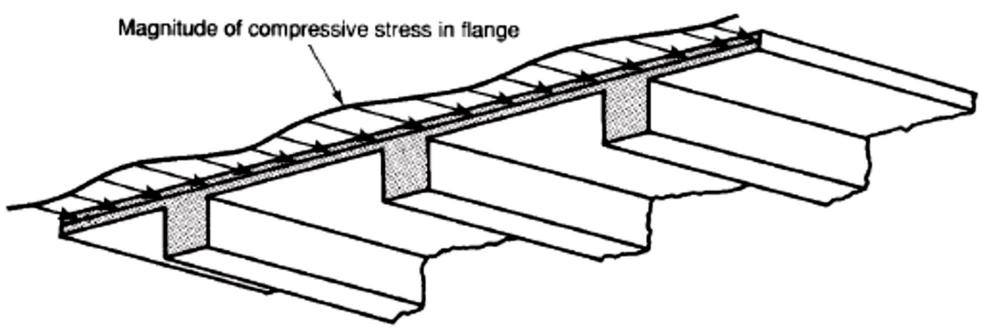
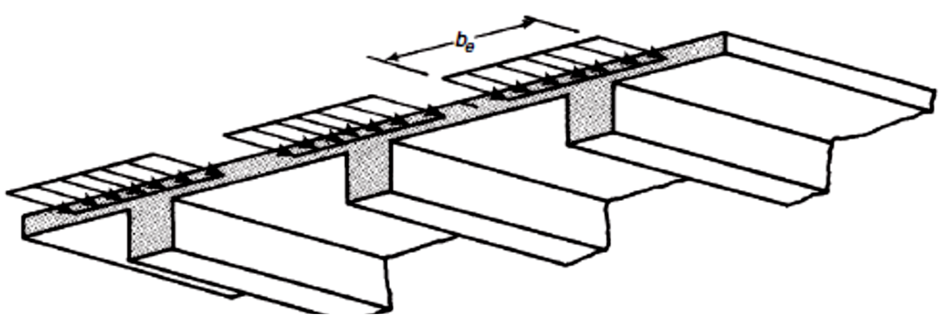
Analysis and design of T – beam

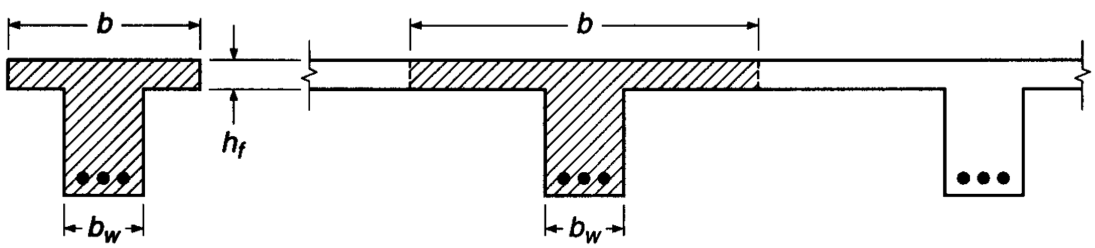
Reinforced concrete floor systems normally consist of slabs and beams that are placed monolithically. As a result, the two parts act together to resist loads. In effect, the beams have extra widths at their tops, called flanges, and the resulting T-shaped beams are called T-beams. The part of a T beam below the slab is referred to as the web or stem. (The beams may be L shaped if the stem is at the end of a slab.) The stirrups in the webs extend up into the slabs, as perhaps do bent-up bars, with the result that they further make the beams and slabs act together.

There is a problem involved in estimating how much of the slab acts as part of the beam. If, however, the flanges are wide and thin, bending stresses will vary quite a bit across the flange due to shear deformations. The farther a particular part of the slab or flange is away from the stem, the smaller will be its bending stress.



(a) Distribution of maximum flexural compressive stresses.



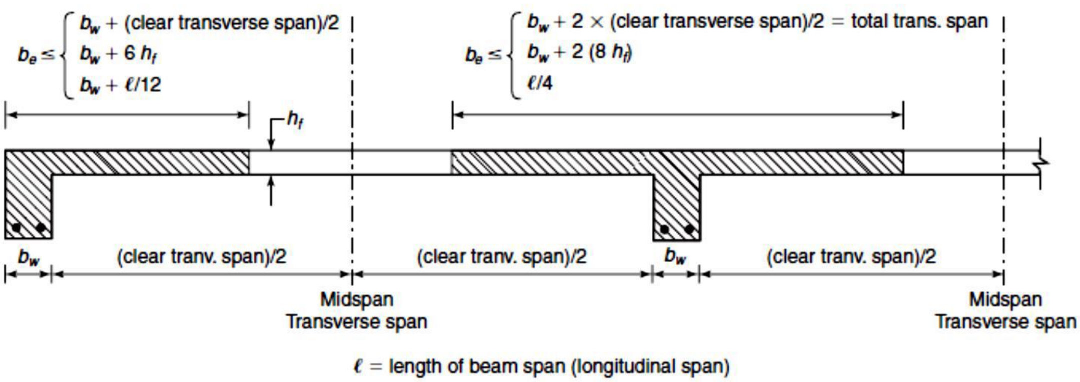


(a) (b)

Effective flange width (**b**) interior beam, code 6.3.2:

b is the smallest of:

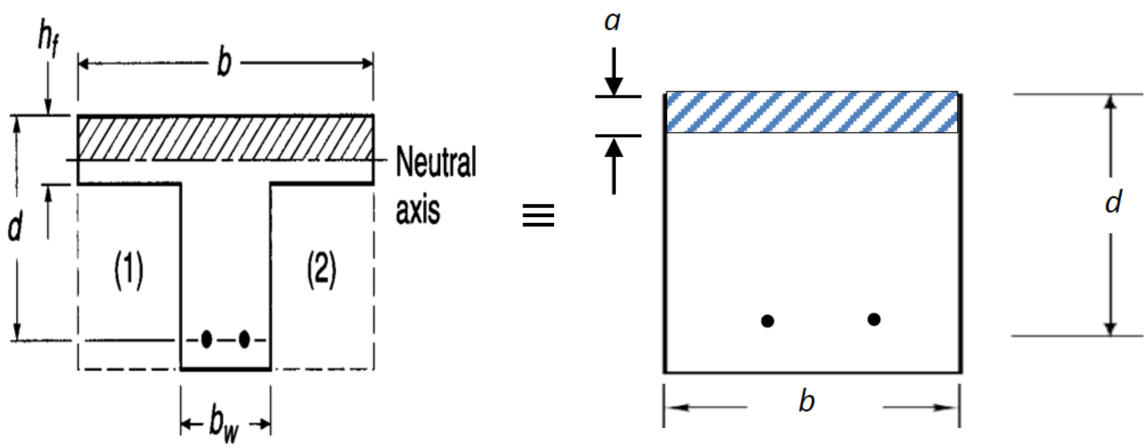
1. $b_w + 16\,h_f$ (bw is web width; h_f is flange thickness)
2. $\ell/4$ (ℓ = span length)
3. $b_w + s$ (s = clear transverse span between beams)



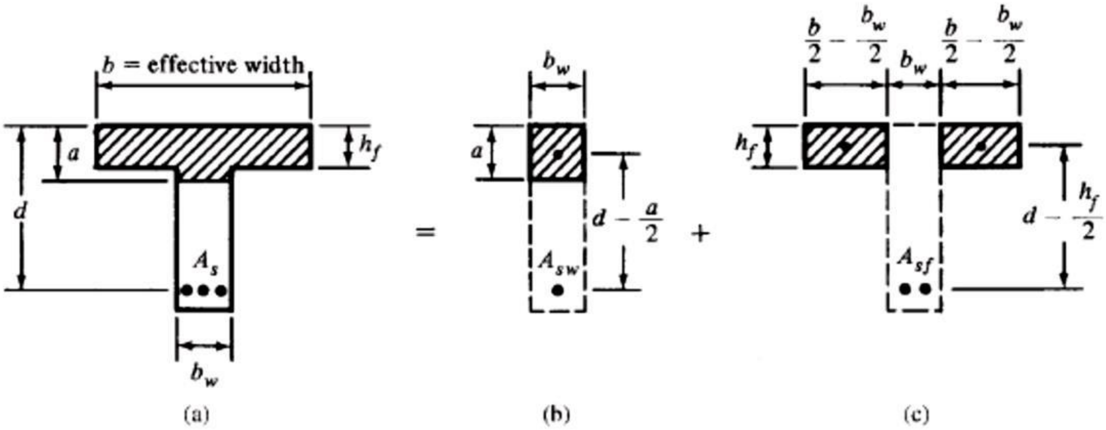
In case of external beam (b) is the **smallest of:**

1. $b_w + 6\,h_f$
2. $b_w + \ell/12$ (ℓ = span length)
3. $b_w + s/2$ (s = clear transverse span)

Usually, the depth of stress block $a < h_f$, in this case the analysis is identical to that of wide beam of width *b*.



If the neutral axis is below the flange, however, as shown for the beam of figure the compression concrete above the neutral axis no longer consists of a single rectangle, and thus the normal rectangular beam expressions do not apply.

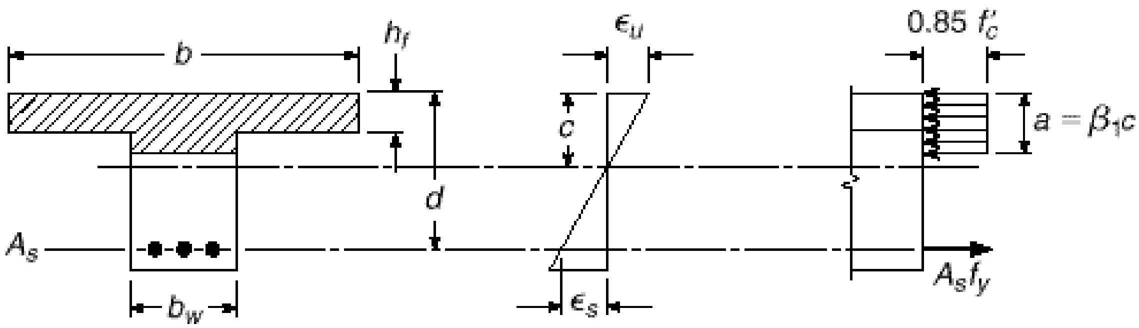


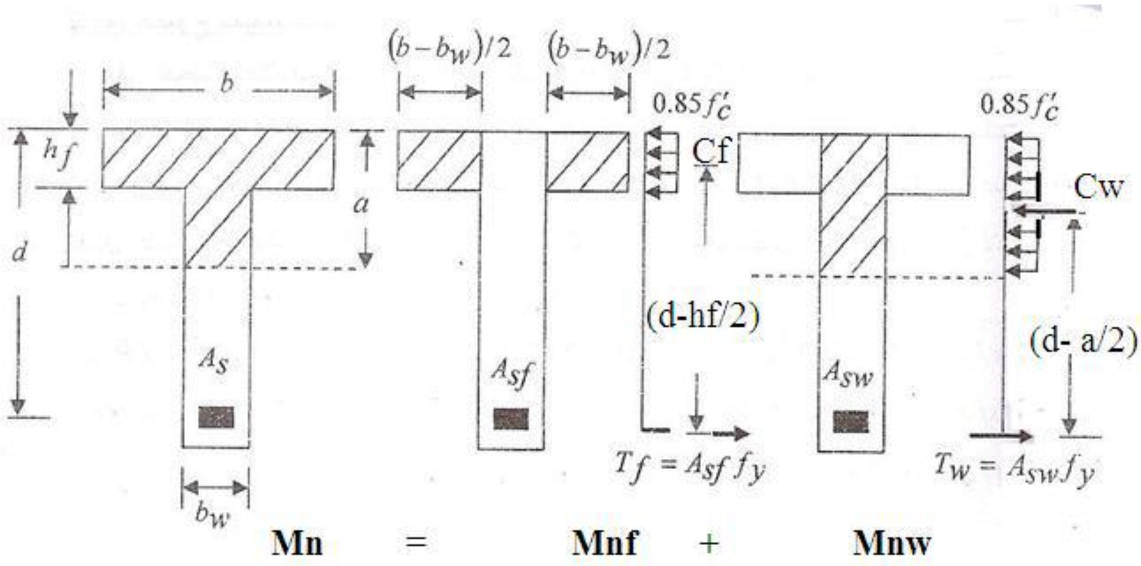
Separation of T beam into rectangular parts

Analyzing T Beams:

First, the value of ***a*** is determined, if it is less than the flange thickness, h_f, rectangular beam and the rectangular beam formulas will apply. And if it is greater than the flange thickness, h_f, then T section.

The beam is divided into a set of rectangular parts consisting of the overhanging parts of the flange and the compression part of the web. The total compression, C_w, in the web rectangle, and the total compression in the overhanging flange, C_f, are computed:





$$\boldsymbol{C_w = 0.85\, f_{c'}\, a\, b_w}$$

$$\boldsymbol{C_f = 0.85\, f_{c'}\, (b - \; b_w) h_f}$$

Then the nominal moment, M_n, is determined from compression forces by multiplying C_w and C_f by their respective lever arms from their centroid to the centroid of the steel:

$$M_n = \; C_w \left(d - \frac{a}{2}\right) + \; \boldsymbol{C_f} \; (\boldsymbol{d} - \frac{\boldsymbol{h_f}}{\boldsymbol{2}})$$

Or from force balanced:

$$A_{sf} \; = \; \frac{0.85\, f_{c'}(b_{eff} - b_w)h_f}{f_y}, \quad \text{and} \quad A_{sw} \; = \; A_s - \; A_{sf}$$

Then:

$$a_{new} = \frac{A_s \; - \; A_{sf}}{0.85\, f_{c\prime}\, b_w}\, f_y$$

$$M_n = \; A_{sw}\, f_y \left(d - \frac{a}{2}\right) + \; \boldsymbol{A_{sf}}\, f_y \; (\boldsymbol{d} - \frac{\boldsymbol{h_f}}{\boldsymbol{2}})$$

As for rectangular beams, the tensile steel should yield prior to sudden crushing of the compression concrete, as assumed in the preceding development. Yielding of the tensile reinforcement and Code compliance are ensured if the net tensile strain is greater than $\varepsilon_t \; \geq 0.004$.

Setting $\varepsilon_u = 0.003$ and $\varepsilon_t = 0.004$ provides a maximum reinforced ratio. This will occur if $\rho_w \; = \; \frac{As}{bw\; d}$ is less than:

$$\rho_{w,max} = \rho_{max} + \rho_f$$

where:

$$\rho_f = \frac{A_{sf}}{b_w d}$$

ρ_{max}: as previously defined for a rectangular cross section.

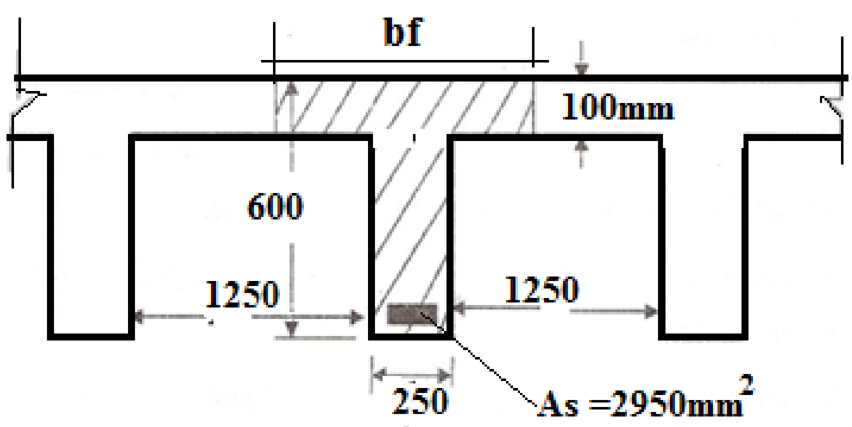
$$\rho_{w,max} = 0.85\,\beta_1 \frac{fc'}{f_y} \frac{\varepsilon u}{\varepsilon u + 0.004} + \frac{A_{sf}}{b_w d}$$

<u>Note:</u> For T or L (ACI-Code - For statically determinate members with a flange in tension, $\boldsymbol{A_{s,\,min}}$ shall not be less than the value given by the equation:

$$A_{S,min} = \frac{0.25\sqrt{f_{c'}}}{f_y}\, b_w d$$

Except that $\boldsymbol{b_w}$ is replaced by either $\boldsymbol{2b_w}$ or the width of the flange, whichever is smaller).

Example: Determine the design strength of the T beam shown in figure below, with f_c' = 25 *MPa* and f_y = 420 *MPa*. The beam has a 10m span and is cast integrally with a floor slab that is 100 mm thick. The clear distance between webs is 1250 mm.



Effective flange width (b):

b is the smallest of:

1. $b_w + 16\,h_f$ = 250 + 16 (100) = 1850 mm
2. $\ell/4 = \frac{10000}{4}$ = 2500 mm
3. $b_w + s$ = 250 + 1250 =**1500 mm**

∴ b_{eff} = 1500 mm

$$A_{S,min} = \frac{0.25\sqrt{f_{c'}}}{f_y}\, b_w d = \frac{0.25\sqrt{25}}{420}\, 250 \; x \; 530 = 394.34\; mm^2$$

But not less than:

$$A_{S,min} = \frac{1.4}{f_y}\, b_w d = \frac{1.4}{420}\, 250\; x\; 530 = \mathbf{441.7\; mm^2} \text{ control}$$

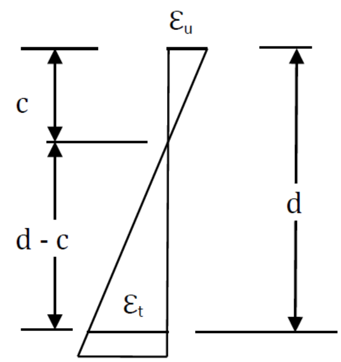
$$A_s > A_{S,min}$$

$$a = \frac{A_s\; f_y}{0.85\; f_{c'}\; b_{eff}} = \frac{2950\; x\; 420}{0.85\; x\; 25\; x\; 1500} = 38.87\; mm < \; h_f$$

∴ Rectangular section

$$M_n = A_s\, f_y\left(d - \frac{a}{2}\right) = 2950\; x\; 420\left(530 - \frac{38.87}{2}\right) = 632.59\; kN.m$$

Calculate the value of ∅:



$$c = \frac{a}{0.85} = 45.73\; mm$$

$$\varepsilon_{ty} = \frac{f_y}{200000} = \frac{420}{200000} = 0.0021$$

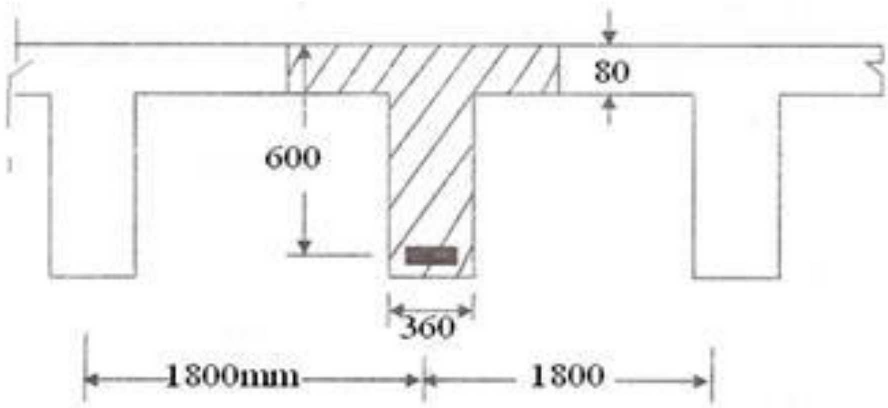
$$\varepsilon_t = \varepsilon u\left(\frac{d-c}{c}\right) = 0.003\left(\frac{530 - 45.73}{45.73}\right) = 0.03177$$

$$\because \varepsilon_t > \varepsilon_{ty} + 0.003$$

∴ ∅ = 0.9

$$M_u = \emptyset M_n = 0.9\; x\; 632.59 = \mathbf{569.33\; kN.m}$$

Example: A concrete slab with 80mm supports on beams the distance between them 1.8 m c/c with simply supported span of 5 m, find **ultimate moment capacity** for the interior beam, using $f_{c'}$ = 20.7 *MPa* and *fy* = 345 *MPa*, d = 600 mm. Use 8 # 32 = 6436 mm^2.



Solution:

Effective flange width (b):

b is the smallest of:

1. $b_w + 16\,h_f$ = 360 + 16 (80) = 1640 mm
2. $\ell/4 = \frac{5000}{4}$ = **1250 mm**
3. b_w + s = 360 + 1440 =1800 mm

$\therefore$ **b_{eff} = 1250 mm**

$$A_{S,min} = \frac{0.25\sqrt{f_{c'}}}{f_y} b_w d = \frac{0.25\sqrt{20.7}}{345} 360 \; x \; 600 = 712.13 \; mm^2$$

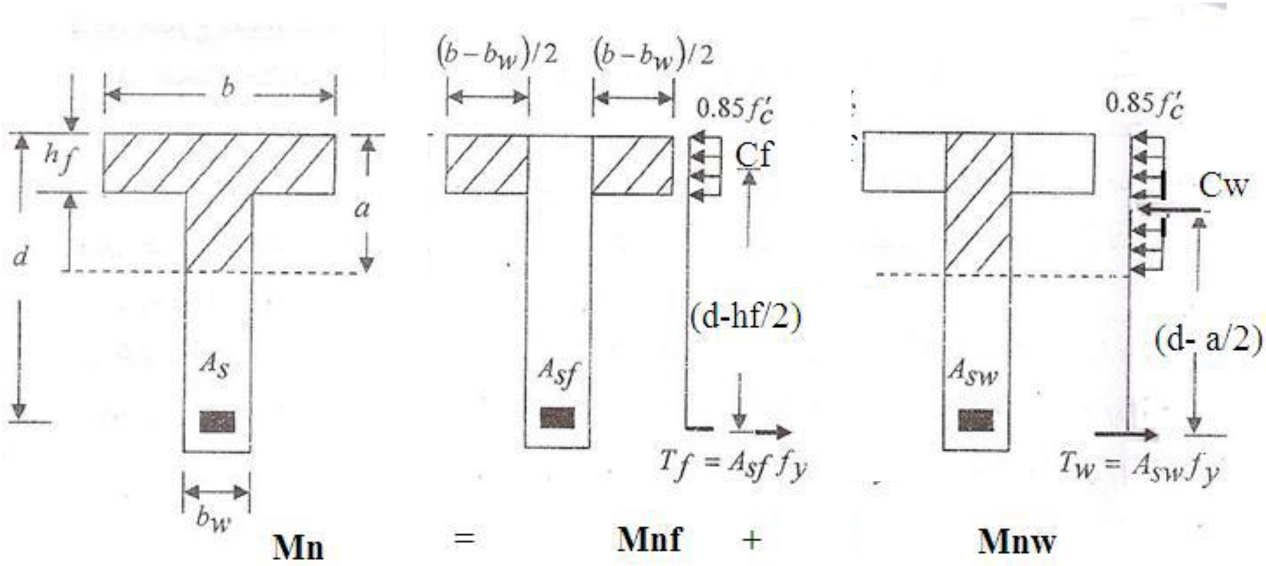
But not less than:

$$A_{S,min} = \frac{1.4}{f_y} b_w d = \frac{1.4}{345} 360 \; x \; 600 = \mathbf{876.5 \; mm^2} \text{ control}$$

$$A_s > A_{S,min}$$

$$a = \frac{A_s \; f_y}{0.85 \; f_{c'} \; b_{eff}} = \frac{6436 \; x \; 345}{0.85 \; 20.7 \; x \; 1250} = 100.956 \; mm > h_f$$

$\therefore$ T- section



$$A_{sf} = \frac{0.85\, f_{c'}(b_{eff} - b_w)h_f}{f_y} = \frac{0.85\ x\ 20.7\ (1250 - 360) x\ 80}{345} = 3631\ mm^2$$

$$A_{sw} = A_s - A_{sf} = 6436 - 3631 = 2805\ mm^2$$

Then:

$$a_{new} = \frac{A_s - A_{sf}}{0.85\ f_{c'}\ b_w} f_y = \frac{6436 - 3631}{0.85\ x\ 20.7\ x\ 360}\ x\ 345 = \mathbf{152.8\ mm}$$

$$\rho_w = \frac{A_s}{b_w\ d} = \frac{6436}{360\ x\ 600} = \mathbf{0.0298}$$

$$\rho_{w,max} = \rho_{max} + \rho_f$$

$$\rho_{w,max} = 0.85\ \beta_1 \frac{fc'}{f_y} \frac{\varepsilon u}{\varepsilon u + 0.004} + \frac{A_{sf}}{b_w d}$$

$$\rho_{w,max} = 0.85\ x\ 0.85\ \frac{20.7}{345} \frac{0.003}{0.003 + 0.004} + \frac{3631}{360\ x\ 600}$$

$$\rho_{w,max} = 0.01858 + 0.01681 = \mathbf{0.032539}$$

$$\rho_w < \rho_{w,max}$$

∴ Under Reinforcement

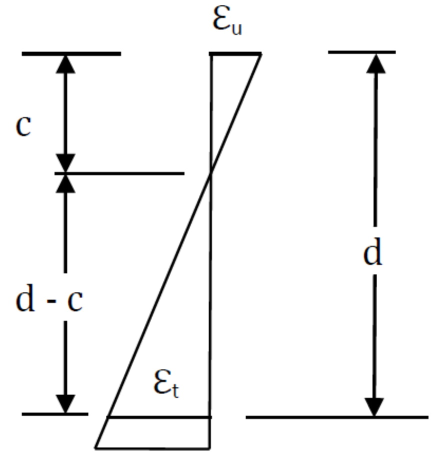
$$M_n = A_{sw}\, f_y \left(d - \frac{a}{2}\right) + A_{sf}\, f_y \left(d - \frac{h_f}{2}\right)$$

$$= 2805\ x\ 345 \left(600 - \frac{152.8}{2}\right) + 3631\ x\ 345 \left(600 - \frac{80}{2}\right)$$

$$= \mathbf{1208.21\ kN.mm}$$

Calculate the value of Ø:

$$c = \frac{a}{0.85} = 179.8\ mm$$



$$\varepsilon_{ty} = \frac{f_y}{200000} = \frac{345}{200000} = 0.0017$$

$$\varepsilon_t = \varepsilon u \left(\frac{d-c}{c}\right) = 0.003 \left(\frac{600 - 179.8}{179.8}\right) = 0.00701$$

$$\because \varepsilon_t > \varepsilon_{ty} + 0.003$$

$$\therefore \emptyset = \mathbf{0.9}$$

$$M_u = \emptyset M_n = 0.9\ x\ 1208.21 = \mathbf{1087.389\ kN.m}$$

Design of L and T beams:

For the design of T or L beams, the flange has normally already been selected in the slab design, as it is for the slab. The size of the web is normally not selected on the basis of moment requirements but probably is given an area based on shear requirements; that is, a sufficient area is used so as to provide a certain minimum shear capacity. It is also possible that the width of the web may be selected on the basis of the width estimated to be needed to put in the reinforcing bars. Sizes may also have been preselected, to simplify formwork for architectural requirements or for deflection reasons.

The flanges of most T beams are usually so large that the neutral axis probably falls within the flange, and thus the rectangular beam formulas apply. Should the neutral axis fall within the web, a trial-and-error process is often used for the design. In this process, a lever arm from the center of gravity of the compression block to the center of gravity of the steel is estimated to equal the larger of **0.9*d*** or ($\boldsymbol{d - \frac{h_f}{2}}$), and from this value, called **z**, a trial steel area is calculated **(**$\boldsymbol{A_s = \frac{M_n}{f_y\, z}}$).

If there is much difference, the estimated value of **z** is revised and a new $\boldsymbol{A_s}$ determined. This process is continued until the change in $\boldsymbol{A_s}$ is quite small.

The bending moment over the support is negative, so the flange is in tension. Also, the magnitude of the negative moment is usually larger than that of the positive moment near mid span. This situation will control the design of the T beam because the depth and web width will be determined for this case. Then, when the beam is designed for positive moment at mid-span, the width and depth are already known.

Example: Determine the **steel area** of T beam, with f_c' = 20 *MPa* and f_y = 400 *MPa*. The beam has a 7.2 m span and is cast integrally with a floor slab that is 75 mm thick. The center distance between webs is 1.2 m. If M_u = 710 kN.m, b_w = 275 mmm and d = 500 mm.

Effective flange width (b):

b is the smallest of:

1. $b_w + 16\, h_f$ = 275 + 16 (75) = 1475 mm
2. $\ell/4 = \frac{7200}{4}$ = 1800 mm
3. $b_w + s$ = 275 + 925 =**1200 mm** ∴ $\mathbf{b_{eff}}$ **= 1200 mm**

Try a = h_f = 75 mm try $\emptyset = 0.9$

$$M_{u,flange} = \emptyset\, M_n = \emptyset\; x\; 0.85\; f_{c'}\; x\; b_{eff}\; x\; h_f \left(d - \frac{h_f}{2}\right)$$

$$= 0.9\; x\; 0.85\; x\; 20\; x\; 1200\; x\; 75 \left(500 - \frac{75}{2}\right) = 636.9\; kN.m$$

$$< 710\; kN.m$$

Then T- section

$$A_{sf} = \frac{0.85\, f_{c'}(b_{eff} - b_w)h_f}{f_y} = \frac{0.85\ x\ 20\ (1200 - 275) x\ 75}{400} = 2949\ mm^2$$

$$M_{u,flange} = \emptyset\ x\ A_{sf} x\ f_y \left(d - \frac{h_f}{2}\right) = 0.9\ x\ 2949\ x\ 400\ \left(500 - \frac{75}{2}\right)$$

$$= 491.0\ kN.m$$

$$M_{u,w} = 710 - 491.0 = 219\ kN.m$$

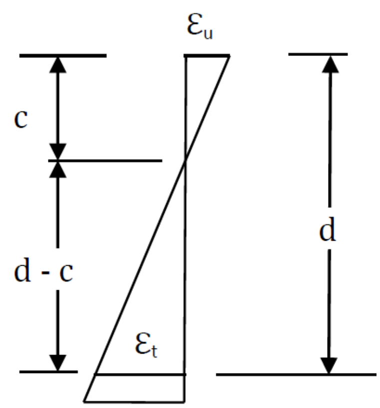
$$M_{u,w} = \emptyset\ x\ 0.85\ f_{c'}\ x\ b_w\ x\ a\ \left(d - \frac{a}{2}\right)$$

$$219\ x\ 10^6 = 0.9\ x\ 0.85\ x\ 20\ x\ 275\ x\ a\ \left(500 - \frac{a}{2}\right)$$

$$a^2 - 1000a + 104099.82 = 0$$

$$a = 118.0\ mm\ >\ h_f = 75\ mm$$

$$A_{sw} = \frac{219\ x\ 10^6}{0.9\ x\ 400\ x\ (500 - \frac{118}{2})} = 1380\ mm^2$$



$A_{s\text{-}total}$ = 2949 + 1380 = **4329 mm^2**

Check the value of ∅:

$$c = \frac{a}{0.85} = 138.82\ mm$$

$$\varepsilon_t = \varepsilon u \left(\frac{d - c}{c}\right) = 0.003\ \left(\frac{500 - 138.82}{138.82}\right) = 0.0078$$

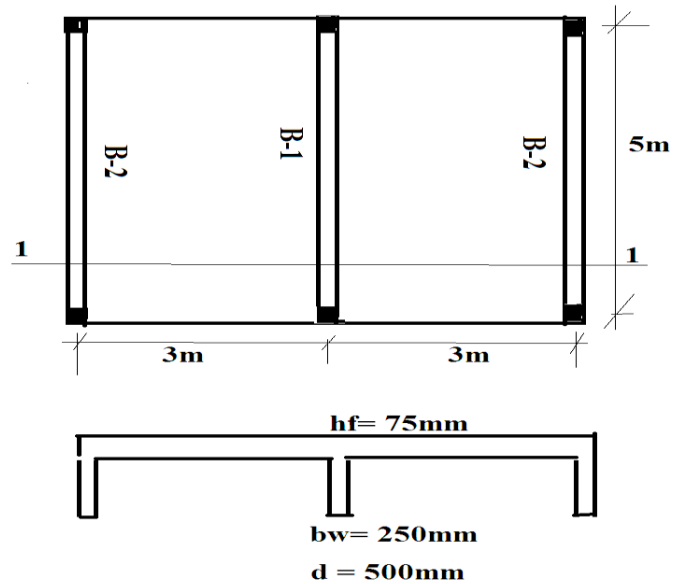
$$\varepsilon_{ty} = \frac{f_y}{200000} = \frac{400}{200000} = 0.002$$

$$\because \varepsilon_t > \varepsilon_{ty} + 0.003$$

∴ ∅ = **0.9**

Example: Find the ***area of steel*** required for the simply supported beams **B-1** and **B-2** for the plan shown, using $f_c' = 20\ MPa$, and $fy = 400\ MPa$.

Loads	B-1	B-2	Nots
W_d kN/m	55	27.5	(including its own weight)
W_L kN/m	100	50	------



Section 1-1

For interior beam B-1:

Effective flange width (b):

b is the smallest of:

4. $b_w + 16\,h_f = 250 + 16\,(75) = 1450$ mm
5. $\ell/4 = \frac{5000}{4} = \mathbf{1250\ mm}$
6. $b_w + s = 250 + 2750 = 3000$ mm

$\therefore$ **b_{eff} = 1250 mm**

$$W_u = 1.2\ x\ 55 + 1.6\ x\ 100 = 226\ kN/m$$

$$M_u = \frac{wl^2}{8} = \frac{226\ x\ 25}{8} = 706.25\ kN.m$$

Try a = h_f = 75 mm try $\emptyset = 0.9$

$$M_{u,flange} = \emptyset\ M_n = \emptyset\ x\ 0.85\ f_{c'}\ x\ b_{eff}\ x\ h_f\left(d - \frac{h_f}{2}\right)$$

$$= 0.9\ x\ 0.85\ x\ 20\ x\ 1250\ x\ 75\left(500 - \frac{75}{2}\right) = 663.4\ kN.m$$

$$< 706.3\ kN.m$$

Then T- section

$$A_{sf} = \frac{0.85\, f_{c'}(b_{eff} - b_w)h_f}{f_y} = \frac{0.85\ x\ 20\ (1250 - 250) x\ 75}{400} = 3188\ mm^2$$

$$M_{u,flange} = \emptyset\ x\ A_{sf} x\ f_y \left(d - \frac{h_f}{2}\right) = 0.9\ x\ 3188\ x\ 400\ \left(500 - \frac{75}{2}\right)$$

$$= 530.8\ kN.m$$

$$M_{u,w} = 706.3 - 530.8 = 175.5\ kN.m$$

$$M_{u,w} = \emptyset\ x\ 0.85\ f_{c'}\ x\ b_w\ x\ a\ \left(d - \frac{a}{2}\right)$$

$$175.5\ x\ 10^6 = 0.9\ x\ 0.85\ x\ 20\ x\ 250\ x\ a\ \left(500 - \frac{a}{2}\right)$$

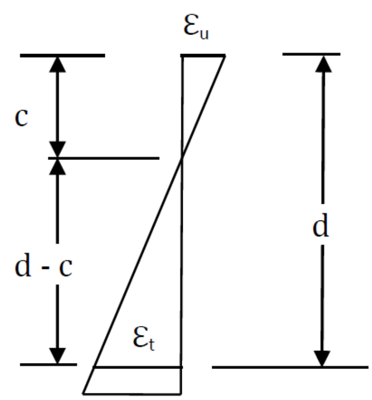
$$a^2 - 1000a + 91764.7 = 0$$

$$a = 102.2\ mm\ > h_f = 75\ mm$$

$$A_{sw} = \frac{175.5\ x\ 10^6}{0.9\ x\ 400\ x\ (500 - \frac{102.2}{2})} = 1086\ mm^2$$

$A_{s\text{-total}}$ = 3188 + 1086 = **4274 mm²**

Check the value of Ø:



$$c = \frac{a}{0.85} = 120.235\ mm$$

$$\varepsilon_t = \varepsilon u \left(\frac{d - c}{c}\right) = 0.003\ \left(\frac{500 - 120.235}{120.235}\right) = 0.00948$$

$$\varepsilon_{ty} = \frac{f_y}{200000} = \frac{400}{200000} = 0.002$$

$$\because \varepsilon_t > \varepsilon_{ty} + 0.003$$

$$\therefore \emptyset = \mathbf{0.9}$$

For exterior beam B-2:

In case of external beam (b) is the smallest of:

1. $b_w + 6\,h_f = 250 + 6(75) = 700$ mm
2. $b_w + \ell/12 = 250 + \frac{5000}{12} = \boldsymbol{667\ mm}$
3. $b_w + s/2 = 250 + \frac{2750}{2} = 1625\ mm$

$$\therefore \mathbf{b_{eff} = 667\ mm}$$

$$W_u = 1.2\ x\ 27.5 + 1.6\ x\ 50 = 113\ kN/m$$

$$M_u = \frac{wl^2}{8} = \frac{113\ x\ 25}{8} = 353.13\ kN.m$$

Try $a = h_f = 75$ mm try $\emptyset = 0.9$

$$M_{u,flange} = \emptyset\ x\ 0.85\ f_{c'}\ x\ b_{eff}\ x\ h_f\ \left(d - \frac{h_f}{2}\right)$$

$$= 0.9\ x\ 0.85\ x\ 20\ x\ 667\ x\ 75\ \left(500 - \frac{75}{2}\right) = 353.98\ kN.m$$

$$\approx 353.13\ kN.m$$

Then rectangular- section

$$M_{u,w} = \emptyset\ x\ 0.85\ f_{c'}\ x\ b\ x\ a\ \left(d - \frac{a}{2}\right)$$

$$353.13\ x\ 10^6 = 0.9\ x\ 0.85\ x\ 20\ x\ 667\ x\ a\ \left(500 - \frac{a}{2}\right)$$

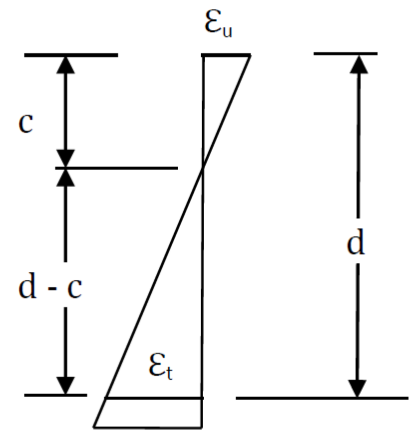
$$a^2 - 1000a + 69206.6 = 0$$

$$a = 74.8\ mm < h_f = 75\ mm \qquad \text{ok.}$$

$$A_{sw} = \frac{0.85\ x\ 20\ x\ 667\ x\ 74.8}{400} = 2121\ mm^2$$

Check the value of ∅:

$$c = \frac{a}{0.85} = 88\ mm$$



$$\varepsilon_t = \varepsilon u \left(\frac{d-c}{c}\right) = 0.003 \left(\frac{500-88}{88}\right) = 0.01405$$

$$\varepsilon_{ty} = \frac{f_y}{200000} = \frac{400}{200000} = 0.002$$

$$\because \varepsilon_t > \varepsilon_{ty} + 0.003$$

∴ ∅ = **0.9**

Analysis and design of doubly reinforced rectangular beam

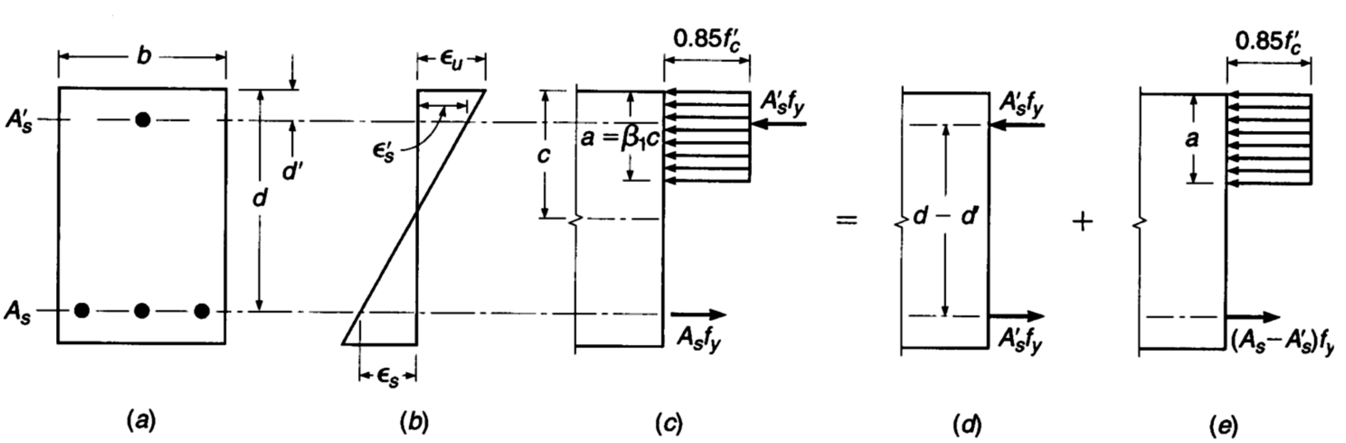
If the beam cross-section is limited and the moment is high, reinforcement is added in the compression zone as well as tension reinforcement. This will permit using reinforcement ratio $\rho > \rho_{max}$ and failure will not be sudden. Also, this compression reinforcement reduces the long-term deflections which result from creep of concrete.

Analysis of doubly reinforced beam (tension and compression reinforcement)

Calculation of resisting Moment of Double Reinforced Beam:

If $\rho < \rho_{max}$ the strength of beam may be approximated by **disregarding (neglecting**) the compression bars and analysis is done as for **single** reinforced beam.

If $\boldsymbol{\rho > \rho_{max}}$ then the total resisting moment can be assumed as sum of two parts:



doubly reinforced rectangular

$\mathbf{M_n = M_{n1} + M_{n2}}$ **$\mathbf{M_{n1}}$** **$\mathbf{M_{n2}}$**

$$M_{n1} = A_{\acute{s}} f_y \, (d - \acute{d})$$

$$M_{n2} = (A_s - A_{\acute{s}}) f_y \, (d - \frac{a}{2})$$

Then total resisting moment $M_n = M_{n1} + M_{n2}$

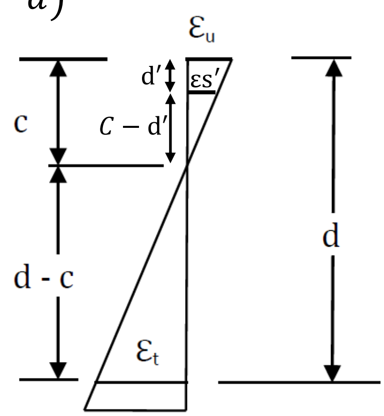
$$M_n = M_{n1} + M_{n2}$$

$$M_n = A_{\acute{s}} f_y \left(d - \acute{d}\right) + (A_s - A_{\acute{s}}) f_y \left(d - \frac{a}{2}\right)$$

$$a = \frac{A_s - A_{s'}}{0.85\, f_{c'}\, b}\, f_y$$

If the compression steel is not yielded at failure (i.e. $\boldsymbol{f_{\acute{s}}} < \boldsymbol{f_y}$) then:

$$M_n = 0.85\, f_{\acute{c}}\, a \,.\, b \left(d - \frac{a}{2}\right) + A_{\acute{s}}\, f_{\acute{s}} \left(d - \acute{d}\right)$$



Where:

$$f_{\acute{s}} = \varepsilon_{\acute{s}}\, E_s$$

$$\frac{\varepsilon_{s'}}{c - d'} = \frac{\varepsilon_u}{c} \quad , \quad \varepsilon_{s'} = \frac{c - d'}{c}\, \varepsilon_u$$

To calculate a:

$$N_t = N_{c1} + N_{c2}$$

$$A_S f_y = 0.85\, f_{c'}\, a \,.\, b + A_{\acute{s}}\, f_{\acute{s}}$$

$$A_S f_y = 0.85\, f_{c'}\, \beta_1 c \,.\, b + A_{\acute{s}}\, \frac{c - d'}{c}\, \varepsilon_u . E_s$$

Solve the above equation using **Quadratic formula** we get **c**.

$$X_{1,2} = \frac{-B \mp \sqrt{B^2 - 4A.C}}{2A}$$

Then calculate a where:

$$a = \beta_1 c$$

Or calculate **c** from the following formula:

$$c = \sqrt{Q + R^2} - R$$

Where: $Q = \frac{600\, d'\, A_{s'}}{0.85 \beta_1\, f_{c'}}$ and $R = \frac{600\, A_{s'} - A_S f_y}{1.7 \beta_1\, f_{c'}\, b}$

To check if compression steel yielded ($f_{\acute{s}} = f_y$) ρ is compared with $\bar{\rho}_{cy}$ where:

$$\bar{\rho}_{cy} = 0.85\,\beta_1 \frac{fc'}{f_y}\frac{d'}{d}\ \frac{\varepsilon u}{\varepsilon u - \varepsilon y} + \rho' \quad \text{, where: } \rho' = \frac{A_{s'}}{bd}$$

If $\rho \geq \bar{\rho}_{cy}$ then compression steel yielded ($f_{\acute{s}} = f_y$)

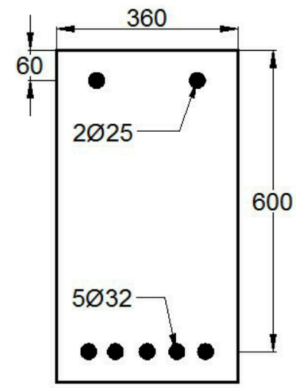
If $\rho < \bar{\rho}_{cy}$ then compression steel ($f_{\acute{s}} < f_y$), $f_{\acute{s}} = \frac{c-d'}{c}\,\varepsilon_u E_s$

To avoid sudden failure ACI code limits $\bar{\rho}_{max}$ to the following limit:

$$\bar{\rho}_{max} = \rho_{max} + \rho' > \rho$$

$$\rho_{max} = 0.85\,\beta_1 \frac{fc'}{f_y}\ \frac{\varepsilon u}{\varepsilon u + 0.004}$$

Example: Compute the design moment capacity for the section below, using f_y = 400 *MPa* and f_c'= 20.7 *MPa*.



$$As = (5)\frac{\pi\, d^2}{4} = (5)\frac{\pi\ 32^2}{4} = 4021\ mm^2$$

$$\rho = \frac{As}{b\ d} = \frac{4021}{360\ x\ 600} = 0.01862$$

$$As' = (2)\frac{\pi\, d^2}{4} = (2)\frac{\pi\ 25^2}{4} = 981.7\ mm^2$$

$$\rho' = \frac{As'}{b\ d} = \frac{981.7}{360\ x\ 600} = 0.00455$$

Check $\boldsymbol{\rho}$ with $\boldsymbol{\rho_{max}}$ to see if the beam can be analyzed as single reinforced.

$$\rho_{max} = 0.85\ x\ 0.85\ \frac{20.7}{400}\ \frac{0.003}{0.003 + 0.004} = 0.01602$$

$$\because \rho > \rho_{max}$$

$\therefore$ The beam must be analyzed as **doubly reinforced**.

Now, check if compression steel stress $f_{s'} = f_y$

$$\bar{\rho}_{cy} = 0.85\,\beta_1 \frac{fc'}{f_y}\frac{d'}{d}\ \frac{\varepsilon u}{\varepsilon u - \varepsilon y} + \rho', \quad \varepsilon_y = \frac{f_y}{E_s} = \frac{400}{200000} = 0.002$$

$$\bar{\rho}_{cy} = 0.85\ x\ 0.85\ \frac{20.7}{400}\frac{60}{600}\ \frac{0.003}{0.003 - 0.002} + 0.00455 = 0.01577$$

$$\rho > \bar{\rho}_{cy}$$

$$\therefore\ f_{s'} = f_y$$

$$\bar{\rho}_{max} = \rho_{max} + \rho' = 0.01602 + 0.00455 = 0.02057 > \rho \quad \therefore \text{ok.}$$

Total resisting moment $M_n = M_{n1} + M_{n2}$

$$M_{n1} = A_{\acute{s}} f_y \left(d - \acute{d}\right) = 981.7\ x\ 400\ (600 - 60) = 212.0\ x\ 10^6\ kN.mm$$

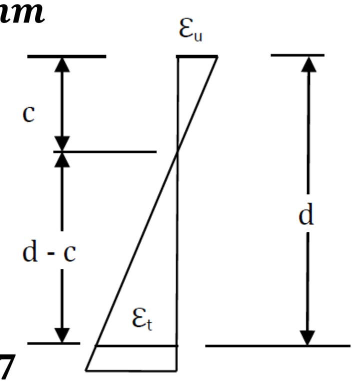
$$M_{n2} = (A_s - A_{\acute{s}}) f_y \left(d - \frac{a}{2}\right)$$

Where:

$$a = \frac{A_s - A_{s'}}{0.85\ f_{c'}\ b}\ f_y = \frac{4021 - 981.7}{0.85\ x\ 20.7\ x\ 360}\ 400 = \mathbf{192\ mm}$$

$$M_{n2} = (4021 - 981.7)\ x\ 400\ x\left(600 - \frac{192}{2}\right) = \mathbf{612.7\ x\ 10^6\ N.mm}$$

$$M_n = 212.0\ x\ 10^6 + 612.7\ x\ 10^6 = \mathbf{824.7\ x\ 10^6\ N.mm}$$



$$c = \frac{192}{0.85} = 225.88\ mm$$

$$\varepsilon_t = \varepsilon u \left(\frac{d - c}{c}\right) = 0.003\ \left(\frac{600 - 225.88}{225.88}\right) = \mathbf{0.00497}$$

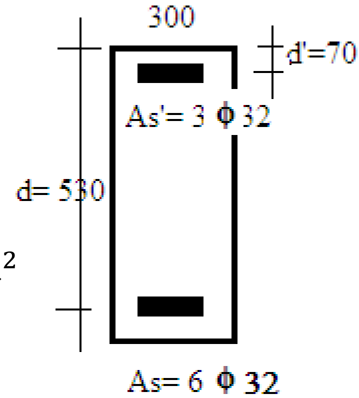
$$\because\ \varepsilon_{ty} < \varepsilon_t < \varepsilon_{ty} + 0.003\ (0.005)$$

$$\therefore\ \emptyset = 0.65 + 0.25\left(\frac{\varepsilon_t - \varepsilon_{ty}}{0.003}\right) = 0.65 + 0.25\left(\frac{0.00497 - 0.002}{0.003}\right)$$

$$\emptyset = 0.898$$

$\therefore$ The design moment $M_u = \phi\ M_n = 0.898\ x\ 824.7 = \mathbf{739.76\ kN.m}$

Example.: Find the ultimate moment capacity for the section below, using, f_y = 345 *MPa* and $fc_{'}$ = 27.6 *MPa*.



$$As = (6)\frac{\pi\ d^2}{4} = (6)\frac{\pi\ 32^2}{4} = 4826\ mm^2$$

$$\rho = \frac{As}{b\ d} = \frac{4826}{300\ x\ 530} = 0.03035$$

$$As' = (3)\frac{\pi\ d^2}{4} = (3)\frac{\pi\ 32^2}{4} = 2413\ mm^2$$

$$\rho' = \frac{As'}{b\ d} = \frac{2413}{300\ x\ 530} = 0.01518$$

Check $\boldsymbol{\rho}$ with $\boldsymbol{\rho}_{max}$ to see if the beam can be analyzed as single reinforced.

$$\rho_{max} = 0.85\ x\ 0.85\ \frac{27.6}{345}\ \frac{0.003}{0.003 + 0.004} = 0.02477$$

$$\boldsymbol{\rho > \bar{\rho}max}$$

$\therefore$ The beam must be analyzed as ***doubly reinforced***.

Now, check if compression steel stress $f_{s\prime} = f_y$

$$\bar{\rho}_{cy} = 0.85\ \beta_1\ \frac{fc'}{f_y}\frac{d'}{d}\ \frac{\varepsilon u}{\varepsilon u - \varepsilon y} + \rho'$$

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{345}{200000} = 0.00173$$

$$\bar{\rho}_{cy} = 0.85\ x\ 0.85\ \frac{27.6}{345}\frac{70}{530}\ \frac{0.003}{0.003 - 0.00173} + 0.01518 = \mathbf{0.03321}$$

$$\rho < \bar{\rho}_{cy}$$

$$\therefore\ \boldsymbol{f_{s'} < f_y}$$

$$M_n = 0.85\ f_{\acute{c}}\ a\ .\ b\ \left(d - \frac{a}{2}\right) + A_{\acute{s}}\ f_{\acute{s}}\ (d - \acute{d})$$

$$A_S f_y = 0.85\ f_{c'}\ a\ .\ b + A_{\acute{s}}\ f_{\acute{s}}$$

$$A_S f_y = 0.85\ f_{c'}\ \beta_1 c\ .\ b + A_{\acute{s}}\ \frac{\mathrm{c - d'}}{\mathrm{c}}\ \varepsilon_{\mathrm{u}}.E_s$$

$$4826\ x\ 345 = 0.85\ x\ 27.6\ x\ 0.85\ c\ x\ 300 + 2413\ x\ \frac{c - 70}{c}\ x\ 0.003\ x\ 200000$$

$$1664970c = 5982.3\ c^2 + 1447800\ c - 101346000$$

$$5982.3\ c^2 - 217170\ c - 101346000 = 0$$

$$c^2 - 36.3\ c - 16940.98 = 0$$

$$X_{1,2} = \frac{-B \mp \sqrt{B^2 - 4A.C}}{2A}$$

$$c_{1,2} = \frac{36.3 \mp \sqrt{36.3^2 - 4x\ (1)\ x(-16940.98)}}{2x\ 1}$$

$$\boldsymbol{c = 149.6\ mm}$$

$$a = \beta_1 c = 0.85\ x\ 149.6 = 127.13\ \boldsymbol{mm}$$

$$f_{\acute{s}} = \varepsilon_{\acute{s}}\ E_s$$

$$f_{\acute{s}} = \frac{\mathrm{c - d'}}{\mathrm{c}}\ \varepsilon_{\mathrm{u}} E_s = \frac{149.6 - 70}{149.6}\ x\ 0.003\ x\ 200000 = 319.25\ \boldsymbol{Mpa}$$

$$M_n = 0.85\ f_{\acute{c}}\ \mathrm{a}\ .\ b\ \left(d - \frac{a}{2}\right) + A_{\acute{s}}\ f_{\acute{s}}\ (d - \acute{d})$$

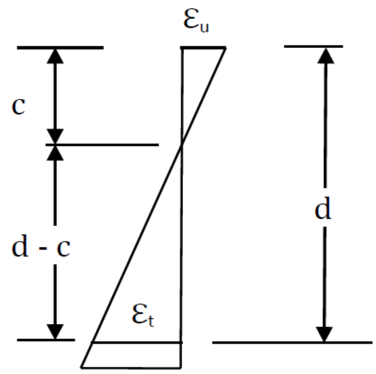
$$M_n = 0.85 \, x \, 27.6 \, x \, 127.13 \, x \, 300 \left(530 - \frac{127.13}{2}\right)$$
$$+ 2413 \, x \, 319.25 \, (530 - 70) = \mathbf{771.7 \, kN.m}$$

$$\varepsilon_t = \varepsilon u \left(\frac{d - c}{c}\right) = 0.003 \left(\frac{530 - 149.6}{149.6}\right) = 0.00763$$

$$\because \varepsilon_t > \varepsilon_{ty} + 0.003 \, (0.004725)$$

$$\therefore \ \emptyset = \mathbf{0.9}$$

$$M_u = 0.9 \, x \, 771.7 = \mathbf{694.53 \, N.mm}$$



Resume

If $\rho > \rho_{max}$ Doubly reinforcement

$f_{\acute{s}} < f_y$

$$M_n = 0.85 \, f_{\acute{c}} \, a \, . \, b \left(d - \frac{a}{2}\right) + A_{\acute{s}} \ f_{\acute{s}} \left(d - \acute{d}\right)$$

$$A_s f_y = 0.85 \, f_{c'} \, a \, . \, b + A_{\acute{s}} \ f_{\acute{s}}$$

$$a = \beta_1 \, c \qquad f_{\acute{s}} = \frac{c - d'}{c} \varepsilon_u E_s$$

$$A_s f_y = 0.85 \, f_{c'} \, \beta_1 c \, . \, b + A_{\acute{s}} \frac{c - d'}{c} \varepsilon_u . E_s$$

$$X_{1,2} = \frac{-B \mp \sqrt{B^2 - 4A.C}}{2A}$$

Or calculate **c** from the following formula:

$$c = \sqrt{Q + R^2} - R$$

$$Q = \frac{600 \, d' \, A_{s'}}{0.85 \beta_1 \, f_{c'} \, b} \quad \text{and} \quad R = \frac{600 \ A_{s'} - A_s f_y}{1.7 \beta_1 \, f_{c'} \, b}$$

$f_{\acute{s}} = f_y$

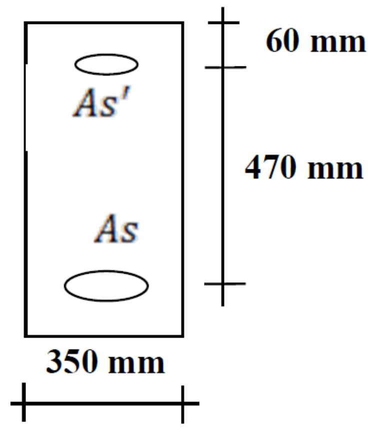
$$M_n = A_{\acute{s}} f_y \left(d - \acute{d}\right) + \left(A_s - A_{\acute{s}}\right) f_y \left(d - \frac{a}{2}\right)$$

$$a = \frac{A_s - A_{s'}}{0.85 \, f_{c'} \, b} f_y$$

Design of Doubly reinforced concrete rectangular beams

Sufficient tensile steel can be placed in most beams so that compression steel is not needed. But if it is needed, the design is usually quite straight forward. The design procedure follows the theory used for analyzing doubly reinforced sections**.**

Example: Design a rectangular beam for M_D = 200 kN.m and M_L = 350 kN.m if $f_{c'}$ = 25 MPa and fy = 410 MPa. The maximum permissible beam dimensions are shown in below.



$$M_u = 1.2\, MD + 1.6\, ML$$

$$M_u = 1.2\, x\, 200 + 1.6\, x\, 350 = 800\, kN.m$$

For checking $\emptyset = 0.9$, then $\varepsilon_t = 0.005$, there is no economic efficiency of using $\varepsilon_t \leq 0.005$ Then use:

$$\rho = 0.85\, \beta_1 \frac{fc'}{f_y} \frac{\varepsilon u}{\varepsilon u + 0.005} = 0.01652$$

$$A_{s1}(A_s) = \rho\, b\, d = 0.01652\, x\, 350\, x\, 530 = \mathbf{3065\, mm^2}$$

$$M_u = \emptyset M_n = \emptyset\, \rho\, f_y\, b\, d^2 \left(1 - 0.59 \frac{\rho\, fy}{fc'}\right) = \emptyset\, k_n b\, d^2$$

$$k_n = \rho\, f_y \left(1 - 0.59 \frac{\rho\, fy}{fc'}\right) = 0.01652\, x\, 410\, (1 - 0.59 \frac{0.01652\, x\, 410}{25})$$
$$= 5.69052$$

$$M_{u1} = \emptyset M_n = \emptyset\, k_n b\, d^2 = 0.9\, x\, 5.69052\, x\, 350\, x\, 530^2 =$$
$$\mathbf{503.52\, kN.mm} < \mathbf{800\ kN.m}$$

∴ Compression reinforcement required

$$a = \frac{A_{s1}\, f_y}{0.85\, fc'\, b} = \frac{3065\ x\ 410}{0.85\ x\ 25\ x\ 350} = 168.96\ mm$$

$$c = \frac{a}{\beta_1} = \frac{168.96}{0.85} = 198.8\ mm$$

$$M_{u2} = 800 - 503.52 = 296.48\ kN.mm$$

$$\frac{\varepsilon s'}{c - d'} = \frac{\varepsilon_u}{c}\ ,\ \varepsilon s' = \frac{c - d'}{c}\ \varepsilon_u = \frac{198.8 - 60}{198.8}\ x\ 0.003 = 0.00209$$

$$f_{\acute{s}} = \varepsilon_{\acute{s}}\ E_s = 0.00209\ x\ 200000 = 418.9\ Mpa$$

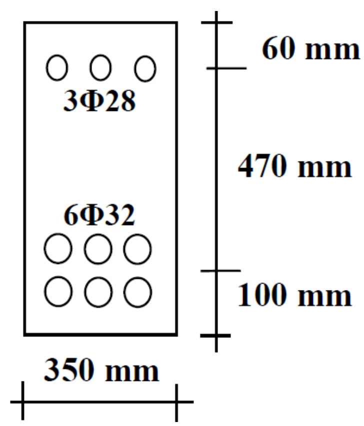
$$f_{\acute{s}} > f_y$$

$$\therefore \boldsymbol{f_{\acute{s}} = f_y = 410\ Mpa}$$

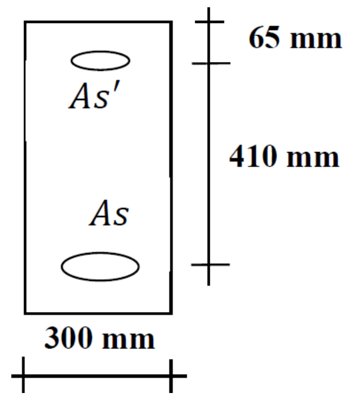
$$A_{s2}\ (A_{s'}) = \frac{M_{u2}}{\emptyset\ f_y (d - d')} = \frac{296.48\ x\ 10^6}{0.9\ x\ 410\ x\ (530 - 60)} = \boldsymbol{1710\ mm^2}$$

For tension reinforcement $A_s = A_{s1}(A_s) + A_{s2}(A_{s'})$

$$= 3065 + 1710 = \boldsymbol{4775\ mm^2}$$



Example: Find **total area of steel** required for the section below which supports M_u = 400 kN.m, using f_y = 420 *MPa*, fc' = 21*MPa* and d'= 65 mm if you need compression reinforcement.



For checking $\emptyset = 0.9$, then $\varepsilon_t = 0.005$, there is no economic efficiency of using $\varepsilon_t \leq 0.005$ Then use:

$$\rho = 0.85\, \beta_1 \frac{fc'}{f_y} \frac{\varepsilon u}{\varepsilon u + 0.005} = 0.01355$$

$$A_{s1}(A_s) = \rho\, b\, d = 0.01355\ x\ 300\ x\ 475 = \mathbf{1931\ mm^2}$$

$$M_u = \emptyset M_n = \emptyset\, \rho\, f_y\, b\, d^2 \left(1 - 0.59 \frac{\rho\, fy}{fc'}\right) = \emptyset\, k_n b\, d^2$$

$$k_n = \rho\, f_y \left(1 - 0.59 \frac{\rho\, fy}{fc'}\right) = 0.01355\ x\ 420\ (1 - 0.59 \frac{0.01355\ x\ 420}{21})$$
$$= 4.781$$

$$M_{u1} = \emptyset M_n = \emptyset\, k_n b\, d^2 = 0.9\ x\ 4.781\ x\ 300\ x\ 475^2 = \mathbf{291.25\ kN.mm}$$

< 400 kN.m

∴ Compression reinforcement required

$$a = \frac{A_{s1}\, f_y}{0.85\, fc'\, b} = \frac{1931\ x\ 420}{0.85\ x\ 21\ x\ 300} = 151.5\ mm$$

$$c = \frac{a}{\beta_1} = \frac{151.5}{0.85} = 178.2\ mm$$

$$M_{u2} = 400 - 291.25 = 108.75\ kN.mm$$

$$\frac{\varepsilon s'}{c - d\prime} = \frac{\varepsilon_u}{c} \quad , \ \varepsilon s' = \frac{c - d\prime}{c}\ \varepsilon_u = \frac{178.2 - 65}{178.2}\ x\ 0.003 = 0.00191$$

$$f_{\acute{s}} = \varepsilon_{\acute{s}}\ E_s = 0.00191\ x\ 200000 = 381.14\ Mpa$$

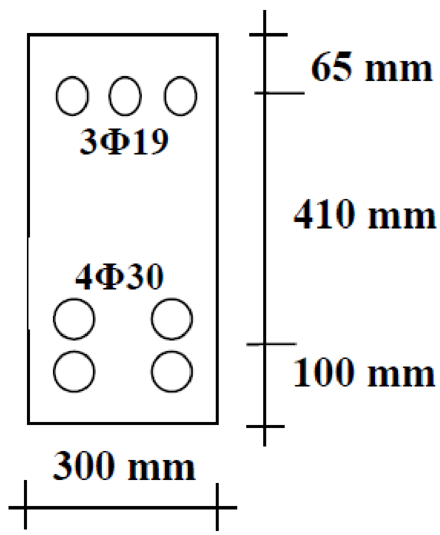
$$f_{\acute{s}} < f_y$$

$$\therefore \boldsymbol{f_{\acute{s}} = 381.14\ Mpa}$$

$$A_{s2}\ (A_{s'}) = \frac{M_{u2}}{\emptyset\ f_{\acute{s}}(d - d')} = \frac{108.75\ x\ 10^6}{0.9\ x\ 381.14\ x\ (475 - 65)} = \boldsymbol{774\ mm^2}$$

For tension reinforcement $A_s = A_{s1}(A_s) + A_{s2}(A_{s'})$

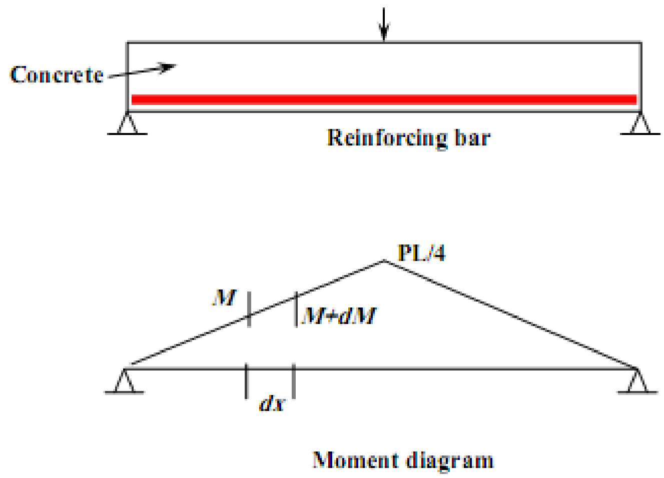
$$= 1\boldsymbol{931 + 774 = 2705\ mm^2}$$

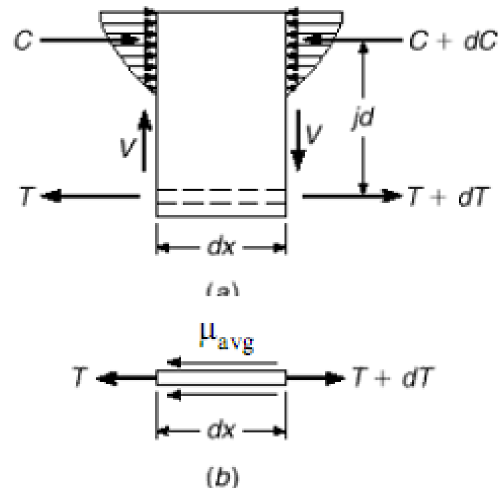


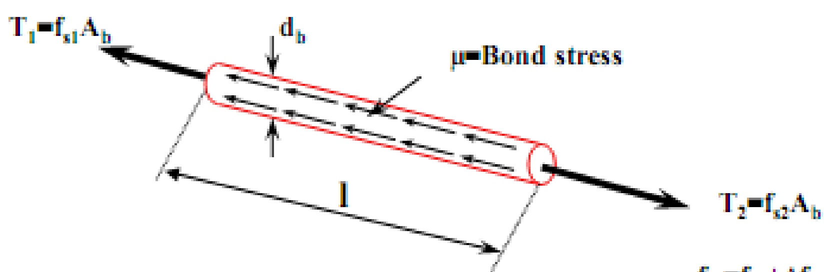
Bond, Anchorage and development length

Concept of Bond Stress:

Bond stresses are existent whenever the tensile stress or force in a reinforcing bar changes from point to point along the length of the bar in order to maintain equilibrium. Without bond stresses, the reinforcement will pull out of the concrete.







$$\sum F = 0$$

$$T_2 - T_1 = F_{bond}$$

If this equation is not true (bond force F_{bond} is not strong enough), the bar will pull out

$$A_{bar}\left(f_{s2} - f_{s1}\right) = \mu_{avg} A_{surface}$$

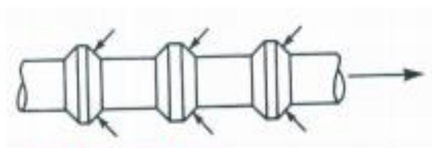
$$\frac{\pi d_b^2}{4}\left(f_{s2} - f_{s1}\right) = \mu_{avg}(\pi d_b)\, l$$

$$\Rightarrow \mu_{avg} = \frac{\left(f_{s2} - f_{s1}\right) d_b}{4l}$$

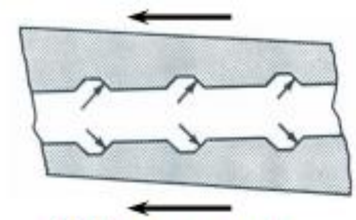
μ_{avg}= average bond stress

Mechanism of Bond Transfer

A smooth bar embedded in concrete develops bond by adhesion between concrete and reinforcement, and a small amount of friction. This is different in a deformed bar. Once adhesion is lost at high bar stress and some slight movement between the reinforcement and the concrete occurs, bond is then provided by friction and bearing on the deformations of the bar. At much higher bar stress, bearing on the deformations of the bar will be the only component contributing to bond strength.



(a) Forces on bar



(b) Forces on concrete

Concept of development Length

The concept of development length of a reinforcing bar which could be defined as that length of embedment necessary to develop the full tensile strength of the bar, controlled by either pullout or splitting.

ACI code 25.4.2.1: Development length ℓ_d for deformed bars and deformed wires in **tension** shall be the greater of (a) and (b):

- **a.** Length calculated in accordance with 25.4.2.3 or 25.4.2.4 using the applicable modification factors of 25.4.2.5
- **b.** 300 mm.

Table 25.4.2.3—Development length for deformed bars and deformed wires in tension

Spacing and cover	No. 19 and smaller bars and deformed wires	No. 22 and larger bars
Clear spacing of bars or wires being developed or lap spliced not less than d_b, clear cover at least d_b, and stirrups or ties throughout ℓ_d not less than the Code minimum or Clear spacing of bars or wires being developed or lap spliced at least $2d_b$ and clear cover at least d_b	$\left(\frac{f_y\psi_t\psi_e\psi_g}{2.1\lambda\sqrt{f'_c}}\right)d_b$	$\left(\frac{f_y\psi_t\psi_e\psi_g}{1.7\lambda\sqrt{f'_c}}\right)d_b$
Other cases	$\left(\frac{f_y\psi_t\psi_e\psi_g}{1.4\lambda\sqrt{f'_c}}\right)d_b$	$\left(\frac{f_y\psi_t\psi_e\psi_g}{1.1\lambda\sqrt{f'_c}}\right)d_b$

$$l_d = \frac{f_y}{1.1\,\lambda\,\sqrt{f_{c'}}}\,\frac{\psi_t\,\psi_e\psi_s\,\psi_g}{\left(\frac{c_b+K_{tr}}{d_b}\right)}\,d_b \qquad 25.4.2.4$$

In which the confinement term $\left(\frac{C_b+K_{tr}}{d_b}\right)$ shall not exceed 2.5, $\left(\frac{\boldsymbol{C_b+K_{tr}}}{\boldsymbol{d_b}}\right) \leq \mathbf{2.5}$ and,

$$K_{tr} = \frac{40\,A_{tr}}{sn}$$

The values of $\sqrt{\boldsymbol{f_{c'}}}$ used to calculate development length **shall not exceed 8.3 MPa**.

$$\sqrt{\boldsymbol{f_{c'}}} \leq \mathbf{8.3}$$

Where n is the number of bars or wires being developed or lap spliced along the plane of splitting. It shall be permitted to use K_{tr} = 0 as a design simplification even if transverse reinforcement is present or required.

K_{tr} = transverse reinforcement index.

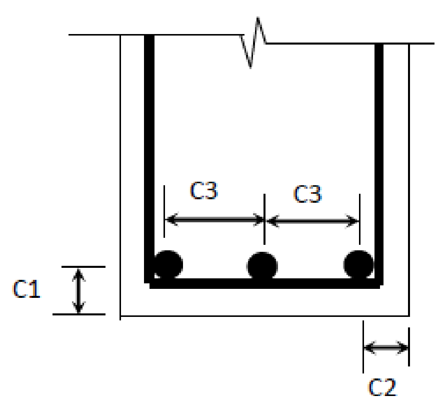
$\boldsymbol{A_{tr}}$: total cross-sectional area of transverse reinforcement that is within the spacing **s** and that crosses plane of splitting through the reinforcement being developed (mm^2)

s: Maximum spacing of transvers reinforcement within l_d center to center (mm)

c_b: Is a factor that represents the least of the **side cover**, the concrete cover to the bar or wire (in both cases measured to the **center** of the bar or wire), or one-half the center-to-center spacing of the bars or wires.

c_b is the lesser of

1. c_1
2. c_2
3. $\frac{c_3}{2}$.



For the calculation of ℓ_d, modification factors shall be in accordance with Table 25.4.2.5.

Table 25.4.2.5—Modification factors for development of deformed bars and deformed wires in tension

Modification factor	Condition	Value of factor
Lightweight λ	Lightweight concrete	0.75
	Normalweight concrete	1.0
Reinforcement grade ψ_g	Grade 280 or Grade 420	1.0
	Grade 550	1.15
	Grade 690	1.3
Epoxy[1] ψ_e	Epoxy-coated or zinc and epoxy dual-coated reinforcement with clear cover less than $3d_b$ or clear spacing less than $6d_b$	1.5
	Epoxy-coated or zinc and epoxy dual-coated reinforcement for all other conditions	1.2
	Uncoated or zinc-coated (galvanized) reinforcement	1.0
Size ψ_s	No. 22 and larger bars	1.0
	No. 19 and smaller bars and deformed wires	0.8
Casting position[1] ψ_t	More than 300 mm of fresh concrete placed below horizontal reinforcement	1.3
	Other	1.0

[1]The product $\psi_t \psi_e$ need not exceed 1.7.

According to ACI code 25.4.10: Reduction in development length l_d shall be permitted by the ratio of:

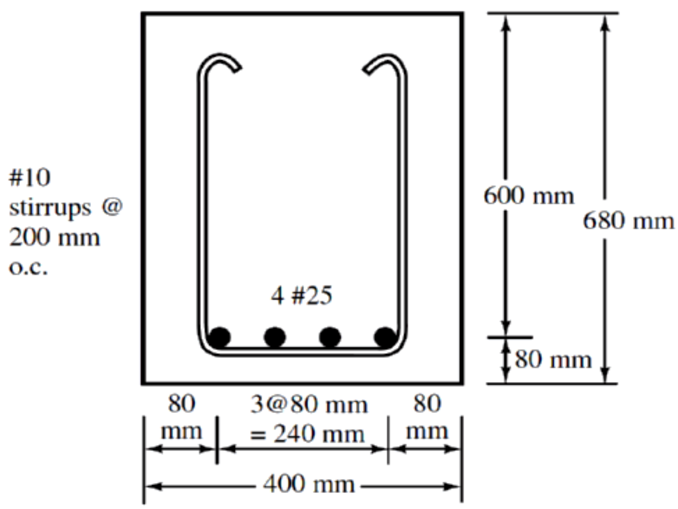
$$\frac{A_s\, required}{A_s\, provided}$$

When the provided tensile flexural reinforcement **exceeds** the required reinforcement, but with exceptions of **five cases**:

a. At noncontiguous supports.
b. At locations where anchorage or development for *fy* is required.
c. Where bars are required to be continuous.
d. For headed and mechanically anchored deformed reinforcement
e. In seismic-force-resisting systems in structures assigned to seismic design categories D, E, or F.
f. Anchorage of concrete piles and concrete filled pipe piles to pile caps in structures assigned to Seismic Design Categories C, D, E, or F.

Example: Determine the development length required for the epoxy coated bottom bars shown in figure.

1. Assuming $k_{tr} = 0$
2. Computing K_{tr} with the appropriate equation, $f_y = 420\ Mpa$ and $f_c' = 21\ Mpa$.



Solution:

$$\sqrt{f_{c'}} = 4.582\ Mpa\ < 8.3\ Mpa \quad \therefore OK.$$

From table 25.4.2.5:

Ψ_g = 1.0 (Grad 420)

Ψ_t = 1.0 (For bottom bars)

Ψ_e = 1.5 (for epoxy – coated bars with clear spacing < $6d_b$)

$\Psi_t\Psi_e$ = 1.0 x 1.5 = 1.5 < 1.7 OK.

Ψ_s = 1.0 (Bar size more than 22)

$\boldsymbol{\lambda}$ = 1.0 (Normal concrete)

cb : The **lesser** of:

1. Side cover of bar = 80 mm
2. Bottom cover of bar = 80 mm
3. ½ the center-to-center spacing of the bars = ½ (80) = **40 mm** (Control)

∴ c_b = 40 mm

1. Assuming k_{tr} = 0

$$\frac{c_b + K_{tr}}{d_b} = \frac{40 + 0}{25} = 1.6\ < 2.5 \quad \therefore OK.$$

$$l_d = \frac{f_y}{1.1\ \lambda \sqrt{f_{c'}}} \frac{\psi_t\ \psi_e \psi_s\ \psi_g}{(\frac{c_b + K_{tr}}{d_b})}\ d_b$$

$$l_d = \frac{420}{1.1\ x\ 1\ x \sqrt{21}} \frac{(1.0)\ x\ (1.5)\ x\ (1.0)\ x\ (1.0)}{1.6}\ x\ 25\ = \mathbf{1952.8}\ \boldsymbol{mm}$$

2. Computing K_{tr} with the appropriate equation:

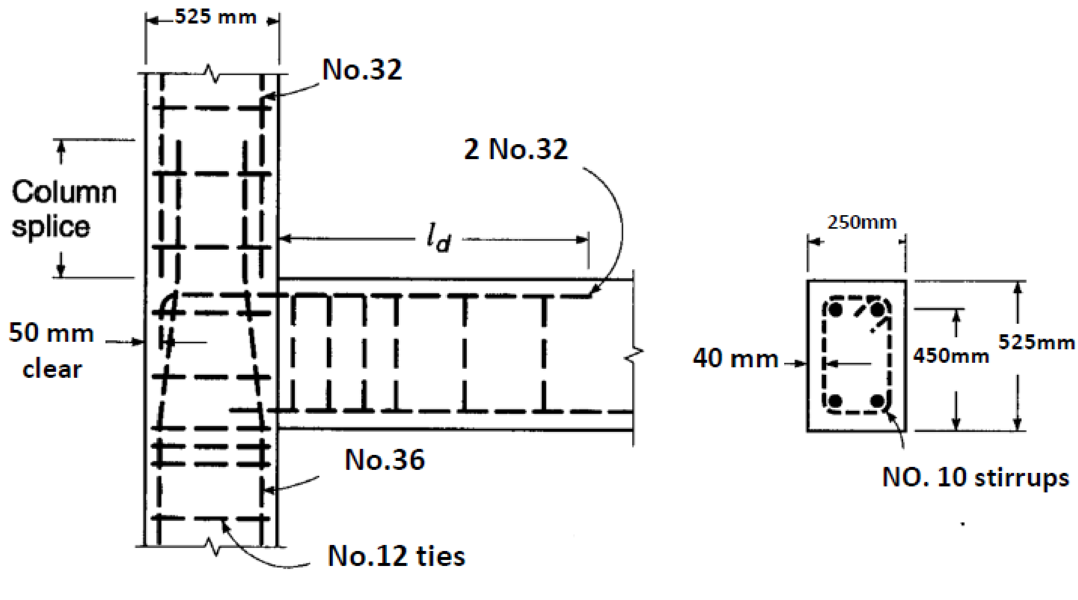
$$K_{tr} = \frac{40\ A_{tr}}{sn} = \frac{40\ x\ 2\ (79)}{4\ x\ 200} = 7.9\ mm$$

$$\frac{c_b + K_{tr}}{d_b} = \frac{40 + 7.9}{25} = 1.916\ < 2.5 \quad \therefore OK.$$

$$l_d = \frac{f_y}{1.1\ \lambda \sqrt{f_{c'}}} \frac{\psi_t\ \psi_e \psi_s\ \psi_g}{(\frac{c_b + K_{tr}}{d_b})}\ d_b$$

$$l_d = \frac{420}{1.1\ x\ 1\ x \sqrt{21}} \frac{(1.0)\ x\ (1.5)\ x\ (1.0)\ x\ (1.0)}{1.916}\ x\ 25\ = \mathbf{1630.732}\ \boldsymbol{mm}$$

Example: The figure below shows a beam-column joint, the negative steel required at the end of the beam is 1300 mm^2, two No. 32 mm bars are used providing 1608 mm^2. The design will include No. 10 stirrups with s =125 mm, f_c' =30 MPa, f_y = 420 MPa. Find the minimum distance $\boldsymbol{l_d}$ at which the negative bars can be cut off.



(a) (b)

Solution:

$$l_d = \frac{f_y}{1.1\,\lambda\sqrt{f_{c'}}}\;\frac{\psi_t\,\psi_e\psi_s\,\psi_g}{\left(\frac{c_b+K_{tr}}{d_b}\right)}\;d_b$$

$\sqrt{f_{c'}}$ = 5.477 ≤ 8.3 Mpa OK.

From table 25.4.2.5:

Ψ_g = 1.0 (Grad 420)

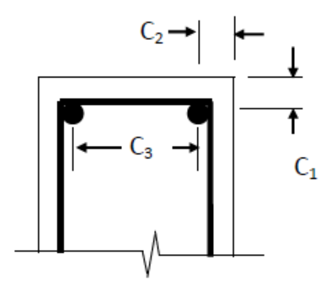
Ψ_t = 1.3 (more than 300 mm concrete below reinforcement)

Ψ_e = 1.0 (Uncoated bars)

$\Psi_t\Psi_e$ = 1.0 x 1.3 = 1.3 < 1.7 OK.

Ψ_s = 1.0 (Bar size more than 22)

$\boldsymbol{\lambda}$ = 1.0 (Normal concrete)



cb : The **lesser** of:

1. C1 = 525 – 450 = 75 mm
2. C2 = 40 + 10 + 32/2 = 66 mm

3. C3 = ½ the center-to-center spacing of the bars = **59 mm** (Control)

$$K_{tr} = \frac{40\ A_{tr}}{sn} = \frac{40\ x\ 2\ (79)}{2\ x\ 125} = 25.28\ mm$$

$$\frac{c_b +\ K_{tr}}{d_b} = \frac{59 + 25.28}{32} = 2.633 > 2.5$$

$$\therefore \textbf{ Take } \frac{c_b + K_{tr}}{d_b} = 2.5$$

$$l_d = \frac{420}{1.1\ x\ 1\ x\ \sqrt{30}} \frac{1.3\ x\ 1.0\ x\ 1\ x\ 1}{2.5}\ x\ 32 = 1160\ mm$$

l_d can be reduced by the ratio ($\frac{As_{req}}{As_{pro.}}$)

$$\therefore\ l_d = 1160\ x\ \frac{1300}{1608} = \mathbf{937.81\ mm}$$

Development of Deformed Bars in Compression:

The development length l_{dc} in compression is smaller than in tension because of two reasons. First, there are no tensile cracks present to encourage splitting, and second, there is some bearing of the ends of the bars on concrete, which also helps develop the load.

ACI 318 -19 (25.4.9.1) Development length ℓ_{dc} for deformed bars and deformed wires in compression shall be the **greater** of **(a)** and **(b)**:

a. Length calculated in accordance with 25.4.9.2

b. 200 mm.

ACI 318 -19 (25.4.9.2) ℓ_{dc} shall be the **greater** of (a) and (b), using the modification factors of 25.4.9.3:

$$\boldsymbol{a}.\, l_{dc} = \left(\frac{0.24 f_y \Psi_r}{\lambda\sqrt{f_{c'}}}\right) d_b$$

$$\boldsymbol{b}.\, l_{dc} = 0.043\, f_y \Psi_r d_b$$

ACI 318 -19 (25.4.9.3) For the calculation of ℓ_{dc}, modification factors shall be in accordance with Table 25.4.9.3, except ψ_r shall be permitted to be taken as 1.0.

Table 25.4.9.3—Modification factors for deformed bars and wires in compression

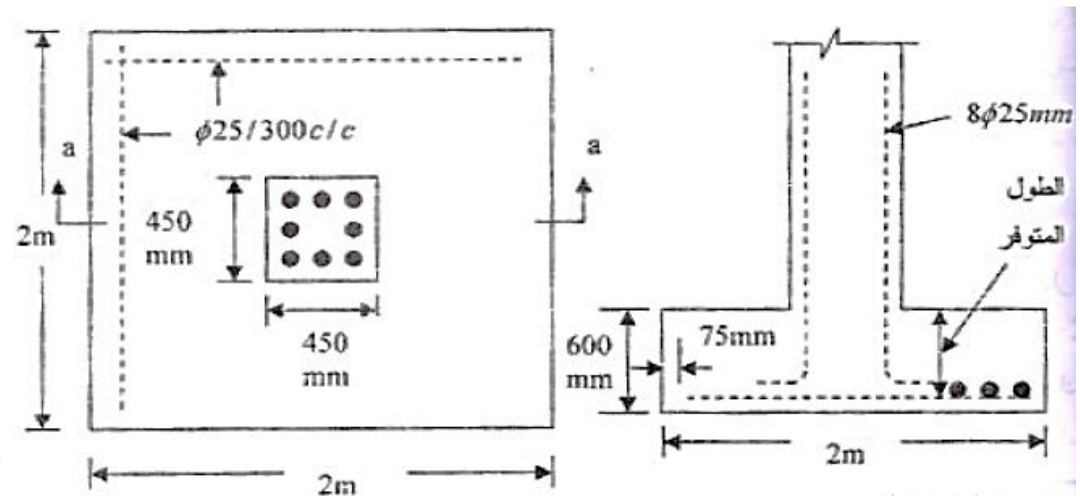
Modification factor	Condition	Value of factor
Lightweight λ	Lightweight concrete	0.75
	Normalweight concrete	1.0
Confining reinforcement ψ_r	Reinforcement enclosed within (1), (2), (3), or (4): (1) a spiral (2) a circular continuously wound tie with $d_b \geq 6$ mm and pitch 100 mm (3) No. 13 bar or MD130 wire ties in accordance with 25.7.2 spaced ≤ 100 mm on center (4) hoops in accordance with 25.7.4 spaced ≤ 100 mm on center	0.75
	Other	1.0

Reduction in development length l_{dc} shall be permitted by the ratio of:

$$\frac{A_s\, required}{A_s\, provided}$$

When the **provided tensile** flexural reinforcement exceeds the **required reinforcement**.

Example: Find the development length for dowel bars for the separate footing shown and check the provided length inside the footing for the bars, use $f_c' = 30\ MPa$ and $f_y = 400\ MPa$.



$$\boldsymbol{a}.\, l_{dc} = \left(\frac{0.24 f_y \Psi_r}{\lambda\sqrt{f_{c'}}}\right) d_b = \left(\frac{0.24\ x\ 400\ x\ 1}{1\ x\ \sqrt{30}}\right) 25 = \mathbf{438.18\ mm}$$

$$\boldsymbol{b}.\, l_{dc} = 0.043\ f_y \Psi_r d_b = 0.043\ x\ 400\ x\ 1\ x\ 25 = 430\ mm$$

$$l_{dc} = 438.18\ mm > 200\ mm$$

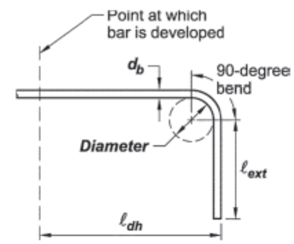
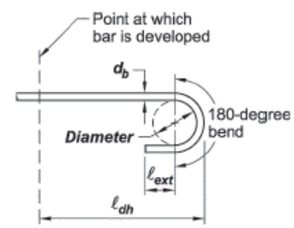
∴ OK.

$L_{provide}$ = 600 – 75 – 25 – 25 = 475 mm > 438.18 mm ∴ OK

Development of standard hooks in tension:

When sufficient space is not available to anchor the tension bars to the required development lengths, hooks maybe used. The figure below shows the standard 90° and 180° hooks as specified in the **ACI 318-19 (25.3.1)**.

Table 25.3.1—Standard hook geometry for development of deformed bars in tension

Type of standard hook	Bar size	Minimum inside bend diameter, mm	Straight extension[1] ℓ_{ext}, mm	Type of standard hook
90-degree hook	No. 10 through No. 25	$6d_b$	$12d_b$	
	No. 29 through No. 36	$8d_b$		
	No. 43 through No. 57	$10d_b$		
180-degree hook	No. 10 through No. 25	$6d_b$	Greater of $4d_b$ and 65 mm	
	No. 29 through No. 36	$8d_b$		
	No. 43 through No. 57	$10d_b$		

[1]A standard hook for deformed bars in tension includes the specific inside bend diameter and straight extension length. It shall be permitted to use a longer straight extension at the end of a hook. A longer extension shall not be considered to increase the anchorage capacity of the hook.

ACI 318-19 (25.4.3.1) Development length ℓ_{dh} for deformed bars in tension terminating in a standard hook shall be the greater of (a) through (c):

$$\textbf{a.}\left(\frac{f_y \Psi_e \Psi_r \Psi_o \Psi_c}{23\,\lambda\sqrt{f_{c'}}}\right) {d_b}^{1.5}$$

$$\textbf{b.}\, 8d_b$$

$$\textbf{c.}\, 150\ mm$$

ACI 318-19 (25.4.3.2) For the calculation of ℓ_{dh}, modification factors ψ_e, ψ_r, ψ_o, ψ_c, and **λ** shall be in accordance with **Table 25.4.3.2.** At discontinuous ends of members, 25.4.3.4 shall apply.

Table 25.4.3.2—Modification factors for development of hooked bars in tension

Modification factor	Condition	Value of factor
Lightweight λ	Lightweight concrete	0.75
	Normalweight concrete	1.0
Epoxy ψ_e	Epoxy-coated or zinc and epoxy dual-coated reinforcement	1.2
	Uncoated or zinc-coated (galvanized) reinforcement	1.0
Confining reinforcement ψ_r	For No. 36 and smaller bars with $A_{th} \geq 0.4A_{hs}$ or $s^{[1]} \geq 6d_b^{[2]}$	1.0
	Other	1.6
Location ψ_o	For No. 36 and smaller diameter hooked bars: (1) Terminating inside column core with side cover normal to plane of hook ≥ 65 mm, or (2) With side cover normal to plane of hook $\geq 6d_b$	1.0
	Other	1.25
Concrete strength ψ_c	For $f_c' < 42$ MPa	$f_c'/105 + 0.6$
	For $f_c' \geq 42$ MPa	1.0

[1]s is minimum center-to-center spacing of hooked bars.

[2]d_b is nominal diameter of hooked bar.

ACI 318 - 19 (25.4.3.3) The total cross-sectional area of ties or stirrups confining hooked bars A_{th} shall consist of **(a)** or **(b)**:

a. Ties or stirrups that enclose the hook and satisfy 25.3.2.

b. Other reinforcement enclosing the hook, that extends at least **$0.75\ell_{dh}$** from the enclosed hook in the direction of the bar in tension, and is in accordance with (1) or (2). For members with confining reinforcement that is both parallel and perpendicular to ℓ_{dh}, it shall be permitted to use the value of A_{th} based on (1) or (2) that results in the lower value of ℓ_{dh}.

1. Two or more ties or stirrups shall be provided **parallel** to ℓ_{dh} enclosing the hooks, evenly distributed with a center-to-center spacing not exceeding **$8d_b$**, and within **$15d_b$** of the centerline of the straight portion of the hooked bars, where d_b is the nominal diameter of the hooked bar.

2. Two or more ties or stirrups shall be provided **perpendicular** to ℓ_{dh}, enclosing the hooked bars, and evenly distributed along ℓ_{dh} with a center-to-center spacing not exceeding $8d_b$, where d_b is the nominal diameter of the hooked bar.

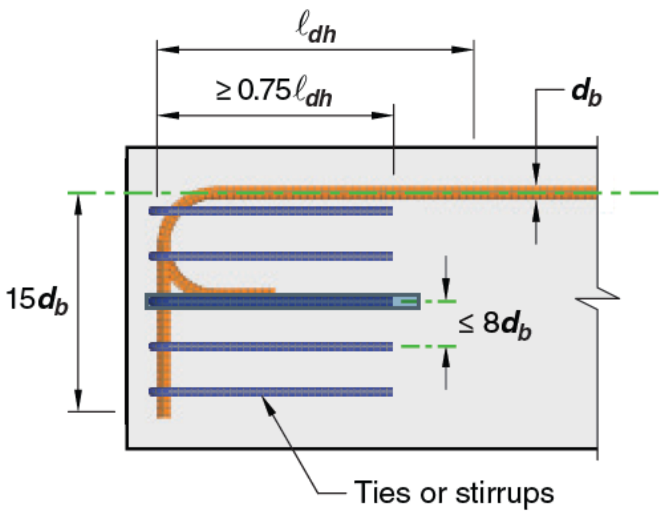


Fig. R25.4.3.3a—Confining reinforcement placed parallel to the bar being developed that contributes to anchorage strength of both 90- and 180-degree hooked bars.

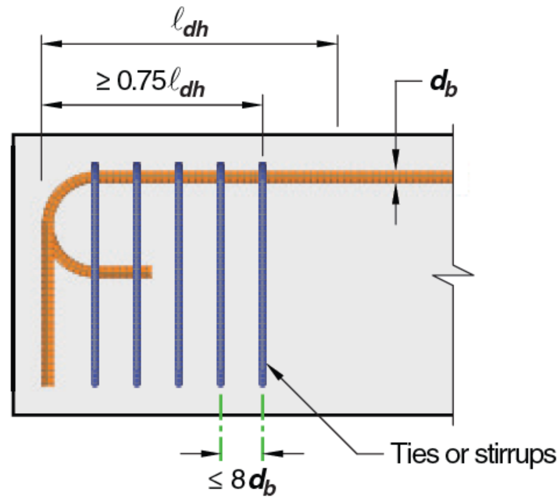
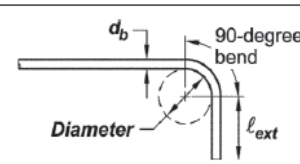
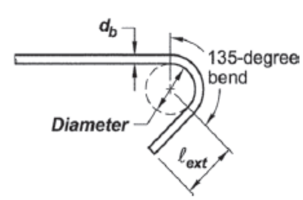
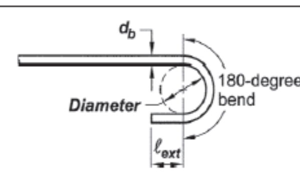


Fig. R25.4.3.3b—Confining reinforcement placed perpendicular to the bar being developed, spaced along the development length ℓ_{dh}, that contributes to anchorage strength of both 90- and 180-degree hooked bars.

ACI 318 – 19 (25.3.2) Minimum inside bend diameters for bars used as transverse reinforcement and standard hooks for bars used to anchor stirrups, ties, hoops, and spirals shall conform to Table 25.3.2. Standard hooks shall enclose longitudinal reinforcement.

Table 25.3.2—Minimum inside bend diameters and standard hook geometry for stirrups, ties, and hoops

Type of standard hook	Bar size	Minimum inside bend diameter, mm	Straight extension[1] ℓ_{ext}, mm	Type of standard hook
90-degree hook	No. 10 through No. 16	$4d_b$	Greater of $6d_b$ and 75 mm	
	No. 19 through No. 25	$6d_b$	$12d_b$	
135-degree hook	No. 10 through No. 16	$4d_b$	Greater of $6d_b$ and 75 mm	
	No. 19 through No. 25	$6d_b$		
180-degree hook	No. 10 through No. 16	$4d_b$	Greater of $4d_b$ and 65 mm	
	No. 19 through No. 25	$6d_b$		

[1]A standard hook for stirrups, ties, and hoops includes the specific inside bend diameter and straight extension length. It shall be permitted to use a longer straight extension at the end of a hook. A longer extension shall not be considered to increase the anchorage capacity of the hook.

ACI 318 – 19 (25.4.1.2) Hooks and heads **shall not** be used to develop bars in **compression.**

ACI 318 – 19 (25.4.3.4) For bars being developed by a standard hook at discontinuous ends of members with both side cover and top (or bottom) cover to hook less than 65 mm, (a) and (b) shall be satisfied:

a. The hook shall be enclosed along ℓ_{dh} within ties or stirrups perpendicular to ℓ_{dh} at $s \leq 3d_b$

b. The first tie or stirrup shall enclose the bent portion of the hook within $2d_b$ of the outside of the bend where d_b is the nominal diameter of the hooked bar.

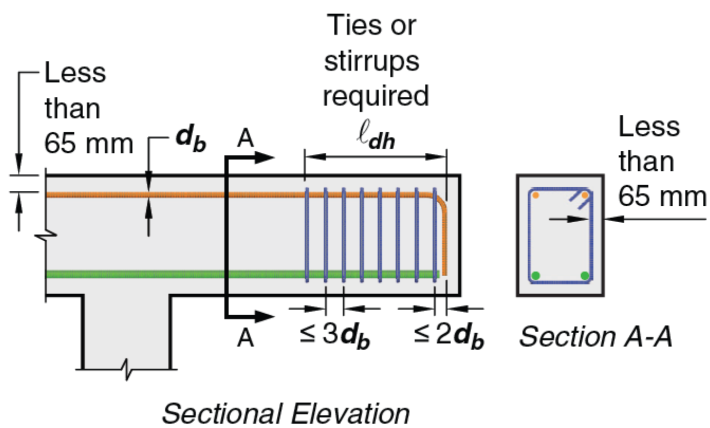
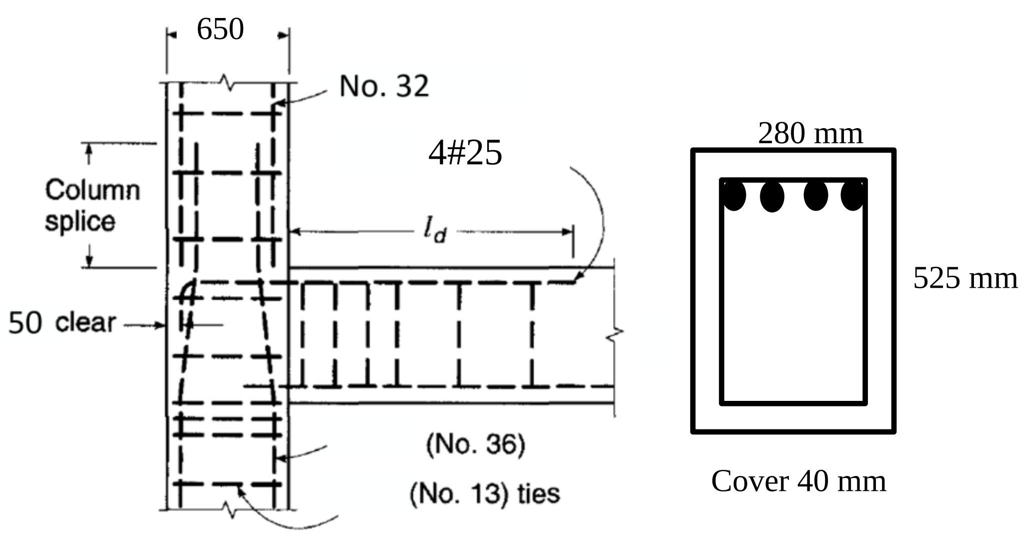


Fig. R25.4.3.4—Concrete cover according to 25.4.3.4.

Example: Referring to the beam-column joint of that is represented below for convenience, the **No. 25 negative** bars are to be extended into the column and terminated in a **standard 90° hook**, keeping 50 mm clear to the outside face of the column. The column width in the direction of beam width is 500 mm. **Find** the minimum length of embedment of the **hook past the column face**, and specify the hook details. A_s required is 1870 mm^2, assume that **normal weight concrete** is to be used, with $f_c' = 28\ MPa$, and $f_y = 420\ MPa$.



Solution:

$$\textbf{A.}\ \boldsymbol{l_{dh}} = \left(\frac{f_y \Psi_e\, \Psi_r\, \Psi_o \Psi_c}{23\, \lambda \sqrt{f_{c'}}}\right) {d_b}^{1.5}$$

$$\Psi_e = 1.0\ (Uncoated)$$
$$\Psi_r = 1.6$$
$$\Psi_o = 1.0$$
$$\Psi_c = 0.87$$
$$\lambda = 1.0\ normal\ concrete$$

$$\boldsymbol{l_{dh}} = \left(\frac{420\ x\ 1.0\ x\ 1.6\ x\ 1.0\ x\ 0.87}{23\ x\ 1\ x\ \sqrt{28}}\right) 25^{1.5} = 600.5\ mm$$

∵ A_s provide > A_s required

∴ Reduction in development length l_{dc} shall be permitted by the ratio of:

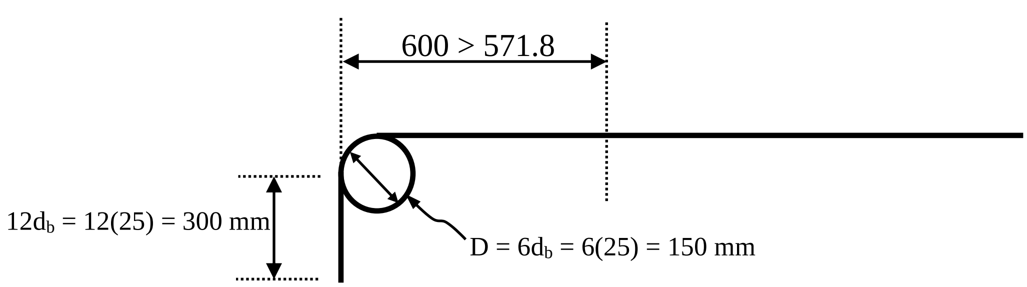
$$\frac{\boldsymbol{A_s\ required}}{\boldsymbol{A_s\ provided}}$$

$$l_{dh} = l_{dh}\ \frac{A_s\ required}{A_s provided} = 600.5\ x\ \frac{1870}{1964} = \boldsymbol{571.8\ mm}$$

$$\textbf{B.}\ \ l_{dh} = 8d_b = 8\ x\ 25 = 200\ mm$$

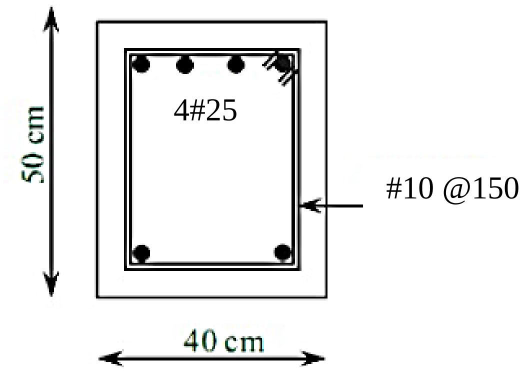
$$\boldsymbol{c}.\ 150\ mm$$

$$\therefore \boldsymbol{l_{dh}} = \boldsymbol{571.8\ mm}$$



Example: Determine the development length or anchorage required for the uncoated top bars of the beam shown in the figure. The beam frames into an exterior 800 mm × 660 mm column (the bars extend parallel to the 800 mm side), $f_c' = 28\ MPa$ and $f_y = 420\ MPa$. Show the details if:

a. Using straight bar.
b. Using 180 - degree hook.
c. Using 90 - degree hook.



Solution:

a. Using straight bar.

$$l_d = \frac{f_y}{1.1\,\lambda\sqrt{f_{c'}}}\,\frac{\psi_t\,\psi_e\psi_s\,\psi_g}{(\frac{c_b + K_{tr}}{d_b})}\,d_b$$

$$\sqrt{f_{c'}} = 5.29\ Mpa\ < 8.3\ Mpa \quad \therefore OK.$$

From table 25.4.2.5:

Ψ_g = 1.0 (Grad 420)
Ψ_t = 1.3 (For more than 300 mm)
Ψ_e = 1.0 (uncoated)
$\Psi_t\Psi_e$ = 1.0 x 1.3 = 1.3 < 1.7 OK.
Ψ_s = 1.0 (Bar size more than 22)
$\boldsymbol{\lambda}$ = 1.0 (Normal concrete)

cb : The **lesser** of:

4. Side cover of bar = 66 mm
5. Bottom cover of bar = 66 mm
6. ½ the center-to-center spacing of the bars = ½ (89.3) = **44.7 mm** (Control)

∴ c_b = 44.7 mm

$$K_{tr} = \frac{40\ A_{tr}}{sn} = \frac{40\ x\ 2\ (113)}{4\ x\ 150} = 15.1mm$$

$$\frac{c_b +\ K_{tr}}{d_b} = \frac{44.7 + 15.1}{28} = 2.135 < 2.5 \quad \therefore OK.$$

$$l_d = \frac{f_y}{1.1\,\lambda\sqrt{f_{c'}}}\,\frac{\psi_t\,\psi_e\psi_s\,\psi_g}{(\frac{c_b + K_{tr}}{d_b})}\,d_b$$

$$l_d = \frac{420}{1.1\,x\,1\,x\,\sqrt{28}}\,\frac{1.3x\,1.0\,x\,1\,x\,1}{2.135}\,x\,28 = \mathbf{1230.215\ \ mm} > 300\,mm\quad \therefore OK.$$

Available length = 800 – 40 = 760 mm

$l_d > 760$ Not good

b. Using 180 - degree hook.

$$\boldsymbol{l_{dh}} = \left(\frac{f_y\Psi_e\,\Psi_r\,\Psi_o\Psi_c}{23\,\lambda\sqrt{f_{c'}}}\right)d_b{}^{1.5}\ ,\ \boldsymbol{l_{dh}} = 8d_b\ ,\ \boldsymbol{l_{dh}} = 150\,mm$$

$$\Psi_e = 1.0\ (uncoated)$$
$$\Psi_r = 1.6$$
$$\Psi_o = 1.0$$
$$\Psi_c = 0.87$$
$$\lambda = 1.0\ normal\ concrete$$

a. $\boldsymbol{l_{dh}} = \left(\frac{420\,x\,1.0\,x\,1.6\,x\,1.0\,x\,0.87}{23\,x\,1\,x\,\sqrt{28}}\right)28^{1.5} = \mathbf{711.7\ mm}$

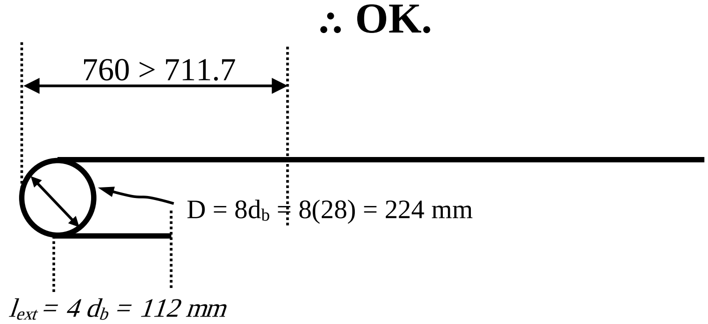
b. $\boldsymbol{l_{dh}} = 8d_b = 8\,x\,28 = 224\,mm$

c. $\boldsymbol{l_{dh}} = 150\,mm$

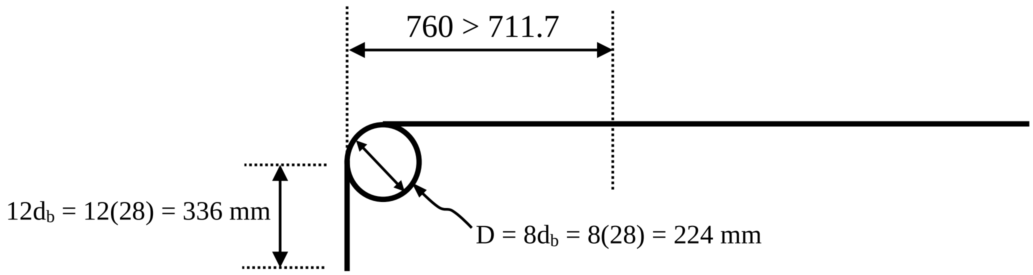
$$\therefore \boldsymbol{l_{dh}} = \mathbf{711.7\ mm}$$

$$\boldsymbol{l_{dh}} < \mathbf{760\ mm\ avialble\ length}$$

∴ OK.



C. Using 90 - degree hook.



Splices of reinforcement

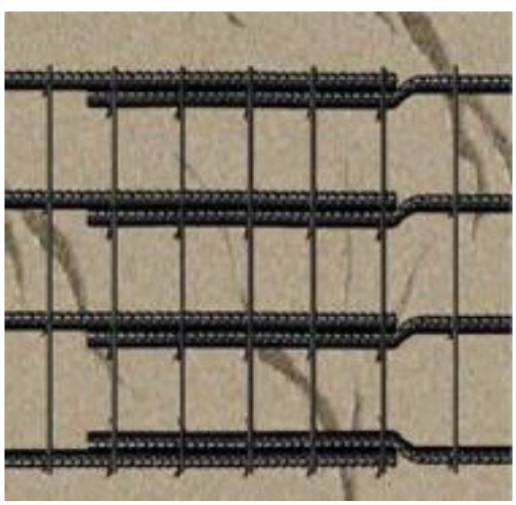
Need for Splices:

In general, reinforcing bars are stocked by suppliers in lengths of 12m. For this reason, and because it is often more convenient to work with shorter bar lengths, it is frequently necessary to splice bars in the field.

Splice Types:

Rebars are spliced to each other by:

- Lap Splices: In this type, rebars are usually made simply by lapping the bars a sufficient distance to transfer stress by bond from one bar to the other. The lapped bars are usually placed in contact and lightly wired so that they stay in position as the concrete is placed.



- Mechanical Splices: Sample of mechanical splice is presented

- Welding Splice: Splice with welding splice, with fillet weld,

ACI318 – 19 (25.5.1.1): Lap splices shall not be permitted for bars larger than No. 36, except as provided in 25.5.5.3 ***(***compression lap splices of No. 43 and No. 57 bars with smaller bars***)***. This because of lack of adequate experimental data on lap splices for larger diameters.

ACI318 – 19 (25.5.1.4): Reduction of development length in accordance with 25.4.10**.**1 **is not permitted** in calculating lap splice lengths

Splice of Tension Reinforcement

ACI318 – 19 (25.5.2.1): Tension lap splice length ℓ_{st} for deformed bars and deformed wires in tension shall be in accordance with Table 25.5.2.1, where ℓ_d shall be in accordance with 25.4.2.1(a).

Table 25.5.2.1—Lap splice lengths of deformed bars and deformed wires in tension

$A_{s,provided}/A_{s,required}$[1] over length of splice	Maximum percent of A_s spliced within required lap length	Splice type	ℓ_{st}	
≥ 2.0	50	Class A	Greater of:	$1.0\ell_d$ and 300 mm
	100	Class B	Greater of:	$1.3\ell_d$ and 300 mm
< 2.0	All cases	Class B		

[1]Ratio of area of reinforcement provided to area of reinforcement required by analysis at splice location.

ACI318 – 19 (25.5.2.2): If bars of different size are lap spliced in tension, ℓ_{st} shall be the greater of ℓ_d of the larger bar and ℓ of the smaller bar.

For calculating ℓ_d for staggered splices, the clear spacing is taken as the minimum distance between adjacent splices, as illustrated in Figure below.

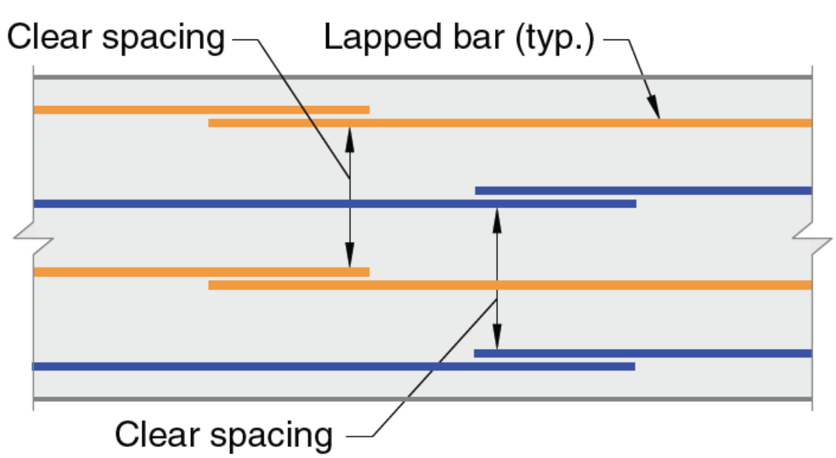
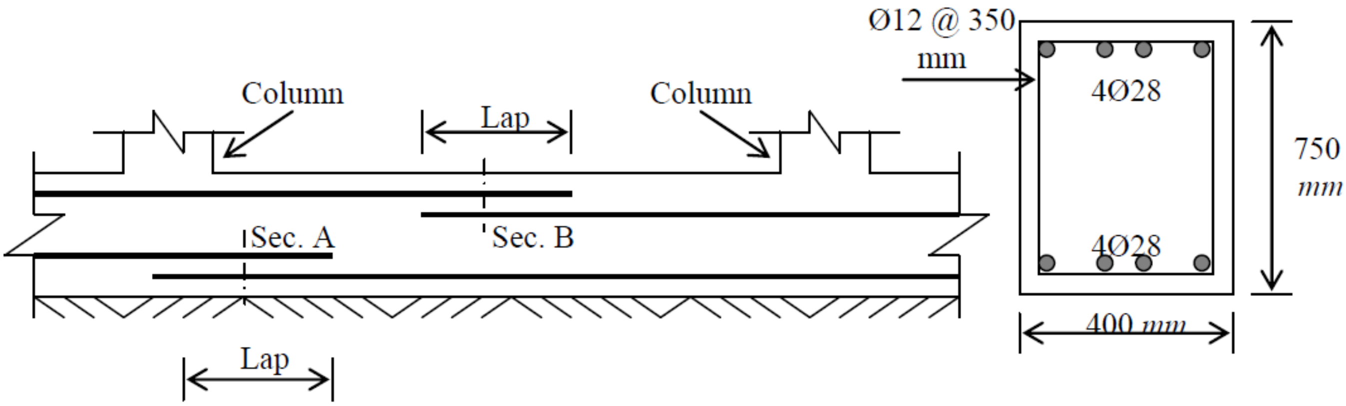


***Fig. R25.5.2.1**—Clear spacing of lap-spliced bars for determination of ℓ_d for staggered splices.*

Example: Design the tension lap splice for the continuous grade beam shown below using the general equation. From beam analysis, the required reinforcement area at section A is 1910 mm^2 and the required reinforcement area at section B is 650 mm^2, fc' = 28 = *MPa* and f_y = 420 *Mpa. Cover = 75 mm.*



Solution:

1- Lap splice of reinforcement at section A:

a- Assuming all bars are spliced at the same location:

$$l_d = \frac{f_y}{1.1\,\lambda\,\sqrt{f_{c'}}}\,\frac{\psi_t\,\psi_e\psi_s\,\psi_g}{(\frac{c_b + K_{tr}}{d_b})}\,d_b$$

$\sqrt{f_{c'}}$ = 5.29 ≤ 8.3 Mpa OK.

From table 25.4.2.5:

Ψ_g = 1.0 (Grad 420)

Ψ_t = 1.0(bottom bars)

Ψ_e = 1.0 (Uncoated bars)

$\Psi_t\Psi_e$ < 1.7 OK.

Ψ_s = 1.0 (Bar size more than 22)

$\boldsymbol{\lambda}$ = 1.0 (Nor,al concrete)

cb : The **lesser** of:

4. C1 = 75 + 12+ 14 = 101 mm
5. C2 = 101 mm
6. C3 = ½ the center-to-center spacing of the bars = ½(66) = **33 mm** (Control)

$$K_{tr} = \frac{40\,A_{tr}}{sn} = \frac{40\ x\ 2\ (113)}{4\ x\ 350} = 6.45\ mm$$

$$\frac{c_b + K_{tr}}{d_b} = \frac{33 + 6.45}{28} = 1.41 < 2.5$$

∴ OK.

$$l_d = \frac{420}{1.1\ x\ 1\ x\ \sqrt{28}}\ \frac{1.0\ x\ 1.0\ x\ 1.0\ x\ 1.0}{1.41}\ x\ 28 = 1433\ mm$$

$$\frac{As_{pro.}}{As_{req.}} = \frac{2463}{1910} = 1.29\ < 2.0$$

∴ Class B splice must be used

∴ splice length = 1.3l_d or 300 mm

Splice = 1.3 x 1433 = 1862.9 mm > 300 mm OK.

∴ The required splice length = **1862.9 mm**

b- Assuming alternate lap spliced bars are staggered by l_d (As spliced = 50 %).

$$l_d = \frac{f_y}{1.1\ \lambda \sqrt{f_{c'}}}\ \frac{\psi_t\ \psi_e \psi_s\ \psi_g}{(\frac{c_b + K_{tr}}{d_b})}\ d_b$$

$\sqrt{f_{c'}}$ = 5.29 ≤ 8.3 Mpa OK.

From table 25.4.2.5:

Ψ_g = 1.0 (Grad 420)

Ψ_t = 1.0(bottom bars)

Ψ_e = 1.0 (Uncoated bars)

$\Psi_t \Psi_e$ < 1.7 OK.

Ψ_s = 1.0 (Bar size more than 22)

$\boldsymbol{\lambda}$ = 1.0 (Nor,al concrete) cb : The **lesser** of:

1. C1 = 75 + 12+ 14 = 101 mm
2. C2 = 101 mm
3. C3 = ½ the center-to-center spacing of the bars = ½ (132) = **66 mm** (Control)

$$K_{tr} = \frac{40\ A_{tr}}{sn} = \frac{40\ x\ 2\ (113)}{2\ x\ 350} = 12.91\ mm$$

$$\frac{c_b +\ K_{tr}}{d_b} = \frac{66 + 12.91}{28} = 2.818 > 2.5$$

$$\therefore\ take\ \frac{c_b + K_{tr}}{d_b} = 2.5$$

$$l_d = \frac{420}{1.1\ x\ 1\ x\ \sqrt{28}} \frac{1.0\ x\ 1.0\ x\ 1.0\ x\ 1.0}{2.5}\ x\ 28 = 808\ mm$$

$$\frac{As_{pro}}{As_{req.}} = \frac{2463}{1910} = 1.29\ < 2.0$$

∴ Class B splice must be used

∴ Splice length = 1.3l_d or 300 mm

Splice = 1.3 x 808 = 1050.6 mm > 300 mm OK.

∴ The required splice length = **1050.4 mm**

∴ Use 1050.4 mm lap splice at section **A** and stagger alternate lap splices.

H.W.

2- Lap splice of reinforcement at section **B**:

Lap splice lengths of deformed bars in compression

ACI318 – 19 (25.5.5.1): Compression lap splice length ℓ_{sc} of No. 36 or smaller deformed bars in compression shall be calculated in accordance with (a), (b), or (c):

a. For $f_y \leq 420$ *MPa*: ℓ_{sc} is the longer of ($0.071\ f_y\ d_b$) and 300 mm.

b. For 420 *MPa* < $f_y \leq 550$ *MPa*: ℓ_{sc} is the longer of ($0.13f_y - 24$) d_b and 300 mm.

c. For f_y **> 550 *MPa***, ℓ_{sc} is the longer of (**$0.13f_y - 24$**) d_b and ℓ_{st} calculated in accordance with 25.5.2.1.

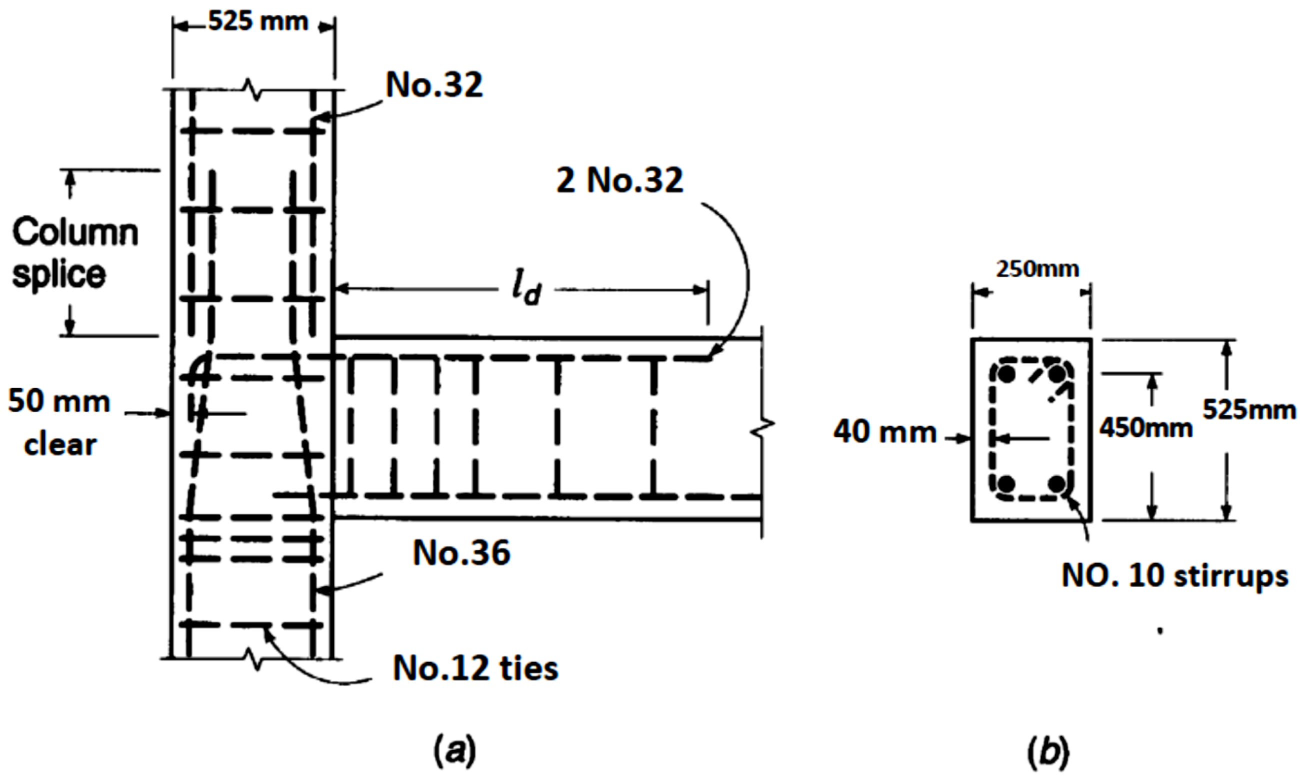
For $fc' < 21$ *MPa*, the length of lap shall be increased by one-third.

ACI318 – 19 (25.5.5.2): Compression lap splices shall not be used for bars larger than No. 36, except as permitted in 25.5.5.3.

ACI318 – 19 (25.5.5.3): Compression lap splices of No. 43 or No. 57 bars to No. 36 or smaller bars shall be permitted and shall be in accordance with 25.5.5.4.

ACI318 – 19 (25.5.5.4): Where bars of different size are lap spliced in compression, ℓ_{sc} shall be the longer of ℓ_{dc} of larger bar calculated in accordance with 25.4.9.1 and ℓ_{sc} of smaller bar calculated in accordance with 25.5.5.1 as appropriate.

Example: In the figure below, 4 No.36mm column bars for the floor below are to be lap spliced with 4 No. 32mm column bars from above. The transverse reinforcement consists of No.12mm ties at 200mm spacing. All vertical bars may be assumed to be fully stressed. Calculate the required splice length of No. 32mm bars $f_c' = 30$ *MPa* and $f_y = 400$ *MPa*.



Solution:

The length of the splice must be larger of the development length of No. 36 bars and the splice length of No.32 bars. For No. 36 bars, the development length is the greater of:

$$\boldsymbol{a}.\, l_{dc} = \left(\frac{0.24 f_y \Psi_r}{\lambda \sqrt{f_{c'}}}\right) d_b = \left(\frac{0.24\ x\ 400\ x\ 1}{1\ x\ \sqrt{30}}\right) 36 = \boldsymbol{631\ mm}$$

$$\boldsymbol{b}.\, l_{dc} = 0.043\ f_y \Psi_r d_b = 0.043\ x\ 400\ x\ 1\ x\ 36 = 619.2\ m$$

$$\boldsymbol{c}.\, l_{dc} = 200\ mm$$

$$\therefore \boldsymbol{l_{dc}} = \boldsymbol{631\ mm}$$

Calculate splice length: $f_y = 400\ Mpa\ < 420\ Mpa$

$\therefore$ splice length for No 32 bars $\boldsymbol{l_{sc}} = 0.071\ f_y\ d_b = 0.071\ x\ 400\ x\ 32 = 908.8\ mm > 300\ mm$ OK.

The required splice length = 908.8 mm.

Development of Flexural Reinforcement – General

ACI318 – 19 (9.7.3.2): Critical locations for development of reinforcement are points of maximum stress and points along the span where bent or terminated tension reinforcement is no longer required to resist flexure.

ACI318 – 19 (9.7.3.3): Reinforcement shall extend beyond the point at which it is no longer required to resist flexure for a distance equal to the greater of $\boldsymbol{d}$ and $\boldsymbol{12d_b}$, except at supports of simply-supported spans and at free ends of cantilevers.

ACI318 – 19 (9.7.3.4): Continuing flexural tension reinforcement shall have an embedment length at least $\boldsymbol{\ell_d}$ beyond the point where bent or terminated tension reinforcement is no longer required to resist flexure.

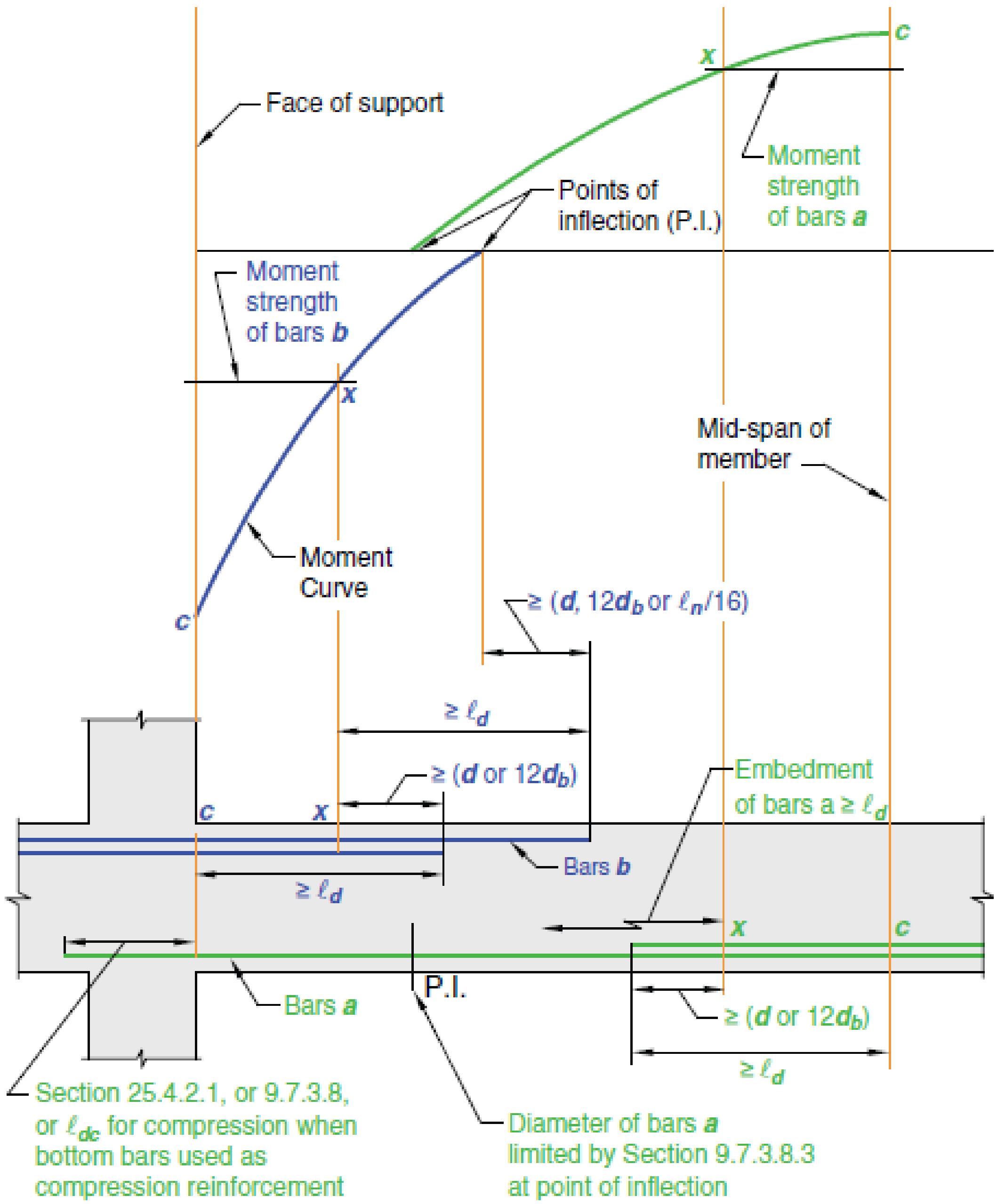
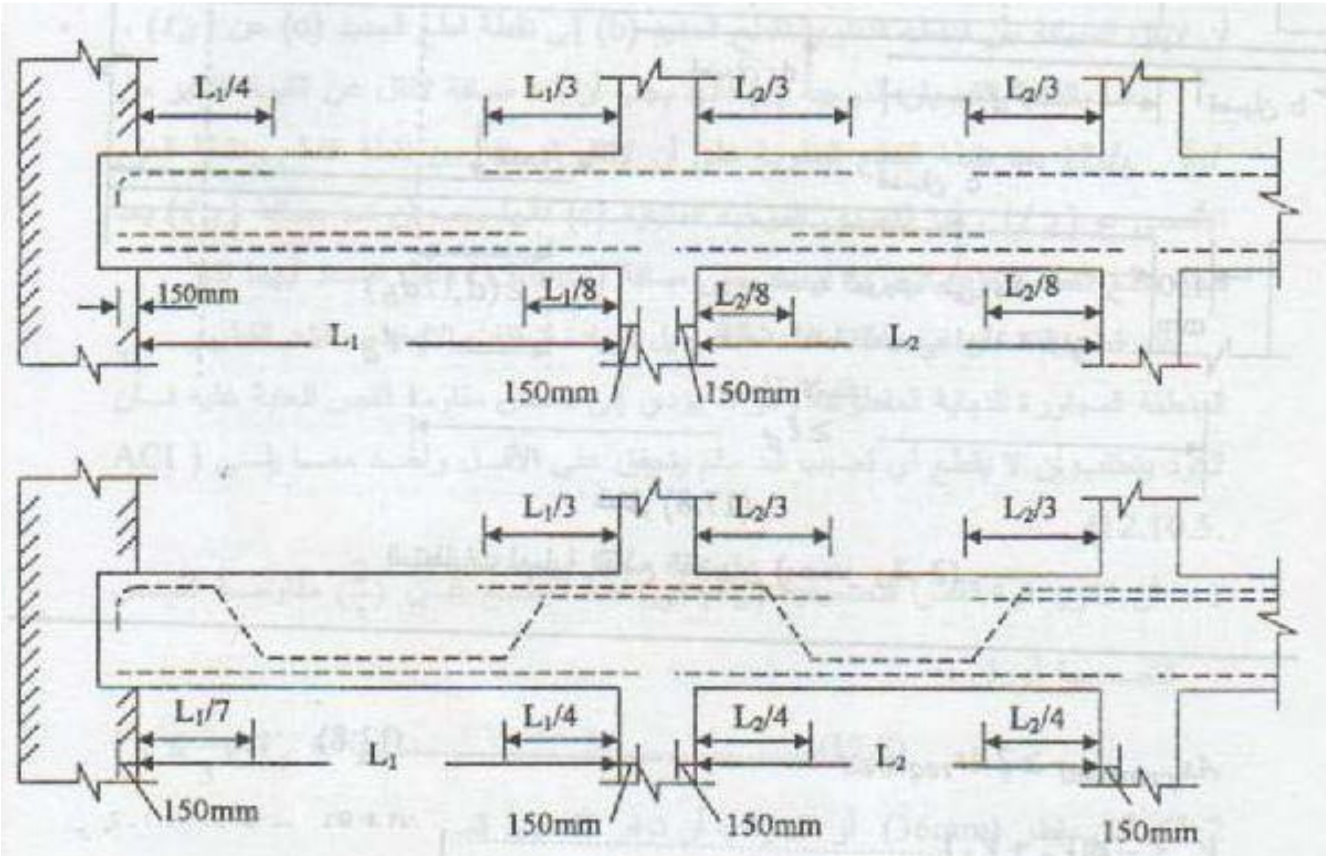


Fig. R9.7.3.2—Development of flexural reinforcement in a typical continuous beam.

ان عملية قطع او ثني الحديد تتطلب حسابات طويلة ودقيقة ومعظم المصممين يعتمدون على الخبرة والشكل ادناه يوضح نقاط القطع التقريبية المعتمدة.



Cutoff and bend points for bars in approximately equal spans with uniformly distributed loads

Design of under singly reinforced rectangular beams

Before the design of an actual beam is attempted, several miscellaneous topics need to be discussed. These include the following:

1. Beam proportions. The most economical beam sections are usually obtained for shorter beams (up to 6.0 m or 7.6 m in length), when the ratio of d to b is in the range of **1.5 to 2**. For longer spans, better economy is usually obtained if deep, narrow sections are used. The depths may be as large as **3*b** or **4*b**. However, today's reinforced concrete designer is often confronted with the need to keep members rather shallow to reduce floor heights. As a result, wider and shallower beams are used more frequently than in the past.

2. Deflections. The ACI Code in its table 9.3.1.1 provides minimum thicknesses of beams for which such deflection calculations are not required. The minimum thicknesses provided apply only to members that are not supporting or attached to partitions or other construction likely to be damaged by large deflection.

CODE

Table 9.3.1.1—Minimum depth of nonprestressed beams

Support condition	Minimum h[1]
Simply supported	$\ell/16$
One end continuous	$\ell/18.5$
Both ends continuous	$\ell/21$
Cantilever	$\ell/8$

[1]Expressions applicable for normalweight concrete and f_y = 420 MPa. For other cases, minimum h shall be modified in accordance with 9.3.1.1.1 through 9.3.1.1.3, as appropriate.

Note: According to (ACI Code318 – 19) 9.3.1.1.1 For *fy* other than 420 *MPa*, the expressions in Table 9.3.1.1 shall be **multiplied** by $(0.4 + \frac{f_y}{700})$.

3. Estimated beam weight. The weight of the beam to be selected must be included in the calculation of the bending moment to be resisted, because the beam must support itself as well as the external loads. For instance, calculate the moment due to the external loads only, select a beam size, and calculate its weight. Another practical method for estimating beam sizes is to assume a minimum overall depth, **h**, equal to the minimum depth specified by [ACI-318-19, Table 9.3.1.1] if deflections are not to be calculated. Then the beam width can be roughly estimated equal to about one-half of the assumed value of hand the weight of this estimated beam calculated = **b *x* h *x* 24** times the concrete weight per cubic meter. After *M* is determined for all of the loads, including the estimated beam weight, the section is selected. If the dimensions of this section are significant different from those initially assumed, it will be necessary to recalculate the weight and *Mu* and repeat the beam selection.

4. Selection of bars. Select an appropriate reinforcement ratio between $\boldsymbol{\rho_{min}}$ and $\boldsymbol{\rho_{max}}$. Often a ratio of about **0.60** $\boldsymbol{\rho_{max}}$, will be an economical and practical choice. Selection of $\rho < \rho_{0.005}$ ensures that $\boldsymbol{\varphi}$ will remain equal to **0.90**. For $\rho_{0.005} < \rho < \boldsymbol{\rho_{max}}$ an iterative solution will be necessary.

After the required reinforcing area is calculated, select diameter of and numbers of bar that provide the necessary area. For the usual situations, bars of sizes and smaller are practical. It is usually convenient to use bars of one size only in a beam, although occasionally two sizes will be used. Bars for compression steel and stirrups are usually a different size.

5. Cover. The reinforcing for concrete members must be protected from the surrounding environment; that are, fire and corrosion protection need to be provided. To do this, the reinforcing is located at **certain minimum distances from the surface of the concrete so that a protective layer of concrete, called cover is provided**. In addition, the cover improves the **bond between the concrete and the steel.** In Section 7.7 of the ACI Code, specified cover is given for reinforcing bars under different conditions. Values are given for reinforced concrete beams, columns, and slabs; for cast-in-place members; for precast members; for pre-stressed members; for members exposed to earth and weather.

Concrete protection for reinforcement

Cast-in-place concrete (non-prestress), cover for reinforcement shall not be less than the following:

No.	Position	Cover, mm
a	Concrete cast against and permanently exposed to earth	75
b	Concrete exposed to earth or weather: bars with diameter 19mm through 57mm Bars with diameter 16 bar, MW200 or MD200 wire, and smaller	 50 40
c	Concrete not exposed to weather or in contact with ground: Slabs, walls, joists: bars with dia. 43mm and 57mm bars with dia. 36 mm and smaller Beams, columns: Primary reinforcement, ties, stirrups, spirals Shells, folded plate members: bars with dia. 19 mm and larger Bars with dia. 16mm, MW200 or Pl D200 wire, and smaller	 40 20 40 20 13

6. Minimum spacing of bars. The code (25.2) states that the clear distance between parallel bars cannot be less than **25 mm** or less than the **nominal bar diameter**. If the bars are placed in more than one layer, those in the upper layers are required to be placed directly over the ones in the lower layers, and the clear distance between the layers must be not less than 25 mm.

Spacing limits for reinforcement

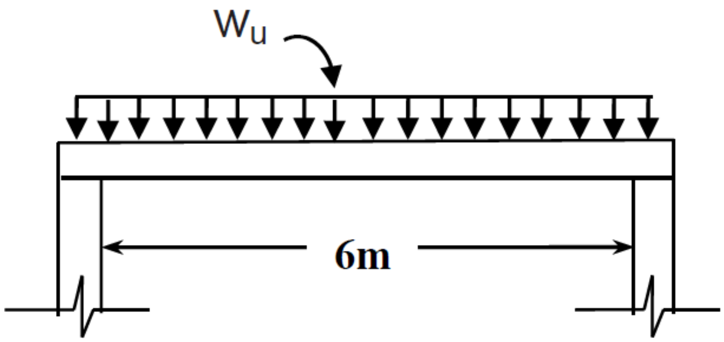
The minimum clear spacing between parallel bars in a layer shall be but not less than **25 mm**, d_b and $(4/3)d_{agg}$.

Where parallel reinforcement is placed in two or more layers, bars in the upper layers shall be placed directly above bars in the bottom layer with clear distance between layers not less than 25 mm.

In walls and slabs other than concrete joist construction, primary flexural reinforcement shall not be spaced farther apart than ($3h$) three times the wall or slab thickness, nor farther apart than **450 mm**.

A major purpose of these requirements is to enable the concrete to pass between the bars. The ACI Code further relates the spacing of the bars to the maximum aggregate sizes for the same purpose. In the code maximum permissible aggregate sizes are limited to the smallest of (a) 1/5 of the narrowest distance between side forms, (b) 1/3 of slab depths, and (c) 3/4 of the minimum clear spacing between bars.

Example: Design the simply supported rectangular beam with clear span of 6 m support service dead load of 10 kN/m and service live load of 22 kN/m, f_c' **= 30** *MPa* and f_y = 420 *MPa*.



Solution:

Assume b = 250 mm , and h = 600 mm

Weigth of beam (W_b) = 0.25 *x* 0.6 *x* 24 = 3.6 kN/m , take W_b = 4.0 kN/m

DL = 10 + 4 = 14 kN/m

$$w_u = 1.2\, DL + 1.6\, LL$$

$$w_u = 1.2\,(14) + 1.6\,(22) = 52.0\; kN/m$$

$$M_u = \frac{w_u\, l^2}{8} = \frac{52.0\; x\; 36}{8} = 234\; kN.m = \mathbf{234\; x\; 10^6\; N.mm}$$

To use the strength reduction factor (***Φ* = 0.9**), we put steel tensile strain $\boldsymbol{\varepsilon_t \geq (\varepsilon_{ty} + 0.003)}$ in the equation below.

$$\rho_{min} = \frac{0.25\sqrt{fc'}}{f_y} = \frac{0.25\sqrt{30}}{420} = 0.00326$$

Or

$$\rho_{min} = \frac{1.4}{f_y} = \frac{1.4}{420} = \mathbf{0.0033} \qquad \textbf{control}$$

$$\rho_{max} = 0.85\,\beta_1\,\frac{fc'}{f_y}\,\frac{\varepsilon u}{\varepsilon u + 0.004}$$

$$\rho_{max} = 0.85\ x\ 0.835\ \frac{30}{420}\ \frac{0.003}{0.003 + 0.004} = 0.0217$$

$$\rho = 0.85\ \beta_1\ \frac{fc'}{f_y}\frac{\varepsilon u}{\varepsilon u + 0.005} = 0.85\ x\ 0.835\frac{30}{420}\frac{0.003}{0.003 + 0.005}$$
$$= 0.01901$$

$$\rho_{min} < \rho < \rho_{max}$$

$$M_u = \emptyset M_n = \emptyset\ \rho\ f_y\ b\ d^2\ \left(1 - 0.59\ \frac{\rho\ fy}{fc'}\right) = \emptyset\ k_n b\ d^2$$

$$k_n = \rho\ f_y\ \left(1 - 0.59\ \frac{\rho\ fy}{fc'}\right) = 0.0194\ x\ 420\ (1 - 0.59\ \frac{0.0194\ x\ 420}{30})$$
$$= 6.842$$
$$M_u = \emptyset M_n = \emptyset\ k_n b\ d^2 = 6.158\ b\ d^2$$

$$M_u = 6.158\ b\ d^2$$

$$234\ x\ 10^6 = 6.158\ b\ d^2$$

$$\boldsymbol{b\ d^2 = 38\ x\ 10^6}$$

$$\boldsymbol{d = \sqrt{\frac{38\ x\ 10^6}{b}}}$$

b mm	d mm	d/b
200	435	2.2
250	**390**	**1.6**
300	356	1.2

∴ Use b = 250 m, d 390 mm

$$h = d + c.cover + dia.of\ stirrups + \frac{1}{2}\ dia.of\ bar$$

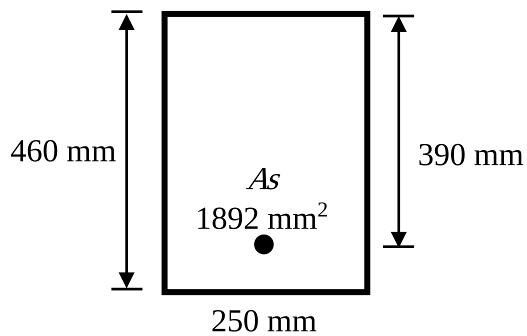
$$h = 390 + 70 = 460\ mm$$

Check the weight of beam

(W_b) = 0.25 *x* 0.46 *x* 24 = 2.76 kN/m < 4.0 kN/m **∴ O.K.**

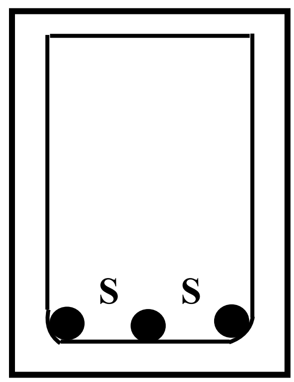
$$\therefore\ A_s = \rho\, b\, d = 0.0194\ x\ 250\ x\ 390 = \mathbf{1892\ mm^2}$$

Use 3#28 mm



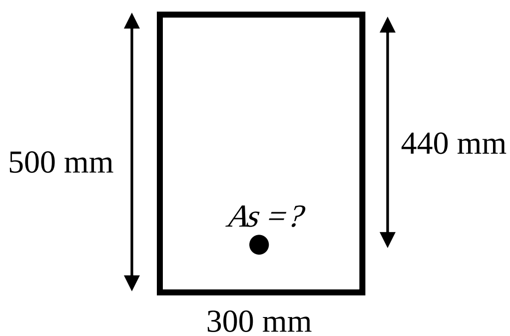
Clear space between bar $S = \frac{250-100-3(28)}{2} = 33\ mm$ > 28 mm O.K. 1 lyer.

1. 25 mm **2. Diameter of bar (d_b) = 28 mm** 3. $\frac{4}{3}$ maximum aggregate size



H.W: (Redesign the beam assuming **ε_t = 0.008**)

Example: Find the steel area (*As*) required to resist a moment M_u = 210 kN.m if the beam dimensions are (b = 300 mm, d = 440 mm and h = 500 mm), f_c' = 25 *MPa* and f_y = 300 *MPa*.



Solution:

$$N_t = Nc$$

$$A_s\, f_y = 0.85\, f_{c'}\, a\, b$$

$$a = \frac{A_s\, f_y}{0.85\, fc'\, b} \ldots\ldots\ldots\ldots\ldots\ldots.(1)$$

$$M_n = Nt\left(d - \frac{a}{2}\right) = As\, fy\, (d - \frac{a}{2})$$

$$As = \frac{M_n}{fy\,(d - \frac{a}{2})} \ldots\ldots\ldots\ldots\ldots\ldots..(2)$$

By solving equations (1) and (2), we get the values of ***a*** and, $\boldsymbol{A_s}$.

Or:

Assume a ≈ (0.15 – 0.3) d

Let *a* = 130 mm then:

$$As = \frac{M_n}{fy\,(d - \frac{a}{2})} = \frac{M_u}{\emptyset\, fy\,(d - \frac{a}{2})} = \frac{210\ x\ 10^6}{0.9\ x\ 300\ x\ (440 - \frac{130}{2})}$$
$$= 2074\ mm^2$$

Check the assumed value of *a*.

$$a = \frac{A_s\, f_y}{0.85\, fc'\, b} = \frac{2074\ x\ 300}{0.85\ x\ 25\ x\ 300} = 97.6\ mm$$

Now assume *a* = 98 mm

$$As = \frac{M_n}{fy\,(d - \frac{a}{2})} = \frac{M_u}{\emptyset\, fy\,(d - \frac{a}{2})} = \frac{210\ x\ 10^6}{0.9\ x\ 300\ x\ (440 - \frac{98}{2})}$$
$$= 1989\ mm^2$$

Check the assumed value of *a*.

$$a = \frac{A_s\, f_y}{0.85\, fc'\, b} = \frac{1989\ x\ 300}{0.85\ x\ 25\ x\ 300} = 93.6\ mm$$

(No further iteration is required), ∴ a = 94 mm

$$\therefore\ As = \frac{M_n}{fy\,(d-\frac{a}{2})} = \frac{210\ x\ 10^6}{0.9\ x\ 300\ x\ (440-\frac{94}{2})} = 1979\ mm^2$$

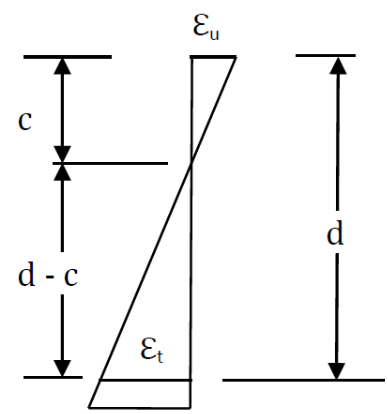
$$\rho = \frac{A_s}{b\ d} = \frac{1979}{300\ x\ 440}\ 0.01499$$

$$\therefore\ \rho_{max} = 0.85\ \beta_1\ \frac{fc'}{f_y}\ \frac{\varepsilon u}{\varepsilon u + 0.004} = 0.85\ x\ 0.85\ \frac{25}{300}\ \frac{0.003}{0.003 + 0.004}$$
$$= 0.0258$$

$$\rho_{min} = \frac{0.25\sqrt{fc'}}{f_y} = \frac{0.25\ x\ 5}{300} = 0.00417$$

$$\rho_{min} = \frac{1.4}{f_y} = \mathbf{0.00467\ Control}$$

$$\rho_{min} \le \rho \le \rho_{max}\quad \therefore \text{O.K.}$$



Check value of ϕ

$$c = \frac{a}{\beta_1} = \frac{94}{0.85} = 110.6\ mm$$

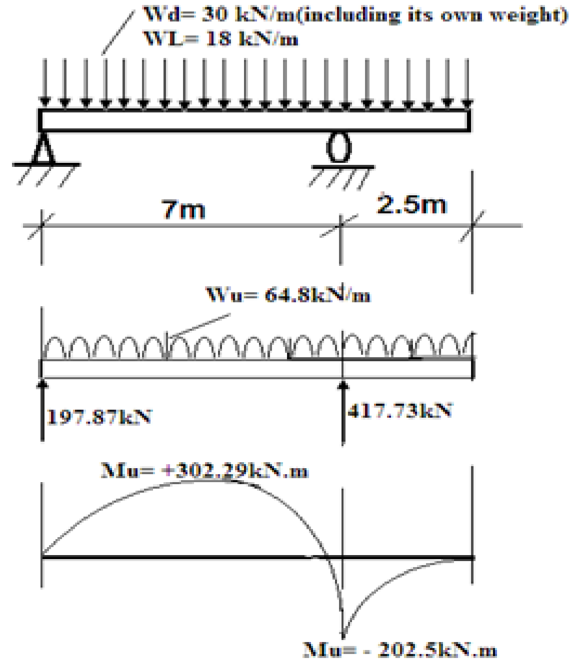
$$\varepsilon_t = \varepsilon u\left(\frac{d-c}{c}\right) = 0.003\ \left(\frac{440-110.6}{110.6}\right) = 0.009\ > \varepsilon_{ty} +\ 0.003$$

∴ ϕ = 0.9 (as assumed)

O.K.

Example: Find the **minimum dimension** of the cross section for the beam shown with b = 300 mm, use f_c' = 25MPa and f_y = 400 MPa, then find area of steel for the whole beam, bars with 25 mm.

Solution:



$$w_u = 1.2\, DL + 1.6\, LL$$

$$w_u = 1.2\,(30) + 1.6\,(18) = 64.8\; kN/m$$

$M_u = 302.29\; kN.m$ = **302.29 x 10^6 N.mm**

To use the strength reduction factor (**Φ = 0.9**), we put steel tensile strain **$\varepsilon_t \geq \varepsilon_{ty}$ + 0.00** in the equation below:

$$\rho = 0.85\, \beta_1 \frac{fc'}{f_y} \frac{\varepsilon u}{\varepsilon u + 0.005} = 0.85\; x\; 0.85 \frac{25}{400} \frac{0.003}{0.003 + 0.005} = 0.01693$$

$$\rho_{min} = \frac{0.25\sqrt{fc'}}{f_y} = \frac{0.25\sqrt{25}}{400} = 0.00313$$

Or

$$\rho_{min} = \frac{1.4}{f_y} = \frac{1.4}{420} = \mathbf{0.0035} \qquad \textbf{control}$$

$$\rho_{max} = 0.85\, \beta_1 \frac{fc'}{f_y} \frac{\varepsilon u}{\varepsilon u + 0.004}$$

$$\rho_{max} = 0.85\; x\; 0.85 \frac{25}{400} \frac{0.003}{0.003 + 0.004} = 0.01935$$

$$\rho_{min} < \rho < \rho_{max}$$

$$M_u = \emptyset M_n = \emptyset\, \rho\, f_y\, b\, d^2 \left(1 - 0.59\,\frac{\rho\, fy}{fc'}\right) = \emptyset\, k_n b\, d^2$$

$$k_n = \rho\, f_y \left(1 - 0.59\,\frac{\rho\, fy}{fc'}\right) = 0.01693\; x\; 400\; (1 - 0.59\;\frac{0.01693\; x\; 400}{25})$$
$$= 5.689$$

$$M_u = \emptyset M_n = \emptyset\, k_n b\, d^2 = 0.9\; x\; 5.689\; x\; 300\; x\; d^2$$

$$302.29\; x\; 10^6 = 1536.2\, d^2$$

d = 443.6 mm

∴ **Use d = 450 mm**

$$h = d + c.cover + dia.of\ stirrups + \frac{1}{2}\ dia.of\ bar$$

$$h = 450 + 70 = 520\,mm$$

$$h_{min} = \frac{L}{18.5}\left(0.4 + \frac{fy}{700}\right) = \frac{7000}{18.5}\left(0.4\ + \frac{400}{700}\right) = 367.6mm$$

$h > h_{min}$ **∴ O.k.**

$$\therefore\ A_s = \rho\, b\, d = 0.01693\; x\; 300\; x\; 450 = \mathbf{2285.55\ mm^2}$$

$$A_{bar(25mm)} = 491\; mm^2\ , no.of\ bar\ req. = \frac{A_s}{A_{bar}} = \frac{2285.55}{491} = \mathbf{4.7}$$

∴ Use 5 # 25 mm

Check one or two layers of steel at tension zone.

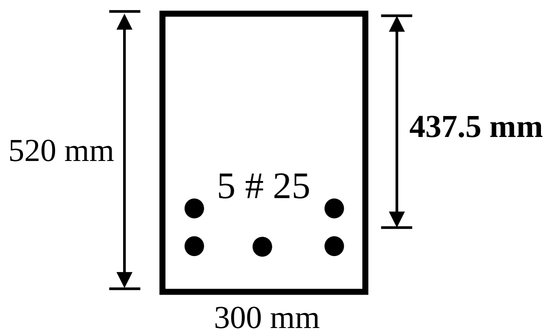
1. 25 mm

2. Diameter of bar (d_b) = 25 mm

3. $\frac{4}{3}$ maximum aggregate size.

$$\frac{300 - 2*40 - 2*10 - 5*25}{4} = 18.75\,mm < 25\,mm\ then\ \boldsymbol{two\ layers}$$

$$\frac{300 - 2*40 - 2*10 - 3*25}{2} = 62.5 > 25mm$$



Example: Find the steel area (***A_s***), if the service dead moment M_d = 100 kN.m (including weight of the beam) and service live moment M_L = 150 kN.m. The beam dimensions are (*b* = 300 mm, *h* = 700 mm), f_c' = 20 *MPa* and f_y = 400 *MPa*.

Solution:

$$M_u = 1.2\ M_d + 1.6\ M_L$$

$$M_u = 1.2\ (100) + 1.6\ (150) = \boldsymbol{360\ kN.m}$$

Assume a ≈ (0.15 – 0.3) d

d = 700 – (70) = 630 mm

Let *a* = 100 mm then:

$$As = \frac{M_n}{fy\ (d - \frac{a}{2})} = \frac{M_u}{\emptyset\ fy\ (d - \frac{a}{2})} = \frac{360\ x\ 10^6}{0.9\ x\ 400\ x\ (630 - \frac{100}{2})}$$
$$= 1724\ mm^2$$

Check the assumed value of *a*.

$$a = \frac{A_s\ f_y}{0.85\ fc'\ b} = \frac{1724\ x\ 400}{0.85\ x\ 20\ x\ 300} = 135.2\ mm$$

Now assume *a* = 136 mm

$$As = \frac{M_n}{fy\ (d - \frac{a}{2})} = \frac{M_u}{\emptyset\ fy\ (d - \frac{a}{2})} = \frac{360\ x\ 10^6}{0.9\ x\ 400\ x\ (630 - \frac{136}{2})}$$
$$= 1780\ mm^2$$

Check the assumed value of *a*.

$$a = \frac{A_s\, f_y}{0.85\, fc'\, b} = \frac{1780\ x\ 400}{0.85\ x\ 20\ x\ 300} = 139.6\, mm$$

(No further iteration is required),

$$\therefore \text{a} = 139.6 \text{ mm}$$

$$\therefore\ As = \frac{M_n}{fy\,(d - \frac{a}{2})} = \frac{360\ x\ 10^6}{0.9\ x\ 400\ x\ (630 - \frac{139.6}{2})} = 1785\ mm^2$$

$$\rho = \frac{A_s}{b\ d} = \frac{1785}{300\ x\ 630} = \mathbf{0.00944}$$

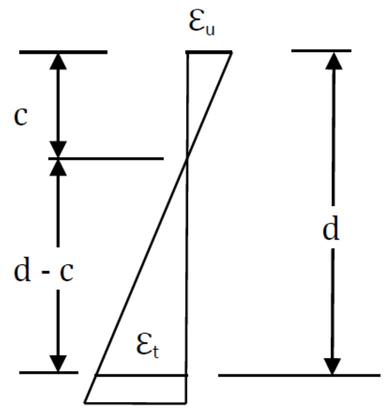
$$\therefore\ \rho_{max} = 0.85\,\beta_1\,\frac{fc'}{f_y}\,\frac{\varepsilon u}{\varepsilon u + 0.004} = 0.85\ x\ 0.85\ \frac{20}{400}\ \frac{0.003}{0.003 + 0.004}$$

$$= \mathbf{0.01548}$$

$$\rho_{min} = \frac{0.25\,\sqrt{fc'}}{f_y} = \frac{0.25\ x\ \sqrt{20}}{400} = 0.0028$$

$$\rho_{min} = \frac{1.4}{f_y} = \mathbf{0.0035\ Control}$$

$$\rho_{min} \le \rho \le \rho_{max} \quad \therefore \text{O.K.}$$



Check value of ϕ

$$c = \frac{a}{\beta_1} = \frac{139.6}{0.85} = 155.1\, mm$$

$$\varepsilon_t = \varepsilon u \left(\frac{d - c}{c}\right) = 0.003 \left(\frac{630 - 155.1}{155.1}\right) = 0.00919 > \varepsilon_{ty} + 0.003$$

$\therefore \phi$ **= 0.9 (as assumed)**

O.K.

Flexural Analysis and Design of Beams

The basic assumptions made in flexural design are:

1. Sections perpendicular to the axis of bending that are plane before bending remains plane after bending.
2. A perfect bond exists between the reinforcement and the concrete such that the strain in the reinforcement is equal to the strain in the concrete at the same level.
3. The strains in both the concrete and reinforcement are assumed to be directly proportional to the distance from the neutral axis.
4. Concrete is assumed to fail when the compressive strain reaches 0.003.
5. The tensile strength of concrete is neglected.
6. The stresses in the concrete and reinforcement can be computed from the strains using stress-strain curves for concrete and steel, respectively.

Structural Design Requirements:

The design of a structure must satisfy three basic requirements:

1. Strength to resist safely the stresses induced by the loads in the various structural members.

2. Serviceability to ensure satisfactory performance under service load conditions, which implies providing adequate stiffness to contain deflections, crack widths and vibrations within acceptable limits.

3. Stability to prevent overturning, sliding or buckling of the structure, or part of it under the action of loads.

There are two other considerations that a sensible designer should keep in mind: **Economy and aesthetics.**

Design Methods (Philosophies)

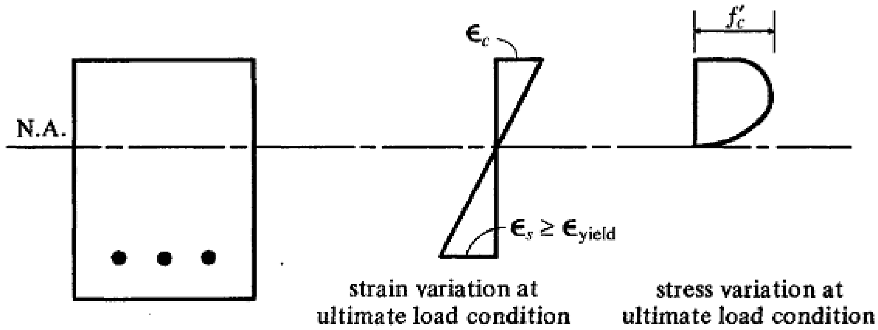
Two methods of design have long prevalent.

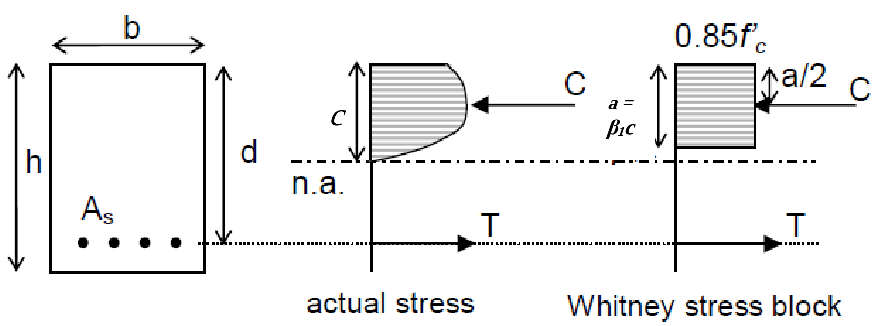
Working Stress Method: focuses on conditions at service loads.

Strength Design Method: focusing on conditions at loads greater than the service loads when failure may be imminent. The Strength Design Method is deemed conceptually more realistic to establish structural safety.

Stress and strain distribution

The compressive stress-strain relationship for concrete may be assumed to be rectangular, trapezoidal, parabolic, or any other shape that results in prediction of strength in substantial agreement with the results of comprehensive tests outlines the use of a rectangular compressive stress distribution which is known as the **Whitney rectangular stress block.**





Whitney replaced the curved stress block with an equivalent rectangular block of intensity $0.85fc'$ and depth $\boldsymbol{a = \beta_1 c}$, as shown in figure above. The area of this rectangular block should equal that of the curved stress block, and the centroids of the two blocks should coincide. Sufficient test results are available for concrete beams to provide the depths of the equivalent rectangular stress blocks. $\boldsymbol{\beta_1}$ shall be in accordance with table 22.2.2.4.3:

Table 22.2.2.4.3—Values of β_1 for equivalent rectangular concrete stress distribution

f_c', MPa	β_1	
$17 \leq f_c' \leq 28$	0.85	(a)
$28 < f_c' < 55$	$0.85 - \frac{0.05(f_c' - 28)}{7}$	(b)
$f_c' \geq 55$	0.65	(c)

Based on these assumptions regarding the stress block, statics equations can easily be written for the sum of the horizontal forces and for the resisting moment produced by the internal couple. These expressions can then be solved separately for a and for the moment M_n.

M_n is defined as the theoretical or nominal resisting moment of a section. It was stated that the usable strength of a member equals its theoretical strength times the strength reduction factor, or, in this case ΦM_n.

The usable flexural strength of a member, ΦM_n must at least be equal to the calculated factored moment, M_u caused by the factored loads $\Phi M_n \geq M_u$.

For writing the beam expressions, reference is made to figure equating the horizontal forces **C** and **T** and solving for a, we obtain:

Internal Equilibrium

N_c = compression ***force*** in concrete = stress *x* area = $0.85\, f_c{}'\, b\, a$

N_t = tension ***force i***n steel = stress x area = $A_s\, f_y$

$$N_c = N_t \quad \text{and} \quad M_n = N_t\,(d - \frac{a}{2})$$

Where:

$f_c{}'$ = concrete compression strength
a = height of stress block
β_1 = factor based on $f_c{}'$
c = location to the neutral axis.
b = width of stress block.
f_y = steel yield strength.
A_s = area of steel reinforcement
d = effective depth of section = depth to axis of reinforcement

$$N_t = Nc$$

$$A_s\, f_y = 0.85\, f_{c'}\, a\, b$$

So, a can be determined with: $$a = \frac{A_S\, f_y}{0.85\, fc'\, b}$$

Check whether the tension steel is yielding. The yield strain for the reinforcing steel is: $$\frac{c}{0.003} = \frac{d-c}{\varepsilon_t} \quad \rightarrow \quad \varepsilon_t = \frac{d-c}{c}\; x\; 0.003$$

Because the reinforcing steel is limited to an amount such that it will yield well before the concrete reaches its ultimate strength, the value of the nominal moment can be written as:

$$M_n = Nt\left(d - \frac{a}{2}\right) = As\, fy\, (d - \frac{a}{2})$$

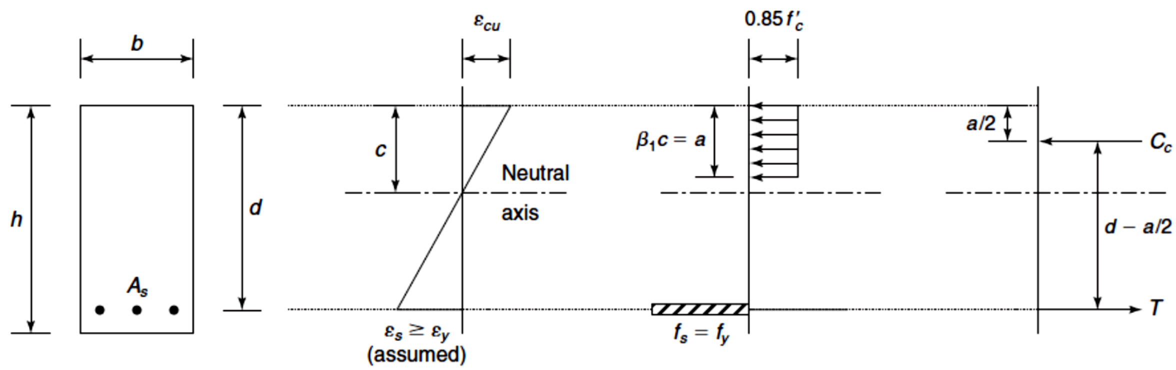
$$\emptyset M_n = \emptyset\, A_s\, f_y\, (d - \frac{a}{2})$$

If we substitute into this expression the value previously obtained for (it was) and equate to we obtain the following expression:

$$\emptyset M_n = Mu = \emptyset\, \rho\, b\, fy\, d^2\, (1 - 0.59 * \rho * \frac{fy}{fc'})$$

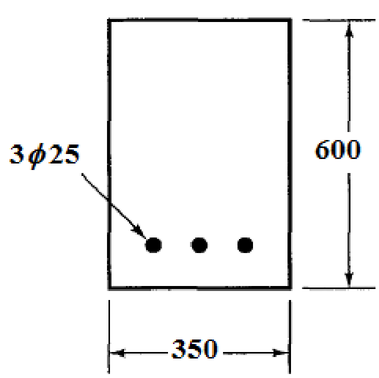
Replacing A_s with ρ b d and letting, $R = \frac{Mu}{\emptyset b d^2}$ we can solve this expression for ρ (the percentage of steel required for a particular beam) with the following results:

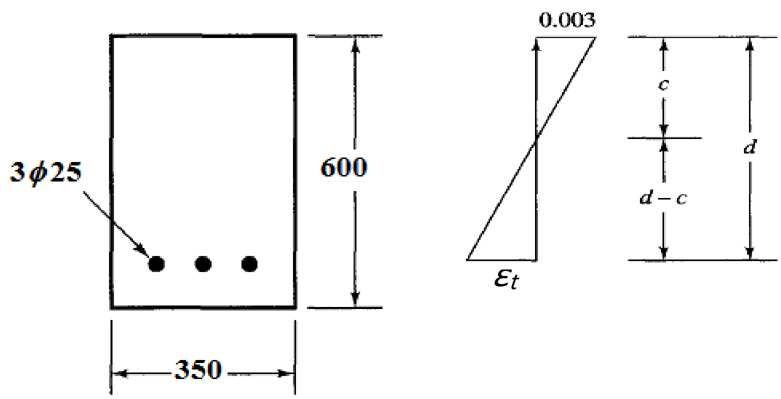
$$\rho = \frac{0.85\, fc'}{fy}\, (1 - \sqrt{1 - \frac{2R}{0.85\, fc'}})$$

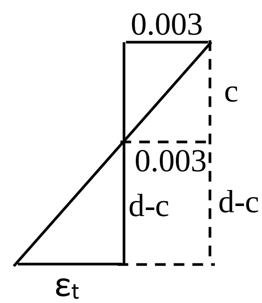


(a) Singly reinforced section. (b) Strain distribution. (c) Stress distribution. (d) Internal forces.

Example: Determine the values **a**, **c**, and ***εt*** for the beam shown in figure below. If $fc' = 21$ *MPa* and *fy* = 420 *MPa*.







$$As = (3)\frac{\pi\, d^2}{4} = (3)\frac{\pi\, 25^2}{4} = 3 * 491 = 1473\, mm^2$$

$$a = \frac{As\, fy}{0.85\, fc'\, b} = \frac{1473\ x\ 420}{0.85\ x\ 21\ x\ 350} = 99.025\, mm$$

$$c = \frac{a}{\beta 1}$$

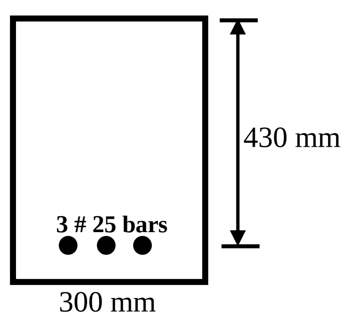
$$fc' = 21\, MPa \ \rightarrow \ \beta 1 = 0.85 \quad (\mathbf{17} \ \leq \ \boldsymbol{fc'} \ \leq \mathbf{28})$$

$$c = \frac{a}{\beta 1} = \frac{99.025}{0.85} = 116.5\, mm$$

$$d = 600 - 40 - 10 - 12.5 = 537.5$$

$$\frac{c}{0.003} = \frac{d-c}{\varepsilon_t} \rightarrow \varepsilon t = \frac{d-c}{c} 0.003 = \frac{537.5 - 116.5}{116.5} 0.003 = \mathbf{0.01084}$$

Example: Determine the nominal moment strength ($\boldsymbol{M_n}$) of the beam shown in figure below if fc' = 28 *MPa* and f_y = 420 *MPa*.



$$As = (3)\frac{\pi\, d^2}{4} = (3)\frac{\pi\, 25^2}{4} = 3 * 491 = 1473\, mm^2$$

$$N_t = N_c,$$

$$A_s\, f_y = 0.85\, f_c'\, b\, a$$

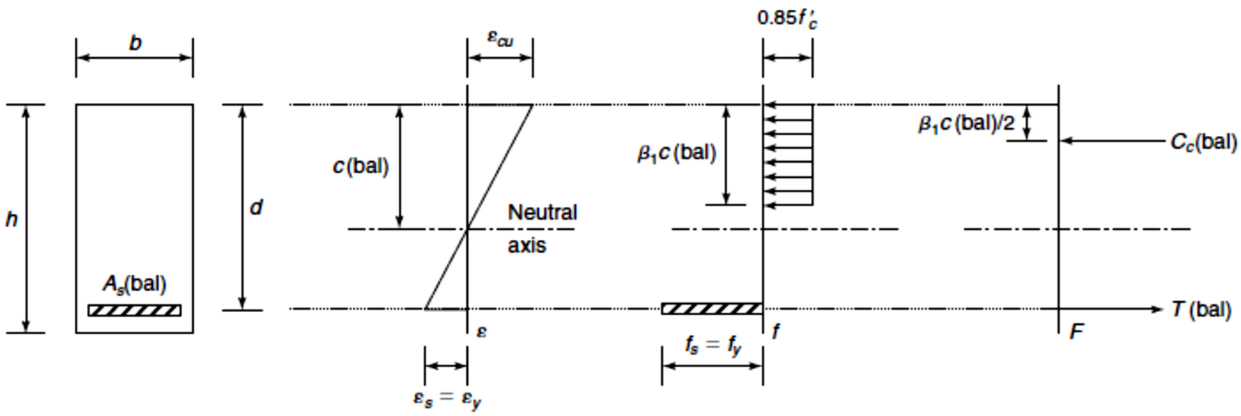
$$a = \frac{As\ fy}{0.85\ fc'\ b} = \frac{1473\ x\ 420}{0.85\ x\ 28\ x\ 300} = 86.6\ mm$$

$$M_n = Nt\left(d - \frac{a}{2}\right) = As\ fy\ (d - \frac{a}{2})$$

$$M_n = 1473\ x\ 420\left(430 - \frac{86.6}{2}\right) = 239.2\ x\ 10^3\ N.mm = 239.2\ kN.m$$

The balanced rectangular beam

A beam that has a balanced steel ratio is one for which the tensile steel will theoretically yield at the time the extreme compression concrete fibers attain a strain equal to **0.003**.



(a) Beam section. (b) Balanced strain distribution. (c) Stress distribution. (d) Internal forces.

From the figure above (using similar triangles):

$$\frac{cb}{\varepsilon u} = \frac{d - cb}{\varepsilon y}$$

$$cb\ \varepsilon y = \varepsilon u\ d - \varepsilon u\ cb$$

$$cb = \frac{\varepsilon u}{\varepsilon u + \varepsilon y}\ d$$

Then from equilibrium requirement we have:

$$N_{tb} = N_{cb}$$

Where: N_{tb} = tension force in steel at balanced condition
N_{cb} = compression force in concrete at balanced condition

$$N_{tb} = A_{sb}\ f_y$$

Putting:

$$\rho = \frac{As}{b\ d}$$

ρ is *called reinforcement ratio*

$$\rho_b = \frac{Asb}{b\ d} \rightarrow \ Asb = \rho_b\ b\ d$$

Where: ρ_b = *balanced reinforcement ratio.*

$$N_{tb} = \ \rho_b b\ d\ f_y$$

$$N_{cb} = 0.85\ fc'\ a_b b$$

$$a_b = \ \beta_1 c_b$$

$$\therefore \ N_{cb} = 0.85\ fc' \beta_1 c_b b$$

$$N_{tb} = N_{cb}$$

$$\rho_b\ b\ d\ f_y = \ 0.85\ fc' \beta_1 c_b b$$

$$\rho_b\ d\ f_y = \ 0.85\ fc' \beta_1 c_b$$

$$\rho_b\ d\ f_y = \ 0.85\ fc' \beta_1 \ \frac{\varepsilon u}{\varepsilon u + \varepsilon y}\ d$$

$$\therefore \ \boldsymbol{\rho_b = \ 0.85\ \beta_1\ \frac{fc'}{f_y}\ \frac{\varepsilon u}{\varepsilon u + \varepsilon y}}$$

Under-reinforced beams:

In actual practice, the reinforcement ratio ρ should be < ρ_b to avoid sudden failure of beam (compression failure). Where lower reinforcement ratio ρ result in gradual failure and give warning before total collapse.

ACI provisions for under-reinforced beams:

To insure under-reinforced behavior, ACI code establishes a minimum net tensile strain ϵ_t = 0.004 at the nominal member strength for members subjected to axial loads < 0.1 $fc'\ A_g$ Where A_g = gross area of cross section. (For comparison ϵ_t = 0.002 for grade 400 steel).

$$\rho = \ 0.85\ \beta_1\ \frac{fc'}{f_y}\ \frac{\varepsilon u}{\varepsilon u + \varepsilon t}$$

Using εt = 0.004 in previous equation provides the maximum reinforcement ratio allowed by ACI code for beams.

$$\therefore \ \rho_{max} = 0.85\, \beta_1 \frac{fc'}{f_y} \frac{\varepsilon u}{\varepsilon u + 0.004}$$

Since $\epsilon_t \geq 0.004$ then $f_s = f_y$ at failure for ordinary reinforcing steel.

∴ The nominal flexural strength is given by:

$$M_n = N_t\, Z$$

$$M_n = A_s\, f_y\, (d - \frac{a}{2})$$

Where:

$$a = \frac{As\, fy}{0.85\, fc'\, b}$$

And:

$$A_s = \rho\, b\, d$$

$$a = \frac{\rho\, b\, d\, fy}{0.85\, fc'\, b} = \frac{\rho\ \ d\, fy}{0.85\, fc'}$$

$$\therefore \ M_n = \rho\, b\, d\, f_y\, (d - \frac{\rho\ \ d\, fy}{2\, x\, 0.85\, fc'})$$

$$\therefore \ M_n = \rho\, f_y\, b\, d^2 \left(1 - 0.59\, \frac{\rho\, fy}{fc'}\right)$$

$$\therefore \ \boldsymbol{M_n = k_n b\, d^2}$$

Where:

$$\therefore \ \boldsymbol{k_n = \rho\, f_y \left(1 - 0.59\, \frac{\rho\, fy}{fc'}\right)}$$

Values of $\boldsymbol{k_n}$ for maximum reinforcement ratio and are given in table below.

Table of limiting constants for rectangular beams

		f_y = 300 MPa			f_y = 400 MPa		
Grade	f_c' (MPa)	k_n	ρ_{max}	a/d	k_n	ρ_{max}	a/d
C20	20	5.053	0.0206	0.363	5.066	0.0155	0.365
C25	25	6.326	0.0258	0.363	6.339	0.0194	0.365
C30	30	7.58	0.0309	0.363	7.586	0.0232	0.365
C35	35	8.525	0.0344	0.347	8.525	0.0258	0.347
C40	40	9.363	0.0374	0.330	9.376	0.0281	0.33

Maximum Reinforcement Ratio ρ for Singly Reinforced Rectangular Beams (tensile strain = 0.005) for which ϕ is permitted to be 0.9

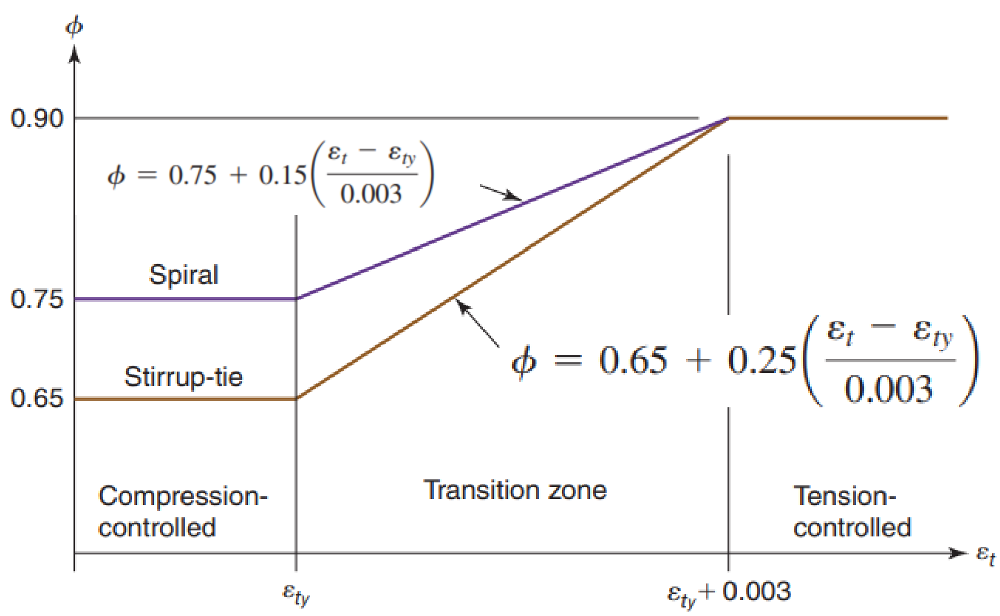
f_y	f'_c = 3000 psi β_1 = 0.85	f'_c = 3500 psi β_1 = 0.85	f'_c = 4000 psi β_1 = 0.85	f'_c = 5000 psi β_1 = 0.80	f'_c = 6000 psi β_1 = 0.75
40,000 psi	0.0203	0.0237	0.0271	0.0319	0.0359
50,000 psi	0.0163	0.0190	0.0217	0.0255	0.0287
60,000 psi	0.0135	0.0158	0.0181	0.0213	0.0239
f_y	f'_c = 20 MPa β_1 = 0.85	f'_c = 25 MPa β_1 = 0.85	f'_c = 30 MPa β_1 = 0.85	f'_c = 35 MPa β_1 = 0.81	f'_c = 40 MPa β_1 = 0.77
300 MPa	0.0181	0.0226	0.0271	0.0301	0.0327
350 MPa	0.0155	0.0194	0.0232	0.0258	0.0281
400 MPa	0.0135	0.0169	0.0203	0.0226	0.0245
500 MPa	0.0108	0.0135	0.0163	0.0181	0.0196

The ACI code encourages the use of lower ρ by allowing higher strength reduction factors (ϕ) in such beams.

If the net tensile strain in the extreme tension reinforcement is sufficiently large (≥ **ε*ty* + 0.003**), the section is defined as **tension-controlled**.

If the net tensile strain in the extreme tension reinforcement is small (≤ **ε*ty***), a brittle **compression failure** condition is expected.

Beams and slabs are usually tension-controlled, whereas columns may be compression-controlled. Some members, such as those with small axial forces and large bending moments, experience net tensile strain in the extreme tension reinforcement between the limits of **ε*ty*** and (**ε*ty* + 0.003**). These sections are in a transition region between compression-controlled and tension-controlled.



Variation of *Φ* with net tensile strain

Example: Calculate the maximum nominal moment (***Mₙ***), design moment (ϕ ***Mₙ***) and maximum reinforcement ratio ρ_{max} for $f_c' = 25\ MPa$, and $f_y = 300\ MPa$.

$$\rho_{max} = 0.85\,\beta_1\,\frac{fc'}{f_y}\,\frac{\varepsilon u}{\varepsilon u + 0.004}$$

$$fc' = 25\,MPa \;\rightarrow\; \beta_1 = 0.85 \qquad (\mathbf{17} \le \boldsymbol{fc'} \le \mathbf{28})$$

$$\rho_{max} = 0.85\,x\,0.85\,\frac{25}{300}\,\frac{0.003}{0.003 + 0.004} = 0.0258$$

$$M_n = \rho\, f_y\, b\, d^2\,\left(1 - 0.59\,\frac{\rho\, fy}{fc'}\right)$$

$$M_n = 0.0258\,x\,300\,x\,b\,d^2\,\left(1 - 0.59\,\frac{0.0258\,x\,300}{25}\right) = 6.326\,bd^2$$

OR:

$$M_n = k_n b\, d^2$$

$k_n = 6.326$ (from table):

$$\therefore M_n = 6.326\,b\,d^2$$

$$\text{Design Moment} = \phi M_n$$

$$\boldsymbol{\phi = ?}$$

$$\boldsymbol{\epsilon_t = 0.004}$$

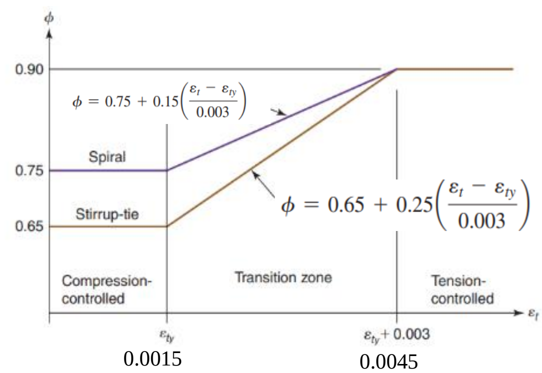
$$\varepsilon_{ty} = \frac{f_y}{E_s} = \frac{300}{200000} = 0.0015$$

$$\varepsilon_{ty} + 0.003 = 0.0015 + 0.003 = 0.0045$$

$$\because \varepsilon_{ty} < \varepsilon_t < \varepsilon_{ty} + 0.003$$

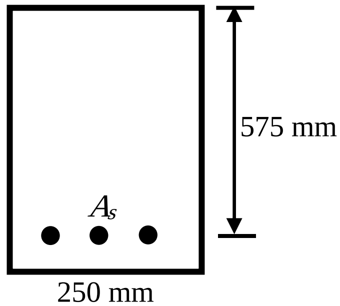
$$\varphi = 0.65 + 0.25\left(\frac{\varepsilon_t - \varepsilon_{ty}}{0.003}\right) = 0.65 + 0.25\left(\frac{0.004 - 0.0015}{0.003}\right) = \mathbf{0.858}$$

$$\therefore \text{Design moment} = \Phi\, Mn = 0.858 \text{ x } 6.326\, b\, d^2 = \mathbf{5.428\, b\, d^2}$$



H.W: Resolve the same example using ϵ_t = 0.005.

Example: Calculate the design moment for the beam section shown in figure below, if the steel reinforcement is 3 No. 25 mm bars. fc' = 30 MPa and f_y = 400 MPa.



$$As = (3)\frac{\pi\, d^2}{4} = (3)\frac{\pi\, 25^2}{4} = 3 * 491 = 1473\; mm^2$$

$$\rho = \frac{As}{b\,d} = \frac{1473}{250\; x\; 575} = 0.01025$$

The maximum reinforcement ratio is given by:

$$\rho_{max} = 0.85\, \beta_1 \frac{fc'}{f_y} \frac{\varepsilon u}{\varepsilon u + 0.004}$$

$$fc' = 30\; MPa \quad (28 < f_c' < 55)$$

$$\therefore \beta_1 = 0.85 - \frac{0.05\,(f_c' - 28)}{7} = \mathbf{0.836}$$

$$\rho_{max} = 0.85\; x\; \mathbf{0.836} \frac{30}{400} \frac{0.003}{0.003 + 0.004} = 0.0228$$

The actual reinforcement ratio ρ = 0.01025 < ρ_{max} **O.K**

∴ The member is **under-reinforced** and will fail by yielding of steel (tension failure)

$$Nc = Nt$$

$$0.85\, fc'\, a\, b = A_s\, f_y$$

$$a = \frac{As\, fy}{0.85\, fc'\, b} = \frac{1473\; x\; 400}{0.85\; x\; 30\; x\; 250} = 92.4\; mm$$

$$M_n = N_t\, Z$$

$$M_n = A_s\, f_y\left(d - \frac{a}{2}\right) = 1473\; x\; 400\left(575 - \frac{92.4}{2}\right) = 311.6\; x\; 10^6\; N.mm$$

$$\boldsymbol{M_n = 311.6\ kN.m}$$

The design moment = $\phi\ M_n$

To find ϕ value, the distance to the neutral axis (c) must be known.

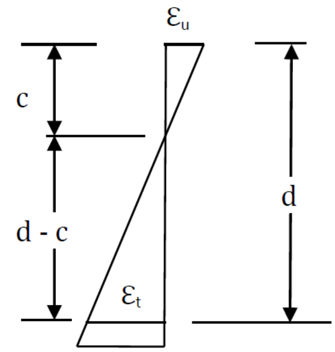
$$c = \frac{a}{\beta_1} = \frac{92.4}{0.836} = \mathbf{110.5\ mm}$$

$$\varepsilon_t = \varepsilon u \left(\frac{d-c}{c}\right) = 0.003 \left(\frac{575-110.5}{110.5}\right) = 0.01261$$

$$\varepsilon_{ty} = \frac{400}{200000} = 0.002$$

$$\because \varepsilon_t > \varepsilon_{ty} + 0.003\ (0.005) \rightarrow \varphi = 0.9$$

$\therefore$ Design moment = $\Phi\ Mn$ = 0.9 x 311.6 = 280.44 kN.m

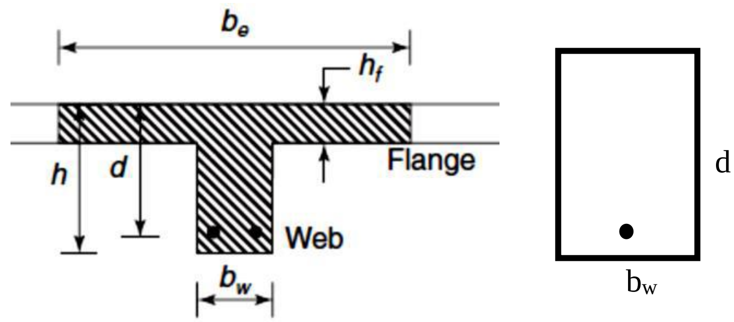


Minimum Reinforcement Ratio

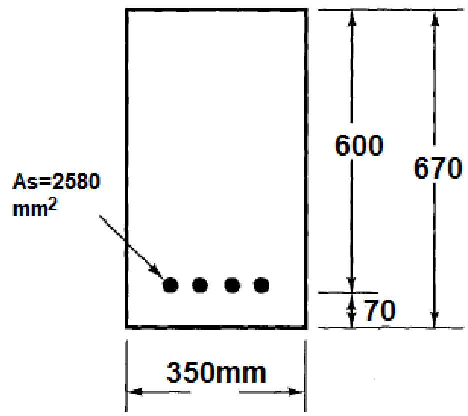
If the flexural strength of the cracked section is less than the moment that produced cracking of the previously uncracked section, the beam will fail immediately and without warning (i.e. sudden failure). Therefore, the **ACI318-19** (9.6) provides minimum $\boldsymbol{A_s}$ for flexural members.

$$A_{s\,min} = \frac{0.25\sqrt{fc'}}{f_y} b_w d \quad \text{but not less than } (\geq) \quad A_{s\,min}\ \frac{1.4}{f_y} b_w d$$

Where: b_w is width of web of beam and, the value of $\boldsymbol{f_y}$ shall be limited to a maximum of 550 *MPa*.



Example: Determine the design moment capacity ΦM_n of the beam shown in figure below, if $fc' = 30\ MPa$ and $f_y = 420\ MPa$.



$$\rho = \frac{As}{b\ d} = \frac{2580}{350\ x\ 600} = 0.01229$$

$$\rho_{min} = \frac{0.25\sqrt{fc'}}{f_y} = \frac{0.25\sqrt{30}}{420} = 0.00326$$

Or

$$\rho_{min} = \frac{1.4}{f_y} = \frac{1.4}{420} = \mathbf{0.00333} \qquad \text{control}$$

$$\rho_{max} = 0.85\,\boldsymbol{\beta_1}\,\frac{fc'}{f_y}\,\frac{\varepsilon u}{\varepsilon u + 0.004}$$

$$fc' = 30\ MPa \quad (28 < f_c' < 55)$$

$$\therefore \beta_1 = 0.85 - \frac{0.05\,(f_c' - 28)}{7} = \mathbf{0.836}$$

$$\rho_{max} = 0.85\ x\ 0.836\ \frac{30}{420}\ \frac{0.003}{0.003 + 0.004} = 0.0217$$

$$\rho_{min} < \rho < \rho_{max}$$

$\therefore$ The beam is under reinforcement.

$$Nc = Nt$$

$$0.85\ fc'\ a\ b = A_s\ f_y$$

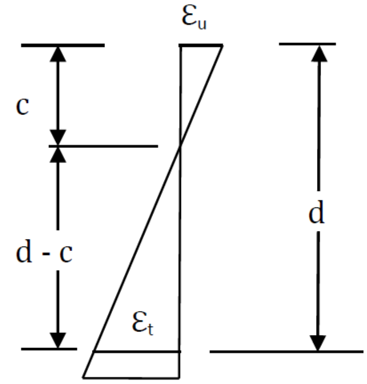
$$a = \frac{As\ fy}{0.85\ fc'\ b} = \frac{2580\ x\ 420}{0.85\ x\ 30\ x\ 350} = 121.4\ mm$$

$$M_n = N_t\ Z$$

$$M_n = A_s\ f_y\ \left(d - \frac{a}{2}\right) = 2580\ x\ 420 \left(600 - \frac{121.4}{2}\right) = 584.4\ x\ 10^6\ N.mm$$

$$\boldsymbol{M_n = 584.4\ kN.m}$$

The design moment = $\phi\ M_n$



$$c = \frac{a}{\beta_1} = \frac{121.4}{0.836} = \mathbf{145.2\ mm}$$

$$\varepsilon_t\ =\ \varepsilon u\ \left(\frac{d-c}{c}\right) = 0.003\ \left(\frac{600-145.2}{145.2}\right) = 0.00939$$

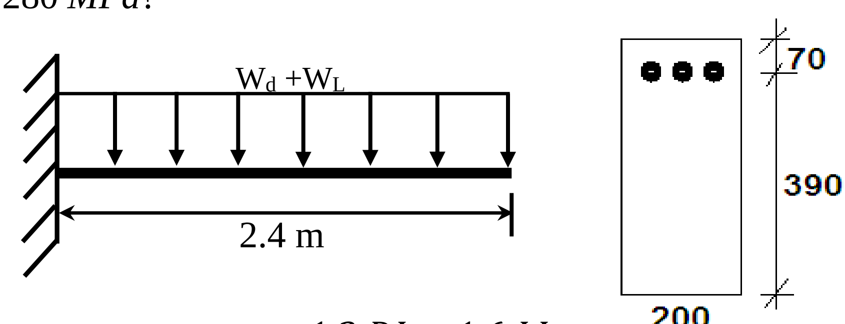
$$\varepsilon_{ty}\ =\ \frac{420}{200000} = 0.0021$$

$$\because \varepsilon_t > \ \varepsilon_{ty} + 0.003\ (0.0051)$$

$$\therefore \varphi = 0.9$$

∴ Design moment = ΦM_n = 0.9 *x* 584.4 = 525.96 kN.m

Example: A 2.4m span cantilever beam has a rectangular section of b = 200 mm and d = 390 mm with 3 bars of 22 mm diameter, carries a uniform dead load including its own weight of 12 kN/m and a uniform distributed live load of 10.5 kN/m. Check the **adequacy** of the section, using f_c' of 28 *MPa* and f_y of 280 *MPa*?



$$w_u\ =\ 1.2\ DL + 1.6\ LL$$

$$w_u = 1.2\,(12) + 1.6\,(10.5) = 31.2\ kN/m$$

$$M_u = \frac{w\,l^2}{2} = \frac{31.2\ x\ 2.4^2}{2} = \mathbf{89.86\ kN.m}$$

$$\rho = \frac{As}{b\,d} = \frac{1140.4}{200\ x\ 390} = 0.0146$$

$$\rho_{min} = \frac{0.25\sqrt{fc'}}{f_y} = \frac{0.25\sqrt{28}}{280} = 0.00472$$

Or

$$\rho_{min} = \frac{1.4}{f_y} = \frac{1.4}{280} = \mathbf{0.005} \qquad \text{control}$$

$$\rho_{max} = 0.85\,\beta_1\,\frac{fc'}{f_y}\,\frac{\varepsilon u}{\varepsilon u + 0.004}$$

$$fc' = 28\ MPa \ \rightarrow\ \beta_1 = 0.85 \qquad (\mathbf{17} \le \boldsymbol{fc'} \le \mathbf{28})$$

$$\rho_{max} = 0.85\ x\ 0.85\ \frac{28}{280}\ \frac{0.003}{0.003 + 0.004} = 0.03096$$

$$\rho_{min} < \rho < \rho_{max}$$

∴ The beam is under reinforcement.

$$Nc = Nt$$

$$0.85\,fc'\,a\,b = A_s\,f_y$$

$$a = \frac{As\,fy}{0.85\,fc'\,b} = \frac{1140.4\ x\ 280}{0.85\ x\ 28\ x\ 200} = 67.1\ mm$$

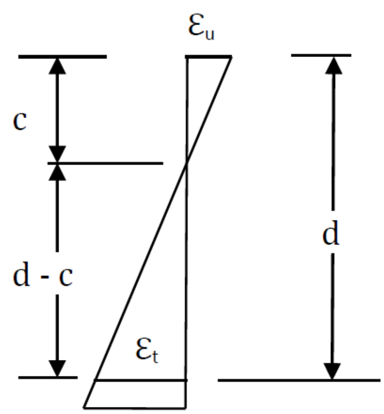
$$M_n = N_t\,Z$$

$$M_n = A_s\,f_y\left(d - \frac{a}{2}\right) = 1140.4\ x\ 280\left(390 - \frac{67.1}{2}\right) = 113.8\ x\ 10^6\ N.mm$$

$$\boldsymbol{M_n = 113.8\ kN.m}$$

The design moment = $\phi\ M_n$

$$\boldsymbol{c = \frac{a}{0.85} = 78.94\ mm}$$



$$\varepsilon_t = \varepsilon u \left(\frac{d-c}{c}\right) = 0.003\ \left(\frac{390-78.94}{78.94}\right) = 0.01182$$

$$\varepsilon_{ty} = \frac{280}{200000} = 0.0014$$

$$\because \varepsilon_t > \varepsilon_{ty} + 0.003\ (0.0044)$$

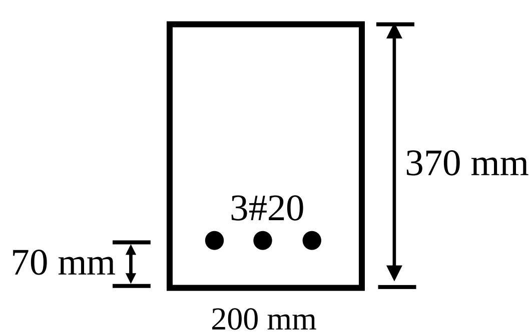
$$\therefore \varphi = 0.9$$

∴ Design moment $\boldsymbol{M_u}$ = ΦM_n = 0.9 x 113.8 = **102.42 kN.m**

∵ $M_{u\ \text{provid}}$ (102.42 kN.m) > $M_{u\ \text{required}}$ (89.86 kN.m)

∴ The section is adequacy

Example: Determine the design moment capacity ΦM_n of the beam shown in Figure below, if f_c' = 21 *MPa* and f_y = 280 *MPa*.



$$As = (3)\frac{\pi\, d^2}{4} = (3)\frac{\pi\ 20^2}{4} = 3 * 314 = 942\ mm^2$$

$$\rho_{act.} = \frac{As}{b\,d} = \frac{942}{200\ x\ 300} = 0.0157$$

$$\rho_{min} = \frac{0.25\sqrt{fc'}}{f_y} = \frac{0.25\sqrt{21}}{280} = 0.00409$$

Or

$$\rho_{min} = \frac{1.4}{f_y} = \frac{1.4}{280} = \mathbf{0.005} \qquad \textbf{control}$$

$$\rho_{max} = 0.85\,\beta_1\,\frac{fc'}{f_y}\,\frac{\varepsilon u}{\varepsilon u + 0.004}$$

$$fc' = 21MPa \ \rightarrow \ \beta_1 = 0.85 \qquad (17 \le fc' \le 28)$$

$$\rho_{max} = 0.85\ x\ 0.85\,\frac{21}{280}\,\frac{0.003}{0.003 + 0.004} = 0.02322$$

$$\rho_{min} < \rho < \rho_{max}$$

∴ The beam is **under** reinforcement.

$$Nc = Nt$$

$$0.85\,fc'\,a\,b = A_s\,f_y$$

$$a = \frac{As\,fy}{0.85\,fc'\,b} = \frac{942\ x\ 280}{0.85\ x\ 21\ x\ 200} = 73.9\ mm$$

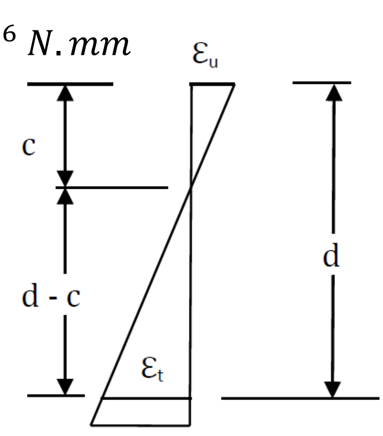
$$M_n = N_t\,Z$$

$$M_n = A_s\,f_y\left(d - \frac{a}{2}\right) = 942\ x\ 280\left(300 - \frac{73.9}{2}\right) = 69.4\ x\ 10^6\ N.mm$$

$$\boldsymbol{M_n = 69.4\ kN.m}$$

The design moment = $\phi\,M_n$

$$\boldsymbol{c = \frac{a}{0.85} = 86.94\ mm}$$



$$\varepsilon_t = \varepsilon u \left(\frac{d-c}{c}\right) = 0.003 \left(\frac{300-86.94}{86.94}\right) = 0.00735$$

$$\varepsilon_{ty} = \frac{280}{200000} = 0.0014$$

$$\because \varepsilon_t > \varepsilon_{ty} + 0.003\ (0.0044)$$

$$\therefore \varphi = 0.9$$

$\therefore$ Design moment = ΦM_n = 0.9 x 69.4 = 62.46 kN.m

H.W.: Resolve the previous example using A_s = 3 #28 mm.