}}{15\text{ m}} = 0.5333 \qquad \alpha = 28.07°$$



- Apply the conditions for static equilibrium to determine the desired member forces.

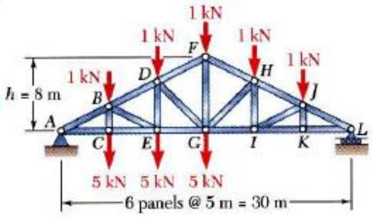
$$\sum M_H = 0$$

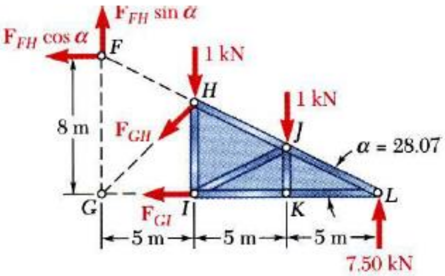
$$(7.50\text{ kN})(10\text{ m}) - (1\text{ kN})(5\text{ m}) - F_{GI}(5.33\text{ m}) = 0$$

$$F_{GI} = +13.13\text{ kN}$$

$$\boxed{F_{GI} = 13.13\text{ kN} \;\; T}$$

Method of Sections: Example Solution



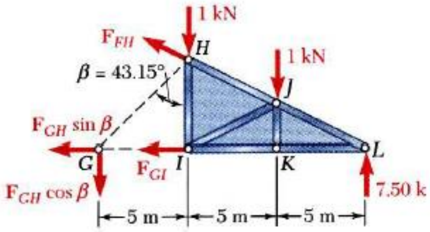


$$\sum M_G = 0$$

$$(7.5\,\text{kN})(15\,\text{m}) - (1\,\text{kN})(10\,\text{m}) - (1\,\text{kN})(5\,\text{m}) + (F_{FH}\cos\alpha)(8\,\text{m}) = 0$$

$$F_{FH} = -13.82\,\text{kN}$$

$$\boxed{F_{FH} = 13.82\,\text{kN}\ \ C}$$



$$\tan\beta = \frac{GI}{HI} = \frac{5\,\text{m}}{\frac{2}{3}(8\,\text{m})} = 0.9375 \qquad \beta = 43.15^\circ$$

$$\sum M_L = 0$$

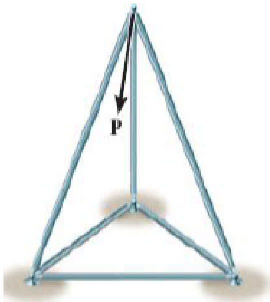
$$(1\,\text{kN})(10\,\text{m}) + (1\,\text{kN})(5\,\text{m}) + (F_{GH}\cos\beta)(15\,\text{m}) = 0$$

$$F_{GH} = -1.371\,\text{kN}$$

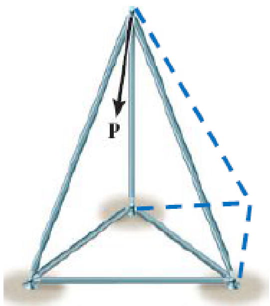
$$\boxed{F_{GH} = 1.371\,\text{kN}\ \ C}$$

Structural Analysis: Space Truss

Space Truss



- 6 bars joined at their ends to form the edges of a tetrahedron as the basic non-collapsible unit
- 3 additional concurrent bars whose ends are attached to three joints on the existing structure are required to add a new rigid unit to extend the structure.



A space truss formed in this way is called a Simple Space Truss

If center lines of joined members intersect at a point
→Two force members assumption is justified
→ Each member under Compression or Tension

Structural Analysis: Space Truss

Static Determinacy of Space Truss

$$\Sigma F_x = 0 \quad \Sigma M_x = 0$$
$$\Sigma F_y = 0 \quad \Sigma M_y = 0$$
$$\Sigma F_z = 0 \quad \Sigma M_z = 0$$

Six equilibrium equations available to find out support reactions
→ if these are sufficient to determine all support reactions
→ The space truss is Statically Determinate Externally

Equilibrium of each joint can be specified by three scalar force equations
→ 3j equations for a truss with "j" number of joints
→ Known Quantities

For a truss with "m" number of two force members, and maximum 6 unknown support reactions → Total Unknowns = m + 6
("m" member forces and 6 reactions for externally determinate truss)
Therefore:
m + 6 = 3j → Statically Determinate Internally
m + 6 > 3j → Statically Indeterminate Internally
m + 6 < 3j → Unstable Truss

A necessary condition for Stability but not a sufficient condition since one or more members can be arranged in such a way as not to contribute to stable configuration of the entire truss

Statical Indeterminacy

It is difference of the unknown forces (internal forces plus external reactions) and the equations of equilibrium.

Kinematic Indeterminacy

It is the number of possible relative displacements of the nodes in the directions of stress resultants.

INDETERMINACY OF STRUCTURAL SYSTEM

The indeterminacy of a structure is measured as statical (∝ s) or kinematical (∝ k)

indeterminacy.

$\propto s = P(M - N + 1) - r = PR - r$

$\propto k = P(N - 1) + r - c$

$\propto s + \propto k = PM - c$

P = 6 for space frames subjected to general loading

P = 3 for plane frames subjected to in plane or normal to plane loading.

N = Number of nodes in structural system.

M = Number of members of completely stiff structure which includes foundation as singly connected system of members. In completely stiff structure there is no release present. In singly connected system of rigid foundation members there is only one route between any two points in which tracks are not retraced. The system is considered comprising of closed rings or loops.

R = Number of loops or rings in completely stiff structure.

r = Number of releases in the system.

c = Number of constraints in the system.

R = (M – N + 1)

For plane and space trusses $\propto s$ reduces to:

$\propto s = M - (NDOF)\ N + P$

M = Number of members in completely stiff truss.

P = 6 and 3 for space and plane truss respectively

N = Number of nodes in truss.

NDOF = Degrees of freedom at node which is 2 for plane truss and 3 for space truss.

For space truss $\propto s = M - 3N + 6$

For plane truss $\propto s = M - 2N + 3$

Test for static indeterminacy of structural system

If $\propto s > 0$ Structure is statically indeterminate

If $\propto s = 0$ Structure is statically determinate

and if $\propto s < 0$ Structure is a mechanism.

It may be noted that structure may be mechanism even if $\propto s > 0$ if the releases are present in such a way so as to cause collapse as mechanism. The situation of mechanism is unacceptable

INTERNALLY INDETERMINATE STRUCTURES:

A truss is statically determinate internally if the total number of members

m=2j – 3

where j= number of joints.

A truss having more than (2j – 3) members is statically indeterminate or redundant, the degree of indeterminacy or redundancy being equal to the number of extra members.

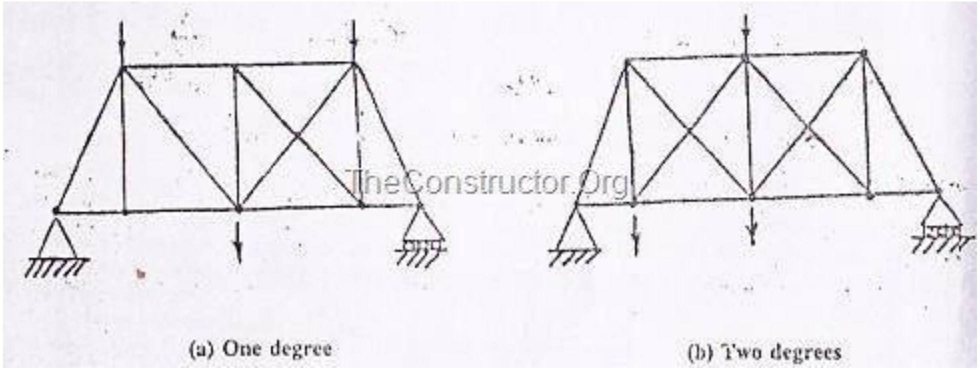


Figure 6

Thus the truss shown in figure 6(a) is statically redundant by one degree because there are 14 members and 8 joints.

Number of redundant members = m= 2j – 3

= 14 – (16 – 3) = 1

Similarly, the truss shown in figure 6(b) is internally redundant by two degrees.

The internally indeterminate trusses can be analysed by **strain energy method**.

EXTERNALLY AND INTERNALLY INDETERMINATE STRUCTURES

A truss is statically determinate, both externally and internally when

(a) All the reactions can be determined from the conditions of equilibrium, namely $\sum H = 0, \sum V = 0, \sum M = 0$, and

(b) The total number of members, m= 2j – 3, where j= number of joints.

The truss shown in figure 7 is externally indeterminate to one degree because the numbers of reactions to be determined are three, and the conditions of equilibrium reduces to two, namely $\sum V = 0, \sum M = 0$. This truss is also internally indeterminate to one degree because there is one extra member.

Number of redundant member = m – (2j – 3)= 22 – (2 x 12 – 3)= 1

Such trusses can be analysed by using **strain energy method**.

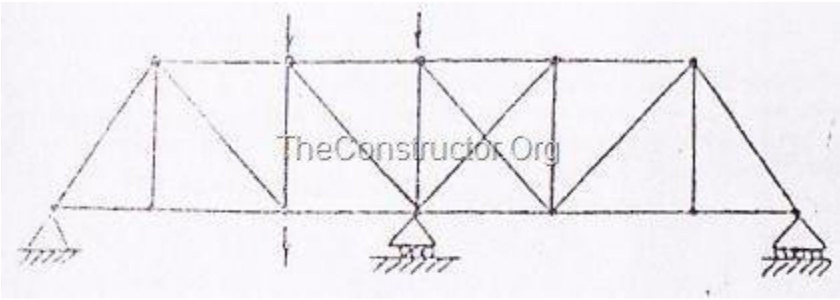


Figure 7

Northern Technical University
Technical College of Mosul
Building & Construction
Technology Engineering Dept.

THEORY OF STRUCTURES

THIRD CLASS

Lecturer:
Dr. Muthanna Adil Najm ABBU

2015 - 2016

Theory of Structures

Text Books:

- Structural Analysis (Eighth Edition) By R. C. Hibbeler Prentice.

References:

- Basic Concepts and Conventional Methods of Structural Analysis by Dr. Mohan, Indian Institute of Technology.
- Theory of Structures (ninth edition) By Dr, B.C. Punmia, Ashok Kumar Jain and Arun Kumar Jain

Units			
	SI	Metric	British
Force	N kN = 1000 N 1 kg = 9.81 N	gm kg = 1000 g Ton = 1000 kg	lb kip = 1000 lb 1 lb = 4.448 N
Length	mm m = 1000 mm mm = 0.1 cm	cm cm = 10 mm m = 100 cm	in ft = 12 in (″) 1 in = 25.4 mm
Stress	$Stress = \frac{Force}{Area} = \frac{N}{m^2} = Pa$ $\frac{kN}{m^2} = kPa$ $\frac{N}{mm^2} = MPa$	$\frac{gm}{cm^2}$ $\frac{kg}{cm^2}$ $\frac{Ton}{m^2}$	$\frac{lb}{in^2} = psi$ $\frac{kip}{in^2} = ksi = 1000\,psi$ $1ksi = 6.895MPa$

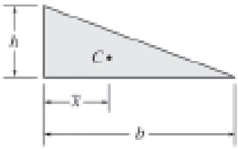
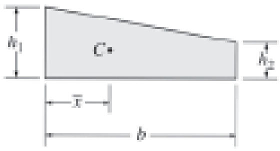
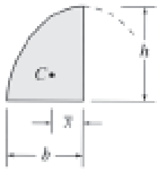
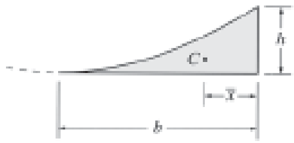
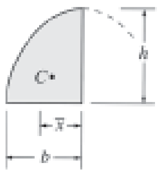
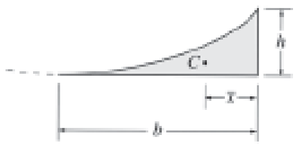
Kilo Pascal = kPa = 10^3 Pa

Mega Pascal = MPa= 10^6 Pa

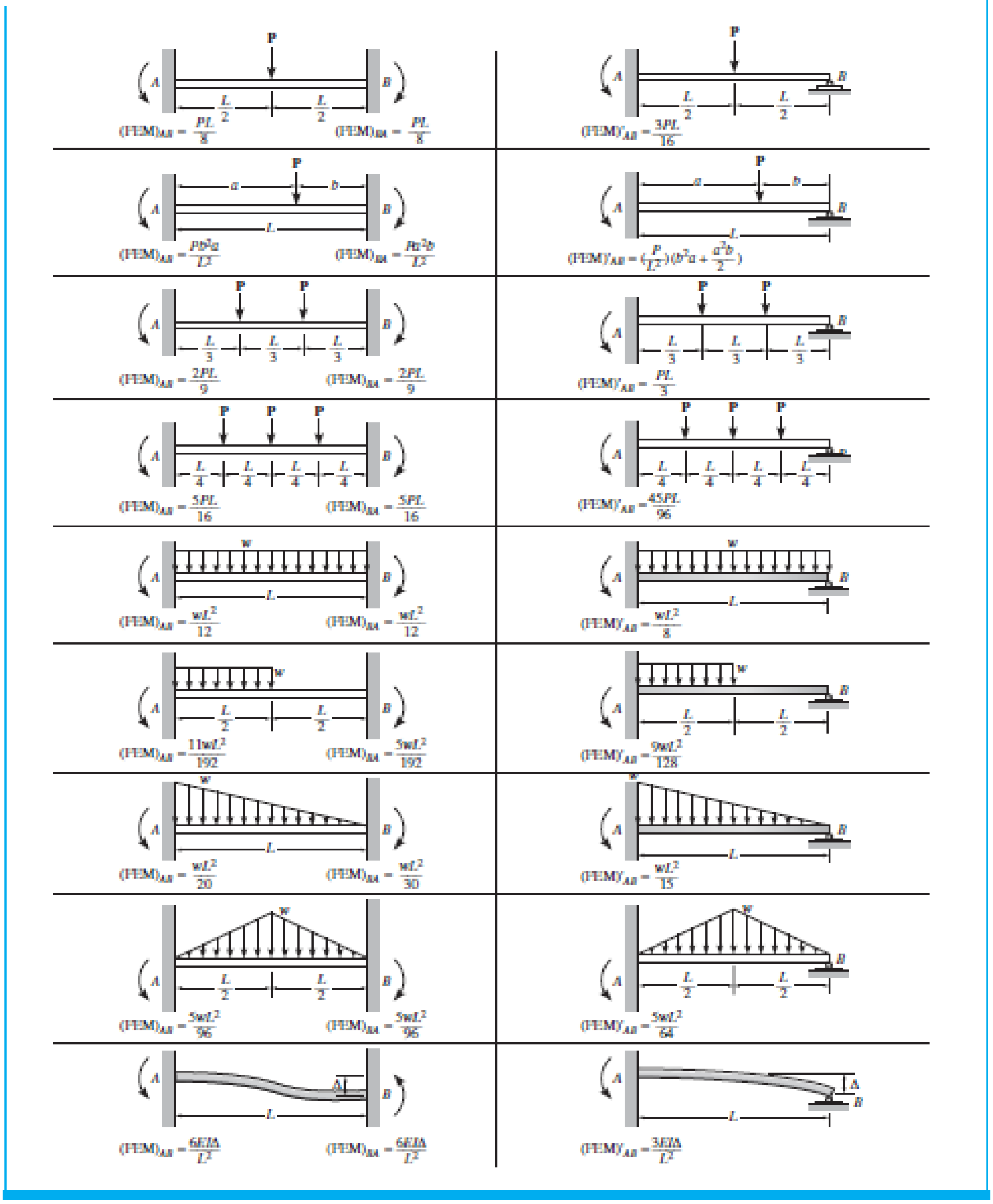
Gega Pascal = GPa = 10^9 Pa

Tera Pascal = TPa = 10^{12} Pa

Geometric Properties of Areas

 Triangle	$A = \frac{1}{2}bh$ $\bar{x} = \frac{1}{3}b$
 Trapezoid	$A = \frac{1}{2}b(h_1 + h_2)$ $\bar{x} = \frac{b(2h_2 + h_1)}{3(h_1 + h_2)}$
 Semi Parabola	$A = \frac{2}{3}bh$ $\bar{x} = \frac{3}{8}b$
 Parabolic spandrel	$A = \frac{1}{3}bh$ $\bar{x} = \frac{1}{4}b$
 Semi-segment of nth degree curve	$A = bh\left(\frac{n}{n+1}\right)$ $\bar{x} = \frac{b(n+1)}{2(n+2)}$
 Spandrel of nth degree curve	$A = bh\left(\frac{1}{n+1}\right)$ $\bar{x} = \frac{b}{(n+2)}$

Fixed End Moments



Theory of Structures

INTRODUCTION

The structural analysis is a mathematical algorithm process by which the response of a structure to specified loads and actions is determined. This response is easured by determining the internal forces or stress resultants and displacements or deformations throughout the structure.

The structural analysis is based on engineering mechanics, mechanics of solids, laboratory research, model and prototype testing, experience and engineering judgment.

The basic methods of structural analysis are flexibility and stiffness methods. The flexibility method is also called force method and compatibility method. The stiffness method is also called displacement method and equilibrium method. These methods are applicable to all type of structures; however, here only skeletal systems or framed structures will be discussed. The examples of such structures are beams, arches, cables, plane trusses, space trusses, plane frames, plane grids and space frames.

The skeletal structure is one whose members can be represented by lines possessing certain rigidity properties. These one dimensional members are also called bar members because their cross sectional dimensions are small in comparison to their lengths. The skeletal structures may be determinate or indeterminate.

CLASSIFICATIONS OF SKELETAL OR FRAMED STRUCTURES

They are classified as under.

1) **Direct force structures** such as pin jointed plane frames and ball jointed space frames which are loaded and supported at the nodes. Only one internal force or stress resultant that is axial force may arise. Loads can be applied directly on the

members also but they are replaced by equivalent nodal loads. In the loaded members additional internal forces such as bending moments, axial forces and shears are produced.

The plane truss is formed by taking basic triangle comprising of three members and three pin joints and then adding two members and a pin node as shown in Figure 1. Sign

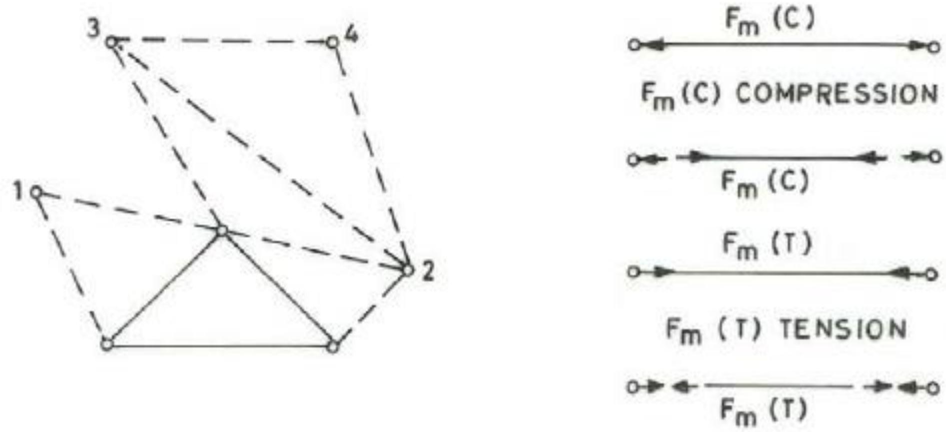


Figure 1 Formation Of Plane Triangulated Truss And Sign Convention For Internal Member Forces.

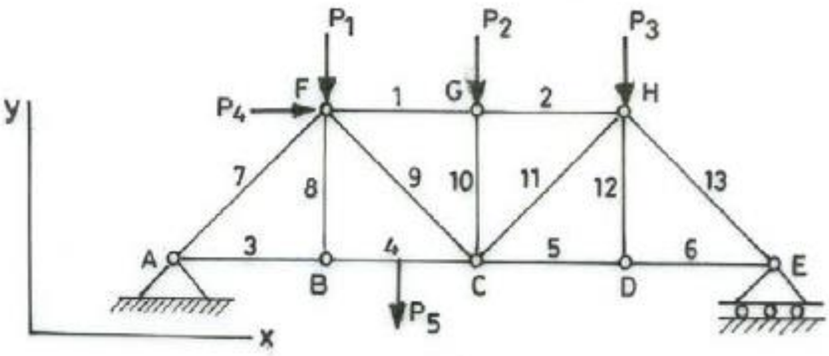


Figure 2 Pin Jointed Plane Truss Subjected To Member And Nodal Loads.

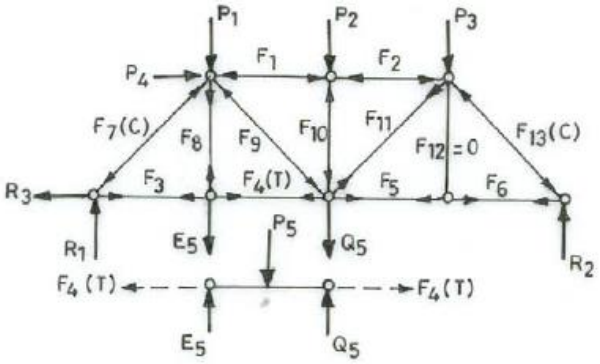


Figure 3 Equivalent Nodal Loads And Free Body Of Loaded Member As Beam.

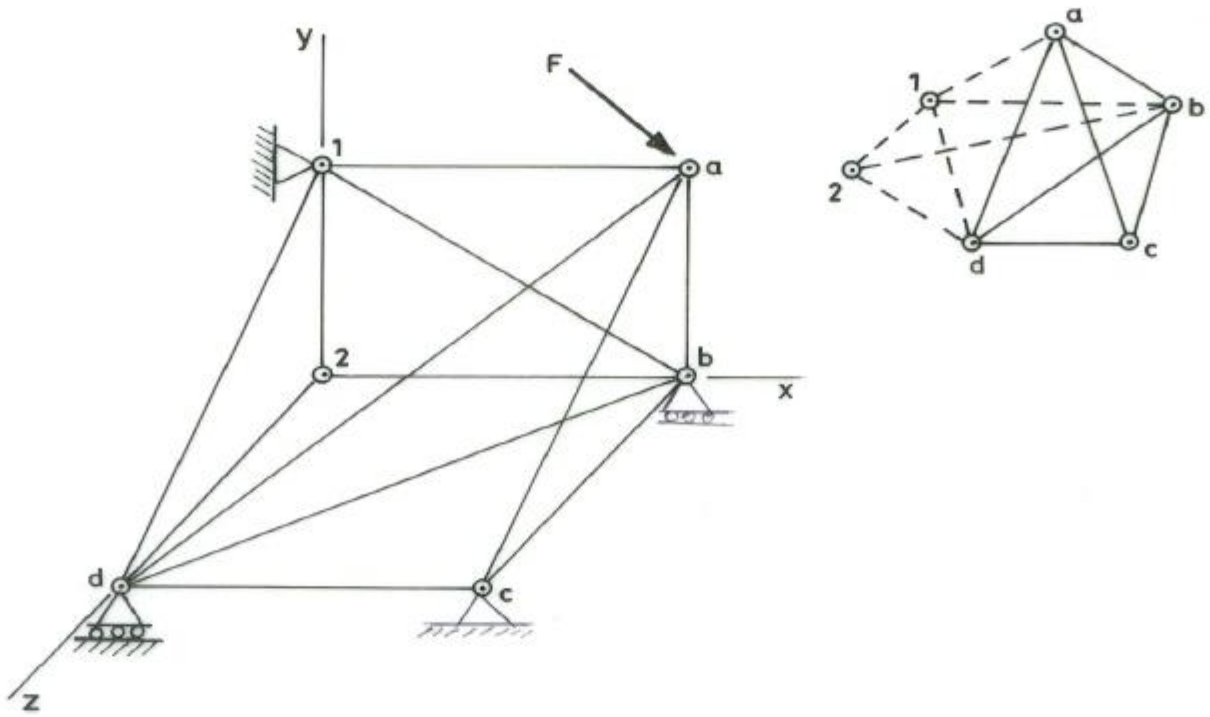


Figure 4 Ball And Socket (Universal) Jointed Space Truss With Nodal Loading.

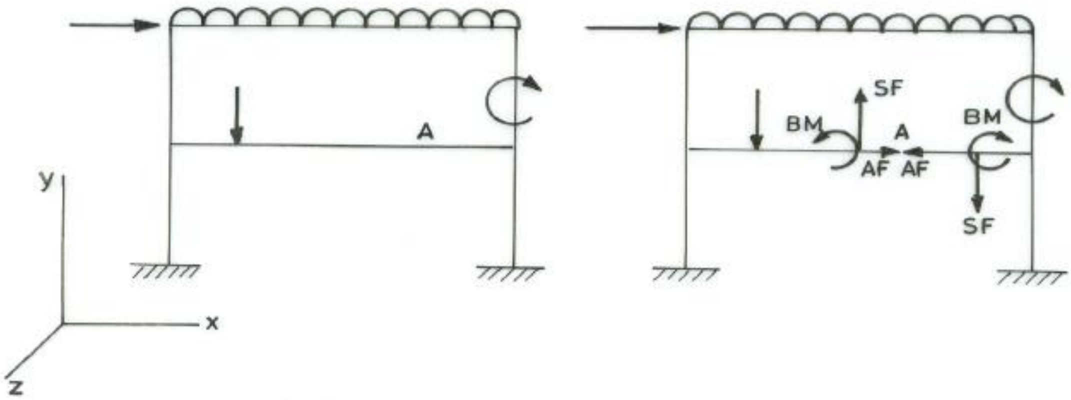


Figure 5 Plane Frame Subjected To In Plane External Loading.

Figure 6 Internal Forces Developed At Section A Due To Applied Loading.

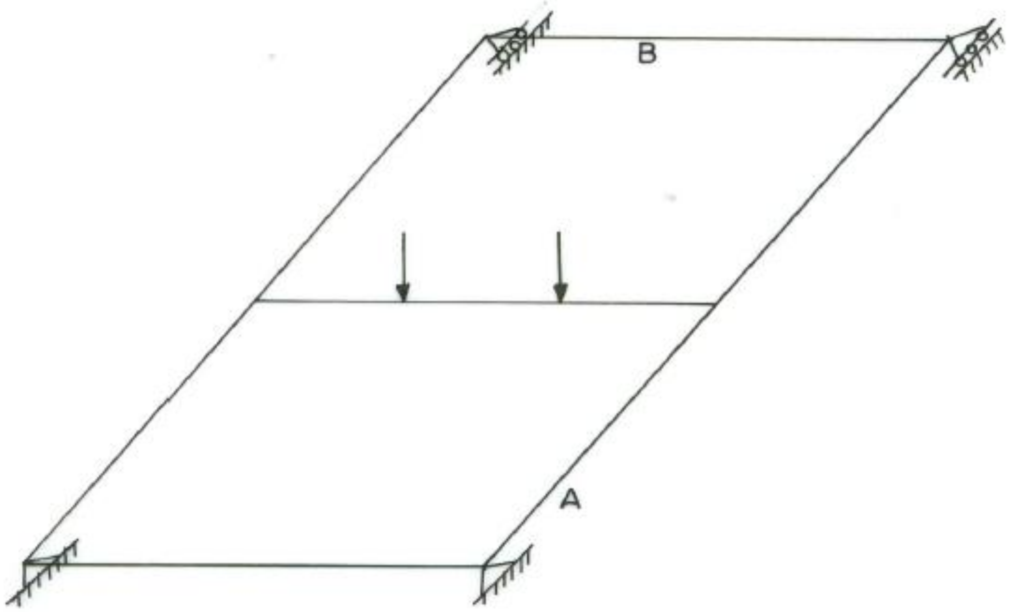


Figure 7 Plane Grid Subjected To Normal To Plane Loading

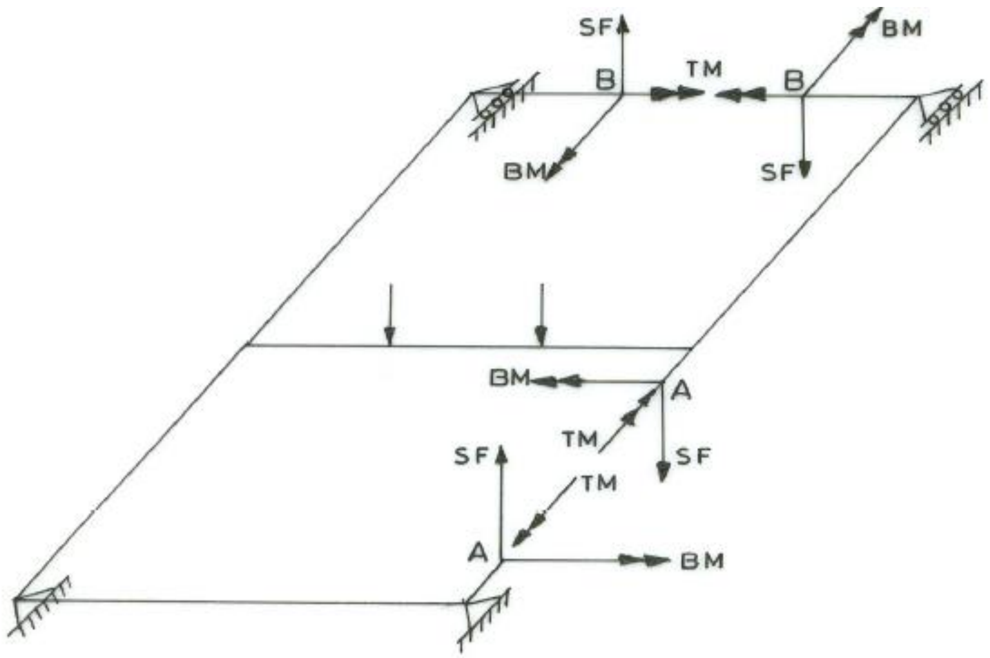


Figure 8 Internal Stress Resltants Developed In Members At A And B Due To Applied Loading-

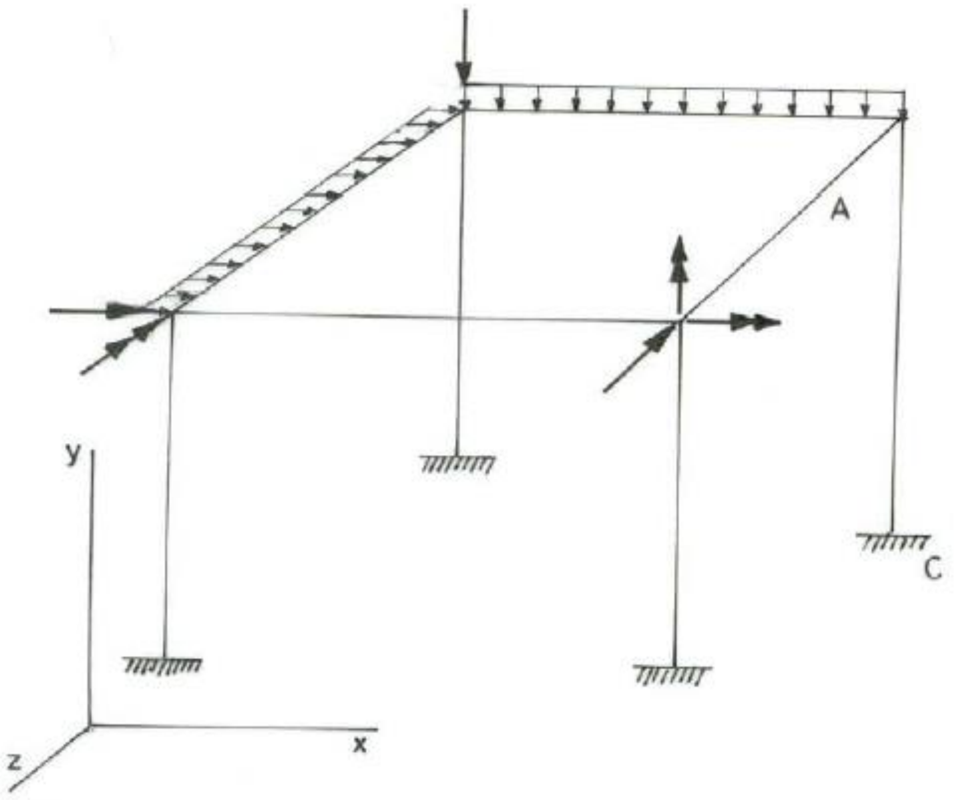


Figure 9 Space Frame Subjected To General External Loading.

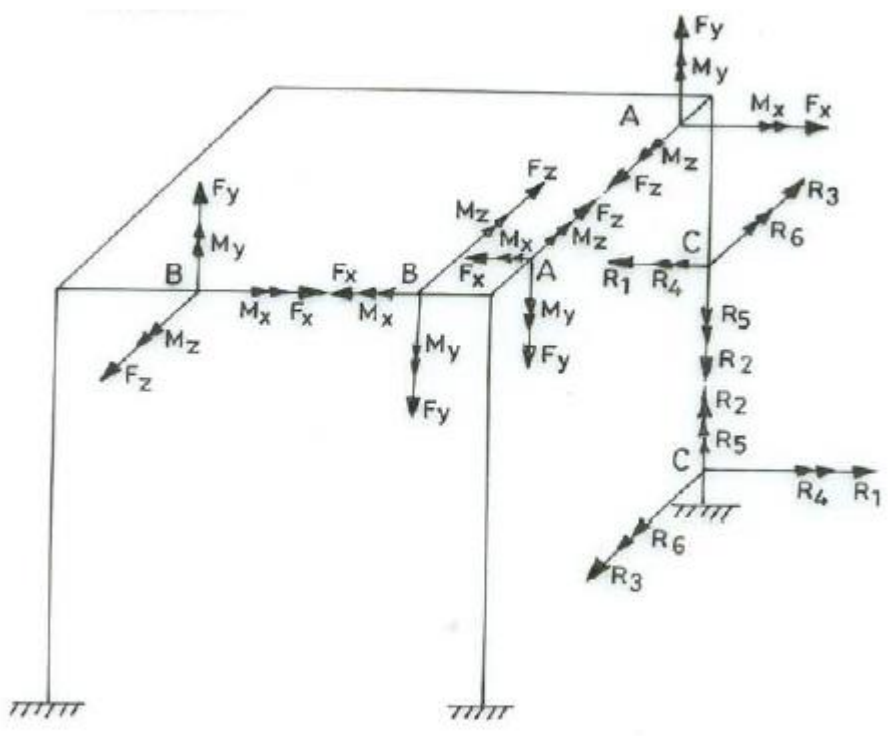


Figure 10 Internal Forces Generated At A And B And React Ion 5 Developed At C Due To External Loading.

Convention for internal axial force is also shown. In Fig 2, a plane triangulated truss with joint and member loading is shown. The replacement of member loading by joint loading is shown in Fig.3. Internal forces developed in members are also shown.

The space truss is formed by taking basic prism comprising of six members and four ball joints and then adding three members and a node as shown in Fig.4.

2) **Plane frames** in which all the members and applied forces lie in same plane as shown in Fig.5. The joints between members are generally rigid. The stress resultants are axial force, bending moment and corresponding shear force as shown in Fig.6.

3) **Plane frames** in which all the members lay in the same plane and all the applied loads act normal to the plane of frame as shown in Fig.7. The internal stress resultants at a point of the structure are bending moment, corresponding shear force and torsion moment as shown in Fig.8.

4) **Space frames** where no limitations are imposed on the geometry or loading in which maximum of six stress resultants may occur at any point of structure namely three mutually perpendicular moments of which two are bending moments and one torsion moment and three mutually perpendicular forces of which two are shear forces and one axial force as shown in figures 9 and 10.

INTERNAL LOADS DEVELOPED IN STRUCTURAL MEMBERS

External forces including moments acting on a structure produce at any section along a structural member certain internal forces including moments which are called stress resultants because they are due to internal stresses developed in the material of member.

The maximum number of stress resultants that can occur at any section is six, the three Orthogonal moments and three orthogonal forces. These may also be described as the axial force F1 acting along x – axis of member, two bending moments F5 and F6 acting about the principal y and z axes respectively of the cross section of the member, two corresponding shear forces F3 and F2 acting along the principal z and y axes respectively and lastly the torsion moment F4 acting about x – axis of member. The stress resultants at any point of centroidal axis of member are shown in Fig. 11 and can be represented as follows.

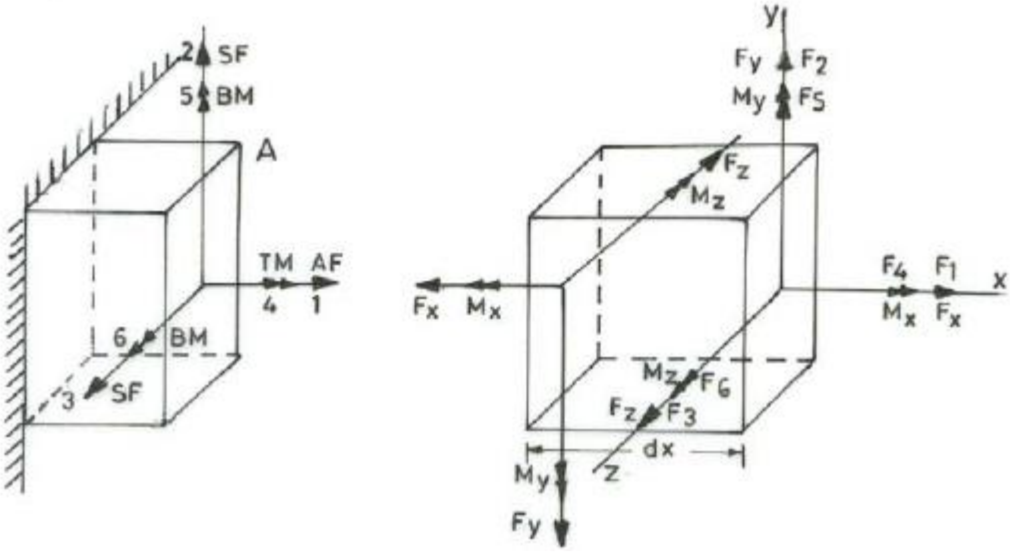


Figure 11 Six Internal Forces At A Section Of Member Under General Loading.

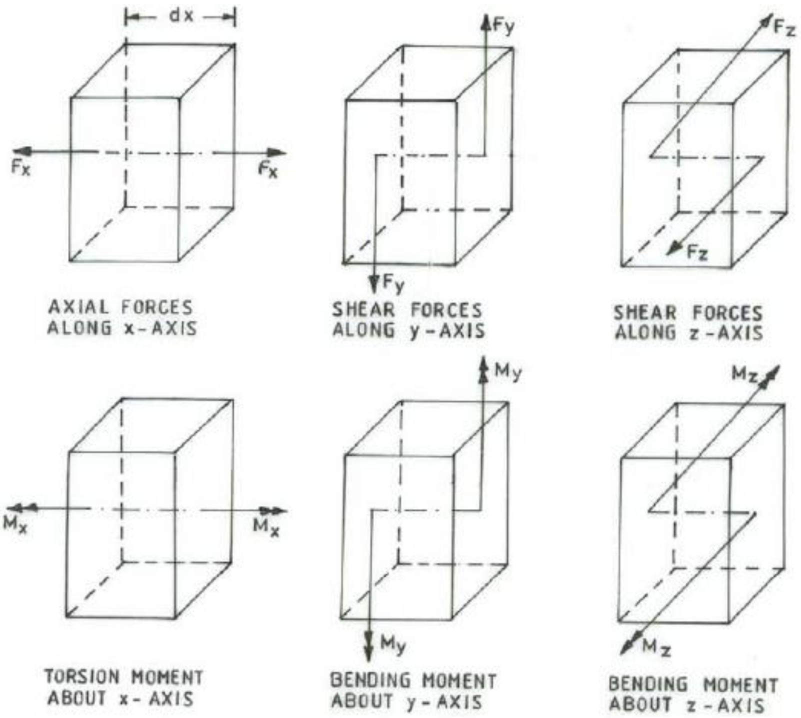


Figure 12 Various Biactions At A Section on an Element

$$\{F\} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{Bmatrix} \quad OR \quad \begin{Bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{Bmatrix}$$

Numbering system is convenient for matrix notation and use of electronic computer.

Each of these actions consists essentially of a pair of opposed actions which causes deformation of an elemental length of a member. The pair of torsion moments cause twist of the element, pair of bending moments cause bending of the element in corresponding plane, the pair of axial loads cause axial deformation in longitudinal direction and the pair of shearing forces cause shearing strains in the corresponding planes. The pairs of biactions are shown in Fig.12.

Primary and secondary internal forces.

In many frames some of six internal actions contribute greatly to the elastic strain energy and hence to the distortion of elements while others contribute negligible amount. The material is assumed linearly elastic obeying Hooke's law. In direct force structures axial force is primary force, shears and bending moments are secondary. Axial force structures do not have torsional resistance. The rigid jointed plane grid under normal loading has bending moments and torsion moments as primary actions and axial forces and shears are treated secondary.

In case of plane frame subjected to in plane loading only bending moment is primary action, axial force and shear force are secondary. In curved members bending moment, torsion and thrust (axial force) are primary while shear is secondary. In these particular cases many a times secondary effects are not considered as it is unnecessary to complicate the analysis by adopting general method.

TYPES OF STRUCTURAL LOADS

Once the structural form has been determined, the actual design begins with those elements that are subjected to the primary loads the structure is intended to carry, and proceeds in sequence to the various supporting members until the foundation is reached. Thus, a building floor slab would be designed first, followed by the supporting beams, columns, and last, the foundation footings. In order to design a structure, it is therefore necessary to first specify the loads that act on it.

For the analysis of structures various loads to be considered are: dead load, live load, snow load, rain load, wind load, impact load, vibration load, water current, centrifugal force, longitudinal forces, lateral forces, buoyancy force, earth or soil pressure, hydrostatic pressure, earthquake forces, thermal forces, erection forces, straining forces etc. How to consider these loads is described in loading standards of various structures.

These loads are idealized for the purpose of analysis as follows.

Concentrated loads: They are applied over a small area and are idealized as point loads.

Line loads: They are distributed along narrow strip of structure such as the wall load or the self weight of member. Neglecting width, load is considered as line load acting along axis of member.

Surface loads: They are distributed over an area. Loads may be static or dynamic, stationary or moving.

Mathematically we have point loads and concentrated moments. We have distributed forces and moments, we have straining and temperature variation forces.

Minimum Design Loads for Buildings and Other Structures, ASCE/SEI 7-10, American Society of Civil Engineers, International Building Code.

EQUILIBRIUM & REACTIONS

To every action there is an equal and opposite reaction. Newton's third law of motion

In the analysis of structures (hand calculations), it is often easier (but not always necessary) to start by determining the reactions. Once the reactions are determined, internal forces are determined next; finally, deformations (deflections and rotations) are determined last. (This is the sequence of operations in the flexibility method which lends itself to hand calculation. In the stiffness method, we determine displacements first, then internal forces and reactions. This method is most suitable to computer implementation.)

Reactions are necessary to determine foundation load. Depending on the type of structures, there can be different types of support conditions.

Roller: provides a restraint in only one direction in a 2D structure, in 3D structures a roller may provide restraint in one or two directions. A roller will allow rotation.

Hinge: allows rotation but no displacements.

Fixed Support: will prevent rotation and displacements in all directions.

Equilibrium

Reactions are determined from the appropriate equations of static equilibrium. Summation of forces and moments, in a static system must be equal to zero2.

Structure Type	Equations					
Beam, no axial forces		ΣF_y				ΣM_z
2D Truss, Frame, Beam	ΣF_x	ΣF_y				ΣM_z
Grid			ΣF_z	ΣM_x	ΣM_y	
3D Truss, Frame	ΣF_x	ΣF_y	ΣF_z	ΣM_x	ΣM_y	ΣM_z
Alternate Set						
Beams, no axial Force	ΣM_z^A	ΣM_z^B				
2 D Truss, Frame, Beam	ΣF_x	ΣM_z^A	ΣM_z^B			
	ΣM_z^A	ΣM_z^B	ΣM_z^C			

The right hand side of the equation should be zero. If your reaction is negative, then it will be in a direction opposite from the one assumed. Summation of all external forces (including reactions) is not necessarily zero (except at hinges and at points outside the structure). Summation of external forces is equal and opposite to the internal ones. Thus the net force/moment is equal to zero. The external forces give rise to the (non-zero) shear and moment diagram.

Equations of Conditions

If a structure has an internal hinge (which may connect two or more ubstructures), then this will provide an additional equation ($\Sigma M = 0$ at the hinge) which can be exploited to determine the reactions.

Northern Technical University
Technical College of Mosul
Building & Construction
Technology Engineering Dept.

THEORY OF STRUCTURES

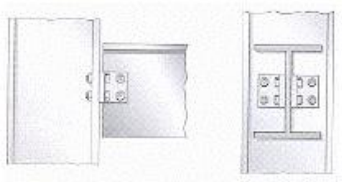
THIRD CLASS

Lecturer:
Dr. Muthanna Adil Najm ABBU

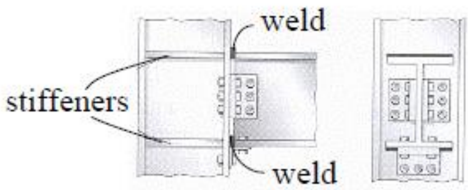
2015 - 2016

Theory of Structures

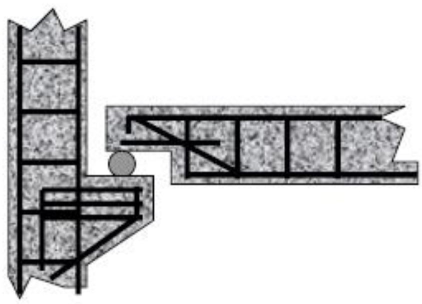
• **Support Connections**



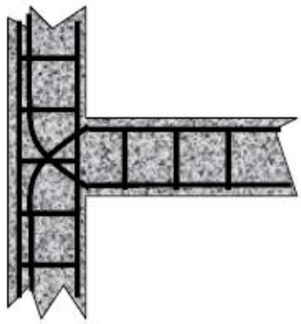
typical "pin-supported" connection (metal)



typical "fixed-supported" connection (metal)

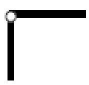


typical "roller-supported" connection (concrete)

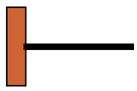


typical "fixed-supported" connection (concrete)

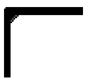
pin support



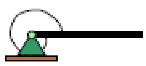
pin-connected joint



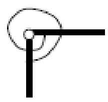
fixed support



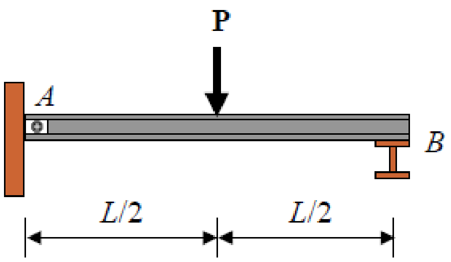
fixed-connected joint



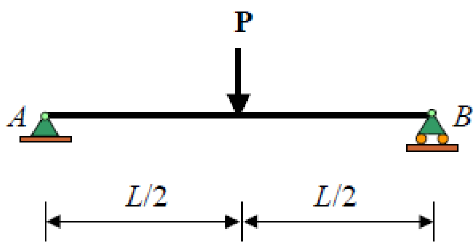
torsional spring support



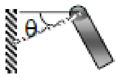
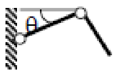
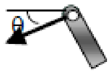
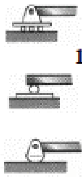
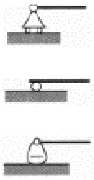
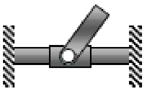
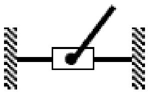
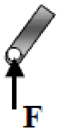
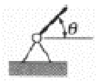
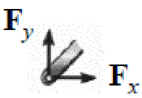
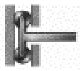
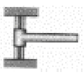
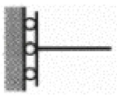
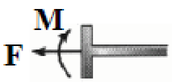
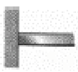
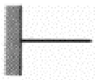
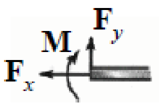
torsional spring joint

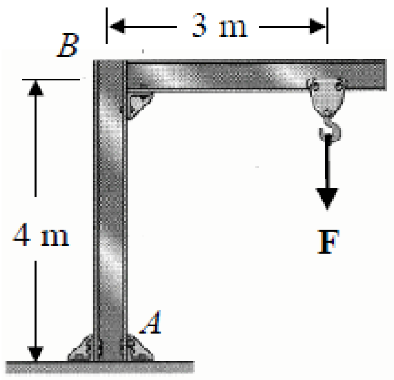


actual beam

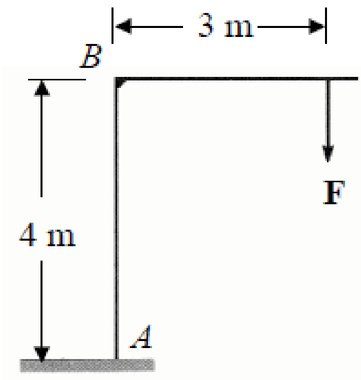


idealized beam

Type of Connection	Idealized Symbol	Reaction	Number of Unknowns
(1)  Light cable			One unknown. The reaction is a force that acts in the direction of the cable or link.
(2)  rollers rockers			One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact
(3) 			One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact
(4) 			One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact
(5)  Smooth pin or hinge			Two unknowns. The reactions are two force components.
(6)  slider  fixed-connected collar			Two unknowns. The reactions are a force and moment.
(7)  fixed support			Three unknowns. The reactions are the moment and the two force components.



actual structure



idealized structure

Fundamentals

A good structural engineer is one who tries to make use of both hemispheres of the brain, the logical analytical left brain, which does all the calculations of bending moment, shear forces and all that based on equations and the intuitive right side of the brain, which can see directly without doing any calculations and it is necessary to correlate these two and that is why structural analysis is a beautiful subject to develop oneself. You develop analytical skills and you also develop intuitive skills.

Introduction

Structure is an assemblage of a number of components like slabs, beams, columns, walls, foundations and so on, which remains in equilibrium. It has to satisfy the fundamental criteria of strength, stiffness, economy, durability and compatibility, for its existence. It is generally classified into two categories as **Determinate** and **Indeterminate structures** or **Redundant Structures**.

Any structure is designed for the stress resultants of bending moment, shear force, deflection, torsional stresses, and axial stresses. If these moments, shears and stresses are evaluated at various critical sections, then based on these, the proportioning can be done. Evaluation of these stresses, moments and forces and plotting them for that structural component is known as analysis. Determination of dimensions for these components of these stresses and proportioning is known as design.

Determinate structures are analysed just by the use of basic equilibrium equations. By this analysis, the unknown reactions are found for the further determination of stresses**. Redundant or indeterminate structures** are not capable of being analysed by mere use of basic equilibrium equations. Along with the basic equilibrium equations, some extra conditions are required to be used like **compatibility conditions** of deformations etc to get the unknown reactions for drawing **bending moment and shear force diagrams**.

Example of determinate structures are: **simply supported beams, cantilever beams, single and double overhanging beams, three hinged arches**, etc.

Examples of indeterminate structures are: **fixed beams, continuous beams, fixed arches, two hinged arches, portals, multistoried frames**, etc.

Special methods like strain energy method, slope deflection method, moment distribution method, column analogy method, virtual work method, matrix methods, etc are used for the analysis of redundant structures.

Indeterminate Structures: a structure is termed as statically indeterminate, if it can not be analysed from principles of statics alone, i.e. $\sum H = 0, \sum V = 0, \sum M = 0$. A statically indeterminate structure may be classified as:

1. Externally indeterminate, (example: continuous beams and frames shown in figure-1(a) and (b)).
2. Internally indeterminate, (example: trusses shown in figure-1(c) and (d)).

3. Both externally and internally indeterminate, (example: trussed beams, continuous trusses shown in figure-1 (e) and (f)).

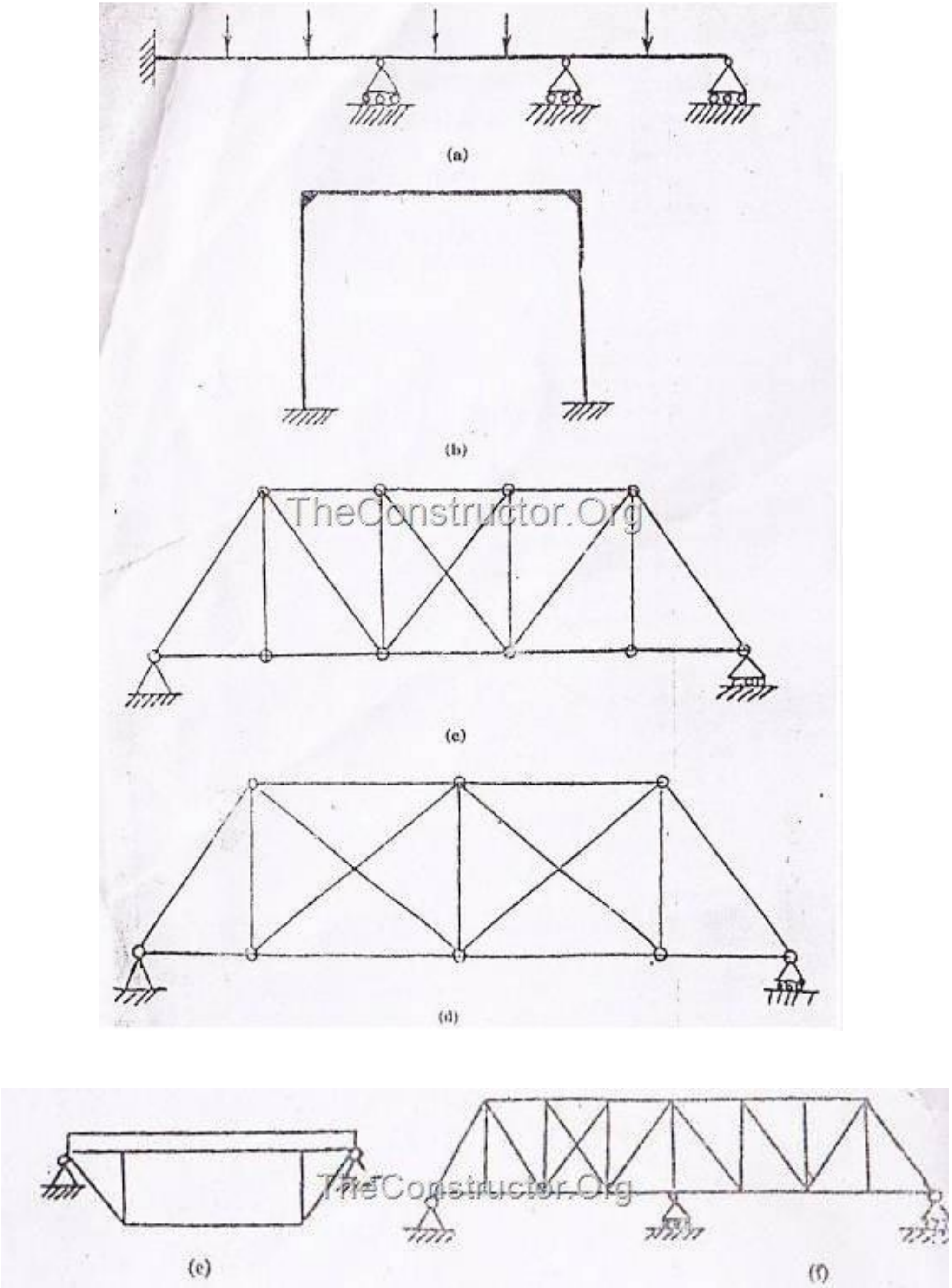


Figure.1

EXTERNALLY INDETERMINATE STRUCTURES:

A structure is usually externally indeterminate or redundant if the reactions at the supports can not be determined by using three equations of equilibrium, i.e. $\sum H = 0, \sum V = 0, \sum M = 0$. In the case of beams subjected to vertical loads only, two reactions can be determined by conditions of equilibrium. Therefore, simply supported cantilever and overhanging beams shown in figure 2 are statically determinate structures.

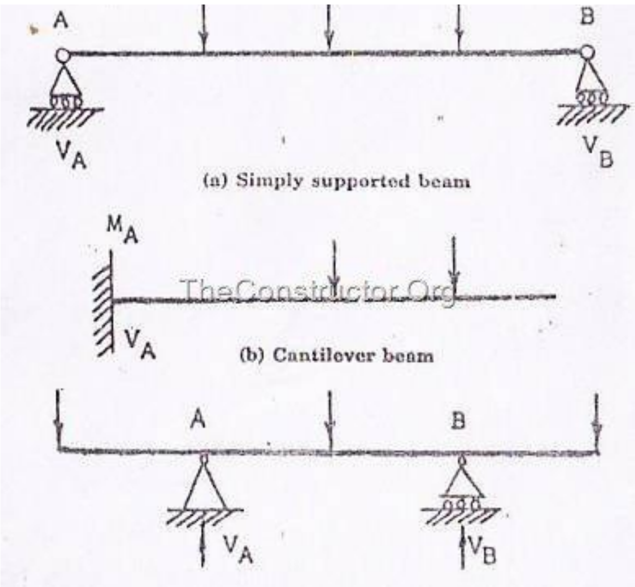


Figure 2

If however a beam rests on more than two supports or in addition any of the end support is fixed, there are more than two reactions to be determined. These reactions can not be determined by conditions of equilibrium alone. The degree of indeterminacy or redundancy is given by the number of extra or redundant reactions to be determined. The beam shown in figure 3 (a) is statically indeterminate to one degree because there are three unknown reactions and statics has only two reactions. The beam in figure 3(b) is statically redundant to two degree. The beam in figure 3(c) is redundant to three degree and the beam in figure 3(d) is redundant to four degrees.

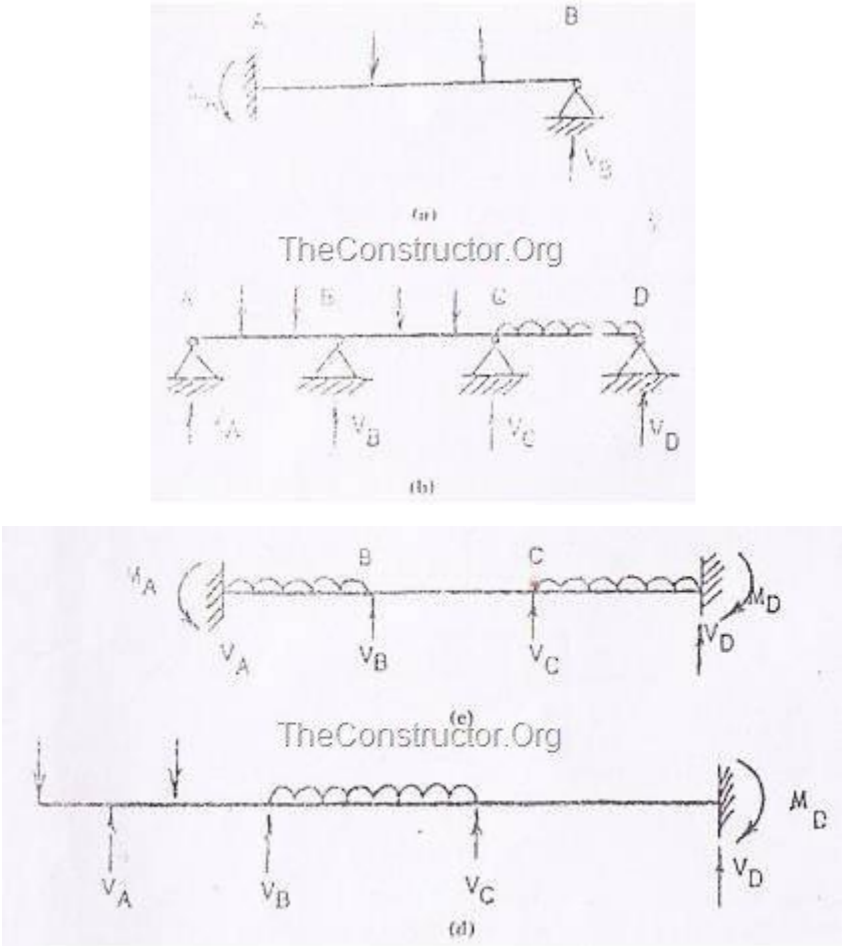


Figure 3

A portal frame is statically determinate if there are only three external reactions, because there are three conditions of equilibrium for such a system. The portal frame shown in figure 4 are statically determinate because there are only three reactions to be determined. If a portal frame has more than three reactions it is statically indeterminate, the degree of indeterminacy or redundancy being equal to the number of redundant or extra reactions to be determined. Therefore, the portal frames of figure 5(a) and (b) are redundant by one degree, that of figure 5(c) is redundant by two degrees, that of figure 5(d) is redundant by three degrees, and that of figure 5(e) is redundant by 5 degrees.

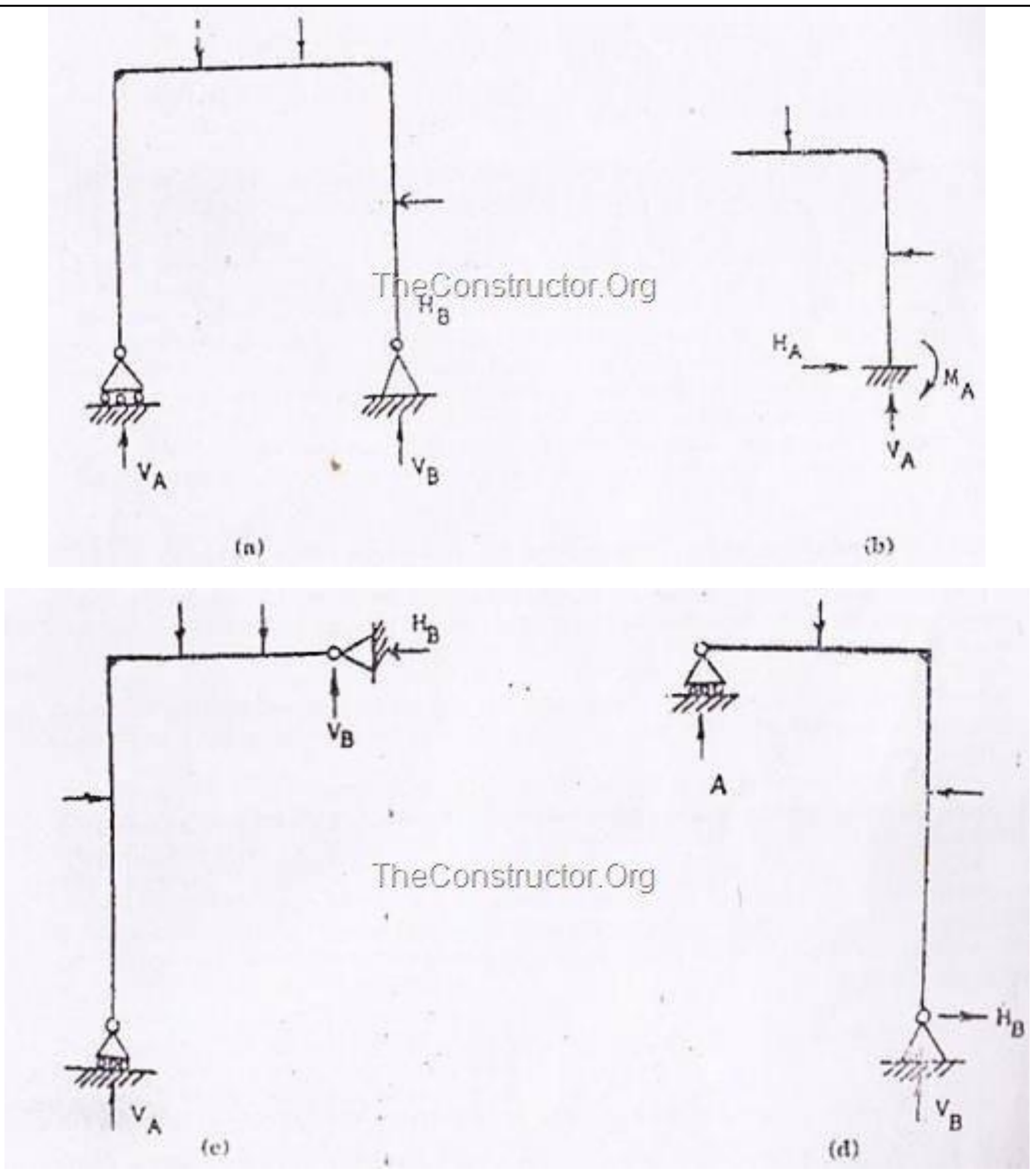
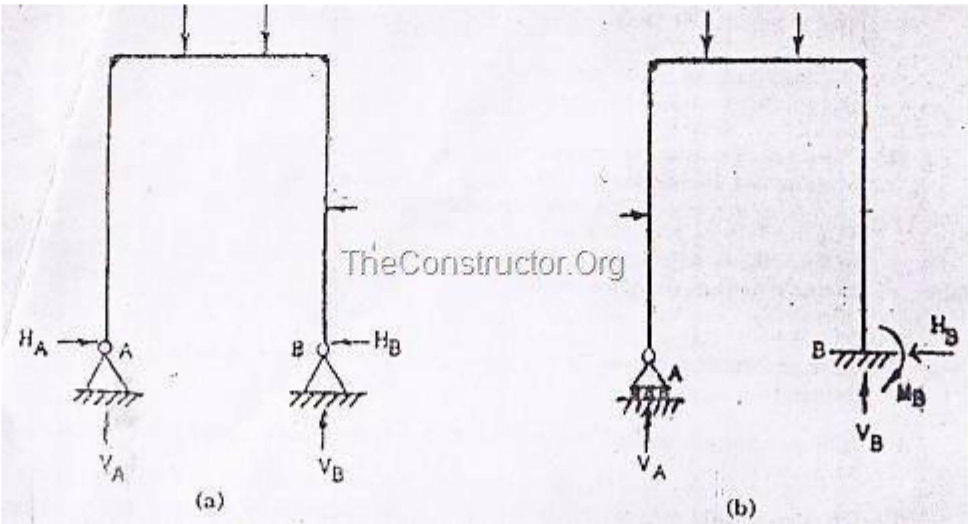

H_B
H_A
M_A
V_A
V_B
(a)
(b)
H_B
V_B
A
V_A
H_B
V_B
(c)
(d)
TheConstructor.Org

Figure 4


H_A
A
B
H_B
V_A
V_B
(a)
A
B
H_B
M_B
V_A
V_B
(b)
TheConstructor.Org

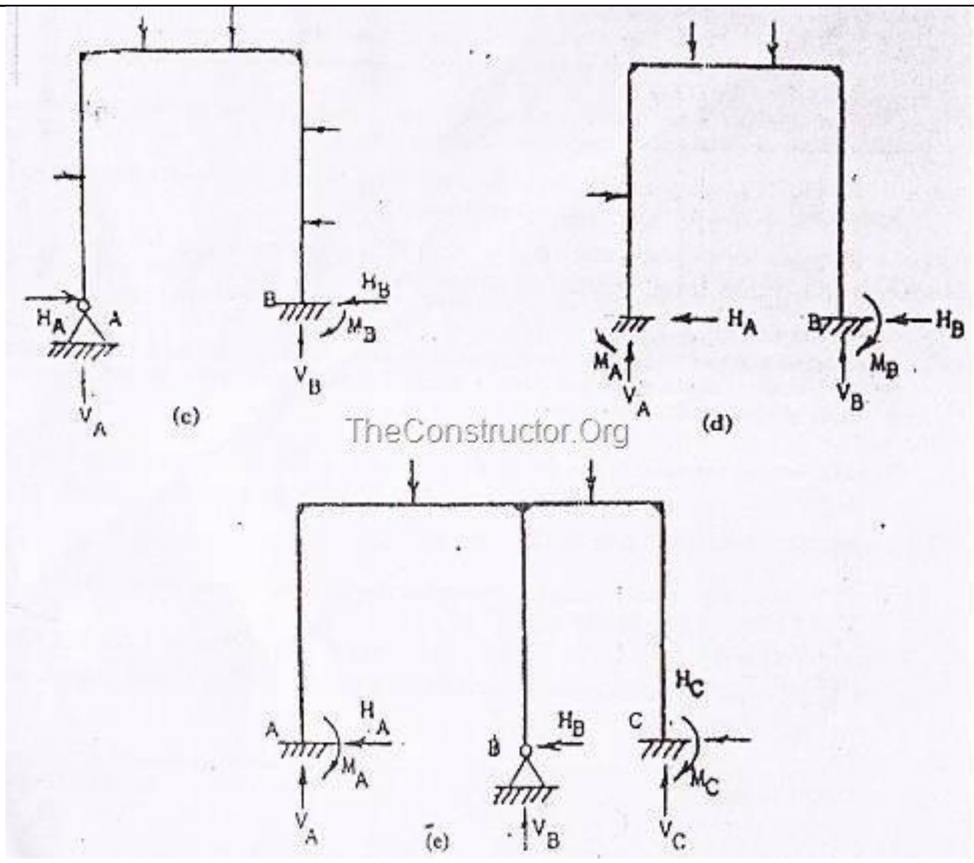


Figure 5

The statically indeterminate beams and frames can be analysed by **strain energy method, three moment equation, slope deflection method or moment distribution method**.

DIFFERENCE BETWEEN DETERMINATE AND INDETERMINATE STRUCTURES

S. No.	Determinate Structures	Indeterminate Structures
1	Equilibrium conditions are fully adequate to analyse the structure.	Conditions of equilibrium are not adequate to fully analyse the structure.
2	Bending moment or shear force at any section is independent of the material property of the structure.	Bending moment or shear force at any section depends upon the material property.
3	The bending moment or shear force at any section is independent of the cross-section or moment of inertia.	The bending moment or shear force at any section depends upon the cross-section or moment of inertia.

4	Temperature variations do not cause stresses.	Temperature variations cause stresses.
5	No stresses are caused due to lack of fit.	Stresses are caused due to lack of fit.
6	Extra conditions like compatibility of displacements are not required to analyse the structure.	Extra conditions like compatibility of displacements are required to analyse the structure along with the equilibrium equations.

DETERMINATE AND INDETERMINATE STRUCTURAL SYSTEMS

If skeletal structure is subjected to gradually increasing loads, without distorting the
initial geometry of structure, that is, causing small displacements, the structure is said to be stable. Dynamic loads and buckling or instability of structural system are not considered here. If for the stable structure it is possible to find the internal forces in all the members constituting the structure and supporting reactions at all the supports provided from statically equations of equilibrium only, the structure is said to be determinate. If it is possible to determine all the support reactions from equations of equilibrium alone the structure is said to be externally determinate else externally indeterminate. If structure is externally determinate but it is not possible to determine all internal forces then structure is said to be internally indeterminate. Therefore a structural system may be:

(1) Externally indeterminate but internally determinate

(2) Externally determinate but internally indeterminate

(3) Externally and internally indeterminate

(4) Externally and internally determinate

A system which is externally **and** internally determinate is said to be determinate system.

A system which is externally **or** internally or externally **and** internally indeterminate is said to be indeterminate system.

Equations of equilibrium

Space frames arbitrarily loaded

$\Sigma Fx = 0$ $\Sigma Mx = 0$

$\Sigma Fy = 0$ $\Sigma My = 0$

$\Sigma Fz = 0$ $\Sigma Mz = 0$

For space frames number of equations of equilibrium is 6. Forces along three orthogonal axes should vanish and moments about three orthogonal axes should vanish.

Plane frames with in plane loading

$\Sigma Fx = 0$ $\Sigma Fy = 0$ $\Sigma Mz = 0$

There are three equations of equilibrium. Forces in x and y directions should vanish and moment about z axis should vanish.

Plane frames with normal to plane loading

There are three equations of equilibrium.

$\Sigma Fy = 0$, $\Sigma Mx = 0$, $\Sigma Mz = 0$

Sum of forces in y direction should be zero. Sum of moments about x and z axes be zero.

Release and constraint

A release is a discontinuity which renders a member incapable of transmitting a stress resultant across that section. There are six releases corresponding to the six stress resultants at a section

Release for Axial Force (AF) F_x: $\begin{Bmatrix} 0 \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{Bmatrix}$

Release for Shear Force (SF) F_y: $\begin{Bmatrix} F_x \\ 0 \\ F_z \\ M_x \\ M_y \\ M_z \end{Bmatrix}$

Release for Shear Force (SF) F_z: $\begin{Bmatrix} F_x \\ F_y \\ 0 \\ M_x \\ M_y \\ M_z \end{Bmatrix}$

Release for Torsion Moment (TM) M_x: $\begin{Bmatrix} F_x \\ F_y \\ F_z \\ 0 \\ M_y \\ M_z \end{Bmatrix}$

Release for Bending Moment (BM) M_y: $\begin{Bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ 0 \\ M_z \end{Bmatrix}$

Release for Bending Moment (BM) M_z: $\begin{Bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ 0 \end{Bmatrix}$

The release may be represented by zero elements of forces

Universal joint (Ball and socket joint) $F = \begin{Bmatrix} F_x \\ F_y \\ F_z \\ 0 \\ 0 \\ 0 \end{Bmatrix}$, Cut $F = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$

release does not necessarily occur at a point, but may be continuous along whole length of member as in chain. On the other hand a constraint is defined as that which prevents any relative degree of freedom between two adjacent nodes connected by a member or when a relative displacement of the nodes does not produce a stress resultant in the member.

Determinacy.

The equilibrium equations provide both the *necessary and sufficient* conditions for equilibrium. When all the forces in a structure can be determined strictly from these equations, the structure is referred to as *statically determinate*. Structures having more unknown forces than available equilibrium equations are called *statically indeterminate*. As a rule, a structure can be identified as being either statically determinate or statically indeterminate by drawing free-body diagrams of all its members, or selective parts of its members, and then comparing the total number of unknown reactive force and moment components with the total number of available equilibrium equations. For a coplanar structure, there are at most *three* equilibrium equations for each part, so that if there is a total of ***n*** parts and ***r*** force and moment reaction components, we have

$r = 3n$, statically determinate

$r > 3n$, statically indeterminate

n = the total parts of structure members.

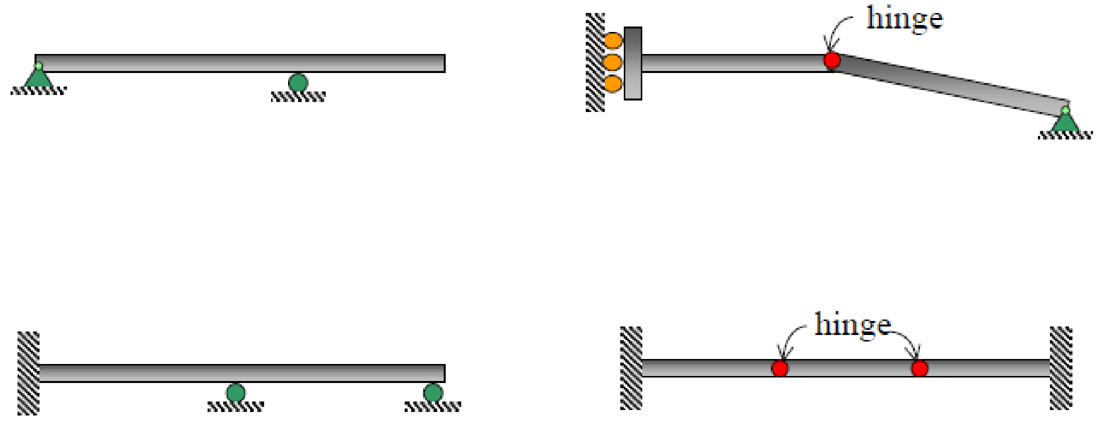
r = the total number of unknown reactive force and moment components

In particular, if a structure is *statically indeterminate,* the additional equations needed to solve for the unknown reactions are obtained by relating the applied loads and reactions to the displacement or slope at different points on the structure. These equations, which are referred to as *compatibility equations,* must be equal in number to the *degree of indeterminacy* of the structure. Compatibility equations involve the

geometric and physical properties of the structure.

Example 1:

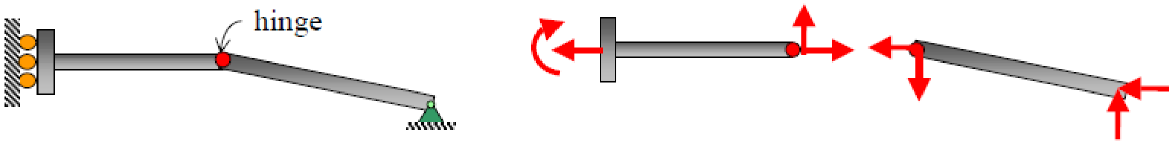
Classify each of the beams shown below as statically determinate or statically indeterminate. If statically indeterminate, report the number of degrees of indeterminacy. The beams are subjected to external loadings that are assumed to be known and can act anywhere on the beams.



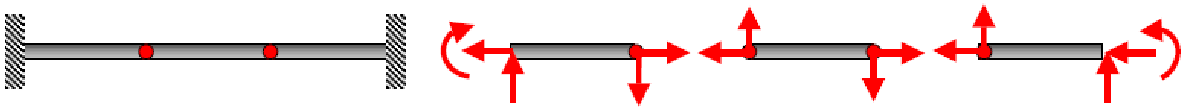
SOLUTION

$r = 3,\ n = 1,\ 3 = 3(1)$ Statically **determinate**

$r = 5,\ n = 1,\ 5 - 3(1) = 2$ Statically **indeterminate** to the second degree



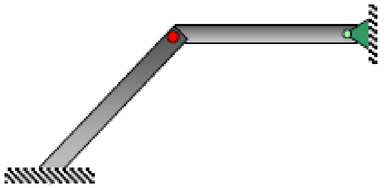
$r = 6,\ n = 2,\ 6 = 3(2)$ Statically **determinate**

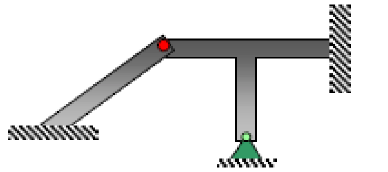


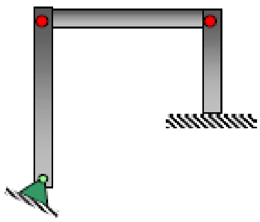
$r = 10,\ n = 3,\ 10 - 3(3) = 1$ Statically **indeterminate** to the first degree

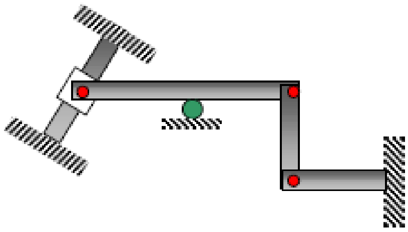
Example 2

Classify each of the pin-connected structures shown in figure below as statically determinate or statically indeterminate. If statically are subjected to arbitrary external loadings that are assumed to be known and can act anywhere on the structures.

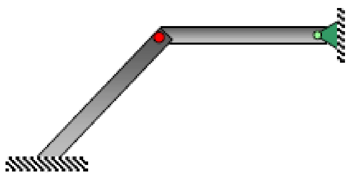


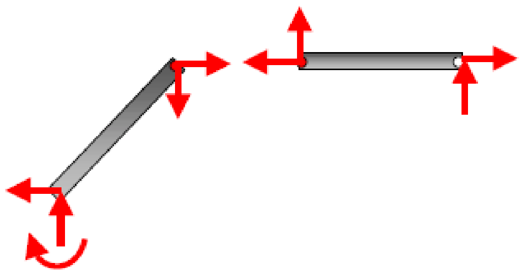






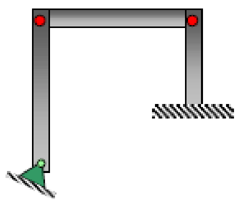
SOLUTION

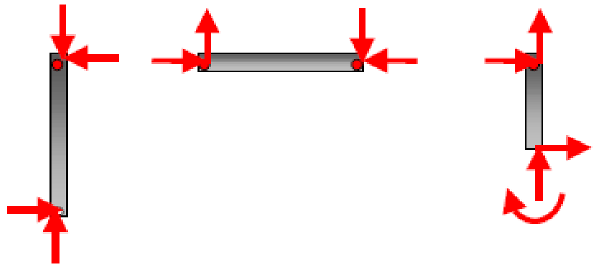




$r = 7$, $n = 2$, $7 - 3(2) = 1$

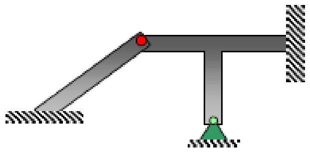
Statically **indeterminate** to the first degree

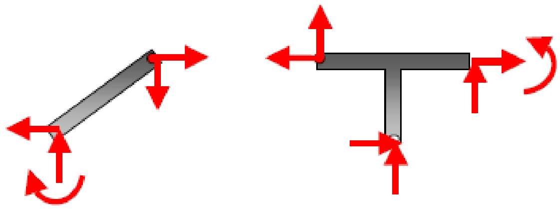




$r = 9$, $n = 3$, $9 = 3(3)$

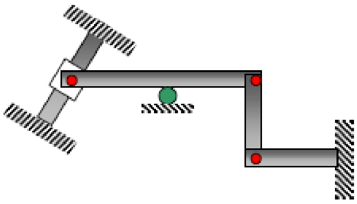
Statically **determinate**

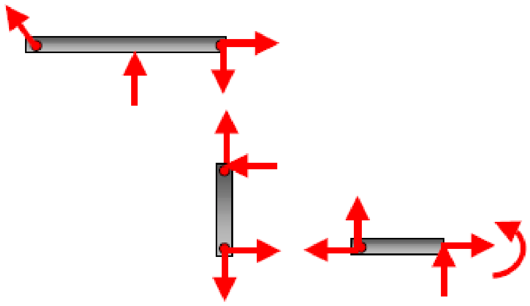




$r = 10$, $n = 2$, $10 - 6 = 4$ degree

Statically **indeterminate** to the fourth

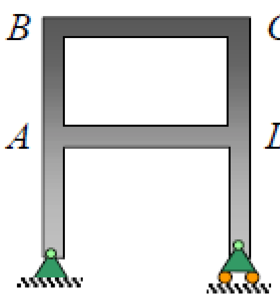


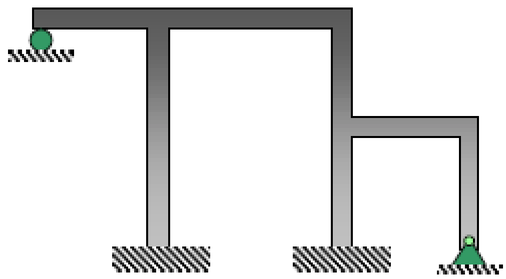


$r = 9$, $n = 3$, $9 = 3(3)$ Statically **determinate**

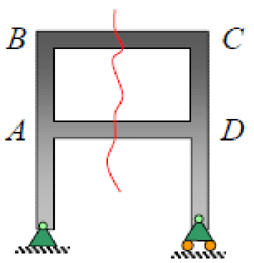
Example 3

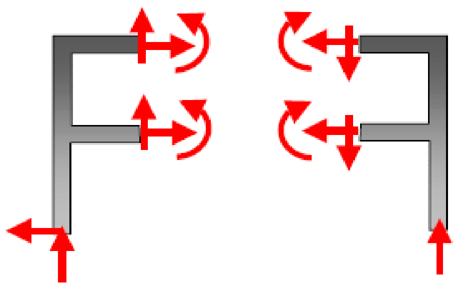
Classify each of the frames shown in figure below as statically determinate or statically indeterminate. If statically indeterminate, report the number of degrees of indeterminacy. The frames are subjected to external loadings that are assumed to be known and can act anywhere on the frames.





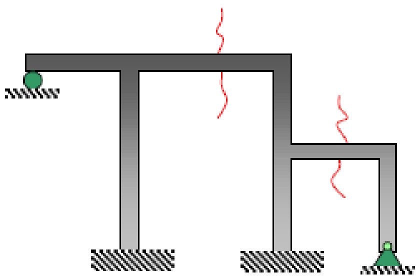
SOLUTION

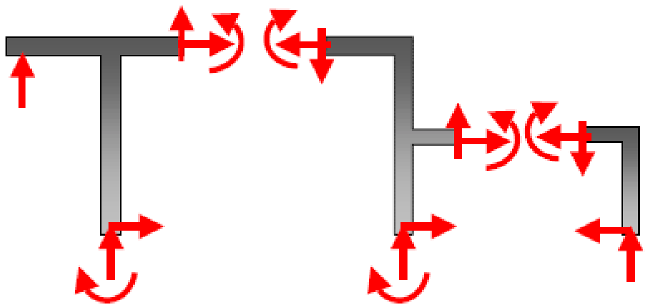




$r = 9$, $n = 2$, $9 - 6 = 3$

Statically **indeterminate** to the third degree





$r = 15$, $n = 3$, $15 - 9 = 6$

Statically **indeterminate** to the sixth degree

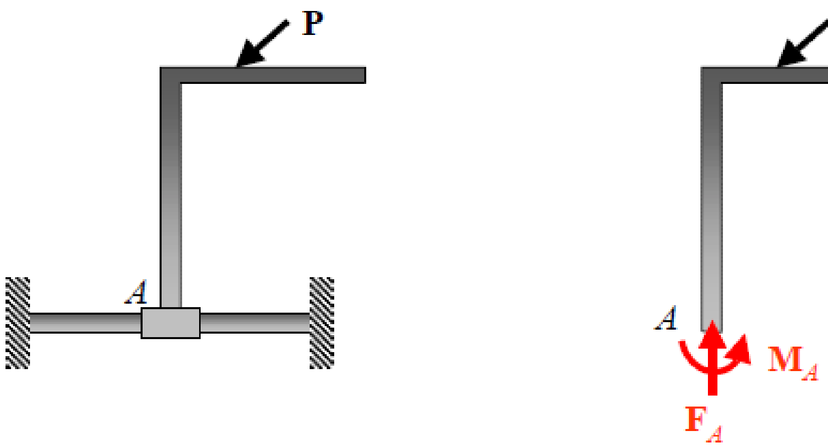
Stability.

To ensure the equilibrium of a structure or its members, it is not only necessary to satisfy the equations of equilibrium, but the members must also be properly held or constrained by their supports.

Two situations may occur where the conditions for proper constraint have not been met.

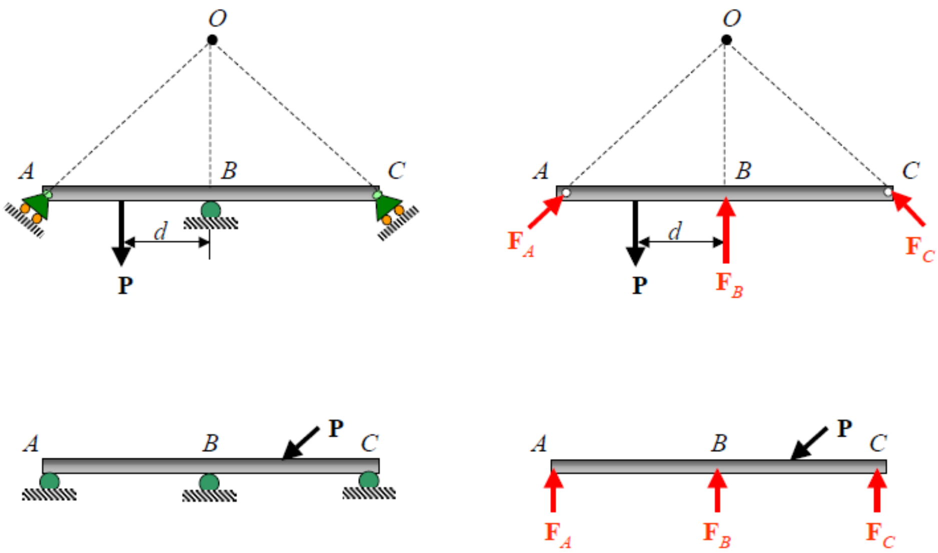
Partial Constraints. In some cases a structure or one of its members may have *fewer* reactive forces than equations of equilibrium that must be satisfied. The structure then becomes only *partially constrained*.

Partial Constrains



Improper Constraints. In some cases there may be as many unknown forces as there are equations of equilibrium; however, *instability* or movement of a structure or its members can develop because of *improper constraining* by the supports. This can occur if all the *support reactions are concurrent* at a point. Another way in which improper constraining leads to instability occurs when the *reactive forces* are all *parallel*.

Improper Constraints



In general, then, a structure will be geometrically unstable—that is, it will move slightly or collapse—if there are fewer reactive forces than equations of equilibrium; or if there are enough reactions, instability will occur if the lines of *action of the reactive forces intersect at a common point or are parallel to one another*.

If the structure consists of several members or components, local instability of one or several of these members can generally be determined *by inspection*. If the members form a collapsible mechanism, the structure will be unstable, for a *coplanar structure* having *n* members or components with *r* unknown reactions. Since three equilibrium equations are available for each member or component, we have

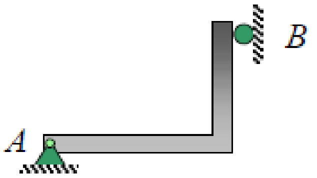
$r < 3n$, unstable

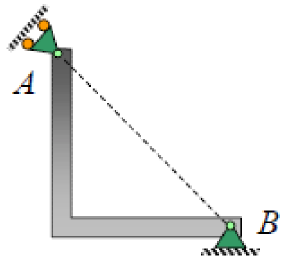
$r \geq 3n$, unstable if member reactions are concurrent or parallel or some of the components form a collapsible mechanism

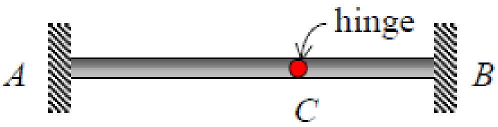
If the structure is unstable, *it does not matter* if it is statically determinate or indeterminate. In all cases such types of structures must be avoided in practice.

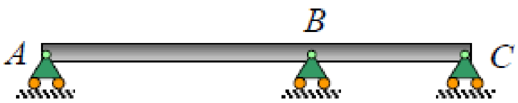
Example 4

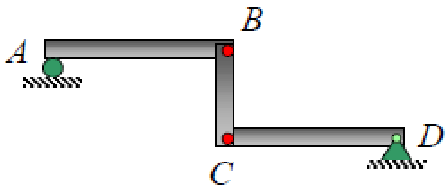
Classify each of the structures in the figure below as stable or unstable. The structures are subjected to arbitrary external loads that are assumed to be known.



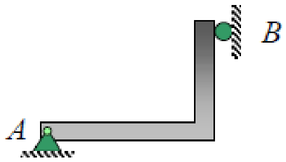


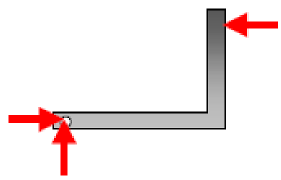




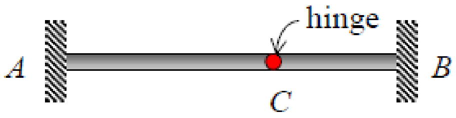


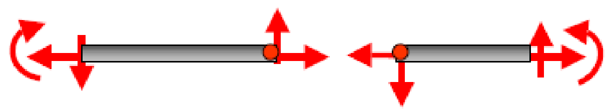
SOLUTION



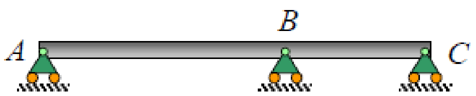


The member is ***stable*** since the reactions are non-concurrent and nonparallel. It is also statically **determinate**.

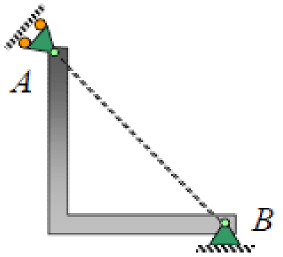


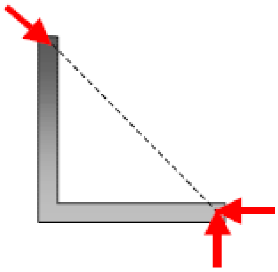


The compound beam is ***stable***. It is also **indeterminate** to the second degree.

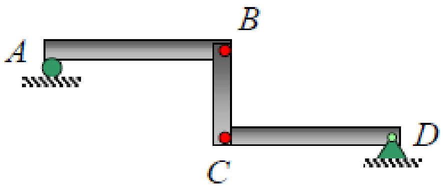


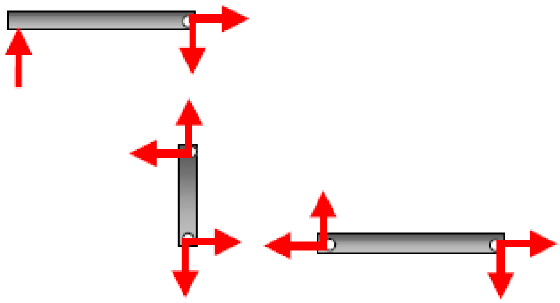
The compound beam is ***unstable*** since the three reactions are all parallel.





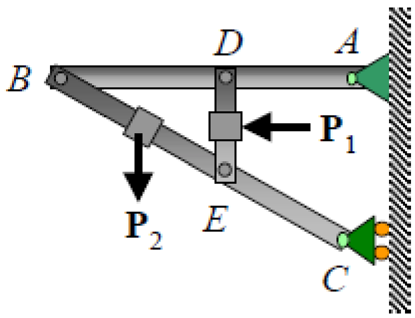
The member is ***unstable*** since the three reactions are concurrent at *B*.

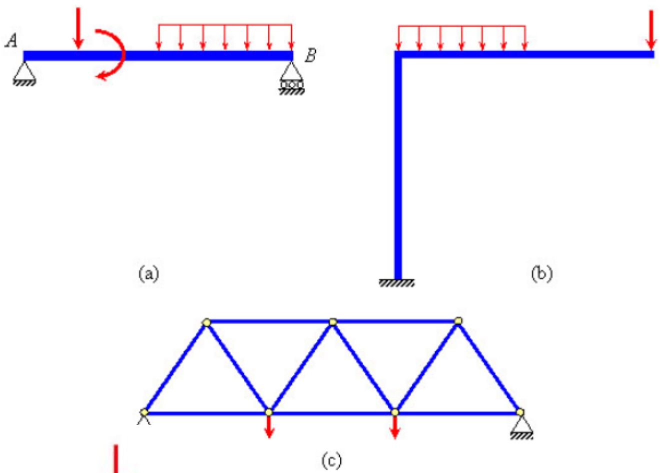


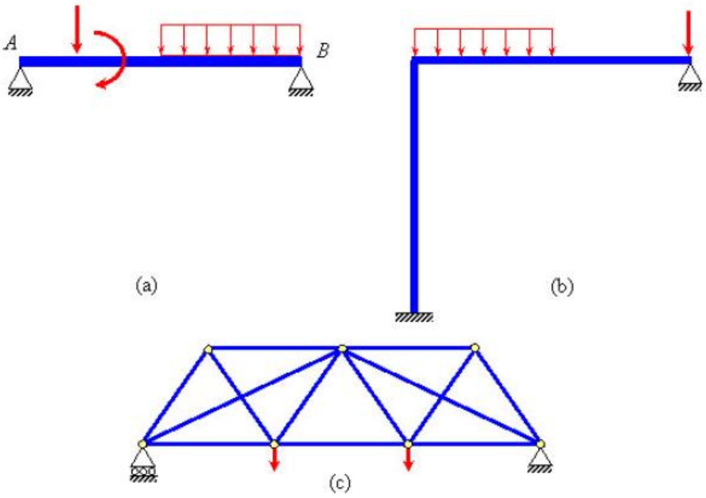


The structure is ***unstable*** since $r = 7$, $n = 3$, so that, $r < 3n$, $7 < 9$. Also, this can be seen by inspection, since *AB* can move horizontally without restraint.

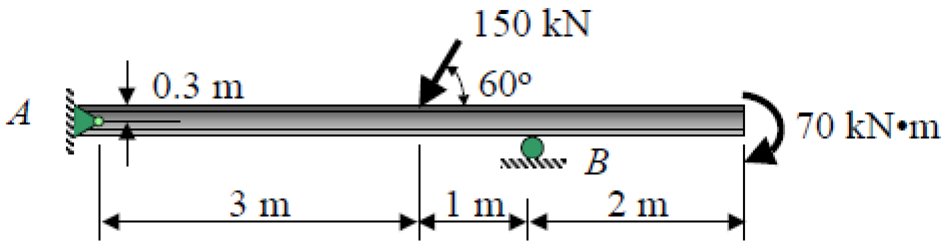
H.W



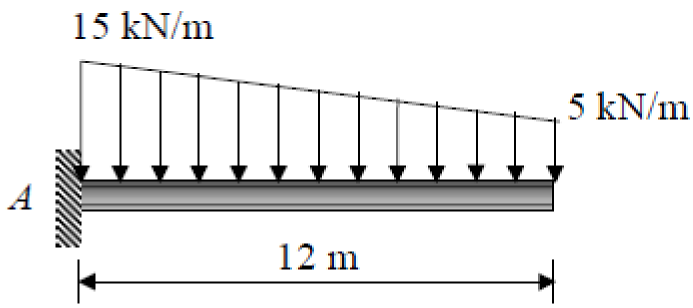




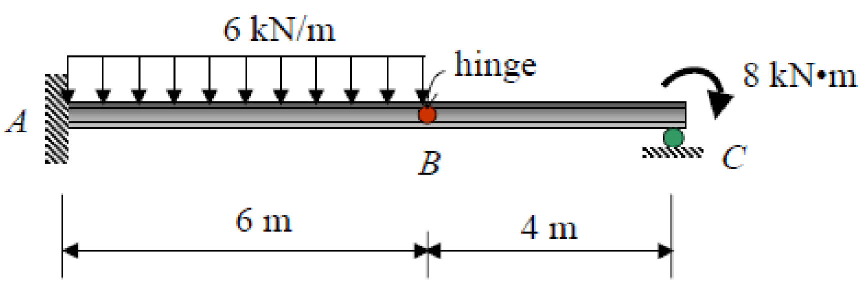
Determine the reactions on the beam shown.



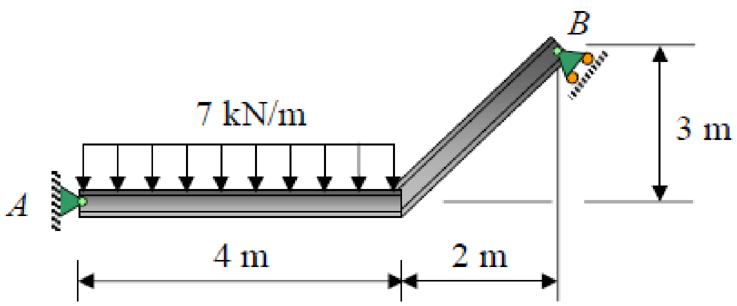
Determine the reactions on the beam shown.



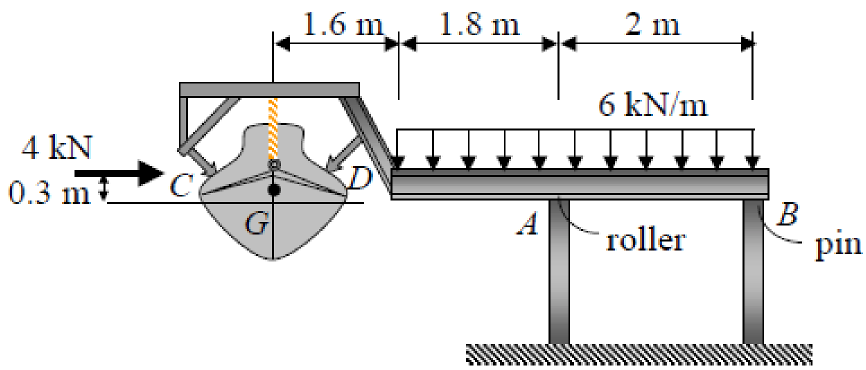
The compound beam in figure below is fixed at *A*. Determine the reactions at *A*, *B*, and *C*. Assume that the connection at pin and *C* is a rooler.



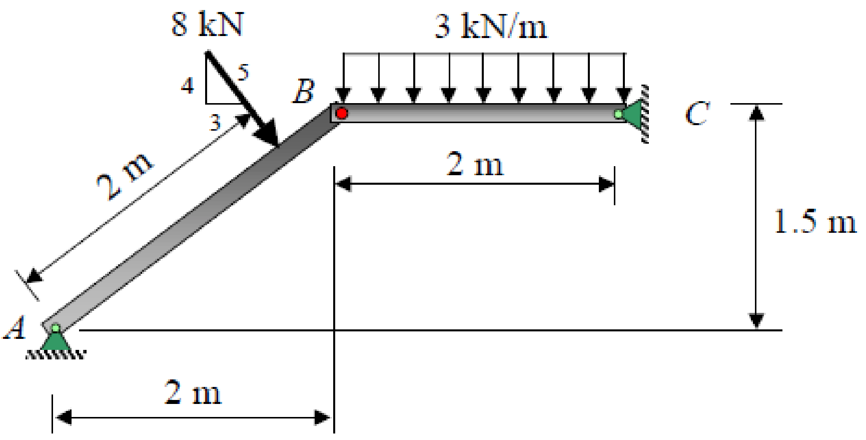
Determine the reactions on the beam shown. Assume *A* is a pin and the support at *B* is a roller (smooth surface).



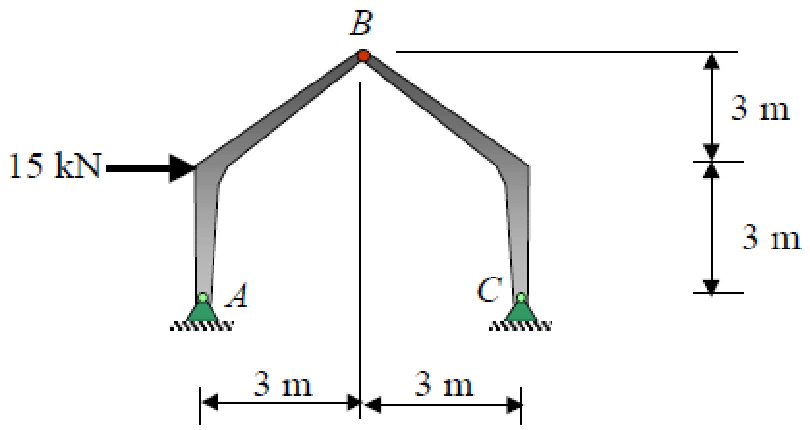
The side girder shown in the photo supports the boat and deck. An idealized model of this girder is shown in the figure below, where it can be assumed *A* is a roller and *B* is a pin. Using a local code the anticipated deck loading transmitted to the girder is 6 kN/m. Wind exerts a *resultant* horizontal force of 4 kN as shown, and the mass of the boat that is supported by the girder is 23 Mg. The boat's mass center is at *G*. Determine the reactions at the supports.



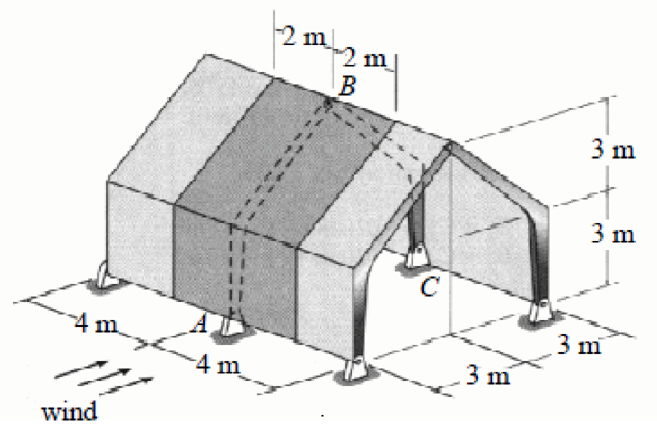
Determine the horizontal and vertical components of reaction at the pins *A*, *B*, and *C* of the two-member frame shown in the figure below.



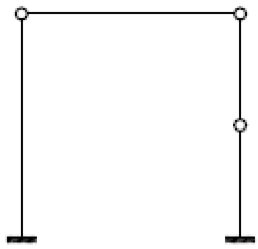
From the figure below, determine the horizontal and vertical components of reaction at the pin connections *A*, *B*, and *C* of the supporting gable arch.

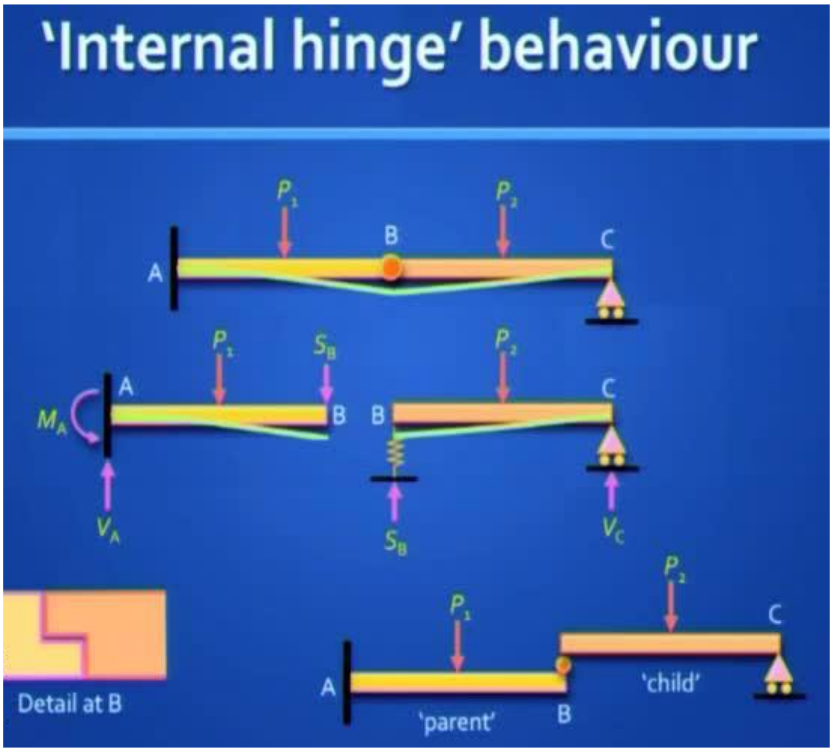


The side of the building in the figure below is subjected to a wind loading that creates a uniform *normal* pressure of 1.5 kPa on the windward side and a suction pressure of 0.5 kPa on the leeward side. Determine the horizontal and vertical components of reaction at the pin connections *A*, *B*, and *C* of the supporting gable arch.



Some structures are built with intermediate hinges, each hinge provides on additional equation of static equilibrium and allows the determination of additional reaction component. For instant, the frame given in figure with three intermediate hinges is a statically determinate structure with six reaction components.





So, you have here a propped cantilever AC, which would be statically indeterminate. But the provision of an internal hinge in the middle at B makes it just rigid and statically determinate. How does this behave? Well, take a look at the deflected shape and this is something I have always emphasized.

So, it is a good practice wherever possible to draw deflected shapes. For example, the curved shape of the deflected diagram must match with the bending moment diagram. So, here you see that at the joint B, there is a relative change in angle. There is no need to satisfy rotational compatibility and you will find that of the two elements AB and BC, one is dependent on the other. Which is dependent on which?

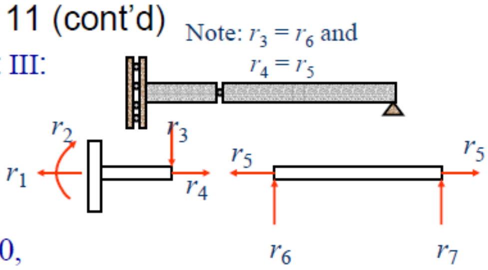
BC is dependent on AB. AB can stand alone on its own. It is a cantilever, but BC is not. BC will fall down because there is no support given at B. So, if we have to separate out these two elements, what kind of support would you provide at B?

Spring.

You will provide a spring support, because you can get a vertical force transfer at B, but B can move. So you should ideally model the correct element and here, a spring element is appropriate and you can see that the load P_2 will be shared if it is right in the middle equally by the forces at B and C and the shear force developed at B gets transmitted to the cantilever AB and that is how it operates. And now it becomes statically determinate. It is very easy to do that. You basically invoked an equation that the bending moment at B is 0. No bending moment can be transmitted from one to the other, but even more important, you can really appreciate how this works. BC is dependent on AB. For example, the load P_1 acting on that structure will go entirely to AB because AB can stand on its own. Nothing gets transmitted to the support at C, but the load P_2 needs a help of AB, so a part of it reaches AB. And if you look at the practical construction these are often used in bridges. You find that it is an articulation like this and there is a bearing provided and clearly you can see from the detail at B, BC is sitting on AB at B and not vice versa. So, another way to look at it is, in this manner, here it is very clear which is a child and which is a parent.

- Example 11 (cont'd)

For part III:



Applying Eq. 30,

$r = 6$, $n = 2$, therefore,

$r = 3n, \Rightarrow 6 = [3(2) = 6] \Rightarrow$ statically determinate