Lateral Earth Pressure

Vertical or near-vertical slopes of soil are supported by retaining walls, cantilever sheetpile walls, and other, similar structures. The proper design of those structures requires an estimation of lateral earth pressure, which is a function of several factors, such as (a) the type and amount of wall movement, (b) the shear strength parameters of the soil, (c) the unit weight of the soil, and (d) the drainage conditions in the backfill. Figure 12.1 shows a retaining wall of height *H*. For similar types of backfill,

a. The wall may be restrained from moving (Figure 12.1a). The lateral earth pressure on the wall at any depth is called the *at-rest earth pressure.*

b. The wall may tilt away from the soil that is retained (Figure 12.1b). With sufficient wall tilt, a triangular soil wedge behind the wall will fail. The lateral pressure for this condition is referred to as *active earth pressure.*

c. The wall may be pushed into the soil that is retained (Figure 12.1c). With sufficient wall movement, a soil wedge will fail. The lateral pressure for this condition is referred to as *passive earth pressure.*

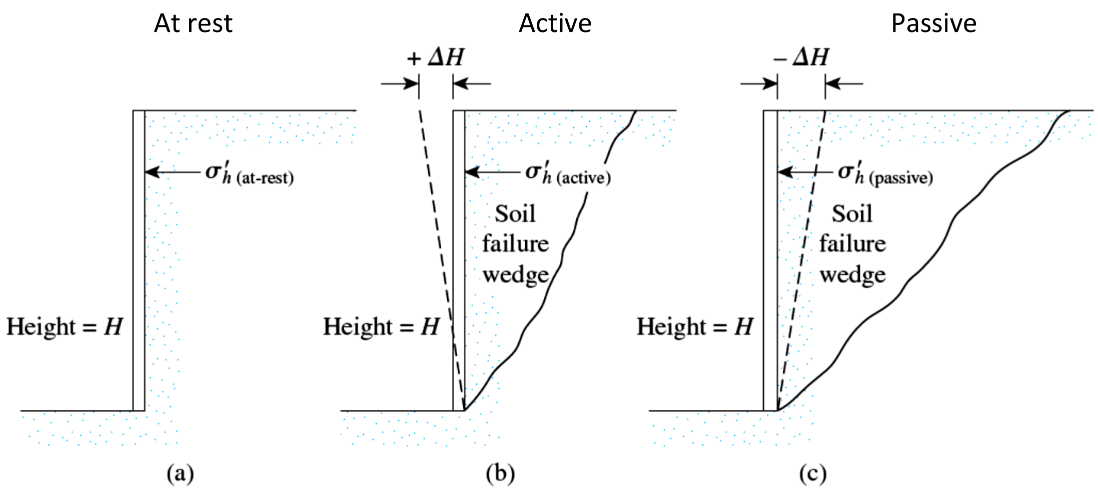


Figure 12.1 Nature of lateral earth pressure on a retaining wall

Lateral Earth Pressure at Rest

Consider a vertical wall of height *H*, as shown in Figure 12.3, retaining a soil having a unit weight of γ. A uniformly distributed load, q/unit area, is also applied at the ground surface. The shear strength of the soil is

$$s = c' + \sigma' \tan \phi'$$

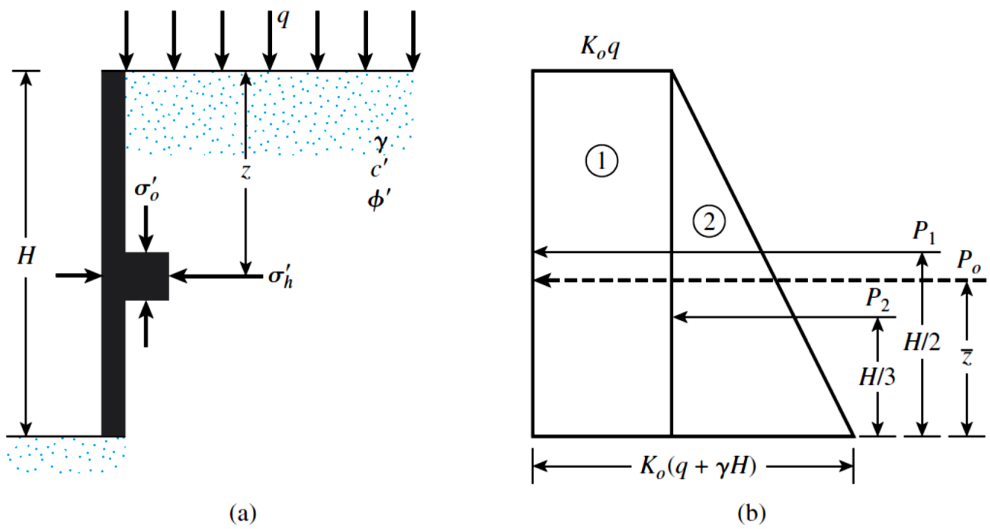


Figure 12.3 At-rest earth pressure

where

c' = cohesion

ϕ' = effective angle of friction

σ'_0 = effective normal stress

At any depth z below the ground surface, the vertical subsurface stress is

$$\sigma'_o = q + \gamma z \qquad \text{.................. } 12.1$$

If the *wall is at rest and is not allowed to move at all*, either away from the soil mass or into the soil mass (i.e., there is zero horizontal strain), the lateral pressure at a depth z is

$$\sigma_h = K_o \sigma'_o + u \qquad \text{................}12.2$$

where

u = pore water pressure

K_o = coefficient of at-rest earth pressure

For normally consolidated soil, the relation for K_o (Jaky, 1944) is

$$K_o = 1 - \sin \phi' \qquad \text{....................................... } 12.3$$

The above equation is an empirical approximation.

For overconsolidated soil, the at-rest earth pressure coefficient may be expressed as:

$$K_o = (1 - \sin \phi')\mathrm{OCR}^{\sin \phi'} \qquad \text{..................}12.4$$

where OCR = overconsolidation ratio.

The total force, *Po* , *per unit length* of the wall given in Figure 12.3a can now be obtained from the area of the pressure diagram given in Figure 12.3b and is

$$P_o = P_1 + P_2 = qK_oH + \tfrac{1}{2}\gamma H^2 K_o \qquad (12.5)$$

where

P_1 = area of rectangle 1

P_2 = area of triangle 2

The location of the line of action of the resultant force, P_o , can be obtained by taking the moment about the bottom of the wall. Thus,

$$\bar{z} = \frac{P_1\left(\frac{H}{2}\right) + P_2\left(\frac{H}{3}\right)}{P_o} \qquad (12.6)$$

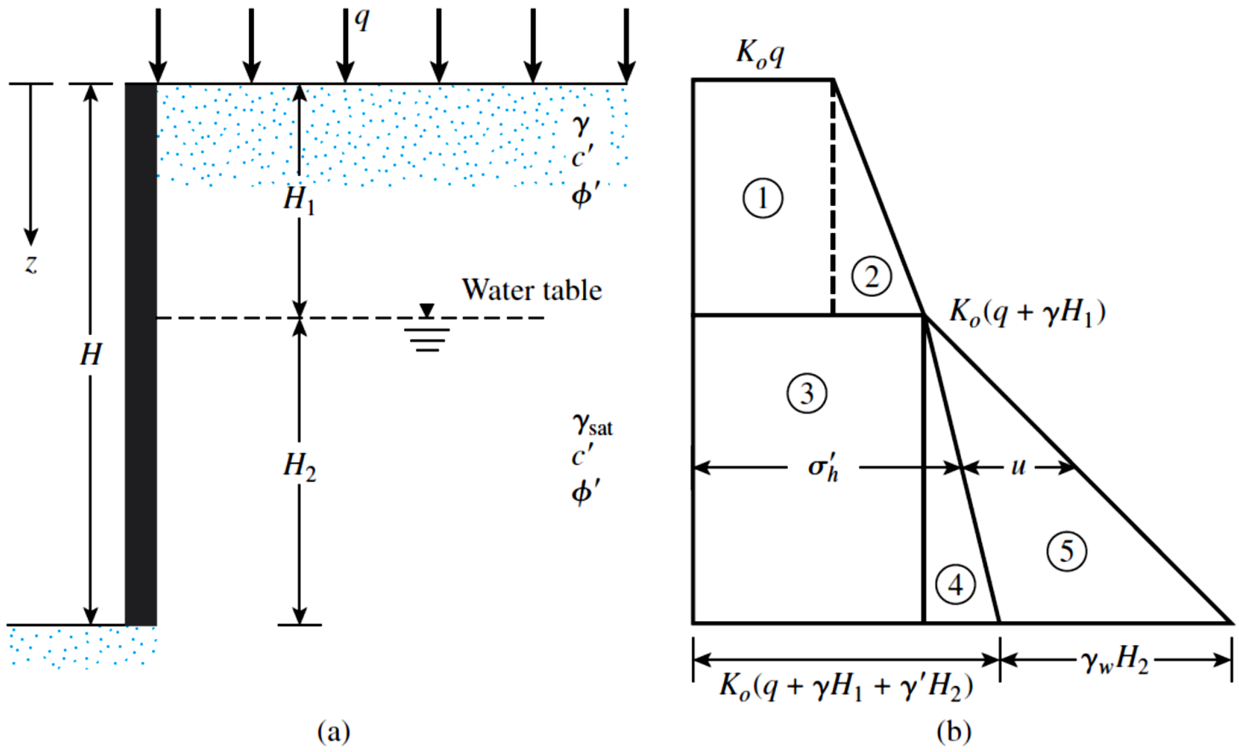


Figure 12.4 At-rest earth pressure with water table located at a depth $z < H$

If the water table is located at a depth $z < H$, the at-rest pressure diagram shown in Figure 12.3b will have to be somewhat modified, as shown in Figure 12.4. If the effective unit weight of soil below the water table equals γ' (i.e., $\gamma_{sat} - \gamma_w$), then

at $z = 0$, $\quad \sigma'_h = K_o\sigma'_o = K_o q$

at $z = H_1$, $\quad \sigma'_h = K_o\sigma'_o = K_o(q + \gamma H_1)$

and

at $z = H_2$, $\quad \sigma'_h = K_o\sigma'_o = K_o(q + \gamma H_1 + \gamma' H_2)$

Note that in the preceding equations, σ'_0 and σ'_h are effective vertical and horizontal pressures, respectively. Determining the total pressure distribution on the wall requires adding the hydrostatic pressure u, which is zero from $z = 0$ to $z = H_1$ and is $H_2\gamma_w$ at $z = H_2$. The variation of σ'_h and u with depth is shown in Figure 12.4b. Hence, the total force per unit length of the wall can be determined from the area of the pressure diagram. Specifically,

$$P_o = A_1 + A_2 + A_3 + A_4 + A_5$$

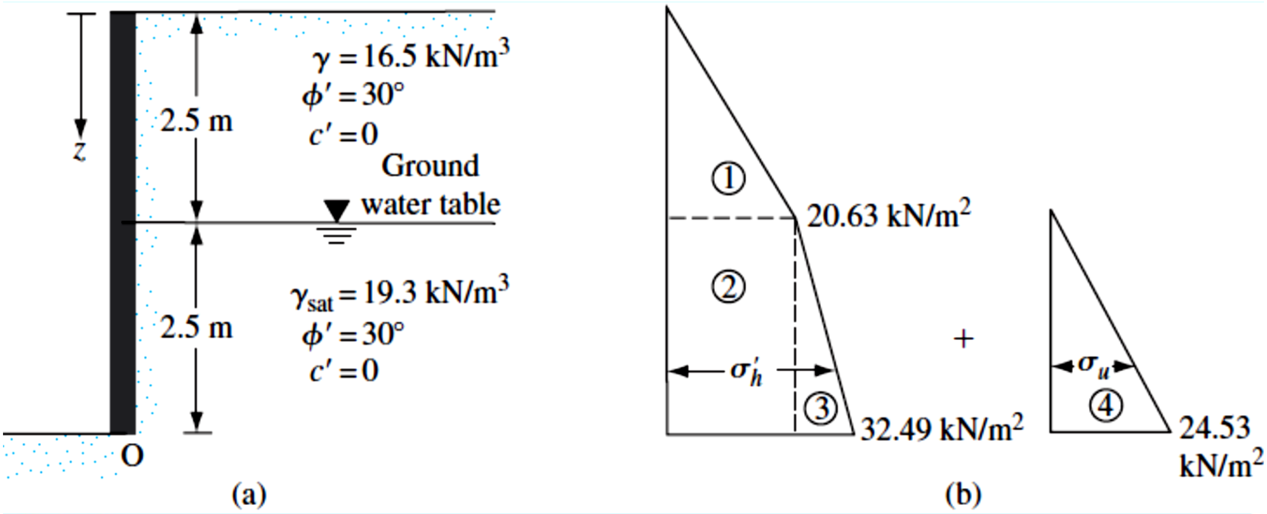
where A = area of the pressure diagram.

So,

$$P_o = K_o q H_1 + \tfrac{1}{2}K_o\gamma H_1^2 + K_o(q + \gamma H_1)H_2 + \tfrac{1}{2}K_o\gamma' H_2^2 + \tfrac{1}{2}\gamma_w H_2^2 \tag{12.7}$$

Example 12.1

For the retaining wall shown in Figure 12.5a, determine the lateral earth force at rest per unit length of the wall. Also determine the location of the resultant force. Assume OCR = 1.



Solution

$$K_o = 1 - \sin\phi' = 1 - \sin 30° = 0.5$$

At $z = 0$, $\sigma'_o = 0$; $\sigma'_h = 0$

At $z = 2.5$ m, $\sigma'_o = (16.5)(2.5) = 41.25$ kN/m²;

$$\sigma'_h = K_o\sigma'_o = (0.5)(41.25) = 20.63 \text{ kN/m}^2$$

At $z = 5$ m, $\sigma'_o = (16.5)(2.5) + (19.3 - 9.81)2.5 = 64.98$ kN/m²;

$$\sigma'_h = K_o\sigma'_o = (0.5)(64.98) = 32.49 \text{ kN/m}^2$$

The hydrostatic pressure distribution is as follows:

From $z = 0$ to $z = 2.5$ m, $u = 0$. At $z = 5$ m, $u = \gamma_w(2.5) = (9.81)(2.5) = 24.53$ kN/m². The pressure distribution for the wall is shown in Figure 12.5b.

The total force per unit length of the wall can be determined from the area of the pressure diagram, or

$$P_o = \text{Area 1} + \text{Area 2} + \text{Area 3} + \text{Area 4}$$

$$= \tfrac{1}{2}(2.5)(20.63) + (2.5)(20.63) + \tfrac{1}{2}(2.5)(32.49 - 20.63)$$

$$+ \tfrac{1}{2}(2.5)(24.53) = \mathbf{122.85\ kN/m}$$

The location of the center of pressure measured from the bottom of the wall (point O) =

$$\bar{z} = \frac{(\text{Area 1})\left(2.5 + \frac{2.5}{3}\right) + (\text{Area 2})\left(\frac{2.5}{2}\right) + (\text{Area 3} + \text{Area 4})\left(\frac{2.5}{3}\right)}{P_o}$$

$$= \frac{(25.788)(3.33) + (51.575)(1.25) + (14.825 + 30.663)(0.833)}{122.85}$$

$$= \frac{85.87 + 64.47 + 37.89}{122.85} = \mathbf{1.53\ m}$$

■

Rankine Active Earth Pressure

If a wall tends to move away from the soil a distance Δx, as shown in Figure 12.6a, the soil pressure on the wall at any depth will decrease. For a wall that is *frictionless,* the horizontal stress, σ'_h, at depth z will equal $K_o\,\sigma'_o = k_o\,\gamma\,z$ when Δx is zero. However, with $\Delta x > 0$, σ'_h will be less than $K_o\,\sigma'_o$.

$$\sigma'_a = \sigma'_o \tan^2\left(45 - \frac{\phi'}{2}\right) - 2c' \tan\left(45 - \frac{\phi'}{2}\right) = \sigma'_o K_a - 2c'\sqrt{K_a} \tag{12.8}$$

where $K_a = \tan^2(45 - \phi'/2)$ = Rankine active-pressure coefficient.

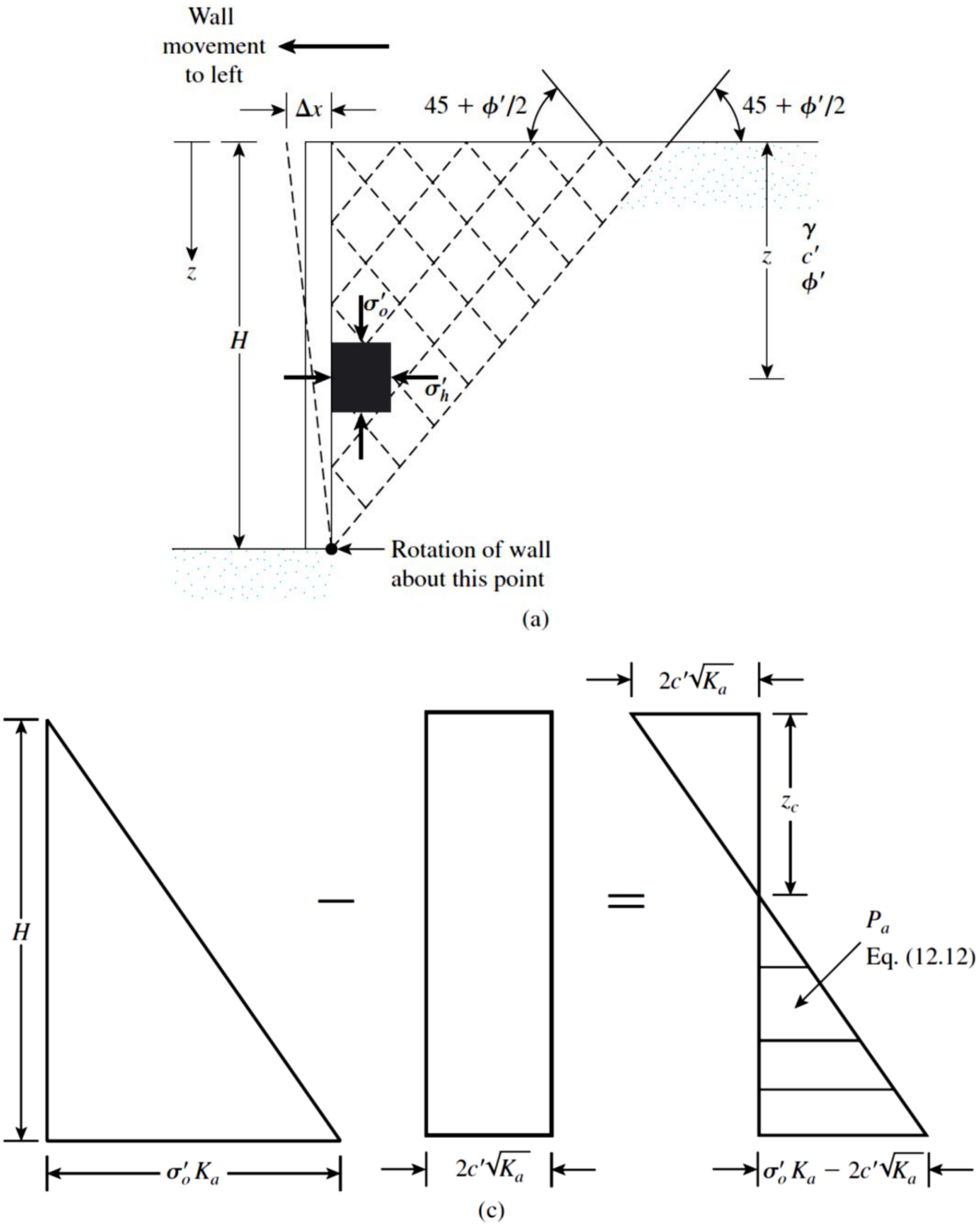


Figure 12.6 Rankine active pressure

The variation of the active pressure with depth for the wall shown in Figure 12.6a is given in Figure 12.6c. Note that $\sigma'_o = 0$ at $z = 0$ and $\sigma'_o = \gamma H$ at $z = H$. The pressure distribution shows that at $z = 0$ the active pressure equals $-2c'\sqrt{K_a}$, indicating a tensile stress that decreases with depth and becomes zero at a depth $z = z_c$, or

$$\gamma z_c K_a - 2c'\sqrt{K_a} = 0$$

and

$$z_c = \frac{2c'}{\gamma\sqrt{K_a}} \tag{12.9}$$

The depth z_c is usually referred to as the *depth of tensile crack,* because the tensile stress in the soil will eventually cause a crack along the soil–wall interface. Thus, the total Rankine active force per unit length of the wall before the tensile crack occurs is

$$P_a = \int_0^H \sigma'_a dz = \int_0^H \gamma z K_a dz - \int_0^H 2c'\sqrt{K_a} dz$$

$$= \tfrac{1}{2}\gamma H^2 K_a - 2c'H\sqrt{K_a} \tag{12.10}$$

After the tensile crack appears, the force per unit length on the wall will be caused only by the pressure distribution between depths $z = z_c$ and $z = H$, as shown by the hatched area in Figure 12.6c. This force may be expressed as

$$P_a = \tfrac{1}{2}(H - z_c)(\gamma H K_a - 2c'\sqrt{K_a}) \tag{12.11}$$

or

$$P_a = \frac{1}{2}\left(H - \frac{2c'}{\gamma\sqrt{K_a}}\right)\left(\gamma H K_a - 2c'\sqrt{K_a}\right) \tag{12.12}$$

However, it is important to realize that the active earth pressure condition will be reached only if the wall is allowed to "yield" sufficiently. The necessary amount of outward displacement of the wall is about $0.001H$ to $0.004H$ for granular soil backfills and about $0.01H$ to $0.04H$ for cohesive soil backfills.

Note further that if the *total stress* shear strength parameters (c, ϕ) were used, an equation similar to Eq. (12.8) could have been derived, namely,

$$\sigma_a = \sigma_o \tan^2\left(45 - \frac{\phi}{2}\right) - 2c \tan\left(45 - \frac{\phi}{2}\right)$$

Example 12.2

A 6-m-high retaining wall is to support a soil with unit weight $\gamma = 17.4$ kN/m^3, soil friction angle $\phi' = 26°$, and cohesion $c' = 14.36$ kN/m^2. Determine the Rankine active force per unit length of the wall both before and after the tensile crack occurs, and determine the line of action of the resultant in both cases.

Solution

For $\phi' = 26°$,

$$K_a = \tan^2\left(45 - \frac{\phi'}{2}\right) = \tan^2(45 - 13) = 0.39$$

$$\sqrt{K_a} = 0.625$$

$$\sigma'_a = \gamma H K_a - 2c'\sqrt{K_a}$$

From Figure 12.6c, at $z = 0$,

$$\sigma'_a = -2c'\sqrt{K_a} = -2(14.36)(0.625) = -17.95 \text{ kN/m}^2$$

and at $z = 6$ m,

$$\sigma'_a = (17.4)(6)(0.39) - 2(14.36)(0.625)$$
$$= 40.72 - 17.95 = 22.77 \text{ kN/m}^2$$

Active Force before the Tensile Crack Appeared: Eq. (12.10)

$$P_a = \tfrac{1}{2}\gamma H^2 K_a - 2c'H\sqrt{K_a}$$
$$= \tfrac{1}{2}(6)(40.72) - (6)(17.95) = 122.16 - 107.7 = \mathbf{14.46\ kN/m}$$

The line of action of the resultant can be determined by taking the moment of the area of the pressure diagrams about the bottom of the wall, or

$$P_a\bar{z} = (122.16)\left(\frac{6}{3}\right) - (107.7)\left(\frac{6}{2}\right)$$

Thus,

$$\bar{z} = \frac{244.32 - 323.1}{14.46} = \mathbf{-5.45\ m.}$$

Active Force after the Tensile Crack Appeared: Eq. (12.9)

$$z_c = \frac{2c'}{\gamma\sqrt{K_a}} = \frac{2(14.36)}{(17.4)(0.625)} = 2.64 \text{ m}$$

Using Eq. (12.11) gives

$$P_a = \frac{1}{2}(H - z_c)(\gamma H K_a - 2c'\sqrt{K_a}) = \frac{1}{2}(6 - 2.64)(22.77) = \mathbf{38.25\ kN/m}$$

Figure 12.6c indicates that the force $P_a = 38.25$ kN/m is the area of the hatched triangle. Hence, the line of action of the resultant will be located at a height $\bar{z} = (H - z_c)/3$ above the bottom of the wall, or

$$\bar{z} = \frac{6 - 2.64}{3} = \mathbf{1.12\ m}$$

■

Example 12.3

Assume that the retaining wall shown in Figure 12.7a can yield sufficiently to develop an active state. Determine the Rankine active force per unit length of the wall and the location of the resultant line of action.

Solution

If the cohesion, c', is zero, then

$$\sigma'_a = \sigma'_o K_a$$

For the top layer of soil, $\phi'_1 = 30°$, so

$$K_{a(1)} = \tan^2\left(45 - \frac{\phi'_1}{2}\right) = \tan^2(45 - 15) = \frac{1}{3}$$

Similarly, for the bottom layer of soil, $\phi'_2 = 36°$, and it follows that

$$K_{a(2)} = \tan^2\left(45 - \frac{36}{2}\right) = 0.26$$

The following table shows the calculation of σ'_a and u at various depths below the ground surface.

Depth, Z (m)	σ'_0 (kN/m^2)	K_a	$\sigma'_a = K_a\sigma'_0$ (kN/m^2)	U (kN/m^2)
0	0	1/3	0	0
3^-	17x3 = 51	1/3	17	0
3^+	51	0.26	13.26	0
6	17x3 + (19-9.8)x3 = 78.6	0.26	20.44	9.81x3 = 29.43

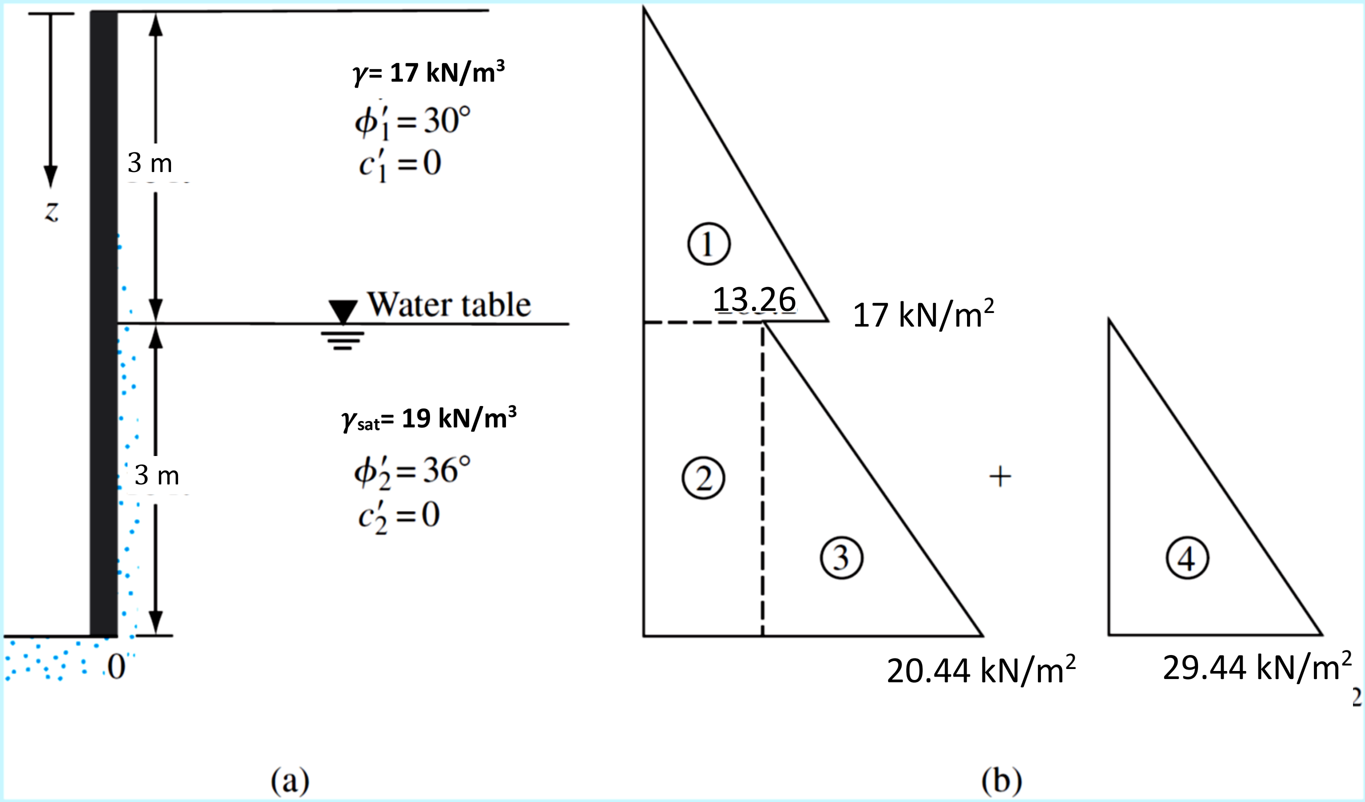


Figure 12.7 Rankine active force behind a retaining wall

The pressure distribution diagram is plotted in Figure 12.7b. The force per unit length is

$$P_a = \text{area 1} + \text{area 2} + \text{area 3} + \text{area 4}$$

$$= \frac{1}{2} \text{ x } 3 \text{ x } 17 + 13.26 \text{ x } 3 + \frac{1}{2}(20.44 - 13.26)\text{x}3 + \frac{1}{2} \text{ x } 29.44 \text{ x}3$$

$$= 25.5 + 39.78 + 10.77 + 44.16 = 120.21 \text{ kN/m}$$

The distance of the line of action of the resultant force from the bottom of the wall can be determined by taking the moments about the bottom of the wall (point O in Figure 12.7a) and is

$$\bar{z} = \frac{(25.5)x\left(3+\frac{3}{3}\right)+(39.78)x\left(\frac{3}{2}\right)+(10.77+44.16)\left(\frac{3}{3}\right)}{120.21} = 1.8 \text{ m}$$

A Generalized Case for Rankine Active Pressure—Granular Backfill

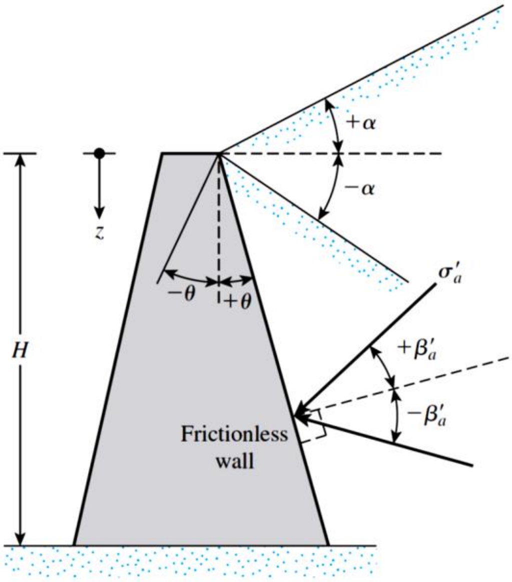


Figure 12.8 General case for a retaining wall with granular backfill

For a Rankine active case, the lateral earth pressure (σ'_a) at a depth z can be given as (Chu, 1991),

$$\sigma'_a = \frac{\gamma z \cos\alpha \sqrt{1 + \sin^2\phi' - 2\sin\phi' \cos\psi_a}}{\cos\alpha + \sqrt{\sin^2\phi' - \sin^2\alpha}} \tag{12.13}$$

$$\text{where } \psi_a = \sin^{-1}\left(\frac{\sin\alpha}{\sin\phi'}\right) - \alpha + 2\theta. \tag{12.14}$$

The pressure σ'_a will be inclined at an angle β'_a with the plane drawn at right angle to the backface of the wall, and

$$\beta'_a = \tan^{-1}\left(\frac{\sin\phi' \sin\psi_a}{1 - \sin\phi' \cos\psi_a}\right) \tag{12.15}$$

The active force P_a for unit length of the wall then can be calculated as

$$P_a = \frac{1}{2}\gamma H^2 K_a \tag{12.16}$$

where

$$K_{a(R)} = \frac{\cos(\alpha - \theta)\sqrt{1 + \sin^2\phi' - 2\sin\phi' \cos\psi_a}}{\cos^2\theta\left(\cos\alpha + \sqrt{\sin^2\phi' - \sin^2\alpha}\right)}$$

$$= \text{Rankine active earth-pressure coefficient for generalized case} \tag{12.17}$$

The location and direction of the resultant force P_a is shown in Figure 12.9. Also shown in this figure is the failure wedge, ABC. Note that BC will be inclined at an angle η. Or

$$\eta_a = \frac{\pi}{4} + \frac{\phi'}{2} + \frac{\alpha}{2} - \frac{1}{2}\sin^{-1}\left(\frac{\sin\alpha}{\sin\phi'}\right) \tag{12.18}$$

Tables 12.1 and 12.2 give the variations of K_a [Eq. (12.17)] and β'_a [Eq. (12.15)] for various values of α, θ, and ϕ'.

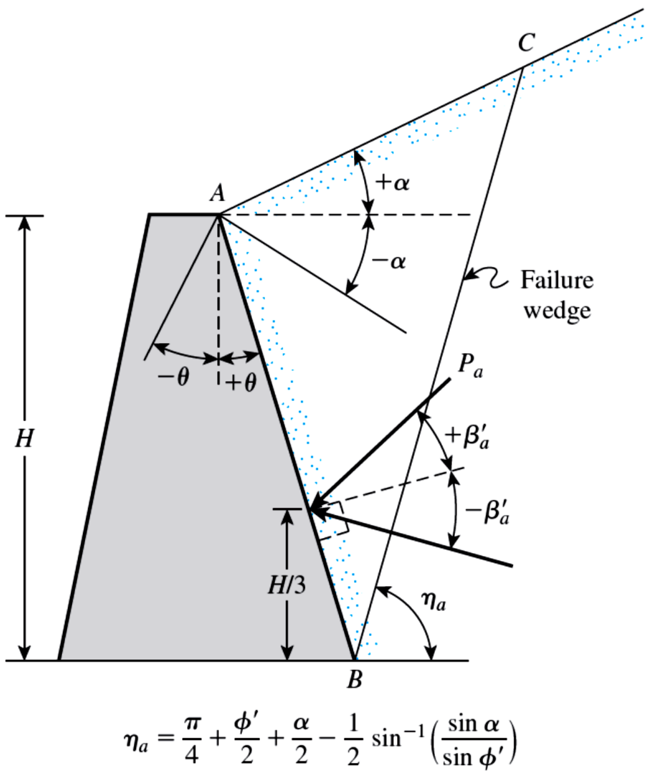


Figure 12.9 Location and direction of Rankine active force

Table 12.1 Variation of $K_{a(R)}$ [Eq. (12.17)]

α (deg)	θ (deg)	$K_{a(R)}$ ϕ' (deg) 28	30	32	34	36	38	40
0	0	0.361	0.333	0.307	0.283	0.260	0.238	0.217
	2	0.363	0.335	0.309	0.285	0.262	0.240	0.220
	4	0.368	0.341	0.315	0.291	0.269	0.248	0.228
	6	0.376	0.350	0.325	0.302	0.280	0.260	0.242
	8	0.387	0.362	0.338	0.316	0.295	0.276	0.259
	10	0.402	0.377	0.354	0.333	0.314	0.296	0.280
	15	0.450	0.428	0.408	0.390	0.373	0.358	0.345
5	0	0.366	0.337	0.311	0.286	0.262	0.240	0.219
	2	0.373	0.344	0.317	0.292	0.269	0.247	0.226
	4	0.383	0.354	0.328	0.303	0.280	0.259	0.239
	6	0.396	0.368	0.342	0.318	0.296	0.275	0.255
	8	0.412	0.385	0.360	0.336	0.315	0.295	0.276
	10	0.431	0.405	0.380	0.358	0.337	0.318	0.300
	15	0.490	0.466	0.443	0.423	0.405	0.388	0.373
10	0	0.380	0.350	0.321	0.294	0.270	0.246	0.225
	2	0.393	0.362	0.333	0.306	0.281	0.258	0.236
	4	0.408	0.377	0.348	0.322	0.297	0.274	0.252
	6	0.426	0.395	0.367	0.341	0.316	0.294	0.273
	8	0.447	0.417	0.389	0.363	0.339	0.317	0.297
	10	0.471	0.441	0.414	0.388	0.365	0.344	0.324
	15	0.542	0.513	0.487	0.463	0.442	0.422	0.404
15	0	0.409	0.373	0.341	0.311	0.283	0.258	0.235
	2	0.427	0.391	0.358	0.328	0.300	0.274	0.250
	4	0.448	0.411	0.378	0.348	0.320	0.294	0.271
	6	0.472	0.435	0.402	0.371	0.344	0.318	0.295
	8	0.498	0.461	0.428	0.398	0.371	0.346	0.323
	10	0.527	0.490	0.457	0.428	0.400	0.376	0.353
	15	0.610	0.574	0.542	0.513	0.487	0.463	0.442
20	0	0.461	0.414	0.374	0.338	0.306	0.277	0.250
	2	0.486	0.438	0.397	0.360	0.328	0.298	0.271
	4	0.513	0.465	0.423	0.386	0.353	0.323	0.296
	6	0.543	0.495	0.452	0.415	0.381	0.351	0.324
	8	0.576	0.527	0.484	0.446	0.413	0.383	0.355
	10	0.612	0.562	0.518	0.481	0.447	0.417	0.390
	15	0.711	0.660	0.616	0.578	0.545	0.515	0.488

Table 12.2 Variation of β'_a [Eq. (12.15)]

α (deg)	θ (deg)	β'_a — φ′ (deg) 28	30	32	34	36	38	40
0	0	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	2	3.525	3.981	4.484	5.041	5.661	6.351	7.124
	4	6.962	7.848	8.821	9.893	11.075	12.381	13.827
	6	10.231	11.501	12.884	14.394	16.040	17.837	19.797
	8	13.270	14.861	16.579	18.432	20.428	22.575	24.876
	10	16.031	17.878	19.850	21.951	24.184	26.547	29.039
	15	21.582	23.794	26.091	28.464	30.905	33.402	35.940
5	0	5.000	5.000	5.000	5.000	5.000	5.000	5.000
	2	8.375	8.820	9.311	9.854	10.455	11.123	11.870
	4	11.553	12.404	13.336	14.358	15.482	16.719	18.085
	6	14.478	15.679	16.983	18.401	19.942	21.618	23.441
	8	17.112	18.601	20.203	21.924	23.773	25.755	27.876
	10	19.435	21.150	22.975	24.915	26.971	29.144	31.434
	15	23.881	25.922	28.039	30.227	32.479	34.787	37.140
10	0	10.000	10.000	10.000	10.000	10.000	10.000	10.000
	2	13.057	13.491	13.967	14.491	15.070	15.712	16.426
	4	15.839	16.657	17.547	18.519	19.583	20.751	22.034
	6	18.319	19.460	20.693	22.026	23.469	25.032	26.726
	8	20.483	21.888	23.391	24.999	26.720	28.559	30.522
	10	22.335	23.946	25.653	27.460	29.370	31.385	33.504
	15	25.683	27.603	29.589	31.639	33.747	35.908	38.114
15	0	15.000	15.000	15.000	15.000	15.000	15.000	15.000
	2	17.576	18.001	18.463	18.967	19.522	20.134	20.812
	4	19.840	20.631	21.485	22.410	23.417	24.516	25.719
	6	21.788	22.886	24.060	25.321	26.677	28.139	29.716
	8	23.431	24.778	26.206	27.722	29.335	31.052	32.878
	10	24.783	26.328	27.950	29.654	31.447	33.332	35.310
	15	27.032	28.888	30.793	32.747	34.751	36.802	38.894
20	0	20.000	20.000	20.000	20.000	20.000	20.000	20.000
	2	21.925	22.350	22.803	23.291	23.822	24.404	25.045
	4	23.545	24.332	25.164	26.054	27.011	28.048	29.175
	6	24.876	25.966	27.109	28.317	29.604	30.980	32.455
	8	25.938	27.279	28.669	30.124	31.657	33.276	34.989
	10	26.755	28.297	29.882	31.524	33.235	35.021	36.886
	15	27.866	29.747	31.638	33.552	35.498	37.478	39.491

Example 12.4

Refer to the retaining wall in Figure 12.9. The backfill is granular soil. Given:

Wall: $H = 3$ m

$\theta = +10^{o}$

Backfill: $\alpha = 15^{o}$

$\phi' = 35^{o}$

$c' = 0$

$\gamma = 18$ kN/m^3

Determine the Rankine active force, P_a, and its location and direction.

Solution

From Table 12.1, for α = 15° and θ = + 10°, the value of K_a $\approx$ 0.42. From Eq. (12.16),

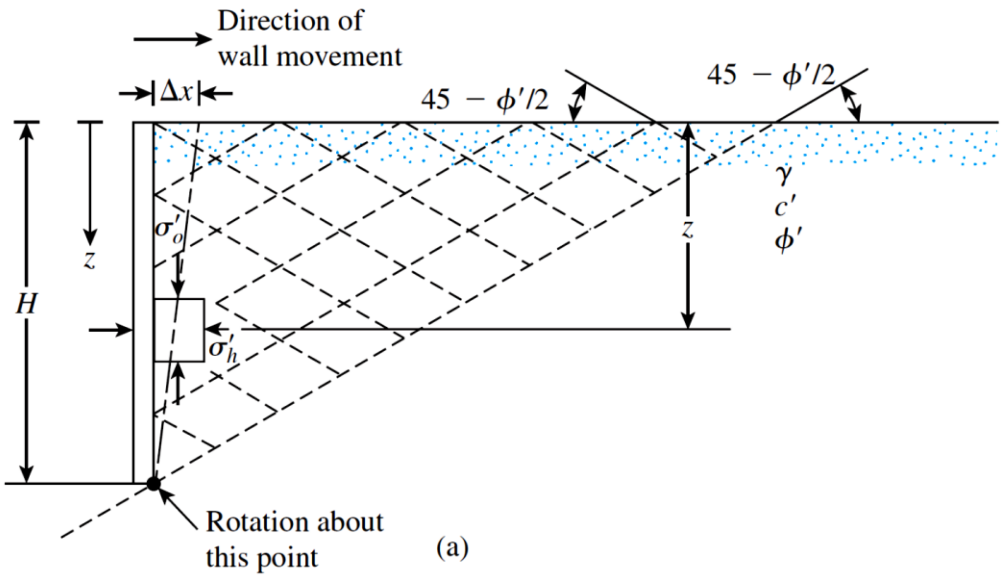
$$P_a = \frac{1}{2}\,\gamma\, H^2\, K_a = \frac{1}{2}\,(18)\,(3)^2\,(0.42) = 102.1 \text{ kN/m}$$

Again, from Table 12.2, for α = 15° and θ = + 10°, β'_a. = 30.5°

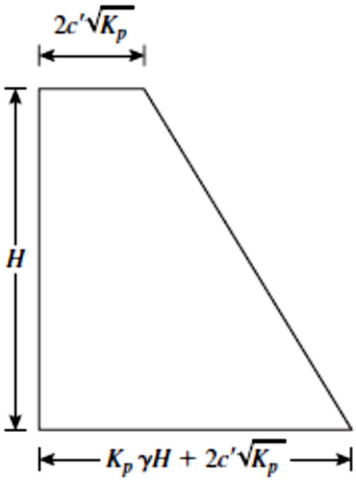
The force P_a will act at a distance of 3.0/3 = 1 m above the bottom of the wall and will be inclined at an angle of +30.5° to the normal drawn to the back face of the wall.

Rankine Passive Earth Pressure

If the wall is pushed into the soil mass by an amount Δx, as shown in Figure below, the vertical stress at depth z will stay the same; however, the horizontal stress will increase. The horizontal stress, σ'_h, at this point is referred to as the ***Rankine passive pressure***, or $\sigma'_h = \sigma'_p$



(a)



(c)

$$\sigma'_p = \sigma'_o \tan^2\left(45 + \frac{\phi'}{2}\right) + 2c' \tan\left(45 + \frac{\phi'}{2}\right) \qquad (12.56)$$

Now, let

$$K_p = \text{Rankine passive earth pressure coefficient} = \tan^2\left(45 + \frac{\phi'}{2}\right) \quad (12.57)$$

Then, from Eq. (12.56), we have

$$\sigma'_p = \sigma'_o K_p + 2c'\sqrt{K_p} \quad (12.58)$$

Equation (12.58) produces (Figure 12.19c), the passive pressure diagram for the wall shown in Figure 12.19a. Note that at $z = 0$,

$$\sigma'_o = 0 \quad \text{and} \quad \sigma'_p = 2c'\sqrt{K_p}$$

and at $z = H$,

$$\sigma'_o = \gamma H \quad \text{and} \quad \sigma'_p = \gamma H K_p + 2c'\sqrt{K_p}$$

The passive force per unit length of the wall can be determined from the area of the pressure diagram, or

$$P_p = \tfrac{1}{2}\gamma H^2 K_p + 2c'H\sqrt{K_p} \quad (12.59)$$

The approximate magnitudes of the wall movements, Δx, required to develop failure under passive conditions are as follows:

Soil type	**Wall movement for passive condition, Δx**
Dense sand	$0.005H$
Loose sand	$0.01H$
Stiff clay	$0.01H$
Soft clay	$0.05H$

If the backfill behind the wall is a granular soil (i.e., $c'=0$), then, from Eq. (12.59), the passive force per unit length of the wall will be

$$P_p = \frac{1}{2}\gamma H^2 K_p \quad (12.60)$$

Example 12.13

A 3-m-high wall is shown in Figure 12.20a. Determine the Rankine passive force per unit length of the wall.

Solution

For the top layer

$$K_{p(1)} = \tan^2\left(45 + \frac{\phi_1'}{2}\right) = \tan^2(45 + 15) = 3$$

From the bottom soil layer

$$K_{p(2)} = \tan^2\left(45 + \frac{\phi_2'}{2}\right) = \tan^2(45 + 13) = 2.56$$

$$\sigma_p' = \sigma_o' K_p + 2c'\sqrt{K_p}$$

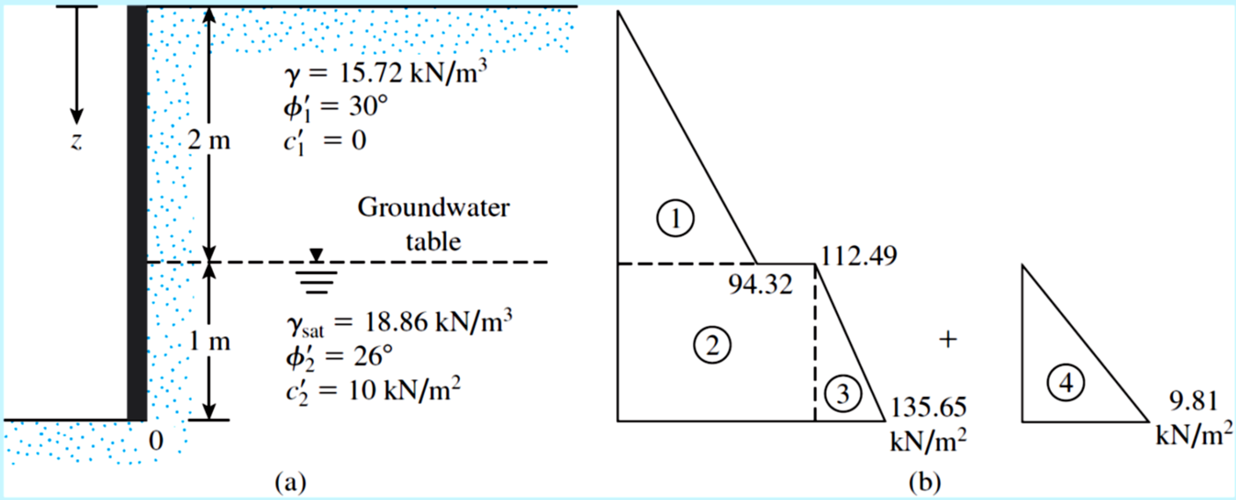


Figure 12.20

where

σ'_o = effective vertical stress
at $z = 0$, $\sigma'_o = 0$, $c'_1 = 0$, $\sigma'_p = 0$
at $z = 2$ m, $\sigma'_o = (15.72)(2) = 31.44$ kN/m², $c'_1 = 0$

So, for the top soil layer

$$\sigma'_p = 31.44K_{p(1)} + 2(0)\sqrt{K_{p(1)}} = 31.44(3) = 94.32 \text{ kN/m}^2$$

At this depth, that is $z = 2$ m, for the bottom soil layer

$$\sigma'_p = \sigma'_o K_{p(2)} + 2c'_2\sqrt{K_{p(2)}} = 31.44(2.56) + 2(10)\sqrt{2.56}$$
$$= 80.49 + 32 = 112.49 \text{ kN/m}^2$$

Again, at $z = 3$ m,

$$\sigma'_o = (15.72)(2) + (\gamma_{sat} - \gamma_w)(1)$$
$$= 31.44 + (18.86 - 9.81)(1) = 40.49 \text{ kN/m}^2$$

Hence,

$$\sigma'_p = \sigma'_o K_{p(2)} + 2c'_2\sqrt{K_{p(2)}} = 40.49(2.56) + (2)(10)(1.6)$$
$$= \mathbf{135.65\ kN/m^2}$$

Note that, because a water table is present, the hydrostatic stress, u, also has to be taken into consideration. For $z = 0$ to 2 m, $u = 0$; $z = 3$ m, $u = (1)(\gamma_w) = 9.81$ kN/m².

The passive pressure diagram is plotted in Figure 12.20b. The passive force per unit length of the wall can be determined from the area of the pressure diagram as follows:

Area no.	Area	
1	$\left(\frac{1}{2}\right)(2)(94.32)$	= 94.32
2	$(112.49)(1)$	= 112.49
3	$\left(\frac{1}{2}\right)(1)(135.65 - 112.49)$	= 11.58
4	$\left(\frac{1}{2}\right)(9.81)(1)$	= 4.905
		$P_P \approx$ 223.3 **kN/m**

Rankine Passive Earth Pressure—Vertical Backface and Inclined Backfill

Granular Soil

For a frictionless vertical retaining wall (Figure 12.10) with a *granular backfill* ($c' = 0$), the Rankine passive pressure at any depth is

$$\sigma'_p = \gamma z K_p \qquad (12.61)$$

and the passive force is

$$P_p = \tfrac{1}{2}\gamma H^2 K_p \tag{12.62}$$

where

$$K_p = \cos\alpha \frac{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi'}}{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi'}} \tag{12.63}$$

As in the case of the active force, the resultant force, P_p, is inclined at an angle α with the horizontal and intersects the wall at a distance $H/3$ from the bottom of the wall. The values of K_p (the passive earth pressure coefficient) for various values of α and ϕ' are given in Table 12.9.

Table 12.9 Passive Earth Pressure Coefficient K_p [from Eq. (12.63)]

	ϕ' (deg)→						
↓α (deg)	28	30	32	34	36	38	40
0	2.770	3.000	3.255	3.537	3.852	4.204	4.599
5	2.715	2.943	3.196	3.476	3.788	4.136	4.527
10	2.551	2.775	3.022	3.295	3.598	3.937	4.316
15	2.284	2.502	2.740	3.003	3.293	3.615	3.977
20	1.918	2.132	2.362	2.612	2.886	3.189	3.526
25	1.434	1.664	1.894	2.135	2.394	2.676	2.987

$c'-\phi'$ Soil

If the backfill of the frictionless vertical retaining wall is a $c-\phi'$ soil (see Figure 12.10), then (Mazindrani and Ganjali, 1997)

$$\sigma_p' = \gamma z K_p = \gamma z K_p' \cos\alpha \tag{12.64}$$

where

$$K_p' = \frac{1}{\cos^2\phi'} \left\{ \begin{aligned} &2\cos^2\alpha + 2\left(\frac{c'}{\gamma z}\right)\cos\phi'\sin\phi' \\ &+ \sqrt{4\cos^2\alpha(\cos^2\alpha - \cos^2\phi') + 4\left(\frac{c'}{\gamma z}\right)^2\cos^2\phi' + 8\left(\frac{c'}{\gamma z}\right)\cos^2\alpha\sin\phi'\cos\phi'} \end{aligned} \right\} - 1 \tag{12.65}$$

The variation of K_p' with ϕ', α, and $c'/\gamma z$ is given in Table 12.10 (Mazindrani and Ganjali, 1997).

Table 12.10 Values of K'_p

ϕ' (deg)	α (deg)	$c'/\gamma z$ 0.025	0.050	0.100	0.500
15	0	1.764	1.829	1.959	3.002
	5	1.716	1.783	1.917	2.971
	10	1.564	1.641	1.788	2.880
	15	1.251	1.370	1.561	2.732
20	0	2.111	2.182	2.325	3.468
	5	2.067	2.140	2.285	3.435
	10	1.932	2.010	2.162	3.339
	15	1.696	1.786	1.956	3.183
25	0	2.542	2.621	2.778	4.034
	5	2.499	2.578	2.737	3.999
	10	2.368	2.450	2.614	3.895
	15	2.147	2.236	2.409	3.726
30	0	3.087	3.173	3.346	4.732
	5	3.042	3.129	3.303	4.674
	10	2.907	2.996	3.174	4.579
	15	2.684	2.777	2.961	4.394

Problems

12.2 Use Eq. (12.3), Figure P12.2, and the following values to determine the at-rest lateral earth force per unit length of the wall. Also find the location of the resultant. $H = 5$ m, $H_1 = 2$ m, $H_2 = 3$ m, $\gamma = 15.5$ kN/m^3, $\gamma_{sat} = 18.5$ kN/m^3, $\phi' = 34°$, $c' = 0$, $q = 20$ kN/m^2, and OCR = 1.

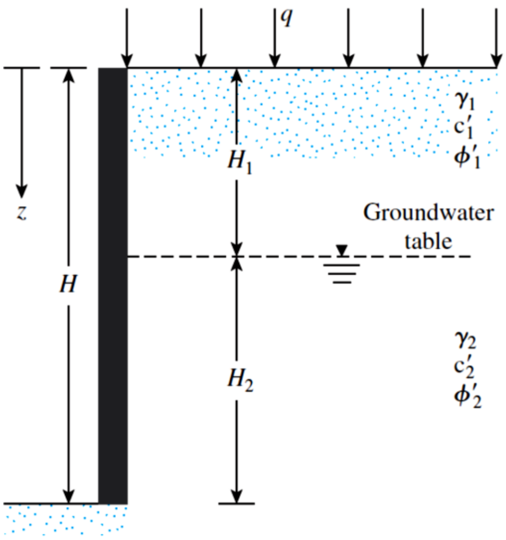


Figure P12.2

12.4 A vertical retaining wall (Figure 12.6a) is 7 m high with a horizontal backfill. For the backfill, assume that $\gamma = 16.5$ kN/m^3, $\phi' = 26°$, and $c' = 18$ kN/m^2. Determine the Rankine active force per unit length of the wall after the occurrence of the tensile crack.

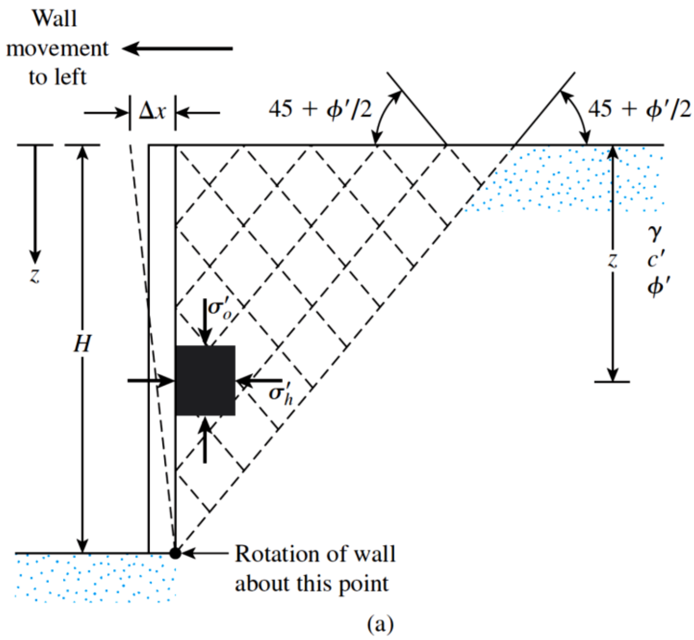


Fig. 12.6 a

12.5 Refer to Problem 12.2. For the retaining wall, determine the Rankine active force per unit length of the wall and the location of the line of action of the resultant.

12.6 Refer to Figure 12.10. For the retaining wall, $H = 8$ m, $\phi' = 36°$, $\alpha = 10°$, $\gamma = 17$ kN/m^3, and $c' = 0$.

a. Determine the intensity of the Rankine active force at $z = 2$ m, 4 m, and 6 m.

b. Determine the Rankine active force per meter length of the wall and also the location and direction of the resultant.

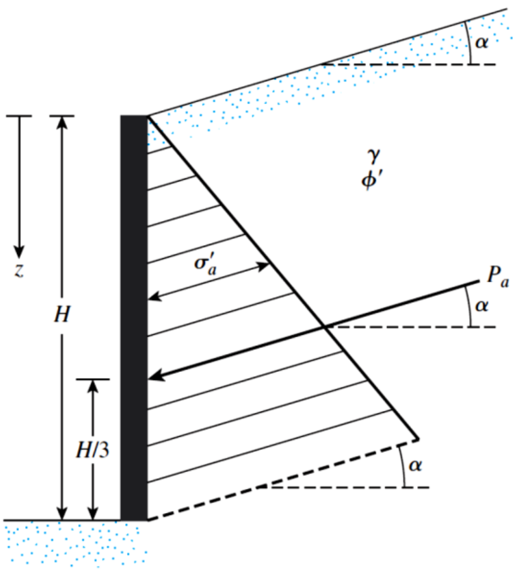


Figure 12.10 Notations for active pressure—Eqs. (12.19), (12.20), (12.21)

12.7 Refer to Figure 12.10. Given: $H = 7$ m, $\gamma = 18$ kN/m^3, $\phi' = 25°$, $c' = 12$ kN/m^2, and $\alpha = 10°$. Calculate the Rankine active force per unit length of the wall after the occurrence of the tensile crack.

12.3 Refer to Figure 12.6a. Given the height of the retaining wall, H is 5.4 m; the backfill is a saturated clay with $\Phi = 0$, $c = 40$ kN/m^2, $\gamma_{sat} = 19.5$kN/m^3,

a. Determine the Rankine active pressure distribution diagram behind the wall.

b. Determine the depth of the tensile crack, z_c.

c. Estimate the Rankine active force per meter length of the wall before and after the occurrence of the tensile crack.

12.13 Refer to Problem 12.3.

a. Draw the Rankine passive pressure distribution diagram behind the wall.

b. Estimate the Rankine passive force per meter length of the wall and also the location of the resultant.

Pile foundation

Pile foundations are used in the following conditions:

1. When one or more upper soil layers are highly compressible and too weak to support the load transmitted by the superstructure, piles are used to transmit the load to underlying bedrock or a stronger soil layer, as shown in Figure 9.1a. When bedrock is not encountered at a reasonable depth below the ground surface, piles are used to transmit the structural load to the soil gradually. The resistance to the applied structural load is derived mainly from the frictional resistance developed at the soil–pile interface. (See Figure 9.1b.)
2. When subjected to horizontal forces (see Figure 9.1c), pile foundations resist by bending, while still supporting the vertical load transmitted by the superstructure. This type of situation is generally encountered in the design and construction of earth-retaining structures and foundations of tall structures that are subjected to high wind or to earthquake forces.
3. In many cases, expansive and collapsible soils may be present at the site of a proposed structure. These soils may extend to a great depth below the ground surface.

Expansive soils swell and shrink as their moisture content increases and decreases, and the pressure of the swelling can be considerable. If shallow foundations are used in such circumstances, the structure may suffer considerable damage. However, pile foundations may be considered as an alternative when piles are extended beyond the active zone, which is where swelling and shrinking occur. (See Figure 9.1d.)

Soils such as loess are collapsible in nature. When the moisture content of these soils increases, their structures may break down. A sudden decrease in the void ratio of soil induces large settlements of structures supported by shallow foundations. In such cases, pile foundations may be used in which the piles are extended into stable soil layers beyond the zone where moisture will change.

4. The foundations of some structures, such as transmission towers, offshore platforms, and basement mats below the water table, are subjected to uplifting forces. Piles are sometimes used for these foundations to resist the uplifting force. (See Figure 9.1e.)
5. Bridge abutments and piers are usually constructed over pile foundations to avoid the loss of bearing capacity that a shallow foundation might suffer because of soil erosion at the ground surface. (Figure 9.1f.)

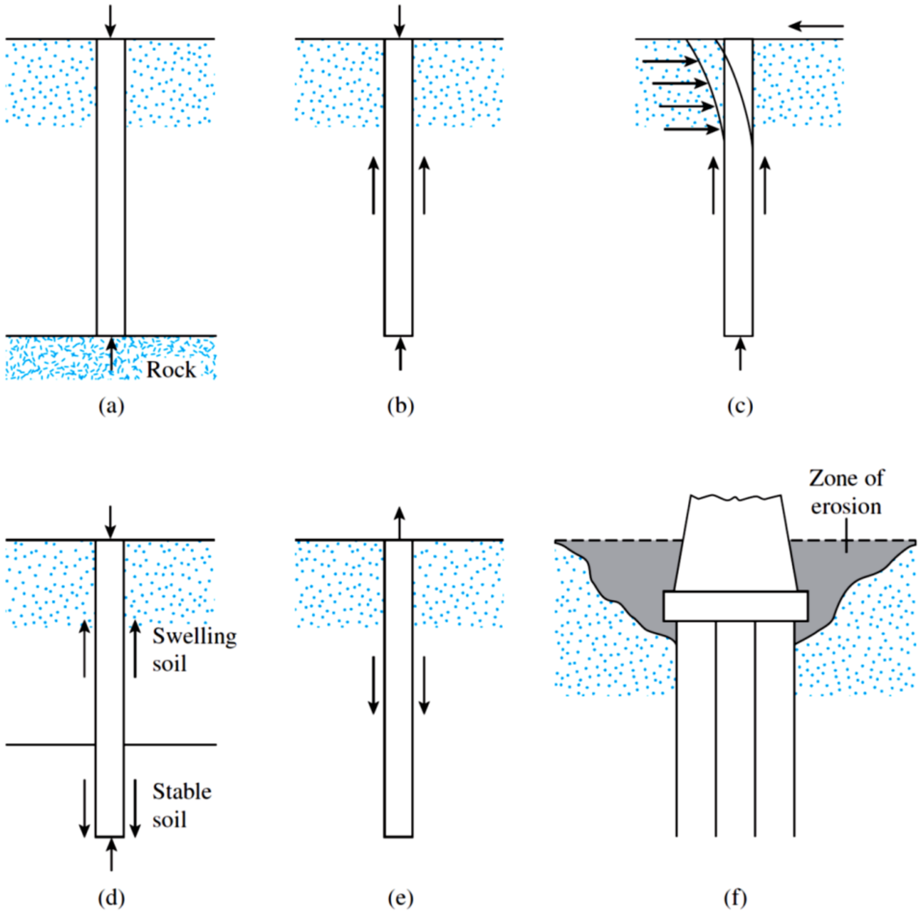


Figure 9.1 Conditions that require the use of pile foundations

Types of Piles Materials and Installation

Concrete piles

Several types of concrete piles are commonly used; these include cast-in-place concrete piles, precast concrete piles. Cast –in-place concrete piles are formed by driving a cylindrical steel shell into the ground to the desired length and then filling the cavity of the shell by fluid concrete. Various types of cast-in-place concrete piles are currently used in construction. These piles may be divided into two broad categories:

(a) cased and **(b)** uncased. Both types may have a pedestal at the bottom.

Cased piles are made by driving a steel casing into the ground with the help of a mandrel placed inside the casing. When the pile reaches the proper depth the mandrel is withdrawn and the casing is filled with concrete. Figure 9.4d shows some examples of cased piles without a pedestal. Figure 9.4e shows a cased pile with a pedestal. The pedestal is an expanded concrete bulb that is formed by dropping a hammer on fresh concrete. Precast concrete piles usually have square or circular or octagonal cross section and are fabricated in construction yard from reinforced or prestressed concrete.

Advantages of concrete piles:

a. Can be subjected to hard driving
b. Corrosion resistant
c. Can be easily combined with a concrete superstructure

Disadvantages:

a. Difficult to achieve proper cutoff

b. Difficult to transport

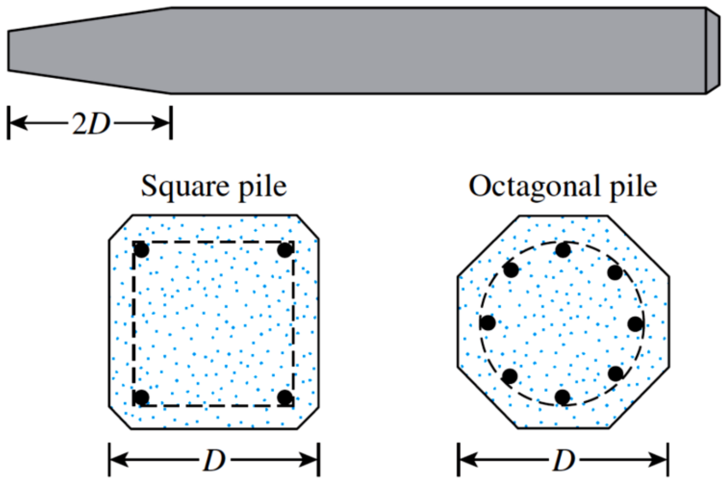


Figure 9.3 Precast piles with ordinary reinforcement

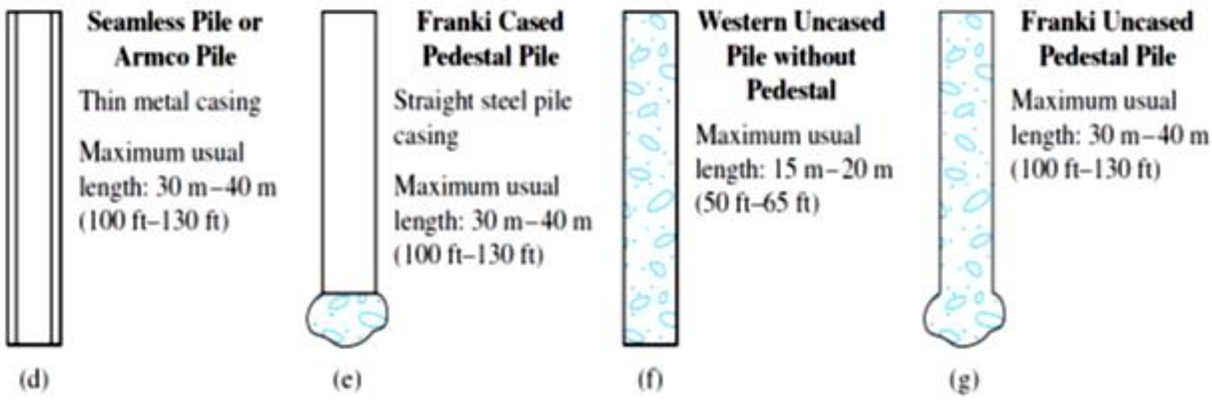


Figure 9.4 Cast-in-place concrete piles

Continuous Flight Auger (CFA) Piles

The continuous flight auger (CFA) piles are also referred to as auger-cast, auger-cast-inplace, and auger-pressure grout piles. CFA piles are constructed by using continuous flight augers and by drilling to the final depth in one continuous process. When the drilling to the final depth is complete, the auger is gradually withdrawn as concrete or sand/cement grout is pumped into the hole through the hollow center of the auger pipe to the base of the auger. Reinforcement, if needed, can be placed in CFA piles immediately after the withdrawal of the auger. The reinforcement is usually confined to the top 10 to 15 m of the pile.

In general, CFA piles are usually 0.3 to 0.9 m in diameter with a length up to about 30 m. In the United States, smaller diameter piles [i.e., 0.3 to 0.5 m] are generally used. However, piles with larger diameters [up to about 1.5 m] have been used. Typical center-to-center pile spacing is kept at 3 to 5 pile diameters. **Advantages of CFA piles are:**

a. Noise and vibration during construction are minimized.

b. Eliminates splicing and cutoff.

● **Disadvantages:**

a. Soil spoils need collection and disposal.

Steel Piles

Steel pile come in various shapes and sizes and include cylindrical seamless pipe, tapered and H –piles which is rolled steel sections , concrete-filled steel pile can be done by replacing the soil inside the tube by concrete to increase the load capacity.

Timber Piles

Timber piles have been used since ancient times with a common length of about 12 meters.

Pile Installation

Piles can be installed in a predrilled hole (bored piles or drilled shafts) by drilling a hole and either inserting a pile into it or, more commonly, filling the cavity with concrete, which produces a pile upon hardening.

Alternatively, the piles can be driven into the ground (driven piles). Driving can be done by:

1. Driving with a steady succession of blows on the top of the pile using a pile hammer. This produces both considerable noise and local vibrations, which may be disallowed by local codes or environmental agencies and, of course, may damage adjacent property.
2. Driving using a vibratory device attached to the top of the pile. This method is usually relatively quiet, and driving vibrations may not be excessive. The Method is more applicable in deposits with little cohesion.
3. Jacking the pile. This technique is more applicable for short stiff members.

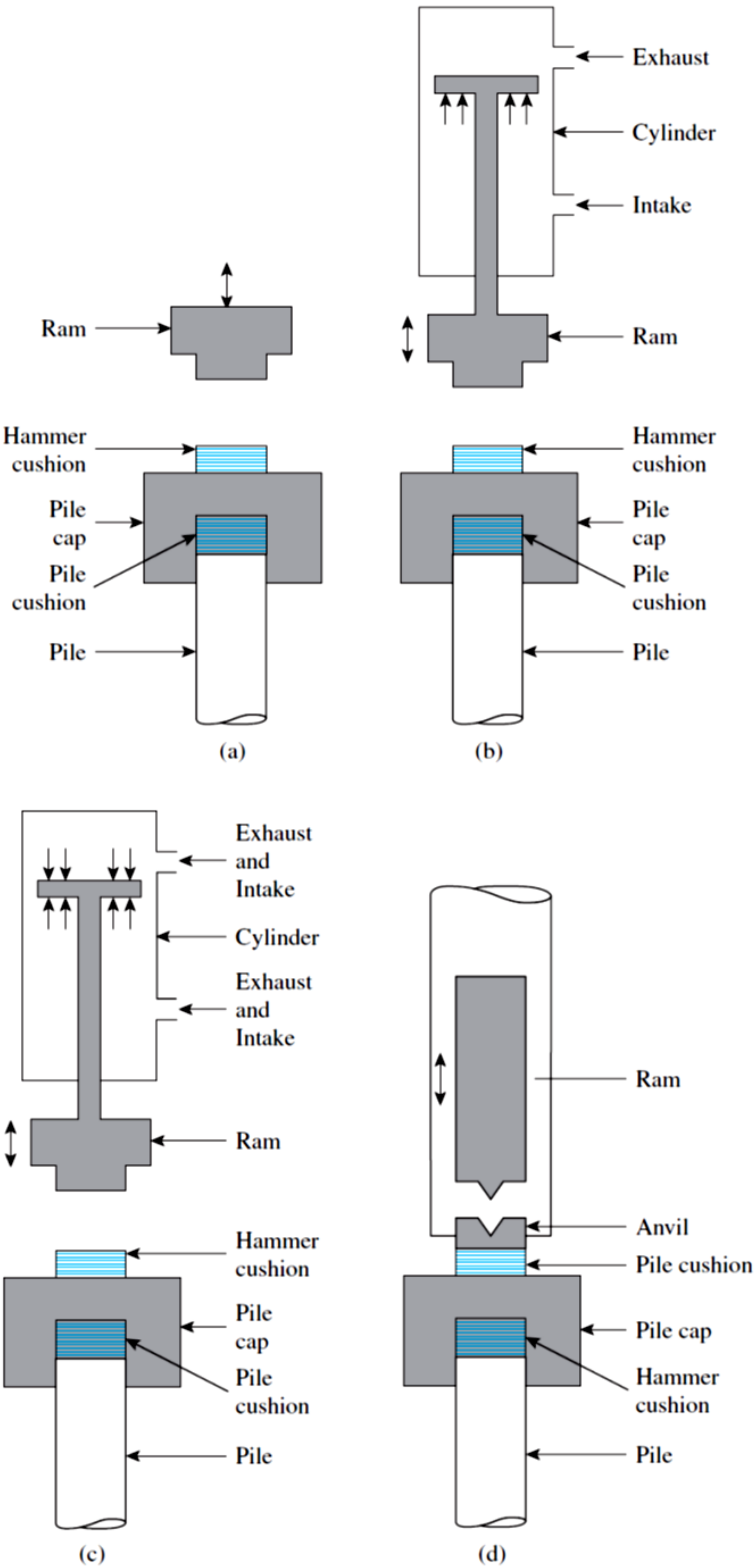
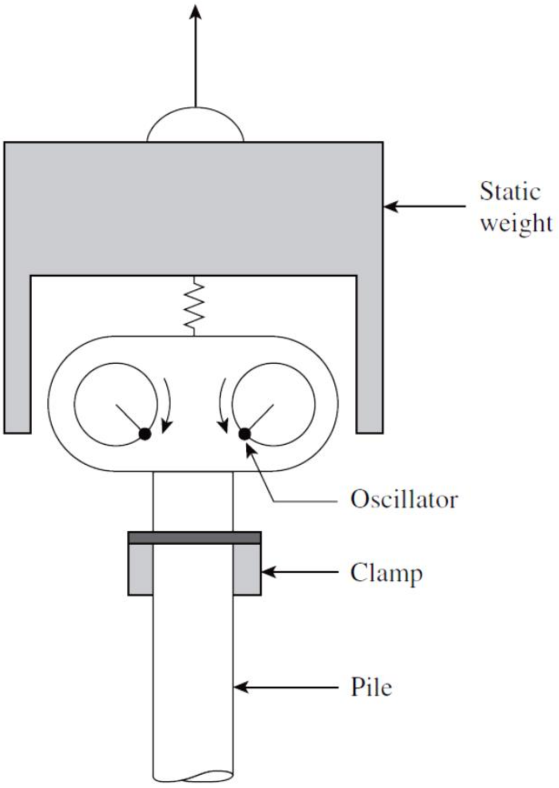


Figure 9.7 Pile-driving equipment: (a) drop hammer; (b) single-acting air or steam hammer; (c) double-acting and differential air or steam hammer; (d) diesel hammer



(e)

(f)

Figure 9.7 (*continued*) Pile-driving equipment: (e) vibratory pile driver; (f) photograph of a vibratory pile driver (*Courtesy of Reinforced Earth Company, Reston, Virginia*)

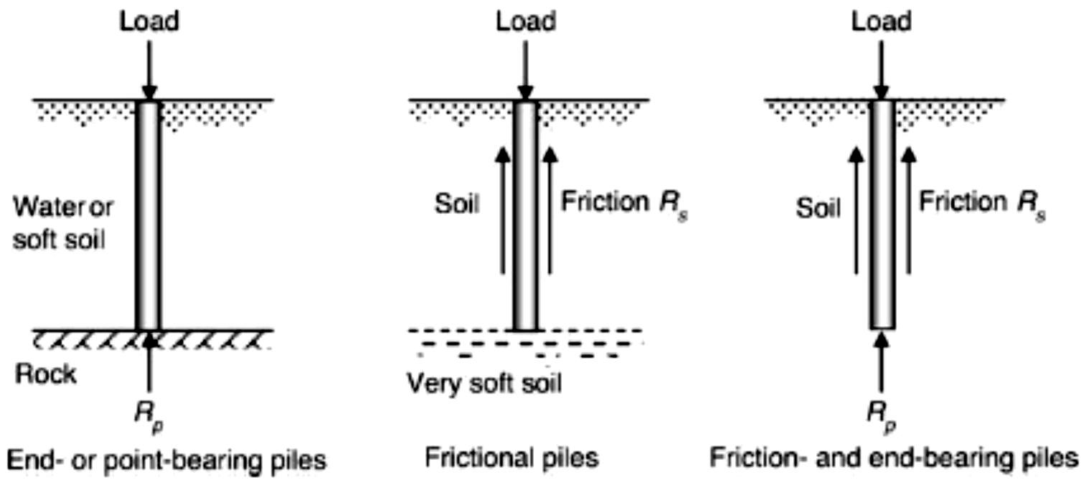
Axial Capacity of Piles in Compression

Axial capacity of piles primarily depends on how and where the applied loads are transferred into the ground. Based on the location of the load transfer in deep foundations, they can be classified as follows:

1. End- or point-bearing piles: The load is primarily distributed at the tip or base of the pile.

2. Frictional piles: The load is distributed primarily along the length of the pile through friction between the pile material and the surrounding soil.

3. Combination of friction and end bearing: The load is distributed both through friction along the length of the pile and at the tip or base of the pile.



$P_{ult} = P_p + P_s$

Pile in Cohesionless Soil

1. Point Capacity

If we incorporate the effect of shape and depth in determination of the N factors, the equation for bearing capacity of shallow foundations may be modified for deep foundations after neglecting the third part because of the small diameter or width of the piles as:

$$q_{ult} = c\,N_c^* + \bar{q}N_q^*$$

$$P_{pu} = (c\,N_c^* + \bar{q}N_q^*)\,A_p$$

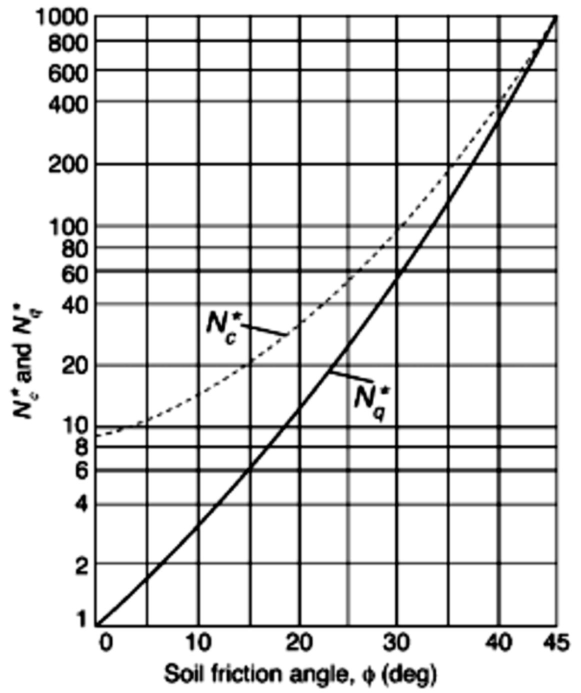


FIGURE 5.16 Meyerhof (1976) bearing capacity factors N_c^* and N_q^* (adapted from Das 1999).

Meyerhof Method Cohesionless soil

$$P_{pu} = \bar{q}N_q^*\;A_p\; < (50\;N_q^*\;tan\emptyset\;)\;A_p\;\;kN$$

2. Skin friction Capacity

Field studies have shown that the unit frictional resistance of piles embedded in **cohesionless soils** increases with depth. However, beyond a certain depth, the unit frictional resistance remains more or less constant, as illustrated; this depth, beyond which the unit frictional resistance does not increase, is called the critical depth and has been observed to vary between 15 to 20 times the pile diameter.

$$\boldsymbol{P_s = \sum A_s\,f_s}$$

Where:

A_s = effective pile surface area on which f_s acts

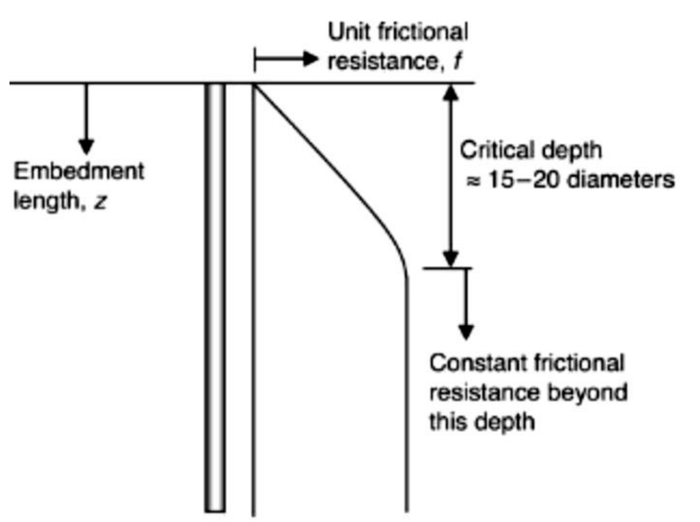
Skin resistance $f_s = K\,\sigma'_v\,tan\delta$

$K = K_0$ Bored or jetted piles

$K = 1.4\,K_0$ Low-displacement driven piles

$K = 1.8K_0$ High-displacement driven piles

where $K_0 = 1 - sin\,\phi$ for sands.



Example:

A concrete pile is 15 m long and 0.4x0.4 m in cross section, the pile is fully embedded in sand for which γ = 15.5 kN/m^3, and ϕ=30°. Calculate;

1. The ultimate point load of the pile?
2. The frictional resistance force if K=1.3 and friction angle between pile and soil δ= 0.8ϕ?
3. The allowable pile load, FS=4?

Solution:

1.

Using Meyerhof Method

$$P_{pu} = \bar{q}N_q^* \ A_p$$

From figure, for =30° N_q^* = 55

P_{pu} = 15.5 x 15 x 55 (0.4x0.4) = 2046 kN

Check with max. limit $(50\ N_q^*\ tan\emptyset\ )\ A_p$ = 50 x 55 x 0.577 x 0.4x0.4 = 254 kN

Use P_{pu} 254 kN

2.

Fs)$_o$ = K $\boldsymbol{\sigma}'_v$ tanδ =0

Critical depth = 20x pie diameter =20x0.4 =8m

Fs)$_8$ = K $\boldsymbol{\sigma}'_v$ tanδ = 1.3x 15.5 x 8 x tan (0.8x30) = 71.7 kN/m^2

$$P_s = \frac{0\ + 71.7}{2}(4 \times 0.4 \times 8)\ \ + 71.7 \times (1.6 \times 7) = 1262\ kN$$

3.

P_{ult}= P_p +P_s = 254 + 1262 = 1516 kN P_{all}= P_{ult}/FS = 1516/4 = 379 kN

Pile in Cohesive Soil

1. Point Capacity

In clay $\boldsymbol{\phi}$**=0** $\boldsymbol{q_u = c\ N_c^*}$

Bearing capacity factor $\boldsymbol{N_c^*}$ is commonly taken as **9**

$$\boldsymbol{P_u = 9\ c\ A_p}$$

2. Skin friction Capacity

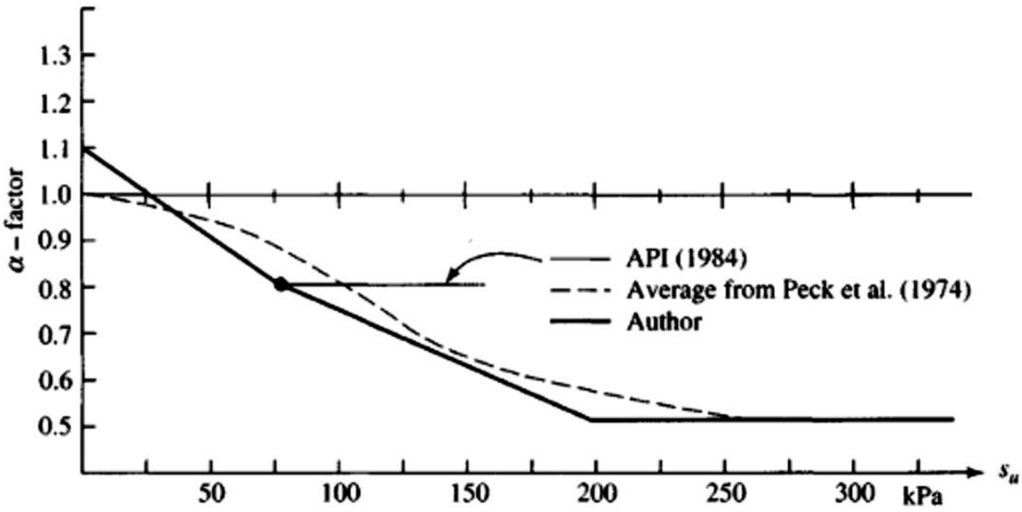
$$\boldsymbol{P_s = \sum A_s\, f_s}$$

$$\boldsymbol{f_s = \alpha\ c}$$

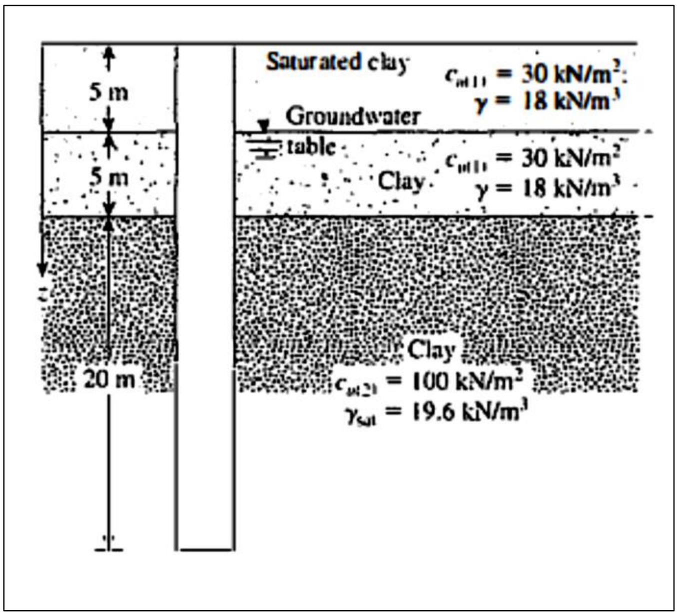
Where

α = coefficient from figure

c = average cohesion (or S_u) for the soil stratum of interest



Example:
A driven-pipe pile in clay is shown in figure.
The pipe has an outside diameter of 406 mm
a. Calculate the net point bearing capacity.
b. Calculate the skin resistance.
c. Estimate the net allowable pile capacity.
Use FS = 4.



Solution:
a.
$\mathbf{P_{pu} = 9\ C\ A_p}$ = 9x 100x (3.14x0.406^2/4)
= 116.5 kN
b.
Perimeter of the pile= 0.406x3.14=1.275 m
$\mathbf{P_s}$=30x0.95x5x1.275 +30x0.95x5x1.275 +100x0.72x20x1.275 =2200 kN
c.
$\mathbf{P_{ult}= P_p + P_s}$ =116.5 +2200= 2316.5 kN
$\mathbf{P_{all}= P_{ult}/FS}$ = 2316.5/4 = 580 kN

Correlations for Calculating Q_p with SPT and CPT Results in Granular Soil

On the basis of field observations, Meyerhof (1976) also suggested that the ultimate point resistance q_p in a homogeneous granular soil ($L = L_b$) may be obtained from standard penetration numbers as

$$q_p = 0.4 p_a N_{60} \frac{L}{D} \leq 4 p_a N_{60} \qquad (9.37)$$

Where
N_{60} = the average value of the standard penetration number near the pile point (about 10D above and 4D below the pile point)
p_a = atmospheric pressure ≈ 100 kN/m^2
Briaud et al. (1985) suggested the following correlation for q_p in granular soil with the standard penetration resistance N_{60}.
$q_p = 19.7 p_a (N60)^{0.36}$ (9.38)

Meyerhof (1956) also suggested that

$q_p \approx q_c$ (9.39)

where q_c = cone penetration resistance.

Example 9.3

Consider a concrete pile that is 0.305 m × 0.305 m in cross section in sand. The pile is 12 m long. The following are the variations of N_{60} with depth.

0

Depth below ground surface (m)	N_{60}
1.5	8
3.0	10
4.5	9
6.0	12
7.5	14
9.0	18
10.5	11
12.0	17
13.5	20
15.0	28
16.5	29
18.0	32
19.5	30
21.0	27

a. Estimate Q_p using Eq. (9.37).
b. Estimate Q_p using Eq. (9.38).

Solution

Part a

The tip of the pile is 12 m below the ground surface. For the pile, $D = 0.305$ m. The average of N_{60} $10D$ above and about $5D$ below the pile tip is

$$N_{60} = \frac{18 + 11 + 17 + 20}{4} = 16.5 \approx 17$$

From Eq. (9.37)

$$Q_p = A_p(q_p) = A_p\left[0.4p_aN_{60}\left(\frac{L}{D}\right)\right] \le A_p(4p_aN_{60})$$

$$A_p\left[0.4p_aN_{60}\left(\frac{L}{D}\right)\right] = (0.305 \times 0.305)\left[(0.4)(100)(17)\left(\frac{12}{0.305}\right)\right] = 2488.8 \text{ kN}$$

$$A_p(4p_aN_{60}) = (0.305 \times 0.305)[(4)(100)(17)] = 632.6 \text{ kN} \approx 633 \text{ kN}$$

Thus, Q_p = **633 kN**

Part b

From Eq. (9.38),

$$Q_p = A_pq_p = A_p[19.7p_a(N_{60})^{0.36}] = (0.305 \times 0.305)[(19.7)(100)(17)^{0.36}]$$

$$= \mathbf{508.2\ kN}$$

Frictional Resistance (Q_s) in Sand

Correlation with Standard Penetration Test Results

Meyerhof (1976) indicated that the average unit frictional resistance, f_{av} , for high-displacement driven piles may be obtained from average standard penetration resistance values as

$$f_{av} = 0.02p_a(\bar{N}_{60}) \qquad (9.45)$$

where

$\bar{N}_{60}$ = average value of standard penetration resistance

p_a = atmospheric pressure ≈100 kN/m^2

For low-displacement driven piles

$$f_{av} = 0.01p_a(\bar{N}_{60}) \qquad (9.46)$$

Briaud et al. (1985) suggested that

$$f_{av} \approx 0.224\ p_a(\bar{N}_{60})^{0.29} \qquad (9.47)$$

Thus,

$$Q_s = pLf_{av}$$

Example 9.4

Refer to the pile described in Example 9.3. Estimate the magnitude of Q_s for the pile.

a. Use Eq. (9.45).
b. Use Eq. (9.47).

c. Considering the results in Example 9.3, determine the allowable load-carrying capacity of the pile based on Meyerhof's method and Briaud's method. Use a factor of safety, FS = 3.

Solution

The average N_{60} value for the sand for the top 12 m is

$$\overline{N}_{60} = \frac{8 + 10 + 9 + 12 + 14 + 18 + 11 + 17}{8} = 12.375 \approx 12$$

Part a

From Eq. (9.45),

$$f_{av} = 0.02p_a(\overline{N}_{60}) = (0.02)(100)(12) = 24 \text{ kN/m}^2$$

$$Q_s = pLf_{av} = (4 \times 0.305)(12)(24) = 351.4 \text{ kN}$$

Part b

From Eq. (9.47),

$$f_{av} = 0.224p_a(\overline{N}_{60})^{0.29} = (0.224)(100)(12)^{0.29} = 46.05 \text{ kN/m}^2$$

$$Q_s = pLf_{av} = (4 \times 0.305)(12)(46.05) = 674.17 \text{ kN}$$

Part c

$$\text{Meyerhof's method: } Q_{all} = \frac{Q_p + Q_s}{FS} = \frac{633 + 351.4}{3} = 328.1 \text{ kN}$$

$$\text{Briaud's method: } Q_{all} = \frac{Q_p + Q_s}{FS} = \frac{508.2 + 674.2}{3} = 394.1 \text{ kN}$$

So the allowable pile capacity may be taken to be about 360 **kN**. ■

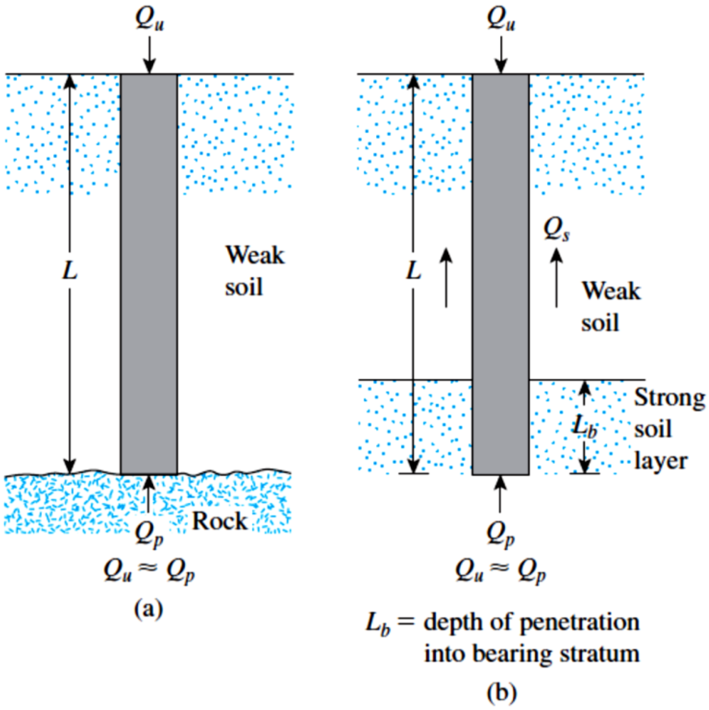


Figure 9.6 (a) and (b) Point bearing piles; (c) friction piles

Correlation with Cone Penetration Test Results

Nottingham and Schmertmann (1975) and Schmertmann (1978) provided correlations for estimating Q_s using the frictional resistance (f_c) obtained during cone penetration tests. According to this method

$$f = \alpha' f_c \tag{9.49}$$

The variations of α' with L/D for electric cone and mechanical cone penetrometers are shown in Figures 9.18 and 9.19, respectively. We have

$$Q_s = \Sigma p(\Delta L) f = \Sigma p(\Delta L)\alpha' f_c \tag{9.50}$$

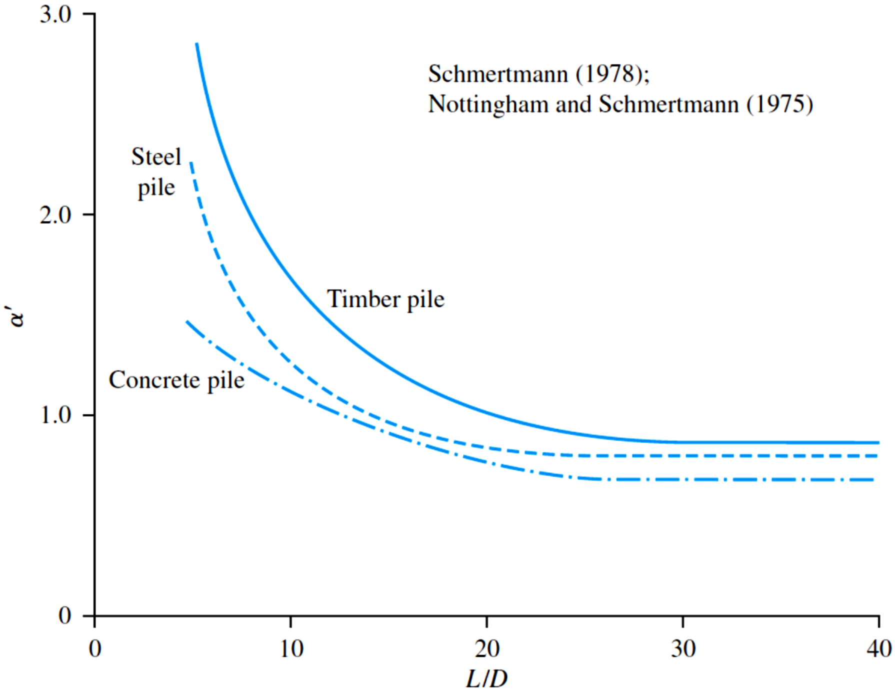


Figure 9.18 Variation of α' with embedment ratio for pile in sand: electric cone penetrometer

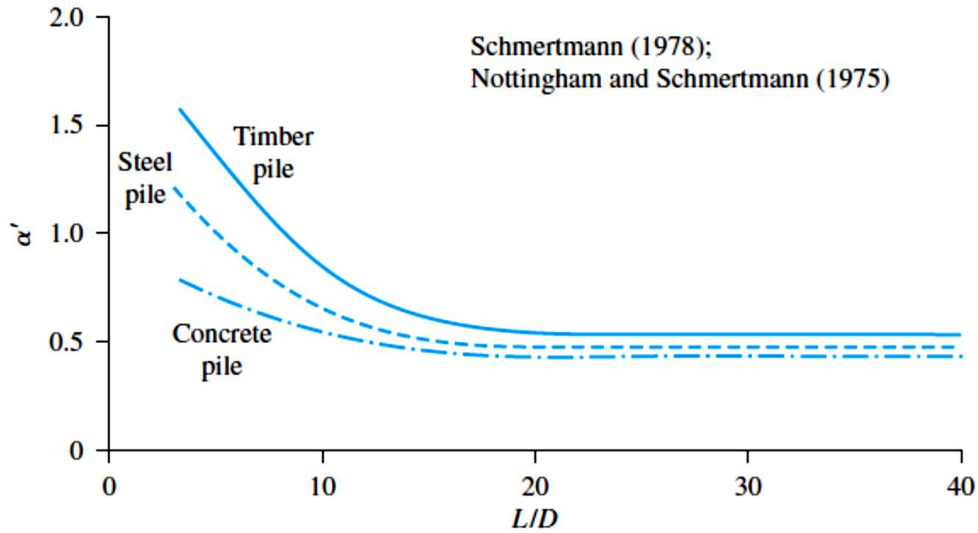


Figure 9.19 Variation of α' with embedment ratio for piles in sand: mechanical cone penetrometer

Example 9.6

Consider an 18-m-long concrete pile (cross section: 0.305 m × 0.305 m) fully embedded in a sand layer. For the sand layer, the following is an approximation of the cone penetration resistance q_c (mechanical cone) and the frictional resistance f_c with depth. Estimate the allowable load that the pile can carry. Use FS = 3.

Depth from ground surface (m)	q_c (kN/m²)	f_c (kN/m²)
0–5	3040	73
5–15	4560	102
15–25	9500	226

Solution

$$Q_u = Q_p + Q_s$$

From Eq. (9.39),

$$q_p \approx q_c$$

At the pile tip (i.e., at a depth of 18 m), $q_c \approx 9500$ kN/m². Thus,

$$Q_p = A_p q_c = (0.305 \times 0.305)(9500) = 883.7 \text{ kN}$$

To determine Q_s, the following table can be prepared. (*Note:* $L/D = 18/0.305 = 59$.)

Depth from ground surface (m)	ΔL (m)	f_c (kN/m²)	α' (Figure 9.19)	$p\Delta L\alpha' f_c$ (kN)
0–5	5	73	0.44	195.9
5–15	10	102	0.44	547.5
15–18	3	226	0.44	363.95

$$Q_s = 1107.35 \text{ kN}$$

Hence,

$$Q_u = Q_p + Q_s = 883.7 + 1107.35 = 1991.05 \text{ kN}$$

$$Q_{\text{all}} = \frac{Q_u}{\text{FS}} = \frac{1991.05}{3} = 663.68 \approx \mathbf{664 \text{ kN}}$$

■

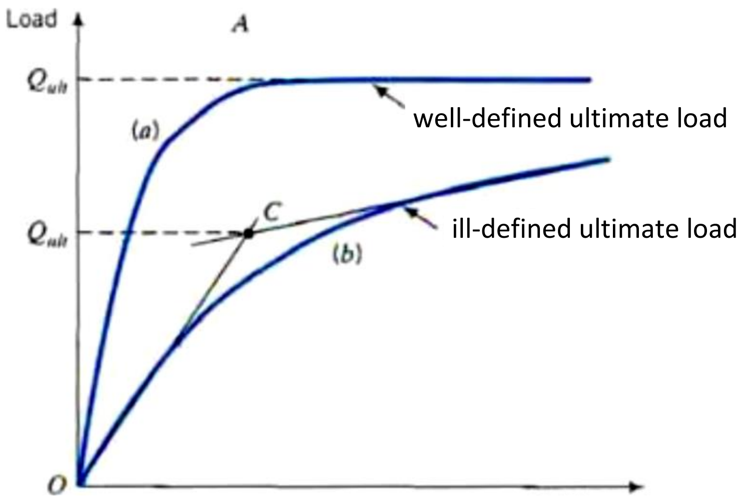
Pile Load Test

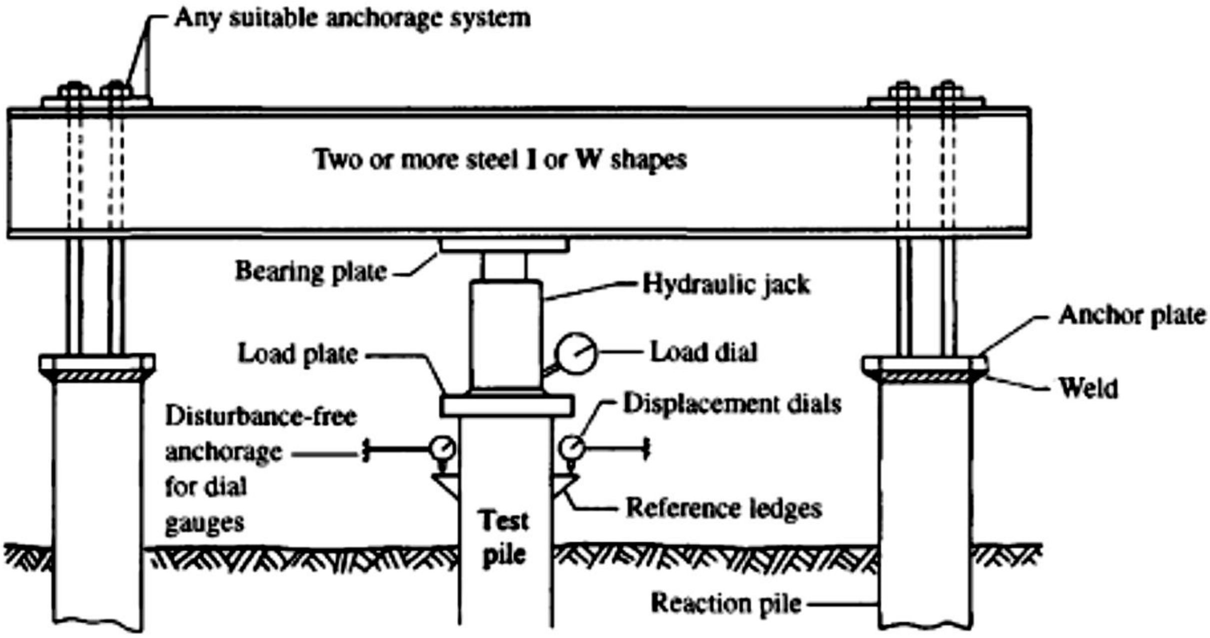
The purposes of a pile load test are:

- To determine the axial load capacity of a single pile.
- To determine the settlement of a single pile at working load.
- To verify the estimated axial load capacity.
- To obtain information on load transfer in skin friction and end bearing.

The allowable bearing capacity is found by dividing the ultimate load, found from the load settlement curve, by a factor of safety, usually 2. An alternative criterion is to determine the allowable pile load capacity for a desired serviceability limit state, for example, a settlement of 10% of the pile diameter. Also pile settlement under double working load should not be more than 25 mm.

Load- settlement curve of pile





(c) Typical pile load test setup using adjacent piles in group for reaction.

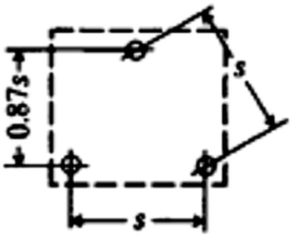
EFFICIENCY OF PILE GROUPS

When several pile butts are attached to a common structural element termed a pile cap the result is a pile group. A question of some concern is whether the pile group capacity is the sum of the individual pile capacities or something different—either more or less. If the capacity is the sum of the several individual pile contributions, the group efficiency Eg = 1.0.

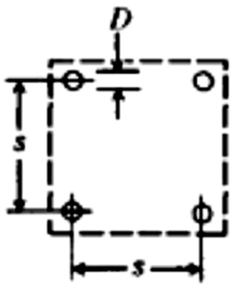
Optimum spacing s seems to be on the order of 2.5 to 3.5D or 2 to 3H for vertical loads where D = pile diameter; H = diagonal of rectangular shape or HP pile. Group efficiency can be estimated using

$$E_g = 1 - \theta \frac{(n-1)m + (m-1)n}{90mn}$$

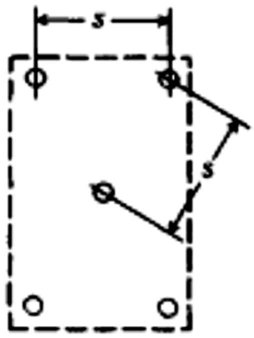
Where ***m, n*** are no. of columns and rows of piles $\boldsymbol{\theta}$**= tan^{-1} D/s** in degrees.



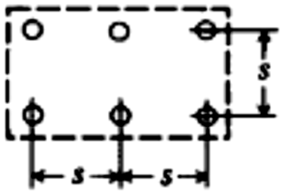
3 piles



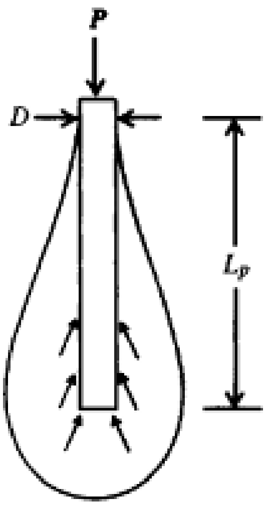
4 piles

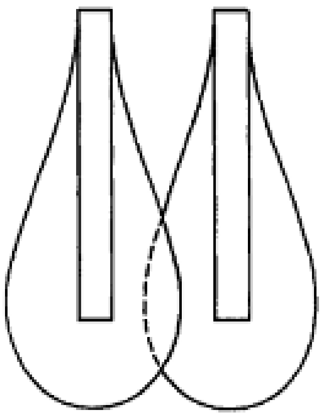


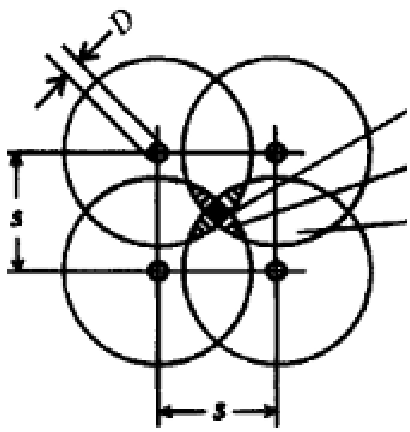
5 piles



6 piles

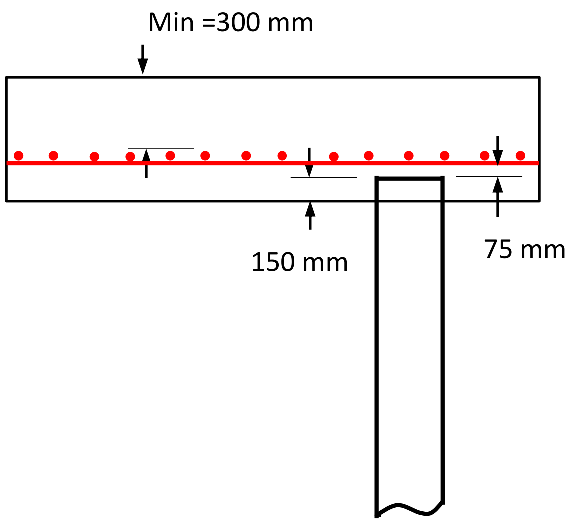






Function of Pile Cap

1. Transfer column load to pile bed.
2. To substitute the ill effect of one pile to others
3. To take any deviation in the location of piles



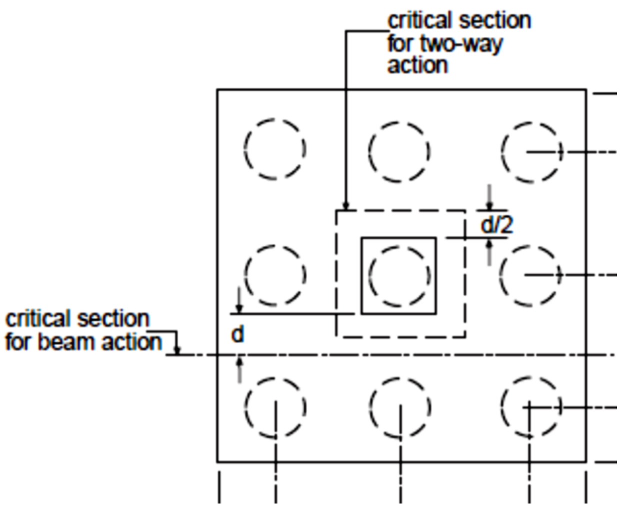
Minimum Total Thickness of Pile Cap

150 mm pile penetration in cap

75 mm concrete cover for cap steel above pile

Twice bar diameter
300 mm minimum concrete thickness above reinforcement

Example:
Estimate the pile group efficiency shown if the load per pile is as follows
P_D = 90 kN, P_L = 45 kN
What should be the minimum required allowable pile capacity of each Individual pile, then design the pile cap for the case shown
Footing size = 2.6 × 2.6 m.
Column size = 0.4× 0.4 m.
Pile diameter = 0.3 m.
c/c spacing between piles= 0.9 m
fc′ = 30 MPa



Solution*:*

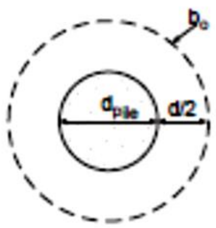
$$E_g = 1 - \theta \frac{(n-1)m + (m-1)n}{90mn}$$

$$\theta = tan^{-1}\frac{0.3}{0.9} = 18.43^\circ$$

$$E_g = 1 - 18.43\frac{(3-1)3+(3-1)3}{90\times3\times3} = 0.72$$

Total working load on each pile = 90 + 45 = 135 kN
Required allowable individual pile capacity = 135/0.72 = 187.5 kN

Design of pile cap:
Ultimate Pile Load
P_u = 1.2 x 90 +1.6 x 45 = 180 kN
Find Depth of Footing Using Shear Strength
1. Wide Beam Shear – Section at d from column face
V_u = 3 x 180 = 540 kN
$\phi V_c = 0.75 \times 0.17 \sqrt{f_c'}\, b_w\, d = 0.75 \times 0.17 \times \sqrt{30} \times 2.6 \times d \times 1000 = 1815\, d$
d = 540/1815 = **0.3 m**
2. Two- Way Shear – Section at d/2 from column face
V_u = 8 x 180 = 1440 kN
$\phi V_c = 0.75 \times 0.33\sqrt{f_c'}\, b_o d = 0.75 \times 0.33 \times \sqrt{30} \times 4(0.4 + d)\, d \times 1000 = 1356\,(1.6d + 4d^2)$
$1440 = 1356\,(1.6d + 4d^2)$
d = 0.35 m
3. Check Punching Shear Strength at Corner pile.
P_u = 180 kN
$\phi V_c = 0.75 \times 0.33\sqrt{f_c'}\, b_o d$
$= 0.75 \times 0.33 \times \sqrt{30} \times 3.14\,(0.3 + d) \times d \times 1000 = 4257\,(0.3d + d^2)$
d = 0.1 m
Use d = 350 mm



Total thickness of pile cap = 150 + 75 + 25+ 350 = 600 mm

EXPLORATION, SAMPLING, AND IN SITU SOIL MEASUREMENTS

The process of identifying the layers of deposits that underlie a proposed structure and their physical characteristics is generally referred to as subsurface exploration. The purpose of subsurface exploration is to obtain information that will aid the geotechnical engineer in

1. Selecting the type and depth of foundation suitable for a given structure.
2. Evaluating the load-bearing capacity of the foundation.
3. Estimating the probable settlement of a structure.
4. Determining potential foundation problems (e.g., expansive soil, collapsible soil, and so on).
5. Determining the location of the water table.
6. Predicting the lateral earth pressure for structures such as retaining walls, sheet pile, and braced cuts.
7. Establishing construction methods for changing subsoil conditions.

Subsurface exploration may also be necessary when additions and alterations to existing structures are contemplated

METHODS OF EXPLORATION

The most widely used method of subsurface investigation is boring holes into the ground, from which samples may be collected for either visual inspection or laboratory testing. Several procedures are commonly used to drill the holes and to obtain the soil samples.

SOIL BORING

Exploratory holes into the soil may be made by hand tools, but more commonly truck- or trailer-mounted power tools are used.

1- Hand Tools

The earliest method of obtaining a test hole was to excavate a test pit using a pick and shovel. Because of economics, the current procedure is to use power excavation equipment such as a backhoe to excavate the pit and then to use hand tools to remove a block sample or shape the site for in situ testing. This is the best method at present for obtaining quality *undisturbed* samples or samples for testing at other than vertical orientation. For small jobs, where the sample disturbance is not critical, hand or powered augers (Fig. 3-1) held by one or two persons can be used. Hand-augered holes are usually drilled to depths of the order of 2 to 5 m, as on roadways or airport runways, or investigations for small buildings.

2- Mounted Power Drills

For numerous borings to greater depths and to collect samples that are *undisturbed,* the only practical method is to use power-driven equipment.

2.1 *Wash boring* is a term used to describe one of the more common methods of advancing a hole into the ground. A hole is started by driving casing (Fig. 3-2) to a depth of 2 to 3.5 m. Casing is simply a pipe that supports the hole, preventing the walls from sloughing off or caving in. The casing is cleaned out by means of a chopping bit fastened to the lower end of the drill rod. Water is pumped through the drill rod and exits at high velocity through holes in the bit. The water rises between the casing and drill rod, carrying suspended soil particles, and overflows at the top of the casing. The hole is advanced by raising, rotating, and dropping the bit into the soil at the bottom of the hole. This method is quite rapid for advancing holes in all but very hard soil strata.

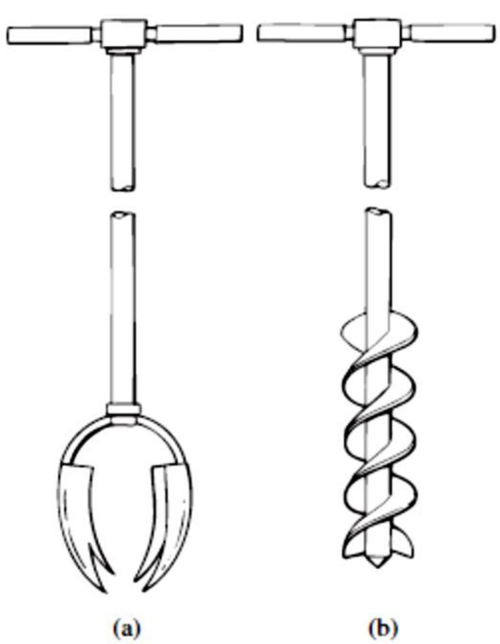


Figure 3-1 Hand tools for soil exploration *(a), (b)* Hand augers

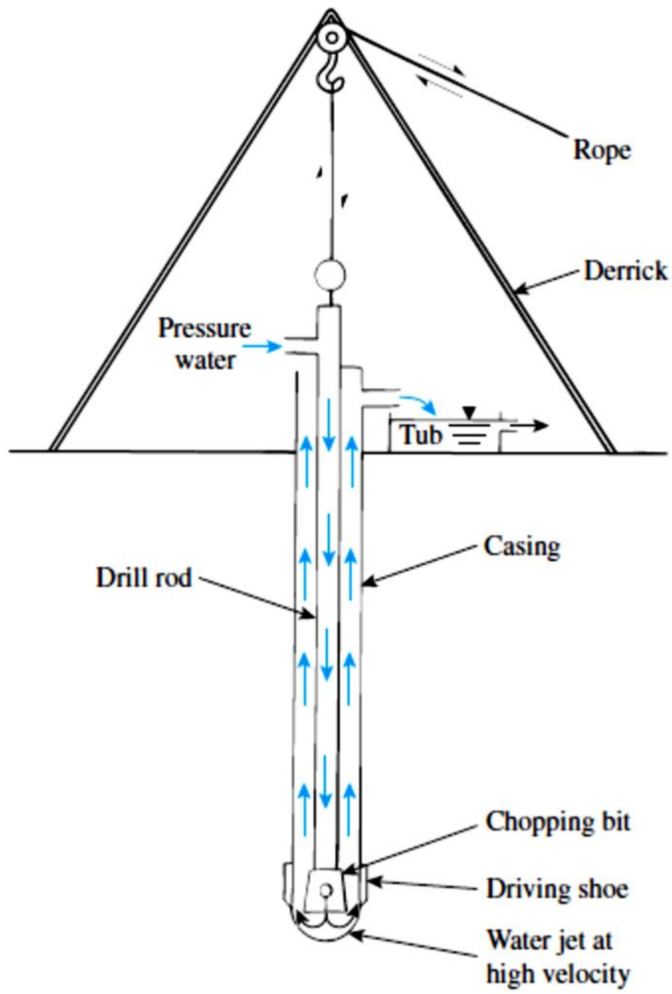


Figure 3.2 **Wash boring**

2.2 Rotary drilling

Rotary drilling is another method of advancing test holes. This method uses rotation of the drill bit, with the application of pressure to advance the hole. Rotary drilling is the most rapid method of advancing holes in rock unless it is badly fissured; however, it can also be used for any type of soil. Drilling mud may be used in soils where the sides of the hole tend to cave in. Drilling mud is usually a water solution of a special kind of clay (such as bentonite), with or without other admixtures, that is forced into the sides of the hole by the rotating drill. The mud cake thus formed provides sufficient strength in conjunction with the hydrostatic pressure of the mud suspension so that the cavity is maintained. When soil samples are needed, the drilling rod is raised and the drilling bit is replaced by a sampler.

2.3 Continuous-flight augers

Continuous flight augers with a rotary drill are probably the most popular method of soil exploration at present (Fig. 3-3). The flights act as a screw conveyor to bring the soil to the surface. The method is applicable in all soils. Borings up to nearly 100 m can be made with these devices, depending on the driving equipment, soil, and auger diameter.

Figure 3-3 Soil drilling using a continuous-flight auger.

SOIL SAMPLING

The most important engineering properties for foundation design are ***strength, compressibility, and permeability***. Reasonably good estimates of these properties for cohesive soils can be made by laboratory tests on *undisturbed* samples, which can be obtained with moderate difficulty. It is nearly impossible to obtain a truly undisturbed sample of soil, so in general usage the term *undisturbed* means a sample where some precautions have been taken to minimize disturbance of the existing soil skeleton. The following represent some of the factors that make an undisturbed sample hard to obtain:

1. *The sample is always unloaded from the in situ confining pressures, with some unknown resulting expansion*

2. Samples collected are disturbed by volume displacement of the tube or other collection device. The presence of gravel greatly aggravates sample disturbance.

3. Sample friction on the sides of the collection device tends to compress the sample during recovery. Most sample tubes are swaged so that the cutting edge is slightly smaller than the inside tube diameter to reduce the side friction.

Cohesionless Soil Sampling

It is nearly impossible to obtain undisturbed samples of cohesionless material for strength testing. Sometimes samples of reasonable quality can be obtained using *thin-walled piston samplers* in medium- to fine-grained sands. In gravelly materials, and in all dense materials, samples with minimal disturbance are obtained only with extreme difficulty. Some attempts have been made to recover cohesionless materials by *freezing the soil*, freezing a zone around the sample (but not the sample), *or injecting asphalt* that is later dissolved from the sample.

Since it is nearly impossible to recover undisturbed samples from cohesionless deposits, density, strength, and compressibility estimates are usually obtained from ***penetration tests*** or other ***in situ methods***. Permeability may be estimated from well pumping tests or, approximately, by bailing the boring and observing the time for the water level to rise some amount.

Disturbed Sampling of All Soils

Disturbed samples are adequate to locate suitable borrow, where compaction characteristics and index tests for classification are usually sufficient. In this case a larger-diameter auger (usually only shallow depths) may be used so that bags of representative soil may be obtained for *laboratory compaction tests, sieve analyses, and Atterberg limits*.

In recognizing the difficulty and expense of obtaining undisturbed samples, it is common practice on most foundation projects to rely on *penetration tests* and, disturbed samples for obtaining an estimate of the soil conditions. The standard penetration test (SPT) is nearly universally used, even though highly disturbed samples are recovered. Other types of tests, particularly cones, are also widely used, although these latter devices do not recover a soil sample. For very complex projects, more than one type of test equipment may be used (such as the standard penetration test together with a cone penetration test).

Figure 3-5 illustrates the *sampling device* (also called a *split spoon)* most commonly used with the SPT. It is made up of a *driving shoe*. The *barrel* consists of a piece of tube split lengthwise (split spoon) with a *coupling* on the upper end to connect the drill rod to the surface. *Inserts* (see Fig. 3-5b) are used when samples of thin mud and sand are to be recovered.

In a test the sampler is driven into the soil a measured distance, using some kind of falling weight producing some number of blows (or drops). The number of blows ***N*** to drive the specified distance is recorded as an indication of soil strength.
The sampler is then slightly twisted to shear the soil at the base of the tube and withdrawn.
The shoe and coupling are unscrewed and the two halves of the barrel are opened to expose the sample (unless a liner is used). If a liner is used, both ends are sealed—usually with melted wax—for later laboratory testing. If a liner is not used, on-site unconfined compression q_u tests are routinely made on cohesive samples. The wall thickness of the driving shoe (Fig. 3-5a) indicates that any samples recovered by this device are likely to be highly disturbed.
Representative samples from the soil in the sampler barrel are stored in sample jars and returned to the laboratory for inspection and classification. The field technician marks the jar with the job and boring number, sample depth, and penetration blow count.
These samples are used for determining the Atterberg limits and natural water content. In routine work these index properties, used with correlation tables and charts and with q_u, are sufficient to select the foundation type, estimate the allowable bearing capacity, and make some kind of estimates of probable settlement.
The penetration number ***N*** (a measure of resistance) is usually sufficient for making estimates of both strength and settlement in cohesionless soils. Where the geotechnical consultant has obtained sufficient experience, strength/settlement predictions made in this manner are quite adequate for about 85 to 90 percent of foundation work.

Unassembled split-spoon sampler after sampling

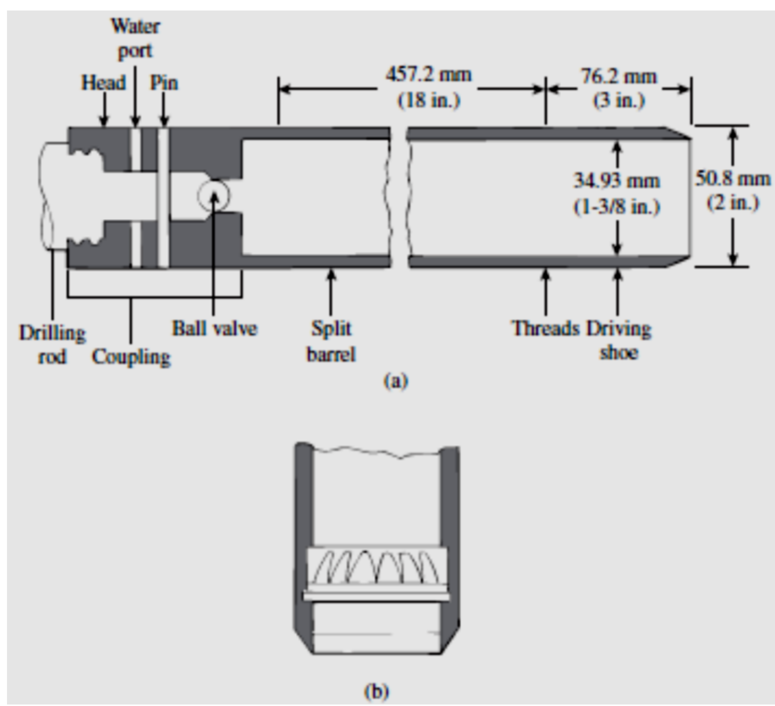


Figure 3.5 **(a) Standard split-spoon sampler; (b) spring core catcher**

Undisturbed Sampling in Cohesive Soils

As the field boring progresses and soft layers are encountered that may influence the foundation selection/design, undisturbed samples are usually taken so that consolidation and more refined laboratory strength tests can be made.

Recovery of "*undisturbed*" samples in cohesive soils is accomplished by replacing the split spoon on the drill rod with specially constructed *thin-walled tubes*, sometimes referred to as *Shelby tubes*. They are made of seamless steel (1.63 to 3.25 mm thick) and are frequently used to obtain undisturbed clayey soils. The most common thin-walled tube samplers have outside diameters of 50.8 mm (2 in.) and 76.2 mm (3 in.). The bottom end of the tube is sharpened. The tubes can be attached to drill rods (Figure 3.6). The drill rod with the sampler attached is lowered to the bottom of the borehole, and the sampler is pushed into the soil. The soil sample inside the tube is then pulled out. The two ends are sealed, and the sampler is sent to the laboratory for testing.

Samples obtained in this manner may be used for consolidation or shear tests. Friction holds the sample in the tube as the sample is withdrawn; however, there is also special valve or piston (Fig. 3-6) arrangement that use a pressure differential (suction) to retain the sample in the tube.

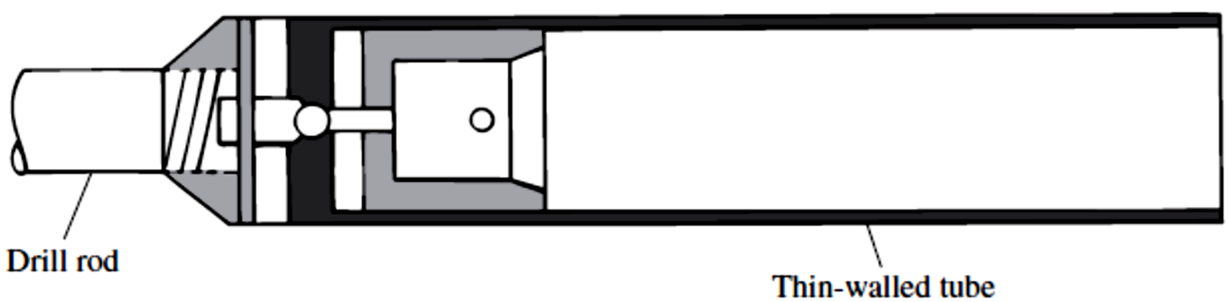


Fig. 3-6 Thin walled tube

The degree of disturbance for a soil sample is usually expressed as

$$A_R(\%) = \frac{D_o^2 - D_i^2}{D_i^2}(100)$$

where

A_R = area ratio (ratio of disturbed area to total area of soil)

D_o = outside diameter of the sampling tube

D_i = inside diameter of the sampling tube

When the area ratio is 10% or less, the sample generally is considered to be undisturbed.

For a standard split-spoon sampler,

$$A_R(\%) = \frac{(50.8)^2 - (34.93)^2}{(34.93)^2}(100) = 111.5\%$$

Hence, these samples are highly disturbed. Split-spoon samples generally are taken at intervals of about 1.5 m.

THE STANDARD PENETRATION TEST (SPT)

The standard penetration test, developed around 1927, is currently the most popular and economical means to obtain subsurface information (both on land and offshore). It is estimated that 85 to 90 percent of conventional foundation design in North and South America is made using the SPT. This test is also widely used in other geographic regions. The method has been standardized as ASTM D 1586. The test consists of the following:

1. Driving the standard split-barrel sampler of dimensions shown in Fig. *3-5a* a distance of 460mm into the soil at the bottom of the boring.
2. Counting the number of blows to drive the sampler the last two 150 mm distances (total = 300 mm) to obtain the ***N*** number.
3. Using a 63.5-kg driving mass (or hammer) falling "free" from a height of 760 mm. Several hammer configurations are shown in Fig. 3-7.

The exposed drill rod is referenced with three chalk marks 150 mm apart, and the guide rod (see Fig. 3-7) is marked at 760 mm (for manual hammers). The assemblage is then seated on the soil in the borehole (after cleaning it of loose cuttings). Next the sampler is driven a distance of 150 mm to seat it on undisturbed soil, with this blow count being recorded. The sum of the blow counts for the next two 150-mm increments is used as the penetration count ***N*** unless the last increment cannot be completed. In this case the sum of the first two 150-mm penetrations is recorded as ***N***.

The boring log shows *refusal* and the test is halted if

a. 50 blows are required for any 150-mm increment.
b. 10 successive blows produce no advance.

It should be evident that the blow count would be directly related to the driving energy, which is theoretically computed as follows:

$$E_{in} = W\,h$$

where W = weight or mass of hammer and h = height of fall.

It was found that the actual input driving energy E_a to the sampler to produce penetration ranged from about 30 to 80 percent. From the several recent studies cited it has been suggested that the SPT be standardized to some energy ratio E_r which should be computed as

$$E_r = \frac{\text{Actual hammer energy to sampler, } E_a}{\text{Input energy, } E_{in}} \times 100$$

For example, N_{70}=25 means that the SPT number (***N***) is 25 for E_r = 70%, and N_{60} = 44 means that means SPT number (***N***) is 44 for E_r = 60% for and so on.

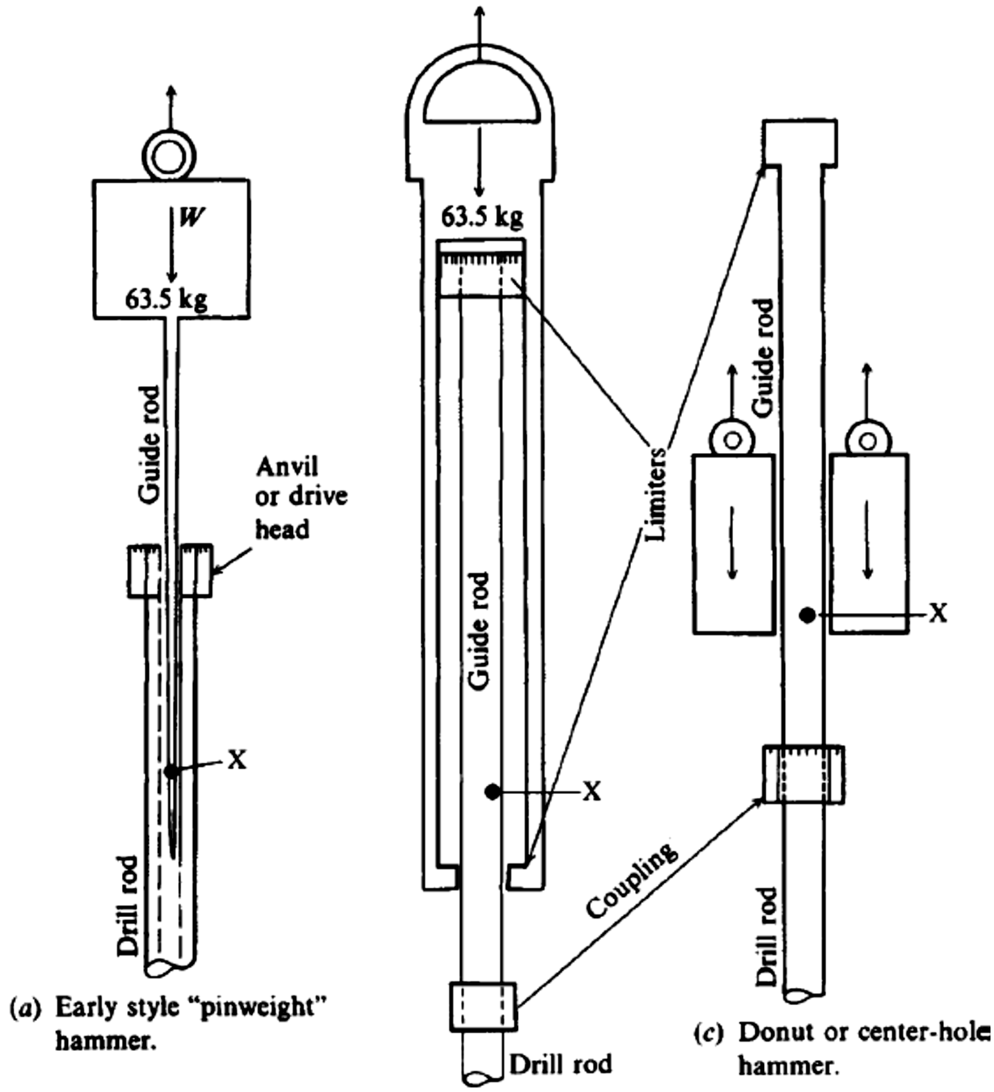


Figure 3-7 Schematic diagrams of the three commonly used hammers. Hammer *(b)* is used about 60 percent; *(a)* and (c) about 20 percent each in the United States. Hammer *(c)* is commonly used outside the United States. Note that the user must be careful with *(b)* and (c) not to contact the limiter and "pull" the sampler out of the soil. Guide rod *X* is marked with paint or chalk for visible height control when the hammer is lifted by rope off the cathead (power takeoff)

In the field, the magnitude of E_r can vary from 30 to 90%. The standard practice now in the U.S. is to express the N-value to an average energy ratio of 60% ≈ (N60).
to correct or standardize the field penetration number as a function of the input driving energy and its dissipation around the sampler into the surrounding soil, we use the following equation:

$$N_{60} = \frac{N\eta_H \eta_B \eta_S \eta_R}{60}$$

where

N_{60} = standard penetration number, corrected for field conditions

N = measured penetration number

η_H = hammer efficiency (%)

η_B = correction for borehole diameter

η_S = sampler correction

η_R = correction for rod length

Values of η_H, η_B, η_S, and η_R, are in tables below.

Table 3.5 Variations of η_H, η_B, η_S, and η_R [Eq. (3.6)]

1. Variation of η_H

Country	Hammer type	Hammer release	η_H (%)
Japan	Donut	Free fall	78
	Donut	Rope and pulley	67
United States	Safety	Rope and pulley	60
	Donut	Rope and pulley	45
Argentina	Donut	Rope and pulley	45
China	Donut	Free fall	60
	Donut	Rope and pulley	50

2. Variation of η_B

Diameter		
mm	in.	η_B
60–120	2.4–4.7	1
150	6	1.05
200	8	1.15

3. Variation of η_S

Variable	η_S
Standard sampler	1.0
With liner for dense sand and clay	0.8
With liner for loose sand	0.9

4. Variation of η_R

Rod length		
m	ft	η_R
>10	>30	1.0
6–10	20–30	0.95
4–6	12–20	0.85
0–4	0–12	0.75

SPT CORRELATIONS

The SPT has been used in correlations for unit weight γ, relative density D_r, angle of internal friction ϕ, and undrained compressive strength q_u. It has also been used to estimate the bearing capacity of foundations and for estimating the stress-strain modulus E_s.

Relative Density

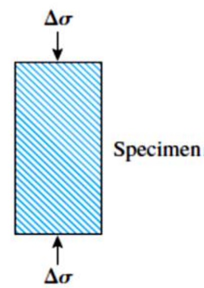
In *granular soils,* the degree of compaction in the field can be measured according to the *relative density,* defined as

$$D_r(\%) = \frac{e_{\max} - e}{e_{\max} - e_{\min}} \times 100 \qquad (2.23)$$

where

$e_{\max}$ = void ratio of the soil in the loosest state
$e_{\min}$ = void ratio in the densest state
e = *in situ* void ratio

Unconfined Compression Test

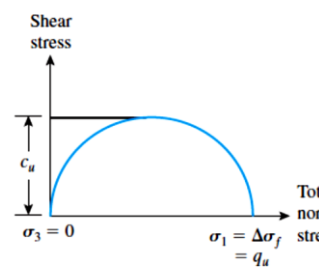


The *unconfined compression test* (Figure 2.29a) is a special type of unconsolidated-undrained triaxial test in which the confining pressure $\sigma_3 = 0$, as shown in Figure 2.29b. In this test, an axial stress $\Delta\sigma$ is applied to the specimen to cause failure (i.e., $\Delta\sigma = \Delta\sigma_f$). The corresponding Mohr's circle is shown in Figure 2.29b. Note that, for this case,

Major principal total stress = $\Delta\sigma_f = q_u$
Minor principal total stress = 0

The axial stress at failure, $\Delta\sigma_f = q_u$, is generally referred to as the *unconfined compression strength.* The shear strength of saturated clays under this condition ($\phi = 0$), from Eq. (2.85), is

$$s = c_u = \frac{q_u}{2} \qquad (2.100)$$



Hara, et al. (1971) also suggested the following correlation between the undrained shear strength of clay (cu) and N_{60}.

$$\frac{c_u}{p_a} = 0.29N_{60}^{0.72} \qquad (3.8)$$

where p_a = atmospheric pressure (≈ 100 kN/m^2; ≈ 2000 lb/in^2).

Cubrinovski and Ishihara (1999) also proposed a correlation between N_{60} and the relative density of sand (D_r) that can be expressed as

$$D_r(\%) = \left[\frac{N_{60}\left(0.23 + \frac{0.06}{D_{50}}\right)^{1.7}}{9} \left(\frac{1}{\frac{\sigma'_o}{p_a}}\right) \right]^{0.5} (100) \qquad (3.23)$$

where

p_a = atmospheric pressure (≈ 100 kN/m^2)

D_{50} = sieve size through which 50% of the soil will pass (mm)

Correlation between Angle of Friction and Standard Penetration Number

The peak friction angle, ϕ', of granular soil has also been correlated with N_{60} by several investigators. Some of these correlations are as follows:

1. Peck, Hanson, and Thornburn (1974) give a correlation between N_{60} and ϕ' in a graphical form, which can be approximated as:

$$\boldsymbol{\phi'(\text{deg}) = 27.1 + 0.3N_{60} - 0.00054[N_{60}]^2} \qquad \textbf{(3.29)}$$

2. Schmertmann (1975) provided the correlation between N_{60}, σ'_0 , and ϕ'. Mathematically, the correlation can be approximated as:

$$\phi' = \tan^{-1}\left[\frac{N_{60}}{12.2 + 20.3\left(\frac{\sigma'_o}{p_a}\right)}\right]^{0.34} \qquad (3.30)$$

where

N_{60} = field standard penetration number

σ'_0 = effective overburden pressure

p_a = atmospheric pressure in the same unit as σ'_0

ϕ' = soil friction angle

The following notes should be considered when standard penetration resistance values are used in the preceding correlations to estimate soil parameters:

1. The equations are approximate.
2. Because the soil is not homogeneous, the values of N_{60} obtained from a given borehole vary widely.
3. In soil deposits that contain large boulders and gravel, standard penetration numbers may be erratic and unreliable.

	*Very loose		Loose		Medium dense		Dense		Very dense	
#D_r (%)	0		15		35		65		85	100
*N_{60}			4		10		30		50	
**ϕ'(deg)			28		30		36		41	

Fig. 3.17 Approximate borderline values for D_r, N_{60}, and ϕ

Example:

Following are the results of a standard penetration test in sand. Note that the water table was not observed within a depth of 10.5 m below the ground surface. Assume that the average unit weight of sand is 17.3 kN/m^3. Using Eq. (3.30), estimate the average soil friction angle, $\emptyset'$. From z = 0 to z = 7.5 m.

Depth, z (m)	N_{60}
1.5	8
3.0	7
4.5	12
6.0	14
7.5	13

Solution

From Eq. (3.30)

$$\phi' = \tan^{-1}\left[\frac{N_{60}}{12.2 + 20.3\left(\frac{\sigma'_a}{p_a}\right)}\right]^{0.34}$$

$$p_a = 100\ \text{kN/m}^2$$

Now the following table can be prepared.

Depth, z (m)	σ'_o (kN/m²)	N_{60}	ϕ' (deg) [Eq. (3.30)]
1.5	25.95	8	37.5
3.0	51.9	7	33.8
4.5	77.85	12	36.9
6.0	103.8	14	36.7
7.5	129.75	13	34.6

Average $\phi' \approx 36°$

TABLE 3-4

Empirical values for ϕ, D_r, and unit weight of granular soils based on the SPT at about 6 m depth and normally consolidated [approximately, $\phi = 28° + 15° D_r$ ($\pm 2°$)]

Description		Very loose	Loose	Medium	Dense	Very dense
Relative density D_r		0	0.15	0.35	0.65	0.85
SPT N'_{70}:	fine	1–2	3–6	7–15	16–30	?
	medium	2–3	4–7	8–20	21–40	> 40
	coarse	3–6	5–9	10–25	26–45	> 45
ϕ:	fine	26–28	28–30	30–34	33–38	
	medium	27–28	30–32	32–36	36–42	< 50
	coarse	28–30	30–34	33–40	40–50	
γ_{wet}, kN/m³		11–16*	14–18	17–20	17–22	20–23

* Excavated soil or material dumped from a truck has a unit weight of 11 to 14 kN/m³ and must be quite dense to weigh much over 21 kN/m³. No existing soil has a $D_r = 0.00$ nor a value of 1.00. Common ranges are from 0.3 to 0.7.

Consistency of saturated cohesive soils*

Consistency			N'_{70}	q_u, kPa	Remarks
Very soft	NC	Young clay	0–2	<25	Squishes between fingers when squeezed
Soft			3–5	25– 50	Very easily deformed by squeezing
Medium			6–9	50– 100	? ?
Stiff	Increasing OCR	Aged/ cemented	10–16	100– 200	Hard to deform by hand squeezing
Very stiff			17–30	200– 400	Very hard to deform by hand squeezing
Hard			>30	>400	Nearly impossible to deform by hand

* Blow counts and OCR division are for a guide—in clay "exceptions to the rule" are very common.

A correlation for ***N*** versus q_u is in the general form of

$$q_u = k\,N$$

Where the value of ***k*** tends to be site-dependent; however, a value of ***k*** = 12 has been used (i.e., for N_{70} = 10, q_u = 120 kPa). Correlations for N_{70} and consistency of cohesive soil deposits (soft, stiff, hard, etc.) are given in Table above.

The overconsolidation ratio, OCR, of a natural clay deposit can also be correlated with the standard penetration number. On the basis of the regression analysis of 110 data points, Mayne and Kemper (1988) obtained the relationship.

$$\mathrm{OCR} = 0.193\left(\frac{N_{60}}{\sigma'_o}\right)^{0.689}$$

where σ'_o = effective vertical stress in MN/m^2.

Vane Shear Test

The *vane shear test* (ASTM D-2573) may be used during the drilling operation to determine the *in situ* undrained shear strength (c_u) of clay soils—particularly soft clays. The vane shear apparatus consists of four blades on the end of a rod, as shown in Figure 3.23. The height, *H*, of the vane is twice the diameter, *D*. The vane can be either rectangular or tapered (see Figure 3.23). The dimensions of vanes used in the field are given in Table 3.8.

The vanes of the apparatus are pushed into the soil at the bottom of a borehole without disturbing the soil appreciably. Torque is applied at the top of the rod to rotate the vanes at a standard rate of 0.18/sec. This rotation will induce failure in a soil of cylindrical shape surrounding the vanes. The maximum torque, *T*, applied to cause failure is measured. Note that

$$T = f\,(c_u, H, \text{ and } D) \qquad (3.33)$$

or

$$c_u = \frac{T}{K} \tag{3.34}$$

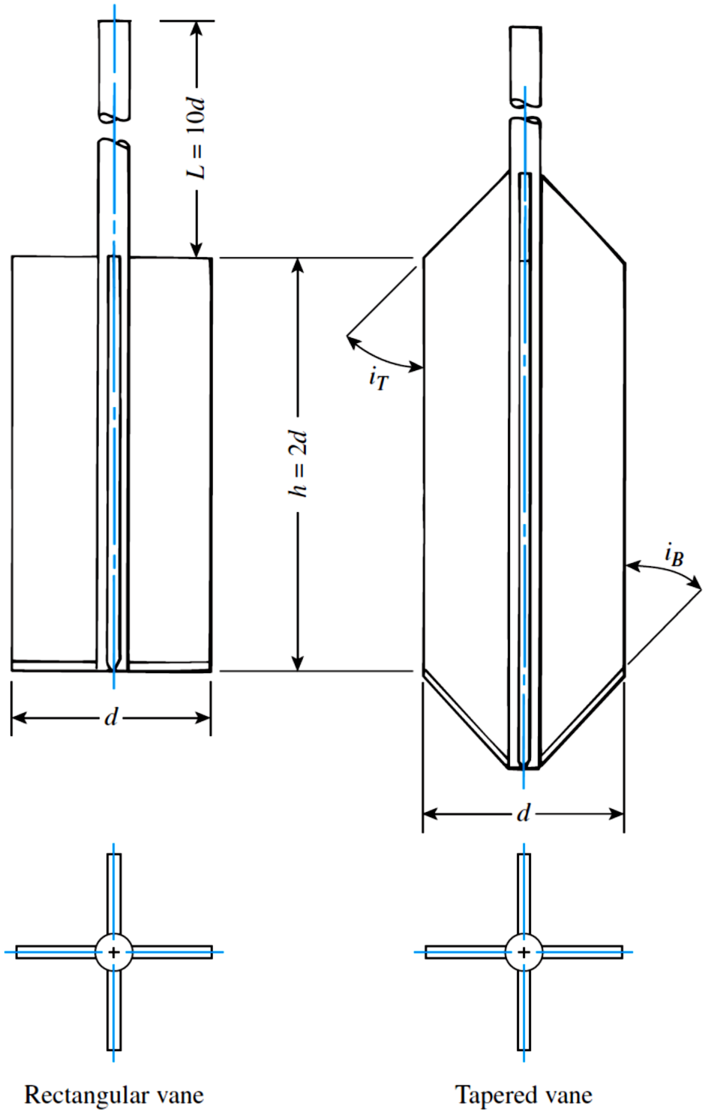


Figure 3.23 Geometry of field vane (After ASTM, 2014) (Based on *Annual Book of ASTM Standards, Vol. 04.08.*)

Table 3.8 ASTM Recommended Dimensions of Field Vanes[a] (Based on *Annual Book of ASTM Standards, Vol. 04.08.*)

Casing size	Diameter, *d* mm (in.)	Height, *h* mm (in.)	Thickness of blade mm (in.)	Diameter of rod mm (in.)
AX	38.1 ($1\frac{1}{2}$)	76.2 (3)	1.6 ($\frac{1}{16}$)	12.7 ($\frac{1}{2}$)
BX	50.8 (2)	101.6 (4)	1.6 ($\frac{1}{16}$)	12.7 ($\frac{1}{2}$)
NX	63.5 ($2\frac{1}{2}$)	127.0 (5)	3.2 ($\frac{1}{8}$)	12.7 ($\frac{1}{2}$)
101.6 mm (4 in.)[b]	92.1 ($3\frac{5}{8}$)	184.1 ($7\frac{1}{4}$)	3.2 ($\frac{1}{8}$)	12.7 ($\frac{1}{2}$)

[a]The selection of a vane size is directly related to the consistency of the soil being tested; that is, the softer the soil, the larger the vane diameter should be.
[b]Inside diameter.

According to ASTM (2014), for rectangular vanes,

$$K = \frac{\pi d^2}{2}\left(h + \frac{d}{3}\right) \tag{3.35}$$

If $h/d = 2$,

$$K = \frac{7\pi d^3}{6} \tag{3.36}$$

Thus,

$$c_u = \frac{6T}{7\pi d^3} \tag{3.37}$$

For tapered vanes,

$$K = \frac{\pi d^2}{12}\left(\frac{d}{\cos i_T} + \frac{d}{\cos i_B} + 6h\right) \tag{3.38}$$

The angles i_T and i_B are defined in Figure 3.23.

For actual design purposes, the undrained shear strength values obtained from field vane shear tests [$c_{u(\text{VST})}$] are too high, and it is recommended that they be corrected according to the equation:

$$c_{u(\text{corrected})} = \lambda\, c_{u(\text{VST})} \tag{3.39}$$

where λ = correction factor.

Several correlations have been given previously for the correction factor λ. The most commonly used correlation for λ is that given by Bjerrum (1972), which can be expressed as:

$$\lambda = 1.7 - 0.54 \log [PI\ (\%)] \qquad (3.40a)$$

Mitchell (1988) derived the following empirical relationship for estimating the preconsolidation pressure of a natural clay deposit:

$$\sigma'_c = 7.04[c_{u(\text{field})}]^{0.83}$$

Here,

σ'_c = preconsolidation pressure (kN/m^2)

$c_{u(field)}$ = field vane shear strength (kN/m^2)

The overconsolidation ratio, OCR, also can be correlated to $c_{u(field)}$ according to the equation

$$\text{OCR} = \beta \frac{c_{u(\text{field})}}{\sigma'_o} \qquad (3.42)$$

where σ'_0 = effective overburden pressure.

The magnitudes of **β** developed by Mayne and Mitchell (1988) is given below.

$$\beta = 22[PI(\%)]^{-0.48}$$

Example 3.3

Refer to Figure 3.23. Vane shear tests (tapered vane) were conducted in the clay layer. The vane dimensions were 63.5 mm (d) x 127 mm (h), and $i_T = i_B = 45^0$. For a test at a certain depth in the clay, the torque required to cause failure was 20 N.m. For the clay, liquid limit was 50 and plastic limit was 18. Estimate the undrained cohesion of the clay for use in the design:

a. Bjerrum's λ relationship (Eq. 3.40a)

b. Estimate the preconsolidation pressure of clay, σ'_c

Solution

Part a

Given: $h/d = 127/63.5 = 2$

From Eq. (3.38),

$$K = \frac{\pi d^2}{12}\left(\frac{d}{\cos i_T} + \frac{d}{\cos i_B} + 6h\right)$$
$$= \frac{\pi(0.0635)^2}{12}\left[\frac{0.0635}{\cos 45} + \frac{0.0635}{\cos 45} + 6(0.127)\right]$$
$$= (0.001056)(0.0898 + 0.0898 + 0.762)$$
$$= 0.000994$$

From Eq. (3.34),

$$c_{u(\text{VST})} = \frac{T}{K} = \frac{20}{0.000994}$$
$$= 20{,}121 \text{ N/m}^2 \approx 20.12 \text{ kN/m}^2$$

From Eqs. (3.40a) and (3.39),

$$c_{u(\text{corrected})} = [1.7 - 0.54 \log (\text{PI\%})]c_{u(\text{VST})}$$
$$= [1.7 - 0.54 \log(50 - 18)](20.12)$$
$$= \mathbf{17.85 \text{ kN/m}^2}$$

Part b

From Eq. (3.41)

$$\sigma'_c = 7.04[c_{u(\text{VST})}]^{0.83} = 7.04(20.12)^{0.83} = \mathbf{85 \text{ kN/m}^2}$$

H.W: Resolve the previous example for the case $i_T = i_B = 30^0$ and the torque was 35 N.m

CONE PENETRATION TEST (CPT)

The CPT is a simple test that is now widely used in lieu of the SPT—particularly for soft clays, soft silts, and in fine to medium sand deposits. The test is not well adapted to gravel deposits or to stiff/hard cohesive deposits. This test has been standardized by ASTM. In outline, the test consists in pushing the standard cone (see Fig. 3-14) into the ground at a rate of 10 to 20 mm/s and recording the resistance. The total cone resistance is made up of side friction on the cone shaft and tip pressure. Data usually recorded are the cone side resistance $\boldsymbol{q_s}$, point resistance $\boldsymbol{q_c}$, and depth. The tip (or cone) usually has a projected cross-sectional area of 10 cm^2.

A CPT allows nearly continuous testing at many sites, which is often valuable and no boreholes are necessary to perform it.

Generally, two types of penetrometers are used to measure q_c and q_s:

1. *Mechanical friction-cone penetrometer*

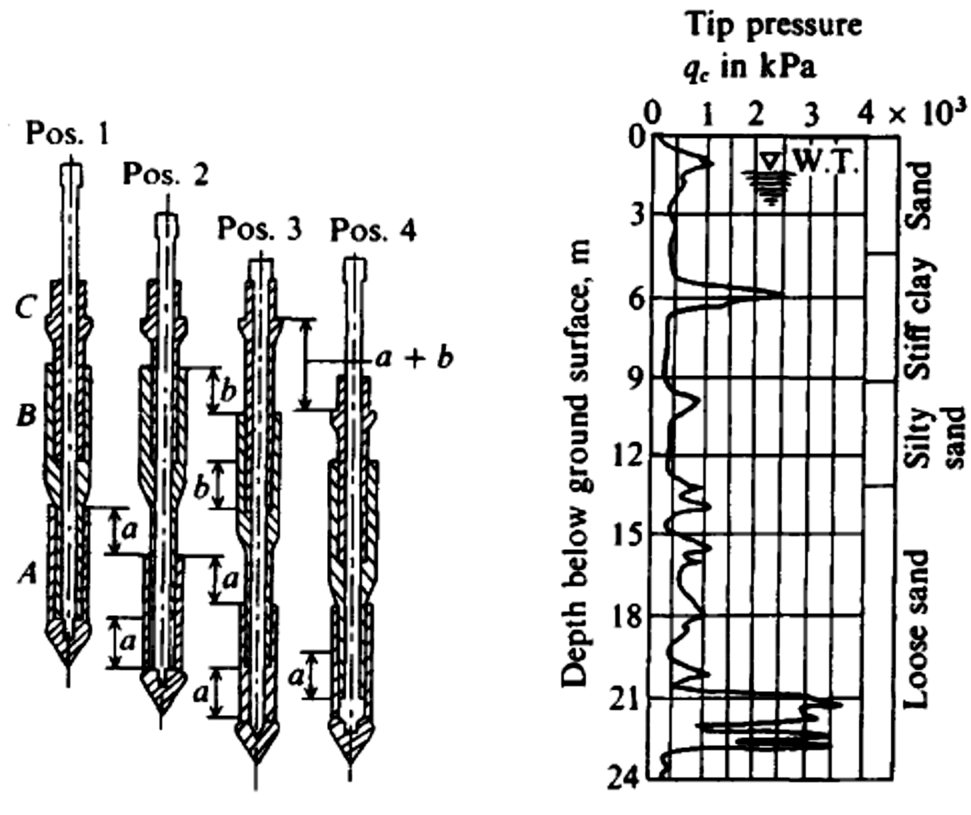
The original mechanical cone test is illustrated in Fig. 3-14b with the step sequence as follows:

A: The cone system is stationary at position 1.

B: The cone is advanced by pushing an inner rod to extrude the cone tip and a short length of cone shaft. This action measures the tip resistance q_c.

C: The outer shaft is now advanced to the cone base, and skin resistance is measured as the force necessary to advance the shaft q_s.

D: Now the cone and sleeve are advanced in combination to obtain position 4 and to obtain a q_{total} which should be approximately the sum of the $q_c + q_s$ just measured. The cone is now positioned for a new position 1.



(*b*) **Positions of the Dutch cone during a pressure record.**

(*c*) **Typical output (usually electronically made).**

Figure 3-14 Mechanical (or Dutch) cone, operations sequence, and tip resistance data.

2. *Electric friction-cone penetrometer*

The tip of this penetrometer is attached to a string of steel rods. The tip is pushed into the ground at the rate of 20 mm/sec. Wires from the transducers are threaded through the center of the rods and continuously measure the cone and side resistances. Figure 3.15 shows a photograph of an electric friction-cone penetrometer.

Figure 3.15 Photograph of an electric friction-cone penetrometer

CPT Correlations for Cohesive Soil

One correlation between the cone bearing resistance q_c and undrained shear strength c_u is based on the bearing capacity equation and is as follows:

$$q_c = N_k c_u + \sigma'_0$$

Solving for the undrained shear strength c_u, one obtains

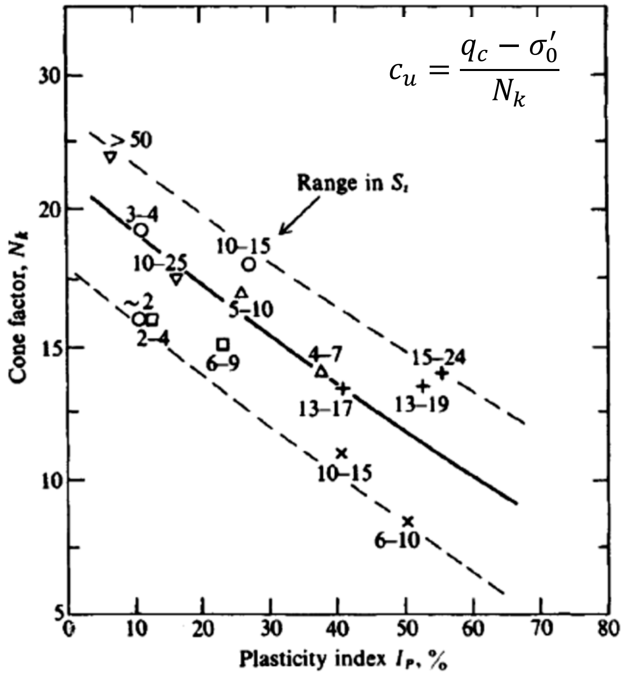
$$c_u = \frac{q_c - \sigma'_0}{N_k}$$

where $\sigma'_0 = \gamma z$ = overburden pressure point where q_c is measured as previously defined and used. This parameter is in the units of q_c.

N_k = cone factor (a constant for that soil). N_k has been found to range from 5 to 75; however, most values are in the 15 to 20 range.

Figure 3-16 is a correlation based on the plasticity index I_p which might be used.

Figure 3-16 Cone factor N_k versus I_P plotted for several soils



CPT Correlations for Cohesionless Soils.

Figure 3-17 is a plot of the correlation between cone pressure $\boldsymbol{q_c}$ and relative density $\boldsymbol{D_r}$.

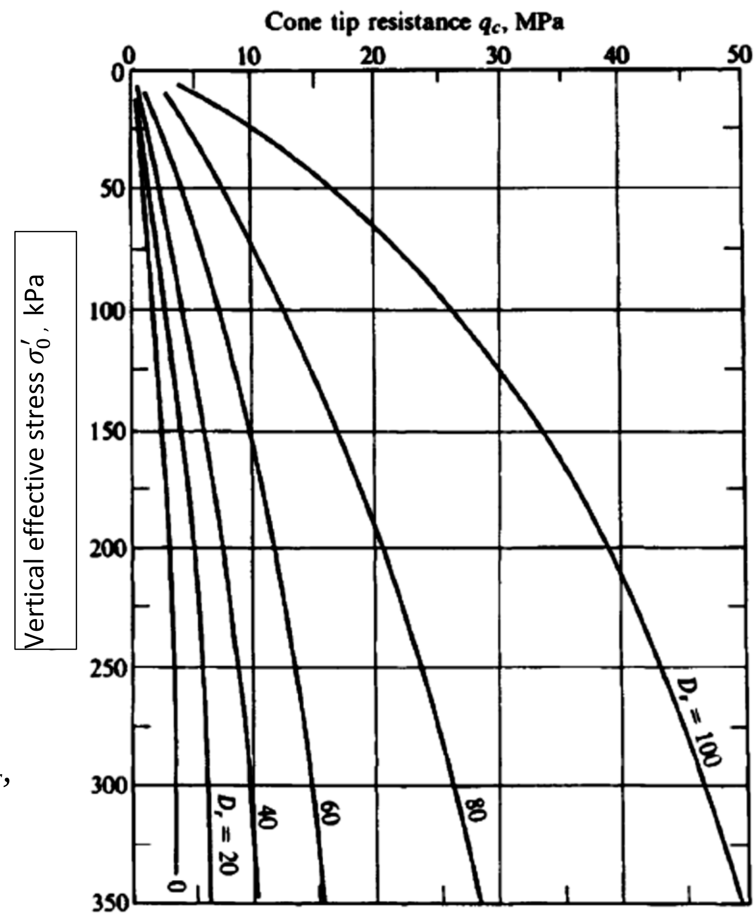


Figure 3-17 Approximate relationship between cone q_c and relative density D_r, for normally consolidated saturated recent (noncemented) deposits.

The relative density of *normally consolidated sand, D_r*, and q_c can be correlated according to the formula:

$$D_r(\%) = 68\left[\log\left(\frac{q_c}{\sqrt{p_a \cdot \sigma_0'}}\right) - 1\right]$$

Where p_a = atmospheric pressure ($\approx$ 100 kN/m^2)
σ_0' = vertical effective stress

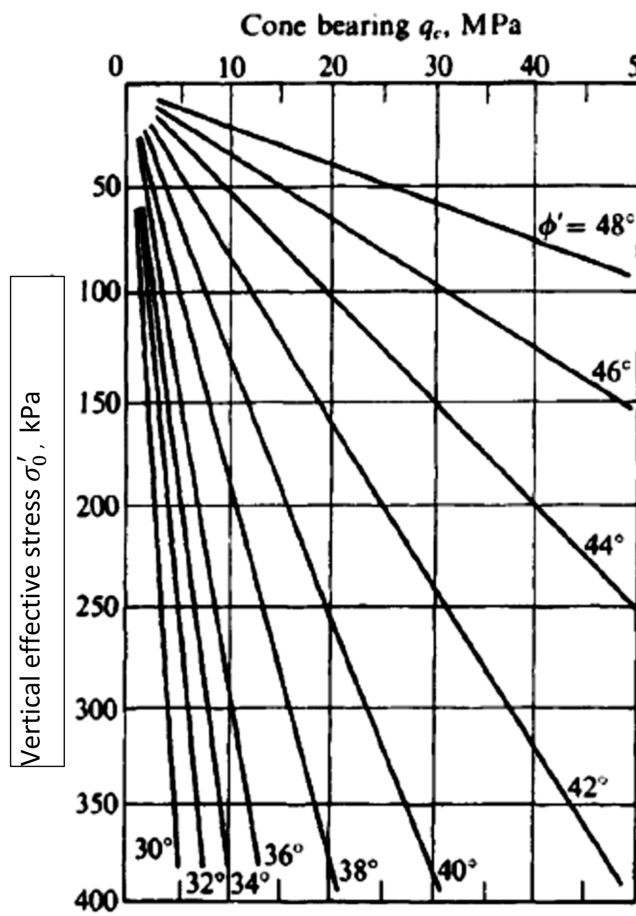
Correlation between q_c and Drained Friction Angle ($\emptyset'$) for Sand

On the basis of experimental results, Robertson and Campanella (1983) suggested the variation of σ_0', and ϕ for normally consolidated quartz sand. This relationship can be expressed as

$$\phi' = \tan^{-1}\left[0.1 + 0.38\log\left(\frac{q_c}{\sigma_o'}\right)\right]$$

The figure below shows graphical correlation between angle ϕ and q_c for uncemented quartz sands.

Figure 3-22 Correlation between peak friction angle ϕ *and* q_c for uncemented quartz sands.



Example:

Given. For CPT test q_c = 200 kg/cm^2 at depth z = *17* m in sand, γ' = 11.15 kN/m^3.

Required. Estimate relative density and angle of internal friction ϕ for the soil

Solution:

σ_0' = *17* X 11.15 = 189.55 kN/ m^2 (kPa) (effective pressure)

q_c = 200 X 98.07 = 19610 kPa (98.07 converts kg/cm^2 to kPa)

$$D_r(\%) = 68\left[\log\left(\frac{q_c}{\sqrt{p_a \cdot \sigma_0'}}\right) - 1\right]$$

$$D_r\,(\%) = 68\left[\log\left(\frac{19610}{\sqrt{100x189.55}}\right) - 1\right] = 78.44\%$$

$$\phi' = \tan^{-1}\left[0.1 + 0.38\log\left(\frac{q_c}{\sigma_o'}\right)\right]$$

$$\phi = \tan^{-1}\left[\ 0.1 + 0.38\log\left(\frac{19610}{189.55}\right)\right] = \tan^{-1} 0.8656 = 40.88^0$$

From Fig. 3-22 and q_c = 200 X 98.07/1000 = 19.61 MPa, we obtain $\phi \approx 41^0$

H.W: Resolve the same example for q_c = 150 kg/ mm^2 and z=22m

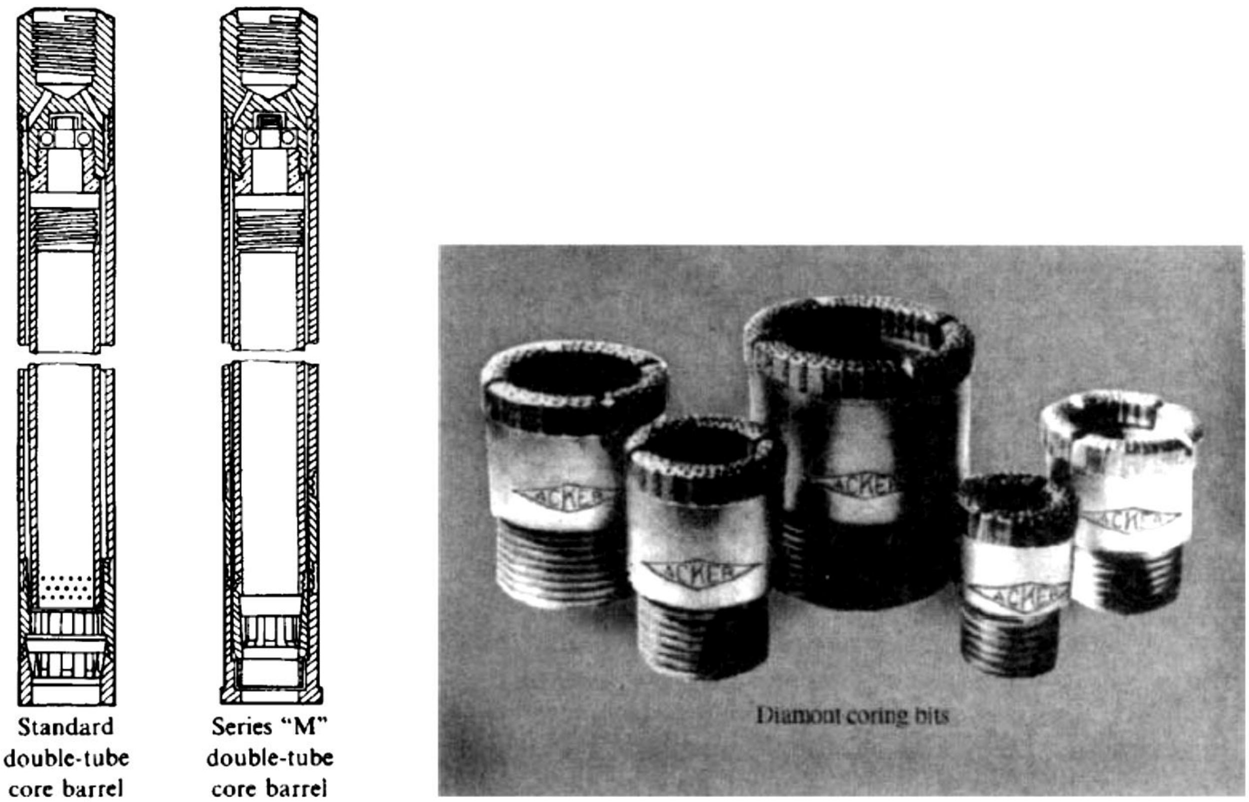
ROCK SAMPLING

In rock, except for very soft or partially decomposed sandstone or limestone, blow counts are at the refusal level (***N*** > **100**). If samples for rock quality or for strength testing are required it will be necessary to replace the soil drill with rock drilling equipment. Of course, if the rock is close to the ground surface, it will be necessary to ascertain whether it represents a competent rock stratum or is only a suspended boulder. Where rock is involved, it is useful to have some background in geology.

Rock cores are necessary if the soundness of the rock is to be established.

Unconfined and high-pressure triaxial tests can be performed on recovered cores to determine the elastic properties of the rock. These tests may give much higher compressive strengths in laboratory testing than the field strength for the rock mass.

The figure below illustrates several commonly used drill bits, which are attached to a piece of hardened steel tube (casing) 0.6 to 3 m long. In the drilling operation the bit and casing rotate while pressure is applied, thus grinding a groove around the core. Water under pressure is forced down the barrel and into the bit to carry the rock dust out of the hole as the water is circulated.



(*a*) Core barrels to collect rock cores (*b*) Coring bits to attach to core barrel. (The Acker Drill Company)

Figure: Rock coring equipment

The term recovery ratio L_r *is* used in estimating the degree of disturbance of a cohesive or rock core sample.

$$L_r = \frac{\text{Actual length of recovered sample}}{\text{Theoretical length of recovered sample}}$$

A recovery ratio near 1.0 usually indicates good-quality rock. In badly fissured or soft rocks the recovery ratio may be 0.5 or less.

Rock quality designation (**RQD**) is an index or measure of the quality of a rock mass used by many engineers. **RQD** is computed from recovered core samples as:

$$\text{RQD} = \frac{\sum \text{Lengths of intact pieces of core} > 100 \text{ mm}}{\text{Length of core advance}}$$

For example, a core advance of 1500 mm produced a sample length of 1310 mm consisting of dust, gravel, and intact pieces of rock. The sum of lengths of pieces 100 mm or larger (pieces vary from gravel to 280 mm) in length is 890 mm. The recovery ratio L_r = 1310/1500 = 0.87 and **RQD** = 890/1500 = **0.59.**

The rating of rock quality may be used to approximately establish the field reduction of modulus of elasticity and/or compressive strength and the following may be used as a guide:

RQD	Rock description	E_f/E_{lab}*
<0.25	Very poor	0.15
0.25–0.50	Poor	0.20
0.50–0.75	Fair	0.25
0.75–0.90	Good	0.3–0.7
>0.90	Excellent	0.7–1.0

* Approximately for field/laboratory compression strengths also.

GROUNDWATER TABLE (GWT) LOCATION

Groundwater affects many elements of foundation design and construction, so the GWT should be established as accurately as possible if it is within the probable construction zone; otherwise, the location within ±0.3 to 0.5 m is usually adequate.

Soil strength (or bearing pressure) is usually reduced for foundations located below the water table. Foundations below the water table will be uplifted by the water pressure, and of course some kind of dewatering scheme must be employed if the foundations are to be constructed "in the dry."

The GWT is generally determined by lowering a weighted tape down the hole until water contact is made. An alternative is to install a *piezometer* (small vertical pipe) with a porous base and a removable top cap in the borehole. Backfill is then carefully placed around the piezometer so that surface water cannot enter the boring. This procedure allows continuous checking until the water level stabilizes.

In theory we might do the following:

Fill the hole and bail it out. After bailing a quantity, observe whether the water level in the hole is rising or falling. The true level is between the bailed depth where the water was falling and the bailed depth where it is rising.

NUMBER AND DEPTH OF BORINGS

There are no criteria for determining directly the number and depth of borings required on a project for subsurface exploration. For buildings a minimum of three borings, where the surface is level and the first two borings indicate regular stratification, may be adequate. Five borings are generally preferable (at building corners and center), especially if the site is not level. On the

other hand, a single boring may be sufficient for an antenna or industrial process tower base in a fixed location with the hole made at the point.
Additional borings may be required in very uneven sites or where fill areas have been made and the soil varies horizontally rather than vertically.
Borings should extend for 2 x the least lateral plan dimensions of the building or 10 m below the lowest building elevation.
If the 2 x width is not practical as, say, for a one-story warehouse or department store, boring depths of 6 to 15 m may be adequate. On the other hand, for important (or high-rise) structures that have small plan dimensions, it is common to extend one or more of the borings to bedrock or to competent (hard) soil regardless of depth.
Summarizing, there are no binding rules on either the number or the depth of exploratory soil borings.

THE SOIL REPORT

When the borings or other field work has been done and any laboratory testing completed, the geotechnical engineer then assembles the data for a recommendation to the client. Computer analyses may be made. The necessary engineering properties of the soil are the following:

1. Soil strength parameters of angle of internal friction ϕ and cohesion c
2. Allowable bearing capacity (considering both strength and probable settlements)
3. Engineering parameters such as E_s, μ .

A plan and profile of the borings may be made as on Fig. 3-37, or the boring information may be compiled from the field and laboratory data sheets as shown on Fig. 3-38.

On the left is the visual soil description as given by the drilling supervisor. The depth scale is shown to identify stratum thickness. The SS indicates that split spoon samples were recovered. The N column shows for each location the blows to seat the sampler 6 in. (150 mm) and to drive it for the next two 6-in. (150-mm) increments. At the 3-ft depth it took five blows to drive the split spoon 6 in., then 10 and 15 each for the next two 6-in. increments— the total N count = 10 + 1 5 = 25 as shown. The next column is the laboratory-determined $Q_u = q_u$ values, and for the 3-ft depth q_u = 7.0 tsf (670 kPa). The GWT appears to be at about elevation 793.6 ft.

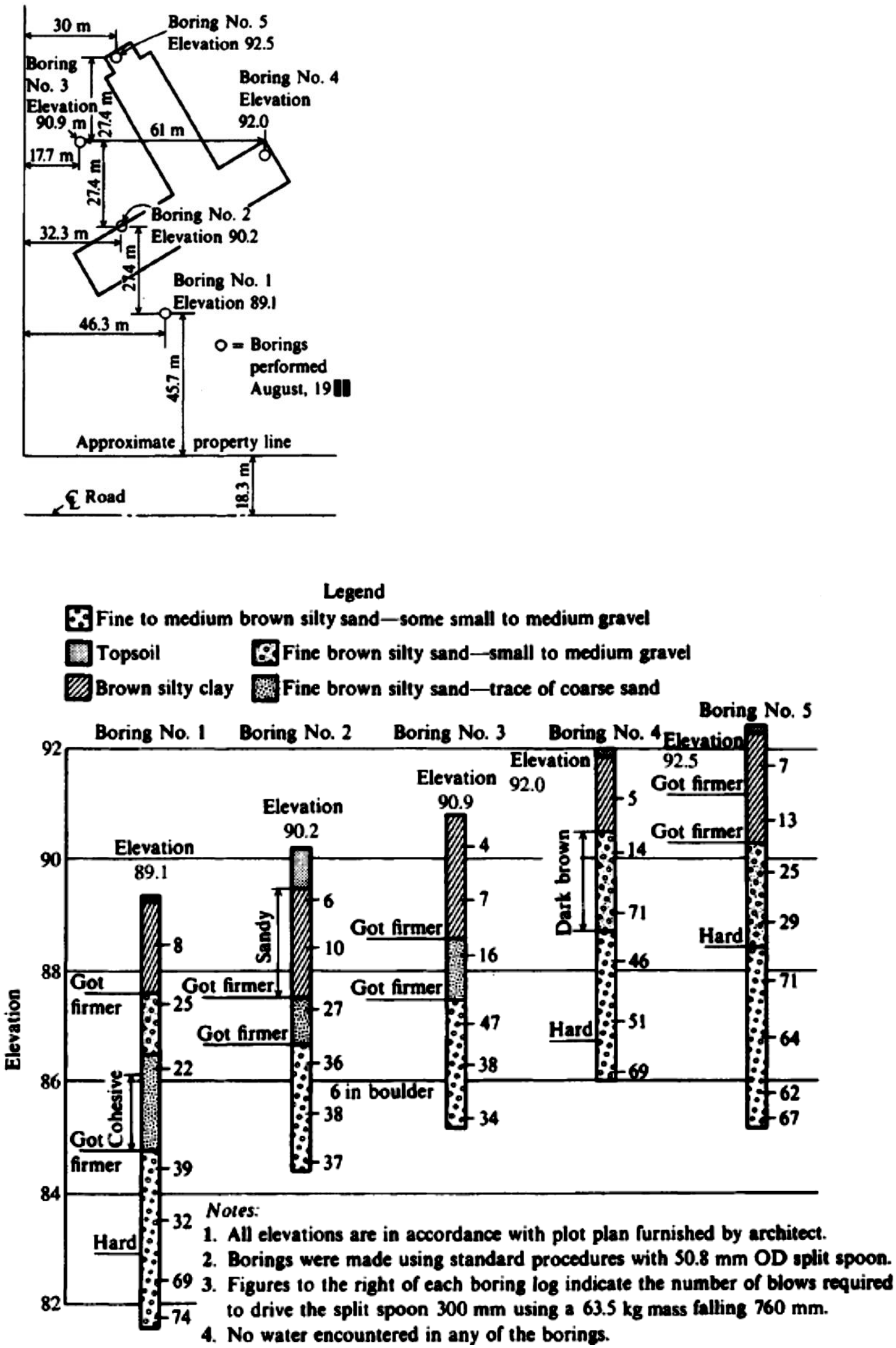


Figure 3-37 A method of presenting the boring information on a project. All dimensions are in meters unless shown otherwise.

WHITNEY & ASSOCIATES
INCORPORATED
2406 West Nebraska Avenue
PEORIA, ILLINOIS 61604

BORING LOG

BORING NO. B-04
DATE 12-03-92
W. & A. FILE NO. 55
SHEET 4 OF 7

PROJECT OHIO-AMERICAN ELEVATED WATER STORAGE TANK
LOCATION Ohio
BORING LOCATION See Plot Plan Sheet
DRILLED BY Winslow
BORING TYPE Hollow-Stem Auger
WEATHER CONDITIONS Partly Cloudy & Cool
SOIL CLASSIFICATION SYSTEM U. S. B. S. C.
SEEPAGE WATER ENCOUNTERED AT ELEVATION None
GROUND SURFACE ELEVATION 804.2
GROUND WATER ELEVATION AT 24+ HRS. 793.6
BORING DISCONTINUED AT ELEVATION 787.2
GROUND WATER ELEVATION AT COMPLETION 793.4

DESCRIPTION	DEPTH IN FEET	SAMPLE TYPE	N	Qp	Qu	Dd	Mc
Brown SILTY CLAY LOAM Organic Topsoil	6"						
Hard, Brown, Weathered GLACIAL SILTY CLAY TILL	03	SS	5 10 15(25)	4.5+	7.0	121	15
	06	SS	8 12 18(30)	4.5+	6.0	118	14
	09	SS	9 14 19(33)	4.5+	5.1	119	15
		SS	8 13 18(31)	4.5+	6.2	124	13
Hard, Gray, Unweathered GLACIAL SILTY CLAY TILL	12	SS	5 7 11(18)	4.5+	5.1	113	18
Very Stiff, Gray, Unweathered GLACIAL SILTY CLAY TILL	15	SS	5 5 8(13)	2.3	2.2	109	20
Hard, Gray LIMESTONE EXPLORATORY BORING DISCONTINUED	18						

N - BLOWS DELIVERED PER FOOT BY A 140 LB. HAMMER FALLING 30 INCHES
SS - SPLIT SPOON SAMPLE
ST - SHELBY TUBE SAMPLE
Qp - CALIBRATED PENETROMETER READING - T.S.F.
Qu - UNCONFINED COMPRESSIVE STRENGTH - T.S.F.
Dd - NATURAL DRY DENSITY - P.C.F.
Mc - NATURAL MOISTURE CONTENT - %

WHITNEY & ASSOCIATES
PEORIA, ILLINOIS

Figure 3-38 Boring log as furnished to client. N = SPT value; Q_p = pocket penetrometer; Q_u = unconfined compression test; D_d = estimated unit weight γ_s; M_c = natural water content w_N in percent.

BEARING CAPACITY OF FOUNDATIONS

INTRODUCTION

The soil must be capable of carrying the loads from any engineered structure placed upon it without *a shear failure* and with the *resulting settlements* being acceptable for that structure.
A soil shear failure can result in excessive building distortion and even collapse whereas excessive settlements can result in structural damage to a building frame.
It is necessary to investigate both base *shear resistance* and *settlements* for any structure.

The recommendation for the allowable bearing capacity q_a to be used for design is based on the *minimum* of either

1. Limiting the settlement to acceptable amount.
2. The ultimate bearing capacity, which considers soil strength, as computed in the following sections.

The allowable bearing capacity based on shear control q_a is obtained by reducing (or dividing) the ultimate bearing capacity q_{ult} (based on soil strength) by a safety factor **SF** that is deemed adequate to avoid a base shear failure to obtain

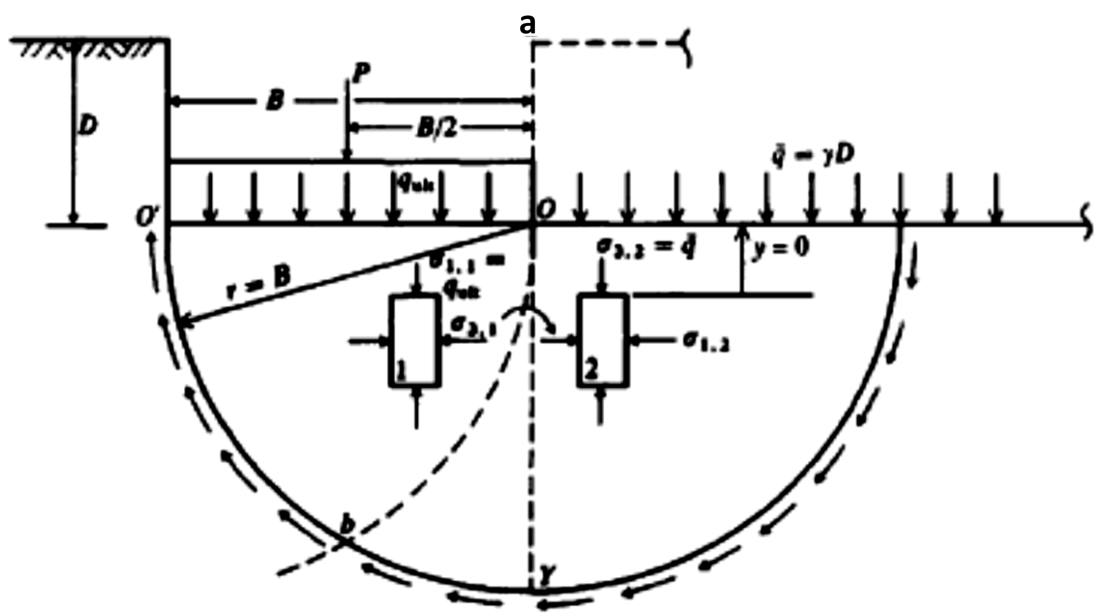
$$q_a = \frac{q_{ult}}{SF}$$

BEARING CAPACITY

From Fig. 4-1*a* and Fig. 4-2 it is evident we have two potential failure modes, where the footing, when loaded to produce the maximum bearing pressure q_{ult}, will do one or both of the following:
a. Rotate as in Fig. 4-*1a* about some center of rotation (probably along the vertical line *Oa)* with shear resistance developed along the perimeter of the slip zone shown as a circle.
b. Punch into the ground as the wedge *agb* of Fig. 4-2 or the approximate wedge *ObO'* of Fig. *4-1a*.

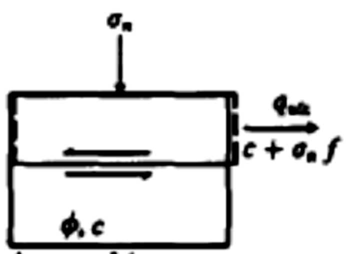
It should be apparent that both modes of potential failure develop the limiting soil shear strength along the slip path according to the shear strength equation given as

$$s = c + \sigma \tan \phi$$



(*a*) Footing on $\phi = 0°$ soil.

Note: $\bar{q} = p'_o = \gamma' D$, but use $\bar{q}$, since this is the accepted symbol for bearing capacity computations.

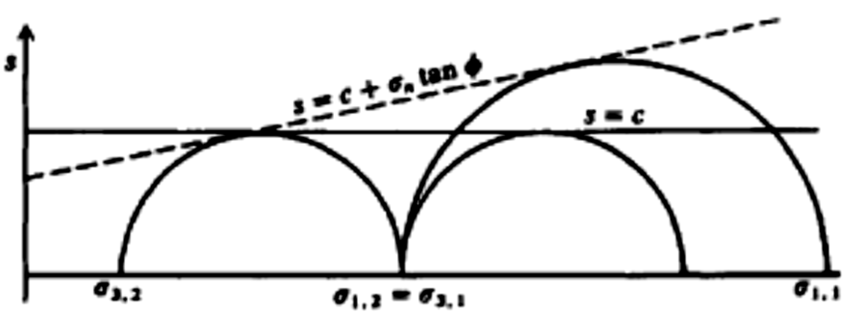


Area = dA

Friction = $\sigma_n \tan\phi = \sigma_n f$

Cohesion = $\frac{c\,dA}{dA}$

(*b*) Physical meaning of Eq. (2-52) for shear strength.



(*c*) Mohr's circle for (*a*) and for a ϕ–c soil.

Figure 4-1 Bearing capacity approximation on a $\phi = 0$ soil.

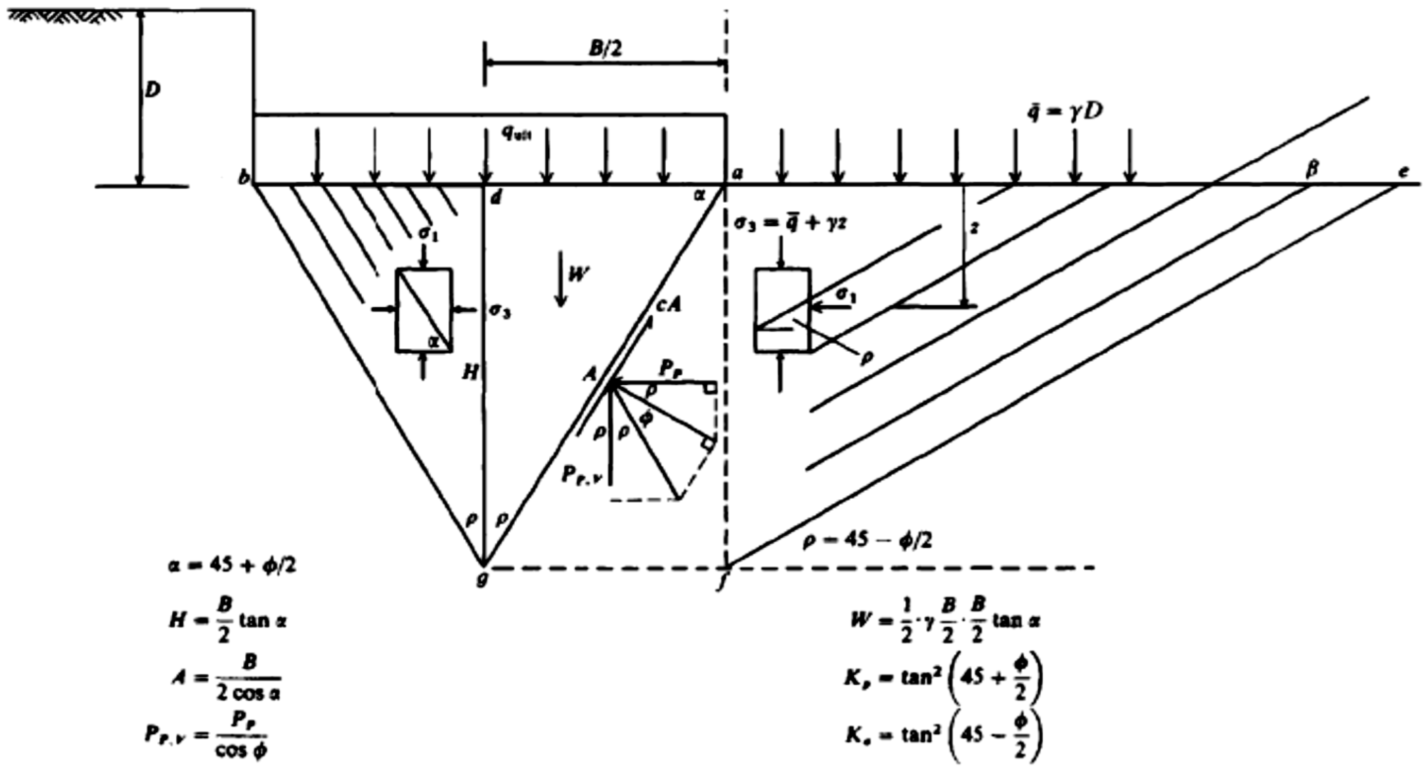


Figure 4-2 Simplified bearing capacity for a ϕ–c soil.

BEARING-CAPACITY EQUATIONS

There is currently no method of obtaining the ultimate bearing capacity of a foundation other than as an estimate.

The Terzaghi Bearing-Capacity Equation

One of the early sets of bearing-capacity equations was proposed by Terzaghi (1943) using the theory of plasticity to analyze the punching of a rigid base into a softer (soil) material as shown in Table 4-1.

Terzaghi's bearing-capacity equations were intended for "shallow" foundations where $D \leq B$

Note that the original equation for ultimate bearing capacity is derived only for the plane-strain case (i.e., for continuous foundations).

Since the soil wedge beneath round and square bases is much closer to a triaxial than plane strain state, the adjustment of ϕ_{tr} to ϕ_{ps} is recommended only when $L/B > 2$

$\phi_{ps} = 1.50\phi_{tr} - 17°$ $\quad (\phi_{tr} > 34°)$

$\phi_{ps} = \phi_{tr}$ $\quad (\phi_{tr} \leq 34°)$

TABLE 4-1

Bearing-capacity equations by the several authors indicated

Terzaghi (1943). See Table 4-2 for typical values and for $K_{p\gamma}$ values.

$$q_{ult} = cN_c s_c + \bar{q}N_q + 0.5\gamma B N_\gamma s_\gamma \qquad N_q = \frac{a^2}{a\cos^2(45 + \phi/2)}$$

$$a = e^{(0.75\pi - \phi/2)\tan\phi}$$

$$N_c = (N_q - 1)\cot\phi$$

$$N_\gamma = \frac{\tan\phi}{2}\left(\frac{K_{p\gamma}}{\cos^2\phi} - 1\right)$$

For:	strip	round	square
$s_c =$	1.0	1.3	1.3
$s_\gamma =$	1.0	0.6	0.8

Meyerhof (1963).* See Table 4-3 for shape, depth, and inclination factors.

$$q_{ult} = cN_c s_c d_c i_c + \bar{q} N_q s_q d_q i_q + 0.5\gamma B N_\gamma s_\gamma d_\gamma i_\gamma$$

$$N_q = e^{\pi\tan\phi}\tan^2\left(45 + \frac{\phi}{2}\right)$$

$$N_c = (N_q - 1)\cot\phi$$

$$N_\gamma = (N_q - 1)\tan(1.4\phi)$$

Hansen (1970).* See Table 4-5 for shape, depth, and other factors.

General:† $q_{ult} = cN_c s_c d_c i_c g_c b_c + \bar{q}N_q s_q d_q i_q g_q b_q + 0.5\gamma B' N_\gamma s_\gamma d_\gamma i_\gamma g_\gamma b_\gamma$

when $\phi = 0$

use $q_{ult} = 5.14 s_u(1 + s'_c + d'_c - i'_c - b'_c - g'_c) + \bar{q}$

$N_q =$ same as Meyerhof above

$N_c =$ same as Meyerhof above

$N_\gamma = 1.5(N_q - 1)\tan\phi$

Vesić (1973, 1975).* See Table 4-5 for shape, depth, and other factors.
Use Hansen's equations above.

$N_q =$ same as Meyerhof above

$N_c =$ same as Meyerhof above

$N_\gamma = 2(N_q + 1)\tan\phi$

*These methods require a trial process to obtain design base dimensions since width B and length L are needed to compute shape, depth, and influence factors.

†See Sec. 4-6 when $i_i < 1$.

Table 4-2 Terzaghi Bearing capacity factors —Eqs. (4.15), (4.13), and (4.11).[a]

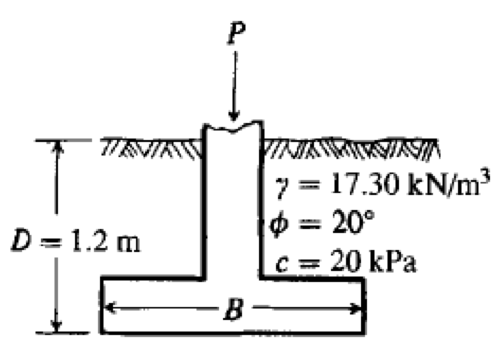
ϕ'	N_c	N_q	N_γ^a	ϕ'	N_c	N_q	N_γ^a
0	5.70	1.00	0.00	26	27.09	14.21	9.84
1	6.00	1.10	0.01	27	29.24	15.90	11.60
2	6.30	1.22	0.04	28	31.61	17.81	13.70
3	6.62	1.35	0.06	29	34.24	19.98	16.18
4	6.97	1.49	0.10	30	37.16	22.46	19.13
5	7.34	1.64	0.14	31	40.41	25.28	22.65
6	7.73	1.81	0.20	32	44.04	28.52	26.87
7	8.15	2.00	0.27	33	48.09	32.23	31.94
8	8.60	2.21	0.35	34	52.64	36.50	38.04
9	9.09	2.44	0.44	35	57.75	41.44	45.41
10	9.61	2.69	0.56	36	63.53	47.16	54.36
11	10.16	2.98	0.69	37	70.01	53.80	65.27
12	10.76	3.29	0.85	38	77.50	61.55	78.61
13	11.41	3.63	1.04	39	85.97	70.61	95.03
14	12.11	4.02	1.26	40	95.66	81.27	115.31
15	12.86	4.45	1.52	41	106.81	93.85	140.51
16	13.68	4.92	1.82	42	119.67	108.75	171.99
17	14.60	5.45	2.18	43	134.58	126.50	211.56
18	15.12	6.04	2.59	44	151.95	147.74	261.60
19	16.56	6.70	3.07	45	172.28	173.28	325.34
20	17.69	7.44	3.64	46	196.22	204.19	407.11
21	18.92	8.26	4.31	47	224.55	241.80	512.84
22	20.27	9.19	5.09	48	258.28	287.85	650.67
23	21.75	10.23	6.00	49	298.71	344.63	831.99
24	23.36	11.40	7.08	50	347.50	415.14	1072.80
25	25.13	12.72	8.34				

[a]From Kumbhojkar (1993)

The bearing capacity factors N_c, N_q, and N_γ are, respectively, the contributions of cohesion, surcharge, and unit weight of soil to the ultimate load-bearing capacity.

BEARING-CAPACITY EXAMPLES

Example 4-0. Compute the allowable bearing pressure using the Terzaghi equation for the square footing and soil parameters shown in Figure below. Use a safety factor of 3 to obtain q_a..



Solution.

Find the bearing capacity. Note that this value is usually what a geotechnical consultant would have to recommend *(B* not known but *D* is).

Since the footing is square (B=L), no adjustment of ϕ value is required.

From Table 4-2 obtain

$N_c = 17.7$ $N_q = 7.4$ $N_\gamma = 3.64$

$s_c = 1.3$ $s_\gamma = 0.8$ (from table 4-1, square footing)

$q_{ult} = cN_c s_c + \overline{q} N_q + 0.5\gamma B N_\gamma s_\gamma$

$= 20 (17.7) (1.3) + 1.2(17.3)(7.4) + 0.5 (17.3) (B)(3.64)(0.8)$

$= (613.8 + 25.2 B)$ kPa

The allowable pressure (a SF = 3 is commonly used when $c > 0$) is

$$q_a = \frac{q_{ult}}{SF}$$

$$= \frac{613.8+25.2B}{3} = (205 + 8.4B) \text{ kPa}$$

Since B is likely to range from 1.5 to 3 m

at B= 1.5 m $q_a = 205 + 8.4(1.5) = 218$ kPa (rounding)

at B = 3m $q_a = 205 + 8.4(3) = 230$ kPa

Recommend $q_a = 215\sim230$ kPa

Example 4.1

A square foundation is 2 m x 2 m in plan. The soil supporting the foundation has a friction angle of $\phi = 25^0$ and $c = 20$ kN/m^2. The unit weight of soil, γ, is 16.5 kN/m^3.

Determine the allowable gross load on the foundation using Terzaghi Bearing Capacity Equations with a factor of safety (FS) of 3.

Assume that the depth of the foundation (D_f) is 1.5 m and that general shear failure occurs in the soil.

<u>Solution</u>

Since the footing is square (B=L), no adjustment of ϕ value is required.

$q_{ult} = cN_c s_c + \overline{q} N_q + 0.5\gamma BN_\gamma s_\gamma$

s_c = 1.3 ***s_γ = 0.8*** (from table 4-1, square footing)

At B=2.0m

From Table 4.1, for $\phi' = 25°$,

$$N_c = 25.13$$

$$N_q = 12.72$$

$$N_\gamma = 8.34$$

Thus,

q_{ult} = (20) (25.13) (1.3) + (1.5x16.5)(12.72) + (0.5)(16.5)(2)(8.34)(0.8)

= 653.38 + 314.82 + 110.09 = 1078.29 kN/m^2

So, the allowable load per unit area of the foundation is

$$q_a = \frac{q_{ult}}{FS} = \frac{1078.29}{3} = 359.5 \text{ kN/m}^2$$

Thus, the total allowable gross load is

$$Q = (359.5)B^2 = (359.5)(2 \times 2) = 1438 \text{ kN}$$

H.W: Resolve the same example assuming the foundation is circular with a diameter of 3m.

Example 4.2

Refer to Example 4.1. Assume that the shear-strength parameters of the soil are the same. A square foundation measuring $B \times B$ will be subjected to an allowable gross load of 1000 kN with FS = 3 and D_f = 1 m. Determine the size B of the foundation.

Solution

Allowable gross load Q =1000 kN with FS =3. Hence, the ultimate load $Q_{ult} = (Q_u)/(FS)$ = (1000)(3) = 3000 kN. So,

$$q_{ult} = \frac{Q_u}{B^2} = \frac{3000}{B^2}$$

$q_{ult} = cN_c s_c + \overline{q} N_q + 0.5\gamma BN_\gamma s_\gamma$

For $\phi' = 25°$, $N_c = 25.13$, $N_q = 12.72$, and $N_\gamma = 8.34$.
Also,

$$q = \gamma D_f = (16.5)(1) = 16.5 \text{ kN/m}^2$$

Now,

q_{ult} = (20) (25.13) (1.3) + (16.5)(12.72) + (0.5)(16.5)(B)(8.34)(0.8)
= 863.26 + 55.04 B (b)

Combining Eqs. (a) and (b),

$$\frac{3000}{B^2} = 863.26 + 55.04B \quad \text{(c)}$$

B (m)	L.H.S	R.H.S
1.0	3000	918.3
1.5	1333	945.8
2.0	750	973.3
Try B=1.75m	979.6	959.6

By trial and error, we have

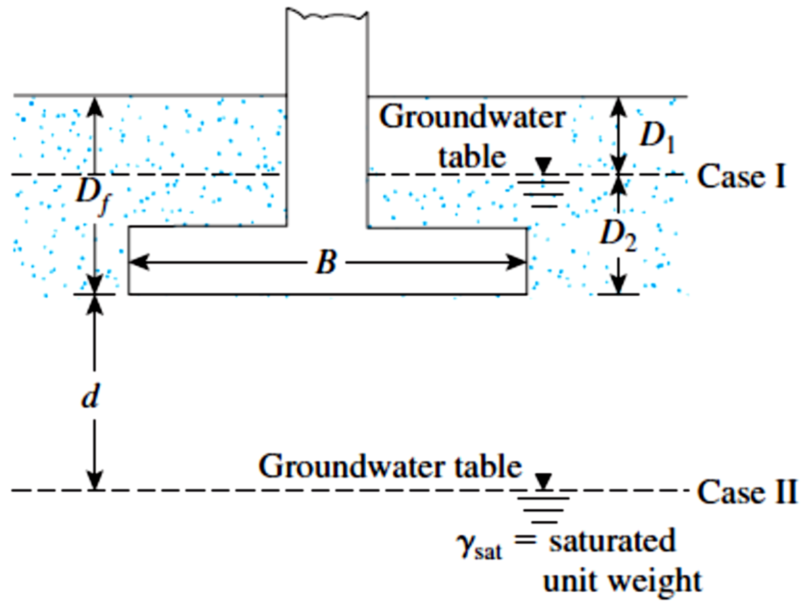
$$B = 1.77 \text{ m} \approx 1.8 \text{ m}$$

■

H.W.: Resolve the same example if the allowable gross load is 2500 kN.

Modification of Bearing Capacity Equations for Water Table

Equations in table 4.1 give the ultimate bearing capacity, based on the assumption that the water table is located well below the foundation. However, if the water table is close to the foundation, some modifications of the bearing capacity equations will be necessary. (See Figure below)



Case I. If the water table is located so that $\mathbf{0 \le D1 \le D_f}$, the factor q in the bearing capacity equations takes the form

$\bar{q}$ = effective surcharge = $D_1 \gamma + D_2 \acute{\gamma}$

where $\acute{\gamma} = \gamma_{sat} - \gamma_w$

γ_{sat} = saturated unit weight of soil

γ_w = unit weight of water = 10 kN/m^3

Also, the value of γ in the last term of the equations has to be replaced by $\acute{\gamma} = \gamma_{sat} - \gamma_w$

Case II. For a water table located so that $0 \le d \le B$,

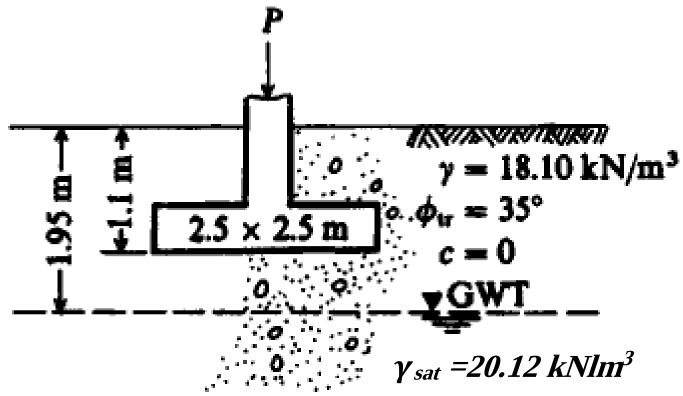
$\bar{q} = \gamma D_f$

In this case, the factor γ in the last term of the bearing capacity equations must be replaced by the factor

$$\bar{\gamma} = \gamma' + \frac{d}{B}(\gamma - \gamma')$$

Case III. When the water table is located so that $d \ge B$, the water will have no effect on the ultimate bearing capacity.

Example 4-8. A square footing that is vertically and concentrically loaded is to be placed on a cohesionless soil as shown in Figure below. The soil and other data are as shown.



Required. What is the allowable bearing capacity using the Terzaghi equation and a SF = 2.5?

Solution:

Since the footing is square (B=L), no adjustment of ϕ value is required.

d = 1.95 – 1.1 = 0.85 m

B= 2.5m and d < B

$$\overline{\gamma} = \gamma' + \frac{d}{B}(\gamma - \gamma')$$

γ =18.1 kN/m³ γ_{sat} =20.12 kNlm³

$\acute{\gamma} = \gamma_{sat} - \gamma_w$

$\acute{\gamma}$ = 20.12 – 10 = 10.12 kN /m³

$\overline{\gamma} = 10.12 + \frac{0.85}{2.5}(18.1-10.12) = 12.83$ kN/m³

$q_{ult} = cN_c s_c + \overline{q} N_q + 0.5\gamma B N_\gamma s_\gamma$

From table 4.2 $N_c = 57.75$ $N_q = 41.44$ $N_\gamma = 45.41$

$s_c = 1.3$ $s_\gamma = 0.8$ (from table 4-1, square footing)

for B = 2.5m

$q_{ult} = 0 + 1.1 \times 18.1 \times 41.44 + 0.5 \times 12.83 \times 2.5 \times 45.41 \times 0.8$

$= 825.1 + 582.6 = 1407.7$ **kN/m²**

$q_a = 1407.7 / 2.5 = 563$ **kN/m² = 563 kPa**

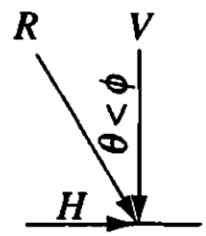
H.W: Resolve the same example assuming the water table is A: 0.5 m below ground level
B: 4.0 m below ground level

Meyerhof 's Bearing-Capacity Equation

Meyerhof (1951, 1963) proposed a bearing-capacity equation similar to that of Terzaghi but included a shape factor s_q with the depth term N_q. He also included depth factors d_i and inclination factors i_i for cases where the footing load is inclined from the vertical. These additions produce equations of the general form shown in Table 4-1, with select N factors computed in Table 4-4.

TABLE 4-3

Shape, depth, and inclination factors for the Meyerhof bearing-capacity equations of Table 4-1

Factors	Value	For
Shape:	$s_c = 1 + 0.2K_p \frac{B}{L}$	Any ϕ
	$s_q = s_\gamma = 1 + 0.1K_p \frac{B}{L}$	$\phi > 10°$
	$s_q = s_\gamma = 1$	$\phi = 0$
Depth:	$d_c = 1 + 0.2\sqrt{K_p}\frac{D}{B}$	Any ϕ
	$d_q = d_\gamma = 1 + 0.1\sqrt{K_p}\frac{D}{B}$	$\phi > 10$
	$d_q = d_\gamma = 1$	$\phi = 0$
Inclination:	$i_c = i_q = \left(1 - \frac{\theta°}{90°}\right)^2$	Any ϕ
	$i_\gamma = \left(1 - \frac{\theta°}{\phi°}\right)^2$	$\phi > 0$
	$i_\gamma = 0 \text{ for } \theta > 0$	$\phi = 0$

Where $K_p = \tan^2(45 + \phi/2)$ as in Fig. 4-2

θ = angle of resultant R measured from vertical without a sign; if $\theta = 0$ all $i_i = 1.0$.

B, L, D = previously defined

TABLE 4-4

Bearing-capacity factors for the Meyerhof, Hansen, and Vesić bearing-capacity equations

Note that N_c and N_q are the same for all three methods; subscripts identify author for N_γ

ϕ	N_c	N_q	$N_{\gamma(H)}$	$N_{\gamma(M)}$	$N_{\gamma(V)}$	N_q/N_c	$2\tan\phi(1-\sin\phi)^2$
0	5.14*	1.0	0.0	0.0	0.0	0.195	0.000
5	6.49	1.6	0.1	0.1	0.4	0.242	0.146
10	8.34	2.5	0.4	0.4	1.2	0.296	0.241
15	10.97	3.9	1.2	1.1	2.6	0.359	0.294
20	14.83	6.4	2.9	2.9	5.4	0.431	0.315
25	20.71	10.7	6.8	6.8	10.9	0.514	0.311
26	22.25	11.8	7.9	8.0	12.5	0.533	0.308
28	25.79	14.7	10.9	11.2	16.7	0.570	0.299
30	30.13	18.4	15.1	15.7	22.4	0.610	0.289
32	35.47	23.2	20.8	22.0	30.2	0.653	0.276
34	42.14	29.4	28.7	31.1	41.0	0.698	0.262
36	50.55	37.7	40.0	44.4	56.2	0.746	0.247
38	61.31	48.9	56.1	64.0	77.9	0.797	0.231
40	75.25	64.1	79.4	93.6	109.3	0.852	0.214
45	133.73	134.7	200.5	262.3	271.3	1.007	0.172
50	266.50	318.5	567.4	871.7	761.3	1.195	0.131

* = $\pi + 2$ as limit when $\phi \rightarrow 0°$.

Slight differences in above table can be obtained using program BEARING.EXE on diskette depending on computer used and whether or not it has floating point.

Hansen's Bearing-Capacity Method

Hansen (1970) proposed the general bearing-capacity case and N factor equations shown in Table 4-1. Hansen's shape, depth, and other factors making up the general bearing capacity equation are given in Table 4-5. The extensions include base factors for situations in which the footing is tilted from the horizontal b_i and for the possibility of a slope β of the ground supporting the footing to give ground factors g_i.

Note that when the base is tilted, ***V*** and ***H*** are perpendicular and parallel, respectively, to the base, compared with when it is horizontal as shown in the sketch with Table 4-5c. The bearing capacity using *N* factors as given in Table 4-4.

The Hansen equation can be used for both shallow (footings) and deep (piles, drilled caissons) bases.

TABLE 4-5a

Shape and depth factors for use in either the Hansen (1970) or Vesić (1973, 1975*b*) bearing-capacity equations of Table 4-1. Use s'_c, d'_c when $\phi = 0$ only for Hansen equations. Subscripts H, V for Hansen, Vesić, respectively.

Shape factors	Depth factors
$s'_{c(H)} = 0.2\dfrac{B'}{L'} \quad (\phi = 0°)$ $s_{c(H)} = 1.0 + \dfrac{N_q}{N_c} \cdot \dfrac{B'}{L'}$ $s_{c(V)} = 1.0 + \dfrac{N_q}{N_c} \cdot \dfrac{B}{L}$ $s_c = 1.0$ for strip	$d'_c = 0.4k \quad (\phi = 0°)$ $d_c = 1.0 + 0.4k$ $k = D/B$ for $D/B \leq 1$ $k = \tan^{-1}(D/B)$ for $D/B > 1$ k in radians
$s_{q(H)} = 1.0 + \dfrac{B'}{L'}\sin\phi$ $s_{q(V)} = 1.0 + \dfrac{B}{L}\tan\phi$ for all ϕ	$d_q = 1 + 2\tan\phi(1 - \sin\phi)^2 k$ k defined above
$s_{\gamma(H)} = 1.0 - 0.4\dfrac{B'}{L'} \quad \geq 0.6$ $s_{\gamma(V)} = 1.0 - 0.4\dfrac{B}{L} \quad \geq 0.6$	$d_\gamma = 1.00 \quad$ for all ϕ

Notes:

1. Note use of "effective" base dimensions B', L' by Hansen but not by Vesić.
2. The values above are consistent with either a vertical load or a vertical load accompanied by a horizontal load H_B.
3. With a vertical load and a load H_L (and either $H_B = 0$ or $H_B > 0$) you may have to compute two sets of shape s_i and d_i as $s_{i,B}$, $s_{i,L}$ and $d_{i,B}$, $d_{i,L}$. For i, L subscripts of Eq. (4-2), presented in Sec. 4-6, use ratio L'/B' or D/L'.

TABLE 4-5b

Table of inclination, ground, and base factors for the Hansen (1970) equations. See Table 4-5c for equivalent Vesić equations.

Inclination factors	Ground factors (base on slope)
$i'_c = 0.5\text{-}0.5\sqrt{1 - \frac{H_i}{A_f C_a}}$	$g'_c = \frac{\beta°}{147°}$
$i_c = i_q - \frac{1 - i_q}{N_q - 1}$	$g_c = 1.0 - \frac{\beta°}{147°}$
$i_q = \left[1 - \frac{0.5H_i}{V + A_f c_a \cot\phi}\right]^{\alpha_1}$ $2 \leq \alpha_1 \leq 5$	$g_q = g_\gamma = (1 - 0.5\tan\beta)^5$
	Base factors (tilted base)
$i_\gamma = \left[1 - \frac{0.7H_i}{V + A_f c_a \cot\phi}\right]^{\alpha_2}$	$b'_c = \frac{\eta°}{147°}$ $(\phi = 0)$
$i_\gamma = \left[1 - \frac{(0.7 - \eta°/450°)H_i}{V + A_f c_a \cot\phi}\right]^{\alpha_2}$ $2 \leq \alpha_2 \leq 5$	$b_c = 1 - \frac{\eta°}{147°}$ $(\phi > 0)$ $b_q = \exp(-2\eta\tan\phi)$ $b_\gamma = \exp(-2.7\eta\tan\phi)$ η in radians

Notes:

1. Use H_i as either H_B or H_L, or both if $H_L > 0$.
2. Hansen (1970) did not give an i_c for $\phi > 0$. The value above is from Hansen (1961) and also used by Vesić.
3. Variable c_a = base adhesion, on the order of 0.6 to 1.0 × base cohesion.
4. Refer to sketch for identification of angles η and β, footing depth D, location of H_i (parallel and at top of base slab; usually also produces eccentricity). Especially note V = force *normal* to base and is not the resultant R from combining V and H_i.

TABLE 4-5c
Table of inclination, ground, and base factors for the Vesic (1973, 1975) bearing-capacity equations. See notes below and refer to sketch for identification of terms.

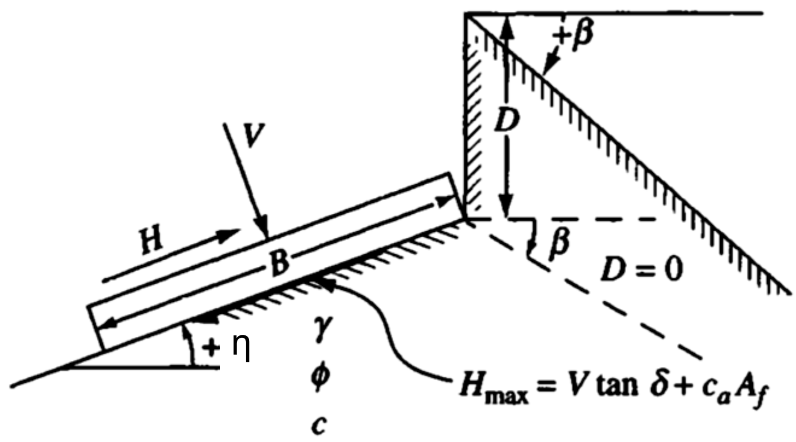
Inclination factors	Ground factors (base on slope)
$i'_c = 1 - \dfrac{mH_i}{A_f c_a N_c} \quad (\phi = 0)$	$g'_c = \dfrac{\beta}{5.14} \quad \beta$ in radians
$i_c = i_q - \dfrac{1 - i_q}{N_q - 1} \quad (\phi > 0)$	$g_c = i_q - \dfrac{1 - i_q}{5.14 \tan\phi} \quad \phi > 0$
i_q, and m defined below	i_q defined with i_c
$i_q = \left[1.0 - \dfrac{H_i}{V + A_f c_a \cot\phi}\right]^m$	$g_q = g_\gamma = (1.0 - \tan\beta)^2$
	Base factors (tilted base)
$i_\gamma = \left[1.0 - \dfrac{H_i}{V + A_f c_a \cot\phi}\right]^{m+1}$	$b'_c = g'_c \quad (\phi = 0)$
$m = m_B = \dfrac{2 + B/L}{1 + B/L}$	$b_c = 1 - \dfrac{2\beta}{5.14 \tan\phi}$
$m = m_L = \dfrac{2 + L/B}{1 + L/B}$	$b_q = b_\gamma = (1.0 - \eta \tan\phi)^2$

Notes:

1. When $\phi = 0$ (and $\beta \neq 0$) use $N_\gamma = -2\sin(\pm\beta)$ in N_γ term.
2. Compute $m = m_B$ when $H_i = H_B$ (H parallel to B) and $m = m_L$ when $H_i = H_L$ (H parallel to L). If you have both H_B and H_L use $m = \sqrt{m_B^2 + m_L^2}$. Note use of B and L, not B', L'.
3. Refer to Table sketch and Tables 4-5*a,b* for term identification.
4. Terms N_c, N_q, and N_γ are identified in Table 4-1.
5. Vesić always uses the bearing-capacity equation given in Table 4-1 (uses B' in the N_γ term even when $H_i = H_L$).
6. H_i term $\leq$ 1.0 for computing i_q, i_γ (always).

Notes: $\beta + \eta \quad 90^{\circ}$ (Both β and η have signs (+) shown.)

$\beta \quad \phi$



For: $L/B \leq 2$ use ϕ_{tr}

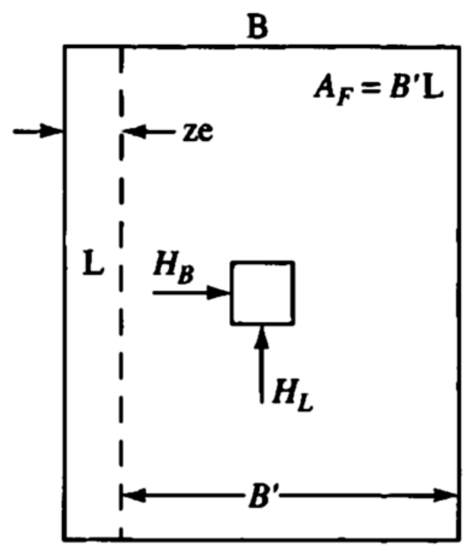
$L/B > 2$ use $\phi_{ps} = 1.5\,\phi_{tr} - 17^{\circ}$

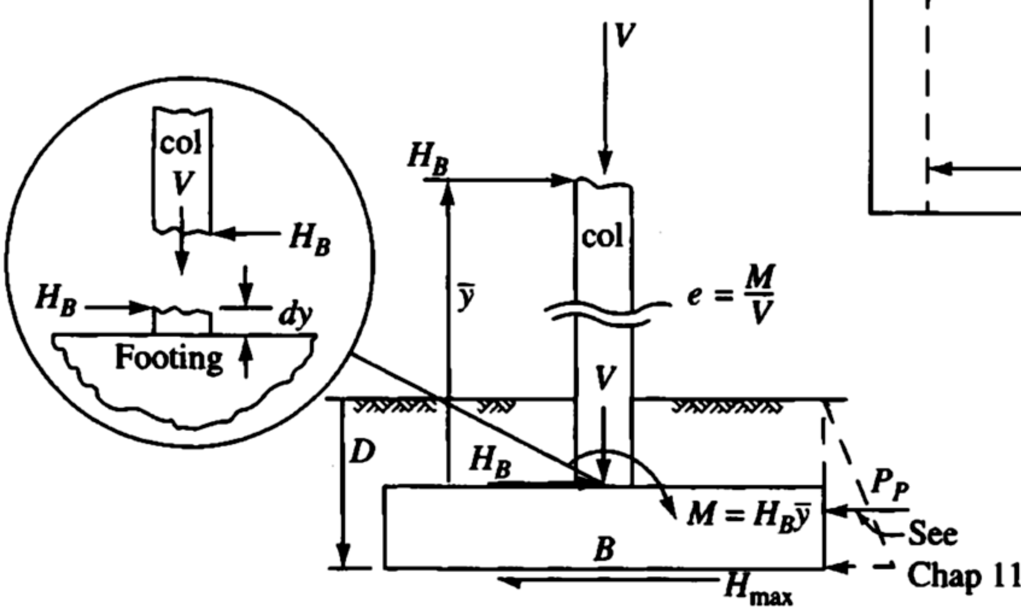
$\phi_{tr} \leq 34^{\circ}$ use $\phi_{tr} = \phi_{ps}$

δ = friction angle between base and soil ($.5\phi \leq \delta \leq \phi$)

$A_f = B'L'$ (effective area)

c_a = base adhesion (0.6 to 1.0c)





$H_{max} + P_p \geq \mathrm{SF} \times (H_B)$

Example 4-2: A footing load test made produced the following data:

D = 0.5 m B = 0.5 m L = 2.0 m
γ' = 9.31 kN/m^3 $\phi_{triaxial}$ = 42.5° Cohesion c = 0
P_{ult} = 1863 kN (measured) $q_{ult} = \frac{P_{ult}}{BL} = \frac{1863}{0.5x2}$ = 1863 kPa (computed)

Required: Compute the ultimate bearing capacity by both Hansen and Meyerhof equations and compare these values with the measured value.
Solution:
a. Since c = 0, any factors with subscript c do not need computing. All g_i and b_i factors are 1.00; with these factors identified, the Hansen equation simplifies to

$$q_{ult} = \gamma' D N_q s_q d_q + 0.5\gamma' B N_\gamma S_\gamma d_\gamma$$

$$L/B = \frac{2}{0.5} = 4 \rightarrow \phi_{ps} = 1.5(42.5) - 17 = 46.75°$$

$$\text{Use } \phi = 47°$$

From a table of ϕ in 1° increments (table not shown) obtain

$$N_q = 187 \qquad N_\gamma = 299$$

Using linear interpolation of Table 4-4 gives 208.2 and 347.2. Using Table 4-5*a* one obtains [get the $2\tan\phi(1-\sin\phi)^2$ part of d_q term from Table 4-4] the following:

$$s_{q(H)} = 1 + \frac{B'}{L'}\sin\phi = 1.18 \qquad s_{\gamma(H)} = 1 - 0.4\frac{B'}{L'} = 0.9$$

$$d_q = 1 + 2\tan\phi(1-\sin\phi)^2\frac{D}{B'} = 1 + 0.155\frac{D}{B'}$$

$$= 1 + 0.155\left(\frac{0.5}{0.5}\right) = 1.155 \qquad d_\gamma = 1.0$$

With these values we obtain

q_{ult} = 9.31(0.5)(187)(1.18)(1.155) + 0.5(9.31)(0.5)(299)(0.9)(l)

= 1812 kPa vs. 1863 kPa measured

b. By the Meyerhof equations of Table 4-1 and 4-3, and ϕ_{ps} = 47°, we can proceed as follows:

Step 1. Obtain $N_q = 187$

$$N_\gamma = (N_q - 1)\tan(1.4\phi) = 413.6 \rightarrow 414$$

$$K_p = \tan^2\left(45 + \frac{\phi}{2}\right) = 6.44 \rightarrow \sqrt{K_p} = 2.54$$

$$s_q = s_\gamma = 1 + 0.1K_p\frac{B}{L} = 1 + 0.1(6.44)\frac{0.5}{2.0} = 1.16$$

$$d_q = d_\gamma = 1 + 0.1\sqrt{K_p}\frac{D}{B} = 1 + 0.1(2.54)\frac{0.5}{0.5} = 1.25$$

Step 2. Substitute into the Meyerhof equation (ignoring any c subscripts):

$$\begin{aligned} q_{ult} &= \gamma' D N_q s_q d_q + 0.5\gamma B N_\gamma s_\gamma d_\gamma \\ &= 9.31(0.5)(187)(1.16)(1.25) + 0.5(9.31)(0.5)(414)(1.16)(1.25) \\ &= 1262 + 1397 = \mathbf{2659}\ \text{kPa} \end{aligned}$$

Example 4-3:

A series of large-scale footing bearing-capacity tests were performed on soft saturated clay (ϕ =0). One of the tests consisted of a 1.05-m-square footing at a depth D = 1.5 m. At a 25 mm. settlement the load was approximately 16.1 tons from interpretation of the given load-settlement curve. Unconfined compression and shear tests gave values as follows:

q_u = 3.0 ton/m^2 c = 1.92 ton/m^2 , the unit weight of soil is 17.5 kN/m^3

Required: Compute the ultimate bearing capacity by the Hansen equations and compare with the load-test value of 16.1 tons.

Solution: Obtain N, s'_i, and d'_i factors. Since ϕ= 0°, we have N_c = 5.14 and N_q= 1.0

$$s'_c = 0.2\frac{B}{L} = 0.2\frac{1}{1} = 0.2$$

$$d'_c = 0.4\tan^{-1}\frac{D}{B} = 0.4\tan^{-1}\frac{1.5}{1.05} = 0.38 \qquad (D > B)$$

q_{ult}= 5.14s_u (1 + s'_c + d'_c) + $\bar{q}$ Table 4-1 for ϕ= 0 case

c = 1.92 x 10 = 19.2 kN/m^2 (10 converts ton to kN)

q_{ult} = 5.14(19.2)(1 + 0.2 + 0.38) + 17.5 x1.5 = 182.2 kN/m^2

From load test, q_{actual} = 16.1/1.05^2 = 14.6 ton/m^2 = 146 kN/m^2

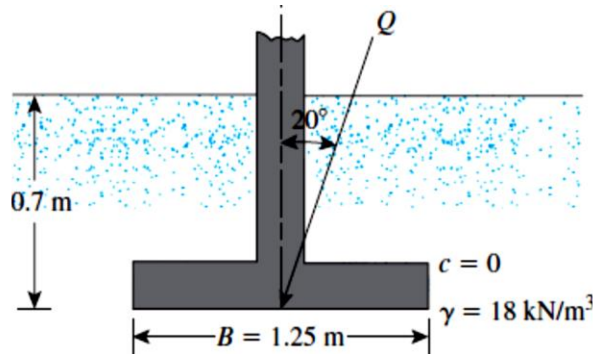
If we use the unconfined compression tests and take $c = q_u/2$, we obtain

q_{ult} = (1.5/ 1.92) x182.2 = 142.4 kN/m^2

Example 4.5

A square column foundation (see figure below) is to be constructed on a fine sand deposit. The allowable load *Q* will be inclined at an angle *β* = 20° with the vertical. The standard penetration numbers N_{70} obtained from the field are as follows.

Depth (m)	N_{70}
1.5	5
3.0	4
4.5	9
6.0	7
7.5	8
9	8



Determine Q using Meyerhof bearing capacity equations, use F.S =3

Solution:

The average SPT number is (5 + 4 + 9 + 7 + 8 + 8) / 6 = 6.83

From table 3-4, the soil can be classified as medium density fine sand and the angle of internal friction (ϕ) is estimated to be = 30^0

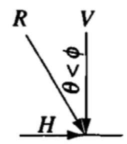
Since the footing is square (B=L), no adjustment of ϕ value is required

The general form of Meyerhof B.C equation is:

$$q_{ult} = cN_cs_cd_ci_c + \bar{q}\, N_qs_qd_qi_q + 0.5\gamma BN_\gamma s_\gamma d_\gamma i_\gamma$$

From table 4-4 and for ϕ = 30^0, we have $N_c = 30.13$, $N_q = 18.4$ and $N_\gamma = 15.7$

Since *c* = 0, any factors with subscript *c* do not need computing.

Factors	Value	For
Shape:	$s_c = 1 + 0.2K_p\frac{B}{L}$	Any ϕ
	$s_q = s_\gamma = 1 + 0.1K_p\frac{B}{L}$	$\phi > 10°$
	$s_q = s_\gamma = 1$	$\phi = 0$
Depth:	$d_c = 1 + 0.2\sqrt{K_p}\frac{D}{B}$	Any ϕ
	$d_q = d_\gamma = 1 + 0.1\sqrt{K_p}\frac{D}{B}$	$\phi > 10$
	$d_q = d_\gamma = 1$	$\phi = 0$
Inclination: 	$i_c = i_q = \left(1 - \frac{\theta°}{90°}\right)^2$	Any ϕ
	$i_\gamma = \left(1 - \frac{\theta°}{\phi°}\right)^2$	$\phi > 0$
	$i_\gamma = 0$ for $\theta > 0$	$\phi = 0$

Where $K_p = \tan^2(45 + \phi/2)$ as in Fig. 4-2
θ = angle of resultant *R* measured from vertical without a sign; if $\theta = 0$ all $i_i = 1.0$.
B, L, D = previously defined

for ϕ > 10^0 $$s_q = s_\gamma = 1 + 0.1K_p\frac{B}{L}$$

where $K_p = \tan^2(45 + \phi/2) = \tan^2(45 + 30/2) = 3.0$

$$\therefore\ s_q = s_\gamma = 1 + 0.1 \times 3 \times \frac{1.25}{1.25} = 1.3$$

for ϕ > 10^0 $$d_q = d_\gamma = 1 + 0.1\sqrt{K_p}\frac{D}{B}$$

$$\therefore\ d_q = d_\gamma = \mathbf{1 + 0.1\sqrt{3}\,\frac{0.7}{1.25} = 1.097 \approx 1.1}$$

for any ϕ $$i_c = i_q = \left(1 - \frac{\theta°}{90°}\right)^2$$

∴ $i_q = (1 - \frac{20}{90})^2 = 0.605$

$$i_\gamma = \left(1 - \frac{\theta°}{\phi°}\right)^2 \qquad \text{for } \phi > 0$$

∴ $i_\gamma = (1 - \frac{20}{30})^2 = 0.111$

$\bar{q} = D \times \gamma = 0.7 \times 18 = 12.6$ kN/m^2

$q_{ult} = 12.6 \times 18.4 \times 1.3 \times 1.1 \times 0.605 + 0.5 \times 18 \times 1.25 \times 15.7 \times 1.3 \times 1.1 \times 0.111 = 200.5 + 28.03$
$= 228.3$ kN/m^2

$q_a = 228.3/3 = 76.2$ kN/m^2

Q = q_a x B x L = 76.2 x 1.25^2 = 119 kN

Example 4-4:

Given: A series of unconfined compression tests in the zone of interest (from SPT samples) from a boring-log give an average q_u = 200 kPa. The soil is fully saturated ($\phi = 0$)

Required: Estimate the allowable bearing capacity for square footings located at somewhat uncertain depths (let D =0 m) and *B* dimensions unknown using both the Meyerhof and Terzaghi bearing-capacity equations. Use safety factor SF = 3.0.

Solution: (The reader should note this is the most common procedure for obtaining the allowable bearing capacity for cohesive soils with limited data.)

a: By Meyerhof equations,
from table 4.1

$$q_{ult} = cN_c s_c d_c + \bar{q} N_q s_q d_q + 0.5\gamma B' N_\gamma s_\gamma d_\gamma$$

c = $q_u/2$ (for both equations)

from table 4.3 $s_c = 1 + 0.2\, K_p \frac{B}{L}$

$K_p = \tan^2(45 + \emptyset / 2) = \tan^2(45) = 1.0$

$s_c = 1.2$

$$d_c = 1 + 0.2\sqrt{K_p}\frac{D}{B}$$

$d_c = 1. + 0 = 1.0$

$$s_q = s_\gamma = 1 \qquad \phi = 0$$

$$d_q = d_\gamma = 1 \qquad\qquad \phi = 0$$

$$q_{ult} = 1.2cN_c + \bar{q}N_q$$

$$q_a = \frac{q_{ult}}{3} = 1.2\frac{q_u}{2}(5.14)\frac{1}{3} + \frac{\bar{q}}{3} = 1.03q_u + 0.3\bar{q}$$

b. By Terzaghi equations, we can take s_c = 1.3 for ϕ= 0.

$$q_a = \frac{q_{ult}}{3} = \frac{q_u}{2}(5.7)(1.3)\frac{1}{3} + \frac{\bar{q}}{3} = 1.24q_u + 0.3\bar{q}$$

FOOTINGS WITH ECCENTRIC OR INCLINED LOADINGS

A footing may be eccentrically loaded from a concentric column with an axial load and moments about one or both axes as in Fig. 4-4. The eccentricity may result also from a column that is initially not centrally located.

Footings with Eccentricity

Research and observation [Meyerhof and Hansen] indicate that *effective* footing dimensions obtained (refer to Fig. 4-4) as

$L' = L - 2e_x \qquad B' = B - 2e_y$

should be used in bearing-capacity analyses to obtain an effective footing area defined as

$\boldsymbol{A_f = B'L'}$

and the center of pressure when using a rectangular pressure distribution of q' is the center of area $B'L'$ at point *A';* i.e., from Fig *4-4a:*

$\boldsymbol{2e_x + L' = L}$

$\boldsymbol{e_x + c = L/2}$

Substitute for *L* and obtain $\boldsymbol{c = L'/2}$. If there is no eccentricity about either axis, use the actual footing dimension for that ***B'** or **L'***.

For design the minimum dimensions (to satisfy ACI 318 code) of a rectangular footing with a central column of dimensions $\boldsymbol{w_x \times w_y}$ are required to be

$\boldsymbol{B_{min} = 4e_y + w_y} \qquad \boldsymbol{B' = 2e_y + w_y}$

$\boldsymbol{L_{min} = 4e_x + w_x} \qquad \boldsymbol{L' = 2e_x + w_x}$

Final dimensions may be larger than B_{min} or L_{min} based on obtaining the required allowable bearing capacity.
The ultimate bearing capacity for footings with eccentricity, using Hansen/Vesic equations, is found by either the Hansen or Vesic bearing-capacity equation given in Table 4-1 with the following adjustments:

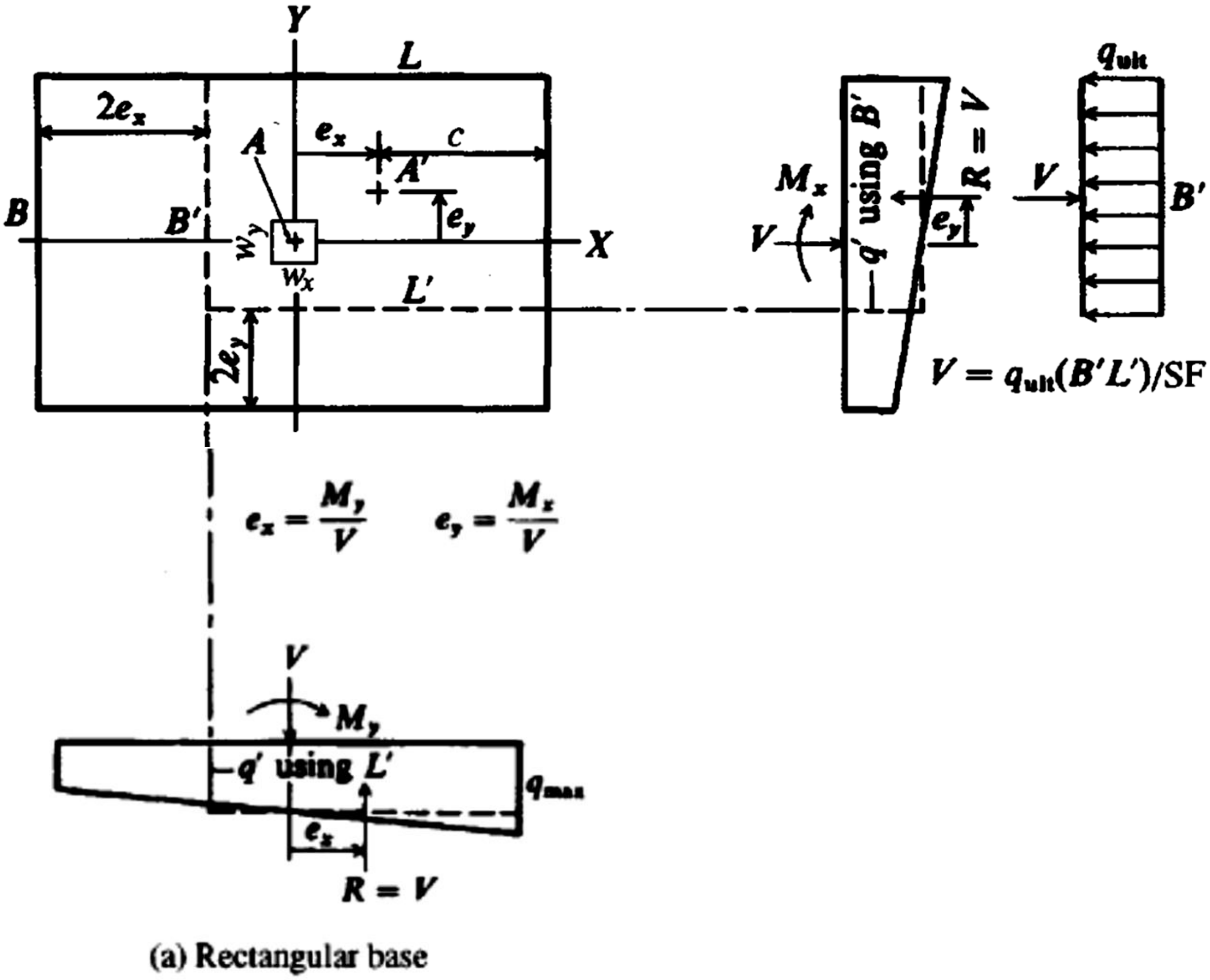


Figure 4-4. Method of computing effective footing dimensions when footing is eccentrically loaded for rectangular bases.

a. Use ***B'*** in the γBN_γ term.
b. Use ***B'*** and ***L'*** in computing the shape factors.
c. Use actual ***B*** and ***L*** for all depth factors.
The computed ultimate bearing capacity q_{ult} is then reduced to an allowable value q_a with an appropriate safety factor **SF** as

$$q_a = q_{ult}/SF \text{ (and } P_a = q_a B'L')$$

Example 4-5. A square footing is 1.8 X 1.8 m with a 0.4 X 0.4 m square column. It is loaded with an axial load of 1800 kN and M_x = 450 kN • m; M_y = 360 kN • m. Undrained triaxial tests (soil not saturated) give ϕ = 36° and c = 20 kPa. The footing depth D = 1.8 m; the soil unit weight γ = 18.00 kN/m^3; the water table is at a depth of 6.1 m from the ground surface.

Required: What is the allowable soil pressure, if SF = 3.0, using the Hansen bearing-capacity equation with *B′*, *L′?*

Solution. See Fig. E4-5.
e_y = 450/1800 = 0.25 m e_x = 360/1800 = 0.20 m

Both values of *e* are < *B/6* = 1.8/6 = 0.30 m. Also
B_{min} = 4(0.25) + 0.4 = 1.4 < 1.8 m given
L_{min} = 4(0.20) + 0.4 = 1.2 < 1.8 m given

Now find

$B' = B - 2e_y$ = 1.8 - 2(0.25) = 1.3 m
$L' = L - 2e_x$ = 1.8 - 2(0.20) = 1.4 m (L' > *B′*)

By Hansen's equation.

From Table 4-4 at ϕ = 36° and rounding to integers, we obtain
$N_c = 51$ $N_q = 38$ $N_\gamma = 40$
$N_q/N_c = 0.746$ $2\tan\phi\,(1-\sin\phi)^2 = 0.247$

Compute *D/B* = 1.8/1.8 = 1.0
Now compute

$s_c = 1 + (N_q/N_C)(B'/L') = 1 + 0.746(1.3/1.4) = 1.69$
$d_c = 1 + 0.4D/B = 1 + 0.4(1.8/1.8) = 1.40$
$s_q = 1 + (B'/L')\sin\phi = 1 + (1.3/1.4)\sin 36° = 1.55$
$d_q = 1 + 2\tan\phi\,(1 - \sin\phi)^2 D/B = 1 + 0.247(1.0) = 1.25$
$s_\gamma = 1 - 0.4\,B'/L' = 1 - 0.4 \times 1.3/1.4 = 0.62 > 0.60$ (O.K.)
$d_\gamma = 1.0$
All $i_i = g_i = b_i = 1.0$ (not 0.0)
The Hansen equation is given in Table 4-1 as
$q_{ult} = c\,N_c\,s_c\,d_c + \bar{q}\,N_q\,s_q\,d_q + 0.5\gamma\,B'\,N_\gamma\,s_\gamma\,d_\gamma$

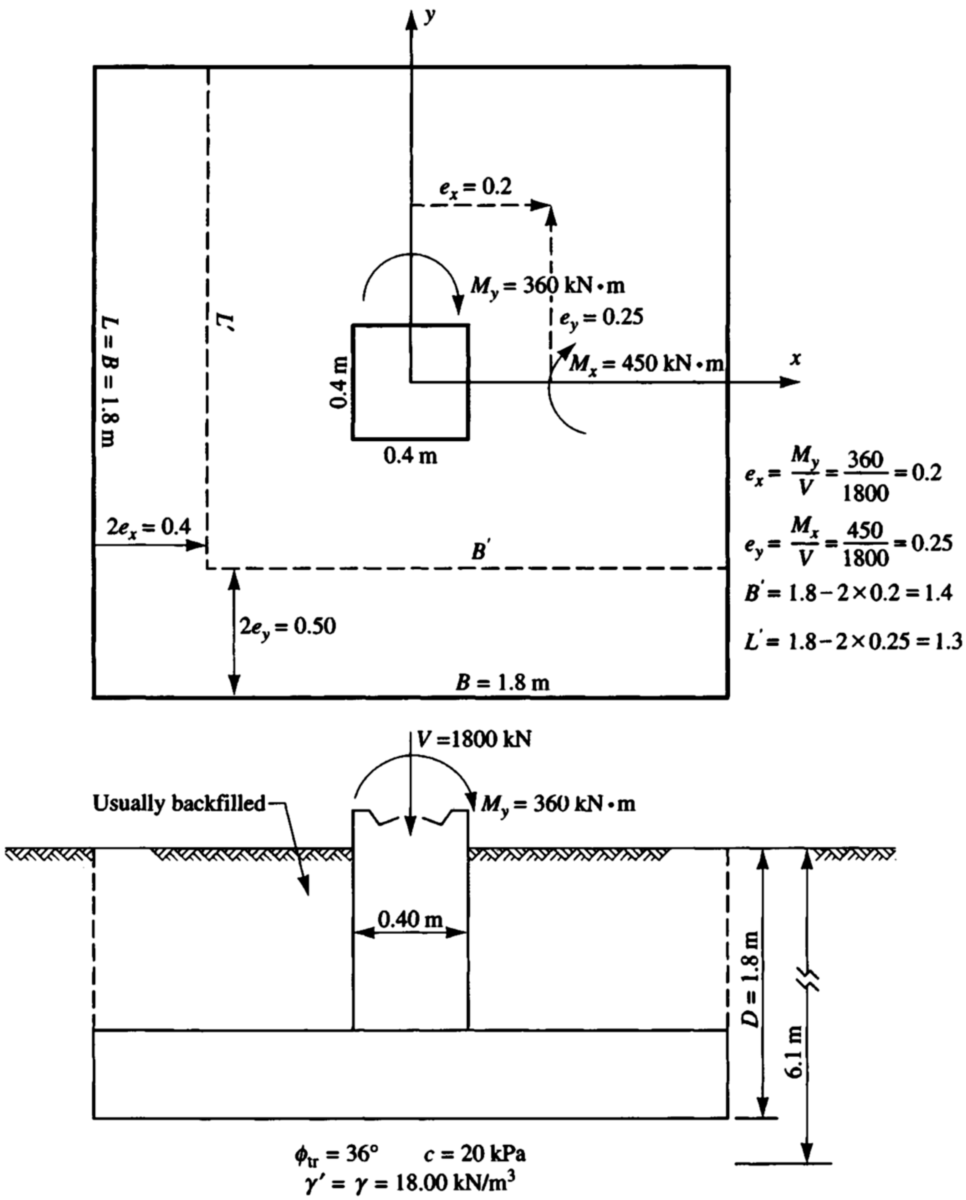


Figure E4-5

Inserting values computed above with terms of value 1.0 not shown (except d_γ) and using

$B' = 1.3$, we obtain

$$q_{ult} = 20(51)(1.69)(1.4) + 1.8(18.0)(38)(1.55)(1.25) + 0.5(18.0)(1.3)(40)(0.62)(1.0)$$

$$= 2413 + 2385 + 290 = 5088 \text{ kPa}$$

For SF = 3.0 the allowable soil pressure q_a is

$q_{all} = 5088/3 = 1696$ kPa → **1700** kPa

The actual soil pressure is

$$q_{act} = \frac{1800}{B' \; L'} = \frac{1800}{1.3x1.4} = 989 \; kPa$$

Note that the allowable pressure q_{all} is very large, and the actual soil pressure q_{act} is also large. With this large actual soil pressure, settlement may be the limiting factor. Some geotechnical consultants routinely limit the maximum allowable soil pressure to around 500 kPa in recommendations to clients for design whether settlement is a factor or not. Small footings with large column loads are visually not very confidence-inspiring during construction.

BEARING CAPACITY FROM SPT

The SPT is widely used to obtain the bearing capacity of soils directly. One of the earliest published relationships was that of Terzaghi and Peck. This has been widely used, but these curves were overly conservative. Meyerhof published equations for computing the allowable bearing capacity for a 25-mm settlement. These were also very conservative.
Joseph E. Bowels adjusted the equations to obtain the following:

$$q_{net}(\text{kN/m}^2) = \frac{N_{60}}{0.05} F_d \left(\frac{S_e}{25}\right) \quad (\text{for } B \leq 1.22 \text{ m})$$

and

$$q_{net}(\text{kN/m}^2) = \frac{N_{60}}{0.08} \left(\frac{B + 0.3}{B}\right)^2 F_d \left(\frac{S_e}{25}\right) \quad (\text{for } B > 1.22 \text{ m})$$

where

q_{net} = allowable bearing pressure for ΔH_0 = 25-mm, kPa

$F_d = 1 + 0.33 \frac{D_f}{B} < 1.33$ [as suggested by Meyerhof]

B = foundation width, in meters

S_e = settlement, in mm.

In these equations the allowable soil pressure is proportional to settlement. In general the allowable pressure for any settlement ΔH_j is

$$q'_a = \frac{\Delta H_j}{\Delta H_o} q_a$$

where ΔH_0 = 25 mm.

Example 4-12

Given. The average N_{60} blow count = 6 in the effective zone for a footing located at D = 1.6 m (blow count average in range from 1- to 4-m depth).

Required. What is the allowable bearing capacity for a 40-mm settlement? Present data as a table of q_a versus ***B.***

Solution. From Figure 3.17 we can see D_r is small, soil is "loose," and settlement may be a problem.
Should one put a footing on loose sand or should it be densified first?
(including F_d) on a programmable calculator or personal computer and obtain the table, which can be plotted as required.

for B = 1 m $F_d = 1 + 0.33\,\frac{1.6}{1}$ =1.528> 1.33 take F_d = 1.33 O.K

∵ B < 1.2 mm

∴ $$q_{net}(\text{kN/m}^2) = \frac{N_{60}}{0.05} F_d \left(\frac{S_e}{25}\right)$$

$$q_{net} = \frac{6}{0.05} \text{ x } 1.33 \text{ x } \left(\frac{40}{25}\right) = 255.36 \text{ kN/m}^2$$

For example for B = 2 m $F_d = 1 + 0.33\,\frac{1.6}{2}$ =1.264 < 1.33 O.K

$$q_{net}(\text{kN/m}^2) = \frac{N_{60}}{0.08}\left(\frac{B+0.3}{B}\right)^2 F_d\left(\frac{S_e}{25}\right) \quad (\text{for } B > 1.22 \text{ m})$$

$$q_{net} = \frac{6}{0.08}\, x \left(\frac{2+0.3}{2}\right)^2 x\ 1.264 x \left(\frac{40}{25}\right) = 200.6 \text{ kN/m}^2$$

for B = 3 m $F_d = 1 + 0.33\,\frac{1.6}{3}$ =1.176 < 1.33 O.K

$$q_{net} = \frac{6}{0.08}\, x \left(\frac{3+0.3}{3}\right)^2 x\ 1.176 x \left(\frac{40}{25}\right) = 170.72 \text{ kN/m}^2$$

for B = 4 m F_d = 1.32 and q_{net} = 157 kN/m^2

B (m)	1	2	3	4
q_{net} (kPa)	255.4	200.6	170.7	157

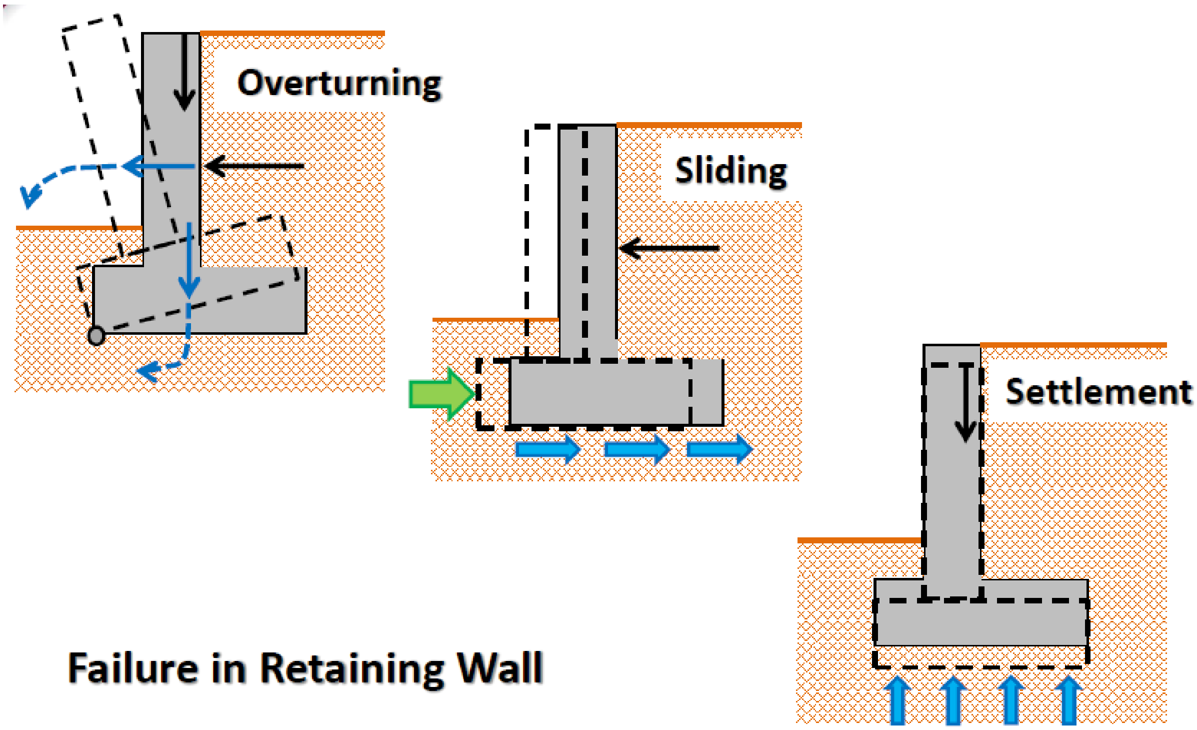
DESIGN OF RETAINING WALLS

Retaining wall is used to retain earth or other material in **vertical (or nearly vertical)** position at locations where an abrupt change in ground level occurs

•Prevent the **retained earth** from assuming its natural angle of repose

•The retained earth **exerts lateral pressure** on the wall –overturn, slide & settlement

•The wall must be designed to be **stable** under the effects of lateral pressure



Types of Retaining Walls

Gravity Wall

•Depends entirely on its **own weight** to provide necessary stability

•Usually constructed of plain concrete or stone masonry

•Plain concrete gravity wall –**height** < **3 m**

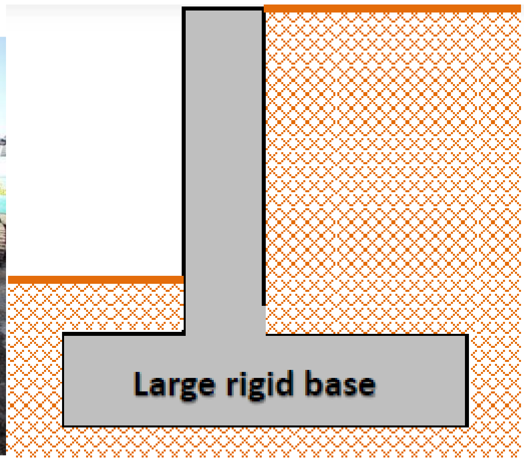
•In designing this wall, must keep the thrust line within the middle third of the base width – **no tensile stress** to be developed

Cantilever Wall

•Economical for height of up to **6 m**

•Structure consist of a vertical cantilever spanning from a large **rigid base slab**

•**Stability** is maintained essentially by the weight of the soil on the base slab + self-weight of structure



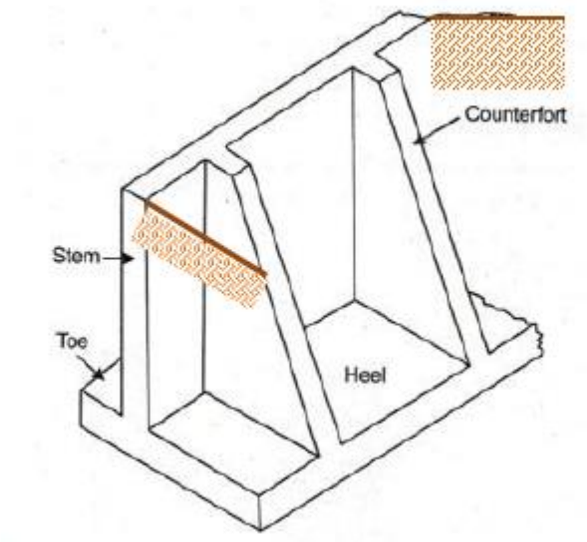
Counterfort Wall

•When the overall height of the wall is **too large** to be constructed economically as a cantilever

•Wall & base are tied together at intervals by **counterfort or bracing** walls

•Bracing in **tension**

•Economical for high wall usually **above 6 –7 m of backfill**

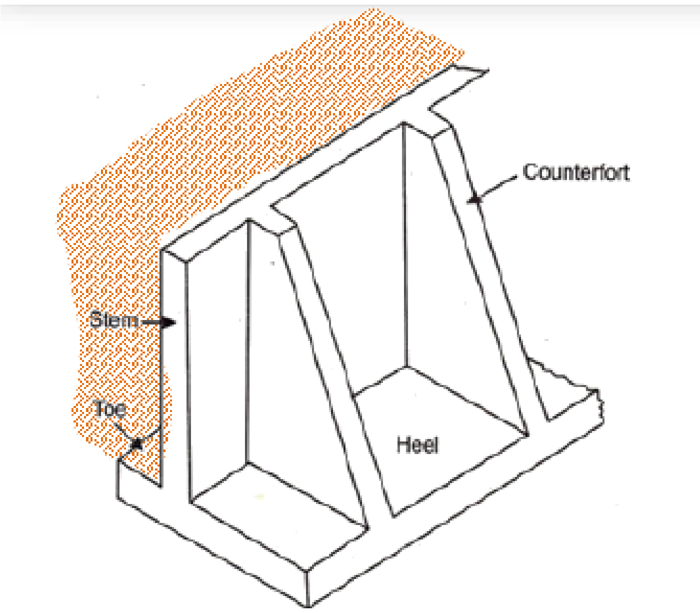


Buttress Wall

•Similar to counterfort wall, but bracing is constructed **in front** of the wall

•Bracing in **compression**

•More efficient than counterforts, but **no usable** space in front of the wall



Gabion Wall

•Made of **rectangular containers**

•Fabricated of heavily galvanized wire, filled with stone and stacked on one another, usually in layers that step back with the slope

•**Advantages:** conform to ground movement, dissipate energy from flowing water & drain-freely


Crib Wall

•**Interlocking** individual boxes made from timber or precast concrete members

•Boxes are filled with crushed stone or other granular materials to create free-draining structure


Tieback Wall

•Tieback is a horizontal wire or rod, or a helical anchor use to reinforce retaining wall for stability

•One end of the tieback is secured to the wall, while the other end is anchored to a stable structure i.e. concrete anchorage driven into the ground or anchored into the earth with sufficient resistance

•Tieback-anchorage structure resists forces that will cause the wall to lean

Keystone Wall

•Made up of segmental block units, made to last

•Based around a system with interlocking fiberglass pins connecting the wall unit and soil reinforcement

•Combination of these resulted in a strong, stable and durable wall system

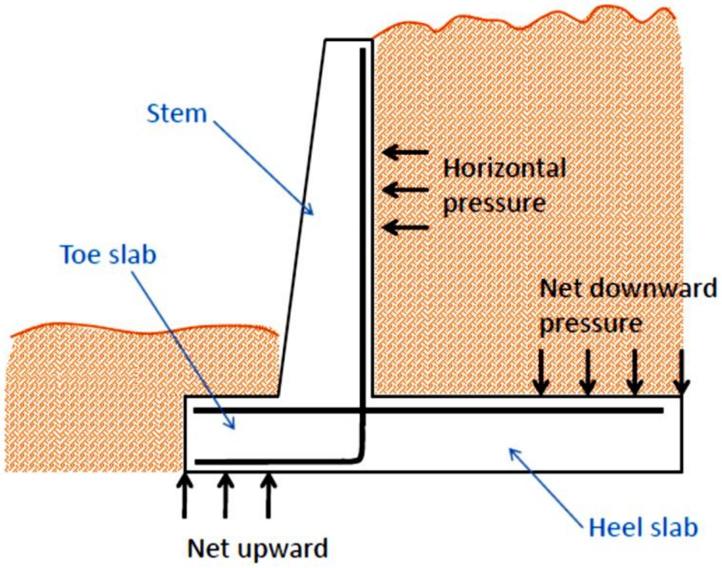
•Offers aesthetic appeal, cost efficiency, easy installation & strength

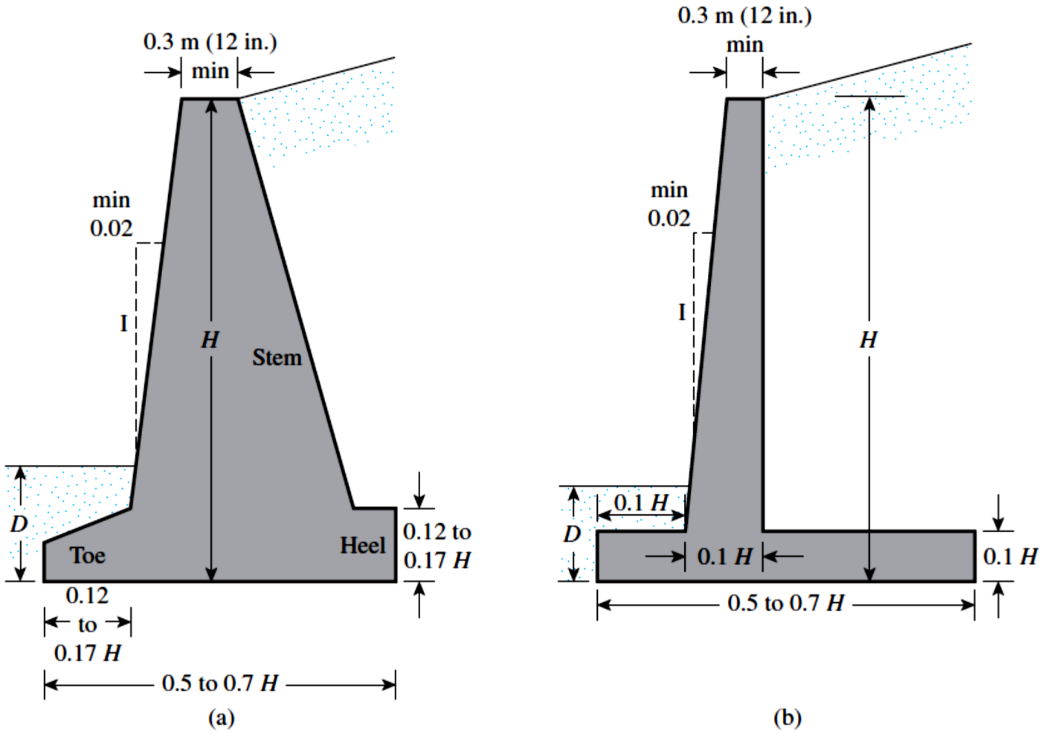
Retaining walls must be designed for lateral earth pressure. The procedures of calculating lateral earth pressure were discussed previously.
Different types of retaining walls are used to retain soil in different places.

Note:
Structural design of cantilever retaining wall depends on separating each part of wall and design it as a cantilever, so it's called cantilever R.W.

Elements of Retaining Walls

Each retaining wall divided into three parts; stem, heel, and toe as shown for the following cantilever footing (as example):





Approximate dimensions for various components of retaining wall for initial stability checks: (a) gravity wall; (b) cantilever wall

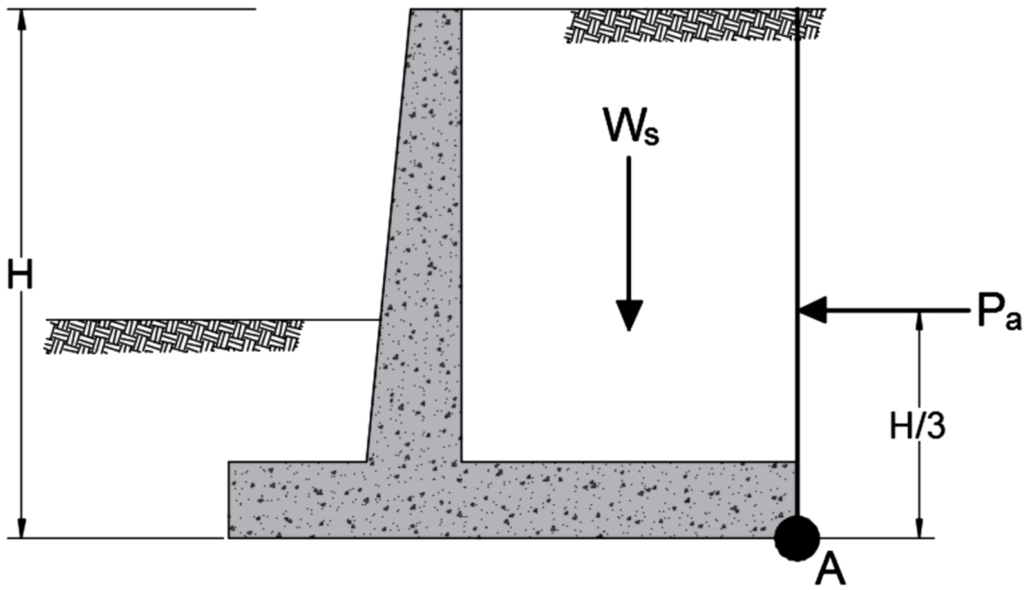
Application of Lateral Earth Pressure Theories to Design

Rankine Theory:

Rankine theory was modified to be suitable for designing a retaining walls.
This modification is drawing a vertical line from the lowest-right corner till intersection with the line of backfill, and then considering the force of soil acting on this vertical line.
The soil between the wall and vertical line is not considered in the value of P_a, so we take this soil in consideration as a vertical weight applied on the heel of the retaining wall as will be explained later.

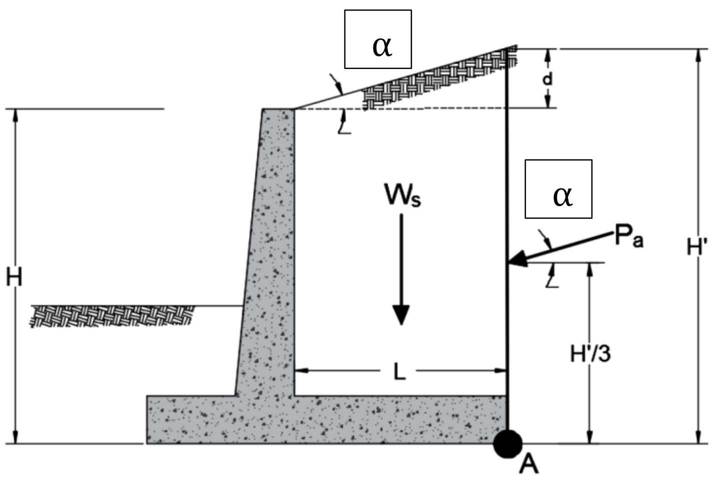
The following are all cases of Rankine theory in designing a retaining wall:

1. **The wall is vertical and backfill is horizontal:**



Here the active force P_a is horizontal and can be calculated as following: $P_a = 0.5\gamma H^2 K_a$, $K_a = \tan^2(45 - \phi/2)$

2. The wall is vertical and the backfill is inclined with horizontal by angle (α):



Here the active force P_a is inclined with angle (α) and can be calculated as following:

$P_a = 0.5\gamma H'^2 K_a$

Why H'?→ Because the pressure is applied on the vertical line (according active theory) not on the wall, so we need the height of this vertical line H'

$H' = H + d$

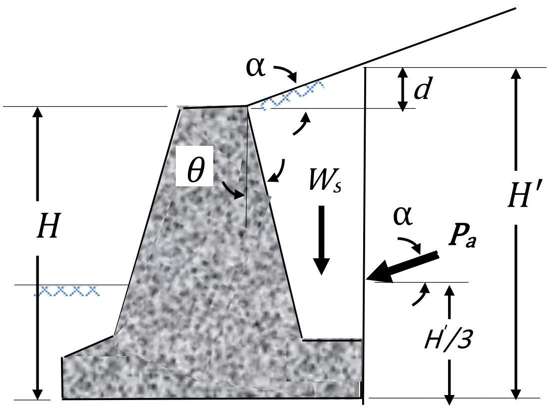
$d = L \tan\alpha$

K_a is calculated from table

Now the calculated value of P_a is inclined with an angle (α), so it is analyzed in horizontal and vertical axes and then we use the horizontal and vertical components in design as will be explained later.

$P_{a,h} = P_a \cos(\alpha)$, $P_{a,v} = P_a \sin(\alpha)$

2. **The wall is inclined by angle (θ) with vertical and the backfill is inclined with horizontal by angle (α):**



Note that the force P_a is inclined with angle (α) and do not depend on the inclination of the wall because the force applied on the vertical line and can be calculated as following:

$P_a = 0.5\gamma H'^2 K_a$

What about K_a ???

K_a depends on the inclination of the wall and inclination of the backfill because it's related to the soil itself and the angle of contact surface with this soil, so K_a can be calculated from the following equation:

$$K_a = \frac{\cos(\alpha-\theta)\sqrt{1+\sin^2\phi - 2\sin\phi\cos\psi_a}}{\cos^2\theta\left(\cos\alpha + \sqrt{\sin^2\phi - \sin^2\alpha}\right)}$$

$$\psi_a = \sin^{-1}\left(\frac{\sin\alpha}{\sin\phi}\right) - \alpha + 2\theta$$

$$P_{a,h} = P_a\cos(\alpha) \qquad , \qquad P_{a,v} = P_a\sin(\alpha)$$

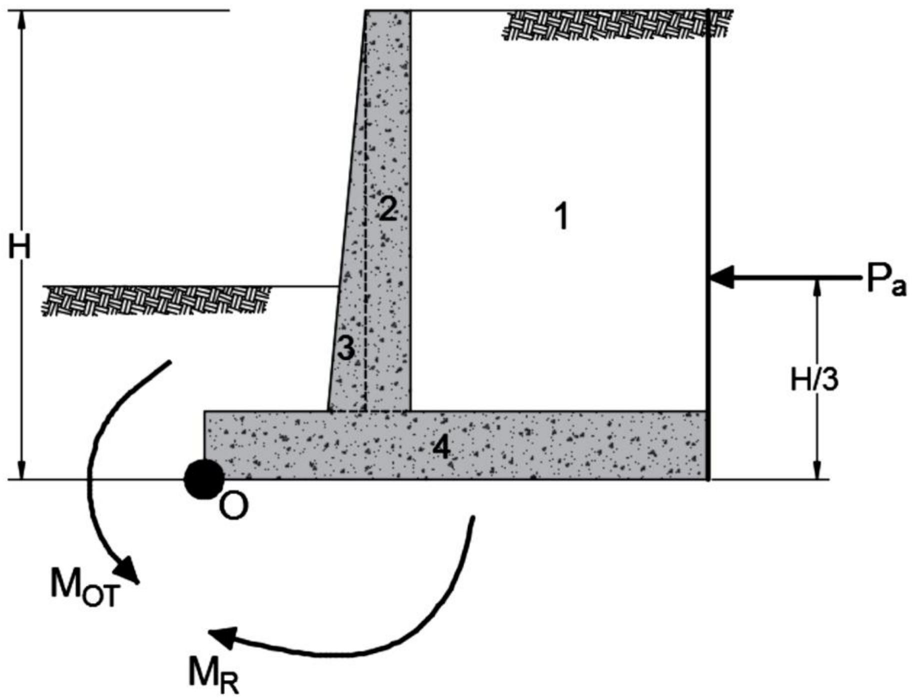
Stability of Retaining Wall

A retaining wall may fail in any of the following:

1. It may overturn about its toe.
2. It may slide along its base.
3. It may fail due to the loss of bearing capacity of the soil supporting the base.
4. It may go through excessive settlement.

We will discuss the stability of retaining wall for the first three types of failure (overturning, sliding and bearing capacity failures).

We will use Rankine theory to discuss the stability of these types of failures



The **horizontal** component of active force will causes overturning on retaining wall about point O by moment called "overturning moment"

$M_{OT} = P_{a,h} \times H/3$

This overturning moment will be resisted by all vertical forces applied on the base of retaining wall:

1. Vertical component of active force $P_{a,v}$ (if exists).
2. Weight of all soil above the heel of the retaining wall.
3. Weight of each element of retaining wall.
4. Passive force (we neglect it in this check for more safety).

Now, to calculate the moment from these all forces (resisting moment) we prepare the following table:

Force=Volume ×unit weight but, we take a strip of 1m length

→Force=Area × unit weight

Section	Area	Weight/unit length of the wall	Moment arm measured from O	Moment about O
1	A_1	$W_1 = A_1 \times \gamma_1$	X_1	M_1
2	A_2	$W_2 = A_2 \times \gamma_c$	X_2	M_2
3	A_3	$W_3 = A_3 \times \gamma_c$	X_3	M_3
4	A_4	$W_4 = A_4 \times \gamma_c$	X_4	M_4
		$P_{a,v}$(if exist).	B	M_V
$\sum$		$\sum V$		$\sum M = M_R$

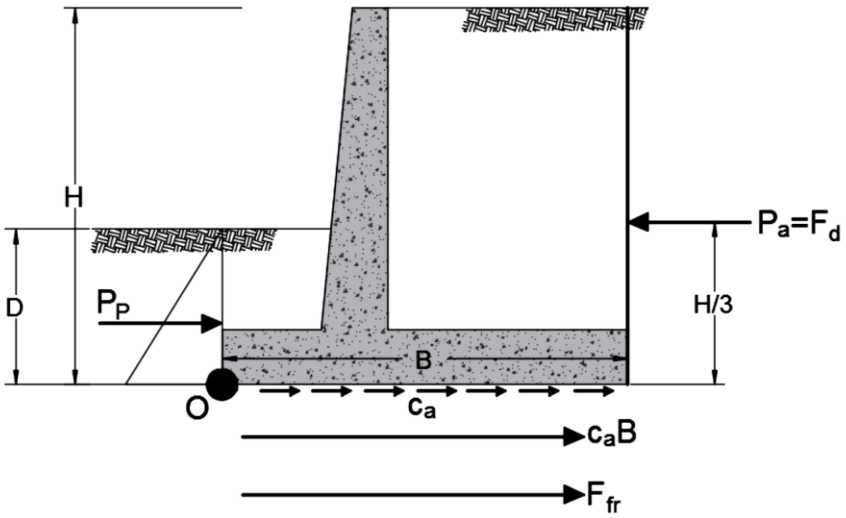
γ_1 = unit weight of the soil above the heel of RW

$$FS_{OT} = \frac{M_R}{M_{OT}} \geq 2$$

Note:

If you asked to consider passive force→ consider it in the resisting moment and the factor of safety remains 2. (So we neglect it here for safety).

Stability for Sliding along the Base



Also, the horizontal component of active force may cause movement of the wall in horizontal direction (i.e. causes sliding for the wall), this force is called driving force

$\mathbf{F_d=P_{a,h}}$

This driving force will be resisted by the following forces:
1. Adhesion between the soil (under the base) and the base of retaining wall:

c_a=adhesion along the base of retaining wall (kN/m)
$C_a=c_a\times B$=adhesion force under the base of retaining wall (kN)
c_a can be calculated from the following relation:
$c_a=K_2c_2$ $\quad$ c_2=cohesion of soil under the base

So adhesion force is:

$\mathbf{Ca=K_2c_2B}$

2. Friction force due to the friction between the soil and the base of retaining wall:

Always friction force is calculated from the following relation: $\mathbf{F_{fr}=\mu_sN}$
Here **N** is the sum of vertical forces calculated in the table of the first check (overturning)
$\rightarrow \mathbf{N=\Sigma V}$ (including the vertical component of active force)

μ_s=coefficient of friction (related to the friction between soil and base)

$\mu_s=\tan(\delta_2)$ $\quad$ $\delta_2=K_1\phi_2$ $\quad$ $\therefore$ $\mu_s=\tan(K_1\phi_2)$

ϕ_2=friction angle of the soil under the base.

$$\rightarrow \mathbf{F_{fr}} = \sum \mathbf{V \times \tan(K_1\phi_2)}$$

Note:

$$\mathbf{K_1 = K_2 = \left(\frac{1}{2} \rightarrow \frac{2}{3}\right)} \text{ if you are not given them} \rightarrow \text{take } \mathbf{K_1 = K_2 = \frac{2}{3}}$$

3. Passive force P_p.(Calculated using rankine theory).

So the total resisting force F_R can be calculated as following:

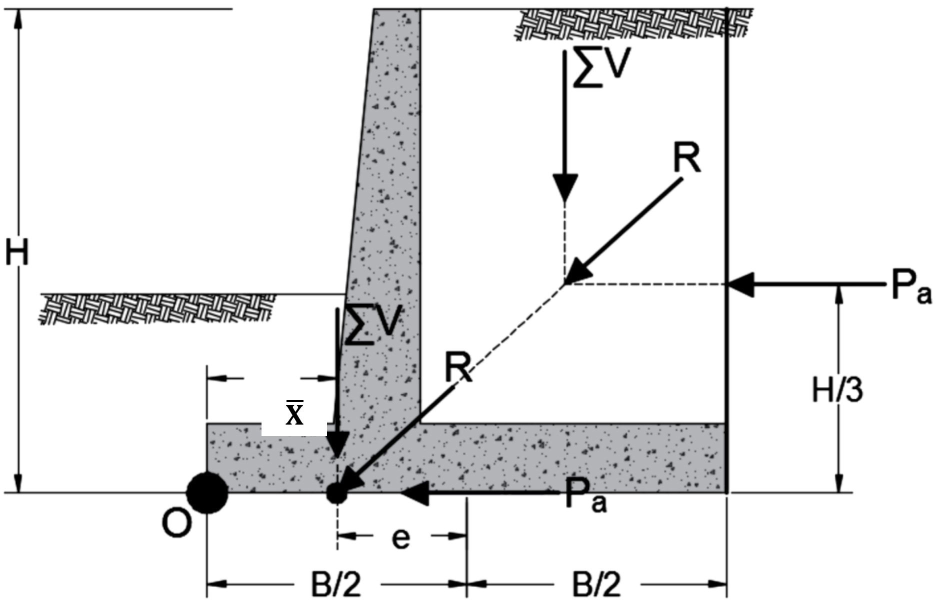
$$F_R = \sum V \times \tan(K_1\phi_2) + K_2c_2B + P_p$$

Factor of safety against sliding:

$$FS_S = \frac{F_R}{F_d} \geq 2 \ \text{(if we consider } P_p \text{ in } F_R\text{)}$$

$$FS_S = \frac{F_R}{F_d} \geq 1.5 \ \text{(if we dont consider } P_p \text{ in } F_R\text{)}$$

Check Stability for Bearing Capacity Failure



As we see, the resultant force (R) is not applied on the center of the base of retaining wall, so there is an eccentricity between the location of resultant force and the center of the base, this eccentricity **may be calculated as following:**

From the figure above, take summation of moments about point O:

$$M_O = \sum V \times \bar{X}$$

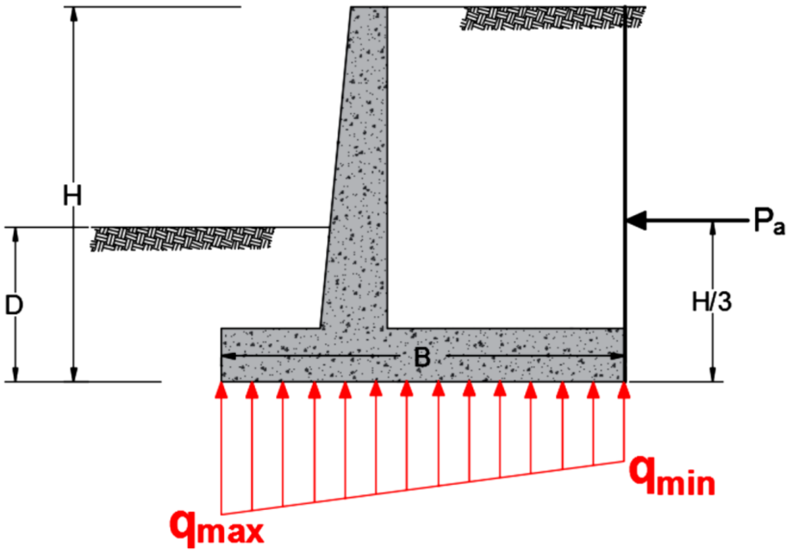
From the first check (overturning) we calculate the overturning moment and resisting moment about point O, so the difference between these two moments gives the net moment at O.

$$M_O = M_R - M_{OT}$$

$$\rightarrow M_R - M_{OT} = \sum V \times \bar{X} \rightarrow\rightarrow \bar{X} = \frac{M_R - M_{OT}}{\sum V}$$

$$e = \frac{B}{2} - \bar{X} = \checkmark \text{ (see the above figure).}$$

Since there exist eccentricity, the pressure under the base of retaining wall is not uniform (there exist maximum and minimum values for pressure).



We calculate $\mathbf{q_{max}}$ and $\mathbf{q_{min}}$ as in the following:
Eccentricity in B-direction and retaining wall can be considered strip footing

If $e < \frac{B}{6}$

$$q_{max} = \frac{\sum V}{B \times 1}\left(1 + \frac{6e}{B}\right)$$

$$q_{min} = \frac{\sum V}{B \times 1}\left(1 - \frac{6e}{B}\right)$$

If $e > \frac{B}{6}$

$$q_{max,new} = \frac{4\sum V}{3 \times 1 \times (B - 2e)}$$

Now, we must check for q_{max}:

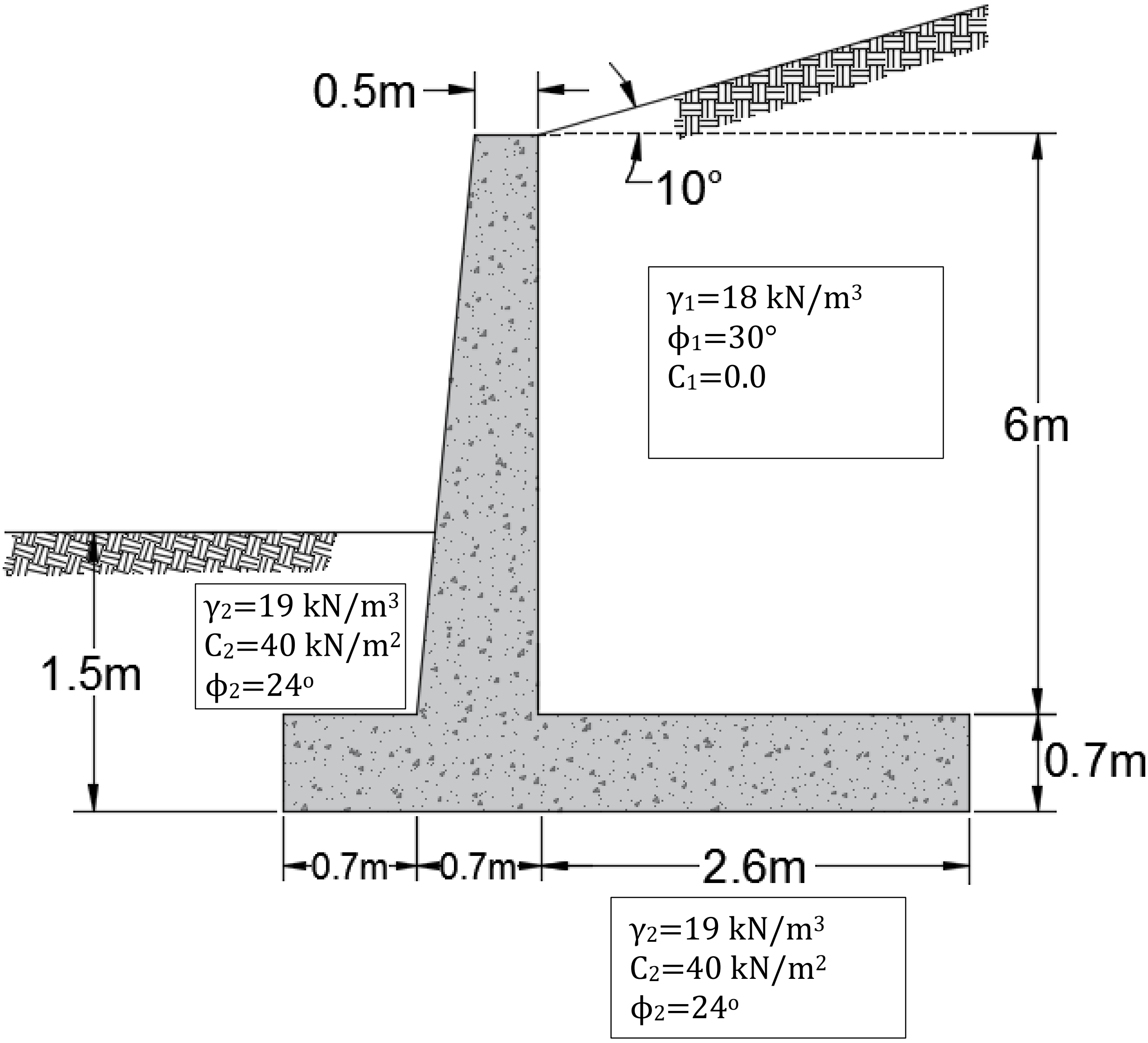
$q_{max} \leq q_{all} \rightarrow q_{max} = q_{all}$ (at critical case)

$$FS_{B.C} = \frac{q_u}{q_{max}} \geq 3$$

Examples:
Example 1:

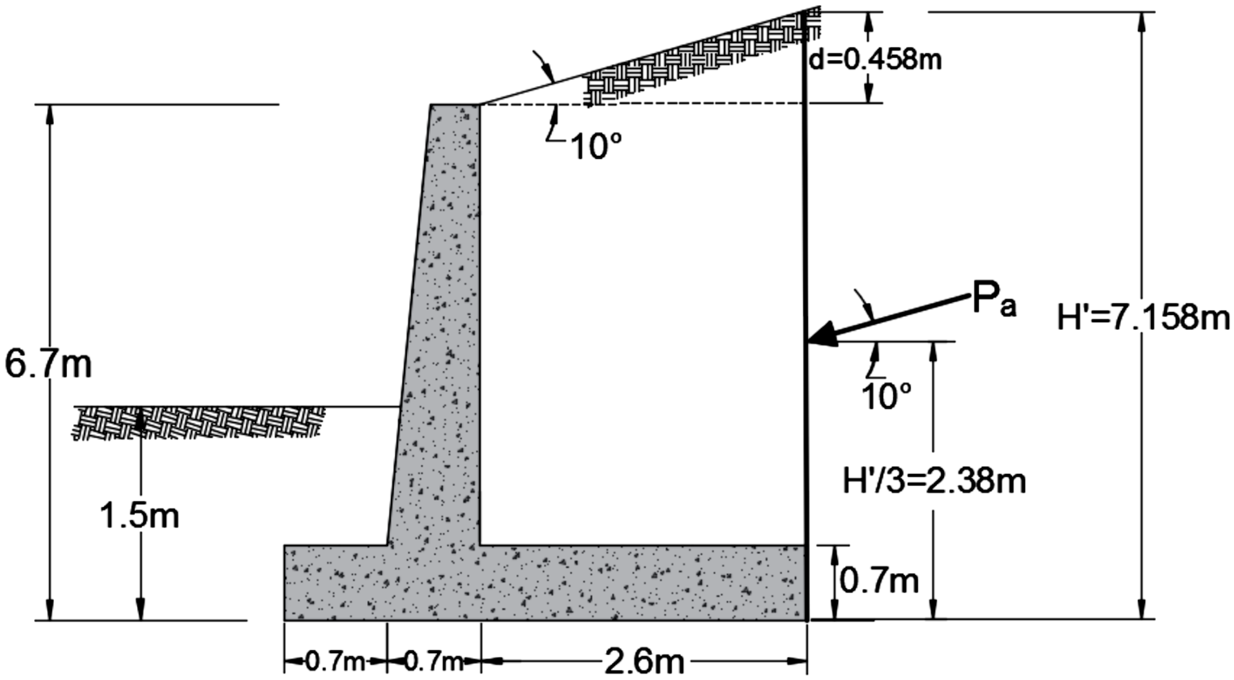
The cross section of the cantilever retaining wall shown below. Calculate the factor of safety with respect to overturning, sliding, and bearing capacity. Assume the ultimate bearing capacity (q_u)= 566.2 kN/m^2

γ_c=24 kN/m^3



Solution

Since it is not specified a method for solving the problem, directly we use Rankine theory. Now draw a vertical line starts from the right-down corner till reaching the backfill line and then calculate active force (P_a):



$$\tan 10 = \frac{d}{2.6} \rightarrow d = 2.6 \times \tan 10 = 0.458m$$
$$H' = 6.7 + d = 6.7 + 0.458 = 7.158m$$

Now we calculate P_a:

$$P_a = \frac{1}{2} \times \gamma_1 \times H'^2 \times K_a$$

Since the backfill is inclined and the wall is vertical, K_a is calculated from **Table** according the values of $\alpha=10$ and $\phi_1=30$: $K_a=0.3495$

$$\rightarrow P_a = \frac{1}{2} \times 18 \times 7.158^2 \times 0.3495 = 161.2 \text{ kN}$$

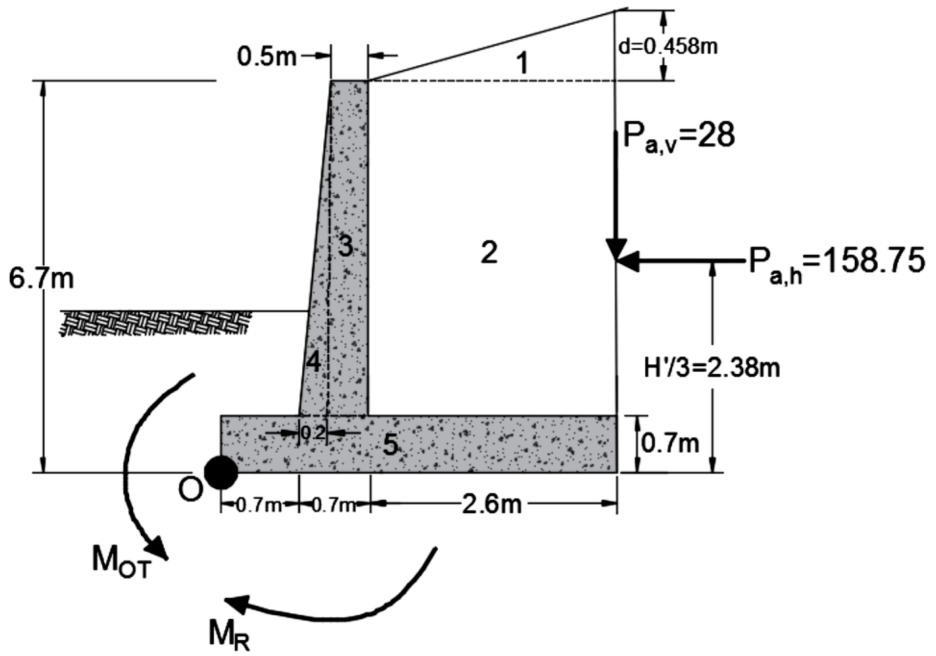
Location of P_a:

Location$= \frac{H'}{3} = \frac{7.158}{3} = 2.38$

The force P_a is inclined with angle $\alpha = 10$ with horizontal:

$P_{a,h} = 161.2 \cos(10) = 158.75$, $P_{a,v} = 161.2 \sin(10) = 28$

Check for Overturning:



M_{OT}=158.75×2.38=337.8 kN.m

Now to calculate M_R we divided the soil and the concrete into rectangles and triangles to find the area easily (as shown above) **and to find the arm from the center of each area to point O** as prepared in the following table:

Section	Area	Weight/unit length of the wall	Moment arm measured from O	Moment about O
1	0.595	$0.595 \times 18 = 10.71$	$4 - \frac{2.6}{3} = 3.13$	33.52
2	15.6	$15.6 \times 18 = 280.8$	$1.4 + 1.3 = 2.7$	758.16
3	3	$3 \times 24 = 72$	$1.4 - 0.25 = 1.15$	82.8
4	0.6	$0.6 \times 24 = 14.4$	$0.9 - \frac{0.2}{3} = 0.833$	12
5	2.8	$2.8 \times 24 = 67.2$	$\frac{4}{2} = 2$	134.4
		$P_{a,v} = 28$	B=4	112
$\sum$		$\sum V = 470.11$		$M_R = 1132.88$

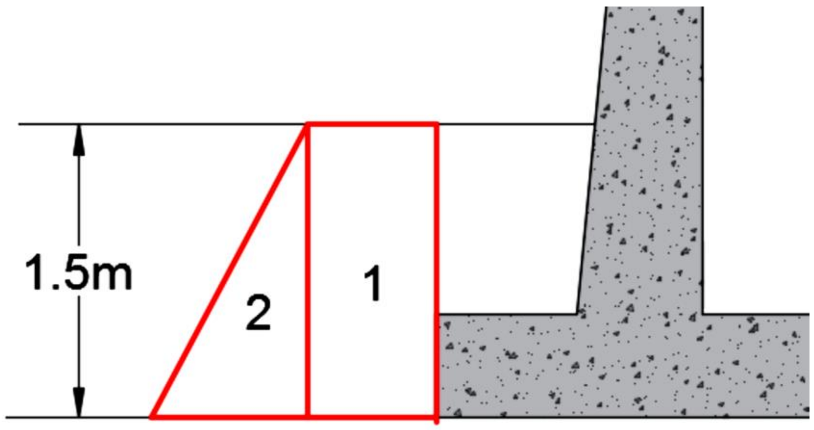
Note that we neglect passive force because it is not obligatory.

$$FS_{OT} = \frac{M_R}{M_{OT}} = \frac{1132.88}{377.8} = 2.99 > 2 \rightarrow \textbf{OK} ✓.$$

Check for Sliding:

$$FS_S = \frac{F_R}{F_d} \geq 2 \ \text{(if we consider } P_P \text{ in } F_R)$$

It is preferable to consider passive force in this check.
Applying Rankine theory on the soil in the left (draw vertical line till reaching the soil surface).



k_P is calculated for the soil using Rankine theory without considering any inclination of the wall, because it is calculated for the soil at L.H.S of wall which is level.

$$k_P = \tan^2\left(45 + \frac{\phi_2}{2}\right) = \tan^2\left(45 + \frac{20}{2}\right) = 2.04$$

$$P_1 = \text{(rectangle area)} = \left(2 \times 40 \times \sqrt{2.04}\right) \times 1.5 = 171.4 \text{ kN}$$

$$P_2 = \text{(triangle area)} = \frac{1}{2} \times (19 \times 1.5 \times 2.04) \times 1.5 = 43.6 \text{ kN}$$

$$P_P = P_1 + P_2 = 171.4 + 43.6 = 215 \text{ kN}$$

$$F_d = P_{a,h} = 158.75 \text{ Kn}$$

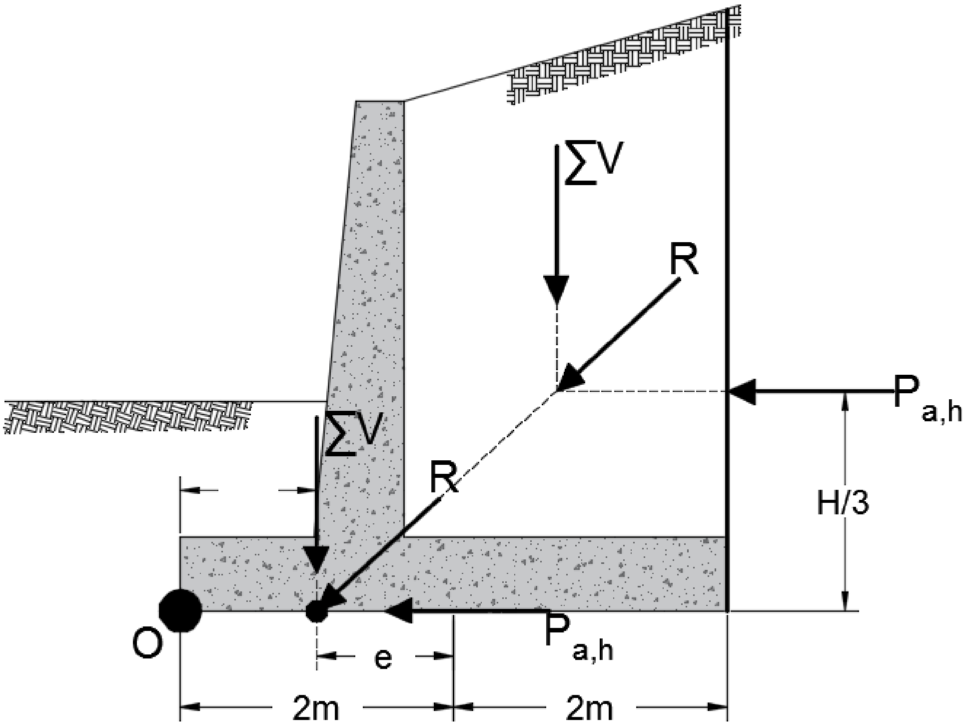
$$F_R = \sum V \times \tan(K_1\phi_2) + K_2 c_2 B + P_P$$

Take $K_1 = K_2 = 2/3$ $\quad \sum V = 470.11$ (from table of first check)

$$F_R = 470.11 \times \tan\left(\frac{2}{3} \times 20\right) + \frac{2}{3} \times 40 \times 4 + 215 = 433.1 \text{ kN}$$

$$\rightarrow FS_S = \frac{433.1}{158.75} = 2.72 > 2 \rightarrow \textbf{OK} \checkmark.$$

Check for Bearing Capacity Failure:



As stated previously, $\bar{X}$ can be calculated as following:

$$\bar{X} = \frac{M_R - M_{OT}}{\sum V} = \frac{1132.88 - 377.8}{470.11} = 1.6 \text{ m}$$

$$e = \frac{B}{2} - \bar{X} = 2 - 1.6 = 0.4\text{m}$$

$$\frac{B}{6} = \frac{4}{6} = 0.667 \rightarrow e = 0.4 < \frac{B}{6} \rightarrow\rightarrow\rightarrow$$

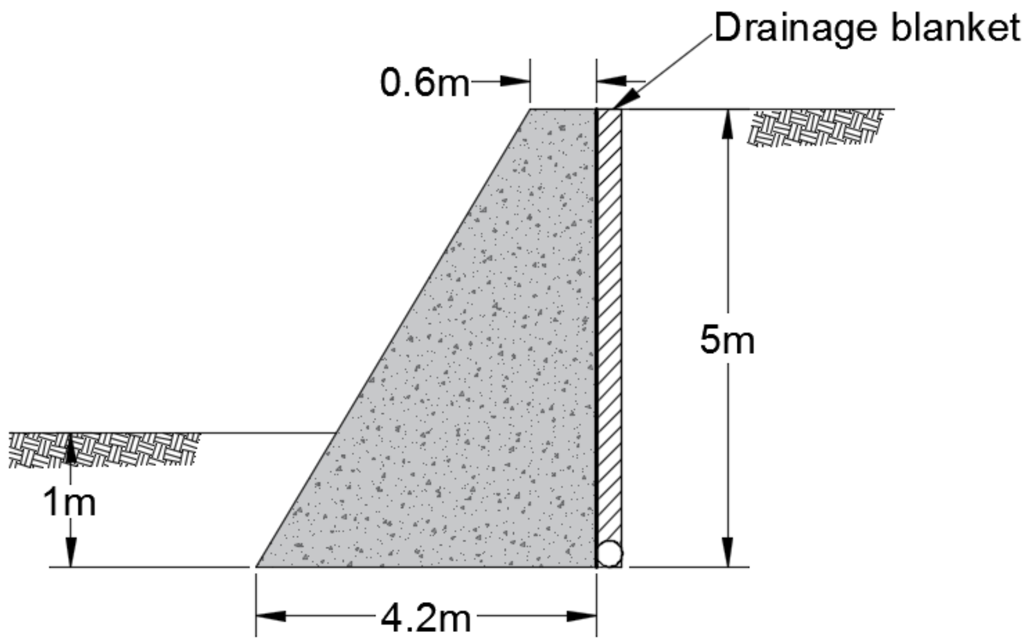
$$q_{max} = \frac{\sum V}{B \times 1}\left(1 + \frac{6e}{B}\right) = \frac{470.11}{4 \times 1}\left(1 + \frac{6 \times 0.4}{4}\right) = 188.04 \text{ kN/m}^2$$

$$q_{min} = \frac{\sum V}{B \times 1}\left(1 - \frac{6e}{B}\right) = \frac{470.11}{4 \times 1}\left(1 - \frac{6 \times 0.4}{4}\right) = 47 \text{ kN/m}^2$$

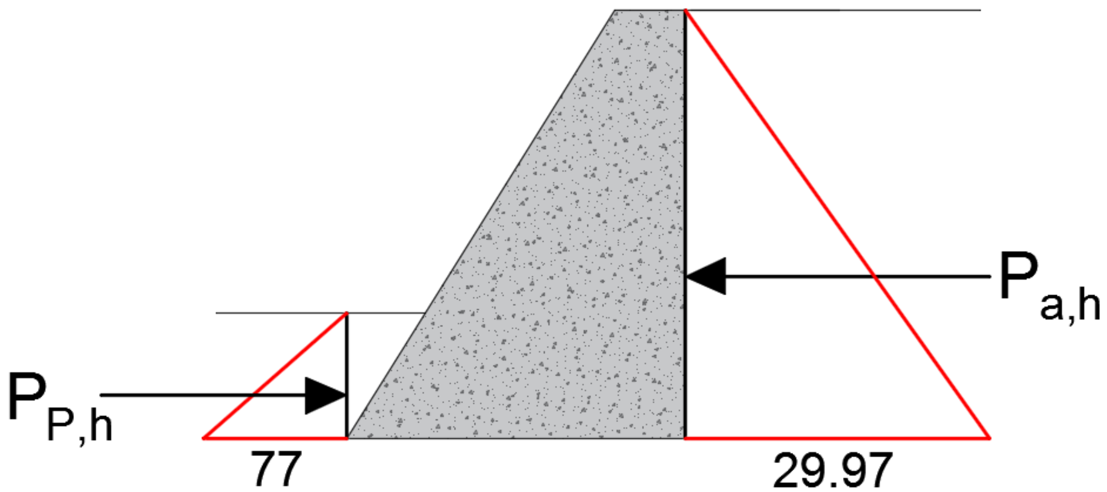
$$FS_{B.C} = \frac{q_u}{q_{max}} = \frac{566.2}{188.04} = 3.01 > 3 \text{ (slightly satisfied)}\textbf{OK} ✓.$$

Example2: A gravity retaining wall shown in the figure below is required to retain 5 m of soil. The backfill is a coarse grained soil with saturated unit weight =18 kN/m^3, and friction angle of ϕ=30°. The existing soil below the base has the following properties; γ_{sat}=20 kN/m^3, ϕ=36°. The wall is embedded 1m into the existing soil, and a drainage system is provided as shown. The ground water table is at 4.5m below the base of the wall. Determine the stability of the wall for the following conditions (assume K_1=K_2 = 2/3):

a- Wall friction angle is zero.
b- The drainage system becomes clogged during several days of rainstorm and the ground water rises to the surface of backfill (use Rankine). $\gamma_{concrete} = 24\ kN/m^3$



a- (wall friction angle $=\delta=0.0$)
Since $\delta=0.0$ (we use Rankine theory).



(The unit weight of the soil (natural) is not given, so we consider the saturated unit weight is the natural unit weight).

$$K_a = \tan^2\left(45 - \frac{\phi}{2}\right) = \tan^2\left(45 - \frac{30}{2}\right) = 0.333 \text{ (for the retained soil)}$$

$$K_P = \tan^2\left(45 + \frac{\phi}{2}\right) = K_P = \tan^2\left(45 + \frac{36}{2}\right) = 3.85 \text{ (for soil below the base)}$$

Calculation of active lateral earth pressure distribution:

$$\sigma_{h,a} = (q + \gamma H)K_a - 2c\sqrt{K_a}$$
$$@z = H = 5m \text{ (right side)}$$
$$\sigma_{h,a} = (0 + 18 \times 5) \times 0.333 - 0 = 29.97 \text{ kN/m}^2$$

Calculation of passive lateral earth pressure distribution:

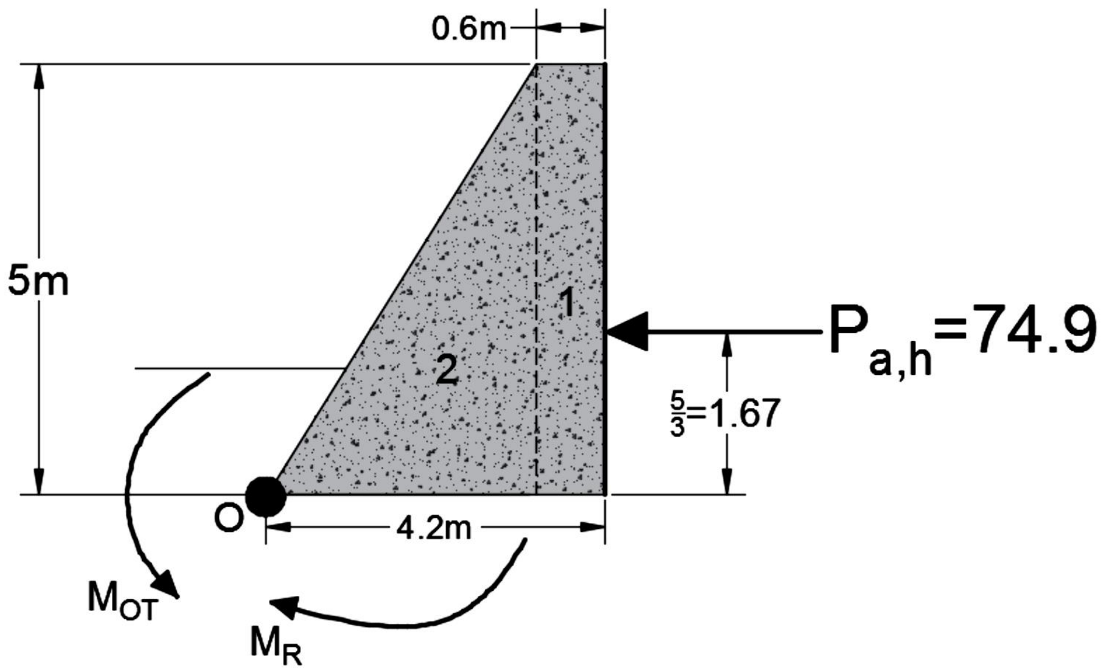
$$\sigma_{h,P} = (q + \gamma H)K_P + 2c\sqrt{K_P}$$
$$@z = 1m\text{(left side)}$$
$$\sigma_{h,P} = (0 + 20 \times 1) \times 3.85 + 0 = 77 \text{ kN/m}^2$$

Calculation of active force:

P_a=(area of right triangle)= $\frac{1}{2}$ ×29.97×5=74.9 kN

Calculation of passive force: P_P=(area of left triangle)= $\frac{1}{2}$ ×77×1=38.5 kN

Overturning Stability:



M_{OT} =74.9×1.67=125.08 kN.m

Now to calculate M_R we divided the soil and the concrete into rectangles and triangles to find the area easily (as shown above) **and to find the arm from the center of each area to point O** as prepared in the following table:
Note that since there is no heel for the wall, the force is applied directly on the wall.

Section	Area	Weight/unit length of the wall	Moment arm measured from O	Moment about O
1	3	$3 \times 24 = 72$	3.9	280.8
2	9	$9 \times 24 = 216$	2.4	518.4
$\sum$		$\sum V = 288$		$M_R = 799.2$

Note that there is no vertical component of active force

$$FS_{OT} = \frac{M_R}{M_{OT}} = \frac{799.2}{125.08} = 6.39 > 2 \rightarrow \textbf{OK} \checkmark.$$

Sliding Stability:

$$FS_S = \frac{F_R}{F_d} \geq 2 \ \text{(if we consider } P_P \text{ in } F_R)$$

$$F_d = P_{a,h} = 74.9 \ kN/m^2$$

$$F_R = \sum V \times \tan(K_1\phi_2) + K_2c_2B + P_P$$

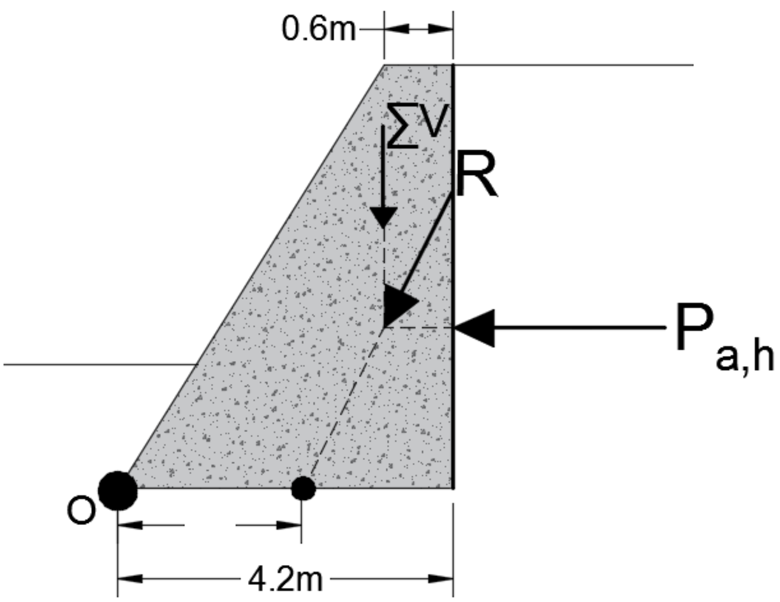
Take $K_2 = K_2 = 2/3$ $\quad \sum V = 288$ (from table of first check)

$P_P = 38.5 \ kN/m^2$ (as calculated above)

$$F_R = 288 \times \tan\left(\frac{2}{3} \times 36\right) + \frac{2}{3} \times 0 \times 4.2 + 38.4 = 166.62 \ kN.$$

$$\rightarrow FS_S = \frac{166.62}{74.9} = 2.2 > 2 \rightarrow \textbf{OK} \checkmark.$$

Bearing capacity check:



$$\bar{X} = \frac{M_R - M_{OT}}{\sum V} = \frac{799.2 - 125.08}{288} = 2.34 \text{ m}$$

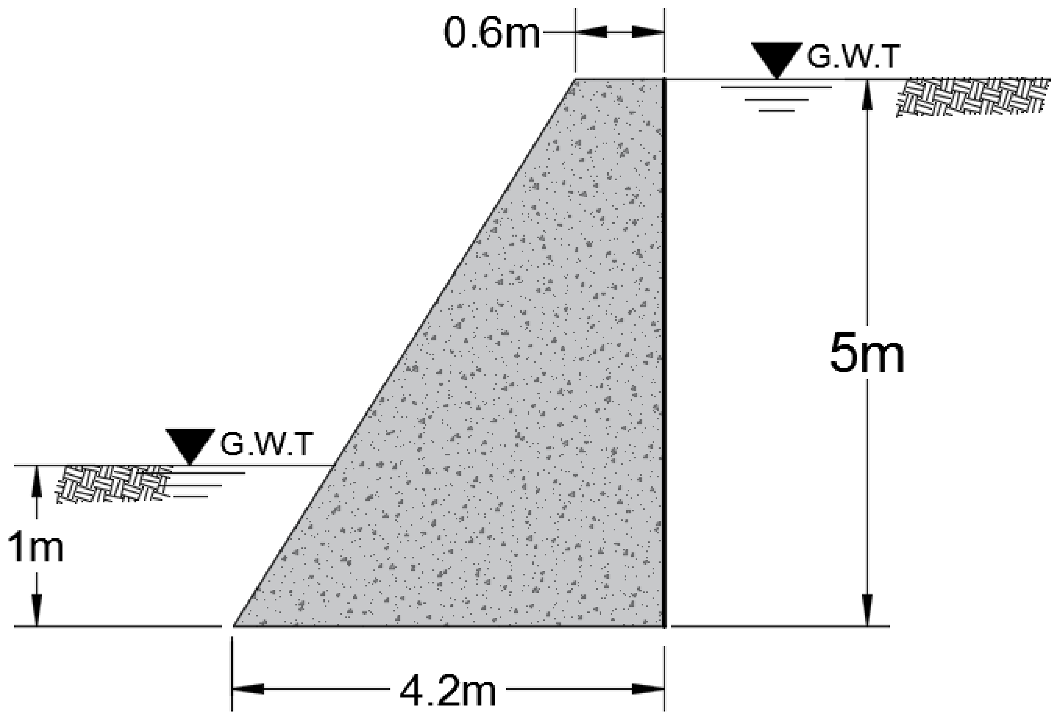
$$e = \frac{B}{2} - \bar{X} = \frac{4.2}{2} - 2.34 = -0.24 \text{ m (R is at right of base center)}$$

$$\frac{B}{6} = \frac{4.2}{6} = 0.7 \rightarrow e = 0.24 < \frac{B}{6} \rightarrow\rightarrow\rightarrow$$

$$q_{max} = \frac{\sum V}{B \times 1}\left(1 + \frac{6e}{B}\right) = \frac{288}{4.2 \times 1}\left(1 + \frac{6 \times 0.24}{4.2}\right) = 92.08 \text{ kN/m}^2$$

$$q_{min} = \frac{\sum V}{B \times 1}\left(1 - \frac{6e}{B}\right) = \frac{288}{4.2 \times 1}\left(1 - \frac{6 \times 0.24}{4.2}\right) = 45.06 \text{ kN/m}^2$$

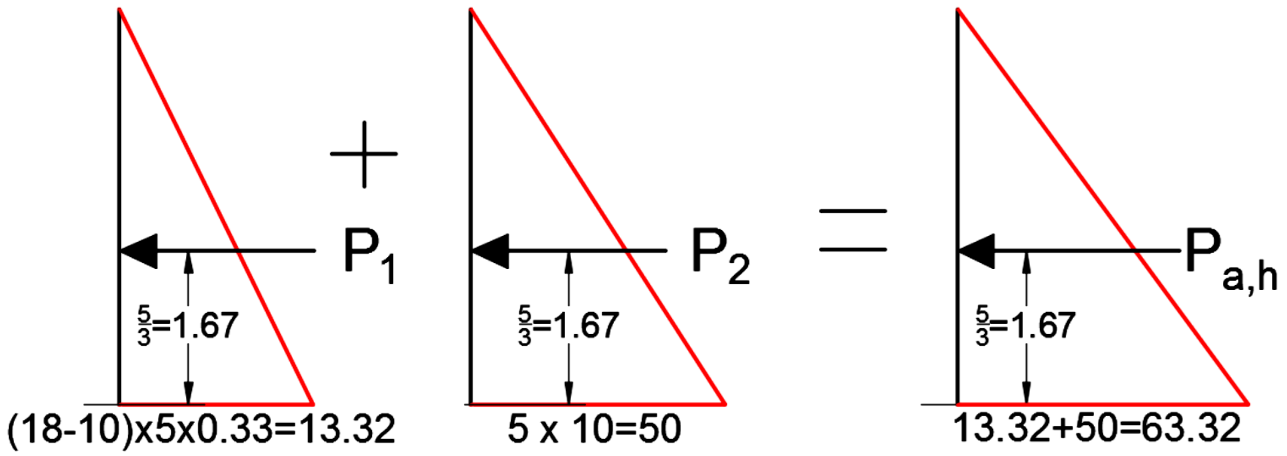
b- When the ground water rises to the surface, the retaining wall is shown below:



What differ???

If we want to use Rankine theory (force from soil is horizontal):

1. Calculation of active force:



Don't forget we calculate effective stress every change, and the we add water alone.

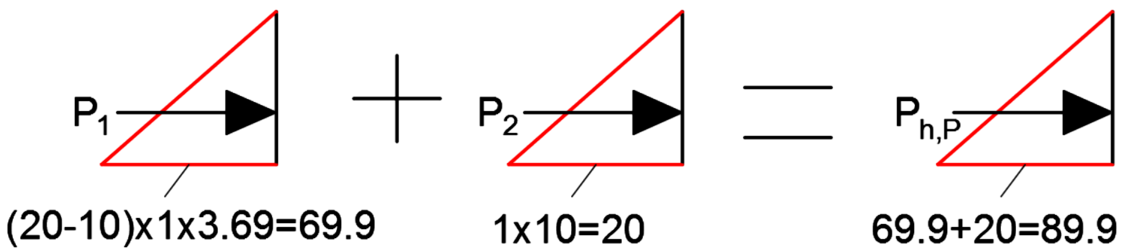
P1=(force due to effective soil)= $\frac{1}{2}\times13.32\times5=33.3$ kN P2=(force due to water)= $\frac{1}{2}\times50\times5=125$ KN

$P_{a,h}=P_1+P_2=33.3+125=158.33$ KN

Loacation of $P_{a,h}$:

Take the moment at the bottom of the wall to get the location, but here the two forces have the same location, so the resultant of the two forces will have the same location (1.67 from base).

2. Calculation of passive force:

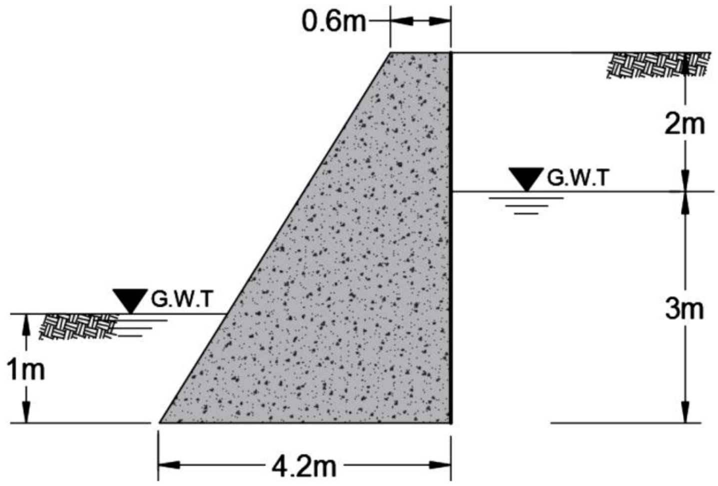


$P_{P,h}=P_1+P_2$

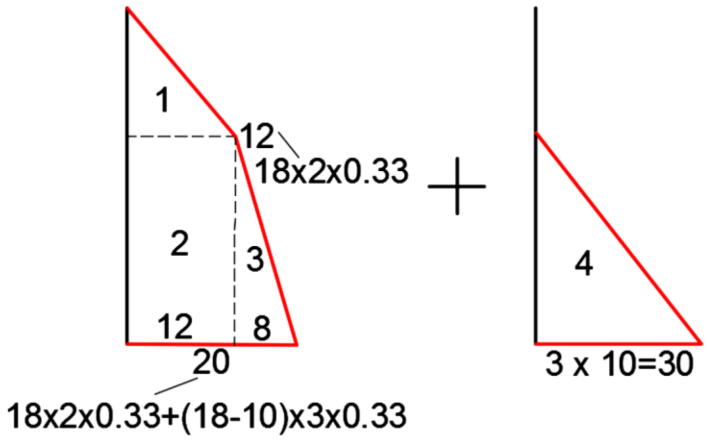
3. In calculation of vertical forces due to the soil weight always take the effective unit weight and multiply it by the area to get the effective force but this is not required in this problem because the force applied directly on the wall.

Now you can complete the solution with the same procedures without any problem

Now, If the water table is at distance 2m below the surface, what's new???



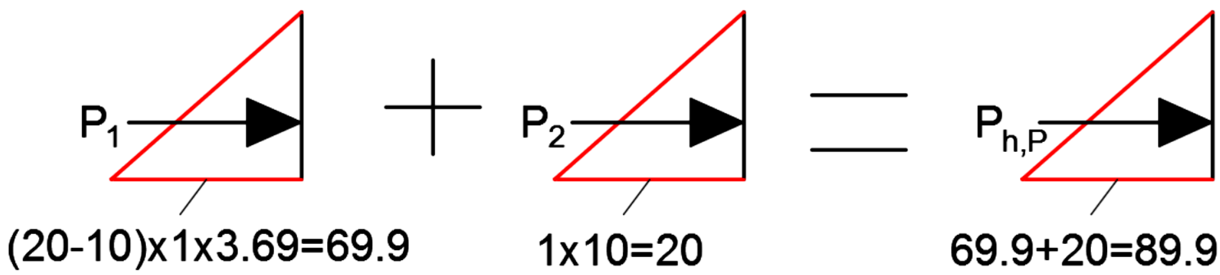
Calculation of Active force:



Here we calculate the effective stress every change, and then added the water alone from its beginning: $P_{a,h}=P_1+P_2+P_3+P_4$

To find the location of $P_{a,h}$ take summation moment at the base of the wall.

Calculation of passive force will not change



The weight of soil above heel (when heel exist), we divide the soil above the heel for two areas, soil above water table and soil below water table. The area of soil above water table is multiplied by natural unit weight, and the area of soil below water table is multiplied by effective unit weight.

Example 13.1

The cross section of a cantilever retaining wall is shown in Figure 13.12. Calculate the factors of safety with respect to overturning, sliding, and bearing capacity.

Solution

From the figure,

$$H' = H_1 + H_2 + H_3 = 2.6 \tan 10° + 6 + 0.7$$
$$= 0.458 + 6 + 0.7 = 7.158 \text{ m}$$

The Rankine active force per unit length of wall $= P_p = \frac{1}{2}\gamma_1 H'^2 K_a$. For $\phi'_1 = 30°$ and $\alpha = 10°$, K_a is equal to 0.3495. (See Table 12.1.) Thus,

$$P_a = \tfrac{1}{2}(18)(7.158)^2(0.3495) = 161.2 \text{ kN/m}$$

$$P_v = P_a \sin 10° = 161.2\,(\sin 10°) = 28.0 \text{ kN/m}$$

and

$$P_h = P_a \cos 10° = 161.2\,(\cos 10°) = 158.75 \text{ kN/m}$$

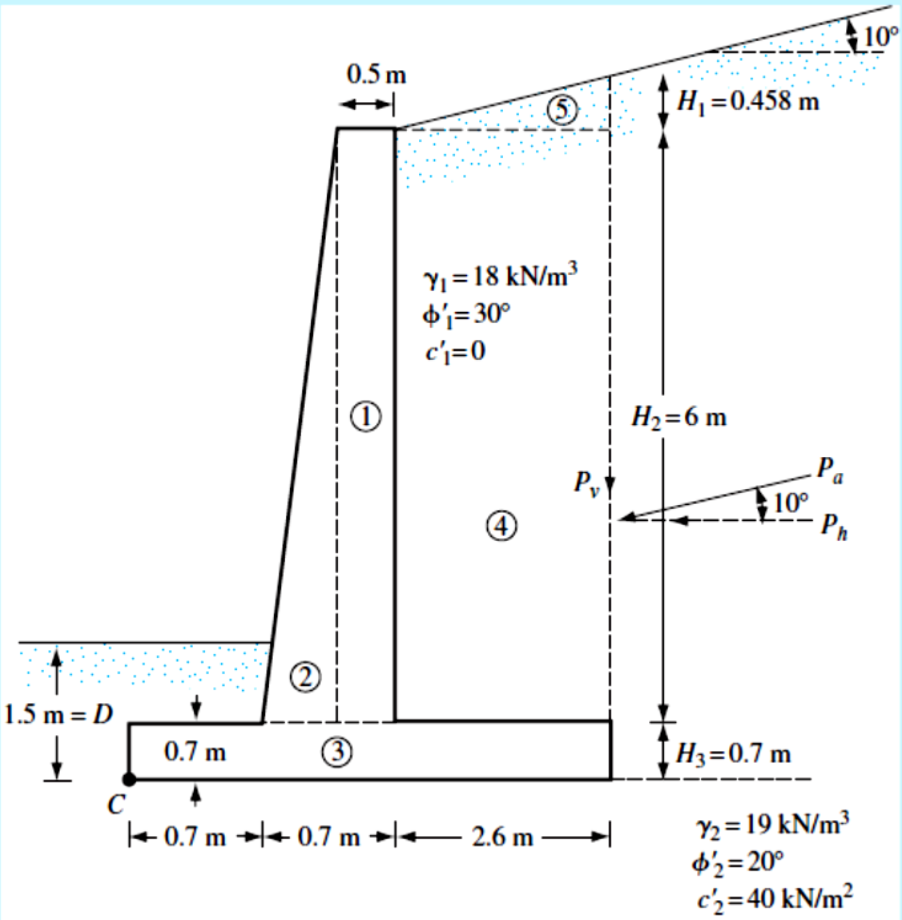


Figure 13.12 Calculation of stability of a retaining wall

Factor of Safety against Overturning

The following table can now be prepared for determining the resisting moment:

Section no.[a]	Area (m^2)	Weight/unit length (kN/m)	Moment arm from point *C* (m)	Moment (kN-m/m)
1	$6 \times 0.5 = 3$	70.74	1.15	81.35
2	$\frac{1}{2}(0.2)6 = 0.6$	14.15	0.833	11.79
3	$4 \times 0.7 = 2.8$	66.02	2.0	132.04
4	$6 \times 2.6 = 15.6$	280.80	2.7	758.16
5	$\frac{1}{2}(2.6)(0.458) = 0.595$	10.71	3.13	33.52
		$P_v = 28.0$	4.0	112.0
		$\Sigma V = 470.42$		$1128.86 = \Sigma M_R$

[a]For section numbers, refer to Figure 13.12

$\gamma_{concrete} = 23.58$ kN/m^3

The overturning moment

$$M_o = P_h\left(\frac{H'}{3}\right) = 158.75\left(\frac{7.158}{3}\right) = 378.78 \text{ kN-m/m}$$

and

$$FS_{(\text{overturning})} = \frac{\Sigma M_R}{M_o} = \frac{1128.86}{378.78} = \mathbf{2.98 > 2, OK}$$

Factor of Safety against Sliding
From Eq. (12.11),

$$FS_{(\text{sliding})} = \frac{(\Sigma V)\tan(k_1\phi'_2) + Bk_2c'_2 + P_p}{P_a\cos\alpha}$$

Let $k_1 = k_2 = \frac{2}{3}$. Also,

$$P_p = \tfrac{1}{2}K_p\gamma_2 D^2 + 2c'_2\sqrt{K_p}D$$

$$K_p = \tan^2\left(45 + \frac{\phi'_2}{2}\right) = \tan^2(45 + 10) = 2.04$$

and

$$D = 1.5 \text{ m}$$

So

$$P_p = \tfrac{1}{2}(2.04)(19)(1.5)^2 + 2(40)(\sqrt{2.04})(1.5)$$
$$= 43.61 + 171.39 = 215 \text{ kN/m}$$

Hence,

$$FS_{(\text{sliding})} = \frac{(470.42)\tan\left(\frac{2 \times 20}{3}\right) + (4)\left(\frac{2}{3}\right)(40) + 215}{158.75}$$

$$= \frac{111.49 + 106.67 + 215}{158.75} = \mathbf{2.73 > 1.5, OK}$$

Note: For some designs, the depth D in a passive pressure calculation may be taken to be *equal to the thickness of the base slab.*

Home work:

13.1 For the cantilever retaining wall shown in Figure P13.1, let the following data be given:

Wall dimensions: $H = 8$ m, $x_1 = 0.4$ m, $x_2 = 0.6$ m, $x_3 = 1.5$ m, $x_4 = 3.5$ m, $x_5 = 0.96$ m, $D = 1.75$ m, $\alpha = 10°$

Soil properties: $\gamma_1 = 16.8$ kN/m³, $\phi'_1 = 32°$, $\gamma_2 = 17.6$ kN/m³, $\phi'_2 = 28°$, $c'_2 = 30$ kN/m²

Calculate the factor of safety with respect to overturning, sliding, and bearing capacity.

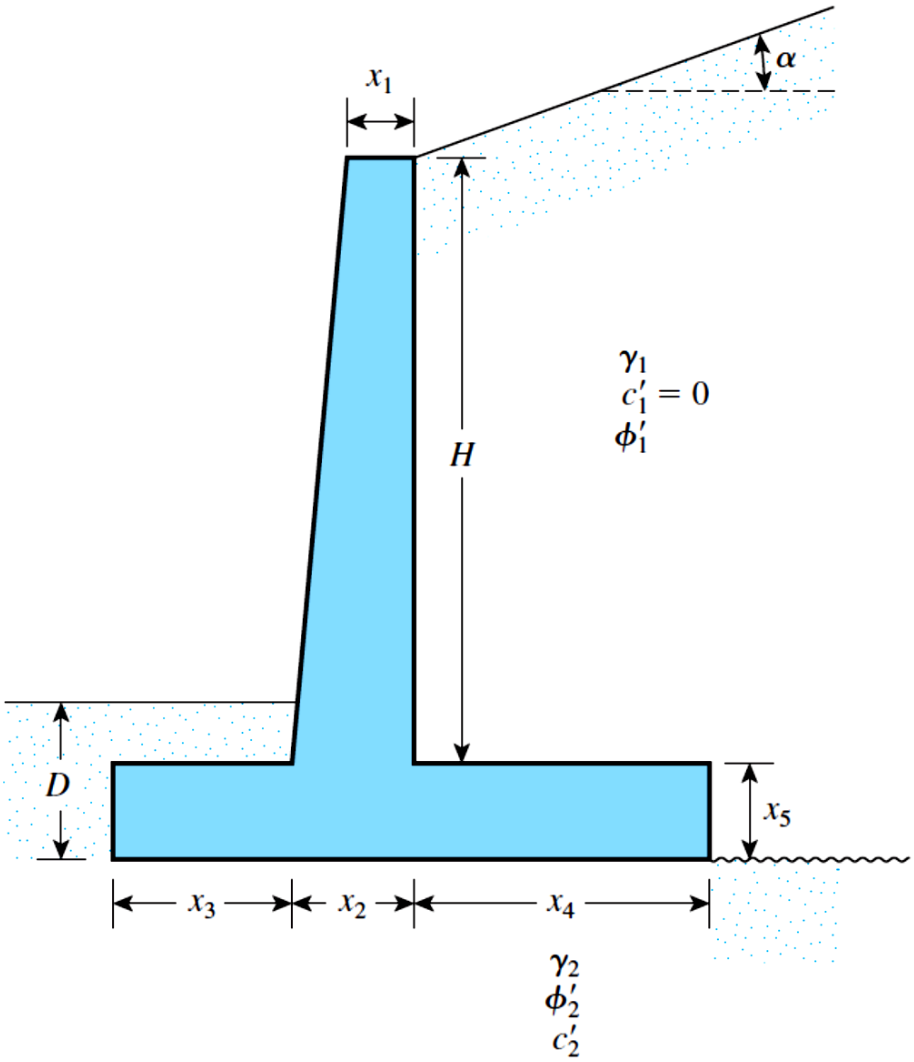


Figure P13.1

13.2 Repeat Problem 13.1 with the following:

Wall dimensions: $H = 6.5$ m, $x_1 = 0.3$ m, $x_2 = 0.6$ m, $x_3 = 0.8$ m, $x_4 = 2$ m, $x_5 = 0.8$ m, $D = 1.5$ m, $\alpha = 0°$

Soil properties: $\gamma_1 = 18.08$ kN/m³, $\phi'_1 = 36°$, $\gamma_2 = 19.65$ kN/m³, $\phi'_2 = 15°$, $c'_2 = 30$ kN/m²

13.3 A gravity retaining wall is shown in Figure P13.3. Calculate the factor of safety with respect to overturning and sliding, given the following data:

Wall dimensions: $H = 6\text{ m}, x_1 = 0.6\text{ m}, x_2 = 2\text{ m}, x_3 = 2\text{ m}, x_4 = 0.5\text{ m}, x_5 = 0.75\text{ m}, x_6 = 0.8\text{ m}, D = 1.5\text{ m}$

Soil properties: $\gamma_1 = 16.5\text{ kN/m}^3, \phi'_1 = 32°, \gamma_2 = 18\text{ kN/m}^3, \phi'_2 = 22°, c'_2 = 40\text{ kN/m}^2$

Use the Rankine active earth pressure in your calculation.

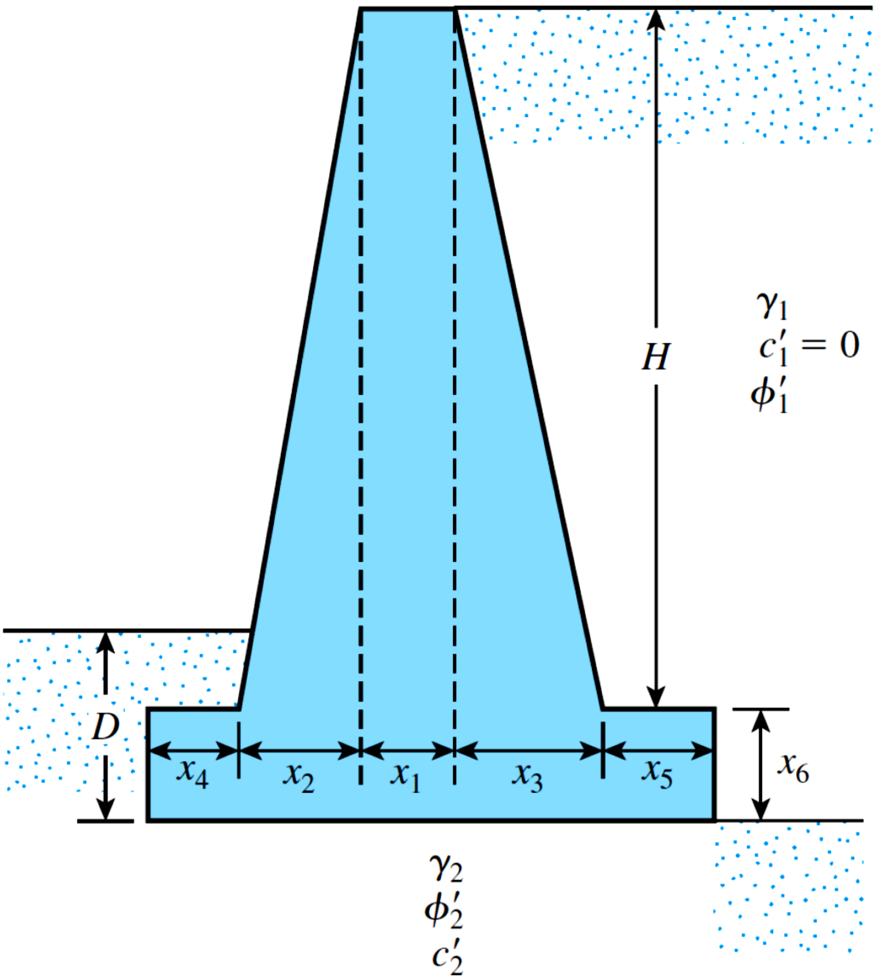


Figure P13.3

Settlement of Shallow Foundations

Introduction

The allowable settlement of a shallow foundation may control the allowable bearing capacity. Thus, the allowable bearing capacity will be the smaller of the following two conditions:

$$q_{all} = \begin{cases} \dfrac{q_u}{FS} \\ \text{or} \\ q_{\text{allowable settlement}} \end{cases}$$

The settlement of a shallow foundation can be divided into two major categories:

(a) elastic, or immediate, settlement and (b) consolidation settlement.

Immediate, or elastic, settlement of a foundation takes place during or immediately after the construction of the structure. Immediate settlement analyses are used for all fine-grained soils including silts and clays with a degree of saturation S ≤ 90 percent and for all coarse-grained soils with a large coefficient of permeability, say, above 10^{-3} m/s.

Consolidation settlement comprises two phases: 1-primary and 2-secondary.

primary consolidation settlement occurs over time. In saturated clays, where the foundation load is gradually transferred from the pore water to the soil skeleton. Immediately after loading, the entire applied normal stress is carried by the water in the voids, in the form of excess pore water pressure. With time, the pore water drains out into the more porous granular soils at the boundaries, thus dissipating the excess pore water pressure and increasing the effective stresses. Secondary consolidation settlement occurs after the completion of primary consolidation caused by slippage and reorientation of soil particles under a sustained load. Primary consolidation settlement is more significant than secondary settlement in inorganic clays and silty soils. The total settlement of a foundation is the sum of the elastic settlement and the consolidation settlement.

Elastic Settlement of Shallow Foundation on Saturated Clay (μ_s= 0.5)

Janbu et al. (1956) proposed an equation for evaluating the average settlement of flexible foundations on saturated clay soils (Poisson's ratio, μ_s =0.5). Referring to Figure 7.1, this relationship can be expressed as

$$S_e = A_1 A_2 \frac{q_o B}{E_s}$$

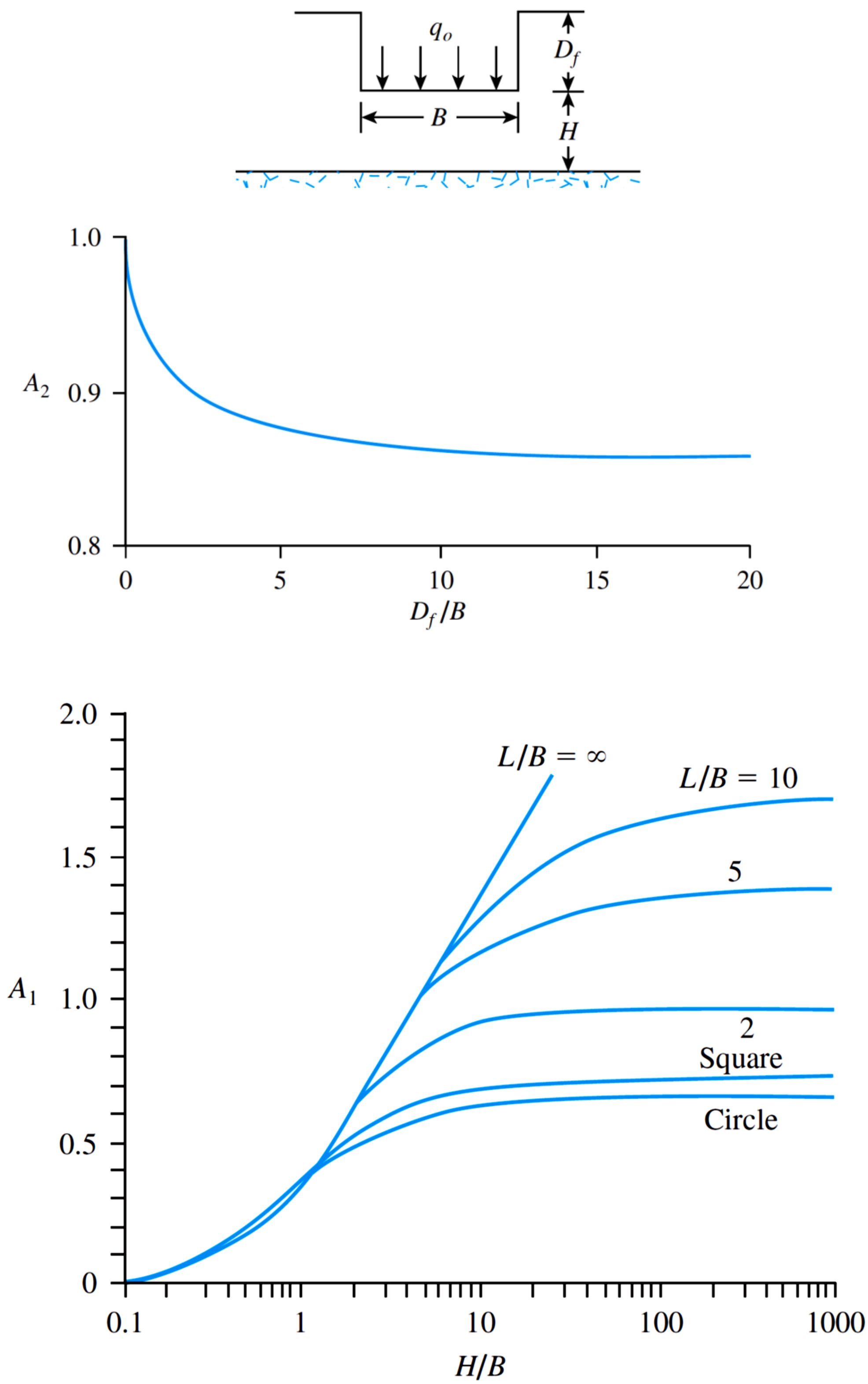


Figure 7.1 Values of A_1 and A_2 for elastic settlement calculation—Eq. (7.1)

where

$A_1 = f(H/B, L/B)$

$A_2 = f(D_f/B)$

L = length of the foundation

B = width of the foundation

D_f = depth of the foundation

H = depth of the bottom of the foundation to a rigid layer

q_o = net load per unit area of the foundation

The modulus of elasticity (E_s) for saturated clays can, in general, be given as

$E_s = \beta c_u$

where c_u = undrained shear strength

The parameter β is primarily a function of the plasticity index and overconsolidation ratio (OCR). Table 7.1 provides a general range for β based on that proposed by Duncan and Buchignani (1976). In any case, proper judgment should be used in selecting the magnitude of β.

Table 7.1 Range of β for Saturated Clay [Eq. (7.2)][a]

Plasticity Index	β				
	OCR = 1	OCR = 2	OCR = 3	OCR = 4	OCR = 5
<30	1500–600	1380–500	1200–580	950–380	730–300
30 to 50	600–300	550–270	580–220	380–180	300–150
>50	300–150	270–120	220–100	180–90	150–75

[a]Based on Duncan and Buchignani (1976)

Natural soil deposits can be *normally consolidated* or *overconsolidated* (or *preconsolidated*). If the present effective overburden pressure $\sigma' = \sigma'_0$ is equal to the preconsolidated pressure σ'_c the soil is *normally consolidated.* However, if $\sigma'_0 < \sigma'_c$, the soil is *overconsolidated.*

$$\text{OCR} = \frac{\text{preconsolidation pressure}, \sigma'_c}{\text{effective overburden pressure}, \sigma'_o}$$

Example 7.1

Consider a shallow foundation 2 m × 1 m in plan in a saturated clay layer. A rigid rock layer is located 8 m below the bottom of the foundation. Given:

Foundation: $D_f = 1$ m, $q_o = 120$ kN/m^2
Clay: $c_u = 150$ kN/m^2, OCR = 2, and Plasticity index, PI = 35

Estimate the elastic settlement of the foundation.

Solution

From Eq. (7.1),

$$S_e = A_1 A_2 \frac{q_o B}{E_s}$$

Given:

$$\frac{L}{B} = \frac{2}{1} = 2$$

$$\frac{D_f}{B} = \frac{1}{1} = 1$$

$$\frac{H}{B} = \frac{8}{1} = 8$$

$$E_s = \beta c_u$$

For OCR = 2 and PI = 35, the value of $\beta \approx 480$ (Table 7.1). Hence,

$$E_s = (480)(150) = 72{,}000 \text{ kN/m}^2$$

Also, from Figure 7.1, $A_1 = 0.9$ and $A_2 = 0.92$. Hence,

$$S_e = A_1 A_2 \frac{q_o B}{E_s} = (0.9)(0.92)\frac{(120)(1)}{72{,}000} = 0.00138 \text{ m} = \mathbf{1.38 \text{ mm}}$$ ■

Elastic Settlement in Granular Soil

Improved Equation for Elastic Settlement

The improved formula for calculating the elastic settlement of foundations takes into account the rigidity of the foundation, the depth of embedment of the foundation, the increase in the modulus of elasticity of the soil with depth, and the location of rigid layers at a limited depth. To use Mayne and Poulos's equation, one needs to determine the equivalent diameter B_e of a rectangular foundation, or

$$B_e = \sqrt{\frac{4BL}{\pi}}$$

where
B = width of foundation
L = length of foundation
For circular foundations,

$$B_e = B$$

where B = diameter of foundation.
Figure 7.5 shows a foundation with an equivalent diameter B_e located at a depth D_f below the ground surface. Let the thickness of the foundation be t and the modulus of elasticity of the foundation material be E_f . A rigid layer is located at a depth H below the bottom of the foundation. The modulus of elasticity of the compressible soil layer can be given as

$$E_s = E_o + kz \qquad (7.16)$$

With the preceding parameters defined, the elastic settlement below the center of the foundation is

$$S_e = \frac{q_o B_e I_G I_F I_E}{E_o}\left(1 - \mu_s^2\right)$$

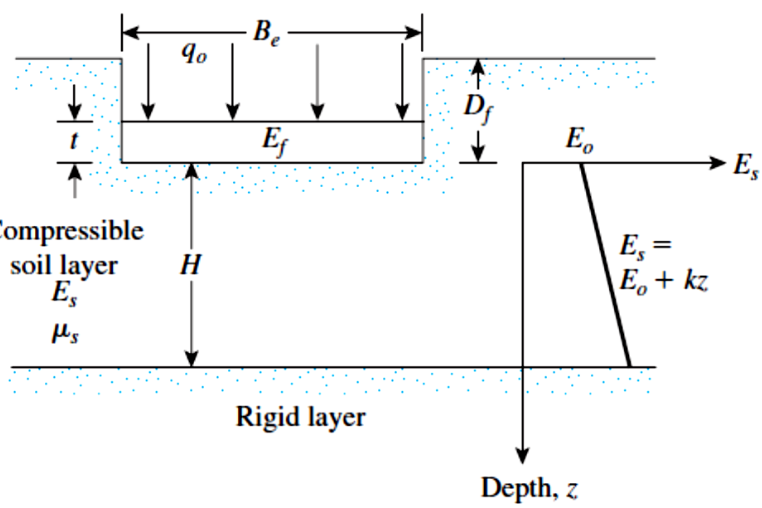


Figure 7.5 Improved equation for calculating elastic settlement: general parameters

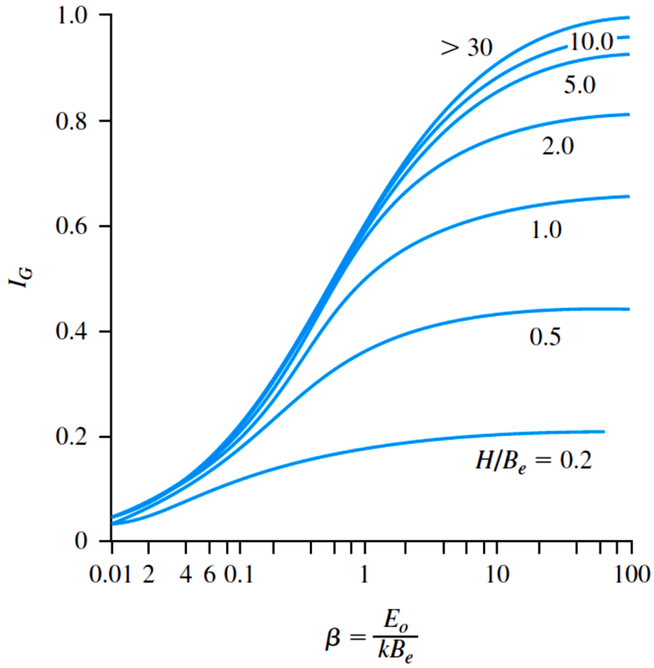


Figure 7.6 Variation of I_G with β

where

I_G = influence factor for the variation of E_s with depth

$$= f\left(\beta = \frac{E_0}{kB_e}, \frac{H}{B_e}\right)$$

I_F = foundation rigidity correction factor

I_E = foundation embedment correction factor

Figure 7.6 shows the variation of I_G with $\beta = E_o/kB_e$ and H/B_e. The foundation rigidity correction factor can be expressed as

$$I_F = \frac{\pi}{4} + \frac{1}{4.6 + 10\left(\frac{E_f}{E_0 + 0.5\, B_e k}\right)\left(\frac{2t}{B_e}\right)^3} \qquad 7.18$$

Similarly, the embedment correction factor is

$$I_E = 1 - \frac{1}{3.5\exp(1.22\mu_s - 0.4)\left(\frac{B_e}{D_f} + 1.6\right)} \qquad (7.19)$$

Figures 7.7 and 7.8 show the variation of I_F and I_E with terms expressed in Eqs. (7.18) and (7.19).

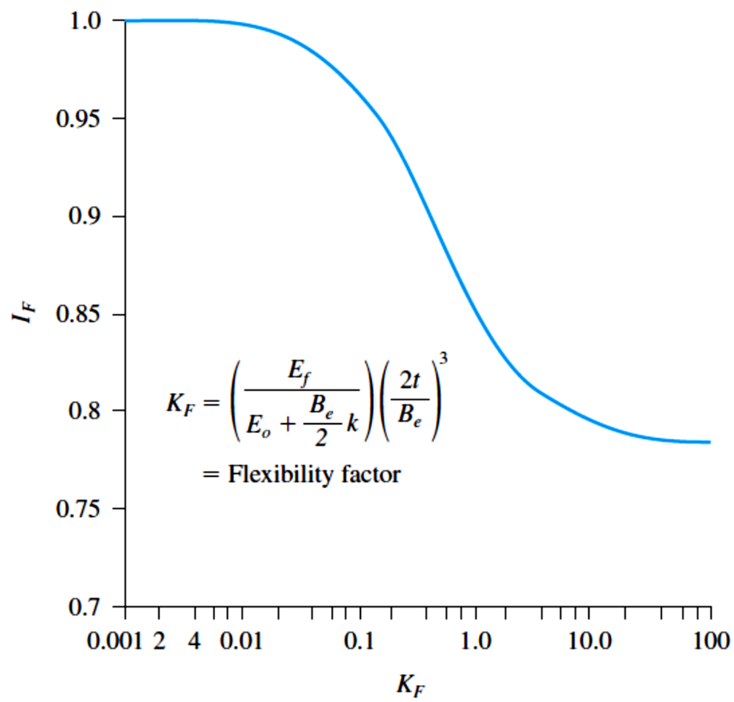


Figure 7.7 Variation of rigidity correction factor I_F with flexibility factor K_F [Eq. (7.18)]

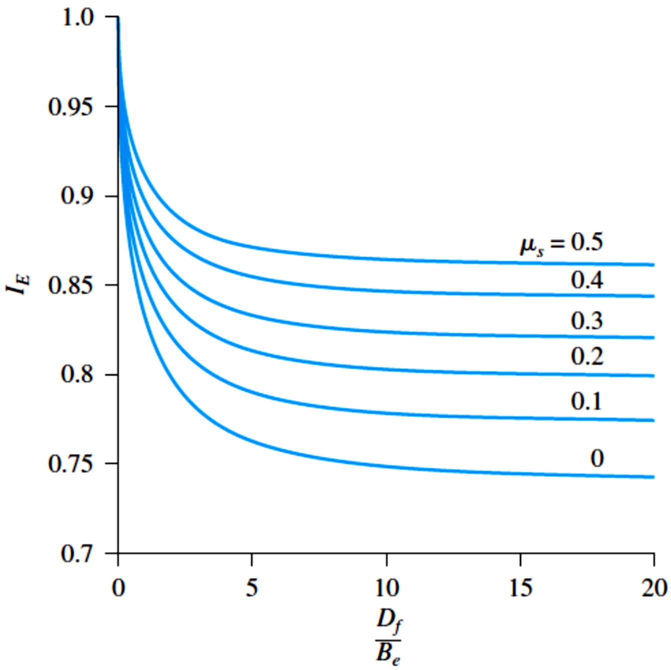


Figure 7.8 Variation of embedment correction factor I_E with D_f/B_e [Eq (7.19)]

Example 7.3

For a shallow foundation supported by a silty sand, as shown in Figure 7.5.

Length = L = 3 m
Width = B = 1.5 m
Depth of foundation = D_f = 1.5 m

Thickness of foundation = t = 0.3 m

Load per unit area = q_o = 240 kN/m^2

$E_f = 16 \times 10^6$ kN/m^2

The silty sand soil has the following properties:

$H = 3.7$ m

$\mu_s = 0.3$

$E_o = 9700$ kN/m^2

$k = 575$ kN/m^2/m

Estimate the elastic settlement of the foundation.

Solution

From Eq. (7.14), the equivalent diameter is

$$B_e = \sqrt{\frac{4BL}{\pi}} = \sqrt{\frac{(4)(1.5)(3)}{\pi}} = 2.39 \text{ m}$$

so

$$\beta = \frac{E_o}{kB_e} = \frac{9700}{(575)(2.39)} = 7.06$$

and

$$\frac{H}{B_e} = \frac{3.7}{2.39} = 1.55$$

From Figure 7.6, for $\beta = 7.06$ and $H/B_e = 1.55$, the value of $I_G \approx 0.7$. From Eq. (7.18),

$$I_F = \frac{\pi}{4} + \frac{1}{4.6 + 10\left(\frac{E_f}{E_0 + 0.5\, B_e k}\right)\left(\frac{2t}{B_e}\right)^3}$$

$$I_F = \frac{3.1416}{4} + \frac{1}{4.6 + 10\left(\frac{16 \times 10^6}{9700 + 0.5 \times 2.39 \times 575}\right)\left(\frac{2 \times 0.3}{2.39}\right)^3} = 0.789$$

From Eq. (7.19),

$$I_E = 1 - \frac{1}{3.5\exp(1.22\mu_s - 0.4)\left(\frac{B_e}{D_f} + 1.6\right)}$$

$$= 1 - \frac{1}{3.5\exp[(1.22)(0.3) - 0.4]\left(\frac{2.39}{1.5} + 1.6\right)} = 0.907$$

From Eq. (7.17),

$$S_e = \frac{q_o B_e I_G I_F I_E}{E_o}(1 - \mu_s^2)$$

so, with q_o = 240 kN/m^2, it follows that

$$S_e = \frac{(240)(2.39)(0.7)(0.789)(0.907)}{9700}(1 - 0.3^2) \approx 0.02696 \text{ m} \approx \mathbf{27\ mm}$$ ■

Settlement of Foundation on Sand Based on Standard Penetration Resistance Meyerhof's Method

Meyerhof (1956) proposed a correlation for the *net bearing pressure* for foundations with the standard penetration resistance, *N*60. The net pressure has been defined as

$$q_{net} = \overline{q} - \gamma D_f$$

where $\overline{q}$ = stress at the level of the foundation.

D_f = depth of foundation

According to Meyerhof's theory, for 25 mm (1 in.) of estimated maximum settlement,

$$q_{net}(\text{kN/m}^2) = \frac{N_{60}}{0.05} F_d\left(\frac{S_e}{25}\right) \quad (\text{for } B \leq 1.22 \text{ m})$$

and

$$q_{net}(\text{kN/m}^2) = \frac{N_{60}}{0.08}\left(\frac{B+0.3}{B}\right)^2 F_d\left(\frac{S_e}{25}\right) \quad (\text{for } B > 1.22 \text{ m})$$

where

F_d = depth factor = ***1 + 0.33(*** D_f/B ***)***

B = foundation width, in meters

S_e = settlement, in mm. Therefore,

$$S_e(\text{mm}) = \frac{1.25 q_{net}(\text{kN/m}^2)}{N_{60} F_d} \quad (\text{for } B \leq 1.22 \text{ m})$$

and

$$S_e(\text{mm}) = \frac{2 q_{net}(\text{kN/m}^2)}{N_{60} F_d}\left(\frac{B}{B+0.3}\right)^2 \quad (\text{for } B > 1.22 \text{ m})$$

The N_{60} referred to in the preceding equations is the standard penetration resistance between the bottom of the foundation and $2B$ below the bottom.

Example 7.6

A shallow foundation measuring 1.75 m x 1.75 m is to be constructed over a layer of sand. Given $D_f = 1$ m; N_{60} is generally increasing with depth; N_{60} in the depth of stress influence = 10, q_{net} =120 kN/m^2. Estimate the elastic settlement of the foundation. Use the Meyerhof's method.

Solution

From Eq. (7.41),

$$S_e = \frac{2q_{net}}{(N_{60})(F_d)}\left(\frac{B}{B+0.3}\right)^2$$

$$F_d = 1 + 0.33(D_f/B) = 1 + 0.33(1/1.75) = 1.19$$

$$S_e = \frac{(2)(120)}{(10)(1.19)}\left(\frac{1.75}{1.75+0.3}\right)^2 = \mathbf{14.7\ mm}$$ ■

Effect of the Rise of Water Table on Elastic Settlement

Terzaghi (1943) suggested that the submergence of soil mass reduces the soil stiffness by about half, which in turn doubles the settlement. In most cases of foundation design, it is considered that, if the ground water table is located $1.5B$ to $2B$ below the bottom of the foundation, it will not have any effect on the settlement. The total elastic settlement (S'_e) due to the rise of the ground water table can be given as

$$S'_e = S_e C_w \qquad (7.59)$$

where

S_e = elastic settlement before the rise of ground water table

C_w = water correction factor

The following are some empirical relationships for Cw (refer to Figure 7.19).

- Peck, Hansen, and Thornburn (1974):

$$C_w = \frac{1}{0.5 + 0.5\left(\dfrac{D_w}{D_f + B}\right)} \geq 1 \qquad (7.60)$$

- Teng (1982):

$$C_w = \frac{1}{0.5 + 0.5\left(\dfrac{D_w - D_f}{B}\right)} \leq 2 \qquad \left(\begin{array}{c}\text{for water table below the}\\ \text{base of the foundation}\end{array}\right) \qquad (7.61)$$

- Bowles (1977):

$$C_w = 2 - \left(\frac{D_w}{D_f + B}\right) \qquad (7.62)$$

In any case, these relationships could be considered approximate, since there is a lack of agreement among geotechnical engineers about the true magnitude of C_w.

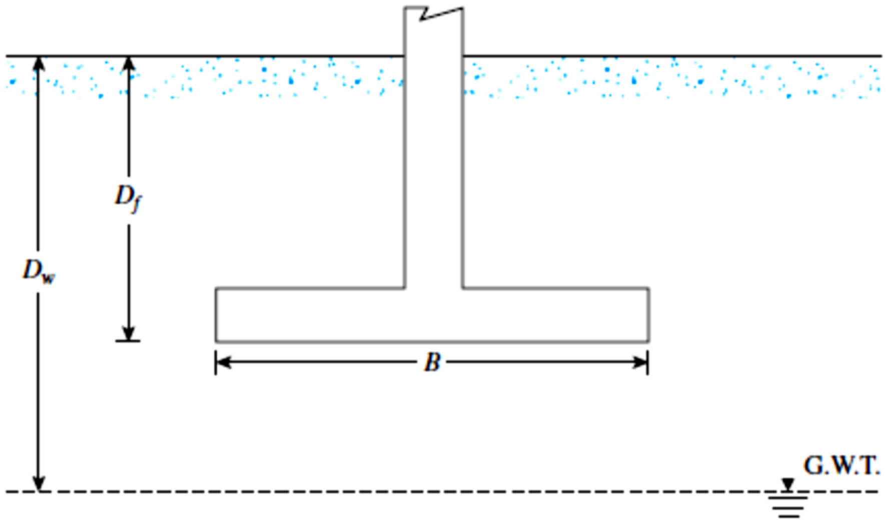


Figure 7.19 Effect of rise of ground water table on elastic settlement in granular soil

Example 7.9

Consider the shallow foundation given in Example 7.6. Due to flooding, the ground water table rose from $D_w = 4$ m to 2 m (Figure 7.19). Estimate the total elastic settlement S'_e after the rise of the water table. Use Eq. (7.60).

Solution

From Eq. (7.59),

$$S'_e = S_e C_w$$

From Eq. (7.60),

$$C_w = \frac{1}{0.5 + 0.5\left(\dfrac{D_w}{D_f + B}\right)} = \frac{1}{0.5 + 0.5\left(\dfrac{2}{1 + 1.75}\right)} = 1.158$$

Hence,

$$S'_e = (11.8 \text{ mm})(1.158) = \mathbf{13.66 \text{ mm}}$$ ■

Problems

1- A flexible foundation measuring 1.5 m x 3 m is supported by a saturated clay. Given: D_f = 1.2 m, H = 3 m, E_s (clay) = 600 kN/m^2, and q_o = 150 kN/m^2. Determine the average elastic settlement of the foundation.

2- A planned flexible load area (see Figure P7.2) is to be 3 m x 4.6 m and carries a uniformly distributed load of 180 kN/m^2. Estimate the elastic settlement below the center of the loaded area. Assume that D_f = 2 m, $H = \infty$.
E_0 = 8500 kN/m^2, k = 700 kN/m^2/m   t= 0.35m and E_f = 18x106 kN/m^2.

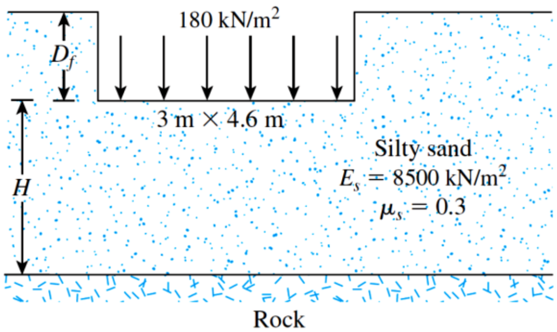


Figure P7.2

3- Redo Problem 2, assuming that D_f = 5 m and H = 3 m.

4- A foundation of 3m x 1.9m resting on a sand deposit. The net load per unit area at the level of the foundation, q_o, is 200kN/m^2. For the sand, μ_s = 0.3, D_f = 0.75 m, and H = 9.5m. Determine the elastic settlement the foundation would undergo.
E_0 = 8500 kN/m^2, k = 700 kN/m^2/m   t= 0.35m and E_f = 18x106 kN/m^2.

5- Repeat Problem 4 for a foundation of size =2.1m x 2.1m, with q_o =230 kN/m^2, D_f = 1.5 m, H = 12 m, and soil conditions of μ_s =0.4, E_0 =16,000 kN/m^2, and *k = 600 kN/m^2/m   t= 0.40m and E_f = 16x106 kN/m^2.*

7.6 A shallow foundation supported by a silty sand is shown in Figure 7.5. Given:

Length: $L = 2$ m
Width: $B = 1$ m
Depth of foundation: $D_f = 1$ m
Thickness of foundation: $t = 0.23$ m
Load per unit area: $q_o = 190$ kN/m^2
$E_f = 15 \times 10^6$ kN/m^2

The silty sand has the following properties:

$H = 2$ m
$\mu_s = 0.4$
$E_o = 9000$ kN/m^2
$k = 500$ kN/m^2/m

Using Eq. (7.17), estimate the elastic settlement of the foundation.

7.13 A shallow foundation measuring 1 m × 2 m in plan is to be constructed over a normally consolidated sand layer. Given: $D_f = 1$ m, N_{60} increases with depth, $\overline{N}_{60}$ (in the depth of stress influence) = 12, and $q_{net} = 153$ kN/m^2. Estimate the elastic settlement using Burland and Burbidge's method (Section 7.6).

7.12 The following are the results of standard penetration tests in a granular soil deposit.

Depth (m)	Standard penetration number, *N*60
1.5	10
3.0	12
4.5	9
6.0	14
7.5	16

What will be the net allowable bearing capacity of a foundation planned to be meyerhof 1.5m x 1.5m? Let $D_f = 0.9$m and the allowable settlement = 25 mm. Use the relationships of Meyerhof presented in Section 7.6.

Average Vertical Stress Increase Due to a Rectangularly Loaded Area

In most cases, the vertical stress below the center of a rectangular area is of importance. This can be given by the relationship

$$\Delta\sigma = q_o I_c \tag{6.14}$$

where

$$I_c = \frac{2}{\pi}\left[\frac{m_1 n_1}{\sqrt{1 + m_1^2 + n_1^2}} \frac{1 + m_1^2 + 2n_1^2}{(1 + n_1^2)(m_1^2 + n_1^2)} + \sin^{-1}\frac{m_1}{\sqrt{m_1^2 + n_1^2}\sqrt{1 + n_1^2}}\right] \tag{6.15}$$

$$m_1 = \frac{L}{B} \tag{6.16}$$

$$n_1 = \frac{z}{\left(\frac{B}{2}\right)} \tag{6.17}$$

The variation of I_c with m_1 and n_1 is given in Table 6.5.

Where L = length of foundation
B = Width of foundation
Z = depth below loaded area

In most practical cases, however, we will need to determine the average stress increase between $z = H_1$ and $z = H_2$ below the center of a loaded area. approximate procedure to determine $\Delta\sigma_{av}$ (H2/H1) is to use the relationship

$$\Delta\sigma_{\text{av}(H_2/H_1)} = \frac{\Delta\sigma_t + 4\Delta\sigma_m + \Delta\sigma_b}{6} \tag{6.29}$$

Table 6.5 Variation of I_c with m_1 and n_1

	m_1									
n_i	1	2	3	4	5	6	7	8	9	10
0.20	0.994	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997
0.40	0.960	0.976	0.977	0.977	0.977	0.977	0.977	0.977	0.977	0.977
0.60	0.892	0.932	0.936	0.936	0.937	0.937	0.937	0.937	0.937	0.937
0.80	0.800	0.870	0.878	0.880	0.881	0.881	0.881	0.881	0.881	0.881
1.00	0.701	0.800	0.814	0.817	0.818	0.818	0.818	0.818	0.818	0.818
1.20	0.606	0.727	0.748	0.753	0.754	0.755	0.755	0.755	0.755	0.755
1.40	0.522	0.658	0.685	0.692	0.694	0.695	0.695	0.696	0.696	0.696
1.60	0.449	0.593	0.627	0.636	0.639	0.640	0.641	0.641	0.641	0.642
1.80	0.388	0.534	0.573	0.585	0.590	0.591	0.592	0.592	0.593	0.593
2.00	0.336	0.481	0.525	0.540	0.545	0.547	0.548	0.549	0.549	0.549
3.00	0.179	0.293	0.348	0.373	0.384	0.389	0.392	0.393	0.394	0.395
4.00	0.108	0.190	0.241	0.269	0.285	0.293	0.298	0.301	0.302	0.303
5.00	0.072	0.131	0.174	0.202	0.219	0.229	0.236	0.240	0.242	0.244
6.00	0.051	0.095	0.130	0.155	0.172	0.184	0.192	0.197	0.200	0.202
7.00	0.038	0.072	0.100	0.122	0.139	0.150	0.158	0.164	0.168	0.171
8.00	0.029	0.056	0.079	0.098	0.113	0.125	0.133	0.139	0.144	0.147
9.00	0.023	0.045	0.064	0.081	0.094	0.105	0.113	0.119	0.124	0.128
10.00	0.019	0.037	0.053	0.067	0.079	0.089	0.097	0.103	0.108	0.112

Foundation engineers often use an approximate method to determine the increase in stress with depth caused by the construction of a foundation. The method is referred to as the *2:1 method.* (See Figure 6.7.) According to this method, the increase in stress at depth z is

$$\Delta\sigma = \frac{q_o \times B \times L}{(B + z)(L + z)}$$

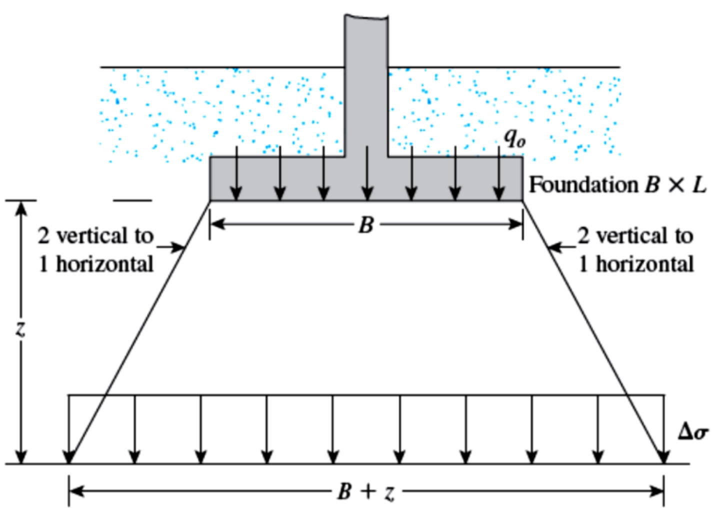


Figure 6.7 2:1 method of finding stress increase under a foundation

Note that Eq. (6.18) is based on the assumption that the stress from the foundation spreads out along lines with a *vertical-to-horizontal slope of 2:1.*

Example 6.2

A flexible rectangular area measures 2.5 m × 5 m in plan. It supports a load of 150 kN/m^2.

Determine the vertical stress increase due to the load at a depth of 6.25 m below the center of the rectangular area.

Solution

From Eq. (6.14),

$$\Delta\sigma = q_o I_c$$

$$m_1 = \frac{L}{B} = \frac{5}{2.5} = 2$$

$$n_1 = \frac{z}{\left(\frac{B}{2}\right)} = \frac{6.25}{\left(\frac{2.5}{2}\right)} = 5$$

From Table 6.5, for $m_1 = 2$ and $n_1 = 5$, the value of $I_c = 0.131$. Thus,

$$\Delta\sigma = (150)(0.131) = \mathbf{19.65\ kN/m^2}$$

Example 6.3

Refer to Figure 6.14. Determine the *average* stress increase below the center of the loaded area between $z = 3$ m to $z = 5$ m (that is, between points A and A').

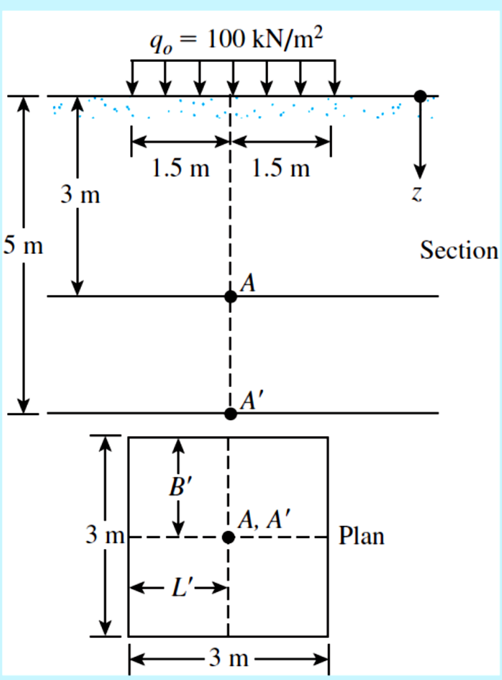


Figure 6.14 Determination of average increase in stress below a rectangular area

Solution

The following table can now be prepared.

z (m)	L (m)	B (m)	m_1	n_1	I_c^*	$q_o I_c^{**}$ (kN/m²)
3	3	3	1	2	0.336	33.6
4	3	3	1	2.67	0.231	23.1
5	3	3	1	3.33	0.155	15.5

*Table 6.5
**$q_o = 100$ kN/m²

From Eq. (6.29),

$$\Delta\sigma_{av(H_2/H_1)} = \frac{33.6 + 4(23.1) + 15.5}{6} = \mathbf{23.58\ kN/m^2}$$

Using the *2:1 method*

Solution

From Eq. (6.18) for a square loaded area,

$$\sigma_t = \frac{q_o B^2}{(B+z)^2} = \frac{(100)(3)^2}{(3+3)^2} = 25\ \text{kN/m}^2$$

$$\sigma_m = \frac{(100)(3)^2}{(3+4)^2} = 18.37\ \text{kN/m}^2$$

$$\sigma_b = \frac{(100)(3)^2}{(3+5)^2} = 14.06\ \text{kN/m}^2$$

$$\Delta\sigma_{av(H_2/H_1)} = \frac{25 + 4(18.37) + 14.06}{6} = \mathbf{18.76\ kN/m^2}$$

Primary Consolidation Settlement Relationships

As mentioned before, consolidation settlement occurs over time in saturated clayey soils subjected to an increased load caused by construction of the foundation. (See Figure 7.20.) On the basis of the one-dimensional consolidation settlement equations, we write

$$S_{c(p)} = \int \varepsilon_z dz$$

where

$$\varepsilon_z = \text{vertical strain} = \frac{\Delta e}{1 + e_o}$$

$$\Delta e = \text{change of void ratio} = f(\sigma'_o, \sigma'_c, \text{ and } \Delta\sigma')$$

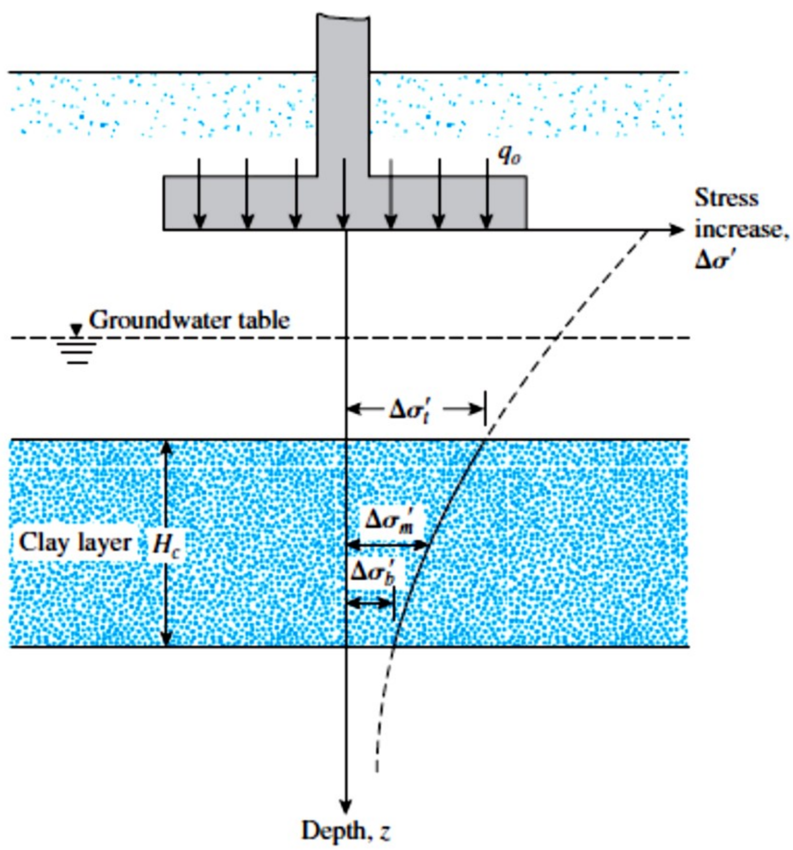


Figure 7.20 Consolidation settlement calculation

$$S_{c(p)} = \frac{C_c H_c}{1 + e_o} \log \frac{\sigma'_o + \Delta\sigma'_{av}}{\sigma'_o}$$ (for normally consolidated clays)

$$S_{c(p)} = \frac{C_s H_c}{1 + e_o} \log \frac{\sigma'_o + \Delta\sigma'_{av}}{\sigma'_o}$$ (for overconsolidated clays with $\sigma'_o + \Delta\sigma'_{av} < \sigma'_c$)

$$S_{c(p)} = \frac{C_s H_c}{1 + e_o} \log \frac{\sigma'_c}{\sigma'_o} + \frac{C_c H_c}{1 + e_o} \log \frac{\sigma'_o + \Delta\sigma'_{av}}{\sigma'_c}$$ (for overconsolidated clays with $\sigma'_o < \sigma'_c < \sigma'_o + \Delta\sigma'_{av}$)

where

σ'_o = average effective pressure on the clay layer before the construction of the foundation

$\Delta\sigma'_{av}$ = average increase in effective pressure on the clay layer caused by the construction of the foundation

σ'_c = preconsolidation pressure

e_o = initial void ratio of the clay layer

C_c = compression index

C_s = swelling index

H_c = thickness of the clay layer

Compression Index

The ***compression index, C_c***, is the slope of the straight-line portion (the latter part) of the loading curve, or

$$C_c = \frac{e_1 - e_2}{\log \sigma'_2 - \log \sigma'_1} = \frac{e_1 - e_2}{\log\left(\frac{\sigma'_2}{\sigma'_1}\right)}$$

where e_1 and e_2 are the void ratios at the end of consolidation under effective stresses σ'_1 and σ'_2, respectively.

The compression index, as determined from the laboratory e–**log** σ' curve, will be somewhat different from that encountered in the field. The primary reason is that the soil remolds itself to some degree during the field exploration. The nature of variation of the e–**log** σ' curve in the field for a normally consolidated clay is shown in Figure below. The curve, generally referred to as the *virgin compression curve,* approximately intersects the laboratory curve at a void ratio of $0.42e_o$

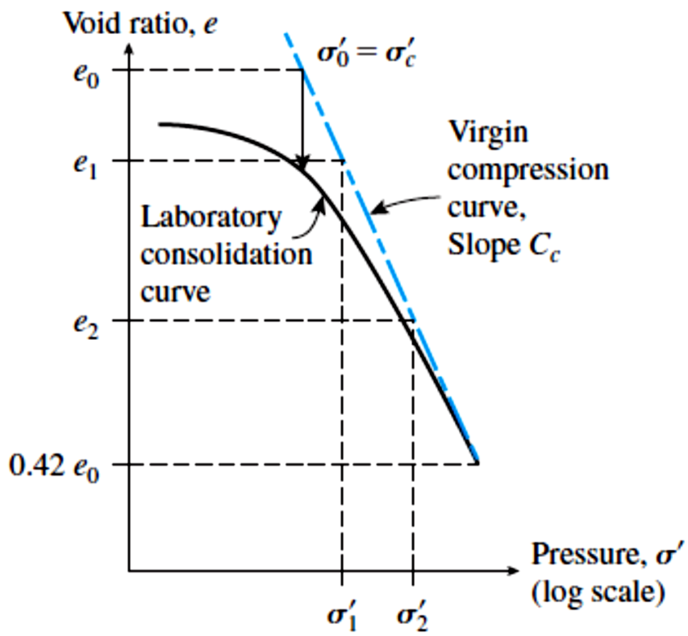


Figure 2.17 **Construction of virgin compression curve for normally consolidated clay**

The value of C_c can vary widely, depending on the soil. Skempton (1944) gave an empirical correlation for the compression index in which

$$C_c = 0.009(LL - 10)$$

where **LL** = liquid limit.

Besides Skempton, several other investigators also have proposed correlations for the compression index. Some of those are given here:

Rendon-Herrero (1983):

$$C_c = 0.141 G_s^{1.2}\left(\frac{1 + e_o}{G_s}\right)^{2.38}$$

Nagaraj and Murty (1985):

$$C_c = 0.2343\left[\frac{LL(\%)}{100}\right]G_s$$

Park and Koumoto (2004):

$$C_c = \frac{n_o}{371.747 - 4.275n_o}$$

where n_o = *in situ* porosity of soil.
Wroth and Wood (1978):

$$C_c = 0.5G_s\left(\frac{\text{PI}(\%)}{100}\right)$$

Swelling Index C_s

The swelling index, C_s, is the slope of the unloading portion of the **e–log** σ' curve. In Figure 2.16b, it is defined as

$$C_s = \frac{e_3 - e_4}{\log\left(\frac{\sigma'_4}{\sigma'_3}\right)}$$

In most cases, the value of the swelling index is **1/4** to **1/5**of the compression index.

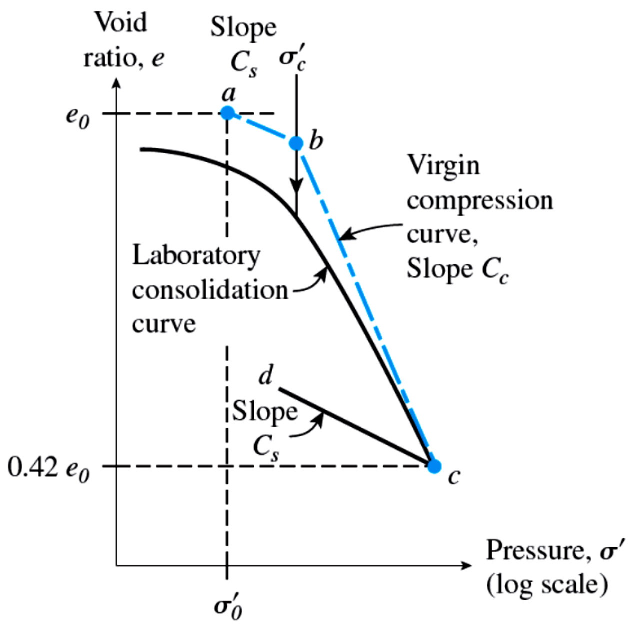


Figure 2.18 Construction of field consolidation curve for overconsolidated clay

The swelling index is also referred to as the *recompression index.* The determination of the swelling index is important in the estimation of consolidation settlement of *overconsolidated clays.*

Using the modified Cam clay model and Eq. (2.58), Kulhawy and Mayne (1990) have shown that

$$C_s = \frac{\text{PI}(\%)}{370} \tag{2.61}$$

Note that the increase in effective pressure, $\Delta\sigma'$, on the clay layer is not constant with depth: The magnitude of $\Delta\sigma'$ will decrease with the increase in depth measured from the bottom of the foundation. However, the average increase in pressure may be approximated by

$$\Delta\sigma'_{\text{av}} = \tfrac{1}{6}(\Delta\sigma'_t + 4\Delta\sigma'_m + \Delta\sigma'_b) \tag{6.29}$$

where $\Delta\sigma_t$, $\Delta\sigma_m$, $\Delta\sigma_b$ = stress increase below the center of the loaded area ($L \times B$), respectively, at depths $z = H_1$, $H_1 + H_2/2$, and $H_1 + H_2$.

Example 7.10

A plan of a foundation 1 m 3 2 m is shown in Figure 7.23. Estimate the consolidation settlement of the foundation,

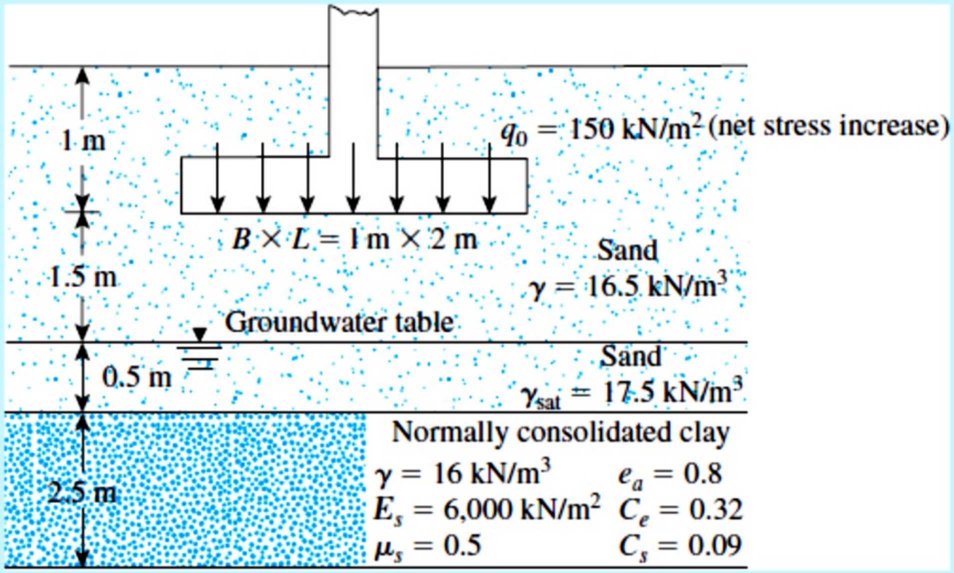


Figure 7.23 Calculation of primary consolidation settlement for a foundation

Solution

The clay is normally consolidated. Thus,

$$S_{c(p)-\text{oed}} = \frac{C_c H_c}{1+e_o} \log \frac{\sigma'_o + \Delta\sigma'_{\text{av}}}{\sigma'_o}$$

so

$$\sigma'_o = (2.5)(16.5) + (0.5)(17.5 - 9.81) + (1.25)(16 - 9.81)$$
$$= 41.25 + 3.85 + 7.74 = 52.84 \text{ kN/m}^2$$

From Eq. (6.29),

$$\Delta\sigma'_{\text{av}} = \tfrac{1}{6}(\Delta\sigma'_t + 4\Delta\sigma'_m + \Delta\sigma'_b)$$

Now the following table can be prepared (*Note:* $L = 2$ m; $B = 1$ m):

$m_1 = L/B$	z(m)	$z/(B/2) = n_1$	I_c[a]	$\Delta\sigma' = q_o I_c$[b]
2	2	4	0.190	$28.5 = \Delta\sigma'_t$
2	2 + 2.5/2 = 3.25	6.5	≈ 0.085	$12.75 = \Delta\sigma'_m$
2	2 + 2.5 = 4.5	9	0.045	$6.75 = \Delta\sigma'_b$

[a]Table 6.5
[b]Eq. (6.14)

Now,

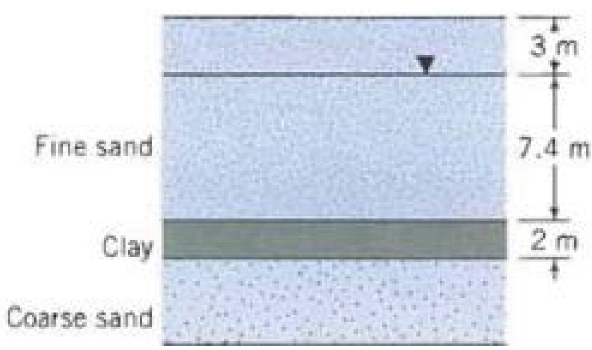
$$\Delta\sigma'_{av} = \tfrac{1}{6}(28.5 + 4 \times 12.75 + 6.75) = 14.38\ \text{kN/m}^2$$

so

$$S_{c(p)-oed} = \frac{(0.32)(2.5)}{1+0.8}\ \log\left(\frac{52.84 + 14.38}{52.84}\right) = 0.0465\ \text{m}$$

$$= 46.5\ \text{mm}$$

Example: The soil profile at a site for a proposed office building consists of a layer of fine Sand 10.4 m thick above a layer of soft normally consolidated clay 2 m thick. Below the soft clay is a deposit of coarse sand. The groundwater table was observed at 3 m below ground level. The void ratio of the sand is 0.76 and the water content of the clay is 43%. The building will impose a vertical stress increase of 140 kPa at the middle of the clay layer. Estimate the primary consolidation settlement of the clay. Assume the soil above the water table to be saturated, C_c = 0.3 and G_s = 2.7.



Solution

For normally consolidated clay

$$S_{c(p)} = \frac{C_c H_c}{1+e_o} \log \frac{\sigma'_o + \Delta\sigma'_{av}}{\sigma'_o}$$

Calculate the current effective stress and void ratio at the middle of the clay layer

Sand layer

$$\gamma_{sat} = \frac{(G_s + e)\gamma_w}{1 + e}$$

$$\gamma_{sat} = \frac{W}{V} = \left(\frac{2.7+0.76}{1+0.76}\right)x9.8 = 19.3 \text{ kN/m}^3$$

$\gamma' = \gamma_{sat} - \gamma_w$ =19.3 -9.8 = 9.5 kN/m^3

Clay layer

$$e = wG_s$$

e = 0.42 x 2.7 = 1.16

$$\gamma_{sat} = \frac{(G_s + e)\gamma_w}{1 + e}$$

$$\gamma_{sat} = \left(\frac{2.7+1.16}{1+1.16}\right)x9.8 = 17.5 \text{ kN/m}^3$$

$\gamma' = \gamma_{sat} - \gamma_w$ = 17.5 - 9.8 = 7.7 kN/m^3

Effective stress σ_0' = 19.3 x 3 + 9.5 x 7.4 + 7.7 x 1 = 135.9 kPa

$$S_{c(p)} = \frac{C_c H_c}{1 + e_o} \log \frac{\sigma_o' + \Delta\sigma_{av}'}{\sigma_o'}$$

$$S_{c(p)} = \frac{0.3\ x\ 2}{1+1.16} \text{ x log} \frac{135.9+140}{135.9} = 0.0854 \text{ m} \quad = 85.4 \text{ mm}$$

Example: Assume the same soil stratigraphy and soil parameters as in previous example except that the clay has an overconsolidation ratio of 1.5, w=38%, C_s=0.05. Determine the primary consolidation settlement of the clay?

Critical Thinking: Since the soil is overconsolidated, you will have to check whether the preconsolidation stress is less than or greater than the sum of the current vertical effective stress and the applied vertical stress at the center of the clay. This check will determine the appropriate equation to use.

Solution:
Clay layer

$e = w\,G_s = 0.38 \times 2.7 = 1.03$

$$\gamma_{sat} = \frac{(G_s + e)\gamma_w}{1 + e}$$

$$\gamma_{sat} = \left(\frac{2.7+1.03}{1+1.03}\right) x 9.8 = 18.0 \text{ kN/m}^3$$

$\gamma' = 18.0 - 9.8 = 8.2 \text{ kN/m}^3$

Effective stresses $\sigma'_o = 19.3 \times 3 + 9.5 \times 7.4 + 8.2 \times 1 = 136.4$ kPa

$\sigma'_o + \Delta\sigma'_{av} = 136.4 + 140 = 276.4$ kPa

Preconsolidation stress $\sigma'_c = 1.5 * 136.4 = 204.6\ k\text{Pa} < \sigma'_o + \Delta\sigma'_{av}$

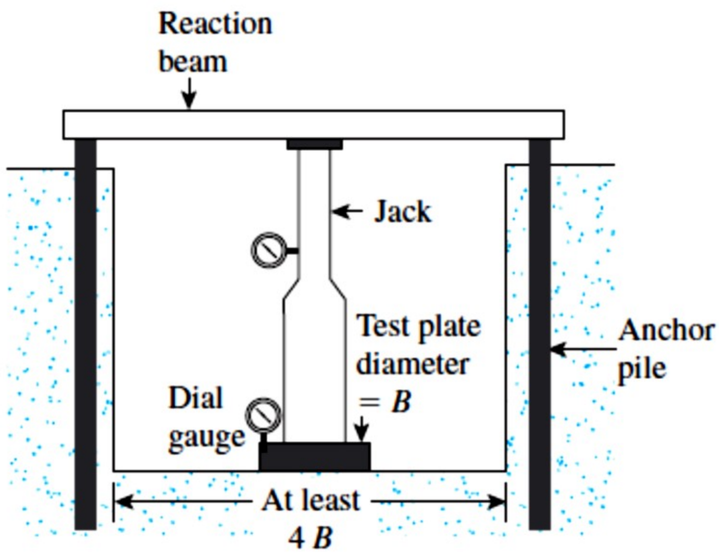
$$S_{c(p)} = \frac{C_s H_c}{1 + e_o} \log \frac{\sigma'_c}{\sigma'_o} + \frac{C_c H_c}{1 + e_o} \log \frac{\sigma'_o + \Delta\sigma'_{av}}{\sigma'_c} \quad \text{(for overconsolidated clays with } \sigma'_o < \sigma'_c < \sigma'_o + \Delta\sigma'_{av}\text{)}$$

$$S_c = \frac{0.05 \, x \, 2}{1+1.03} \text{ x } \log \frac{204.6}{136.4} + \frac{0.3 \, x \, 2}{1+1.03} \text{ x } \log \frac{276.4}{204.6} = 0.047 \text{ m} = 47 \text{ mm}$$

Field Load Test

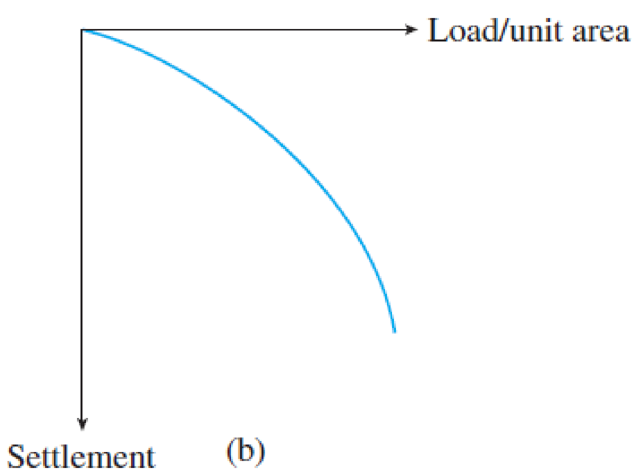
The ultimate load-bearing capacity of a foundation, as well as the allowable bearing capacity based on tolerable settlement considerations, can be effectively determined from the field load test, generally referred to as the *plate load test.* The plates that are used for tests in the field are usually made of steel and are 25 mm (1 in.) thick and 150 mm to 762 mm in diameter. Occasionally, square plates that are 305 mm x 305 mm are also used.

To conduct a plate load test, a hole is excavated with a minimum diameter of 4*B* (*B* is the diameter of the test plate) to a depth of D_f , the depth of the proposed foundation. The plate is placed at the center of the hole, and a load that is about 1/4 to 1/5 of the estimated ultimate load is applied to the plate in steps by means of a jack.



(a)
Plate load test arrangement

During each step of the application of the load, the settlement of the plate is observed on dial gauges. At least one hour is allowed to elapse between each application. The test should be conducted until failure, or at least until the plate has gone through 25 mm (1 in.) of settlement.



(b)

Nature of load–settlement curve

For tests in clay,

$$q_{ult(F)} = q_{ult(P)}$$

where

$q_{ult(F)}$ = ultimate bearing capacity of the proposed foundation

$q_{ult(P)}$ = ultimate bearing capacity of the test plate

the ultimate bearing capacity in clay is virtually independent of the size of the plate.

For tests in sandy soils,

$$q_{ult(F)} = q_{ult(P)} \frac{B_F}{B_P}$$

where

B_F = width of the foundation

B_P = width of the test plate

The allowable bearing capacity of a foundation, based on settlement considerations and for a given intensity of load, q_o , is

$$S_F = S_P \frac{B_F}{B_P} \qquad \text{for clayey soil}$$

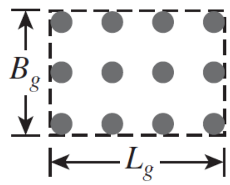
and

$$S_F = S_P\left(\frac{2B_F}{B_F + B_P}\right)^2 \quad \text{(for sandy soil)}$$

Figure 7.26 Plate load test in the field (*Courtesy of Braja M. Das, Henderson, Nevada*)

Consolidation Settlement of Group Piles

The consolidation settlement of a group pile in clay can be estimated by using the 2:1 stress distribution method. The calculation involves the following steps (see Figure below).



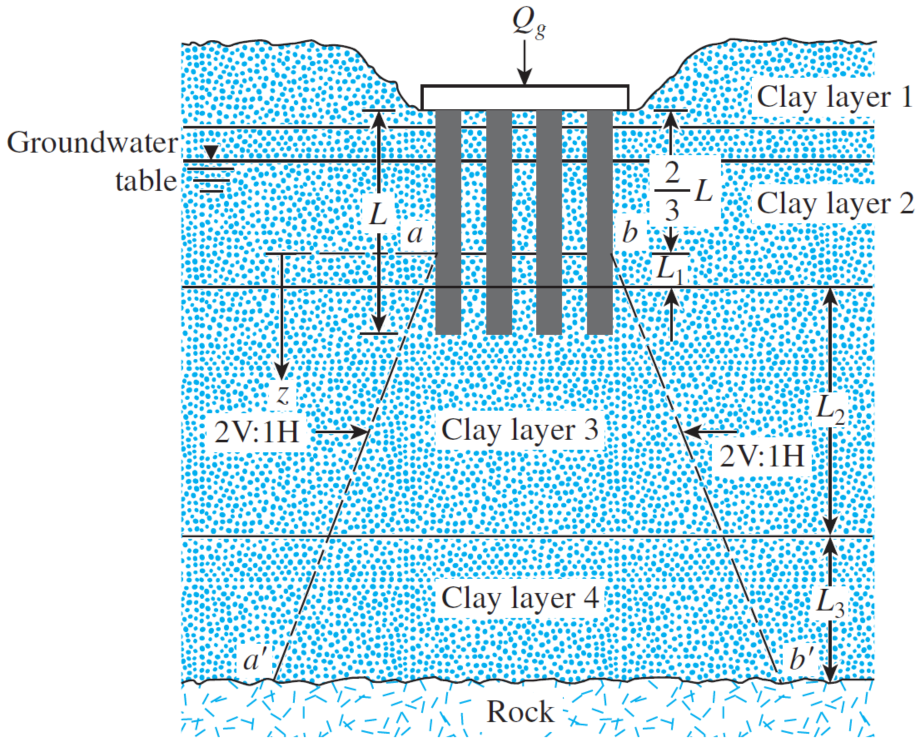


Figure 9.50 Consolidation settlement of group piles

Step 1. Let the depth of embedment of the piles be *L.* The group is subjected to a total load of Q_g. If the pile cap is below the original ground surface, Q_g equals the total load of the superstructure on the piles, minus the effective weight of soil above the group piles removed by excavation.

Step 2. Assume that the load Q_g is transmitted to the soil beginning at a depth of 2*L*/3 from the top of the pile, as shown in the figure. The load Q_g spreads out along two vertical to one horizontal line from this depth. Lines ***aa'*** and ***bb'*** are the two 2:1 lines.

Step 3. Calculate the increase in effective stress caused at the middle of each soil layer by the load Q_g. The formula is

$$\Delta\sigma'_i = \frac{Q_g}{(B_g + z_i)(L_g + z_i)} \qquad (9.138)$$

where

$\Delta\sigma'_i$ = increase in effective stress at the middle of layer *i*

L_g , B_g = length and width, respectively of the planned group piles

z_i = distance from $z = 0$ to the middle of the clay layer *i*

For example, in Figure 9.50, for layer 2, $z_i = L_1/2$; for layer 3, $z_i = L_1 + L_2/2$; and for layer 4, $zi = L_1 + L_2 + L_3/2$. Note, however, that there will be no increase in stress in clay layer 1, because it is above the horizontal plane ($z = 0$) from which the stress distribution to the soil starts.

Step 4. Calculate the consolidation settlement of each layer caused by the increased stress. The formula is

$$\Delta s_{c(i)} = \left[\frac{\Delta e_{(i)}}{1 + e_{o(i)}}\right] H_i \qquad (9.139)$$

where

$\Delta \boldsymbol{s}_{\boldsymbol{c}(\boldsymbol{i})}$ = consolidation settlement of layer *i*

$\Delta \boldsymbol{s}_{\boldsymbol{e}(\boldsymbol{i})}$ = change of void ratio caused by the increase in stress in layer *i*

$\boldsymbol{e}_{\mathbf{0}(\boldsymbol{i})}$ = initial void ratio of layer *i* (before construction)

$\boldsymbol{H_i}$ = thickness of layer *i* (*Note:* In Figure 9.50, for layer 2, $H_i = L_1$; for layer 3, $H_i = L_2$; and for layer 4, $H_i = L_3$.)

Step 5*. The total consolidation settlement of the group piles is then*

$$\Delta s_{c(g)} = \Sigma \Delta s_{c(i)} \qquad (9.140)$$

Example 9.23

A group pile in clay is shown in Figure 9.51. Determine the consolidation settlement of the piles. All clays are normally consolidated.

Solution

Because the lengths of the piles are 15 m each, the stress distribution starts at a depth of 10 m below the top of the pile. We are given that $Q_g = 2000$ kN.

Calculation of Settlement of Clay Layer 1

For normally consolidated clays,

$$\Delta s_{c(1)} = \left[\frac{(C_{c(1)}H_1)}{1 + e_{o(1)}}\right]\log\left[\frac{\sigma'_{o(1)} + \Delta\sigma'_{(1)}}{\sigma'_{o(1)}}\right]$$

$$\Delta\sigma'_{(1)} = \frac{Q_g}{(L_g + z_1)(B_g + z_1)} = \frac{2000}{(3.3 + 3.5)(2.2 + 3.5)} = 51.6 \text{ kN/m}^2$$

and

$$\sigma'_{o(1)} = 2(16.2) + 12.5(18.0 - 9.81) = 134.8 \text{ kN/m}^2$$

So

$$\Delta s_{c(1)} = \frac{(0.3)(7)}{1 + 0.82}\log\left[\frac{134.8 + 51.6}{134.8}\right] = 0.1624 \text{ m} = \mathbf{162.4\ mm}$$

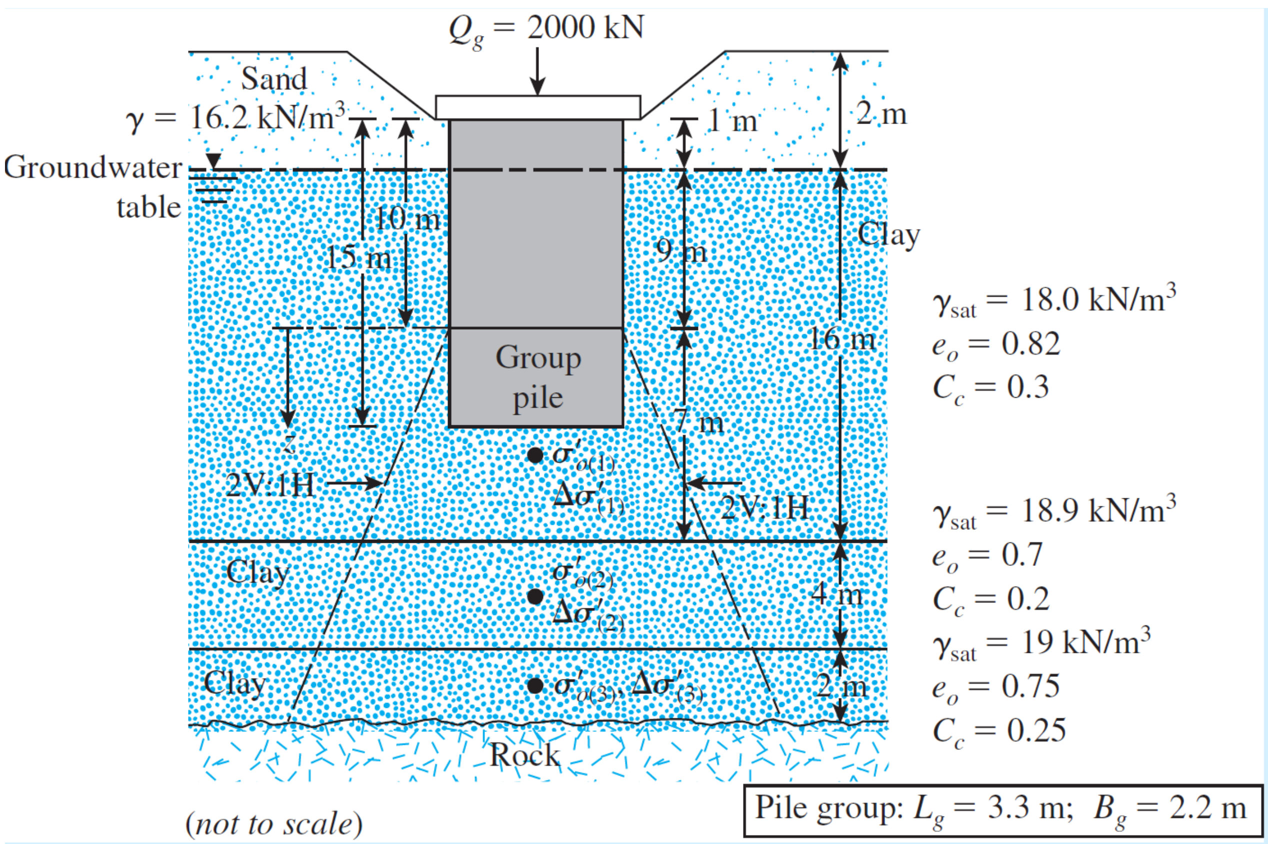


Figure 9.51 Consolidation settlement of a pile group

Settlement of Layer 2

As with layer 1,

$$\Delta s_{c(2)} = \frac{C_{c(2)}H_2}{1 + e_{o(2)}} \log\left[\frac{\sigma'_{o(2)} + \Delta\sigma_{(2)}}{\sigma'_{o(2)}}\right]$$

$$\sigma'_{\sigma(2)} = 2(16.2) + 16(18.0 - 9.81) + 2(18.9 - 9.81) = 181.62 \text{ kN/m}^2$$

and

$$\Delta\sigma'_{(2)} = \frac{2000}{(3.3 + 9)(2.2 + 9)} = 14.52 \text{ kN/m}^2$$

Hence,

$$\Delta s_{c(2)} = \frac{(0.2)(4)}{1 + 0.7} \log\left[\frac{181.62 + 14.52}{181.62}\right] = 0.0157 \text{ m} = \mathbf{15.7 \text{ mm}}$$

Settlement of Layer 3

Continuing analogously, we have

$$\sigma'_{o(3)} = 181.62 + 2(18.9 - 9.81) + 1(19 - 9.81) = 208.99 \text{ kN/m}^2$$

$$\Delta\sigma'_{(3)} = \frac{2000}{(3.3 + 12)(2.2 + 12)} = 9.2 \text{ kN/m}^2$$

$$\Delta s_{c(3)} = \frac{(0.25)(2)}{1 + 0.75} \log\left(\frac{208.99 + 9.2}{208.99}\right) = 0.0054 \text{ m} = \mathbf{5.4 \text{ mm}}$$

Hence, the total settlement is

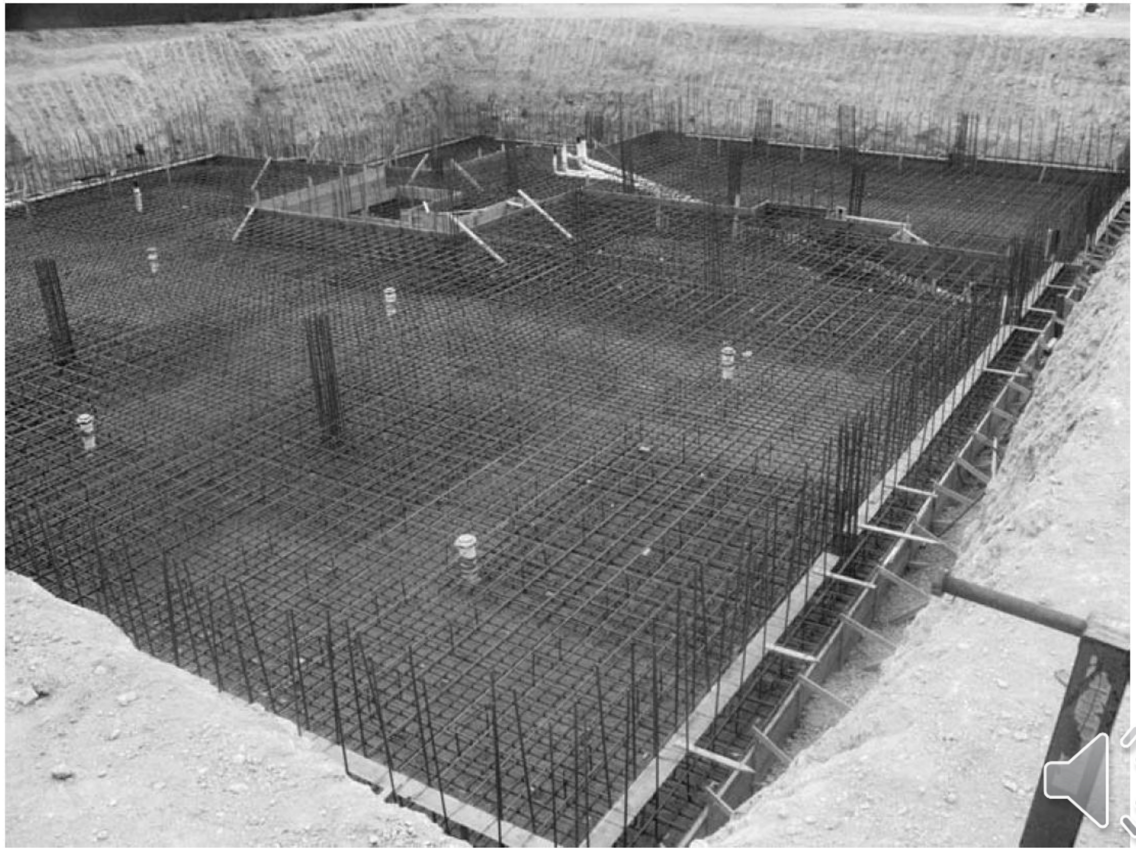
$$\Delta s_{c(g)} = 162.4 + 15.7 + 5.4 = \mathbf{183.5 \text{ mm}}$$

■

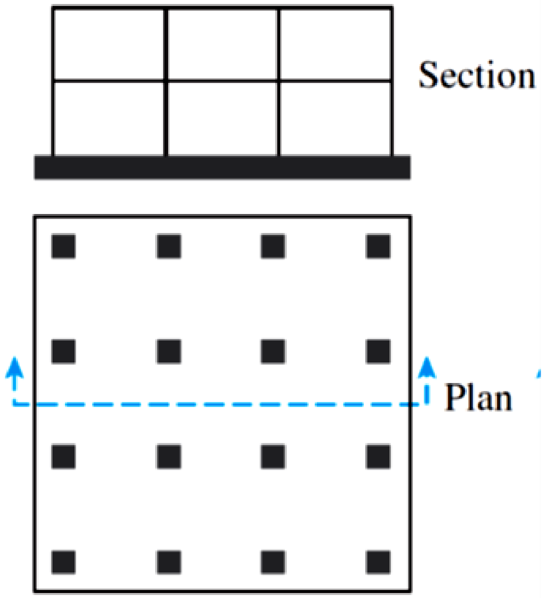
Common Types of Mat Foundations

The mat foundation, which is sometimes referred to as a raft foundation, is a combined footing that may cover the entire area under a structure supporting several columns and walls. Mat foundations are sometimes preferred for soils that have low load-bearing capacities, but that will have to support high column or wall loads.

Under some conditions, spread footings would have to cover more than half the building area, and mat foundations might be more economical.

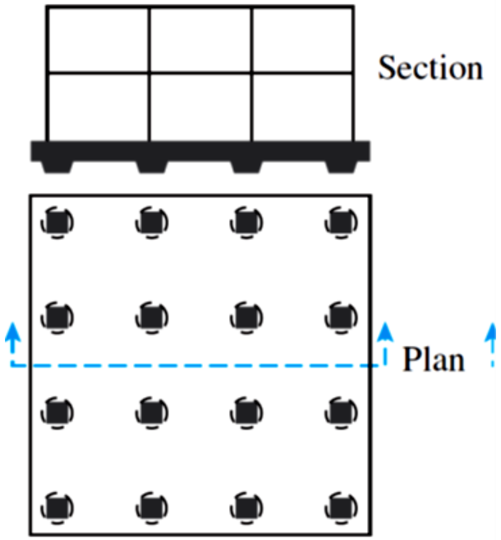


Several types of mat foundations are used currently. Some of the common ones are shown schematically in Figure and include the following:



(a)

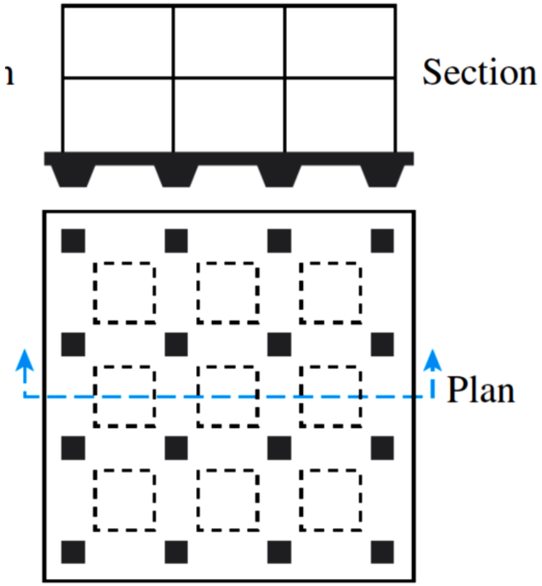
1. Flat plate (Figure a). The mat is of uniform thickness.



(b)

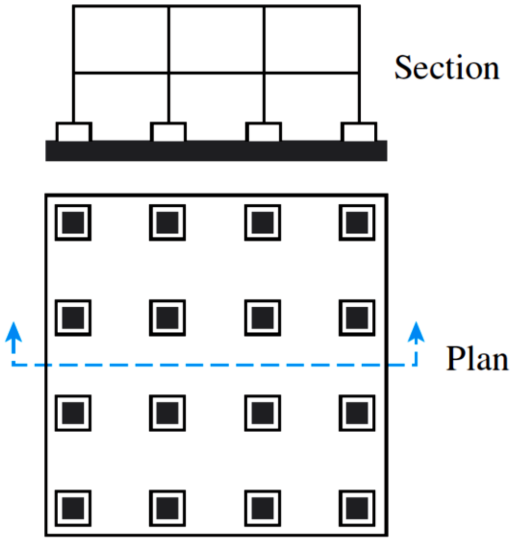
2. Flat plate thickened under columns (Figure b).

3. Beams and slab (Figure c). The beams run both ways, and the columns are located at the intersection of the beams.



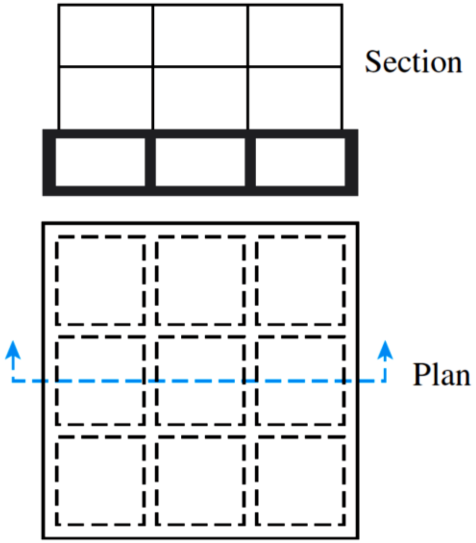
(c)

4. Flat plates with pedestals (Figure d).



(d)

5. Slab with basement walls as a part of the mat (Figure 8.4e). The walls act as stiffeners for the mat.



(e)

Mats may be supported by piles, which help reduce the settlement of a structure built over highly compressible soil. Where the water table is high, mats are often placed over piles to control buoyancy.

Bearing Capacity of Mat Foundations

The gross ultimate bearing capacity of a mat foundation can be determined by the same equation used for shallow foundations, or

$$q_{ult} = cN_c s_c d_c i_c + \bar{q}\, N_q s_q d_q i_q + 0.5\gamma B N_\gamma s_\gamma d_\gamma i_\gamma \qquad \text{……….4.26}$$

The term B in Eq. (4.26) is the smallest dimension of the mat. The net ultimate capacity of a mat foundation is

$$q_{net(u)} = q_{ult} - \bar{q}$$

A suitable factor of safety should be used to calculate the net allowable bearing capacity. For mats on clay, the factor of safety should not be less than 3. For mats constructed over sand, a factor of safety of 3 should normally be used.

The net pressure applied on a foundation (see Figure below) may be expressed as

$$q_{act} = \frac{Q}{A} - \gamma D_f \qquad \ldots\ldots \quad 8.17$$

where

Q = dead weight of the structure and the live load

A= area of the raft

In all cases, $\mathbf{q_{act}}$ should be less than or equal to allowable $\mathbf{q_{net(all)}}$.

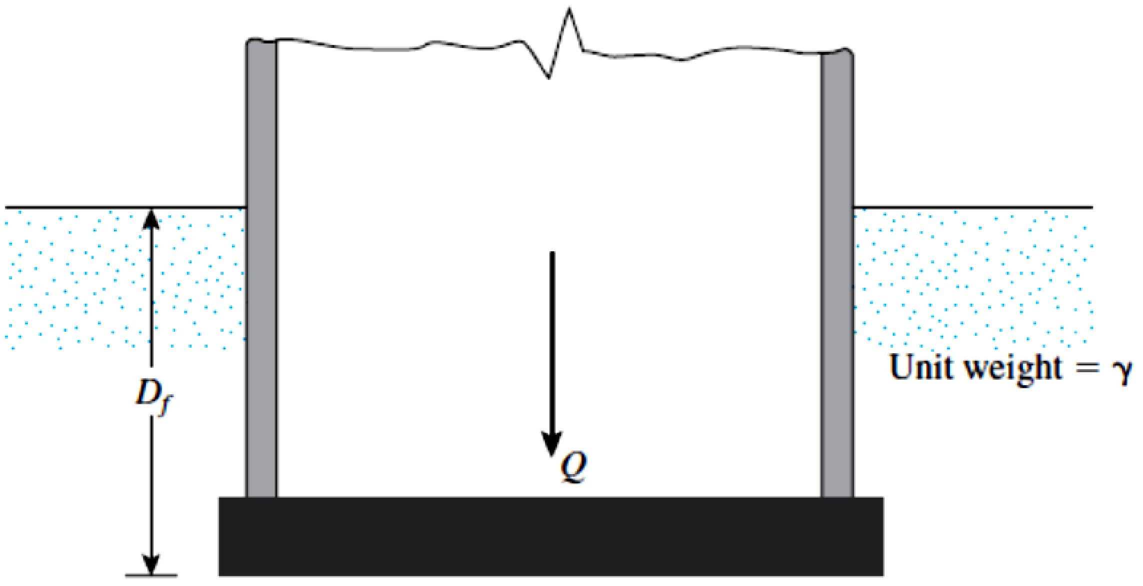


Figure 8.7 Definition of net pressure on soil caused by a mat foundation

For saturated clays with $\emptyset = 0$ and a vertical loading condition where c_u = undrained cohesion. (*Note:* $N_c = 5.14$, $N_q = 1$, and $N_\gamma = 0$.)

The net ultimate bearing capacity of raft foundation is

$$q_{net(u)} = q_u - q = 5.14c_u\left(1 + \frac{0.195B}{L}\right)\left(1 + 0.4\frac{D_f}{B}\right) \quad (8.12)$$

The net allowable bearing capacity for mats constructed over granular soil deposits can be adequately determined from the standard penetration resistance numbers.

$$q_{net}(\text{kN/m}^2) = \frac{N_{60}}{0.08}\left(\frac{B + 0.3}{B}\right)^2 F_d\left(\frac{S_e}{25}\right) \quad [\text{Eq. } (7.39)]$$

where

N_{60} = standard penetration resistance

B = width (m)

$F_d = 1 + 0.33(D_f/B) \leq 1.33$

S_e = settlement, (mm)

When the width B is large, the preceding equation can be approximated as

$$
\begin{aligned}
q_{\text{net}}(\text{kN/m}^2) &= \frac{N_{60}}{0.08} F_d \left(\frac{S_e}{25}\right) \\
&= \frac{N_{60}}{0.08}\left[1 + 0.33\left(\frac{D_f}{B}\right)\right]\left[\frac{S_e(\text{mm})}{25}\right] \\
&\leq 16.63 N_{60}\left[\frac{S_e(\text{mm})}{25}\right]
\end{aligned}
\qquad (8.14)
$$

Example 8.3

Determine the net ultimate bearing capacity of a mat foundation measuring 20 m × 8 m on a saturated clay with $c_u = 85$ kN/m^2, $\phi = 0$, and $D_f = 1.5$ m.

Solution

From Eq. (8.12),

$$q_{\text{net}(u)} = 5.14c_u\left[1 + \left(\frac{0.195B}{L}\right)\right]\left[1 + 0.4\,\frac{D_f}{B}\right]$$

$$= (5.14)(85)\left[1 + \left(\frac{0.195 \times 8}{20}\right)\right]\left[1 + \left(\frac{0.4 \times 1.5}{8}\right)\right]$$

$$= \mathbf{506.3\ kN/m^2}$$

Compensated Foundation

Figure 8.7 and Eq. (8.17) indicate that the net pressure increase in the soil under a mat foundation can be reduced by increasing the depth D_f of the mat. This approach is generally referred to as the compensated foundation design and is extremely useful when structures are to be built on very soft clays. In this design, a deeper basement is made below the higher portion of the superstructure, so that the net pressure increase in soil at any depth is relatively uniform. (See Figure below) From Eq. (8.17) and Figure 8.7, the net average applied pressure on soil is

$$\boldsymbol{q_{act}} = \frac{\boldsymbol{Q}}{\boldsymbol{A}} - \boldsymbol{\gamma D_f}$$

For no increase in the net pressure on soil below a mat foundation, $\boldsymbol{q_{act}}$ should be zero. Thus,

$$D_f = \frac{Q}{A\gamma} \qquad (8.21)$$

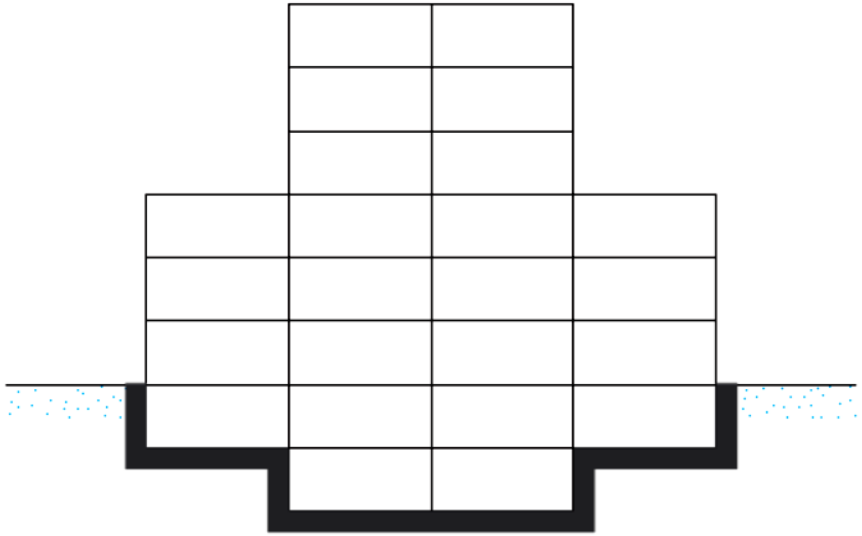


Figure 8.8 Compensated foundation

$$D_f = \frac{Q}{A\gamma} \quad (8.21)$$

This relation for $\boldsymbol{D_f}$ is usually referred to as the depth of a fully compensated foundation. The factor of safety against bearing capacity failure for partially compensated foundations (i.e., $\boldsymbol{D_f} < \boldsymbol{Q}/\boldsymbol{A}\gamma$) may be given as

$$FS = \frac{q_{net(u)}}{q_{act}} = \frac{q_{net(u)}}{\frac{Q}{A} - \gamma\, D_f}$$

where $q_{net(u)}$ = net ultimate bearing capacity.

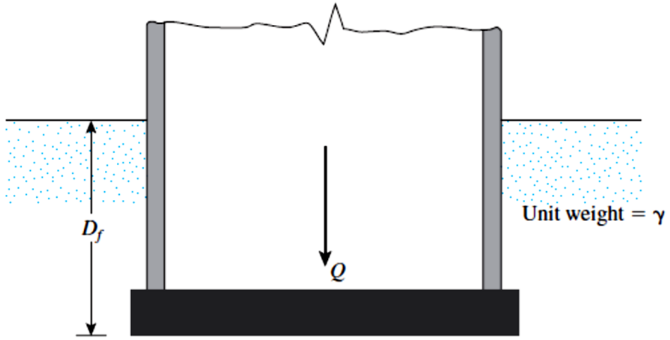


Figure 8.7 Definition of net pressure on soil caused by a mat foundation

$$FS = \frac{q_{net(u)}}{q_{act}} = \frac{q_{net(u)}}{\frac{Q}{A} - \gamma\, D_f}$$

Example 8.5

The mat shown in Figure 8.7 has dimensions of 20 m × 30 m. The total dead and live load on the mat is 110 MN. The mat is placed over a saturated clay having a unit weight of 18 kN/m^3 and c_u = 140 kN/m^2. Given that D_f = 1.5 m, determine the factor of safety against bearing capacity failure.

Solution

From Eq. (8.23), the factor of safety

$$\text{FS} = \frac{5.14c_u\left(1 + \frac{0.195B}{L}\right)\left(1 + 0.4\frac{D_f}{B}\right)}{\frac{Q}{A} - \gamma D_f}$$

We are given that c_u = 140 kN/m^2, D_f = 1.5 m, B = 20 m, L = 30 m, and γ = 18 kN/m^3. Hence,

$$\text{FS} = \frac{(5.14)(140)\left[1 + \frac{(0.195)(20)}{30}\right]\left[1 + 0.4\left(\frac{1.5}{20}\right)\right]}{\left(\frac{110{,}000\ \text{kN}}{20 \times 30}\right) - (18)(1.5)} = \mathbf{5.36}$$

MAT SETTLEMENTS

Mat foundations are commonly used where settlements may be a problem, for example, where a site contains erratic deposits or lenses of compressible materials, suspended boulders, etc. The settlement tends to be controlled via the following:

1. Use of a larger foundation to produce lower soil contact pressures.
2. Displaced volume of soil (flotation effect); theoretically if the weight of excavation equals the combined weight of the structure and mat, the system "floats" in the soil mass and no settlement occurs.
3. Bridging effects attributable to mat rigidity and contribution of superstructure rigidity to the mat.
4. Allowing somewhat larger settlements, say, 50 instead of 25 mm.

A problem of more considerable concern is differential settlement. Again the mat tends to reduce this value. Mat continuity results in a somewhat lower assumed amount of differential settlement relative to the total expected settlement versus a spread footing as follows

Foundation type	Expected maximum settlement, mm	Expected differential settlement, mm
Spread	25	20
Mat	50	20

Computer methods that incorporate frame-foundation interaction can allow one to estimate both total and differential settlements. The total settlements will be only as good as the soil data.