Fluid Mechanics

Lecture – 1

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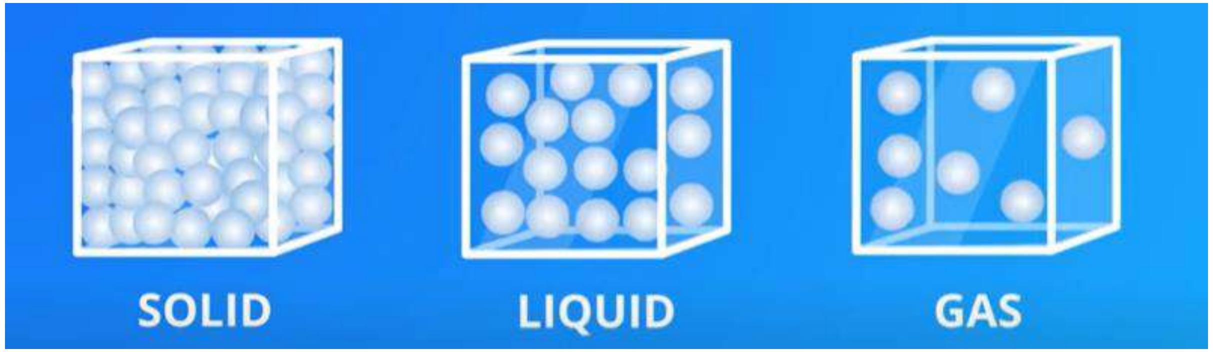
Definition of Fluid

- **A fluid** is a substance which deforms **CONTINUOUSLY** when subjected to external shearing force.
- **A fluid** is a substance which is capable of flowing.
- **Or**, it has no definite shape of its own, but conforms to the shape of the containing vessel.

Solid, and Fluid (Liquid & Gas)

- In ***solids***, the molecules are **very closely spaced** whereas in ***liquids*** (such as water, oil, and gasoline) the spacing between the different molecules is **relatively large** and in ***gases*** (such as CO2 and methane) the spacing between the molecules is still **large**.



Fluid characteristics _ Ideal fluid

An **ideal fluid** is one which has no viscosity and surface tension and is incompressible. In true sense no such fluid exists in nature.

- However fluids which have low viscosities such as water and air can be treated as ideal fluids under certain conditions.
- The assumption of ideal fluids helps in simplifying the mathematical analysis.

Fluid Mechanics Classification

The fluid mechanics may be divided into three parts:

- **Fluid Statics.** The study of incompressible fluids under static conditions is called hydrostatics, and that dealing with the compressible static gases is termed as aerostatics.
- **Fluid Dynamics.** It deals with the relations between velocities, accelerations of fluid *with the forces or energy causing them*.
- **Fluid Kinematics.** It deals with the velocities, accelerations and the patterns of flow only. **Forces or energy** causing velocity and acceleration are ***not* dealt** under this heading.

Fluid Mechanics

Lecture – 2

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Dimensions and Units

Table 1.1 Basic Dimensions and Their Units

Quantity	Dimension	SI Units	English Units
Length l	L	meter m	foot ft
Mass m	M	kilogram kg	slug slug
Time t	T	second s	second sec
Temperature T	Θ	kelvin K	Rankine R
Plane angle		radian rad	radian rad

K = $°C + 273.15$

°C is Celsius

F = $\frac{9}{5}$ °C + 32

F is Fahrenheit

Important terms and relationships

1. **Velocity (v)**

V = Length / Time OR v = L / T OR $V = L\ T^{-1}$

m / sec, ft / sec

2. **Discharge or Flowrate (Q)**

Q = Volume / Time (Q = Vol. / T) (Volume also can be denoted by V̶)

Q = Velocity . Area (Q = v . A)

$Q = L^3\ T^{-1}$

m^3 / sec , cm^3 / sec

Important terms and relationships

3. Acceleration (a)

a = Velocity / Time OR a = v / T OR $a = V\,T^{-1}$

but $V = L\,T^{-1}$ so $a = L\,T^{-2}$

m / sec^2 , m / sec^2

4. Force (F)

Force = Mass × Acceleration (F = M × a)

$F = M\,L\,T^{-2}$

N = kg . m / sec^2

Dyne = gm . cm / sec^2

Ibf = slug . ft / sec^2

Kg_w = slug . ft / sec^2

Important terms and relationships

5. Pressure (P) , Shear stress (τ)

P = Force / Area OR P or τ = F / A

P or τ = $F L^{-2}$ = $M L^{-1} T^{-2}$ (Prove it)

Pa = N / m^2 (Pascal)

psi = Ib / $inch^2$ (Pound per square inch)

psf = Ib / $inch^2$ (Pound per square foot)

1 psi = 144 psf

Important terms and relationships

6. Momentum

Momentum = Mass × Velocity

Momentum = M LT^{-1}

- $Momentum = M\,\frac{L}{T}$
- $Momentum = M\,\frac{L}{T} \times \frac{\boldsymbol{T}}{\boldsymbol{T}}$
- $Momentum = (M\,\frac{L}{T^2}) \times \boldsymbol{T}$

$Momentum = F \times \boldsymbol{T} =$ **Force × Time**

Important terms and relationships

7. Work (W)

Work = Force × Distance

W = F L $= M L^2 T^{-2}$ **(Prove it)**

If the force is being exerted at an angle θ to the displacement, the work done is $W = FL \cos\theta$.

The unit of Work (W) **is** Joule (**J**)

One Joule is equivalent to one Newton of force causing a displacement of one meter.

8. Power ($\dot{W}$)

$\dot{W}$ = Work / Time

$\dot{W}$ = (Force × Length) / Time

$\dot{W} = F L T^{-1}$ $= M L^2 T^{-3}$ **(Prove it)**

Watt (W) = N.m / sec or (J / sec), Horse power HP = Watt / 735

Also, Power = Force × Velocity

Some conservation units

Length (L)

1 ft = 12 inch.

1 yard = 3 ft

1 m = 100 cm

1 inch = 2.54 cm

1 mile = 1760 yard

Volume (Vol.)

1 m^3 = 1000 liter

1 gallon = 3.785 liter

Gravitational acceleration (g)

g = 9.81 m/sec^2

g = 32.2 ft/sec^2

Force (F)

1 N = 10^5 dyne

1 Kg_w = 9.81 N

1 Ibf = 4.45 N

1 Kg_w = 2.205 Ib

Mass (M)

1 slug = 14.594 kg

1 pound = 0.4536 kg

Fluids AND THEIR PROPERTIES

- **Density,**
- **Specific weight,**
- **Viscosity,**
- **Compressibility,**
- **Surface tension,**
- **Capillarity,**
- **Cohesion,**
- **Adhesion etc.**

FLUIDS AND THEIR PROPERTIES

1. Density

a. Mass density (ρ)

Mass density *(also known as **specific mass**, or **Density**)*

*Defined as mass per unit volume (**mass/Volume**), at a standard temperature and pressure.*

It is usually denoted by ***ρ***.

Units is **kg/m³** (in SI Units)

FLUIDS AND THEIR PROPERTIES

1. Density

b. Weight density

Weight density *(also known as* ***Specific weight****)*

*Defined as weight per unit volume (****weight/Volume****), at a standard temperature and pressure.*

It is usually denoted by γ. Mathematically is (ρg)

Units is **kN/m³** (in SI Units),

The specific weight of water is taken as follows:

In S.I. Units: γ = 9.81 kN/m³ (or 9.81× 10–6 N/mm³)

1. Density

c. Specific volume

*Defined as volume per unit mass (**Volume/mass**)*

It is usually denoted by ***v***, mathematically is (**1/*ρ***)

Unit is **m³/kg** (in SI Units)

2. Specific gravity

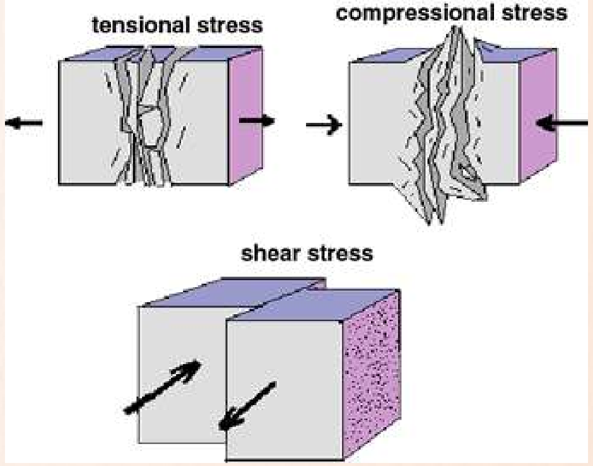
Specific gravity defined as is the ratio of the specific weight of the liquid to the specific weight of a standard fluid.

$$\text{Specific gravity} = \frac{\text{Specific weight of liquid}}{\text{Specific weight of pure water}} = \frac{w_{liquid}}{w_{water}}$$

It is usually denoted by **s.g** or **sp.gr** or **S**

It is **dimensionless** and has no units.

FLUIDS AND THEIR PROPERTIES
3. VISCOSITY (μ)



- *Viscosity* may be defined as the *property of a fluid which determines its resistance to shearing stresses.*

- It is a measure of the internal fluid friction which causes resistance to flow (shearing stresses between the moving layers of fluid)

- Viscosity of fluids is due to cohesion and interaction between particles.

FLUIDS AND THEIR PROPERTIES

Factors Effecting Viscosity (μ)

Temperature

- The viscosity of *liquids* ($\mu_{liquids}$) *decreases* with *increase in temperature (***T***)*. But, the viscosity of *gases* (μ_{gases}) increases with *increase in temperature (***T***).*

This is due to the reason that in *liquids* the shear stress is due to the inter-molecular cohesion which *decreases* with increase of temperature.

As **T** $\uparrow$ **Cohesive force** $\downarrow$ then $\mu_{liquids}$ $\downarrow$

- In gases the inter-molecular cohesion is negligible and the shear stress is due to exchange of momentum of the molecules. The molecular activity increases with rise in temperature and so does the viscosity of gas.

- As **T** $\uparrow$ **Cohesive force** (**Negligible**), **Exchange of momentum of the molecules** $\uparrow$

then μ_{gases} $\uparrow$

FLUIDS AND THEIR PROPERTIES

3. VISCOSITY

$F \propto V$

$F \propto \frac{1}{y}$

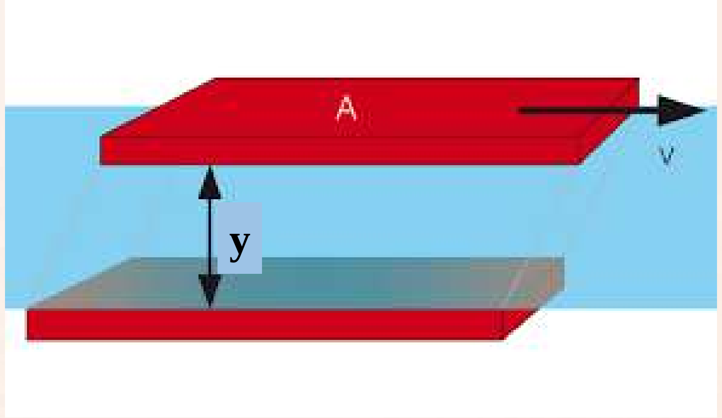
$F \propto A$

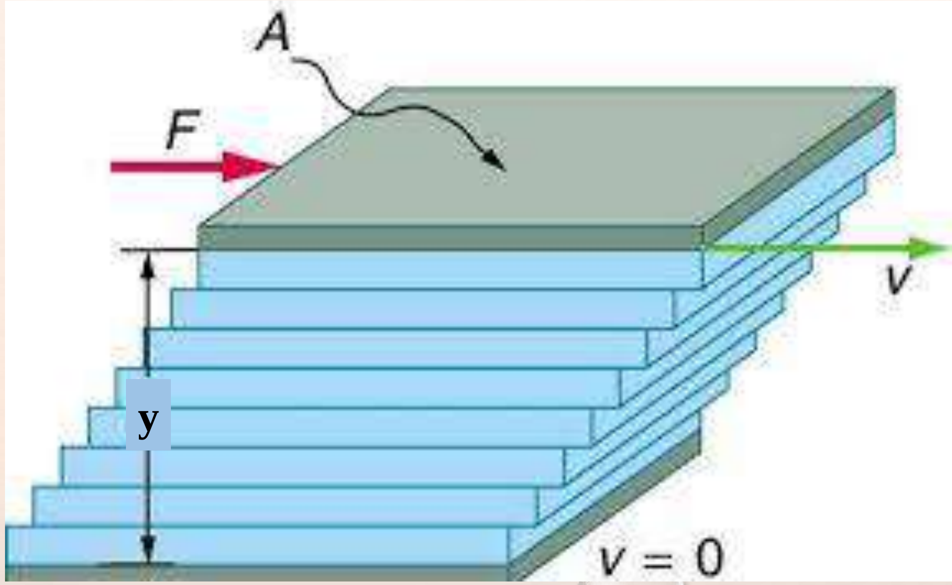
$F \propto \frac{V}{y} A$

$\mathbf{F} = \mu \frac{V}{y} A$

$\frac{F}{A} = \mu \frac{V}{y}$

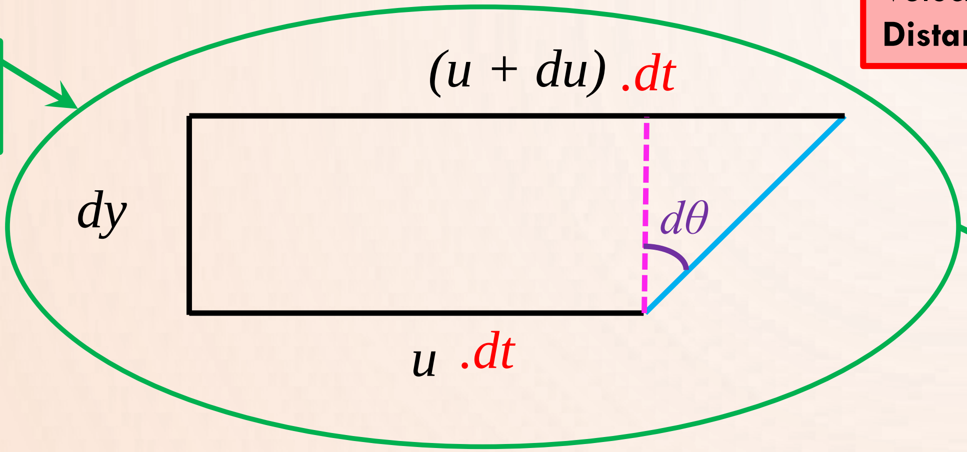
$\tau = \mu \frac{V}{y}$





Newton's Law Of Viscosity

For very small layer



Velocity = Distance / Time
Distance = Velocity . Time

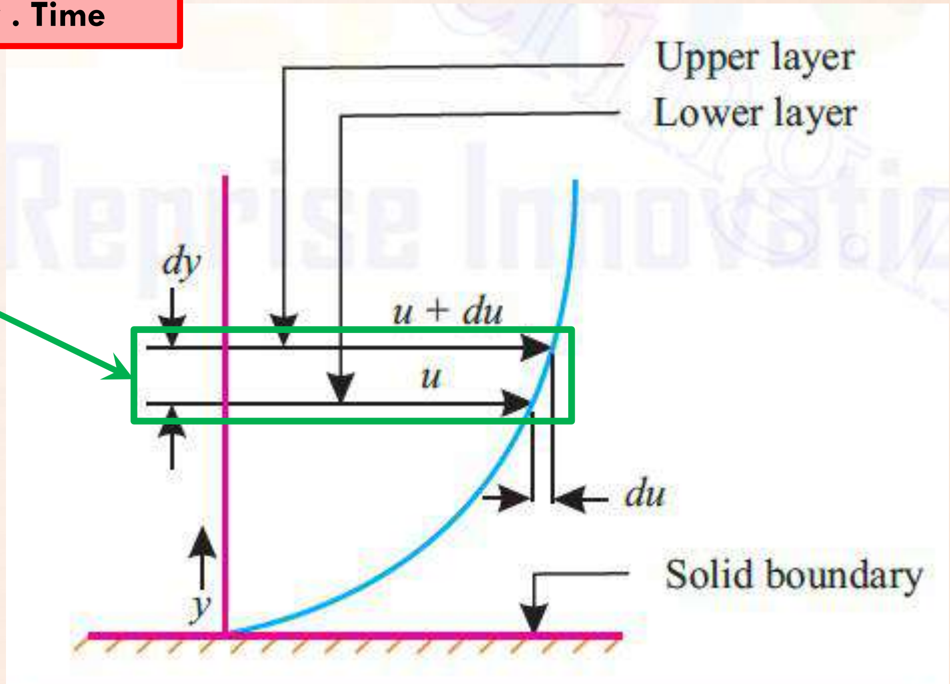


Fig. 1.1 Velocity variation near a solid boundary.

$$Tan\ (d\theta) = \frac{(u+du).dt - u.dt}{dy}$$

$$Tan\ (d\theta) = \frac{du\ .dt}{dy}$$

- For small angles $Tan\ \theta = \theta$
- $d\theta = \frac{du.dt}{dy}$
- $d\theta$ is angular deformation

FLUIDS AND THEIR PROPERTIES / 3.VISCOSITY

Newton's Law Of Viscosity

$$\frac{d\theta}{dt} = \frac{du}{dy}$$

$\frac{d\theta}{dt}$: Rate of angular deformation (Or Rate of shear stress)

$\frac{du}{dy}$: Velocity gradient (Change in velocity w.r.t. distance)

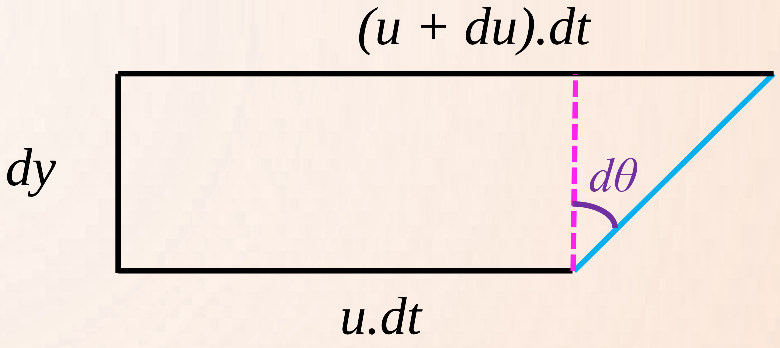
Shear force per unit area on a surface is proportional to the rate of angular deformation.

$$\frac{F}{A} \propto \frac{d\theta}{dt}$$

$$\tau \propto \frac{d\theta}{dt}$$

$$\tau \propto \frac{du}{dy}$$

- $\boldsymbol{\tau = \mu \frac{du}{dy}}$



- ***μ is Dynamic viscosity***

$\boldsymbol{\mu = \tau \frac{dy}{du}} = \frac{N}{m^2} \frac{m}{\frac{m}{sec}} =$ pa . sec (SI units)

1 pa.sec (SI units) = 10 Poise (CGS units)

10^{-1} pa.sec = 1 poise

$\boldsymbol{\mu}_{water} = 8.90 \times 10^{-4}$ pa·s at about 25 °c

3. VISCOSITY

- **Kinematic viscosity ν**
- $\nu = = \frac{\mu}{\rho}$
- ν in SI units: m^2/sec (prove it)
- ν in British units: ft^2/sec
- 1 stoke = 10^{-4} m^2/sec
- $\nu_{water}=10^{-6}$ m^2/sec = 10^{-2} stokes

3. VISCOSITY - EXAMPLES

EXAMPLE 1.3. *A plate 0.05 mm distant from a fixed plate moves at 1.2 m/s and requires a force of 2.2 N/m^2 to maintain this speed. Find the viscosity of the fluid between the plates.*

Solution: Velocity of the moving plate, $u = 1.2$ m/s

Distance between the plates, $dy = 0.05 \text{ mm} = 0.05 \times 10^{-3}$ m

Force on the moving plate, $F = 2.2 \text{ N/m}^2$

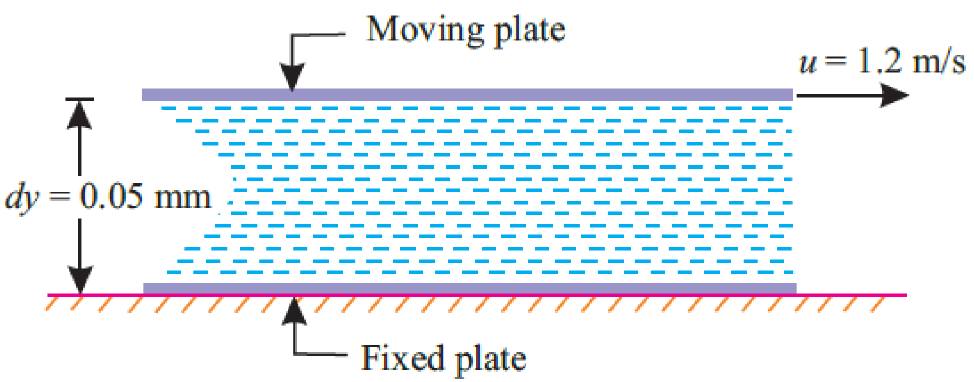


Fig. 1.3

Viscosity of the fluid, μ:

We know, $\tau = \mu . \frac{du}{dy}$

where τ = shear stress or force per unit area = 2.2 N/m^2,

du = change of velocity = $u - 0 = 1.2$ m/s and

dy = change of distance = 0.05×10^{-3}m.

$$\therefore \quad 2.2 = \mu \times \frac{1.2}{0.05 \times 10^{-3}}$$

$$\mu = \frac{2.2 \times 0.05 \times 10^{-3}}{1.2} = 9.16 \times 10^{-5} \text{N.s/m}^2$$

3. VISCOSITY - EXAMPLES

Example 1.4. *A plate having an area of 0.6 m² is sliding down the inclined plane at 30° to the horizontal with a velocity of 0.36 m/s. There is a cushion of fluid 1.8 mm thick between the plane and the plate. Find the viscosity of the fluid if the weight of the plate is 280 N.*

Solution: Area of plate, $A = 0.6\ m^2$

Weight of plate, $W = 280$ N

Velocity of plate, $u = 0.36$ m/s

Thickness of film, $t = dy = 1.8\ mm = 1.8 \times 10^{-3}\ m$

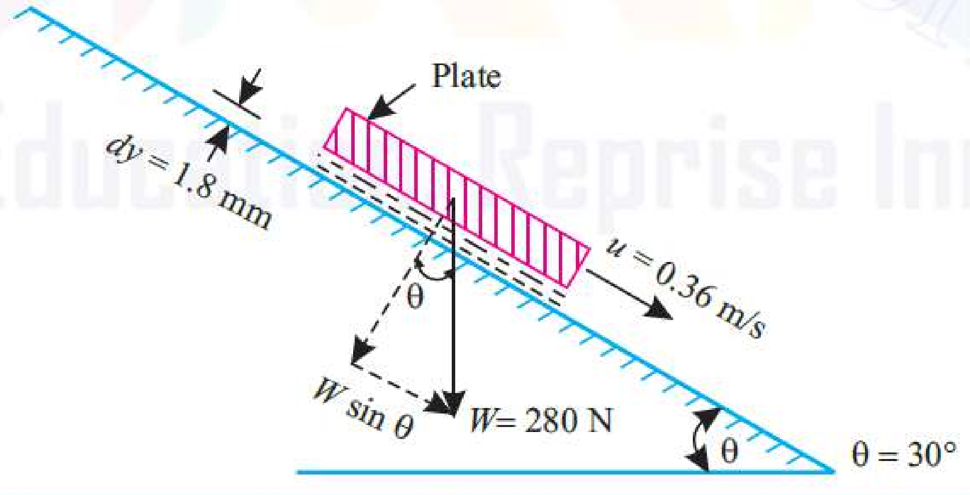


Fig. 1.4

Viscosity of the fluid, μ:

Component of W along the plate $= W \sin\theta = 280 \sin 30° = 140$ N

$$\tau = \frac{F}{A} = \frac{140}{0.6} = 233.33 N/m^2$$

We know,

$$\tau = \mu \cdot \frac{du}{dy}$$

Where, du = change of velocity $= u - 0 = 0.36$ m/s

$dy = t = 1.8 \times 10^{-3}$ m

$$\therefore \quad 233.33 = \mu \times \frac{0.36}{1.8 \times 10^{-3}}$$

$$\mu = \frac{233.33 \times 1.8 \times 10^{-3}}{0.36} = 1.166 N.s/m^2$$

Fluid Mechanics

Lecture – 3

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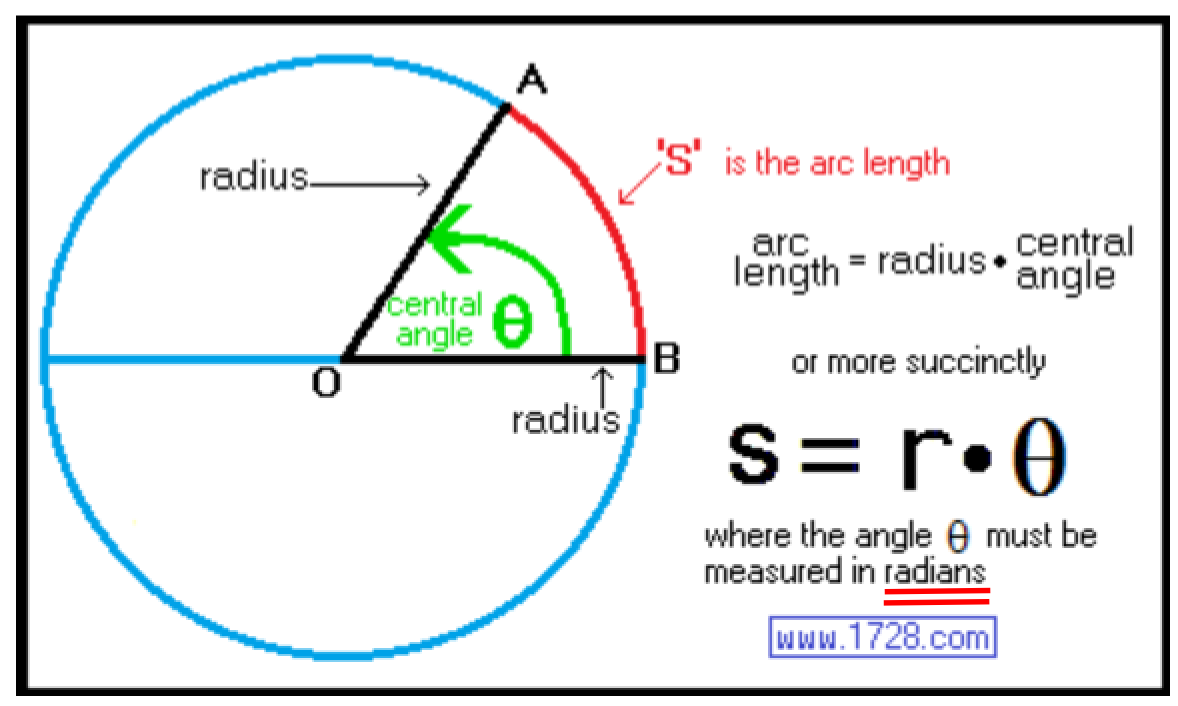
Important terms and relationships

10. Arc Length Formula



arc length = radius x angle

If θ is measured in degrees then

$$\text{arc length} = \frac{\theta}{360^\circ} \times 2\pi r$$

If θ is measured in radians then

$$\text{arc length} = \theta r$$

Important terms and relationships

11. **Linear velocity or speed (v)** (*v is a linear displacement per unit time*)

$v = \Delta x / \Delta t$ (*m/sec, ft/sec, km/hr, mph, etc.*)

12. **Angular velocity or speed (ω)** (ω is an angular displacement per unit time)

$\omega = \Delta \theta / \Delta t$ (*rad./sec (SI units), °/sec, rpm, etc.*)

(1 **rpm** = $2\pi/60$ rad/s.)

$\omega = 2\pi N/60$ (Where N is number of rpm)

Counter-clock wise (CCW) rotation is Positive angular velocity (CCW is +Ve ω)
Clockwise (CW) rotation is Negative angular velocity (CW is -Ve ω)

13. **Relation between Angular and Linear velocity**

$v = r\,\omega$ (*Where r is radius*)

Important terms and relationships

Relation between Angular and Linear velocity

$v = r\,\omega$ (*Where r is radius*) How?

$v = S / \Delta t$ (*where S is arc length*) -------Eq. 1

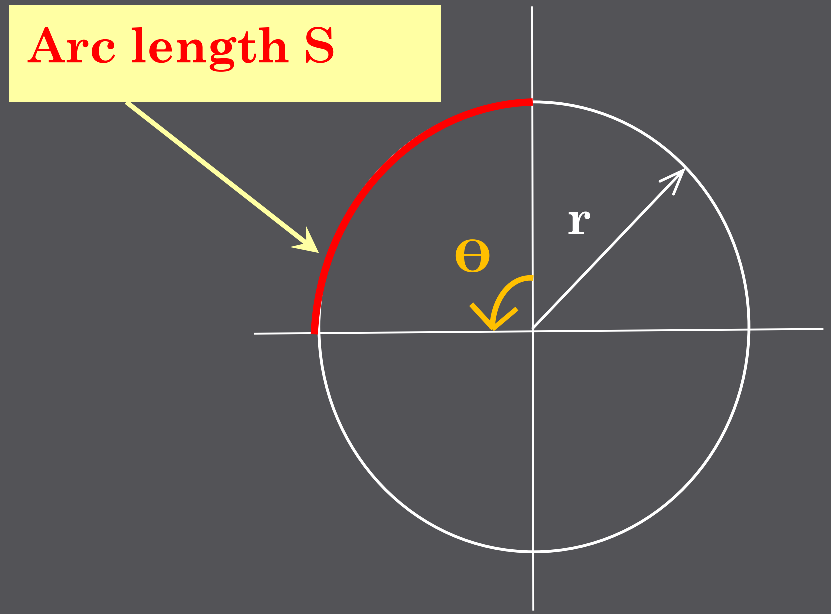
$S = r\,\Delta\theta$ ----------- Eq. 2

Substituting Eq. 2 into Eq. 1 yields:

$v = r\,\Delta\theta / \Delta t$

But $\omega = \Delta\theta / \Delta t$ -------- Eq.3

$\therefore\ v = r\,\omega$



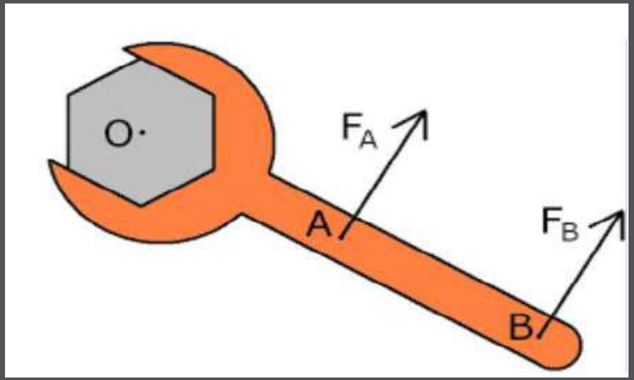
Important terms and relationships

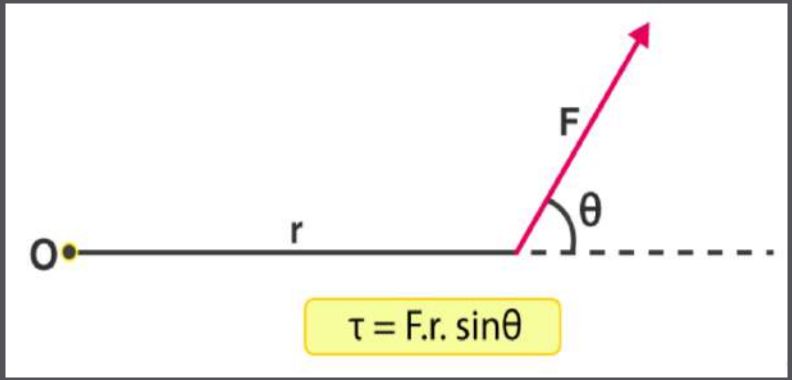
14. <u>Angular acceleration (α)</u> (**α** is a change in angular velocity per unit time)

$\alpha = \Delta \omega / \Delta t$ (rad/sec^2)

15. <u>Torque (M)</u> (M or T is a measure of how much a force acting on an object causes that object to rotate.)

$M = F\ r \sin \Theta$ (**N.m**, *Where **r** is radius*)





3. VISCOSITY

Two concentric cylinders _ Linear movement

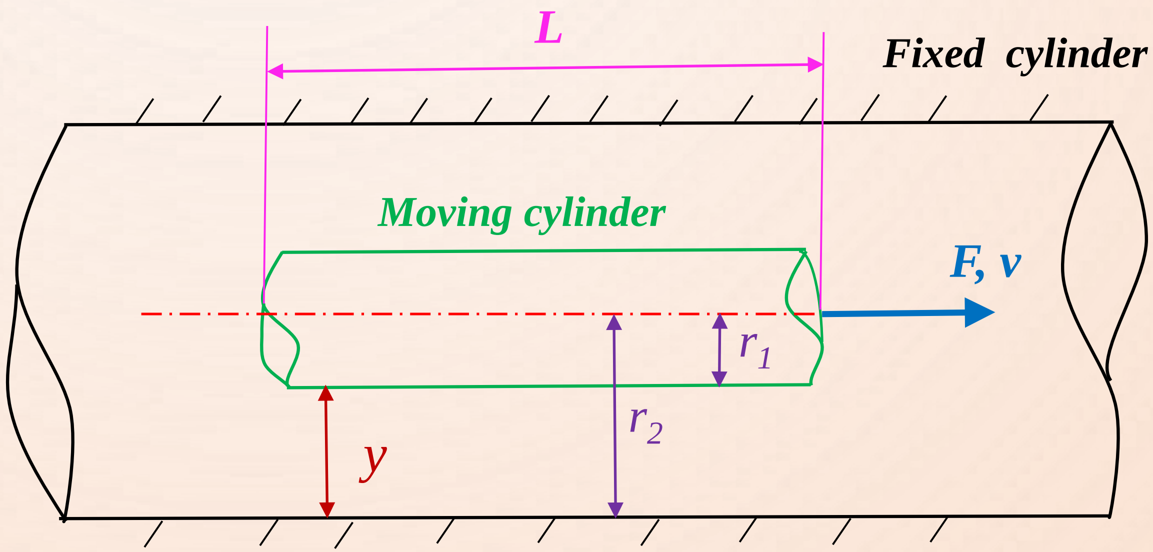
1. Inner cylinder moving with uniform linear velocity

$$F = \mu \frac{v}{y} A$$

$$A = 2 \pi r_1 L$$

$$y = r_2 - r_1$$

$$F = \mu \frac{v}{r_2 - r_1} 2 \pi r_1 L$$



3. VISCOSITY - EXAMPLES

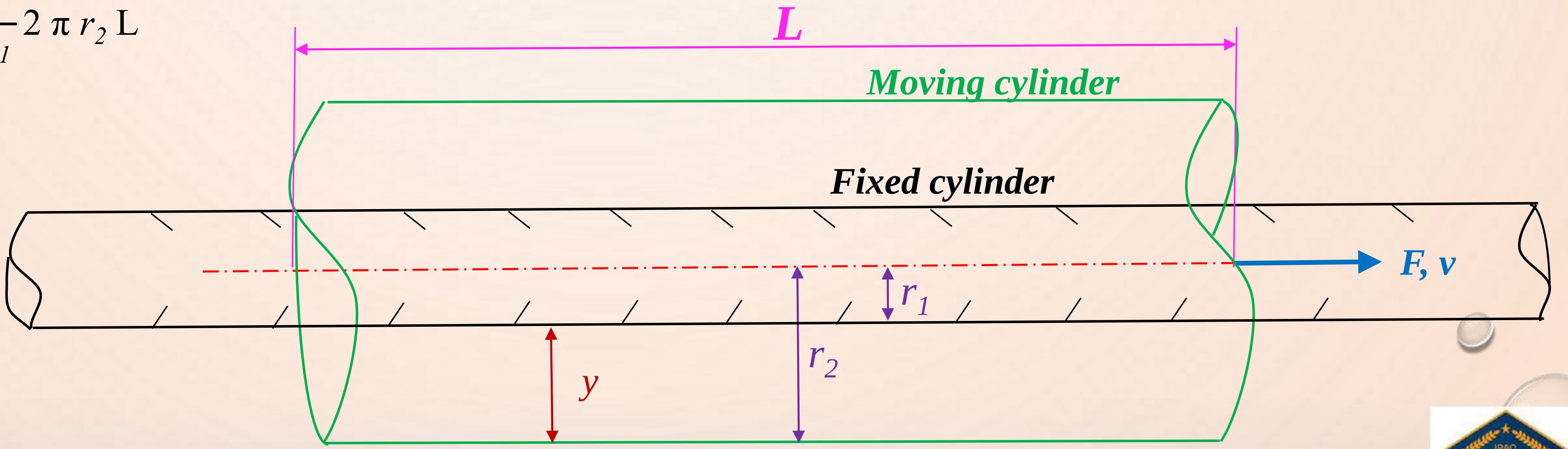
Two concentric cylinders _ Linear movement
2. Outer cylinder moving with uniform linear velocity while inner cylinder fixed

$$F = \mu \frac{V}{y} A$$

$$A = 2 \pi r_2 L$$

$$y = r_2 - r_1$$

$$F = \mu \frac{V}{r_2 - r_1} 2 \pi r_2 L$$



Fluid Mechanics

Lecture – 4

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FLUIDS AND THEIR PROPERTIES

4. Cohesion: Cohesion means intermolecular attraction between *molecules of the same liquid*. Cohesion is a tendency of the liquid to remain as one *assemblage of particles*.

5. Adhesion: *Adhesion* means *attraction between the molecules of a liquid and the molecules of a solid boundary surface in contact with the liquid*. This property enables a liquid to stick to *another body*.

6. Surface Tension

Surface tension is caused by the force of cohesion at the free surface.

At liquid–air interfaces, surface tension results from the greater attraction of liquid molecules to each other (due to cohesion) than to the molecules in the air (due to adhesion).

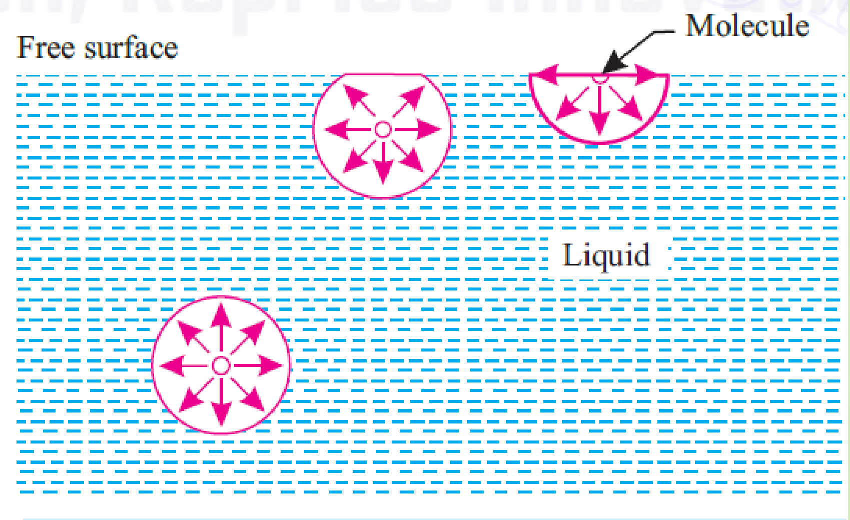
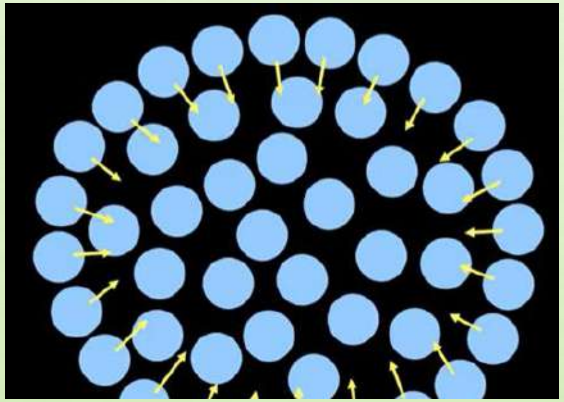
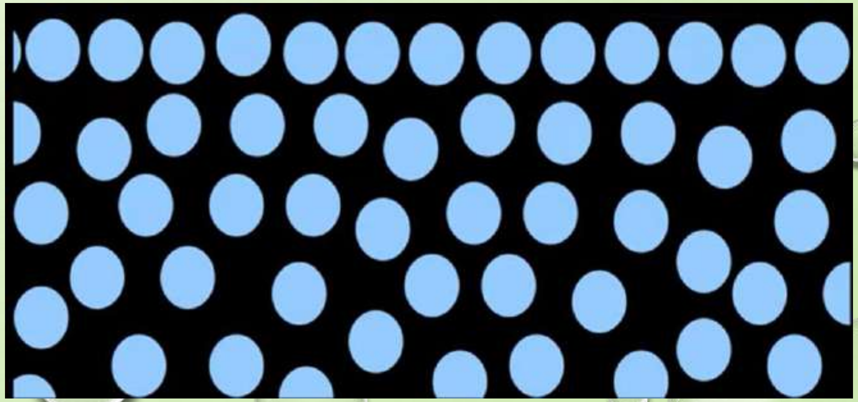


Fig. 1.18





6. Surface Tension

Pressure Inside a Water Droplet, Soap Bubble and a Liquid Jet

Case I. Water droplet:

Let, p = Pressure inside the droplet above outside pressure (*i.e.*, $\Delta p = p - 0 = p$ above atmospheric pressure)

d = Diameter of the droplet and

σ = Surface tension of the liquid. $\frac{F}{L}$

From free body diagram (Fig. 1.19 *d*), we have:

(*i*) Pressure force $= p \times \frac{\pi}{4} d^2$, and

(*ii*) Surface tension force acting around the circumference $= \sigma \times \pi d$.

Under equilibrium conditions these two forces will be equal and opposite,

i.e., $$p \times \frac{\pi}{4} d^2 = \sigma \times \pi d$$

$$\therefore \quad p = \frac{\sigma \times \pi d}{\frac{\pi}{4} d^2} = \frac{4\sigma}{d}$$

The equation above shows that, $\mathbf{P \propto \frac{1}{d}}$

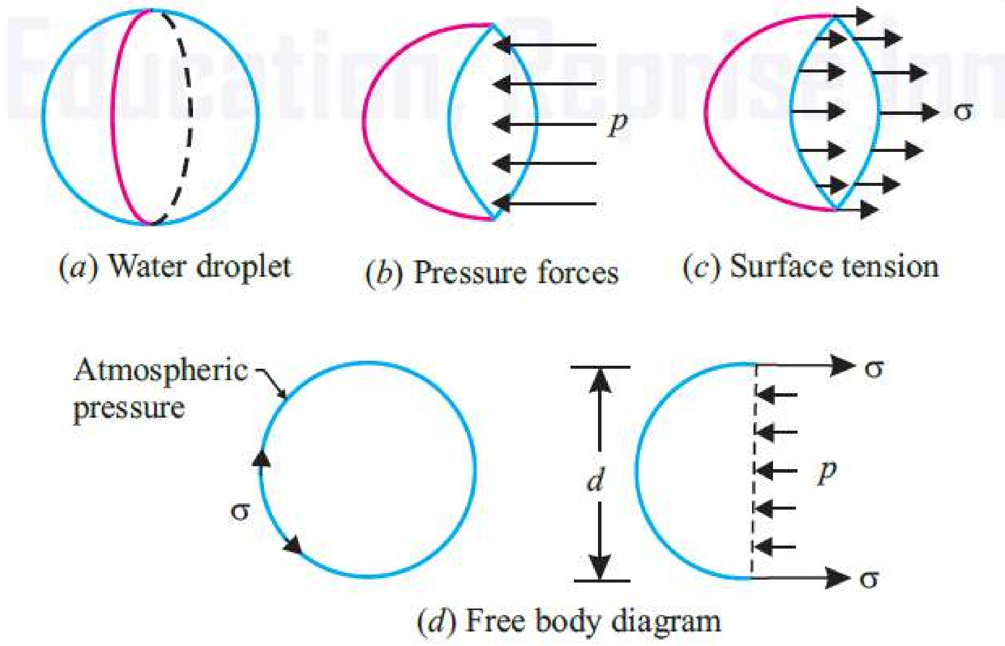


Fig. 1.19. Pressure inside a water droplet.

6. Surface Tension

Case II. Soap (or hollow) bubble:

Soap bubbles have two surfaces on which surface tension σ acts.

From the free body diagram (Fig. 1.20), we have

$$p \times \overleftarrow{\frac{\pi}{4}} d^2 = 2 \times (\vec{\sigma} \times \pi d)$$

$$\therefore \quad p = \frac{2\sigma \times \pi d}{\frac{\pi}{4} d^2} = \frac{8\sigma}{d} \qquad \ldots(1.18)$$

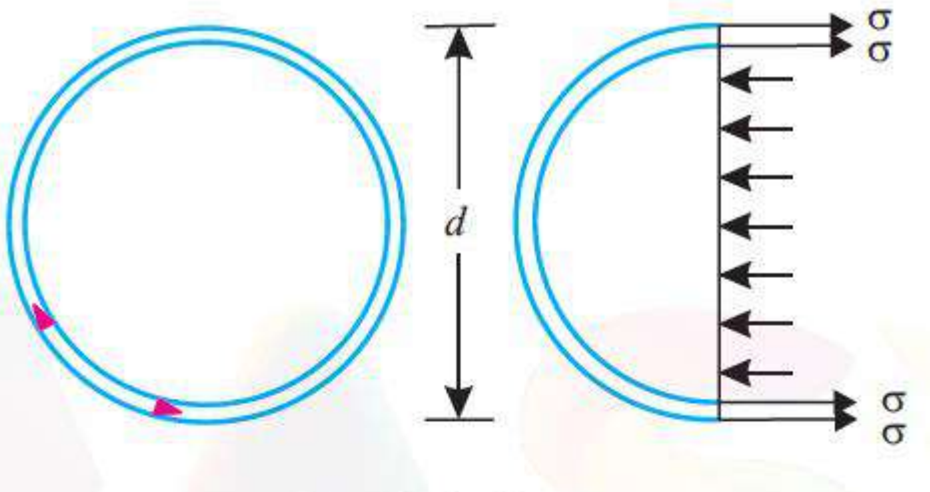


Fig. 1.20. Pressure inside a soap bubble.

6. Surface Tension

Case III. A Liquid jet:

Let us consider a cylindrical liquid jet of diameter d and length l.

Fig. 1.21 shows a semi-jet.

Pressure force $= p \times l \times d$

Surface tension force $= \sigma \times 2l$

Equating the two forces, we have:

$$p \times l \times d = \sigma \times 2l$$

$$p = \frac{\sigma \times 2l}{l \times d} = \frac{2\sigma}{d}$$

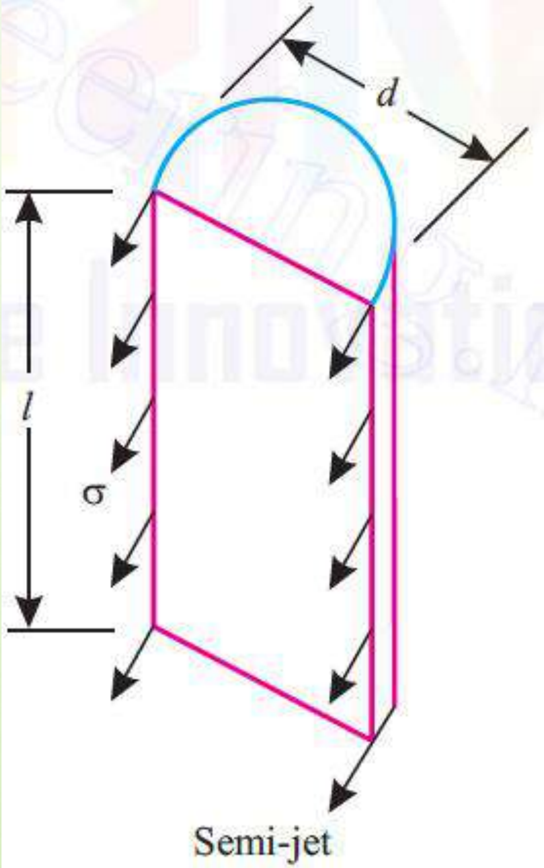


Fig. 1.21. Forces on liquid jet.

7. Capillarity

- ***Capillarity*** *is a phenomenon by which a liquid (depending upon its specific gravity) rises into a thin glass tube above or below its general level. This phenomenon is due to the combined effect of Cohesion and Adhesion of liquid particles.*

Fig. 1.22 shows the phenomenon of rising water in the tube of *smaller* diameters.

Let,
- d = Diameter of the capillary tube,
- θ = Angle of contact of the water surface,
- σ = Surface tension force for unit length, and
- w = Weight density (ρg).

Now, upward surface tension force (lifting force) = weight of the water column in the tube (*gravity force*)

$$\pi d.\sigma \cos\theta = \frac{\pi}{4} d^2 \times h \times w$$

$w = \gamma$

usually use the symbol γ to refer to the weight density

$$\therefore \quad h = \frac{4\sigma\cos\theta}{wd}$$

For water and glass: $\theta \simeq 0$.

Hence the capillary rise of water in the glass tube,

$$h = \frac{4\sigma}{wd} \quad \text{...(1.21)}$$

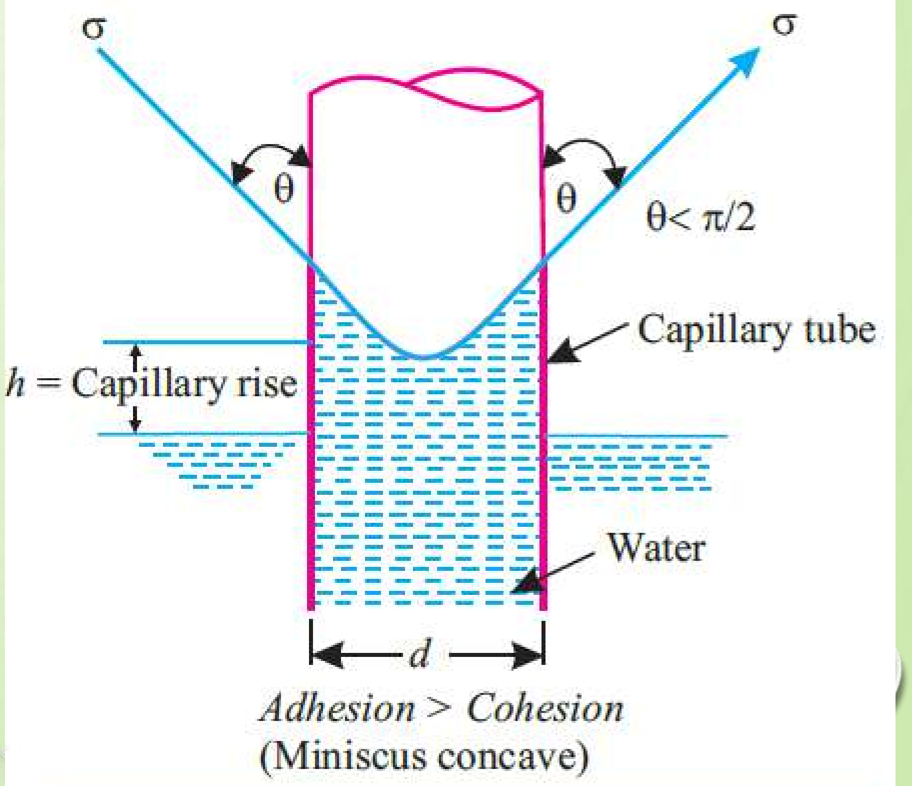


Fig. 1.22. Effect of capillarity.

FLUIDS AND THEIR PROPERTIES

7. Capillarity

In case of mercury there is a capillary depression as shown in Fig. 1.23, and the angle of depression is $\theta \simeq 140°$. (It may be noted that here $\cos\theta = \cos 140°$ $= \cos(180 - 40°) = -\cos 40°$, therefore, h is *negative* indicating capillary depression).

$$\cos(180° - x) = -\cos(x)$$

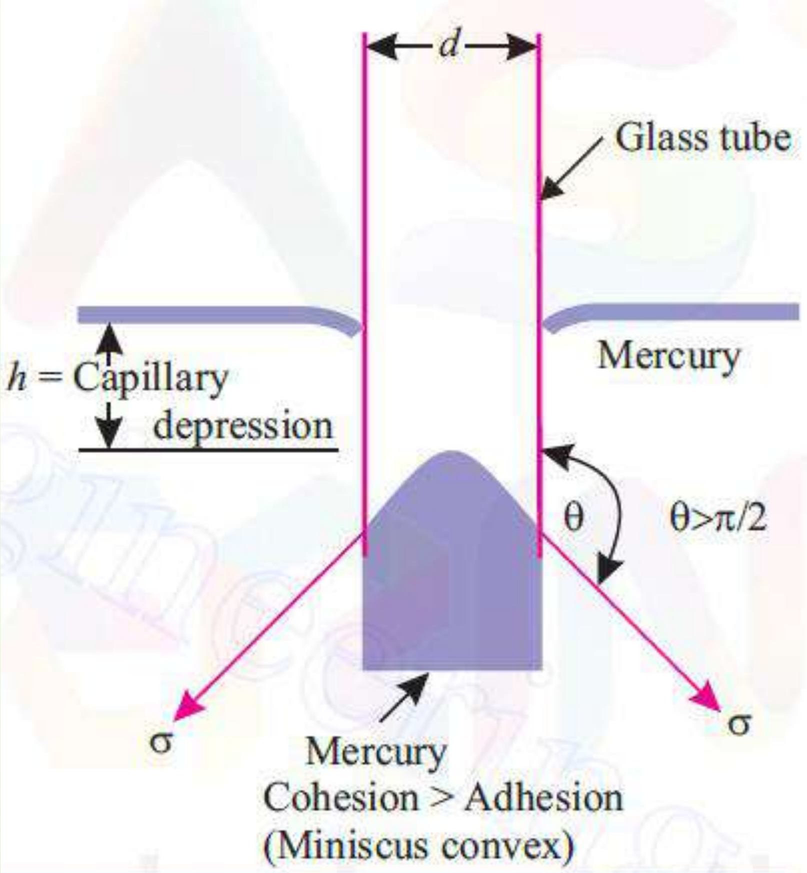


Fig. 1.23

Pressure Measurement

When a fluid is contained in a vessel, it exerts force at all points on the sides and bottom and top of the container.
The *force per unit area is called* **pressure.**

$$P = \frac{F}{A}$$

F = *The force (N), and*

A = *Area on which the force acts* (m^2), *and*

P = *Pressure (or intensity of pressure)*

Pressure head of liquid:

A liquid is subjected to pressure due to its own weight, this pressure increases as the depth of the liquid increases.

Let, ***h*** = *Height of liquid in the cylinder,*

A = *Area of the cylinder base,*

γ = *Specific weight of the liquid, and*

P = *Intensity of pressure.*

Total pressure <u>force</u> on the base of the cylinder = <u>Weight</u> of liquid in the cylinder

i.e.,

$$PA = \gamma Ah$$

$$P = \frac{\gamma Ah}{A}$$

$$P = \gamma h$$

$$P = (\rho g) h$$

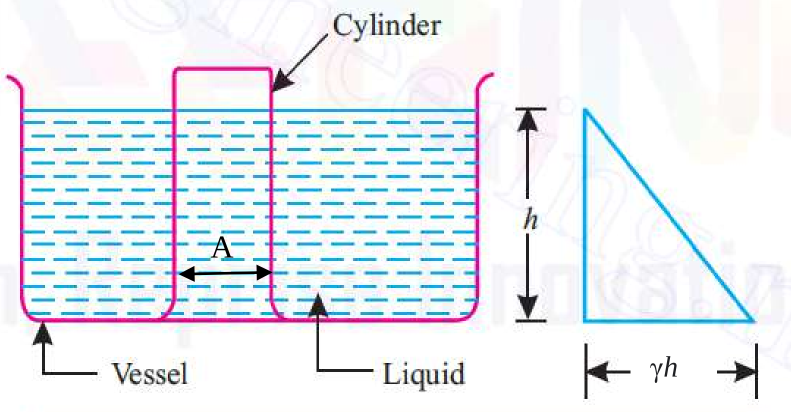


Fig. 2.1. Pressure head.

Pressure Measurement

- As the pressure at any point in a liquid depends on height of the free surface above that point, it is sometimes convenient to express a ***liquid pressure*** by the ***height of the free surface*** which would cause the pressure, *i.e*., $h = P / \gamma$

  The height of the free surface above any point is known as the ***static head*** at that point. In this case, static head is h.
  Hence, the intensity of pressure of a liquid may be expressed in the following two ways:
  1. As a force per unit area (*i.e.*, **N/m²**, **N/mm²**, **Pa**), and
  2. As an equivalent static head (*i.e.*, **m**, **mm** or **cm** of liquid).

Pressure Measurement – Hydrostatic Paradox

According to the hydrostatic equation ***P = (ρ g) h****, pressure (p) depends* ***ONLY on the height of the column and NOT at all upon the size of the column****.*

Thus, in all these vessels of different shapes and sizes, ***the same*** *intensity of pressure would be exerted on the bottom of each of these vessels.*

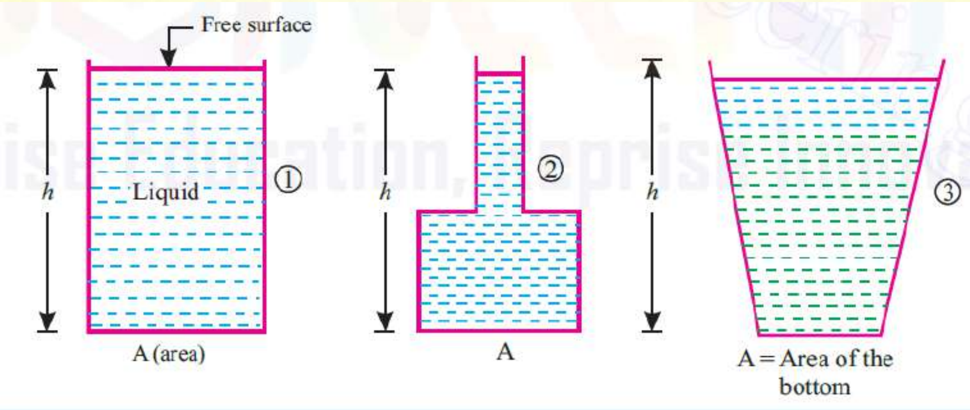
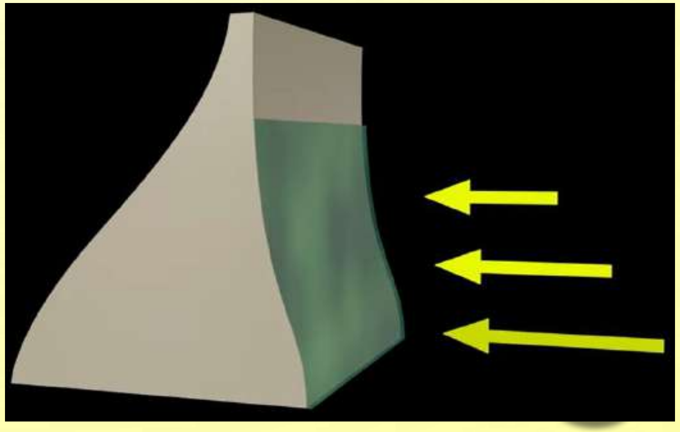


Fig. 2.8. Hydrostatic paradox.



Pressure Measurement

ATMOSPHERIC, ABSOLUTE AND GAUGE PRESSURES

Atmospheric pressure ($P_{atm.}$), *also known as '**Barometric pressure**'.*

*The atmospheric pressure at sea level is called '**Standard atmospheric pressure**'. local atmospheric pressure may be a little lower than these values if the place is higher than sea level, and higher values if the place is lower than sea level, due to the corresponding decrease or increase of the column of air standing, respectively.*

Standard atmosphere
101.3 kPa
14.7 psi
30 in Hg
760 mm Hg
1.013 bar
34 ft water

Absolute pressure ($P_{abs.}$): *Pressure measurement with respect to a zero pressure reference, and it is also called **Total pressure**.*

Gauge pressure (P_{guage}): *Pressure measurement higher than P_{atm}, and referenced to P_{atm}.*

Vacuum pressure ($P_{vac.}$) *Pressure measurement lower than P_{atm}, and referenced to P_{atm}.*

Fluid Mechanics

Lecture – 5

Dr Mohammed Tareq Khaleel

year 2

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MEASUREMENT OF PRESSURE

1. Manometers:

Barometer is a type of close-end manometer.

A **barometer** is a scientific instrument used to measure **atmospheric pressure** which is called **barometric pressure**.

$P_A = P_B$ (The intensity of pressure is the same for the same fluid at the rest, connected, and have the same horizontal level.)

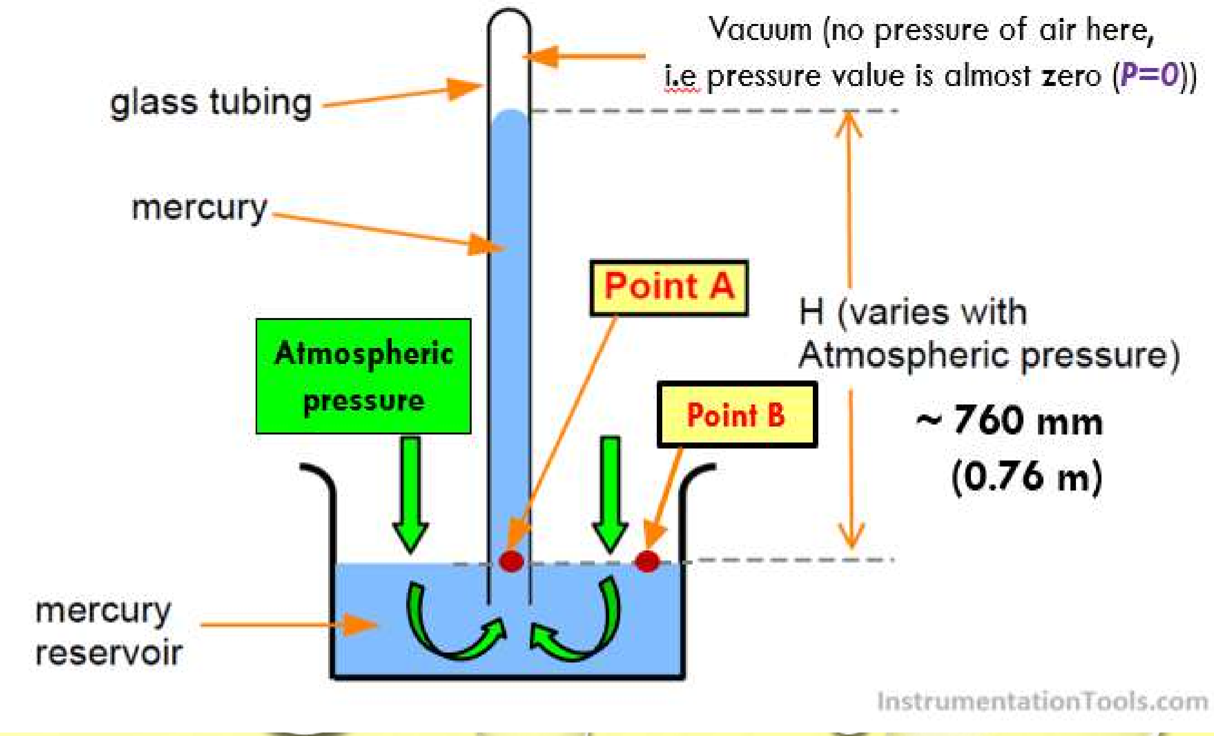
$\rho g h + 0 = P_{atm}$

$13600 \times 9.81 \times 0.76 + 0 = P_{atm}$

$P_{atm} = 101.3 \times 10^3 \quad N/m^2 \, (Pa)$

$P_{atm} = 1.013 \times 10^5 \quad N/m^2 \, (Pa)$

$P_{atm} = 1.013 \quad bar$



MEASUREMENT OF PRESSURE

1. Manometers:

Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of liquid.

These are classified as follows:

(*a*) **Simple manometers:**

i. Piezometer,
ii. U-tube manometer, and
iii. Single column manometer.

(*b*) **Differential manometers.**

i. U-tube differential manometer
ii. Inverted U-tube differential manometer

FYI https://www.youtube.com/watch?v=MpXhXVF9-HM

Pressure Measurement

a. Simple manometers _ i. Piezometers

Piezometers measure *gauge pressure only* (at the surface of the liquid), since the surface of the liquid in the tube is subjected to atmospheric pressure. A piezometer tube **is *not suitable* for measuring *negative pressure***; as in such a case the air will enter in pipe through the tube.

P = $\rho g h$ (or γh)

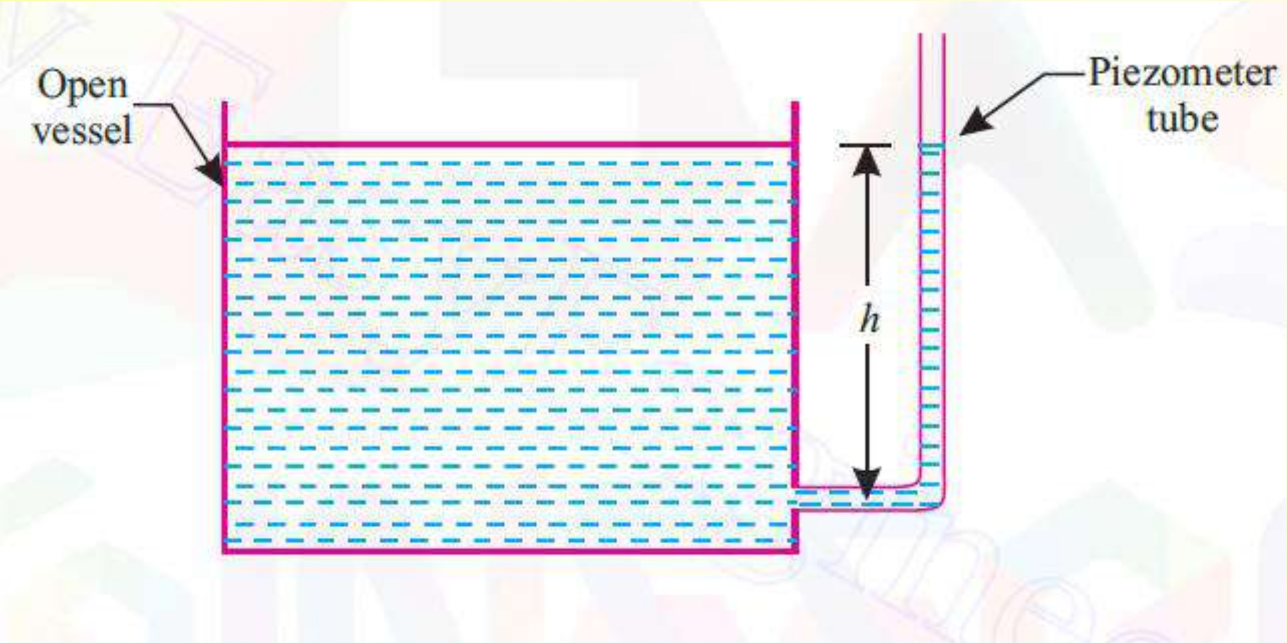


Fig. 2.10. (*a*) Piezometer tube fitted to open vessel.

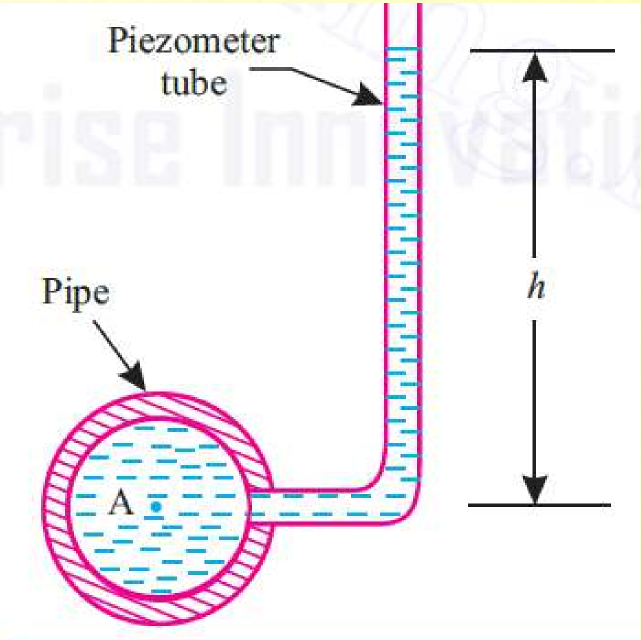
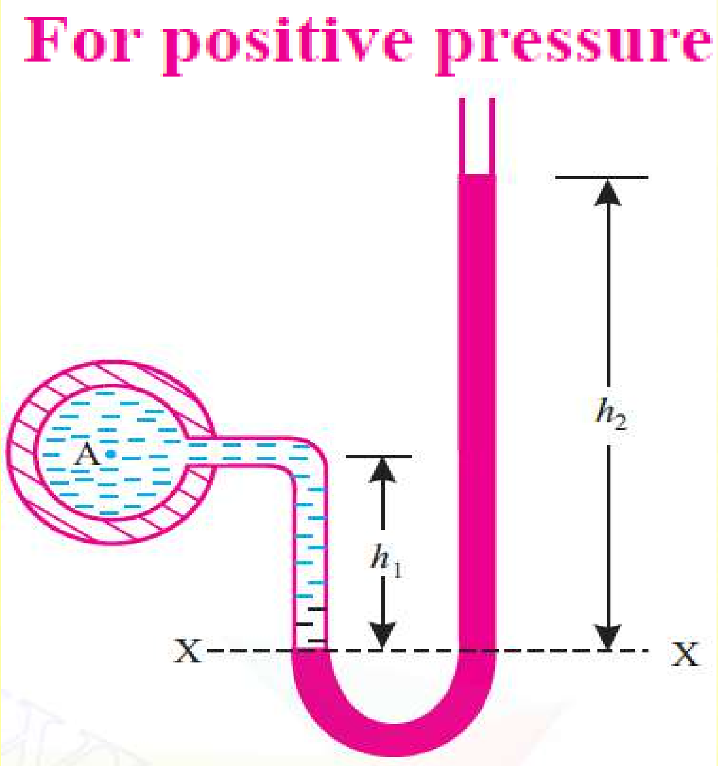


Fig. 2.10. (*b*) Piezometer tube fitted to a closed pipe.

Pressure Measurement

a. Simple manometers _ ii. U-tube manometer:

Piezometers cannot be employed when **large pressures** in the *lighter liquid*s are to be measured, since this would require *very long tubes*, which cannot be handled conveniently. Furthermore gas pressures cannot be measured by the piezometers because a *gas forms no free atmospheric surface*. These limitations can be overcome by the use of U-tube manometers.



h_1 = Height of the light liquid in the left limb above the datum line,

h_2 = Height of the heavy liquid in the right limb above the datum line,

h = Pressure in pipe, expressed in terms of head, (***h*** *is pressure head at* ***A***)

S_1 = Specific gravity of the light liquid, and

S_2 = Specific gravity of the heavy liquid.

(The intensity of pressure is the same for the same fluid at the rest, connected, and have the same horizontal level.)

Assume the head above the datum is +Ve and the head below the datum is - Ve

Pressure head above X–X in the *left limb* $= h + h_1 S_1$

Pressure head above X–X in the *right limb* $= h_2 S_2$

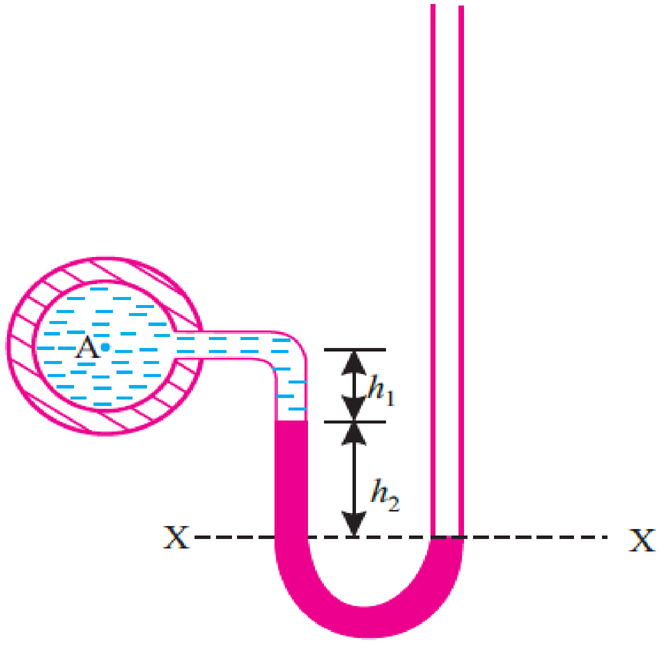
Equating these two pressures, we get:

$$h + h_1 S_1 = h_2 S_2 \quad \text{or} \quad h = h_2 S_2 - h_1 S_1$$

Pressure Measurement

Simple manometers _ ii. U-tube manometer:

For negative pressure



Pressure head above $X–X$ in the *left limb* $= h + h_1 S_1 + h_2 S_2$

Pressure head above $X–X$ in the *right limb* $= 0$.

Equating these two pressures, we get:

$$h + h_1 S_1 + h_2 S_2 = 0 \quad \text{or} \quad h = -(h_1 S_1 + h_2 S_2)$$

*(**h** is pressure head at **A**)*

(The intensity of pressure is the same for the same fluid at the rest, connected, and have the same horizontal level.)

Assume the head above the datum is +Ve and the head below the datum is - Ve

Fluid Mechanics

Lecture – 6

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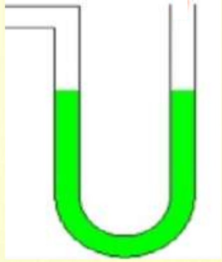
Pressure Measurement

a. Simple manometers _ iii. Single column manometer

The U-tube manometer described above usually requires reading of fluid levels at two or more points since a change in pressure causes a rise of liquid in one limb of the manometer and a drop in the other. This difficulty is however overcome by using single column manometers.



A single column manometer is a modified form of a U-tube manometer in which a shallow reservoir having a large cross-sectional area (about 100 times) as compared to the area of the tube is connected to one limb of the manometer, as shown in Fig. 2.18.

Area of Reservoir >> Area of Limb

For any variation in pressure, the change in the liquid level in the reservoir will be so small that it may be **neglected**, and the pressure is indicated by the height of the liquid in the other limb. As such only one reading in the narrow limb of the manometer need be taken for all pressure measurements. The narrow limb may be vertical or inclined.

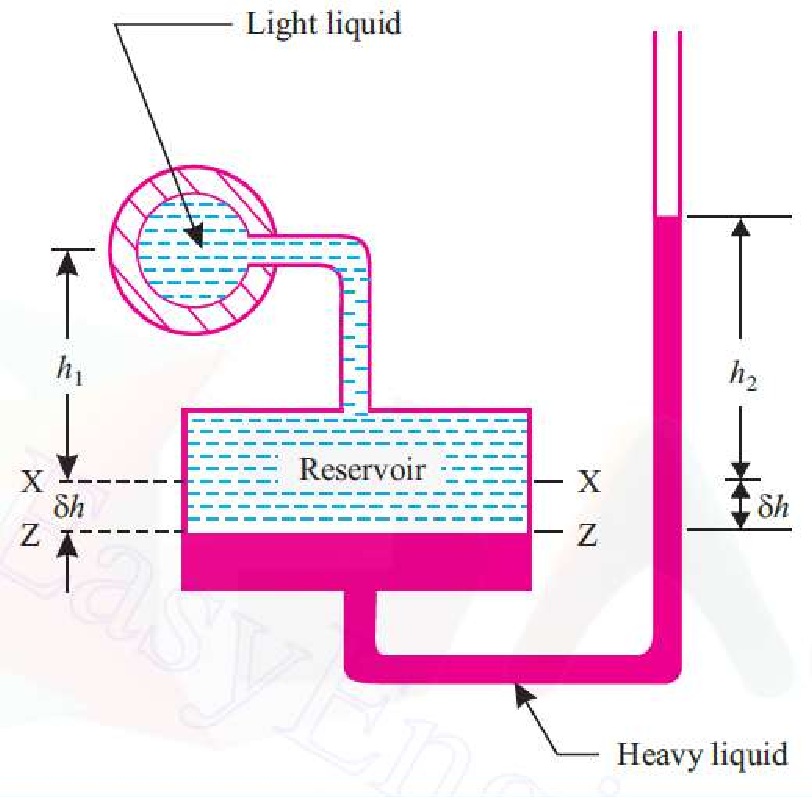


Fig. 2.18. Vertical single column manometer.

Pressure Measurement

a. Simple manometers _ iii. Single column manometer

Let ***X–X*** be the datum line in the reservoir when the single column manometer is not connected to the pipe. Now consider that the manometer is connected to a pipe containing light liquid under a very high pressure. The pressure in the pipe will force the light liquid to push the heavy liquid in the reservoir downwards. As the area of the reservoir is very large, the fall of the heavy liquid level will be very small. This downward movement of the heavy liquid, in the reservoir, will cause a considerable rise of the heavy liquid in the right limb.

h_1 = Height of the centre of the pipe above X–X,
h_2 = Rise of heavy liquid (after experiment) in the right limb,
δh = Fall of heavy liquid level in the reservoir,
h = Pressure in the pipe, expressed in terms of head of water,
A = Cross-sectional area of the reservoir,
a = Cross-sectional area of the tube (right limb),
S_1 = Specific gravity of light liquid in pipe, and
S_2 = Specific gravity of the heavy liquid.

Delta ***h*** appears at the right limb because we changed the datum line from x-x to z-z.

We know that fall of heavy liquid in reservoir will cause a rise of heavy liquid level in the right limb.

Thus, $A \times \delta h = a \times h_2$ or $\delta h = \dfrac{a \times h_2}{A}$

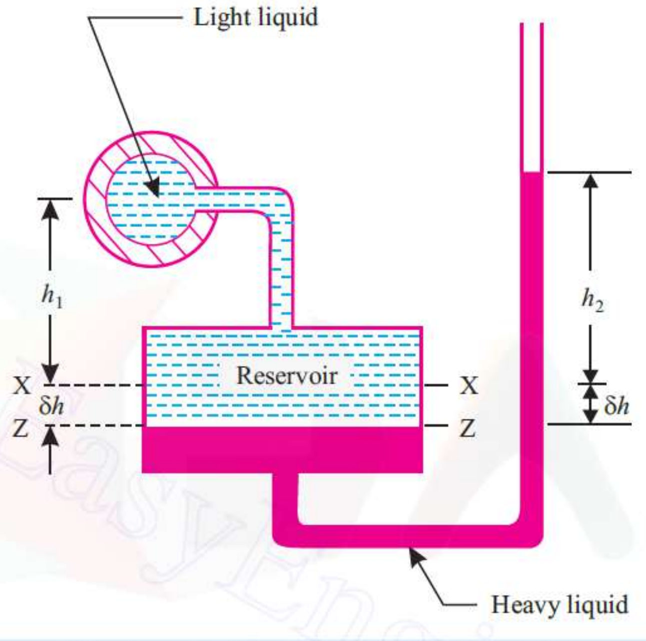


Fig. 2.18. Vertical single column manometer.

Pressure Measurement

a. Simple manometers _ iii. Single column manometer

Let us now consider pressure heads above the datum line $Z–Z$ as shown in Fig. 2.18.

Pressure head in the left limb $= h + (h_1 + \delta h)S_1$

Pressure head in the right limb $= (h_2 + \delta h)S_2$

Assume the head above the datum is +Ve and the head below the datum is - Ve

Equating the pressure heads, we get:

$$h + (h_1 + \delta h)S_1 = (h_2 + \delta h)S_2$$

$$h = (h_2 + \delta h)\, S_2 - (h_1 + \delta h)\, S_1$$

$$= \delta h\,(S_2 - S_1) + h_2 S_2 - h_1 S_1$$

Delta ***h*** appears at the right limb because we changed the datum line from x-x to z-z.

But, $\delta h = \dfrac{a \times h_2}{A}$

$$h = \frac{a \times h_2}{A}(S_2 - S_1) + h_2 S_2 - h_1 S_1 \quad \ldots(2.8)$$

When the area A is very large as compared to a, then the ratio $\frac{a}{A}$ becomes very small, and thus is neglected. Then the above equation becomes

$$h = h_2 S_2 - h_1 S_1 \quad \ldots(2.9)$$

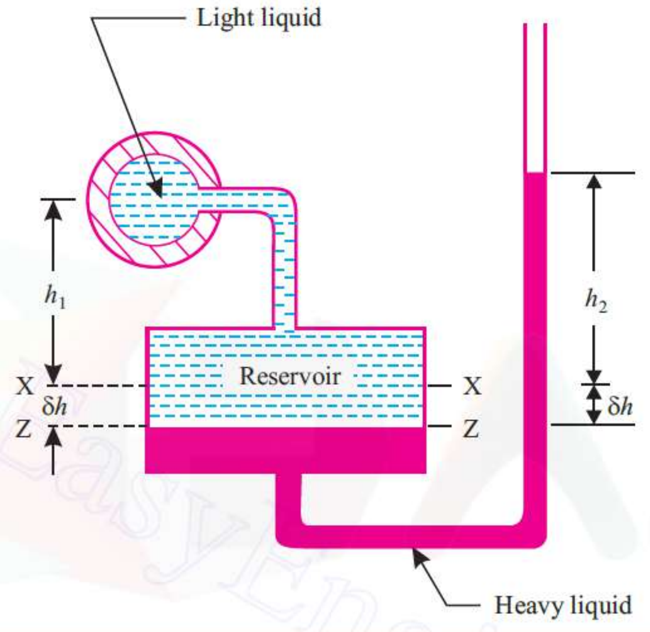


Fig. 2.18. Vertical single column manometer.

Pressure Measurement

b. Differential manometers

is used to measure the difference in pressures between two points in a pipe, or in two different pipes. In its simplest form a differential manometer consists of a U-tube, containing a heavy liquid, whose two ends are connected to the points, whose difference of pressures is required to be found out.

Following are the most commonly used types of differential manometers:

i. U-tube differential manometer.

ii. Inverted U-tube differential manometer.

i. U-tube differential manometer.

Case I. Fig. 2.21 (*a*) shows a differential manometer whose two ends are connected with two different points A and B at the **same level** and containing **same liquid**.

i.e., Difference of pressure head,

$$h_A - h_B = h\,(S - S_1)$$

H.W: Prove it.

Assume the head above the datum is +Ve
and the head below the datum is - Ve

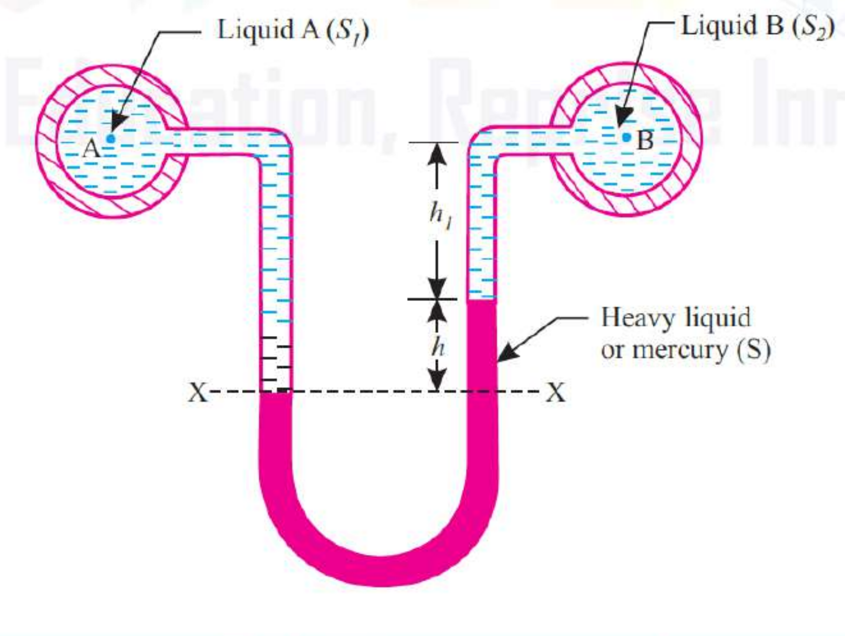


Fig. 2.21. (*a*) Two pipes at same level.

Pressure Measurement

b. Differential manometers

i. U-tube differential manometer.

Case II. Fig. 2.21 (*b*) shows a differential manometer whose two ends are connected to two different points *A* and *B* at **<u>different levels</u>** and containing **<u>different liquids</u>**.

Difference of pressure heads at *A* and *B*,

$$h_A - h_B = h(S - S_1) + h_2 S_2 - h_1 S_1$$

H.W: Prove it.

Assume the head above the datum is +Ve
and the head below the datum is - Ve

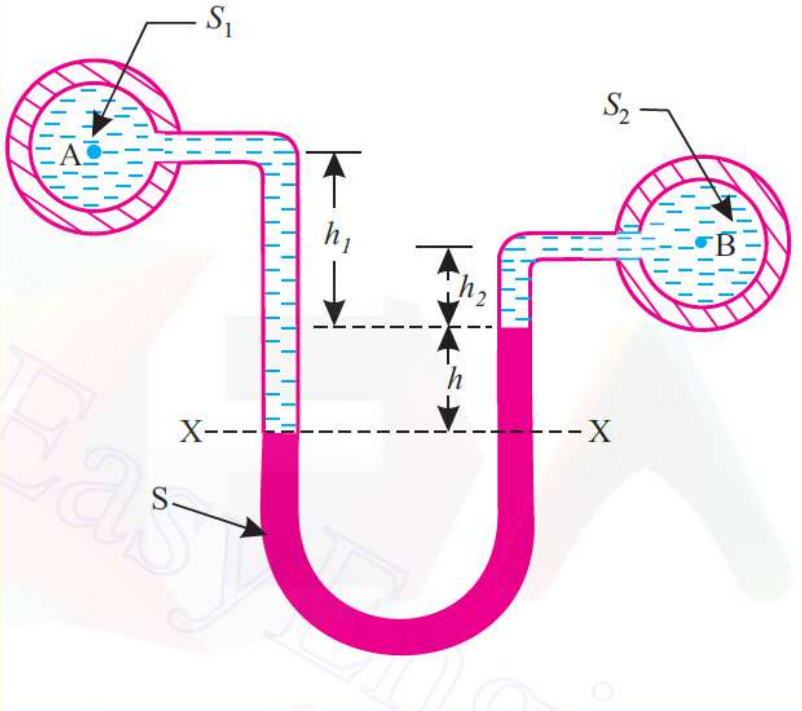


Fig. 2.21. (b) U-tube differential manometers.

Pressure Measurement

Example 2.27. *From the Fig. 2.30 determine the absolute pressure in pipe A that contains oil of specific gravity = 0.88. Take $Z_1 = 0.66$ m, $Z_2 = 0.33$ m, $Z_3 = 0.165$ m and $Z_4 = 0.11$ m.*

Assume an atmospheric pressure 105 kPa. **(Madras University)**

Solution. Starting from F.W.S (free water surface) in tank (at atmospheric pressure), we get

Fall +Ve
Rise -Ve

$$p_{atm} + w_w Z_1 - w_w Z_2 - w_m Z_3 + w_0 (Z_3 + Z_4) = p_A$$

$$105 + 9.81 \times 0.66 - 9.81 \times 0.33 - 13.6 \times 9.81 \times 0.165 + 0.88 \times 9.81 \times (0.165 + 0.11) = p_A$$

$$p_A = 105 + 6.475 - 3.237 - 22.014 + 2.374$$

$$= \mathbf{88.6\ kN/m^2}\ (absolute)\ \textbf{(Ans.)}$$

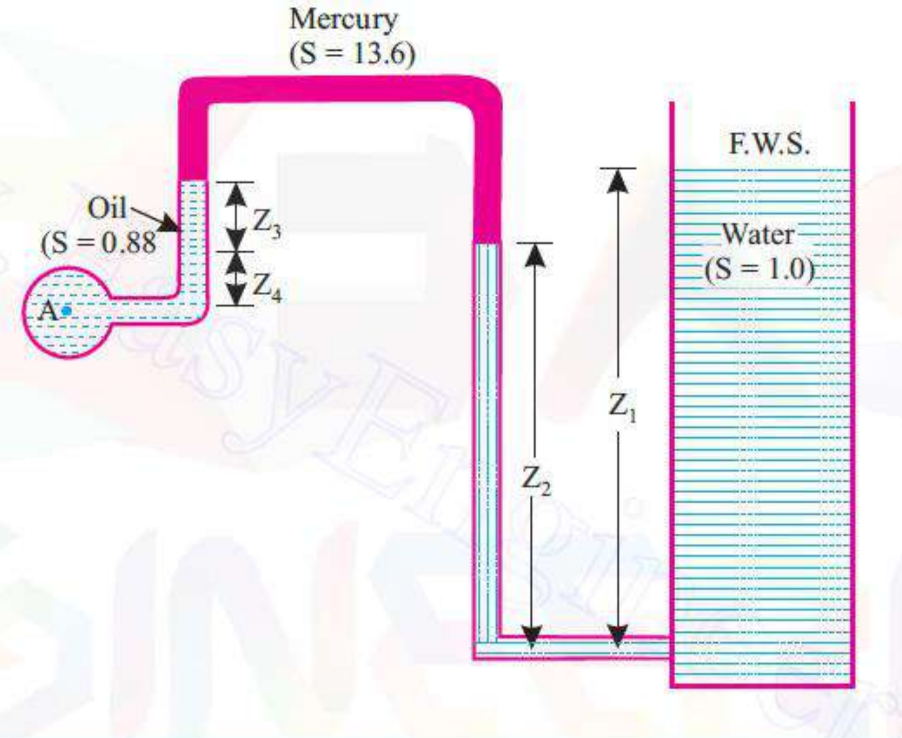


Fig. 2.30

Pressure Measurement

Example 2.28. *Find the pressure difference between L and M in Fig. 2.31.*

Solution. $p_L - p_M$:

$$\frac{p_L}{w} + h \times 1.5 - 0.15 \times 0.8 + (0.15 + 0.2 - h) \times 1.5 = \frac{p_M}{w}$$

$$\frac{p_L}{w} + 1.5\,h - 0.12 + 0.525 - 1.5\,h = \frac{p_M}{w}$$

$$\frac{p_L - p_M}{w} = -0.405 \text{ m}$$

Fall +Ve
Rise -Ve

Negative sign indicates $p_M > p_L$

i.e., $p_M - p_L = 0.405 \times 9.81$

$= \mathbf{3.97\ kN/m^2}$ **(Ans.)**

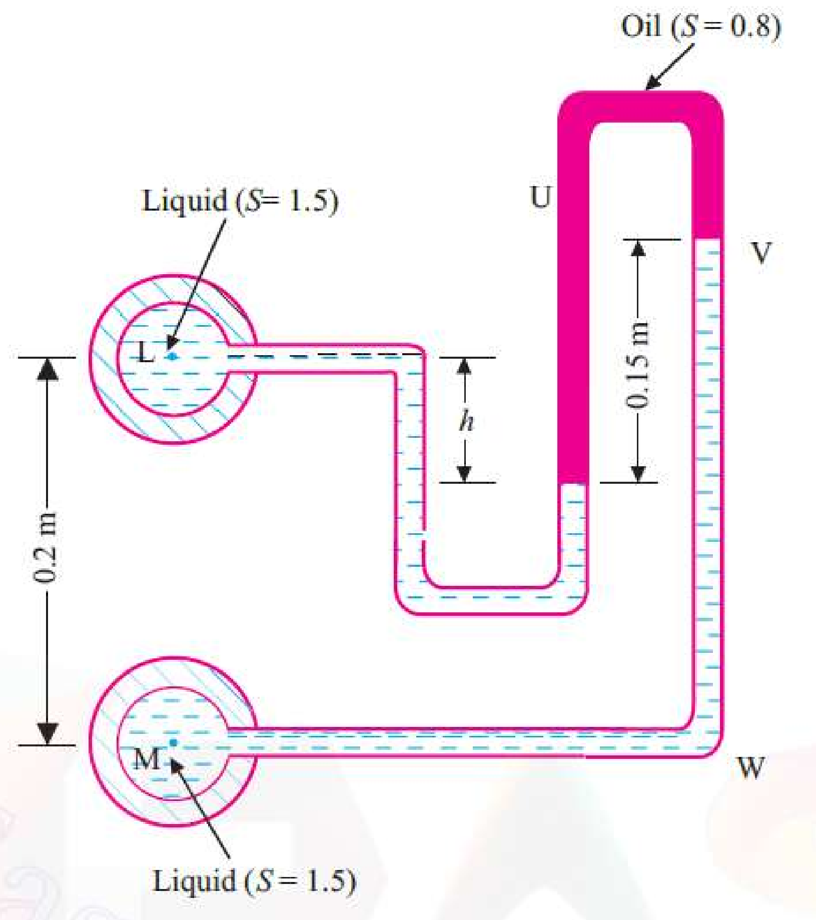


Fig. 2.31

Pressure Measurement

Example 2.29. *In the Fig. 2.32, if the local atmospheric pressure is 755 mm of mercury (sp. gravity = 13.6), calculate:*

(*i*) *Absolute pressure of air in the tank;*

(*ii*) *Pressure gauge reading at L.*

Solution. (*i*) Absolute pressure of air, $(p_{abs})_{air}$:

Starting from the open end, we have:

$0 - (13.6 \times w) \times 0.6 = p_{air}$ (pressure of air)

i.e., $p_{air} = -13.6 \times 9.81 \times 0.6 = -80$ kN/m^2

$p_{atm.}$ = (atmospheric pressure)

$= \frac{755}{1000} \times 13.6 \times 9.81 = 100.73$ kN/m^2

$(p_{abs.})_{air} = p_{air} + p_{atm.}$

$= -80 + 100.73 = 20.73$ kN/m^2

Hence, $(p_{abs.})_{air}$ = **20.73 kN/m^2 (Ans.)**

(*ii*) Pressure gauge reading at L:

Pressure at $L = p_{abs.}$ (air) $+ wh$

$p_L = 20.73 + 9.81 \times 2 = 40.35$ kN/m^2 *abs.*

Now, $40.35 = p_{gauge} + p_{atm.}$

$p_{gauge(L)} = 40.35 - p_{atm.}$

$= 40.35 - 100.73$

$= -60.38$ kN/m^2

Fall +Ve
Rise -Ve

OR:

$P_{gauge (L)}$ =
- 80 + 2 * 9.81
= - 60.38 KN/m^2

i.e., Vacuum pressure = 60.38 kN/m^2

Hence, *pressure gauge reading at L* = **60.38 kN/m^2 (vacuum) (Ans.)**

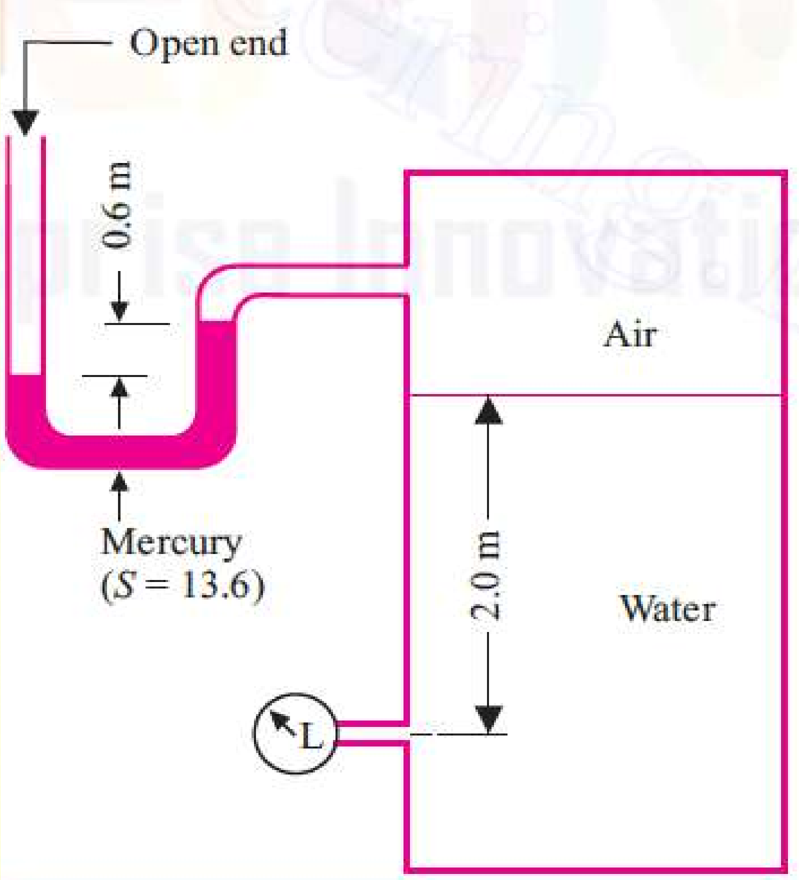


Fig. 2.32

FLUID MECHANICS

LECTURE 7

BY: MOHAMMED TAREQ KHALEEL

HYDROSTATIC FORCES ON SURFACES

TOTAL PRESSURE AND CENTRE OF PRESSURE

- **Total pressure.** It is defined as the ***force*** *exerted by static fluid on a surface (either plane or curved) when the fluid comes in contact with the surface. This force is always at right angle (or normal) to the surface.*

- **Centre of pressure.** It is defined as the point of application of the total pressure on the surface.

The immersed surfaces may be:

1. Horizontal plane surface;
2. Vertical plane surface;
3. Inclined plane surface;
4. Curved surface.

HORIZONTALLY IMMERSED SURFACE

Total Pressure (P):

Refer to Fig. 3.1. Consider a plane horizontal surface immersed in a liquid.

Let, A = Area of the immersed surface,

$\bar{x}$ = Depth of horizontal surface from the liquid, and

w = Specific weight of the liquid.

The total pressure on the surface,

Total Force

P = Weight of the liquid above the immersed surface

= Specific weight of liquid × volume of liquid

= Specific weight of liquid × area of surface × depth of liquid

$= wA\bar{x}$

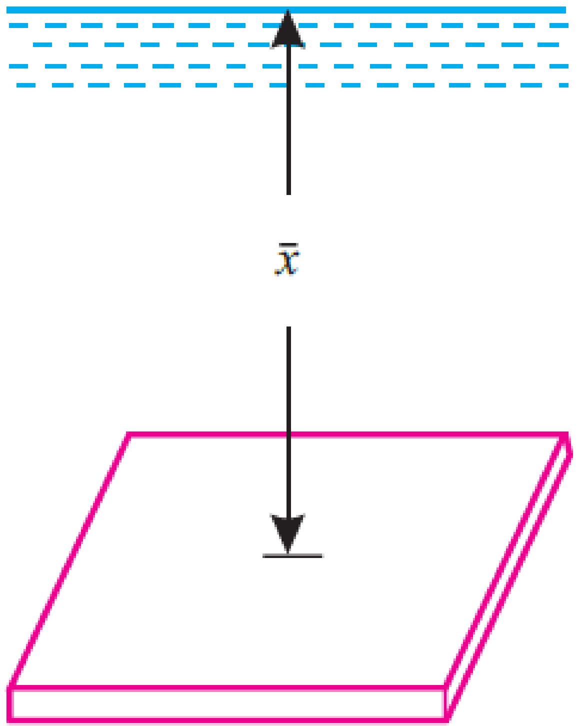


Fig. 3.1. Horizontally immersed surface.

VERTICALLY IMMERSED SURFACE

Consider a plane vertical surface of arbitrary shape immersed in a liquid as shown in Fig. 3.2.

Let, A = Total area of the surface,

G = Centre of the area of the surface,

$\bar{x}$ = Depth of centre of area,

OO = Free surface of liquid, and

$\bar{h}$ = Distance of centre of pressure from free surface of liquid.

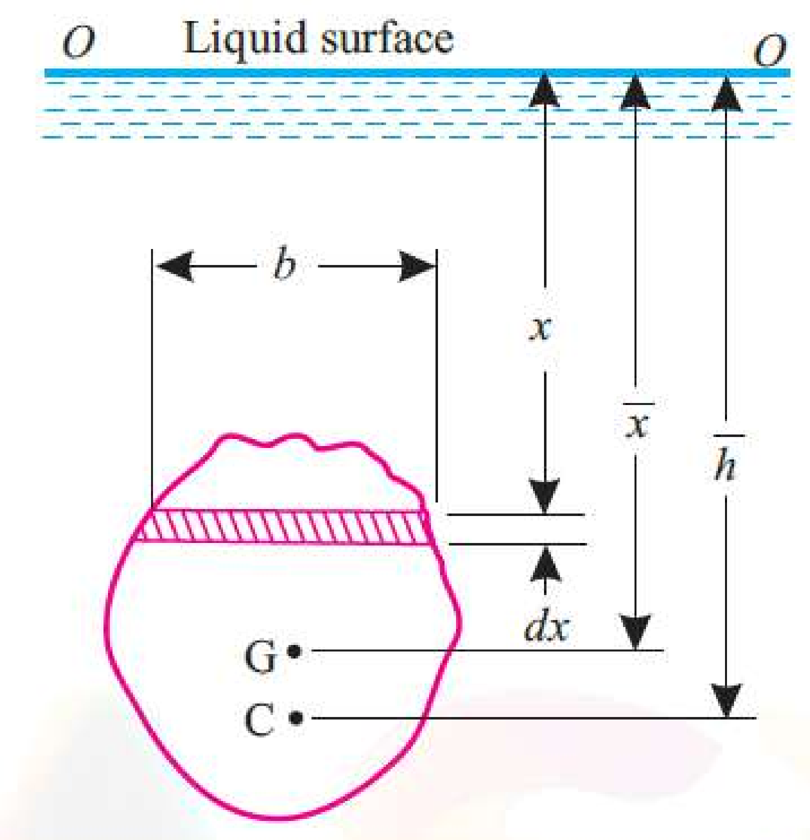


Fig. 3.2. Vertically immersed surface.

(a) Total pressure (P):

Consider a thin horizontal strip of the surface of thickness dx and breadth b. Let the depth of the strip be x. Let the intensity of pressure on strip be p; this may be taken as uniform as the strip is extremely small. Then,

$$p = wx$$ Pressure

where, w = specific weight of the liquid.

Total pressure on the strip $= p.b.dx.$ Force

$= wx \,.\, bdx$

Total pressure on the whole area, $P = \int wx \,.\, bdx = w \int bdx \,.\, x$ Force

But, $\int bdx \,.\, x$ = Moment of the surface area about the liquid level $= A\bar{x}$

$\therefore \quad P = w A\bar{x}$ ...[same as in Art. 3.3] Force

Note that
The Intensity of Pressure = Pressure, (KN/m^2)
Total Pressure = Total Force, (KN)

VERTICALLY IMMERSED SURFACE

$P = w.A\bar{x}$

or, the total pressure on a surface is equal to the *area multiplied by the intensity of pressure at the centre of area of the figure.*

The eqn., $P = wA\bar{x}$ holds good for all surfaces whether flat or curved.

(b) Centre of pressure ($\bar{h}$):

The intensity of pressure on an immersed surface is not uniform, but *increases with depth*. As the pressure is greater over the lower portion of the figure, therefore the resultant pressure, on any immersed surface will act at some point, below the centre of gravity of the immersed surface and towards the lower edge of the figure. *The point through which this resultant pressure acts is known as '**centre of pressure**'* and is always expressed in terms of *depth* from the liquid surface.

Referring to Fig. 3.2, let C be the centre of pressure of the immersed figure. Then the resultant pressure P will act through the point.

Let, $\bar{h}$ = Depth of centre of pressure below free liquid surface, and

I_0 = Moment of inertia of the surface about OO.

Consider the horizontal strip of thickness dx. Total pressure on strip $= w.x.b.dx$

Moment of this pressure about free surface $OO = (w.x.\ b.dx)\ x = w.x^2.b.dx$

Total moment of all such pressures for whole area, $M = \int w.x^2.b.dx. = w \int x^2\ .b.dx$

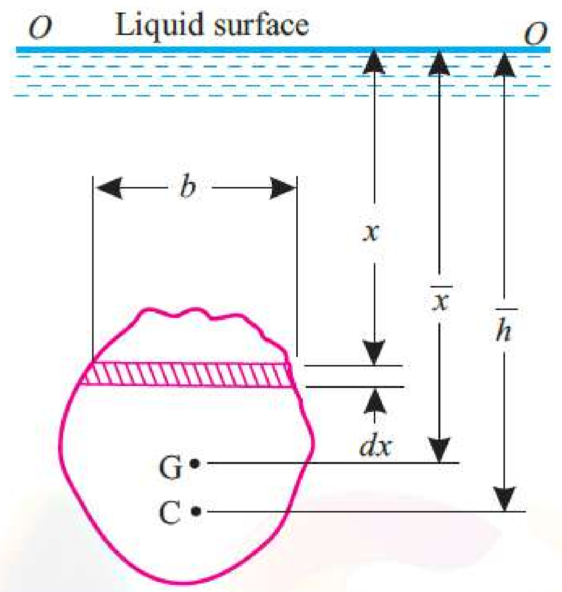


Fig. 3.2. Vertically immersed surface.

Note that

The Intensity of Pressure = Pressure, (KN/m^2)

Total Pressure = Total Force, (KN)

Resultant Pressure = Resultant Force, (KN)

VERTICALLY IMMERSED SURFACE

But, $\int x^2 . b . dx = I_0$ = Moment of inertia of the surface about the free surface OO (or second moment of area)

$$M = wI_0 \qquad ...(i)$$

The sum of the moments of the pressure is also equal to $P \times \bar{h}$ ...(ii)

Now equating eqns. (i) and (ii), we get:

$$P \times \bar{h} = wI_0.$$

$$wA\bar{x} \times \bar{h} = wI_0 \qquad (\because P = wA\bar{x})$$

$$\bar{h} = \frac{I_0}{A\bar{x}} \qquad ...(iii)$$

Also, $I = I_G + Ah^2$ (Theorem of parallel axis)

where, I_G = Moment of inertia of the figure about horizontal axis through its centre of gravity, and

h = Distance between the free liquid surface and the centre of gravity of the figure ($\bar{x}$ in this case)

Thus rearranging equation (iii), we have

$$\bar{h} = \frac{I_G + A\bar{x}^2}{A\bar{x}} = \frac{I_G}{A\bar{x}} + \bar{x}$$

Hence, *centre of pressure*, $\bar{h} = \dfrac{I_G}{A\bar{x}} + \bar{x}$

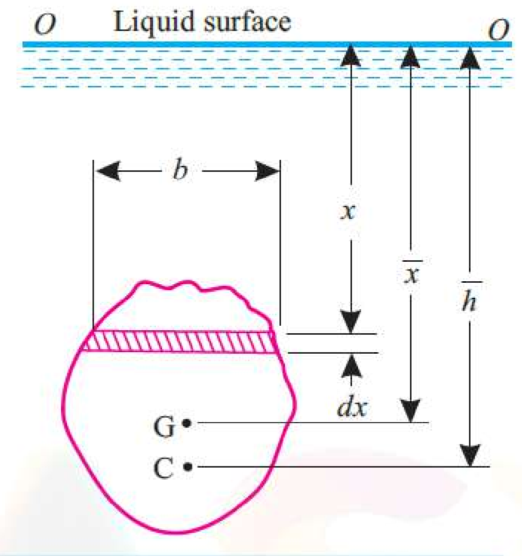


Fig. 3.2. Vertically immersed surface.

Example 3.3. *An isosceles triangular plate of base 3 m and altitude 3 m is immersed vertically in an oil of specific gravity 0.8. The base of the plate coincides with the free surface of oil. Determine:*

(i) Total pressure on the plate; *(ii) Centre of pressure.*

Solution. Base of the plate, $b = 3$ m

Height of the plate, $h = 3$ m

Area, $$A = \frac{b \times h}{2} = \frac{3 \times 3}{2} = 4.5 \text{ m}^2$$

Specific gravity of oil, $S = 0.8$

The distance of C.G. from the free surface of oil,

$$\bar{x} = \frac{1}{3}h = \frac{1}{3} \times 3 = 1 \text{ m}$$

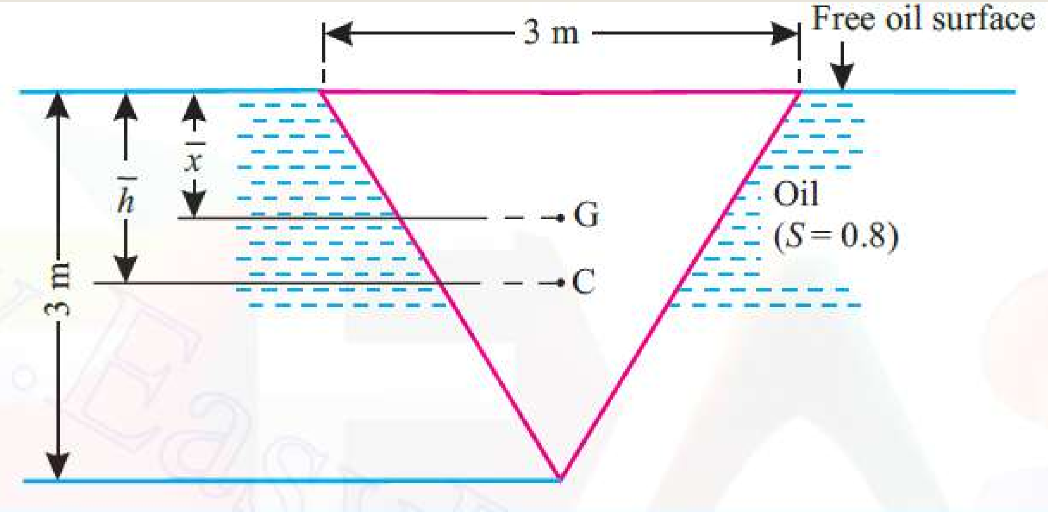


Fig. 3.9

(i) Total pressure on the plate, P:

We know that, $$P = wA\bar{x}$$

$$= (0.8 \times 9.81) \times 4.5 \times 1$$

$$P = \mathbf{35.3 \text{ kN}} \text{ (Ans.)}$$

(ii) Centre of pressure, $\bar{h}$:

Centre of pressure is given by the relation:

$$\bar{h} = \frac{I_G}{A\bar{x}} + \bar{x} = \frac{(bh^3/36)}{A\bar{x}} + \bar{x}$$

$$= \frac{(3 \times 3^3/36)}{4.5 \times 1} + 1$$

$$\bar{h} = \mathbf{1.5 \text{ m}} \text{ (Ans.)}$$

FLUID MECHANICS

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LECTURE 8

BY: MOHAMMED TAREQ KHALEEL

Example 3.14. *An opening in a dam is covered by the use of a vertical sluice gate. The opening is 2 m wide and 1.2 m high. On the upstream of the gate the liquid of specific gravity 1.45 lies upto a height of 1.5 m above the top of the gate, whereas on the downstream side the water is available upto a height touching the top of the gate. Find:*

(*i*) *The resultant force acting on the gate and position of centre of pressure;*

(*ii*) *The force acting horizontally at the top of the gate which is capable of opening it.*

Assume that the gate is hinged at the bottom.

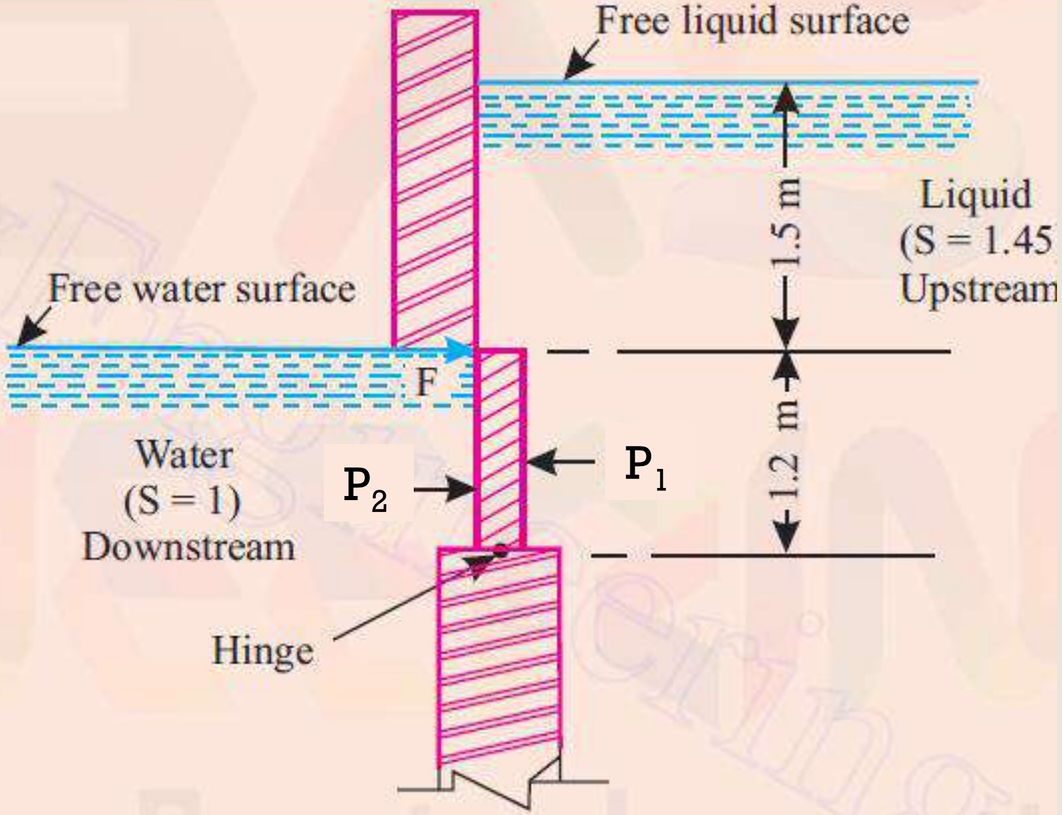


Fig. 3.22

(Rajasthan University

Solution. Width of the gate, $b = 2$ m

Depth of the gate, $d = 1.2$ m

Area, $A = b \times d = 2 \times 1.2 = 2.4 \text{ m}^2$

Specific gravity of liquid $= 1.45$

Let, P_1 = Force exerted by the liquid of sp. gravity 1.45 on the gate, and

P_2 = Force exerted by water on the gate.

(*i*) Resultant force, P:

$$P_1 = wA\bar{x}_1$$

$$w = 9.81 \times 1.45 = 14.22 \text{ kN/m}^3,$$

$$A = 2 \times 1.2 = 2.4 \text{ m}^2$$

$$\bar{x}_1 = 1.5 + \frac{1.2}{2} = 2.1 \text{ m}$$

$$P_1 = 14.22 \times 2.4 \times 2.1 = 71.67 \text{ kN}.$$

Similarly,

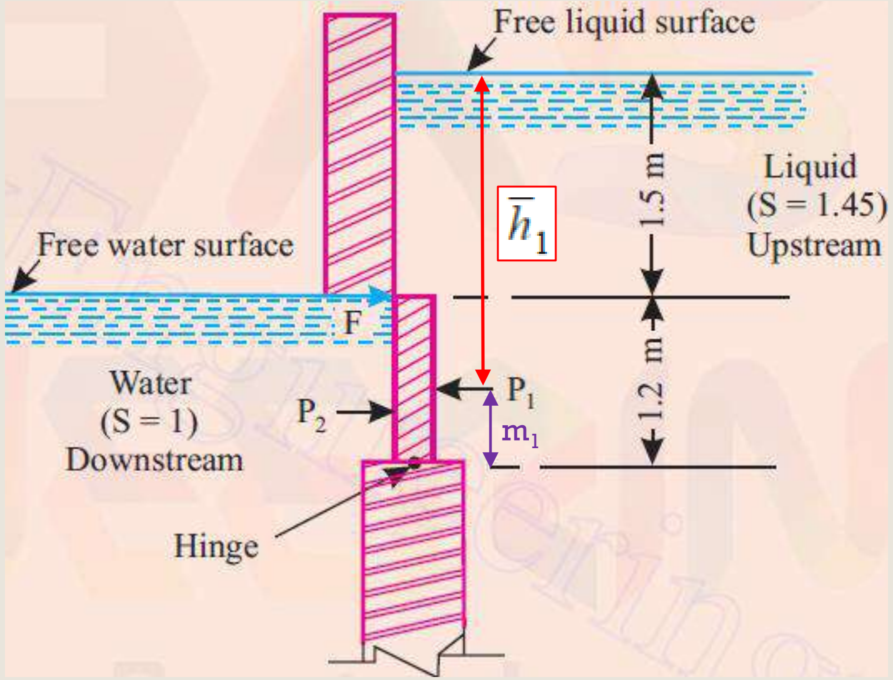
$$P_2 = wA\bar{x}_2$$

$$w = 9.81 \text{ kN/m}^3.$$

$$A = 2.4 \text{ m}^2,$$

$$\bar{x}_2 = \frac{1.2}{2} = 0.6 \text{ m}$$

$$P_2 = 9.81 \times 2.4 \times 0.6 = 14.13 \text{ kN}.$$



Resultant force, $P = P_1 - P_2 = 71.67 - 14.13$

$= $ **57.54 kN (Ans.)**

The force P_1 acts at a depth of $\bar{h}_1$ from free liquid surface, which is given by:

$$\bar{h}_1 = \frac{I_G}{A\bar{x}_1} + \bar{x}_1$$

where, $I_G = \frac{bd^3}{12} = \frac{2 \times 1.2^3}{12} = 0.288 \text{ m}^4$ $\quad A = 2.4 \text{ m}^2,$ $\quad \bar{x} = 1.5 + \frac{1.2}{2} = 2.1 \text{ m}$

$$\bar{h}_1 = \frac{0.288}{2.4 \times 2.1} + 2.1 = 2.157 \text{ m}$$

$\therefore$ Distance of P_1 from the hinge $= (1.5 + 1.2) - \bar{h}_1$

$= 2.7 - 2.157 = 0.543$ m $\quad (m_1)$

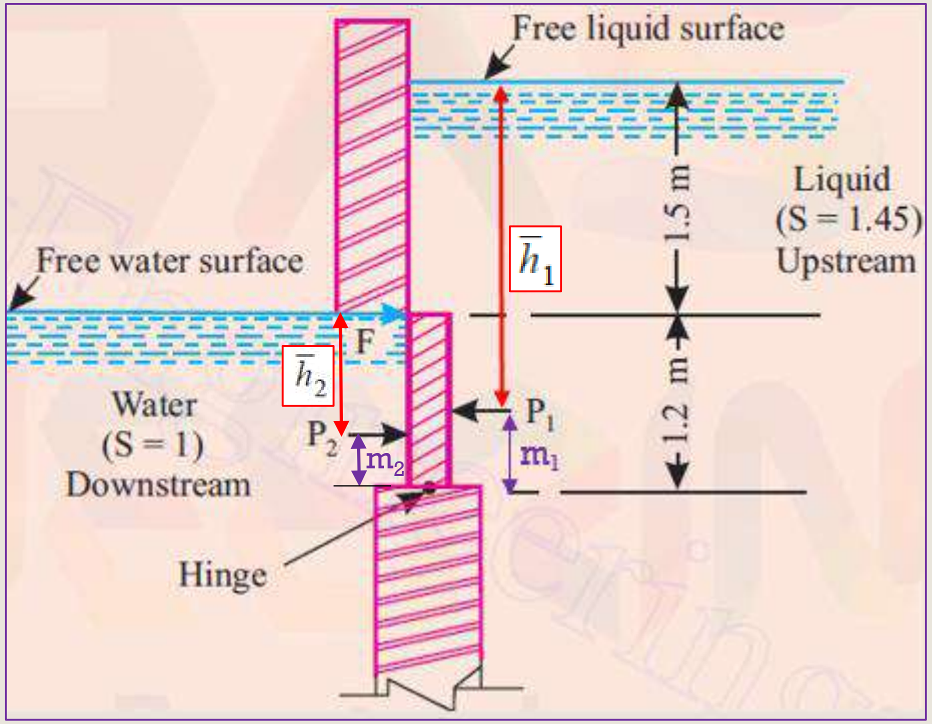
Similarly the force P_2 acting at a depth of $\bar{h}_2$ from the liquid surface is given by:

$$\bar{h}_2 = \frac{I_G}{A\bar{x}_2} + \bar{x}_2$$

$I_G = 0.288\ m^4$ (as above); $\bar{x}_2 = \frac{1.2}{2} = 0.6\ m$; $A = 2.4\ m^2$

$$\bar{h}_2 = \frac{0.288}{2.4 \times 0.6} + 0.6 = 0.8\ m$$

$\therefore$ Distance of P_2 from the hinge $= 1.2 - 0.8 = 0.4\ m$ (m_2)



Now the resultant force will act at a distance given by:

$$P \times m = P_1 \times m_1 - P_2 \times m_2$$

$$\frac{71.67 \times 0.543 - 14.13 \times 0.4}{57.54} = \textbf{0.578 m above the hinge (Ans.)}$$

(ii) Force required to open the gate, F:

Taking moments of P_1, P_2 and F about the hinge, we get:

$$F \times 1.2 + P_2 \times 0.4 = P_1 \times 0.543$$

$$F \times 1.2 + 14.13 \times 0.4 = 71.67 \times 0.543$$

$$F = \frac{71.67 \times 0.543 - 14.13 \times 0.4}{1.2} = \textbf{27.72 kN (Ans.)}$$

FLUID MECHANICS

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LECTURE 9

BY: MOHAMMED TAREQ KHALEEL

HYDROSTATIC FORCES ON SURFACES

INCLINED IMMERSED SURFACE

A = Area of the surface,

$\bar{x}$ = Depth of centre of gravity of immersed surface from the free liquid surface,

θ = Angle at which the immersed surface is inclined with the liquid surface, and

w = Specific weight of the liquid.

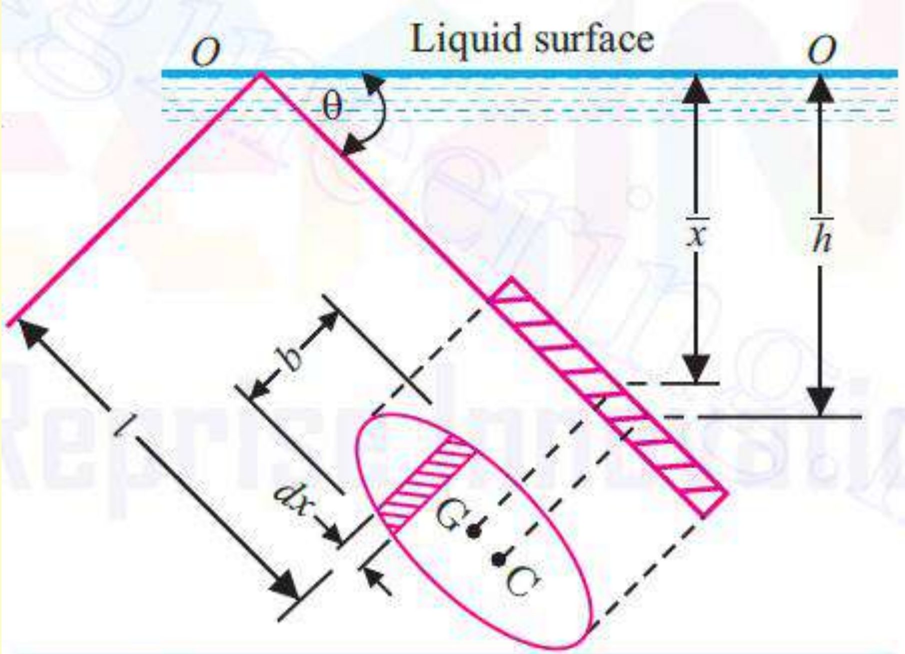


Fig. 3.27. Inclined immersed surface.

(a) Total pressure (P):

Consider a strip of thickness dx, width b at a distance l from O (A point, on the liquid surface, where the immersed surface will meet, if produced).

The intensity of pressure on the strip

$= wl \sin\theta$

Area of the strip $= b.dx$

Pressure on the strip

$=$ Intensity of pressure × *area*

$= wl \sin\theta \, . \, b. \, dx$

Now total pressure on the surface,

$$P = \int wl \sin\theta \, . \, b \, . \, dx = w \sin\theta \int l \, . \, b \, . \, dx$$

$$P = \int wl \sin\theta \, . \, b \, . \, dx = w \sin\theta \int l \, . \, b \, . \, dx$$

But, $\int l \, . \, b \, . \, dx$ = moment of surface area about 00

$$= \frac{A\bar{x}}{\sin\theta},$$

$$\therefore \quad P = w \sin\theta \, . \, \frac{A\bar{x}}{\sin\theta} = wA\bar{x}$$ (same as in Arts. 3.3 and 3.4)

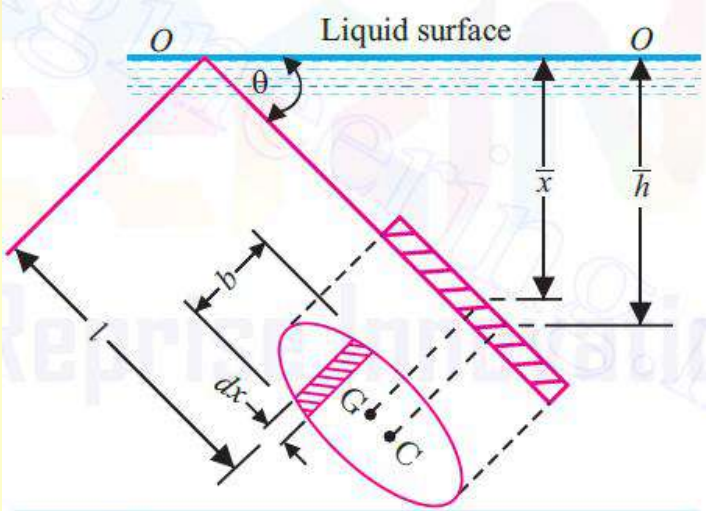


Fig. 3.27. Inclined immersed surface.

(b) Centre of pressure ($\bar{h}$):

Referring to Fig 3.27, let C be the centre of pressure of the inclined surface.

Let,
$\bar{h}$ = Depth of centre of pressure below free liquid surface,
I_G = Moment of inertia of the immersed surface about OO,
$\bar{x}$ = Depth of centre of gravity of the surface from the liquid surface,
θ = Angle at which the immersed surface is inclined with the liquid surface, and
A = Area of the surface.

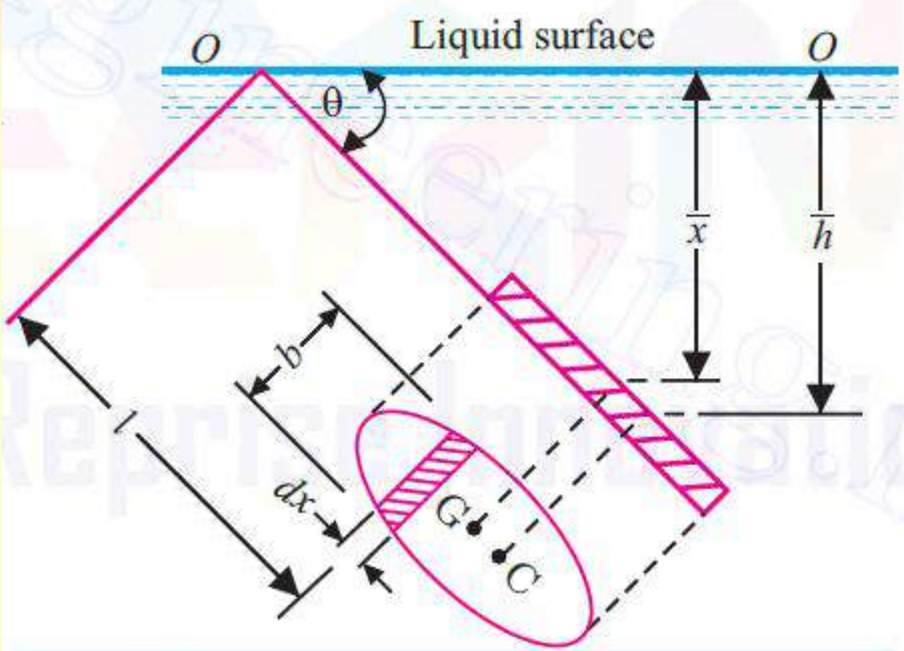


Fig. 3.27. Inclined immersed surface.

Consider a strip of thickness of dx, width b and at distance l from OO.

The intensity of pressure on the strip = $wl \sin\theta$

Area of strip = $b \,.\, dx$

$\therefore$ Pressure on the strip = Intensity of pressure × area = $wl \sin\theta \, b \,.\, dx$

Moment of the pressure about OO = $(wl \sin\theta \,.\, b.dx)\, l = wl^2 \sin\theta \,.\, b \,.\, dx$

Now sum of moments of all such pressures about O,

$$M = \int wl^2 \sin\theta \,.\, b\, dx = w \sin\theta \int l^2 \,.\, b \,.\, dx$$

But, $\int l^2 \,.\, b \,.\, dx = I_0$ = moment of inertia of the surface about the point 0 (or the second moment of area)

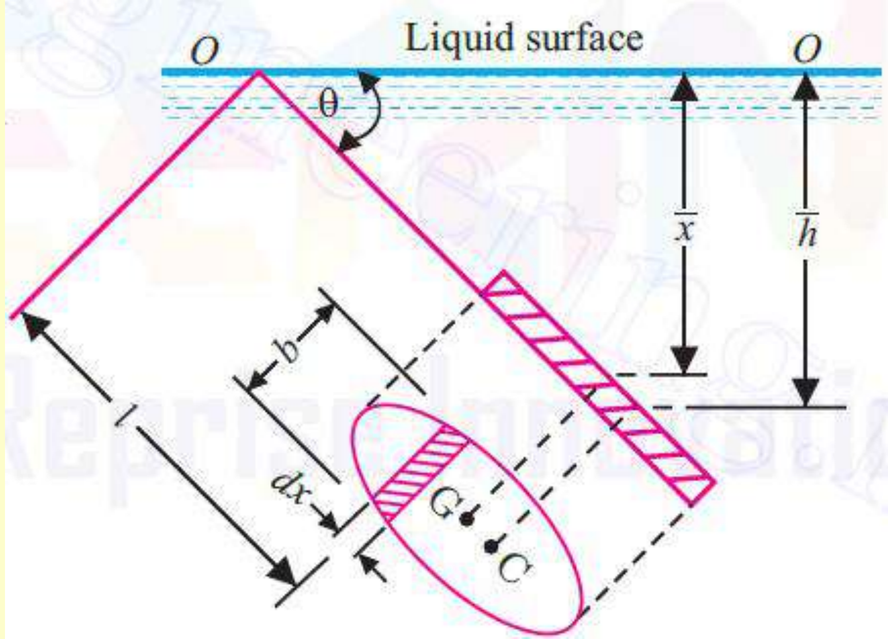
$$M = w \sin\theta \,.\, I_0 \qquad ...(i)$$

$M = w \sin\theta \, . \, I_0$ ...(*i*)

The sum of moments of all such pressures about *O* is also equal to $\frac{P\bar{h}}{\sin\theta}$ ...(*ii*)

$\sin\theta = \frac{\bar{h}}{CO}$

where, *P* is the total pressure on the surface.



Equating eqns. (*i*) and (*ii*), we get:

$$\frac{P\bar{h}}{\sin\theta} = w \sin\theta \, . \, I_0$$

$$\frac{wA\bar{x}\bar{h}}{\sin\theta} = w \sin\theta \, . \, I_0 \qquad (\because P = wA\bar{x})$$

$$\bar{h} = \frac{I_0 \sin^2\theta}{A\bar{x}} \qquad ...(iii)$$

Also, $I_0 = I_G + Ah^2$ ...Theorem of parallel axes.

where, I_G = Moment of inertia of figure about horizontal axis through its centre of gravity, and

h = Distance between 0 and the centre of gravity of the figure $= l \left(= \frac{\bar{x}}{\sin\theta} \right)$ in this case.

$h = l$

Rearranging equation (*iii*), we have:

$$\bar{h} = \frac{\sin^2\theta}{A\bar{x}} (I_G + Al^2)$$

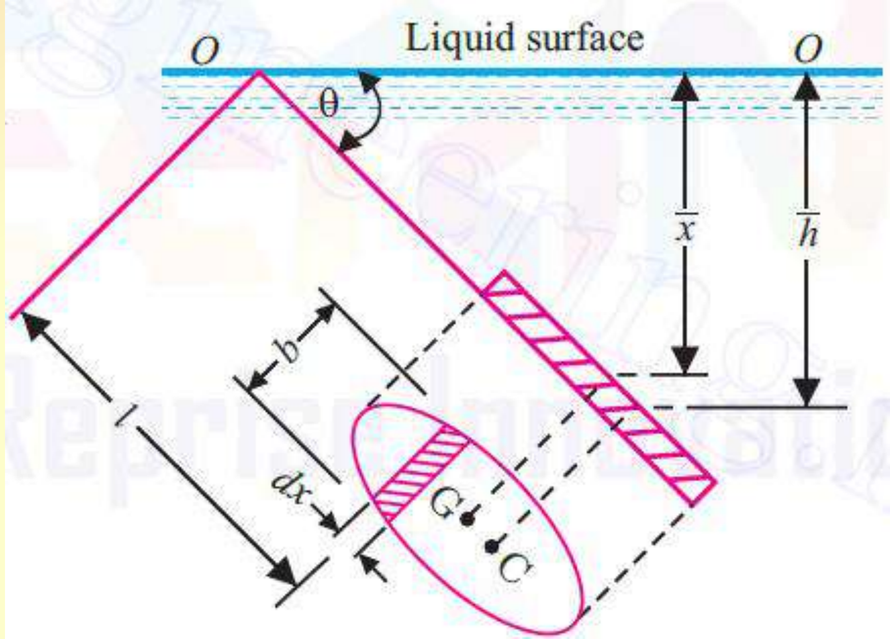
$$\bar{h} = \frac{\sin^2\theta}{A\bar{x}}(I_G + Al^2)$$

$$= \frac{\sin^2\theta}{A\bar{x}}\left[I_G + A\left(\frac{\bar{x}}{\sin\theta}\right)^2\right] = \frac{I_G\sin^2\theta}{A\bar{x}} + \bar{x}$$

Hence, *centre of pressure* $\bar{h} = \dfrac{I_G\sin^2\theta}{A\bar{x}} + \bar{x}$

It will be noticed that if θ = 90° eqn (3.3) becomes the same as equation (3.2).

$$\bar{h} = \frac{I_G}{A\bar{x}} + \bar{x} \qquad \text{...(3.2)}$$



Example 3.24. *An inclined rectangular sluice gate AB 1.2 m by 5 m size as shown in Fig. 3.33 is installed to control the discharge of water. The end A is hinged. Determine the force normal to the gate applied at B to open it.*

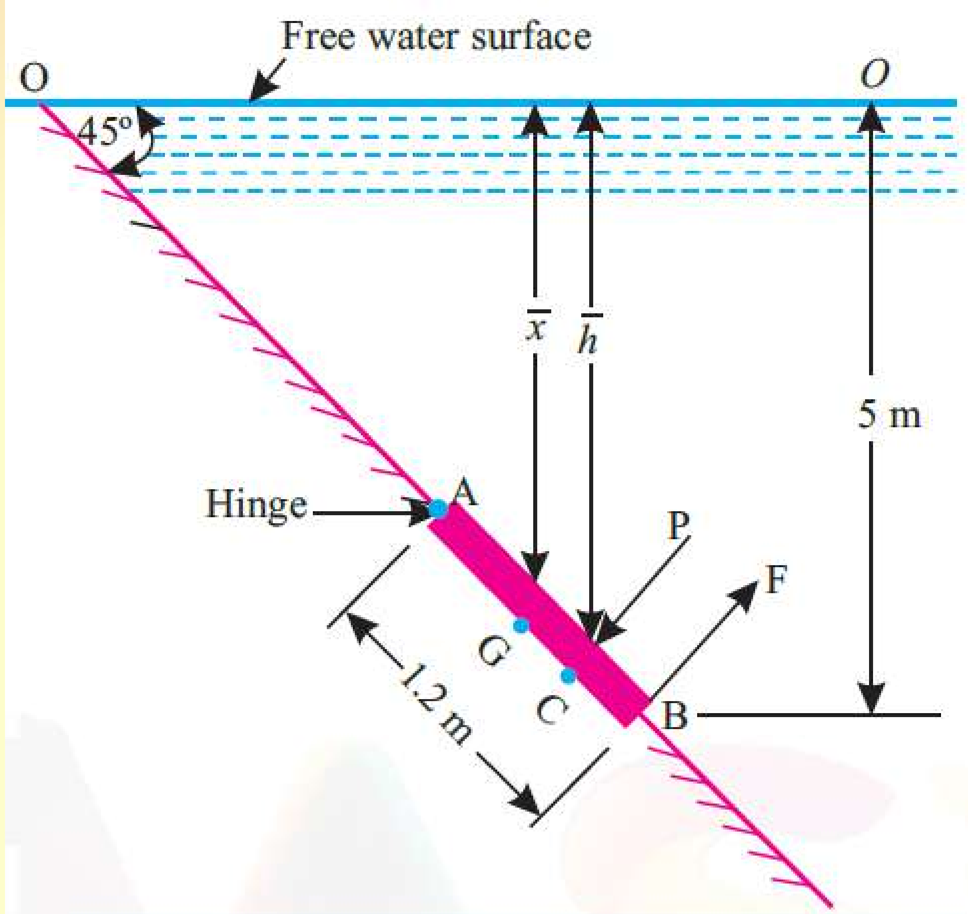


Fig. 3.33

Area of the gate = 1.2 × 5 = 6 m²

Depth of c.g. of the gate from free water surface,

$$\bar{x} = 5 - BG \sin 45°$$
$$= 5 - 0.6 \times 0.707 = 4.576 \text{ m}$$

The total pressure force (P) acting on the gate,

$$P = wA\bar{x}$$
$$= 9.81 \times 6 \times 4.576 = 269.3 \text{ kN}$$

This force acts at a depth $\bar{h}$, given by the relation:

$$\bar{h} = \frac{I_G \sin^2\theta}{A\bar{x}} + \bar{x}$$

$$I_G = M.O.I. \text{ of gate} = \frac{bd^3}{12} = \frac{5 \times 1.2^3}{12} = 0.72 \text{ m}^4.$$

$$\bar{h} = \frac{0.72 \times \sin^2 45°}{6 \times 4.576} + 4.576 = 4.589 \text{ m}$$

From Fig. 3.33, we have $\frac{\bar{h}}{OC} = \sin 45°$

$$\text{Distance, } OC = \frac{\bar{h}}{\sin 45°} = \frac{4.589}{0.707} = 6.49 \text{ m;}$$

$$\text{Distance, } OB = \frac{5}{\sin 45°} = 7.072 \text{ m}$$

$$\therefore \quad \text{Distance, } BC = OB - OC = 7.072 - 6.49 = 0.582 \text{ m}$$

$$\text{Distance, } AC = AB - BC = 1.2 - 0.582 = 0.618 \text{ m}$$

Taking moments about the hinge A, we get:

$$F \times AB = P \times AC$$

$$F = \frac{P \times AC}{AB}$$

$$= \frac{269.3 \times 0.618}{1.2} = \mathbf{138.69 \text{ kN}} \textbf{ (Ans.)}$$

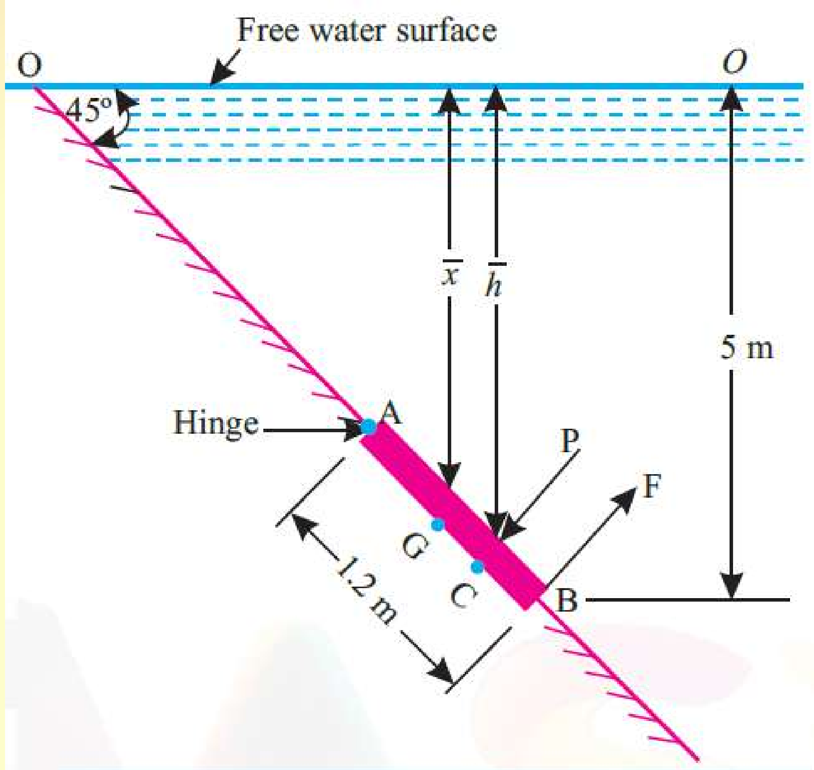


Fig. 3.33

Example 3.25. *A 6 m × 2 m rectangular gate is hinged at the base and is inclined at an angle of 60° with the horizontal. The upper end of the gate is kept in position by a weight of 60 kN acting at angle of 90° as shown in Fig. 3.34. Neglecting the weight of the gate, find the level of water when the gate begins to fall.*

Solution. Length of the gate, $l = 6$ m

Width of the gate, $b = 2$ m

Inclination, $\theta = 60°$

Weight, $W = 60$ kN

Level of water when the gate begins to fall:

Let, h = Height of free water surface from the bottom when the gate just begins to fall.

Then, length of gate in the shape of plate, submerged in water,

$$AD = \frac{AC}{\sin\theta} = \frac{h}{\sin 60°} = \frac{h}{0.866} = 1.1547\,h$$

∴ Area of the gate immersed in water,

$$A = AD \times \text{width}$$
$$= 1.1547\,h \times 2 = 2.309\,h\ \text{m}^2$$

Also depth of *c.g.* of the immersed area,

$$\bar{x} = \frac{h}{2} = 0.5\,h$$

Total pressure on the gate,

$$P = wA\bar{x} = 9.81 \times 2.309\,h \times 0.5\,h$$
$$= 11.326\,h^2\ \text{kN}$$

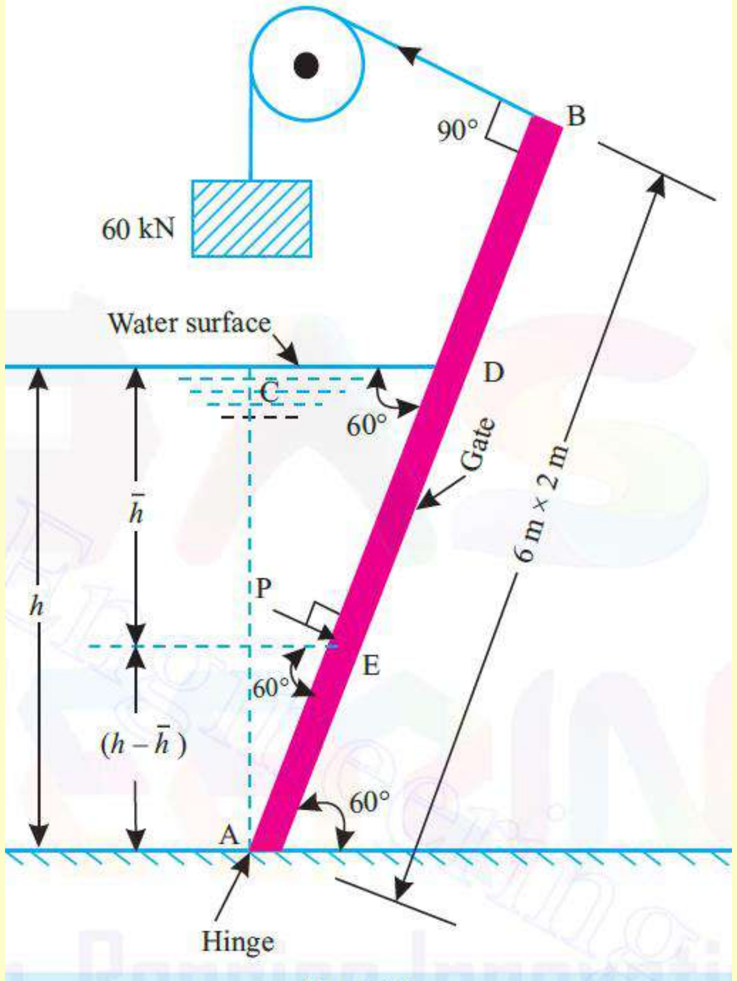


Fig. 3.34

FLUID MECHANICS

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Lecture – 10

Dr Mohammed Tareq Khaleel

Sophomore (year 2)

Building and Construction Technology Engineering Department

HYDROSTATIC FORCES ON SURFACES

CURVED IMMERSED SURFACE

At any point on the curved surface, the pressure acts normal to the surface.

Thus if ***dA*** is the area of a small element of the curved surface lying at a vertical depth of ***h*** from surface of the liquid, then the total pressure on the elemental area is,

$$dp = p \times dA = (wh) \times dA \quad ...(3.4)$$

This force *dP* acts normal to the surface.

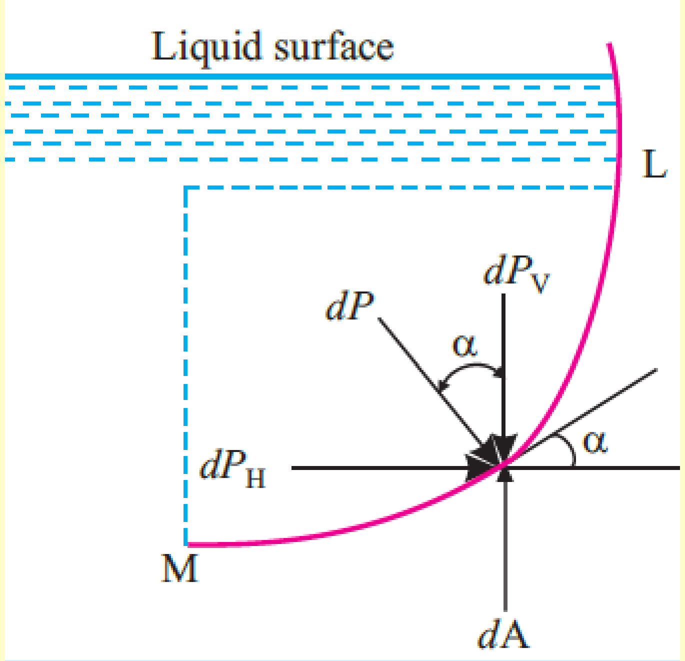


Fig. 3.42

integration of eqn. (3.4) would provide the total pressure on the curved surface and hence,

$$P = \int wh\,dA \quad ...(3.5)$$

But, is case of curved surface the direction of the total pressures on the elementary areas are not in the same direction (varies from point to point).

Thus the integration of eqn. (3.5) for curved surface is **impossible**.

The problem, however, can be solved by resolving the force P into horizontal and vertical components P_H and P_V. Then total force on the curved surface is,

$$P = \sqrt{P_H^2 + P_V^2} \quad \text{...(3.6)}$$

The direction of the resultant force P with the horizontal is given by: $\tan\theta = \frac{P_V}{P_H}$

$$\theta = \tan^{-1}\left(\frac{P_V}{P_H}\right) \quad \text{...(3.7)}$$

P_H = Total pressure force on the projected area of the curved surface on vertical plane, and

P_V = Weight of the liquid supported by the curved surface upto free surface of liquid.

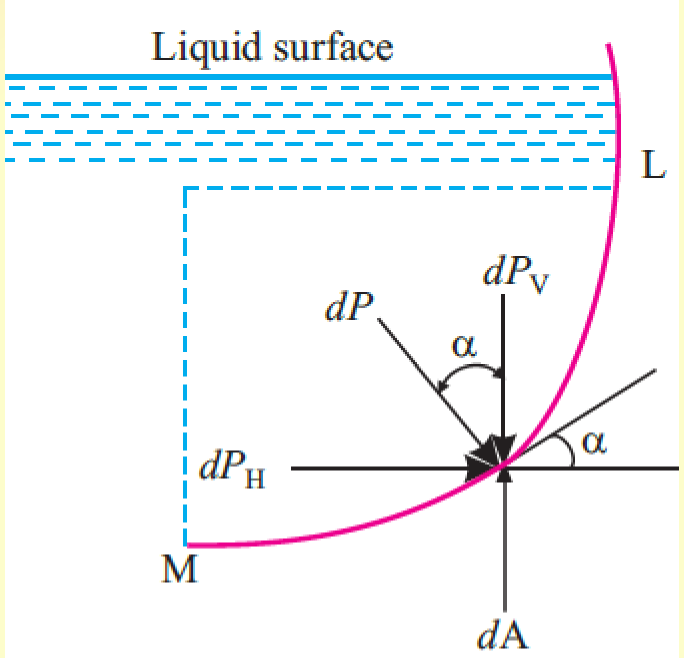


Fig. 3.42

Example 3.34. *Fig. 3.45. shows a curved surface LM, which is in the form of a quadrant of a circle of radius 3 m, immersed in the water. If the width of the gate is unity, calculate the horizontal and vertical components of the total force acting on the curved surface.*

Solution. Radius of the gate = 3 m

Width of the gate = 1m

Refer to Fig. 3.45.

Distance $LO = OM = 3$ m

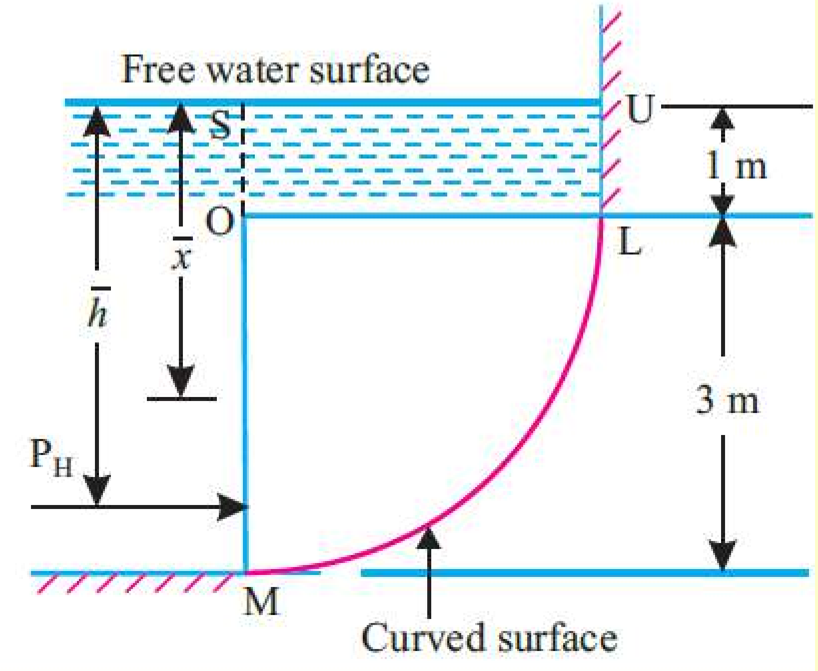


Fig. 3.45. Curved surface (gate).

Horizontal component of total force, P_H:

Horizontal force (P_H) exerted by water on gate is given by,

P_H = Total pressure force on the projected area of curved surface *LM* on vertical plane

= Total pressure force on *OM*

(projected area of curved surface on vertical plane

$= wA\bar{x}$

$A = OM \times 1 = 3 \times 1 = 3\text{m}^2$

$\bar{x} = 1 + \frac{3}{2} = 2.5$ m

$P_H = 9.81 \times (3 \times 1) \times 2.5 = \mathbf{73.57\ kN}$ **(Ans.)**

The point of application of P_H is given by:

$$\bar{h} = \frac{I_G}{A\bar{x}} + \bar{x}$$

$$I_G = M.O.I. \text{ of } OM \text{ about its } c.g. = \frac{bd^3}{12} = \frac{1 \times 3^3}{12} = 2.25 \text{ m}^4$$

$$\bar{h} = \frac{2.25}{(3 \times 1) \times 2.5} + 2.5 = \mathbf{2.8 \text{ m from water surface}} \text{ (Ans.)}$$

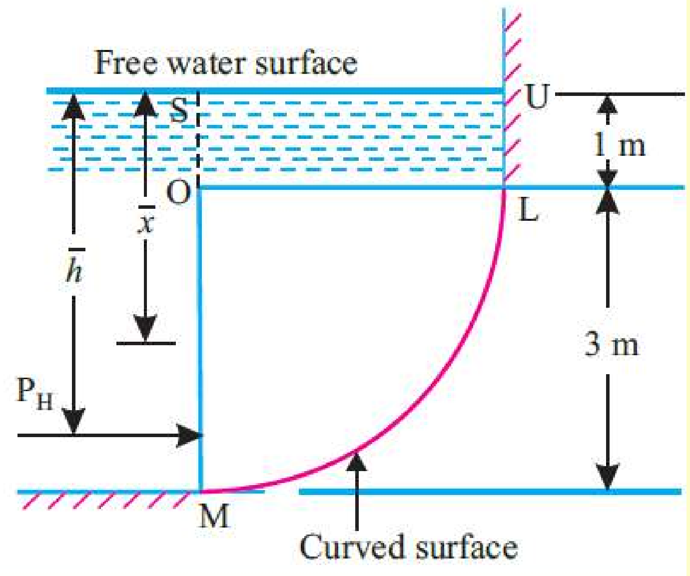


Fig. 3.45. Curved surface (gate).

Vertical component of total force, P_V:

Vertical force (P_V) exerted by water is given by:

P_V = Weight of water supported by LM upto free surface

= weight of portion $ULMOS$

= weight of $ULOS$ + weight of water in LOM

= w (volume of $ULOS$ + volume of LOM)

$$= 9.81\left[UL \times LO + \frac{\pi \times (LO)^2}{4} \times 1\right] = 9.81\left[1 \times 3 + \frac{\pi \times 3^2}{4} \times 1\right]$$

= 9.81 (3 + 7.068) kN = **98.77 kN** (Ans.)

Example 3.38. *Fig. 3.49 shows a radial gate. If it is 3 m long, find the magnitude and direction of the resultant force acting on it.*

Solution. Length of radial gate = 3 m

Refer to Fig. 3.49.

$MU = 3 \sin 60° = 2.6$ m

Horizontal force on the curved surface,

$$P_H = wA\bar{x}$$

$$= 9.81 \times (2.6 \times 3) \times \frac{2.6}{2}$$

$$= 99.47 \text{ kN}$$

It will act at $\frac{2.6}{3}$ or 0.867 m above M.

OR $\bar{h} = \frac{I_G}{A\bar{x}} + \bar{x}$

ST

Vertical force, P_V = Weight of water displaced

= weight of volume equal to $LMU \times 3$.

Now, Area LMU = area LOM – area MUO

$$= \pi R^2 \times \frac{60^0}{360^0} - \frac{1}{2} \times 2.6 \times 3 \cos 60°$$

$$= \pi \times 3^2 \times 1/6 - \frac{1}{2} \times 2.6 \times 3 \times 0.5 = 4.712 - 1.95 = 2.762 \text{ m}^2$$

If θ is measured in degrees then

area of sector $= \frac{\theta}{360°} \times \pi r^2$

$P_V = 2.762 \times 3 \times 9.81 = 81.28$ kN;

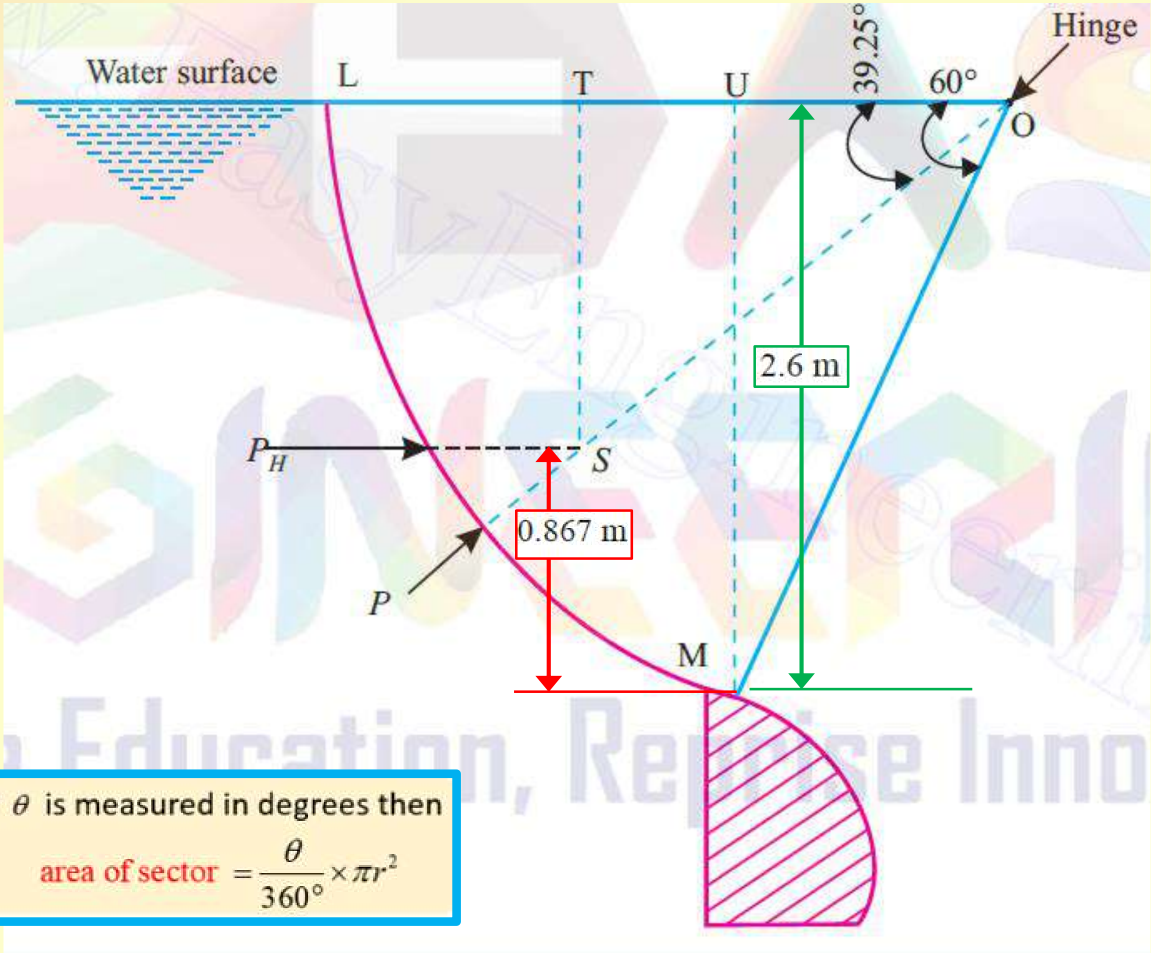


Fig. 3.49

$$P = \sqrt{P_H^2 + P_V^2} = \sqrt{99.47^2 + 81.28^2} = 128.45 \text{ kN}$$

θ = Inclination of P with horizontal.

$$\tan\theta = \frac{P_V}{P_H} = \frac{81.28}{99.47} = 0.817 \quad \text{or} \quad \theta = \mathbf{39.25^\circ} \text{ (Ans.)}$$

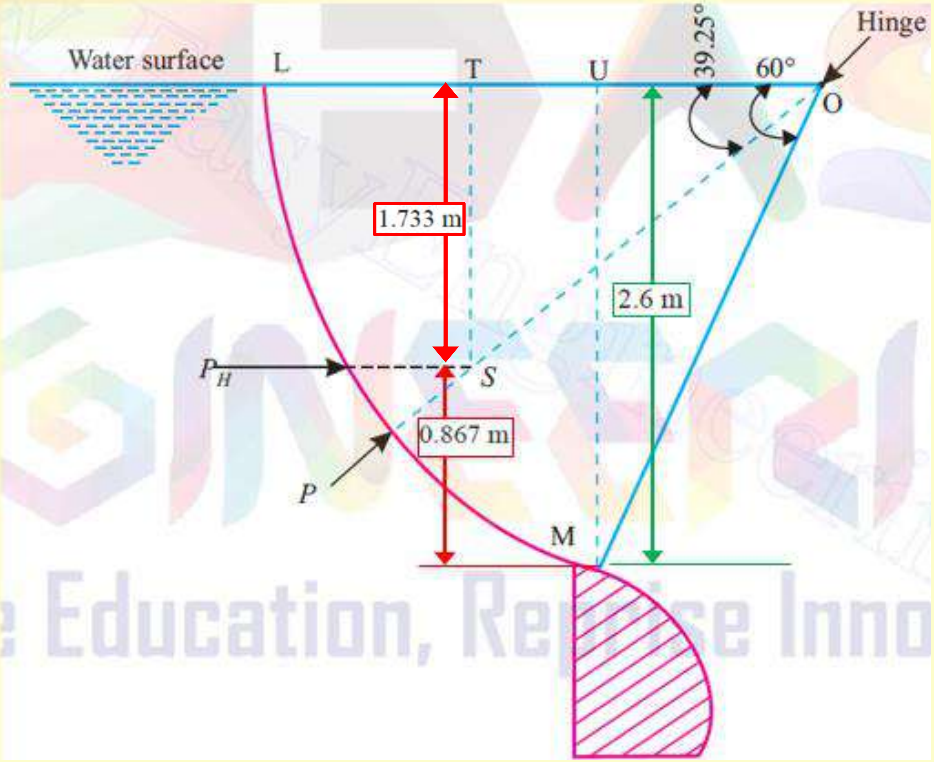
and *P must pass through O.*

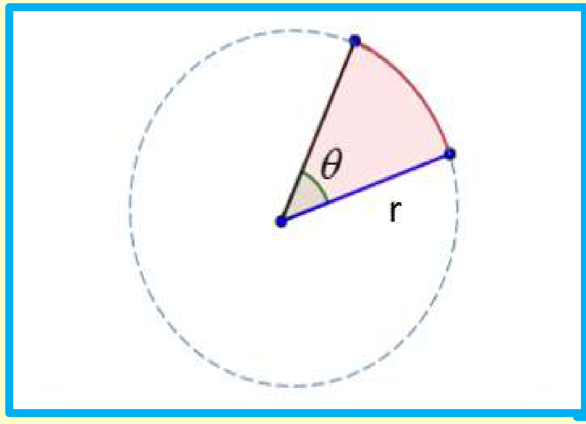
As P_H acts at (2.6 – 0.867) = 1.733 m below water surface,

$$OT = \frac{ST}{\tan 39.25^\circ} = \frac{1.733}{0.817} = 2.12 \text{ m, and}$$

$$UT = OT - OU = 2.12 - 3\cos 60^\circ = 0.62 \text{ m}$$

Hence point of application of P is *0.62 m to the left of MU and 1.733 m below water surface.*





If θ is measured in degrees then

$$\text{area of sector} = \frac{\theta}{360^\circ} \times \pi r^2$$

If θ is measured in radians then

$$\text{area of sector} = \frac{1}{2} r^2 \theta$$

FLUID MECHANICS

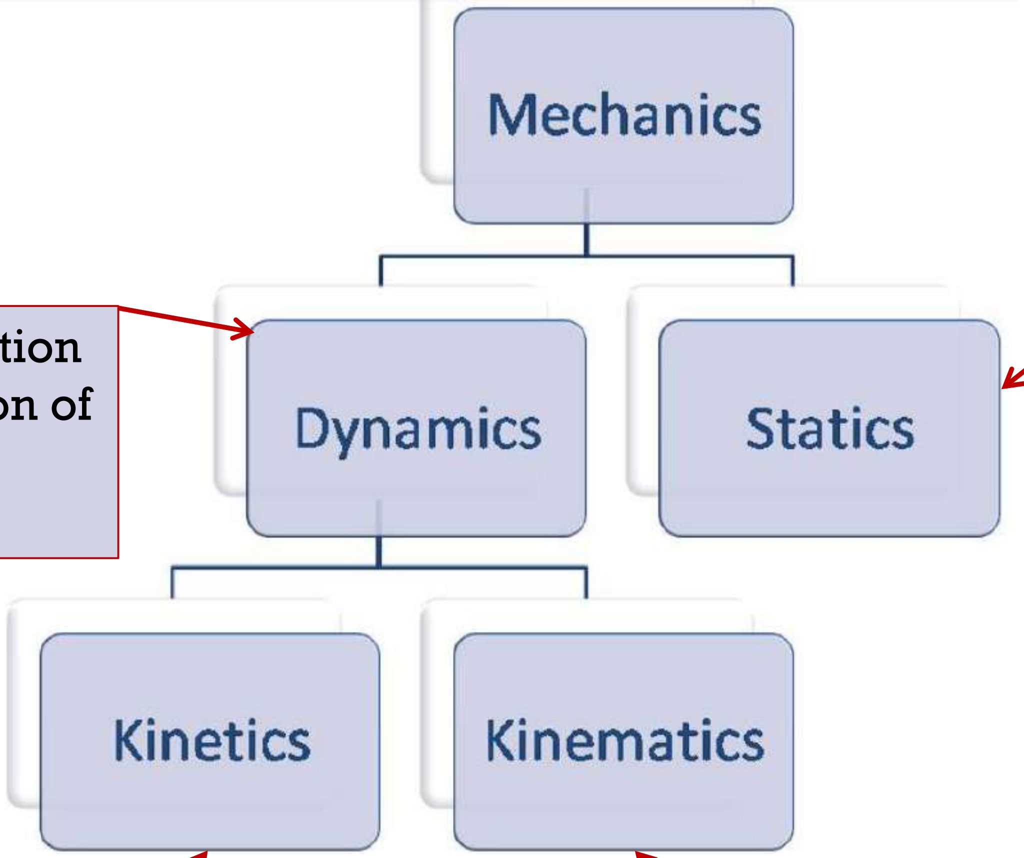
Lecture – 11

Dr Mohammed Tareq Khaleel

Year 2

Building and Construction Technology Engineering Department

BACKGROUND



The study of fluids in motion involves the consideration of ***both*** *the* ***kinetics*** *and* ***kinematics***

When the fluids are at **rest**, the only fluid property of significance is the **specific weight** of the fluids.

The science which deals with ***the action of the forces in producing or changing motion of fluids*** *is known as "****hydrokinetics****" (or simply* ***kinetics****).*

The science which deals with the geometry of motion of fluids ***without reference to the forces causing the motion*** *is known as "****hydrokinematics****" (or simply* ***kinematics****).*

TYPES OF FLOW

1. Steady and Unsteady Flows

- **Steady flow:** The type of flow in which the fluid characteristics like velocity, pressure, density, etc. at a point ***do not change* with time** is called *steady flow*. Mathematically, we have:

$$\left(\frac{\partial u}{\partial t}\right)_{x_0, y_0, z_0} = 0; \left(\frac{\partial v}{\partial t}\right)_{x_0, y_0, z_0} = 0; \left(\frac{\partial w}{\partial t}\right)_{x_0, y_0, z_0} = 0$$

$$\left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} = 0; \left(\frac{\partial \rho}{\partial t}\right)_{x_0, y_0, z_0} = 0; \text{ and so on}$$

where (x_0, y_0, z_0) is a fixed point in a fluid field where these variables are being measured *w.r.t.* time.

where u, v and w are velocity components in x, y and z directions respectively.

Example. *Flow through a prismatic or non-prismatic conduit at a constant flow rate Q m^3/s is steady.*

- **Unsteady flow:** It is that type of flow in which the velocity, pressure or density at a point ***change w.r.t. time***. Mathematically, we have:

$$\left(\frac{\partial u}{\partial t}\right)_{x_0, y_0, z_0} \neq 0; \left(\frac{\partial v}{\partial t}\right)_{x_0, y_0, z_0} \neq 0; \left(\frac{\partial w}{\partial t}\right)_{x_0, y_0, z_0} \neq 0$$

$$\left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} \neq 0; \left(\frac{\partial \rho}{\partial t}\right)_{x_0, y_0, z_0} \neq 0; \text{ and so on}$$

Example. *The flow in a pipe whose valve is being opened or closed gradually*

TYPES OF FLOW

2. Uniform and Non-uniform Flows

- **Uniform flow:** The type of flow, in which the **velocity at any given time** *does not change* with **respect to space** is called *uniform flow*. Mathematically, we have:

$$\left(\frac{\partial V}{\partial s}\right)_{t = \text{constant}} = 0$$

where, ∂V = Change in velocity, and

∂s = Displacement in any direction.

Example. *Flow through a straight prismatic conduit (i.e. flow through a straight pipe of constant diameter).*

- **Non-uniform flow:** It is that type of flow in which the **velocity at any given time** *changes with respect to space*. Mathematically,

$$\left(\frac{\partial V}{\partial s}\right)_{t = \text{constant}} \neq 0$$

Example. *(i) Flow through a non-prismatic conduit.*

(ii) Flow around a uniform diameter pipe-bend or a canal bend.

TYPES OF FLOW

3. One, Two and Three Dimensional Flows:

***i.* One dimensional flow.** It is that type of flow in which the flow parameter such as velocity is a function of time and one space co-ordinate only.

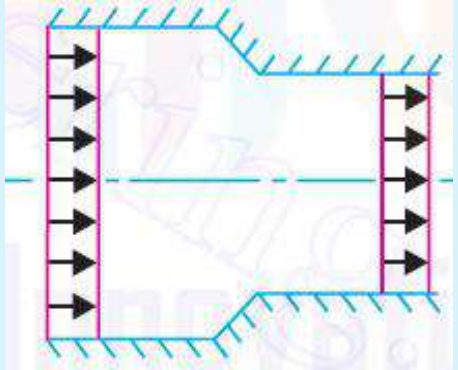


Fig. 5.2. One dimensional flow.

Mathematically:

$$u = f(x),$$
$$v = 0$$
$$w = 0$$

where u, v and w are velocity components in x, y and z directions respectively.

Example: Flow in a pipe considered 1-D when the change (variation) of flow parameters (such as velocity, or pressure, etc.) occur along the length of the pipe, but any change (or variation) over the cross-section is assumed negligible.)

TYPES OF FLOW

***ii.* Two dimensional flow:** The flow in which the velocity is a function of time and two rectangular space coordinates is called *two dimensional flow*.

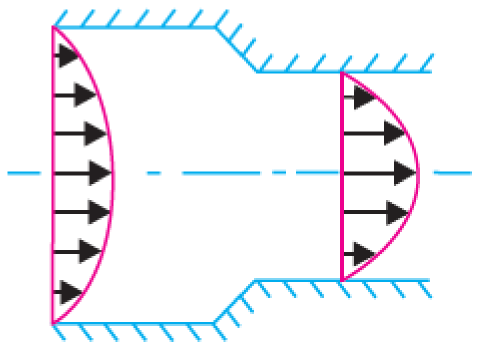


Fig. 5.3. Two dimensional flow.

Mathematically:

$$u = f_1(x,y)$$
$$v = f_2(x,y)$$
$$w = 0$$

Examples. (*i*) *Flow between parallel plates of infinite extent.*

(*ii*) *Flow in the main stream of a wide river.*

TYPES OF FLOW

***iii.* Three dimensional flow:** It is that type of flow in which the Velocity is a function of time and three mutually perpendicular directions.

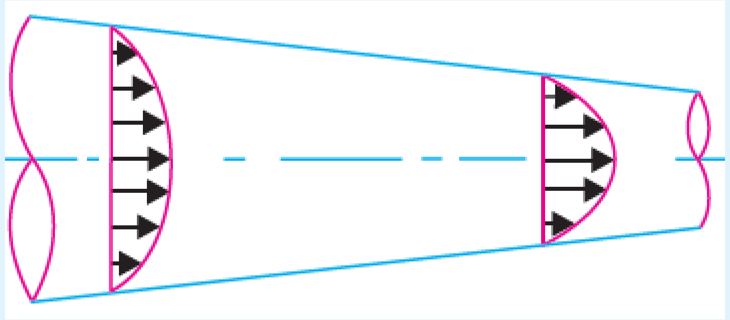


Fig. 5.4. Thee dimensional flow.

Mathematically:

$$u = f_1(x, y, z)$$
$$v = f_2(x, y, z)$$
$$w = f_3(x, y, z)$$

Examples. *(i) Flow in a converging or diverging pipe or channel.*

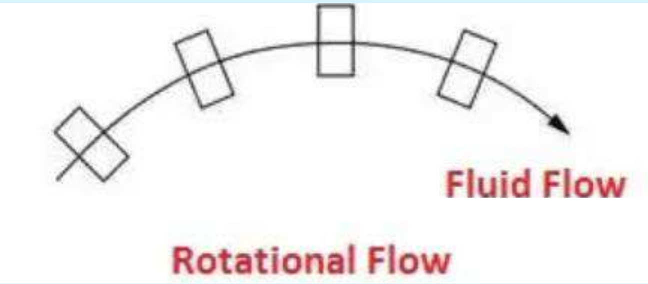
(ii) Flow in a prismatic open channel in which the width and the water depth are of the same order of magnitude.

TYPES OF FLOW

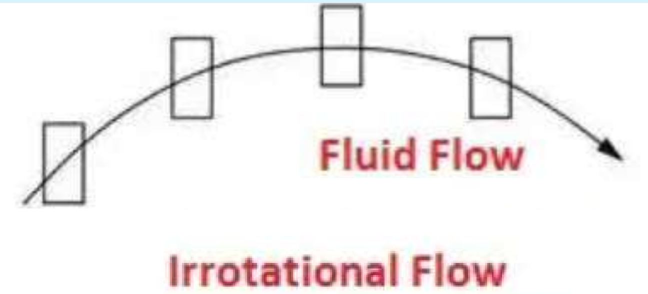
4. Rotational and Irrotational Flows

i. **Rotational flow.** A flow is said to be *rotational* if the fluid particles while moving in the direction of flow *rotate* about their mass centres.

Example. *Motion of liquid in a rotating tank*.



ii. **Irrotational flow.** A flow is said to be *irrotational* if the fluid particles while moving in the direction of flow *do not rotate* about their mass centres.



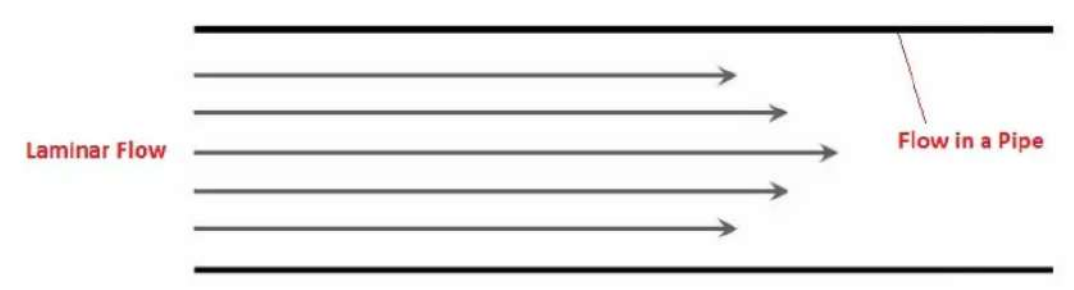
Example. *Flow above a drain hole of a stationary tank or a wash basin.*

TYPES OF FLOW

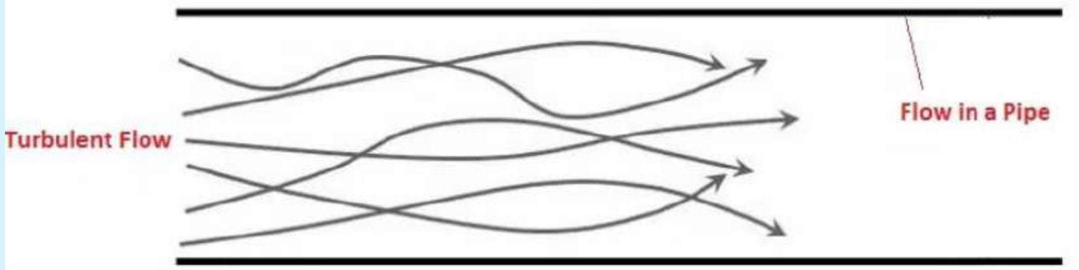
5. Laminar and Turbulent Flows

- **Laminar flow:** A laminar flow is one in which *paths taken by the individual particles do not cross one another and move along well defined paths* (Fig. 5.5), This type of flow is also called *stream-line flow or viscous flow*.

Examples. (i) *Flow through a capillary tube.*
(ii) *Flow of blood in veins and arteries.*
(iii) *Ground water flow.*



- **Turbulent flow:** A turbulent flow is that flow in which fluid *particles move in a zig zag way. Due to the movement of fluid particles in a zigzag manner, eddies are formed which are responsible for the high energy loss.*



Example. *High velocity flow in a conduit of large size. Nearly all fluid flow problems encountered in engineering practice have a turbulent character.*

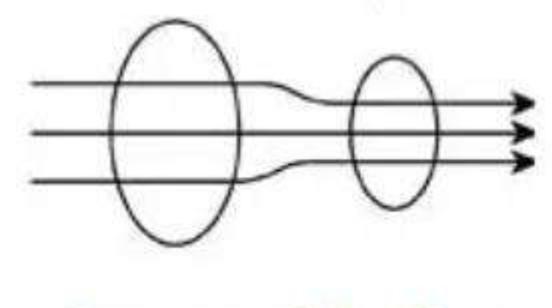
TYPES OF FLOW

6. Compressible and Incompressible Flows

- **Compressible flow:** It is that type of flow in which the *density* (ρ) *of the fluid changes* from point to point (or in other words *density is not constant for this flow*).

Mathematically: $\rho \neq \text{constant.}$

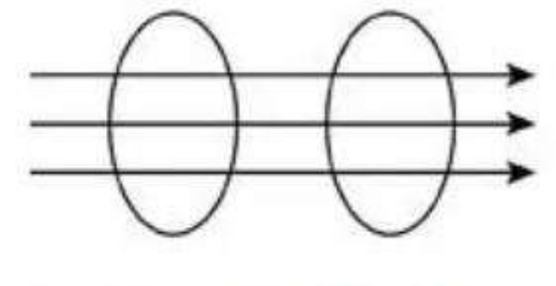
Example. *Flow of gases through orifices, nozzles, gas turbines, etc.*



- **Incompressible flow:** *It is that type of flow in which density is constant for the fluid flow. Liquids are generally considered flowing incompressibly*.

Mathematically: $\rho = \text{constant.}$

Example. *Subsonic aerodynamics.*



TYPES OF FLOW LINES

Whenever a fluid is in motion, its innumerable particles move along certain lines depending upon the conditions of flow.

1. **Path line:** A path line (Fig. 5.7) is the *path followed by a fluid particle in motion*.

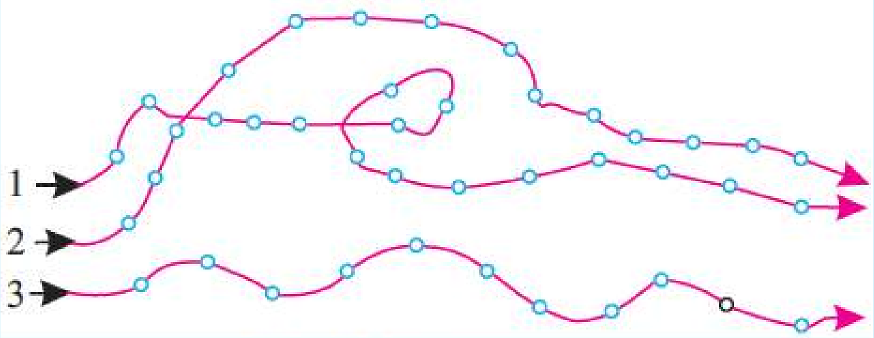


Fig. 5.7. Path lines.

2. Stream line: A ***stream line*** way be defined as *an imaginary line within the flow so that the tangent at any point on it indicates the velocity at that point*.

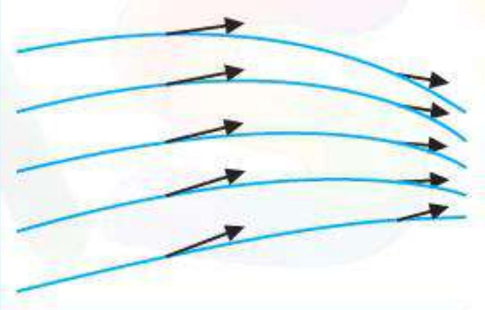


Fig. 5.8. Stream line.

TYPES OF FLOW LINES

STREAM LINE

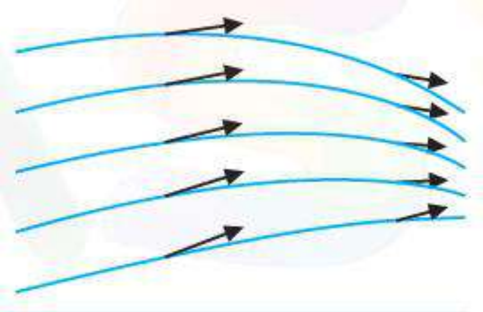


Fig. 5.8. Stream line.

Following *points* about **streamlines** are *worth noting*:

1. A streamline cannot intersect itself, nor two streamlines can cross.
2. There cannot be any movement of the fluid mass across the streamlines.
3. Streamline spacing varies inversely as the velocity; *converging of streamlines in any particular direction shows accelerated flow in that direction*.
4. Whereas a *path line* gives the path of *one particular particle* at successive instants of time, *a streamline* indicates the direction of a *number of particles* at the same instant.
5. The series of streamlines represent the flow pattern at an instant.

- In *steady flow*, the pattern of streamlines remains invariant with time. The path lines and streamlines will then be identical.
- In *unsteady flow*, the pattern of streamlines may or may not remain the same at the next instant.

TYPES OF FLOW LINES

3. Stream Tube: A ***stream tube*** is a *fluid mass bounded by a group of streamlines*.

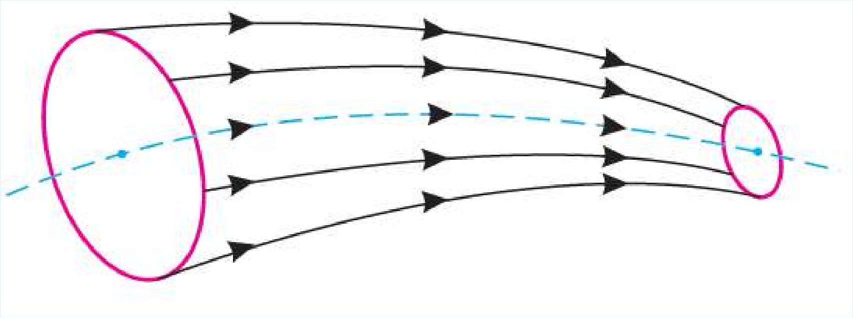


Fig. 5.9. Stream tube.

***Examples of stream tube*:** *Pipes* and *nozzles*.

Following *points* about stream tube are *worth noting*:

1. The stream tube has finite dimensions.
2. As there is no flow perpendicular to stream lines, therefore, there is no flow across the surface (called *stream surface*) of the stream tube. The stream surface functions as if it were a solid wall.
3. The shape of a stream tube changes from one instant to another because of change in the position of streamlines.

RATE OF FLOW (OR DISCHARGE)

- **Rate of flow** (or **discharge**) *is defined as* the *quantity of a liquid flowing per second through a section of pipe or a channel.* It is generally denoted by Q. Let us consider a liquid flowing through a pipe.

Let,

A = Area of cross-section of the pipe, and

V = Average velocity of the liquid.

$\therefore$ Discharge, Q = Area × average velocity *i.e.,* $Q = A.V$ ...(5.21)

If area is in m^2 and velocity is in m/s, then the discharge,

$Q = m^2 \times m/s = m^3/s$ = cumecs.

CONTINUITY EQUATION

- The **continuity equation** *is based on the principle of* ***conservation of mass.*** It states as follows:

"If no fluid is added or removed from the pipe in any length then the mass passing across different sections shall be same."

Consider two cross-sections of a pipe as shown in Fig 5.13

Let, A_1 = Area of the pipe at section 1–1,
V_1 = Velocity of the fluid at section 1–1,
ρ_1 = Density of the fluid at section 1–1,

and A_2, V_2, ρ_2 are corresponding values at sections 2–2.

The total quantity of fluid passing through section 1–1= $\rho_1 A_1 V_1$
and, the total quantity of fluid passing through section 2–2 = $\rho_2 A_2 V_2$

From the law of conservation of mass (theorem of continuity), we have:

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \quad ...(5.22)$$

Eqn. (5.22) is applicable to the compressible as well as incompressible fluids and is called ***Continuity Equation.*** In case of *incompressible fluids,* $\rho_1 = \rho_2$ and the continuity eqn. (5.21) reduces to:

$$A_1 V_1 = A_2 V_2 \quad ...(5.23)$$

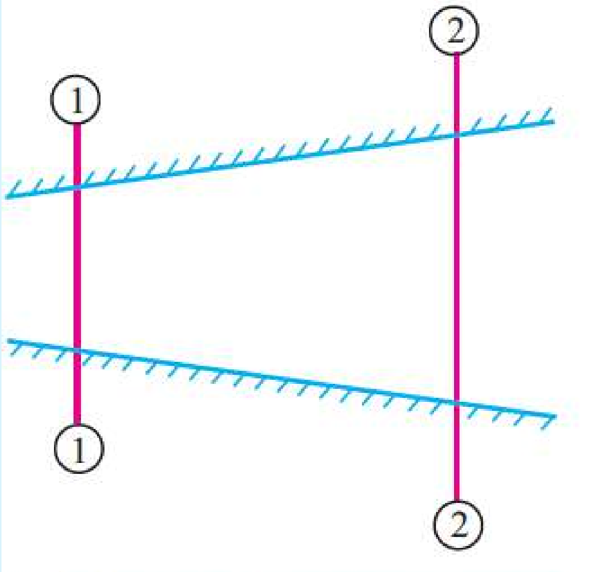


Fig. 5.13. Fluid flow through a pipe.

CONTINUITY EQUATION

Example 5.11. *The diameters of a pipe at the sections 1-1and 2-2 are 200 mm and 300 mm respectively. If the velocity of water flowing through the pipe at section 1-1 is 4m/s, find:*

(i) Discharge through the pipe, and

(ii) Velocity of water at section 2-2

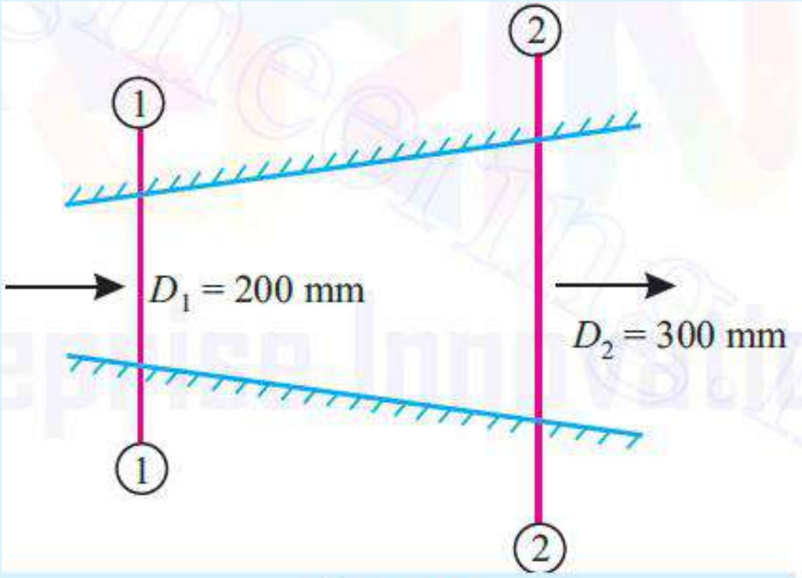


Fig. 5.14

Solution. Diameter of the pipe at *section* 1-1,

$$D_1 = 200 \text{ mm} = 0.2 \text{ m}$$

$$\therefore \quad \text{Area, } A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times 0.2^2 = 0.0314 \text{ m}^2$$

$$\text{Velocity, } V_1 = 4 \text{ m/s}$$

Diameter of the pipe at *section* 2-2,

$$D_2 = 300 \text{ mm}$$

$$\therefore \quad \text{Area, } A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} \times 0.3^2 = 0.0707 \text{ m}^2$$

(i) Discharge through the pipe, Q:

Using the relation,

$Q = A_1 V_1$, we have:

$$Q = 0.0314 \times 4 = \mathbf{0.1256 \text{ m}^3/\text{s}}$$

(ii) Velocity of water at section 2-2, V_2:

Using the relation,

$A_1 V_1 = A_2 V_2$, we have:

$$V_2 = \frac{A_1 V_1}{A_2} = \frac{0.0314 \times 4}{0.0707}$$

$$= \mathbf{1.77 \text{ m/s}} \text{ (Ans.)}$$

FLUID MECHANICS

Lecture – 12

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DIFFERENT TYPES OF HEADS (OR ENERGIES) OF A LIQUID IN MOTION

There are three types of energies or heads of flowing liquids:

1. Potential head or potential energy:

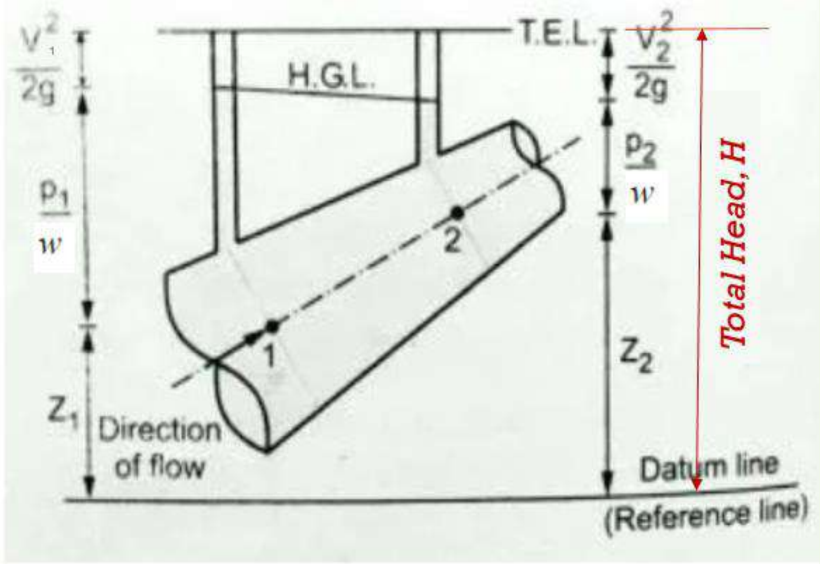
This is due to configuration or position above some suitable datum line. It is denoted by z.

2. Velocity head or kinetic energy:

This is due to velocity of flowing liquid and is measured as $\frac{V^2}{2g}$ where, V is the velocity of flow and 'g' is the acceleration due to gravity ($g = 9.81$)

3. Pressure head or pressure energy:

This is due to the pressure of liquid and reckoned as $\frac{p}{w}$ where, p is the pressure, and w is the weight density of the liquid.



Total head/energy:

Total head of a liquid particle in motion is the sum of its potential head, kinetic head and pressure head. Mathematically,

$$\text{Total head, } H = z + \frac{V^2}{2g} + \frac{p}{w} \text{ m of liquid} \qquad ...[6.1\ (a)]$$

FYI https://www.youtube.com/watch?v=CxqM_kkwgU4

DIFFERENT TYPES OF HEADS (OR ENERGIES) OF A LIQUID IN MOTION

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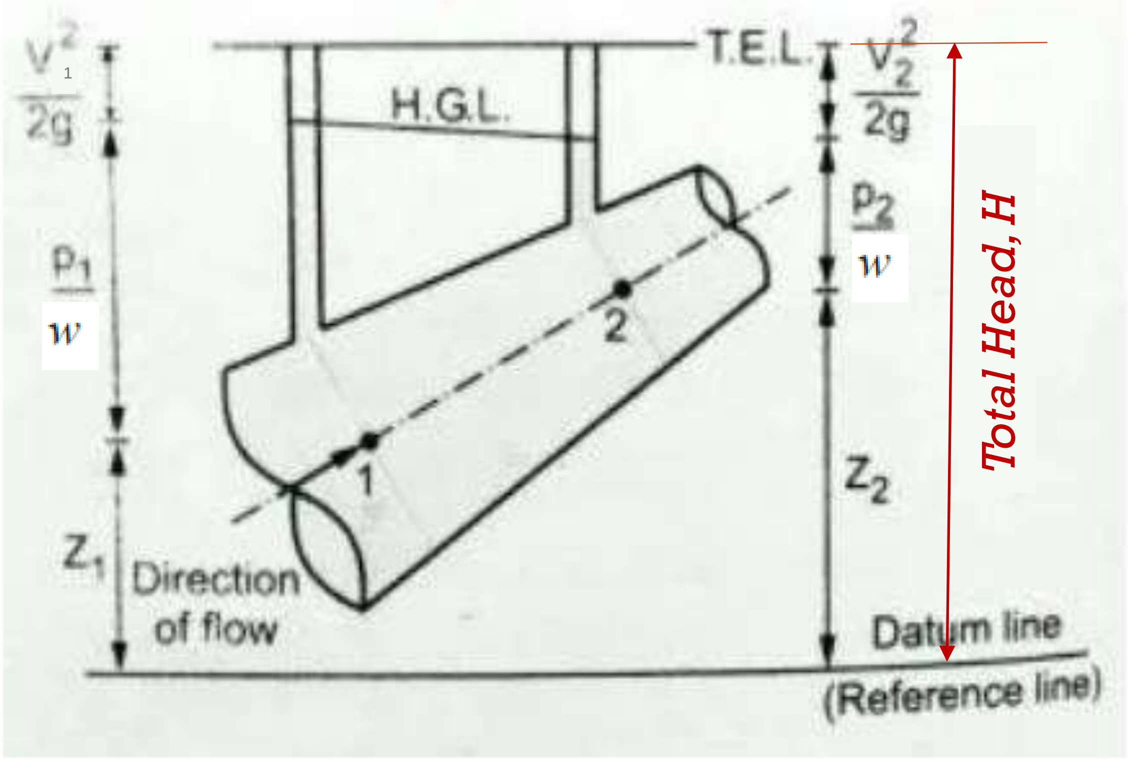


Diagram of HGL and TEL

Total energy line (**T.E.L**) – Line represents the sum of pressure head, potential head, and velocity head. $z + \frac{V^2}{2g} + \frac{p}{w}$

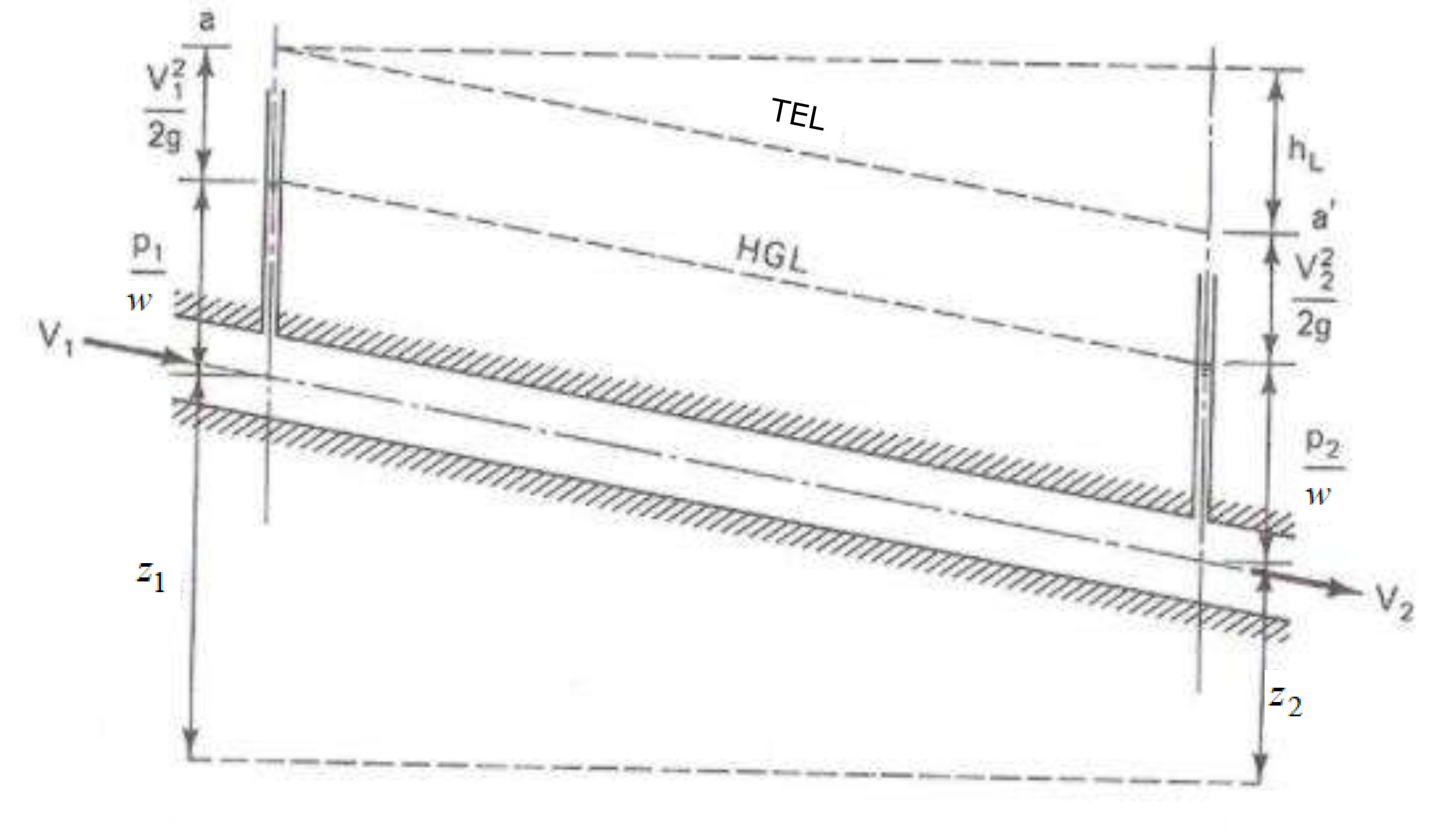
Hydraulic Grade Line **H.G.L** represents the sum of pressure head and potential head $\frac{p}{w} + z$

In ideal condition, the **T.E.L** is Horizontal (means that there is NO Losses)

Hydraulic Grade Line (**H.G.L**)
Total Energy Line (**T.E.L**) 0r Energy Grade Line (E.G.L)

https://www.youtube.com/watch?v=moI4DQNirAw

DIFFERENT TYPES OF HEADS (OR ENERGIES) OF A LIQUID IN MOTION



Energy Head and Head Loss in Real Fluid

https://www.youtube.com/watch?v=moI4DQNirAw

DIFFERENT TYPES OF HEADS (OR ENERGIES) OF A LIQUID IN MOTION

Example 6.1. *In a pipe of 90 mm diameter water is flowing with a mean velocity of 2 m/s and at a gauge pressure of 350 kN/m². Determine the total head, if the pipe is 8 metres above the datum line. Neglect friction.*

Solution. Diameter of the pipe = 90 mm

Pressure, $p = 350 \text{ kN/m}^2$

Velocity of water, $V = 2 \text{ m/s}$

Datum head, $z = 8 \text{ m}$

Specific weight of water, $w = 9.81 \text{ kN/m}^3$

Total head of water, H:

$$H = z + \frac{V^2}{2g} + \frac{p}{w}$$

$$= 8 + \frac{2^2}{2 \times 9.81} + \frac{350}{9.81} = 43.88 \text{ m}$$

$$H = \mathbf{43.88 \text{ m}}$$

BERNOULLI'S EQUATION

Bernoulli's equation states as follows:

"In an ideal incompressible fluid when the flow is steady and continuous, the sum of pressure energy, kinetic energy and potential (or datum) energy is constant along a stream line."

Mathematically,

$$\frac{p}{w} + \frac{V^2}{2g} + z = \text{constant}$$

where,

$$\frac{p}{w} = \text{Pressure energy,}$$

$$\frac{V^2}{2g} = \text{Kinetic energy, and}$$

$$z = \text{Datum (or elevation) energy.}$$

PROOF OF BERNOULLI'S EQUATION

Consider an ideal incompressible liquid through a non-uniform pipe as shown in Fig 6.1.
Let us consider two sections LL and MM and assume that the pipe is running full and there is continuity of flow between the two sections;

Let,
p_1 = Pressure at LL,
V_1 = Velocity of liquid at LL,
z_1 = Height of LL above the datum,
A_1 = Area of pipe at LL, and
p_2, V_2, z_2, A_2 = Corresponding values at MM.

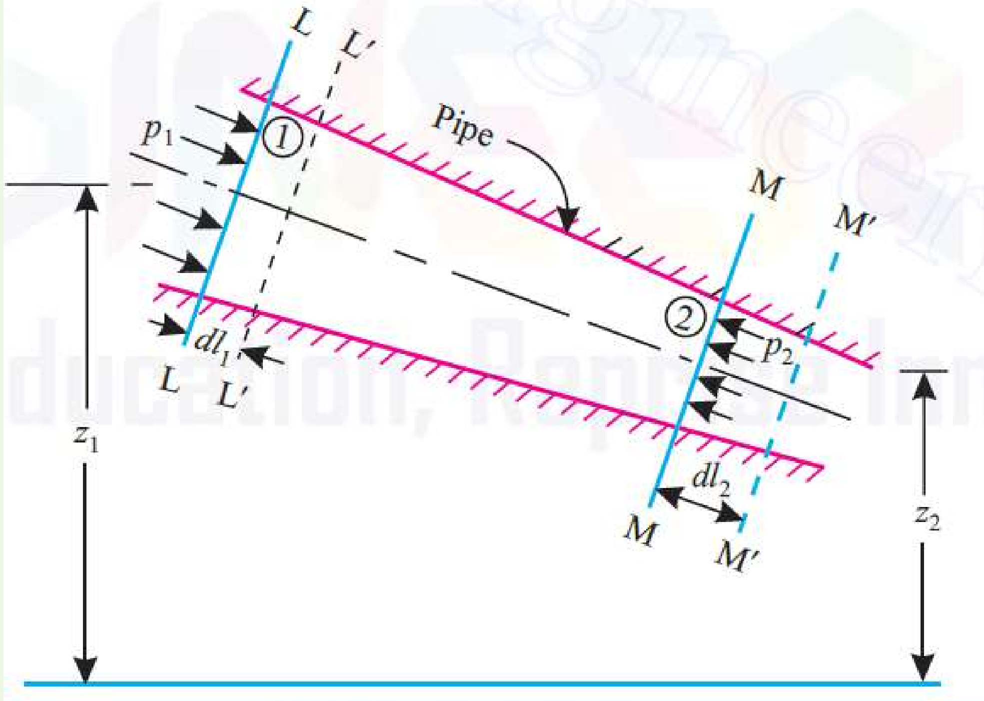


Fig. 6.1. Bernoulli's equation.

Assume the entering liquid between the two sections *LL* and *L'L'* equals to the leaving liquid between the two sections *MM and M'M'*, where dl_1 and dl_2 are the small lengths at these sections, respectively, as shown in Fig. 6.1.

$$\therefore \quad A_1 \, . \, dl_1 = A_2 . dl_2$$

$A \,.\, dl = \Delta$ volume

Work done by pressure at LL, in moving the liquid to $L' L'$

$$= \text{Force} \times \text{distance} = p_1 \, . \, A_1 \, . \, dl_1$$

Similarly, work done by the pressure at MM in moving the liquid to $M'M'$ $= - p_2 . A_2 \, . \, dl_2$
(– ve sign indicates that direction of p_2 is *opposite* to that of p_1)

PROOF OF BERNOULLI'S EQUATION

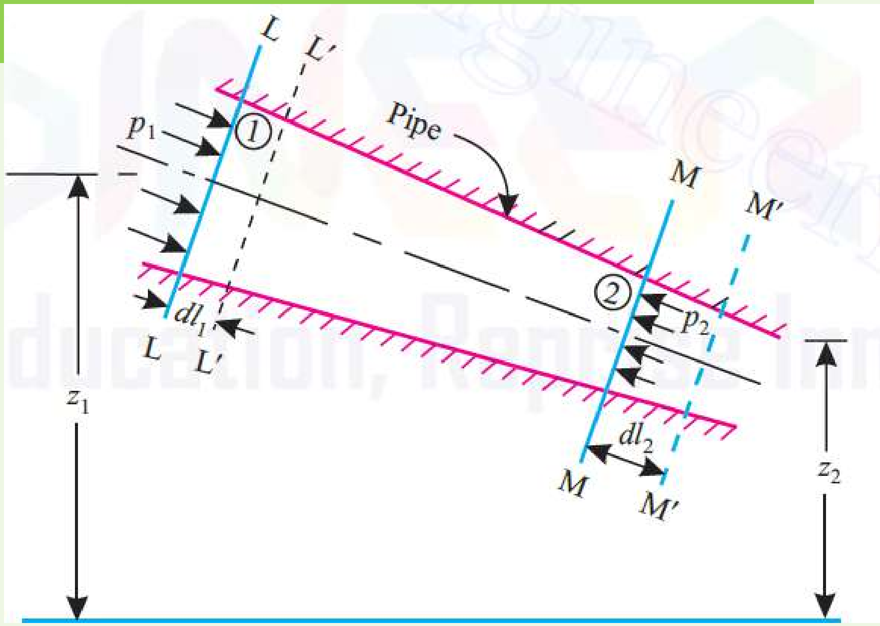
$\therefore$ Total work done by the pressure

$$= p_1 . A_1 dl_1 - p_2 A_2 dl_2$$

$$= p_1 . A_1 dl_1 - p_2 A_1 dl_1 \qquad (\because A_1 dl_1 = A_2 dl_2)$$

$$= A_1 . dl_1 (p_1 - p_2)$$

$$= \frac{W}{w}(p_1 - p_2) \qquad \left(\because A_1 . dl_1 = \frac{W}{w}\right)$$



Loss of potential energy $= W(z_1 - z_2)$

*Potential Energy (P.E.) = (**m.g**).**z** = W.z*

Gain in kinetic energy $= W\left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g}\right) = \frac{W}{2g}(V_2^2 - V_1^2)$

Kinetic Energy*(K.E.)*
= ½ m v^2 (but, m = W/g)
= ½ (W/g) v^2
= W (v^2 /2g)

Also, Loss of potential energy + work done by pressure = Gain in kinetic energy

$$\therefore W(z_1 - z_2) + \frac{W}{w}(p_1 - p_2) = \frac{W}{2g}(V_2^2 - V_1^2)$$

or, $$(z_1 - z_2) + \left(\frac{p_1}{w} - \frac{p_2}{w}\right) = \left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g}\right)$$

or, $$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 \qquad \text{...(6.2)}$$

which proves *Bernoulli's equation.*

OR , can say that,
Work done by pressure =
Δ Kinetic energy + Δ Potential energy

BERNOULLI'S EQUATION

Assumptions:

It may be mentioned that the following *assumptions* are made in the derivation of Bernoulli's equation:

1. The liquid is ideal and incompressible.
2. The flow is steady and continuous.
3. The flow is along the stream line, *i.e.,* it is one-dimensional.
4. The velocity is uniform over the section and is equal to the mean velocity.
5. The only forces acting on the fluid are the *gravity forces* and the *pressure forces*.

Example 6.6. *Water flows in a circular pipe. At one section the diameter is 0.3 m, the static pressure is 260 kPa gauge, the velocity is 3 m/s and the elevation is 10 m above ground level. The elevation at a section downstream is 0 m, and the pipe diameter is 0.15 m. Find out the gauge pressure at the downstream section.*

Frictional effects may be neglected. Assume density of water to be 999 kg/m³.

Solution. Refer to Fig. 6.7. $D_1 = 0.3$ m; $D_2 = 0.15$ m; $z_1 = 0$; $z_2 = 10$ m; $p_1 = 260$ kPa, $V_1 = 3$ m/s; $\rho = 999$ kg/m^3.

From continuity equation, $A_1 V_1 = A_2 V_2$,

$$V_2 = \frac{A_1}{A_2} \times V_1 = \left(\frac{\frac{\pi}{4} D_1^2}{\frac{\pi}{4} D_2^2} \right) \times V_1$$

$$= \left(\frac{D_1}{D_2} \right)^2 \times V_1 = \left(\frac{0.3}{0.15} \right)^2 \times 3 = 12 \text{ m/s}$$

Weight density of water, $w = \rho g = 999 \times 9.81 = 9800.19$ N/m^3

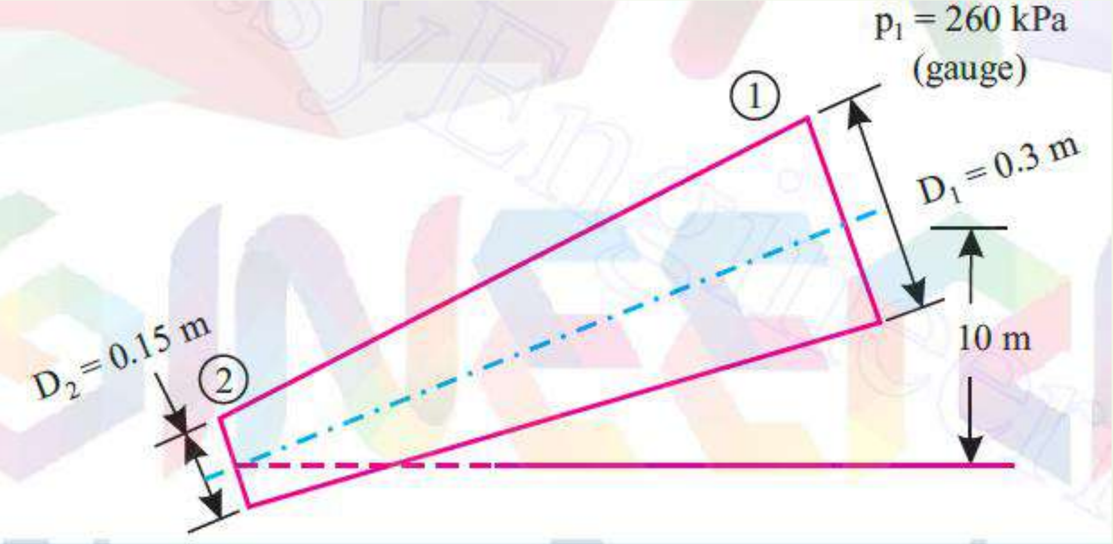


Fig. 6.7

FLUID DYNAMICS - BERNOULLI'S EQUATION

From Bernoulli's equation between sections 1 and 2 (neglecting friction effects as given), we have:

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2$$

$$\frac{260 \times 1000}{9800.19} + \frac{(3)^2}{2 \times 9.81} + 10 = \frac{p_2}{9800.19} + \frac{(12)^2}{2 \times 9.81} + 0$$

$$26.53 + 0.459 + 10 = \frac{p_2}{9800.19} + 7.34$$

$$p_2 = 290566 \text{ N/m}^2$$

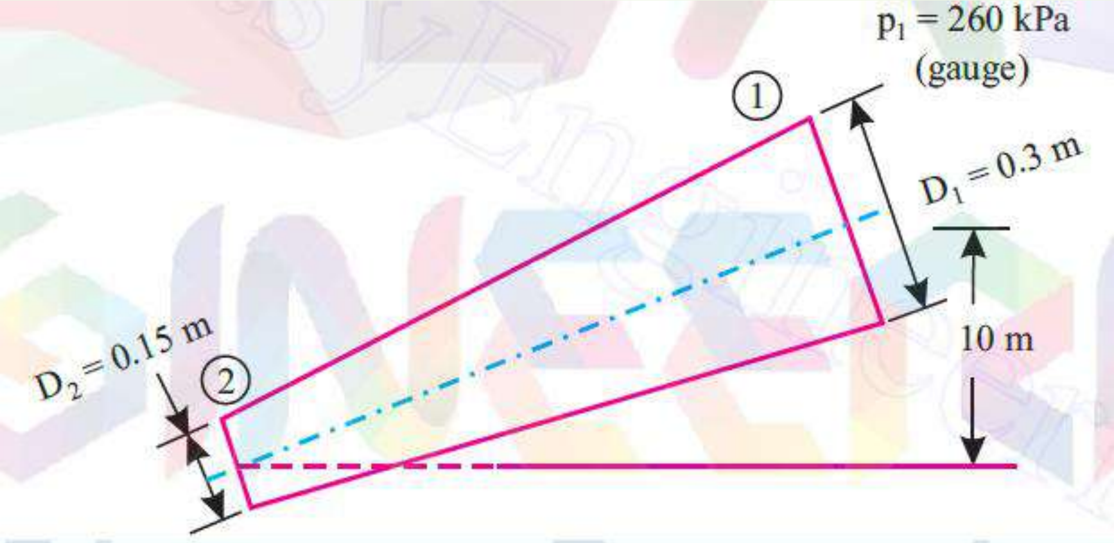


Fig. 6.7

FLUID MECHANICS

Lecture – 13

Dr Mohammed Tareq Khaleel

Year 2

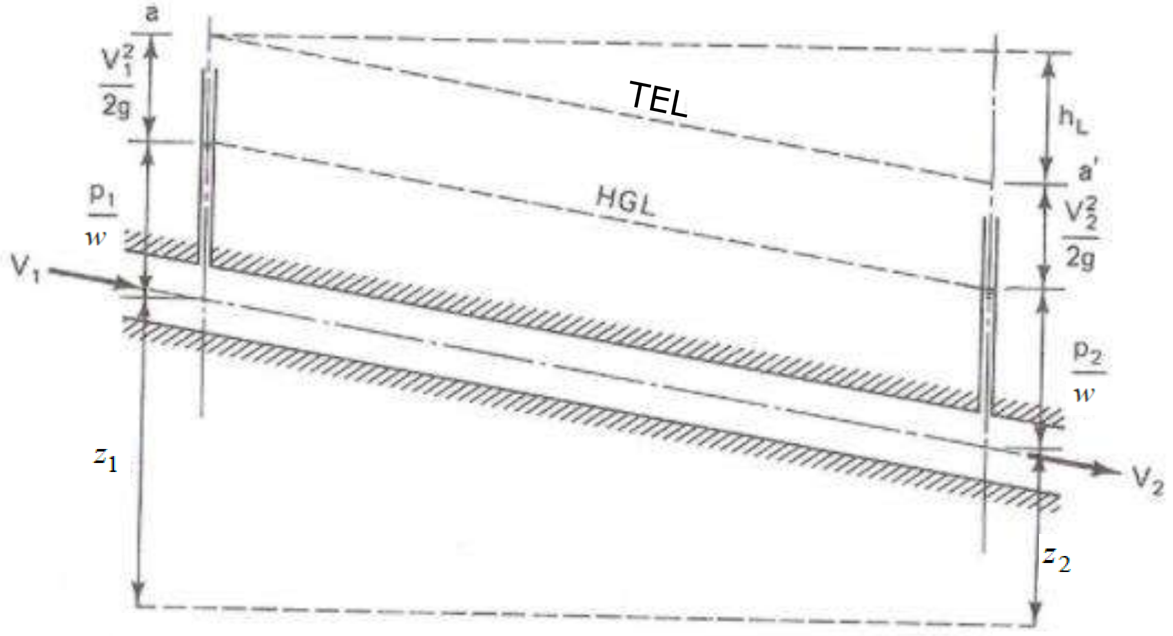
Building and Construction Technology Engineering Department

FLUID DYNAMICS _ BERNOULLI'S EQUATION FOR REAL FLUID

Bernoulli's equation earlier derived was based on the assumption that fluid is non-viscous and therefore frictionless. Practically, all fluids are real (and not ideal) and therefore are viscous as such there are always some losses in fluid flows. These losses have, therefore, to be taken into consideration in the application of Bernoulli's equation which gets modified (between sections 1 and 2) for *real fluids* as follows:

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + h_L \quad ...(6.4)$$

where, h_L = Loss of energy between sections 1 and 2.



Energy Head and Head Loss in Real Fluid

Example 6.14. *In a smooth inclined pipe of uniform diameter 250 mm, a pressure of 50 kPa was observed at section 1 which was at elevation 10 m. At another section 2 at elevation 12 m, the pressure was 20 kPa and the velocity was 1.25 m/s. Determine the direction of flow and the head loss between these two sections. The fluid in the pipe is water. The density of water at 20°C and 760 mm Hg is 998 kg/m³.* **(PTU)**

Solution. *Given*:

$D = 250 \text{ mm} = 0.25 \text{ m},$

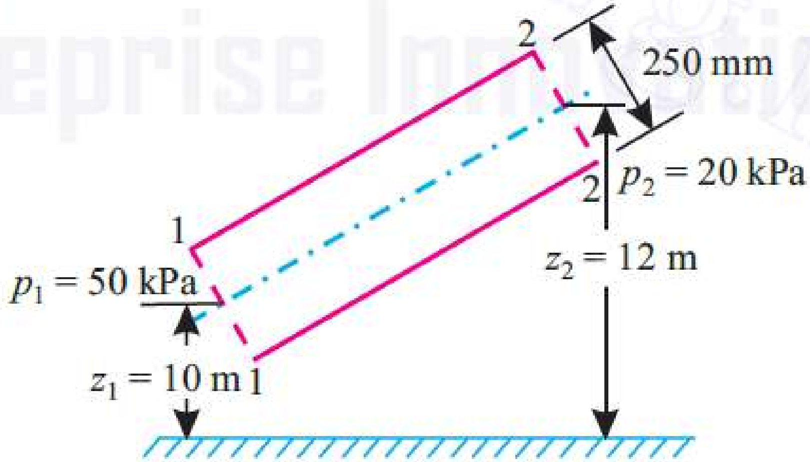
$p_1 = 50 \text{ kPa} = 50 \times 10^3 \text{ N/m}^2;$

$z_1 = 10 \text{ m}; z_2 = 12 \text{ m};$

$p_2 = 20 \text{ kPa} = 20 \times 10^3 \text{ N/m}^2,$

$V_1 = V_2 = 1.25 \text{ m/s}, \rho = 998 \text{ kg/m}^3.$

Refer to Fig. 6.15.



Total energy at *section 1–1*,

$$E_1 = \frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{50 \times 10^3}{998 \times 9.81} + \frac{1.25^2}{2 \times 9.81} + 10 = 15.187 \text{ m}$$

Total energy of *section 2–2*,

$$E_2 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 = \frac{20 \times 10^3}{998 \times 9.81} + \frac{1.25^2}{2 \times 9.81} + 12 = 14.122 \text{ m}$$

∴ Loss of head, $h_L = E_1 - E_2 = 15.187 - 14.122 = \mathbf{1.065 \text{ m}}$

Direction of flow:

Since $E_1 > E_2$ *direction of flow is from* **section 1-1 to section 2-2.**

Example 6.19. *A siphon consisting of a pipe of 12cm diameter is used to empty kerosene oil (Sp. gr. = 0.8) from the tank A. The siphon discharges to the atmosphere at an elevation of 1.2 m. The oil surface in the tank is at an elevation of 4.2 m. The centre line of the siphon pipe at its highest point C is at an elevation of 5.7 m. Determine:*

(*i*) *The discharge in the pipe.*

(*ii*) *The pressure at point C.*

The losses in the pipe may be assumed to be 0.45 m up to summit and 1.25 m from the summit to the outlet.

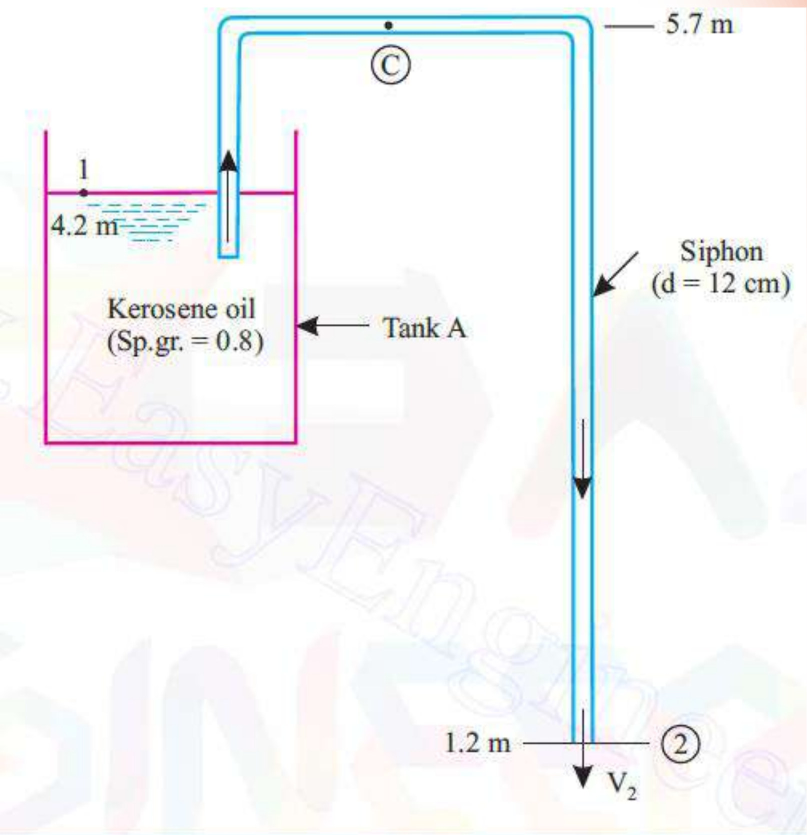


Fig. 6.20

Solution. Consider points 1 and 2 at the surface of the oil in the tank A and at the outlet as shown in Fig. 6.20. The velocity V_1 can be assumed to be zero. Applying Bernoulli's equation at points 1 and 2, we get:

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_1 + h_{f(1-2)} \text{ (losses)}$$

$$0 + 0 + 4.2 = 0 + \frac{V_2^2}{2g} + 1.2 + (0.45 + 1.25)$$

$$V_2 = 5.05 \text{ m/s}$$

(*i*) The discharge in the pipe, Q:

$$Q = A_2 V_2 = \frac{\pi}{4} \times (0.12)^2 \times 5.05 = \mathbf{0.057\ m^3/s}$$

(ii) The pressure at point C:

Applying Bernoulli's equation at points 1 and *C*, we get:

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_C}{w} + \frac{V_C^2}{2g} + z_C + h_{f_{(1-C)}}$$

$$0 + 0 + 4.2 = \frac{p_C}{w} + \frac{(5.05)^2}{2 \times 9.81} + 5.7 + 0.45$$

$$\frac{p_C}{w} = -3.25 \text{ m}$$

$$p_C = (0.8 \times 9.81) \times (-3.25)$$

$$= -25.5 \text{ kN/m}^2 \text{ or } -\mathbf{25.5\ kPa\ (gauge)}$$

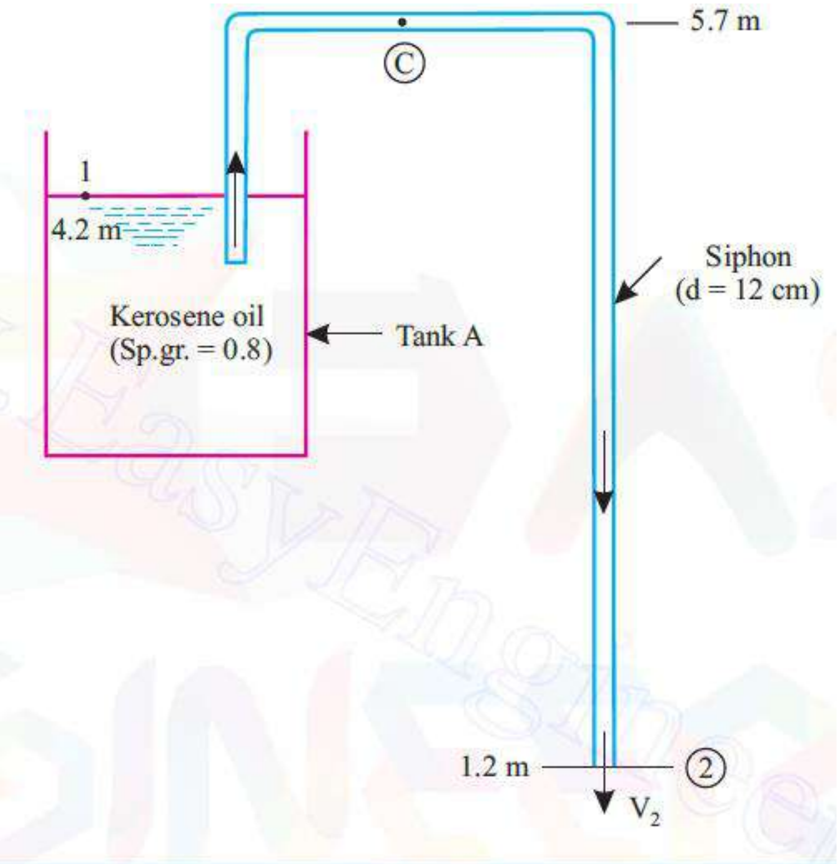


Fig. 6.20

Example 6.25. *Fig. 6.26 shows a pump drawing a solution (specific gravity =1.8) from a storage tank through an 8 cm steel pipe in which the flow velocity is 0.9 m/s. The pump discharges through a 6 cm steel pipe to an overhead tank, the end of discharge is 12 m above the level of the solution in the feed tank. If the friction losses in the entire piping system are 5.5 m and pump efficiency is 65 per cent, determine:*

(*i*) *Power rating of the pump.*

(*ii*) *Pressure developed by the pump.*

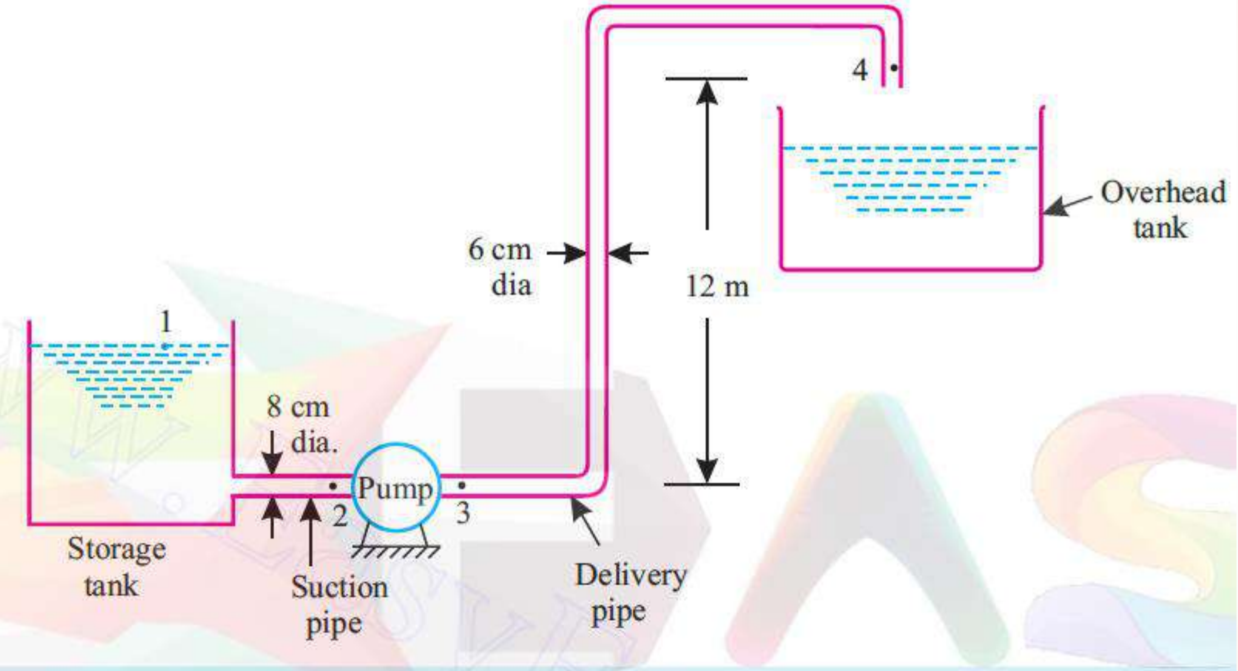


Fig. 6.26

Solution. *Given*: $d_2 = 8$ cm or 0.08 m; $d_3 = 6$ cm or 0.06 m;

$V_2 = 0.9$ m/s, $\eta_{pump} = 65\%$

(*i*) Power rating of the pump:

From continuity equation, we have:

$$A_2V_2 = A_3V_3$$

or,
$$V_3 (=V_4) = \frac{A_2V_2}{A_3} = \frac{\frac{\pi}{4} \times (0.08)^2 \times 0.9}{\frac{\pi}{4} \times (0.06)^2} = 1.6 \text{ m/s}$$

Applying Bernoulli's equation between points 1 and 4, we get:

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 + H_P = \frac{p_4}{w} + \frac{V_4^2}{2g} + z_4 + \text{Losses}$$

(where, H_P = Energy added by the pump per unit weight of liquid in Nm/N or m of the liquid pumped)

$$0 + 0 + 0 + H_P = 0 + \frac{(1.6)^2}{2 \times 9.81} + 12 + 5.5$$

$$H_P = 17.63 \text{ m of liquid}$$

$$\therefore \text{Power rating of the pump} = \frac{wQH_P}{\eta_{pump}}$$

$$= \frac{(9.81 \times 1.8) \times \left(\frac{\pi}{4} \times 0.08^2 \times 0.9\right) \times 17.63}{0.65} = \mathbf{2.167\ kW}$$

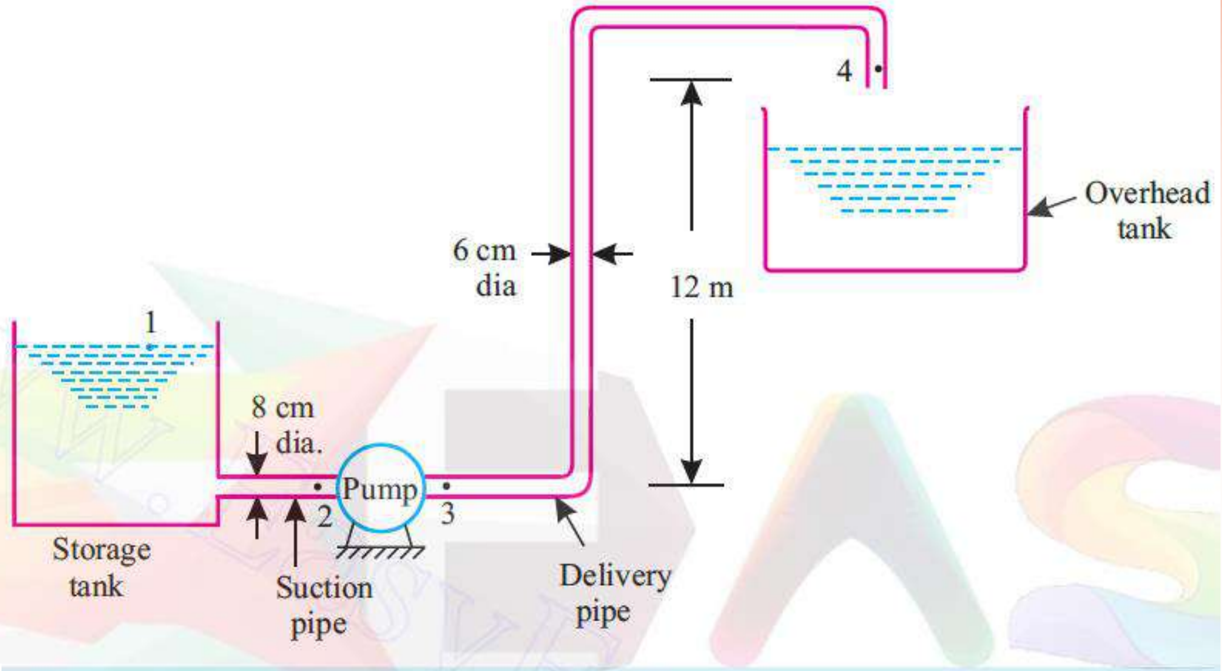


Fig. 6.26

(*ii*) Pressure developed by the pump, $(p_3 - p_2)$:

Applying Bernoulli's equation between points 2 and 3, we have:

$$\frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + H_P = \frac{p_3}{w} + \frac{V_3^2}{2g} + z_3$$

$$\left(\frac{p_3 - p_2}{w}\right) = \left(\frac{V_2^2 - V_3^2}{2g}\right) + H_P \qquad ...(\because z_2 = z_3)$$

$$= \frac{(0.9)^2 - (1.6)^2}{2 \times 9.81} + 17.63 \qquad = 17.54 \text{ m}$$

$$p_3 - p_2 = 17.54 \times (9.81 \times 1.8) \qquad = \mathbf{309.72\ kN/m^2 \text{ or } kPa}$$

Example 6.26. *A pump is 2.2 m above the water level in the sump and has a pressure of –20 cm of mercury at the suction side. The suction pipe is of 20 cm diameter and the delivery pipe is short 25 cm diameter pipe ending in a nozzle of 8 cm diameter. If the nozzle is directed vertically upwards at an elevation of 4.2 m above the water sump level, determine:*

(*i*) *The discharge.*

(*ii*) *The power input into the flow by the pump.*

(*iii*) *The elevation, above the water sump level, to which the jet would reach.*

Neglect all losses.

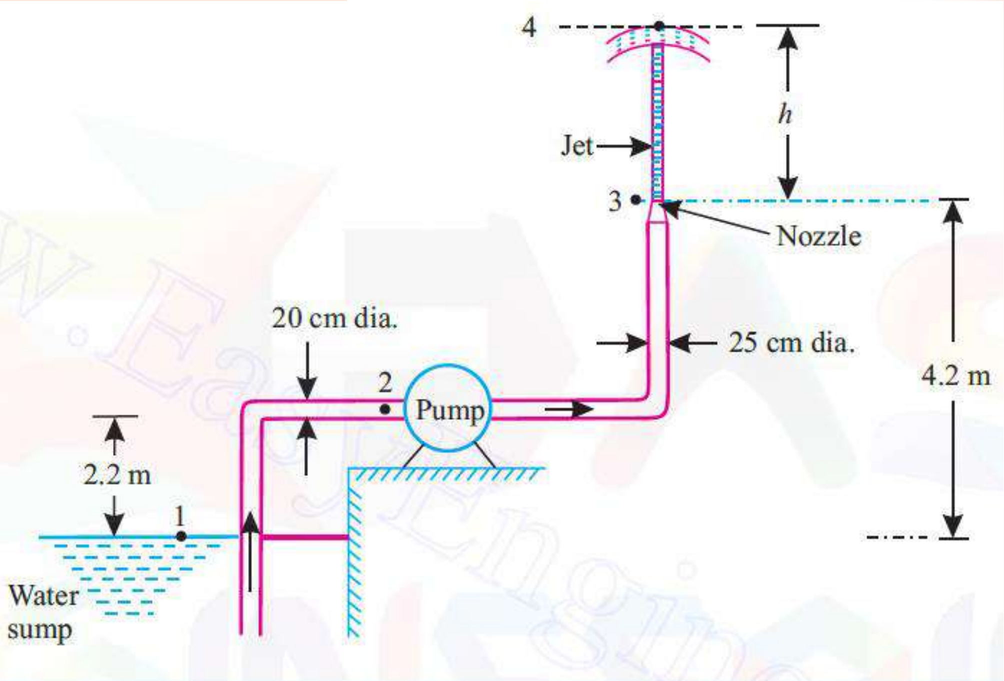


Fig. 6.27

Solution. (*i*) The discharge, *Q*:

Applying Bernoulli's equation to points 1 and 2 (Fig 6.27), we get

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2$$

$$0 + 0 + 0 = (-0.2 \times 13.6) + \frac{V_2^2}{2g} + 2.2 \Longrightarrow V_2 = 3.194 \text{ m/s}$$

$$\text{Discharge, } Q = \frac{\pi}{4} \times 0.21^2 \times 3.194 = \mathbf{0.1\ m^3/s}$$

(*ii*) The elevation, to which the jet will reach, *h*:

$$Q = A_2V_2 = A_3V_3$$

$$\frac{\pi}{4} \times (0.2)^2 \times 3.194 = \frac{\pi}{4} \times (0.08)^2 \times V_3 \implies V_3 = 19.962 \text{ m/s}$$

$$\frac{V_3^2}{2g} = \frac{(19.962)^2}{2 \times 9.81} = 20.31 \text{ m}$$

Hence, the height to which the jet will reach, h = **20.31 m**

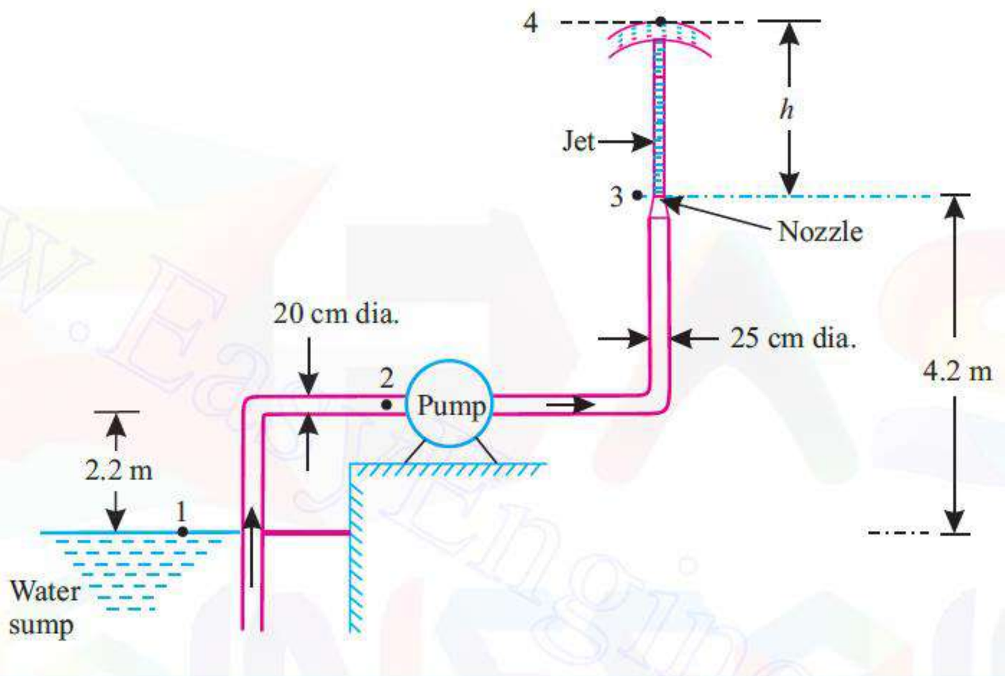


Fig. 6.27

(*iii*) The power input to the flow by the pump, P:

Applying Bernoulli's equation to points 1 and 3, we get:

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 + H_P = \frac{p_3}{w} + \frac{V_3^2}{2g} + z_3$$

$$0 + 0 + 0 + H_P = 0 + 20.31 + 4.2 \implies H_P = 24.51\text{m}$$

OR

The elevation of point 4, the summit of the jet, is

$= 4.2 + 20.31 = 24.51$ m

Power delivered by the pump, $p = wQH_p$

$= 9.81 \times 0.1 \times 24.51 =$ **24.04 kW**

FLUID MECHANICS

Lecture – 14

Dr Mohammed Tareq Khaleel

Year 2

Building and Construction Technology Engineering Department

الجامعة التقنية الشمالية
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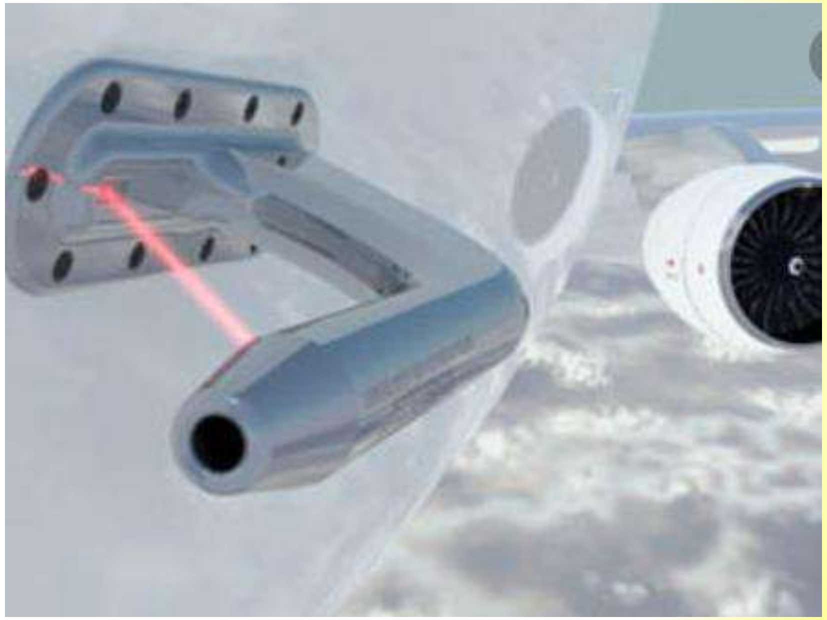
We now to consider the application of Bernoulli's equation to fluid flow through a few devices used in measuring the flow parameters:

1. **Venturimeter**

2. **Orificemeter**



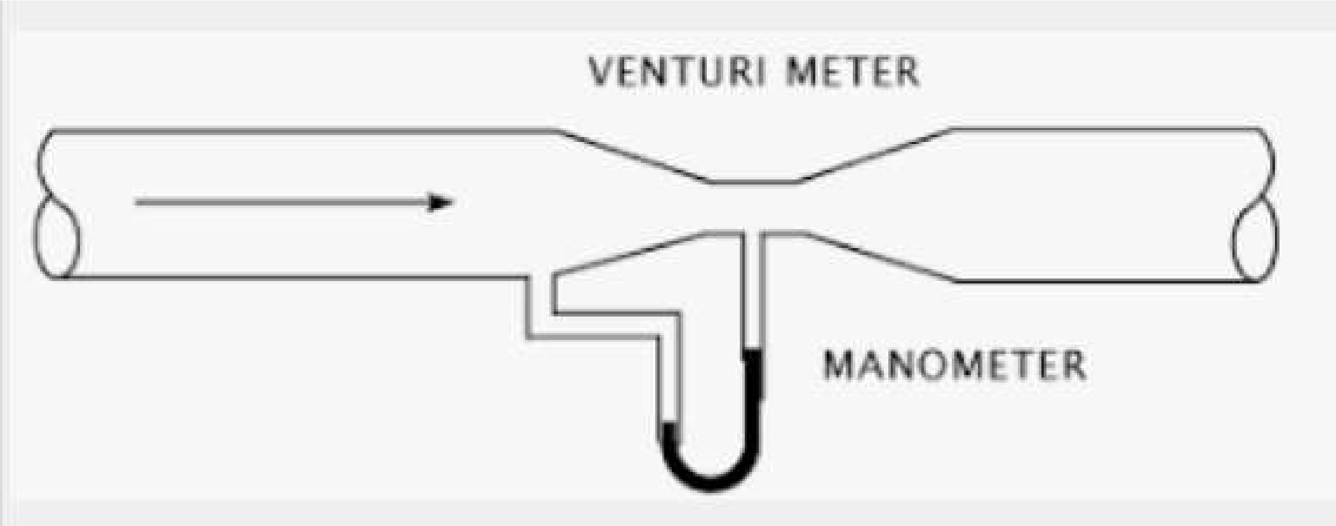
3. **Pitot tube**

For your information:

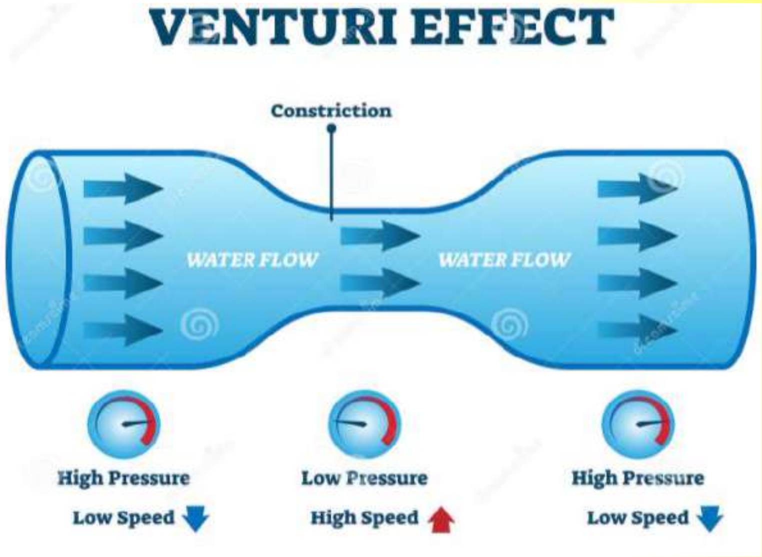
https://www.youtube.com/watch?v=oUd4WxjoHKY

https://www.youtube.com/watch?v=3zEdtkuNYLU

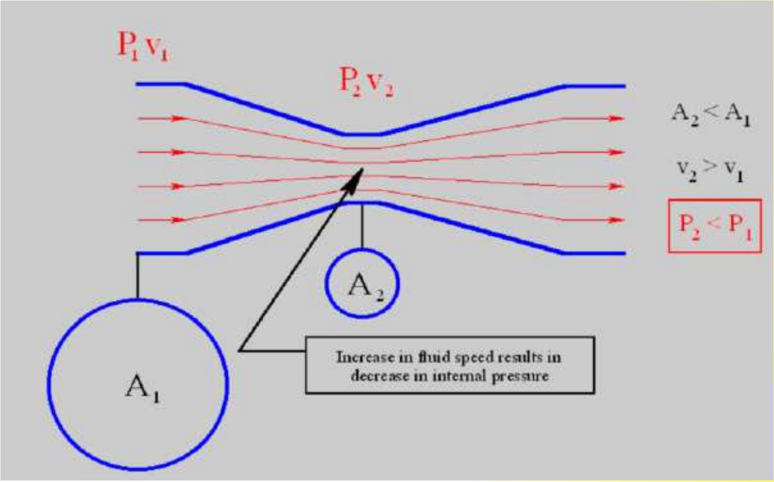
1. **Venturimeter** *It is an instrument used to measure the rate of discharge in a pipeline and is often fixed permanently at different sections of the pipeline to know the discharges there.*



A **venturimeter** has been named after the 18th century Italian engineer *Venturi, which* was the discoverer of the Venturi Effect .



The Venturi Effect is the reduction in fluid pressure that results when a fluid flows through a constricted section (or choke) of a pipe.



i. Horizontal venturimeters

Applying Bernoulli's equation at sections 1 and 2, we get:

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 \quad ...(i)$$

Here, $z_1 = z_2$ ... since the pipe is horizontal.

$$\therefore \quad \frac{p_1}{w} + \frac{V_1^2}{2g} = \frac{p_2}{w} + \frac{V_2^2}{2g}$$

or,

$$\frac{p_1 - p_2}{w} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} \quad ...(ii)$$

But, $\frac{p_1 - p_2}{w}$ = Difference of pressure heads at sections 1 and 2 and is equal to h.

i.e.,

$$\frac{p_1 - p_2}{w} = h$$

Substituting this value of $\frac{p_1 - p_2}{w}$ in eqn. (*ii*), we get:

$$h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} \quad ...(iii)$$

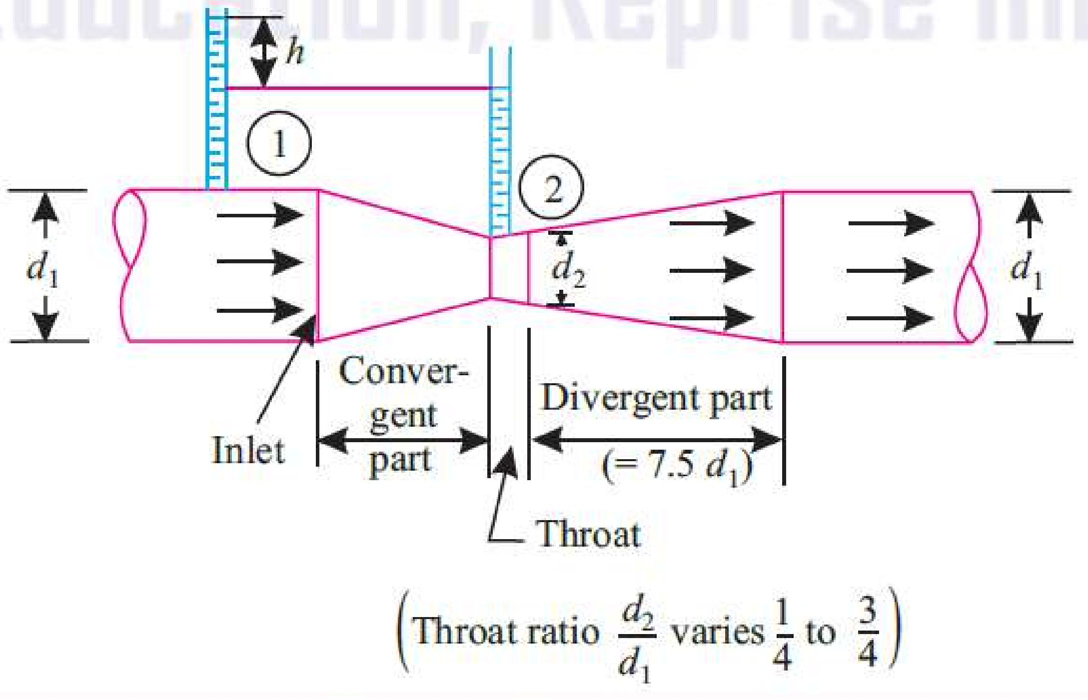


Fig. 6.29. Venturimeter.

Applying continuity equation at sections 1 and 2, we have:

$$A_1V_1 = A_2V_2 \quad \text{or} \quad V_1 = \frac{A_2V_2}{A_1}$$

Substituting the value of V_1 in eqn. (*iii*), we get:

$$h = \frac{V_2^2}{2g} - \frac{\left(\frac{A_2V_2}{A_1}\right)^2}{2g} = \frac{V_2^2}{2g}\left(1 - \frac{A_2^2}{A_1^2}\right)$$

or,
$$h = \frac{V_2^2}{2g}\left(\frac{A_1^2 - A_2^2}{A_1^2}\right) \quad \text{or} \quad V_2^2 = 2gh\left(\frac{A_1^2}{A_1^2 - A_2^2}\right)$$

or,
$$V_2 = \sqrt{2gh\left(\frac{A_1^2}{A_1^2 - A_2^2}\right)} = \frac{A_1}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gh}$$

∴ Discharge,
$$Q = A_2V_2 = A_2\frac{A_1}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh}$$

or,
$$Q = \frac{A_1A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh} \quad \text{...(6.5)}$$

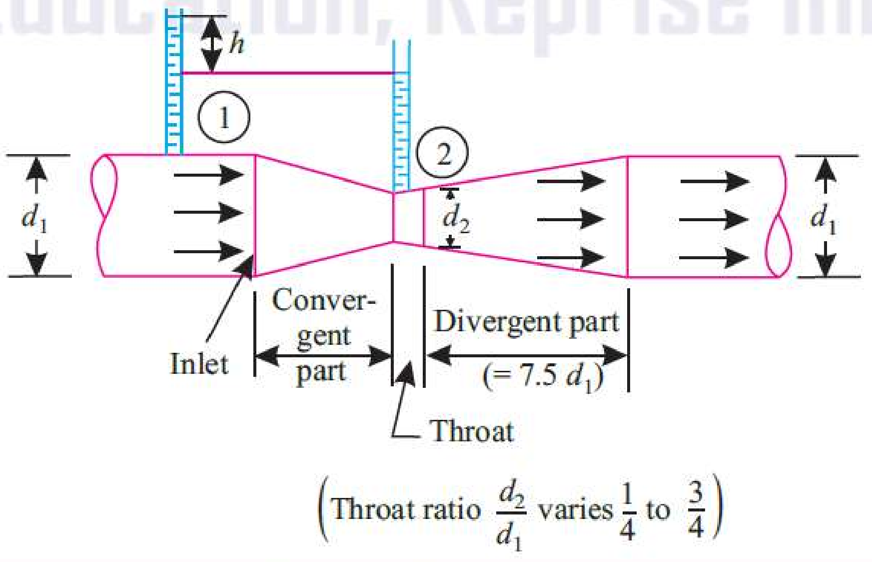


Fig. 6.29. Venturimeter.

or, $$Q = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh} \quad ...(6.5)$$

or, $$Q = C\sqrt{h}$$

where, C = constant of venturimeter

$$= \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2g}$$

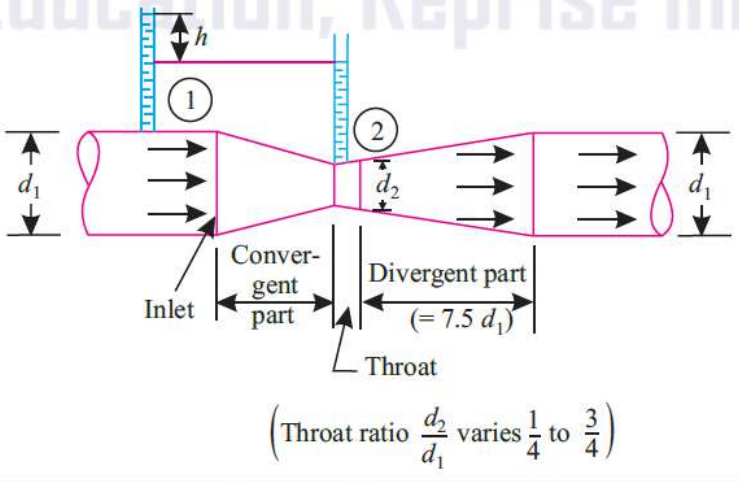


Fig. 6.29. Venturimeter.

Eqn. (6.5) gives the discharge under ideal conditions and is called *theoretical discharge*. Actual discharger (Q_{act}) which is less than the theoretical discharge ($Q_{th.}$) is given by:

$$Q_{act} = C_d \times \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh} \quad ...(6.6)$$

where, C_d = *Co-efficient of venturimeter* (or co-efficient of discharge) and its value is *less than unity* (varies between 0.96 and 0.98)

- *Due to variation of C_d venturimeters are not suitable for very low velocities.*

Value of 'h' by differential U-tube manometer:

Case. I. *Differential manometer containing a liquid heavier than the liquid flowing through the pipe.*

Let, S_{hl} = Sp. gravity of heavier liquid,

S_p = Sp. gravity of liquid flowing through pipe, and

y = Difference of the heavier liquid column in U-tube.

Then
$$h = y\left[\frac{S_{hl}}{S_p} - 1\right] \quad \text{...(6.7)}$$

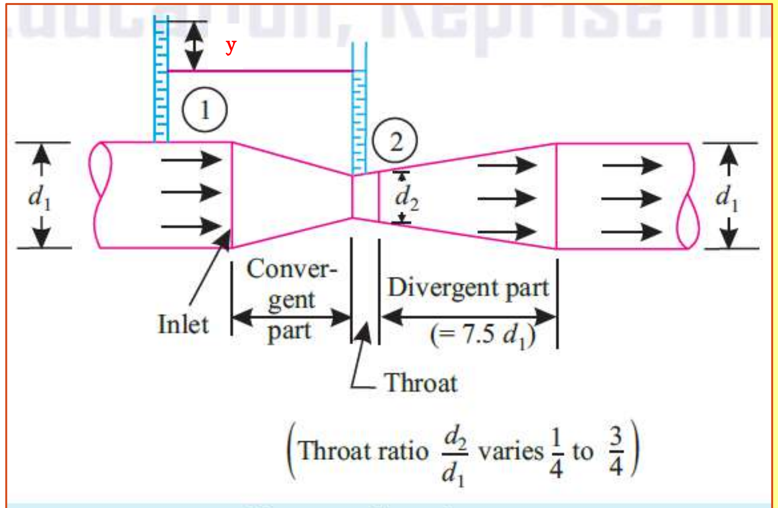


Fig. 6.29. Venturimeter.

Case. II. *Differential manometer containing a liquid lighter than the liquid flowing through the pipe.*

Let, S_{ll} = Sp. gravity of lighter liquid,

S_p = Sp. gravity of liquid flowing through pipe, and

y = Difference of lighter liquid column in U-tube.

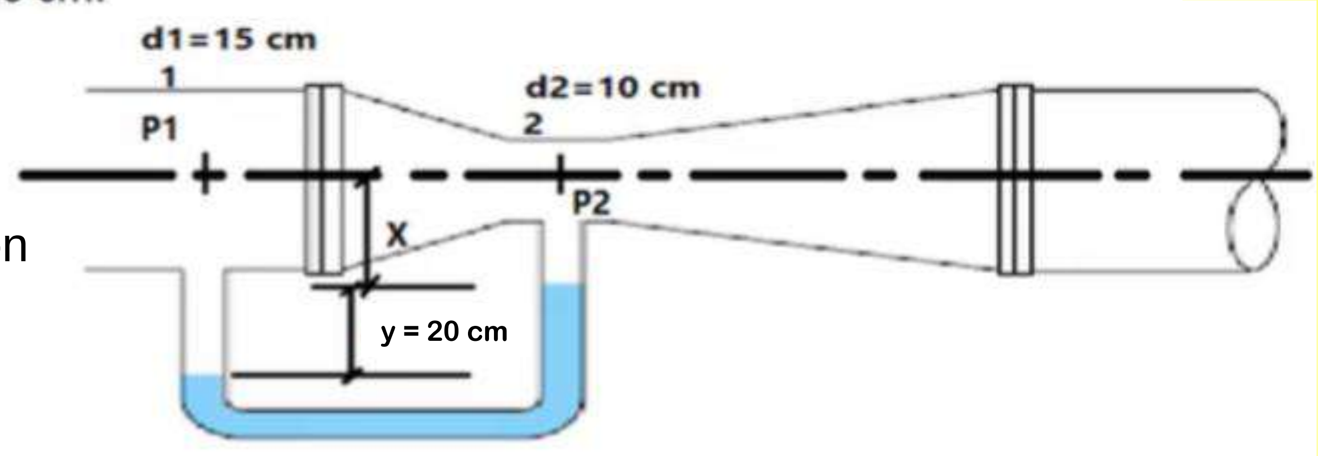
Then,
$$h = y\left[1 - \frac{S_{ll}}{S_p}\right] \quad \text{...(6.8)}$$

Example :

A venturi meter of 15 cm inlet diameter and 10 cm throat is laid horizontally in a pipe to measure the flow of water (1 specific gravity). The reading of a mercury manometer is 20 cm.

1-Calculate the Velocity at throat ?

2-Determine the discharge?



Solution:– (I)

To find difference of pressure head (*h*) using the relation

$$h = y\left[\frac{S_{hl}}{S_p} - 1\right]$$

$$h = 0.2\left[\frac{13.6}{1} - 1\right] = 2.52\ \text{m}$$

Find Q using the equation $Q = \dfrac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh}$ ...(6.5)

$A_1 = (\pi/4)\, d_2^2 = (\pi/4)\,(0.15)^2 = 0.01767\ \text{m}^2$

$A_2 = (\pi/4)\, d_1^2 = (\pi/4)\,(0.10)^2 = 0.00785\ \text{m}^2$

$h = 2.52$ m

$$\therefore Q = \frac{0.01767 * 0.00785}{\sqrt{0.01767^2 - 0.00785^2}} \times \sqrt{2 \times 9.81 \times 2.52}$$

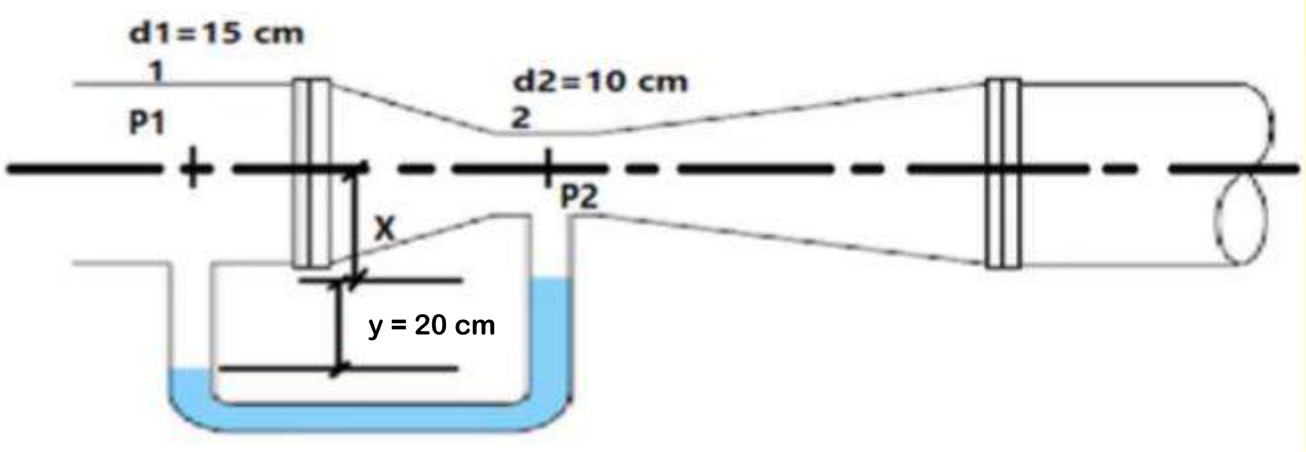
$= 0.0616\ \text{m}^3/\text{s}$

Find Velocity at throat using the equation

$$\text{Velocity, } V_2 = \frac{Q}{A_2}$$

$$V_2 = \frac{0.0616}{0.00785} = 7.85\ \text{m/s}$$

Example :
A venturi meter of 15 cm inlet diameter and 10 cm throat is laid horizontally in a pipe to measure the flow of water (1 specific gravity). The reading of a mercury manometer is 20 cm.
1-Calculate the Velocity at throat ?
2-Determine the discharge?



Solution:– (II)

Applying Bernoulli's equation between point 1 and 2, we get:

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2$$

$$\frac{p_1}{w} - \frac{p_2}{w} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$\frac{p_1 - p_2}{w} = \frac{V_2^2 - V_1^2}{2g} \quad \text{.................... Eq.1}$$

Note that $\frac{p_1 - p_2}{w}$ is h

From manometer equation find $\frac{p_1 - p_2}{w}$ or h

Fall +Ve
Rise -Ve

$$P_1 + w_{water}(x) + w_{water}(y) - w_{mercury}(y) - w_{water}(x) = P_2$$

$$P_1 - P_2 = w_{mercury}(y) - w_{water}(y) \qquad \div w_{water}$$

$$\frac{p_1 - p_2}{w} = S_{mercury}(y) - S_{water}(y)$$

$$\frac{p_1 - p_2}{w} = 13.6\,(0.2) - 1\,(0.2) = 2.52 \text{ m}$$

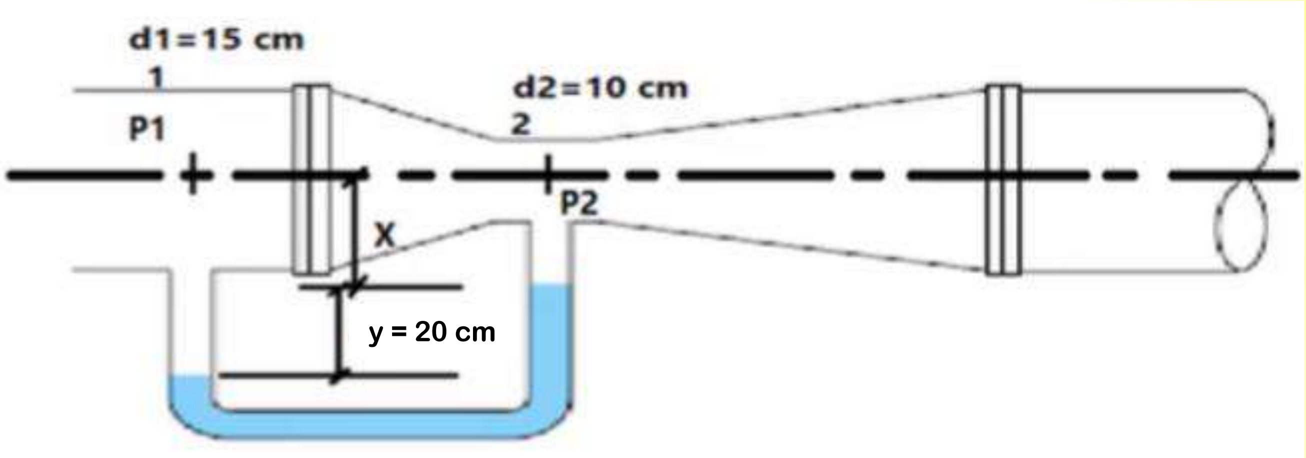
$$\frac{p_1 - p_2}{w} = h = 2.52 \text{ m}$$

substituting the value of h into Eq.1 gives

$$2.52 = \frac{V_2^2 - V_1^2}{2g} \quad \text{.................... Eq.2}$$

Applying continuity equation at sections 1 and 2, we have:

$$A_1 V_1 = A_2 V_2$$



But $A_1 = (\pi/4)\, d_1^2$,and $A_2 = (\pi/4)\, d_2^2$

$\therefore\ V_1 (\pi/4)\, d_1^2 = V_2 (\pi/4)\, d_2^2$

$V_1 d_1^2 = V_2 d_2^2$

$V_1 = V_2 (d_2^2/d_1^2)$

$V_1 = V_2 ((0.10)^2/(0.15)^2)$

$V_1 = 0.444\, V_2$ ………………… Eq.3

Substituting Eq.3 into Eq.2 gives

$$2.52 = \frac{V_2^2 - (0.444V_2)^2}{2 * 9.81}$$

⇨ $V_2 = 7.85$ m/s (velocity at throat)

$Q = A_2 V_2$

$= (\pi/4)\, d_1^2\, (7.85)$

$= (\pi/4)\, (0.1)^2\, (7.85)$

$= 0.0616\ m^3/s$

2. Orificemeter

Orificemeter or orifice plate is *a device* (*cheaper* than a venturimeter) *employed for measuring the discharge of fluid through a pipe.* It also *works on the same principle of a venturimeter*.

It consists of a flat circular plate having a circular sharp edged hole (called orifice) concentric with the pipe. A differential manometer is connected at sections (1) and (2).

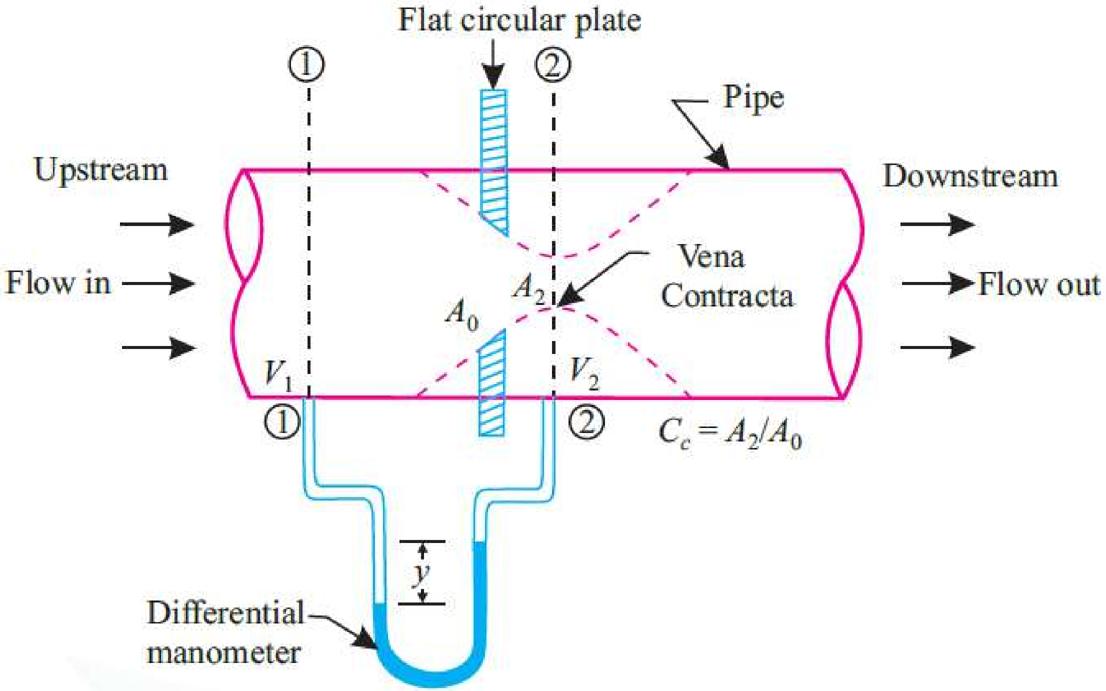
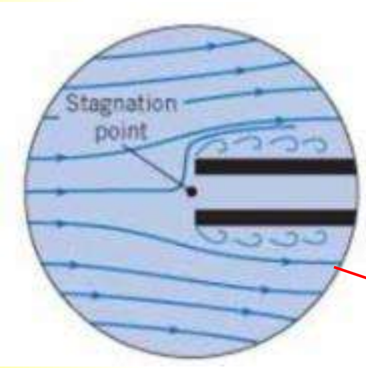


Fig. 6.35. Orificemeter

2. Pitot Tube

Pitot tube *is one of the most accurate devices for velocity measurement.* It works on the *principle that if the velocity of flow at a point becomes zero, the pressure there is increased due to conversion of kinetic energy into pressure.*

It consists of a glass tube in the form of a 90° bend of short length open at both its ends. It is placed in the flow with its bent leg directed upstream so that a ***stagnation point*** is created immediately in front of the opening (Fig. 6.38). The kinetic energy at this point gets converted into pressure energy (dynamic pressure) causing the liquid to rise in the vertical limb, to a height equal to the stagnation pressure.



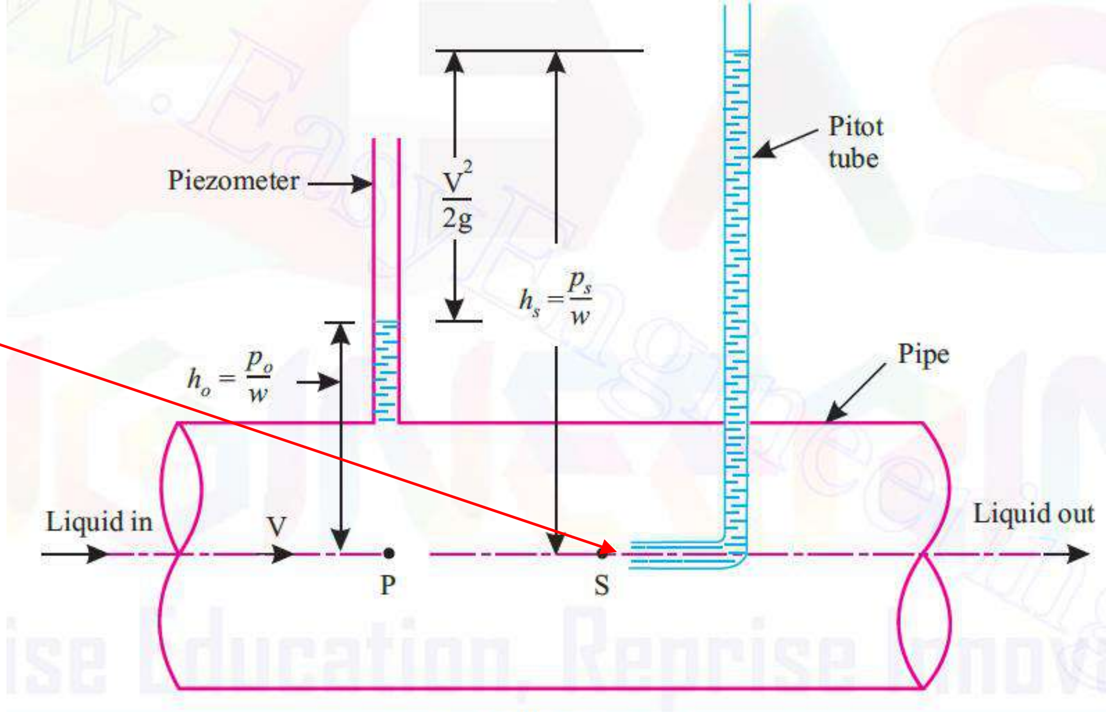


Fig. 6.38. Pitot tube.

For your information:
https://www.youtube.com/watch?v=3zEdtkuNYLU

Applying Bernoulli's equation between stagnation point (S) and point (P) in the undisturbed flow at the same horizontal plane, we get:

$$\frac{p_0}{w} + \frac{V^2}{2g} = \frac{p_s}{w} \quad \text{or} \quad h_0 + \frac{V^2}{2g} = h_s$$

or,
$$V = \sqrt{2g\,(h_s - h_0)} \quad \text{or} \quad \sqrt{2g\,\Delta h} \qquad ...(1)$$

p_0 = Pressure at point 'P', *i.e.* static pressure,
V = Velocity at point 'P', *i.e.* free flow velocity,
p_s = Stagnation pressure at point 'S', and
Δh = Dynamic pressure
= Difference between stagnation pressure head (h_s) and static pressure head (h_0).

The height of liquid rise in the Pitot tube indicates the stagnation head. The static pressure head may be measured separately with a piezometer (Fig. 6.38).

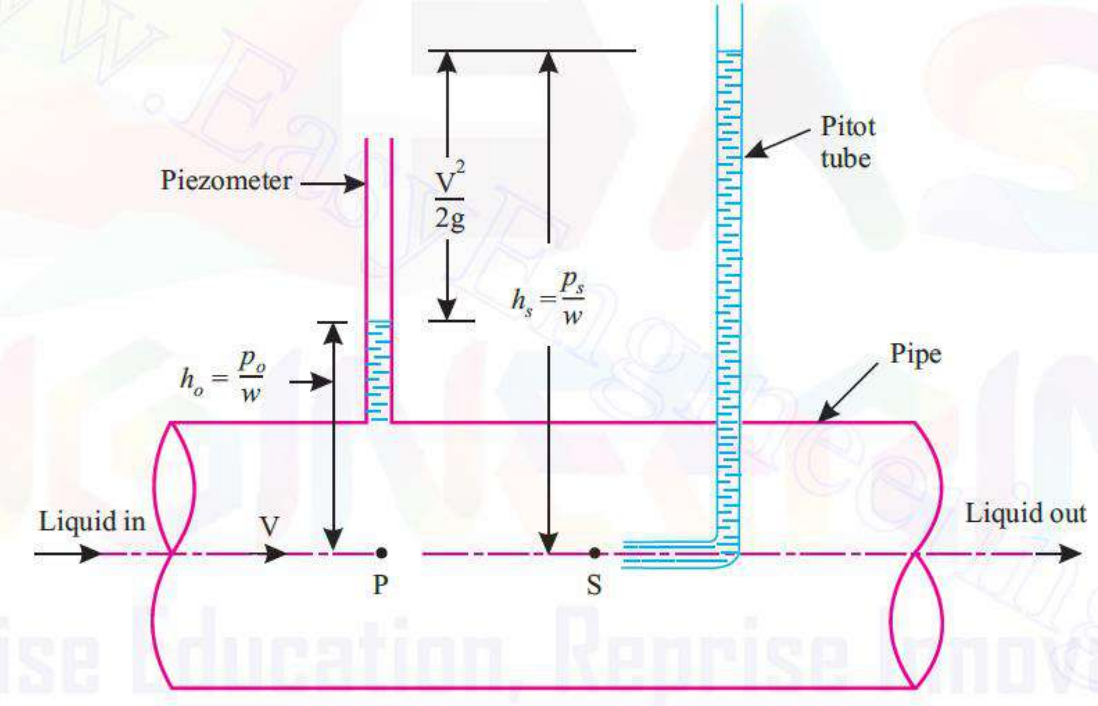


Fig. 6.38. Pitot tube.

Both the static pressure as well as stagnation pressure can be measured in a device known as **Pitot static tube.** (Fig. 6.39).

If a differential manometer is connected to the tubes of a Pitot static tube it will measure the dynamic pressure head.

If y is the manometric difference, then

$$\Delta h = y\left(\frac{S_m}{S} - 1\right)$$

S_m = Specific gravity of manometric liquid, and
S = Specific gravity of the liquid flowing through the pipe.

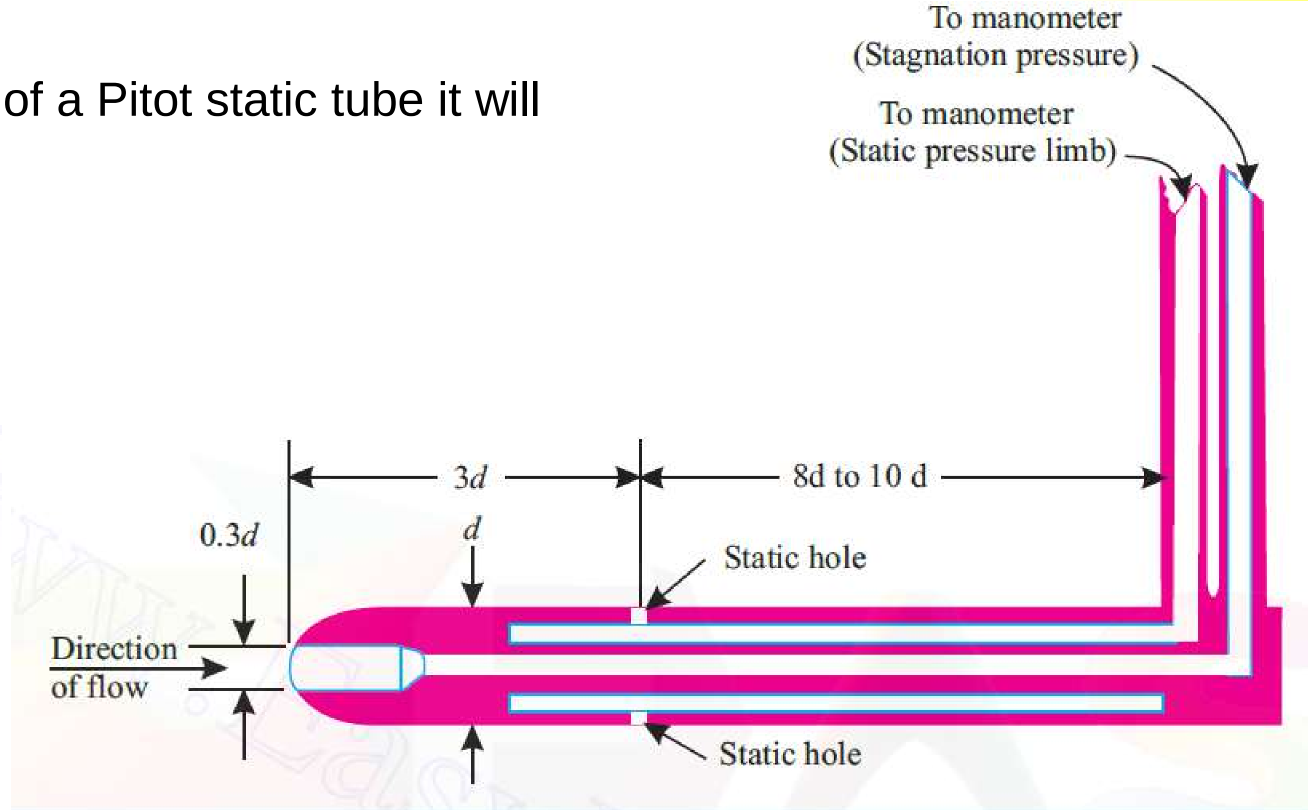


Fig. 6.39. Pitot static tube.

$$V = C\sqrt{2g\Delta h} \quad \ldots(2)$$

where, C = A connective coefficient which takes into account the effect of stem and bent leg.

Fluid Mechanics

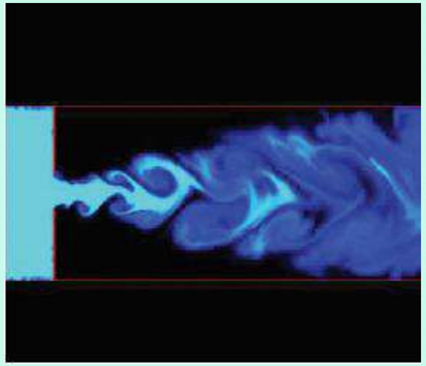
Lecture – 15

Dr Mohammed Tareq Khaleel

Year 2

Building and Construction Technology Engineering Department

FLOW THROUGH ORIFICES AND MOUTHPIECES



Orifices as well as mouthpieces are used to *measure the discharge.*

An **orifice** *is an opening in the wall or base of a vessel through which the fluid flows.* The *top edge* of the orifice is always below *the free surface* (If the free surface is below the top edge of the orifice, becomes a *weir*)

A **mouthpiece** *is an attachmnent in the form of a small tube or pipe fixed to the orifice (the length of pipe extension is usually 2 to 3 times the orifice diameter) and is used to increase the amount of dischrge.*

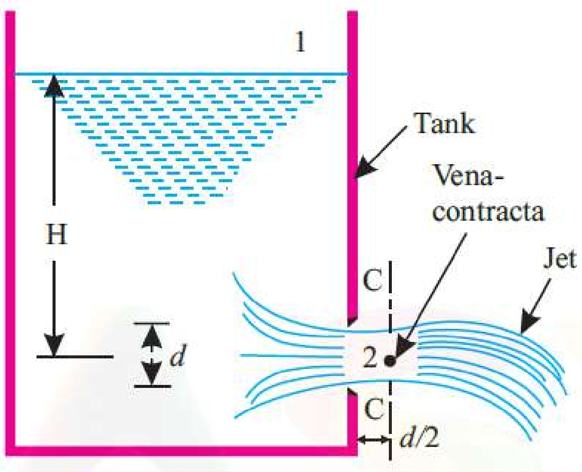


Fig. 8.1. Orifice discharging free.

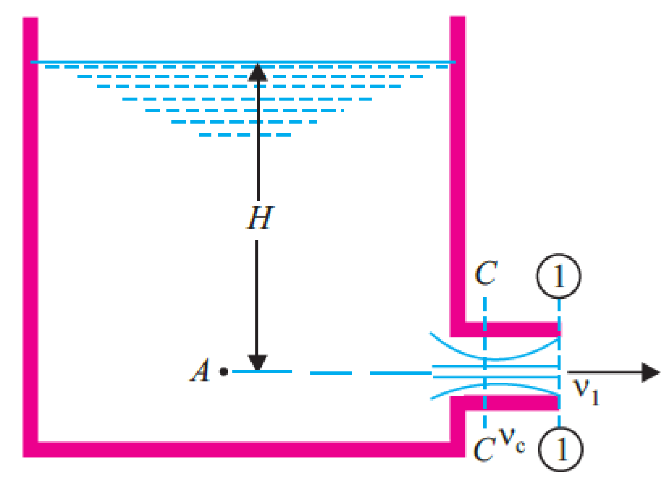


Fig. 8.23. External cylindrical mouthpiece.

FLOW THROUGH ORIFICES

Considering points 1 and 2 as shown in Fig. 8.1 and applying Bernoulli's theorem, we have:

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2$$

But, $p_1 = p_2 = p_a$ (p_a = atmospheric pressure)

$z_1 = z_2 + H$ Or $Z_1 - Z_2 = H$

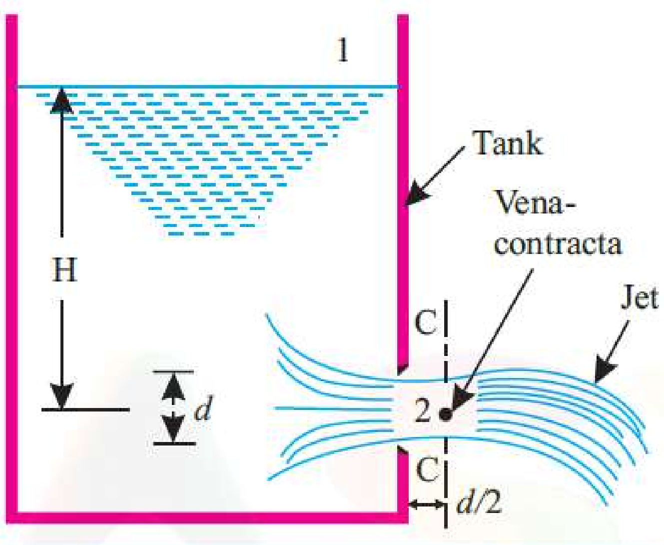


Fig. 8.1. Orifice discharging free.

Further, if the cross-sectional area of the tank is very large, the liquid at point 1 is practically standstill and hence $V_1 = 0$

Thus, $$\frac{V_2^2}{2g} = H$$

or, $$V_2 = \sqrt{2gH} \quad \text{... (8.1)}$$

Equation (8.1) is know as **Torricelli's theorem.**

HYDRAULIC CO-EFFICIENTS

The hydraulic co-efficients (or orifice co-efficients) are enumerated and discussed below :

1. Co-efficient of contraction, C_c
2. Co-efficient of velocity, C_v
3. Co-efficient of discharge, C_d
4. Co-efficient of resistance, C_r

8.4.1. Co-efficient of Contraction (C_c)

The ratio of the area of the jet at vena-contracta to the area of the orifice is known as **Co-efficient of contraction.** It is denoted by C_c.

Let, a_c = Area of jet at vena contracta, and

a = Area of orifice.

Then,
$$C_c = \frac{a_c}{a} \quad \text{... (8.2)}$$

The value of C_c varies slightly with the available head of the liquid, size and shape of the orifice; in practice it varies from 0.613 to 0.69 but the average value is taken as 0.64.

FLOW THROUGH ORIFICES - HYDRAULIC CO-EFFICIENTS

8.4.2. Co-efficient of Velocity (C_v)

The ratio of actual velocity (V) of the jet at vena-contracta to the theoretical velocity (V_{th}) is known as **Co-efficient of velocity.** It is denoted by C_v and mathematically, C_v is given as:

$$C_v = \frac{\text{Actual velocity of jet at vena contracta } (V)}{\text{Theoretical velocity } (V_{th})}$$

$$C_v = \frac{V}{\sqrt{2gH}} \qquad \ldots (8.3)$$

where, V = Actual velocity, and

H = Head under which the fluid flows out of the orifice

The value of C_v varies from 0.95 to 0.99,depending upon the shape of orifice and the head of liquid under which the flow takes place. For sharp-edged orifices the value of C_v is taken as 0.98.

FLOW THROUGH ORIFICES - HYDRAULIC CO-EFFICIENTS

8.4.3. Co-efficient of Discharge

The ratio of actual discharge (Q) through an arifice to the theorerical discharge, (Q_{th}) is known as **Co-efficient of discharge.** It is dinoted by C_d.

Mathematically, $$C_d = \frac{\text{Actual discharge } (Q)}{\text{Theoretical discharge } (Q_{th.})}$$

$$= \frac{\text{Actual area} \times \text{actual velocity}}{\text{Theoretical area} \times \text{theoretical velocity}}$$

$$= \frac{\text{Actual area}}{\text{Theoretical area}} \times \frac{\text{actual velocity}}{\text{theoretical velocity}}$$

$$C_d = C_c \times C_v \quad \text{... (8.4)}$$

The value of C_d varies from 0.62 to 0.65 depending upon size and the shape of the orifice and the head of liquid under which the flow takes place.

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FLOW THROUGH ORIFICES - HYDRAULIC CO-EFFICIENTS

8.4.4. Co-efficient of Resistance (C_r)

The ratio of loss of head (or loss of kinetic energy) in the orifice to the head of water (actual kinetic energy) available at the exit of the orifice is known as **Co-efficient of resistance.** It is denoted by C_r.

Mathematically,
$$C_r = \frac{\text{Loss of head in the orifice}}{\text{Head of water}}$$

The loss of head in the orifice takes place, because the walls of the orifice offer some resistance to the liquid, as it comes out. While solving numerical problems C_r is *generally neglected*.

FLOW THROUGH ORIFICES

Example 8.12. *A closed tank, having an orifice of diameter 20 mm at the bottom of the tank, is partially filled with water upto a height of 2.5 m. The air is pumped into the upper part of the tank. Determine the pressure required for a discharge of 5 litres per second through the orifice, Take discharge co-fficient, $C_d = 0.6$ for the orifice.*

Solution. Height of water above orifice, $H = 2.5$ m

Dia. of the orifice, $d = 20$ mm $= 0.02$ m

$\therefore$ Area of the orifice, $A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (0.02)^2 = 0.000314 \text{ m}^2$

Discharge through the orifice, $Q = 5$ litres/sec.

$$= \frac{5}{1000} = 0.005 \text{ m}^3/s$$

Co-efficient of discharge, $C_d = 0.6$

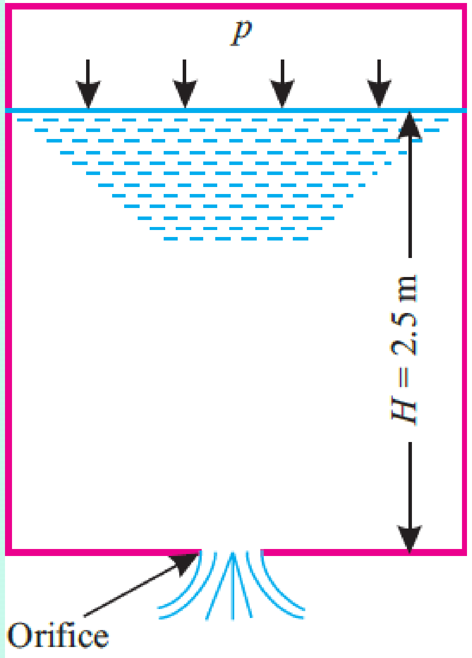


Fig. 8.8

Pressure required

Let p is the intensity of presure required above water surface in kN/m^2.

Then, pressure head of air $= \frac{p}{w} = \frac{p}{9{\cdot}81} = 0.102p$ metres of water.

If V is the velocity at outlet of orifice, then:

$$V = \sqrt{2g\left(H + \frac{p}{w}\right)} = \sqrt{2 \times 9.81\,(2.5 + 0.102p)}$$

$\therefore$ Discharge, $Q = C_d \times a \times V$

or $0.005 = 0.6 \times 0.000314 \times \sqrt{2 \times 9.81\,(2.5 + 0.102p)}$

$p = 326.4 \text{kN/m}^2$

FLOW THROUGH ORIFICES

DISCHARGE THROUGH A LARGE RECTANGULAR ORIFICE

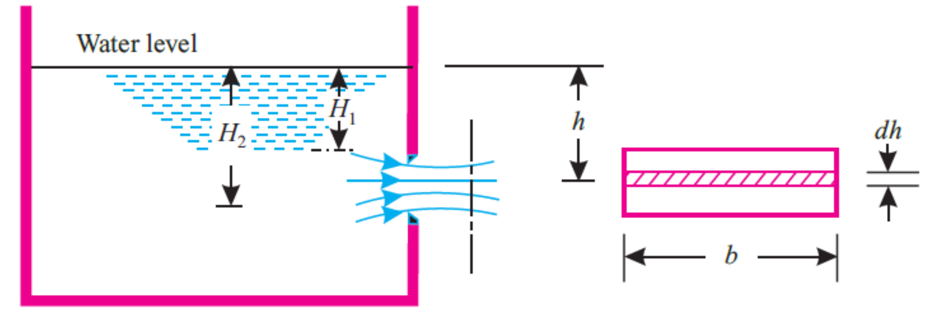


Fig. 8.9. Large rectangular orifice.

$$Q = 2/3\, C_d\, .b\, \sqrt{2g}\, (H_2^{3/2} - H_1^{3/2}) \qquad ...(8.9)$$

Example 8.13. *Find the discharge through a rectangular orifice 3.0 m wide and 2.0 m deep fitted to a water tank. The water level in the tank is 4·0 m above the top edge of the orifice. Take C_d = 0.62.*

Solution. Width of the orifice, $b = 3.0$ m

Depth of the orifice, $d = 2.0$ m

Height of water above the top of the orifice, $H_1 = 4.0$ m

∴ Height of the water above the bottom of the orifice, $H_2 = 4 + d = 4 + 2 = 6$ m

Co-efficient of discharge, $C_d = 0.62$

Discharge through the orifice, *Q*:

Using the relation:

$$Q = \frac{2}{3} C_d \cdot b\sqrt{2g}\, (H_2^{3/2} - H_1^{3/2}) \qquad \text{with usual notations}$$

$$= \frac{2}{3} \times 0.62 \times 3.0 \times \sqrt{2 \times 9.81}\left(6^{3/2} - 4^{3/2}\right) = 36.78 \text{ m}^3/\text{s}$$

i.e. $\mathbf{Q = 36.78\ m^3/s}$ **(Ans.)**

FLOW THROUGH ORIFICES

DISCHARGE THROUGH FULLY SUBMEROGED ORIFICE

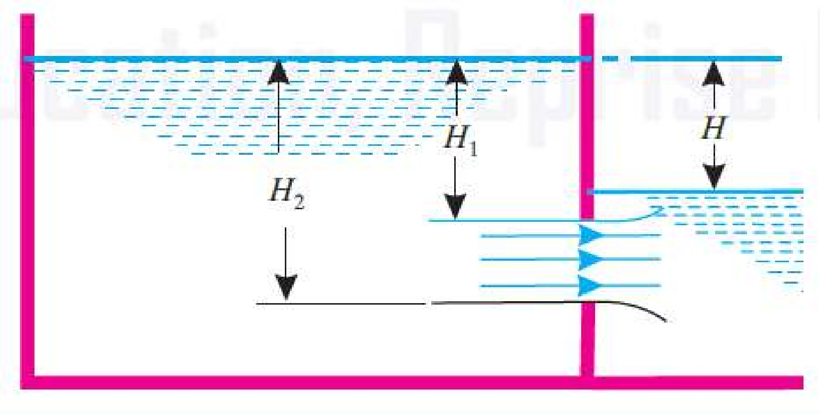


Fig. 8.10. Fully submerged orifice.

H_1 = Height of water (on the upstream side) above the top of the orifice,
H_2 = Height of water (on the upstream side) above the bottom of the orifice,
H = Difference between the two water levels on either side of the orifice,
b = Width of orifice, and
C_d = Co-efficent of discharge.

$$Q = C_d \cdot b\,(H_2 - H_1) \times \sqrt{2gH} \quad \text{...(8.10)}$$

Sometimes, depth of submerged orifice (d) is given instead of H_1 and H_2. In such cases, the discharge,

$$Q = C_d \cdot b \cdot d\,\sqrt{2gH} \quad \text{...(8.11)}$$

DISCHARGE THROUGH FULLY SUBMEROGED ORIFICE

Example 8.15. *Find the discharge through a totally drowned orifice 1.5 m wide and 1 m deep, if the difference of water levels on both the sides of the orifice be 2.5 m. Take $C_d = 0.62$.*

Solution. Width of the orifice, $b = 1.5$ m

Difference of water levels, $H = 2.5$m

Depth of the orifice, $d = 1$ m

Co-efficient of dicsharge, $C_d = 0.62$

Discharge, Q:

Using the relation,

$$Q = C_d \,.\, b \,.\, d \sqrt{2gH}$$

$$= 0.62 \times 1.5 \times 1 \times \sqrt{2 \times 9.81 \times 2.5} = 6.513 \text{ m}^3/\text{s}$$

i.e., $\mathbf{Q = 6.513 \ m^3/s}$ **(Ans.)**

DISCHARGE THROUGH AN EXTERNAL MOUTHPIECE

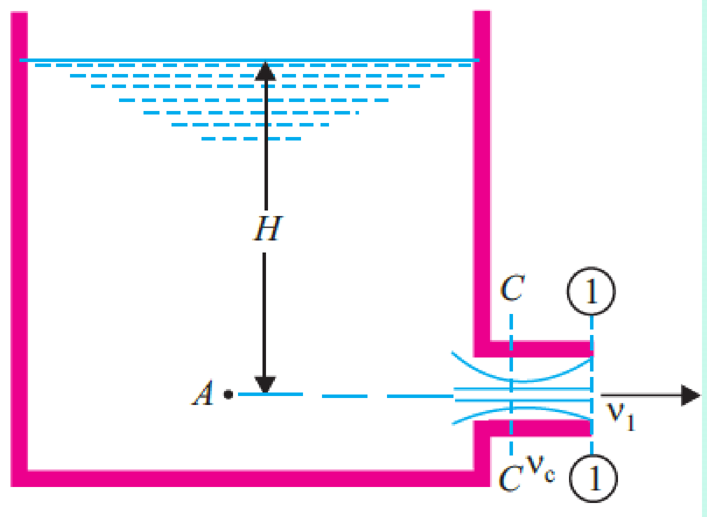


Fig. 8.23. External cylindrical mouthpiece.

a_1 = Area of mouthpiece at outlet,

v_1 = Velocity of liquid at outlet,

a_c = Area of flow at vena-contracta,

v_c = Velocity of liquid at *C-C* section,

H = Height of liquid above the centre of the mouthpiece, and

C_c = Co-efficient of contraction.

$$Q = C_d \times a \times \sqrt{2gH}$$

$C_d = 0.855$

Example 8.29. *Find the discharge from a 80 m diameter external mouthpiece, fitted to a side of a large vessel, if the head over the mouthpiece is 6 m.*

Solution. Dia. of the mouthpiece = 80 mm = 0.08 mm

$\therefore$ Area, $a = \frac{\pi}{4} \times 0.08^2 = 0.005026 \text{ m}^2$

Head over the mouthpiece, $H = 6$ m

C_d for the mouthpiece = 0.855

$\therefore$ Discharge, $Q = C_d \times \text{area} \times \text{velocity}$

$= C_d \times a \times \sqrt{2gH}$

$= 0.855 \times 0.005026 \times \sqrt{2 \times 9.81 \times 6} = \mathbf{0.0466 \text{ m}^3/\text{s}}$ **(Ans.)**

DISCHARGE THROUGH AN INTERNAL MOUTHPIECE (OR RE-ENTRANT OR BORDA'S MOUTHPIECE)

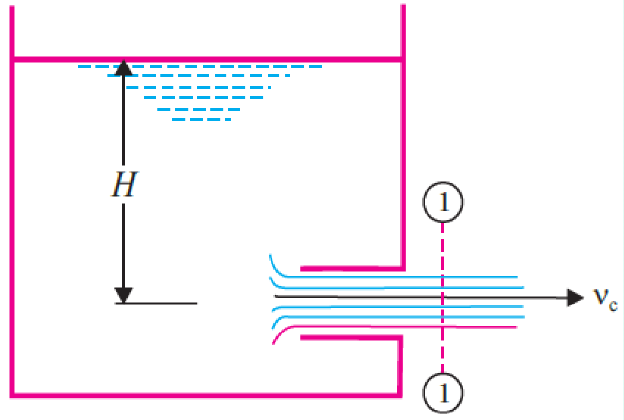


Fig. 8.26. Mouthpiece running free.

H = Height of the liquid above the mouthpiece,
a = Area of orifice or mouthpiece,
a_c = Area of contracted jet, and
v_c = Velocity through mouthpiece.

If the length of the tube is *equal to diameter,* the jet of liquid comes out from mouthpiece without touching the sides of the tube, the mouthpiece is known as ***running free***.

$$Q = C_d \times a \times \sqrt{2gH}$$

$$= 0.5 \times a \times \sqrt{2gH} \quad \text{...(8.21)}$$

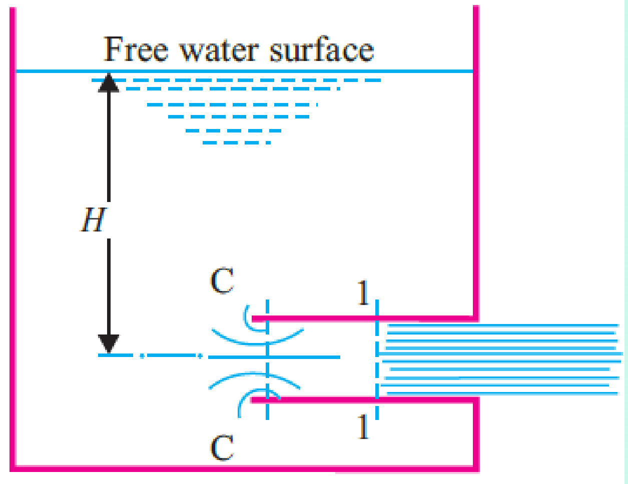


Fig. 8.27. Mouthpiece running full.

But *if the length of the tube is about 3 times its diameter*, the *jet comes out with its diameter equal to the diameter of mouthpiece at the outlet*, *the mouthpiece is said to be* ***running full***.

$$Q = 0.707 \times a \times \sqrt{2gH} \quad \text{...(8.22)}$$

DISCHARGE THROUGH AN INTERNAL MOUTHPIECE (OR RE-ENTRANT OR BORDA'S MOUTHPIECE)

Example 8.34. *An internal mouthpiece of 100 mm diameter is discharging water under a constant head of 5 m. Find the discharge through mouthpiece, when :*

(*i*) *The mouthpiece is running free, and*

(*ii*) *The mouthpiece is running full.*

Solution. Dia. of mouthpiece, $d = 100 \text{ mm} = 0{\cdot}1 \text{ m}$

$\therefore$ Area, $a = \frac{\pi}{4} \times 0.1^2 = 0.00785 \text{ m}^2$

Constant head, $H = 5 \text{ m}$

Discharge, Q:

(*i*) *When mouthpiece is running free:*

Using the relation:

$$Q = 0.5 \times a \times \sqrt{2gH} \qquad \text{...[Eqn. (8.21)]}$$

$$= 0.5 \times 0.00785 \times \sqrt{2 \times 9.81 \times 5}$$

$$= \mathbf{0{\cdot}0388 \text{ m}^3/s} \text{ (Ans.)}$$

(*ii*) *When mouthpiece is running full:*

Using the relation:

$$Q = 0.707 \times a \times \sqrt{2gH} \qquad \text{...[Eqn. (8.22)]}$$

$$= 0.707 \times 0.00785 \times \sqrt{2 \times 9.81 \times 5}$$

$$= \mathbf{0.0549 \text{ m}^3/s} \text{ (Ans.)}$$

Fluid Mechanics

Lecture – 16

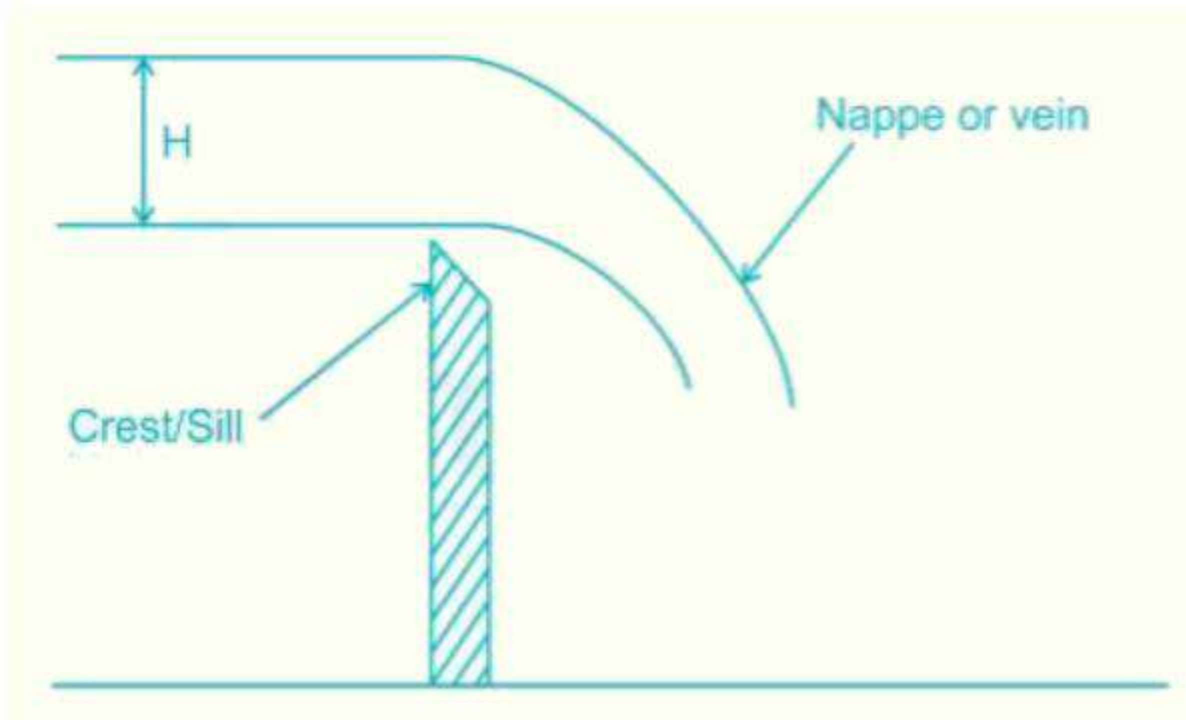
Dr Mohammed Tareq Khaleel

Year 2

Building and Construction Technology Engineering Department

FLOW OVER NOTCHES AND WEIRS

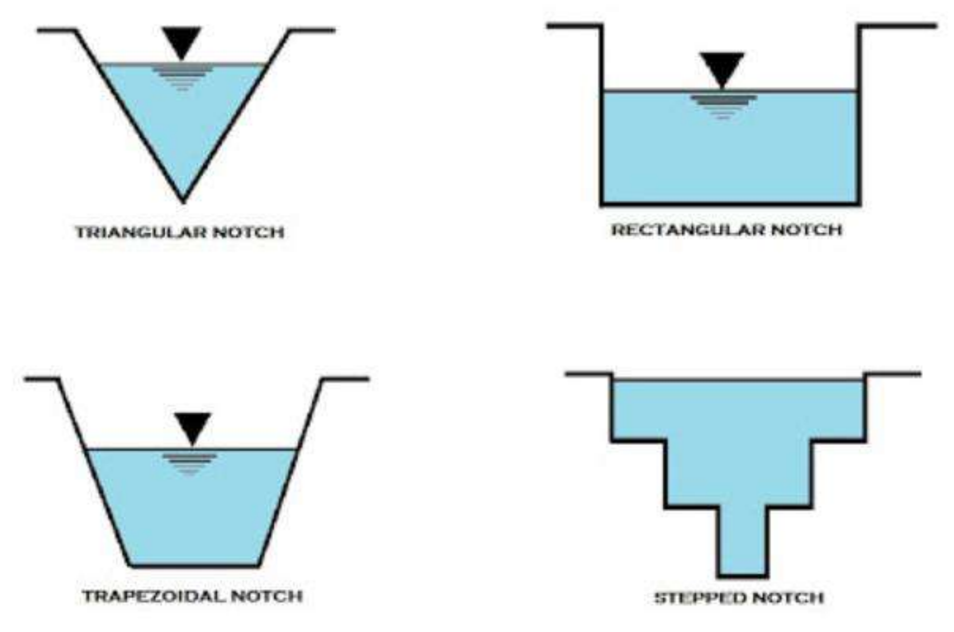
1. **Nappe or Vein.** The sheet of water flowing through a notch or over a weir is called Nappe or Vein.
2. **Crest or Sill.** The bottom edge of a notch or a top of a weir over which the water flows, is known as the sill or crest.



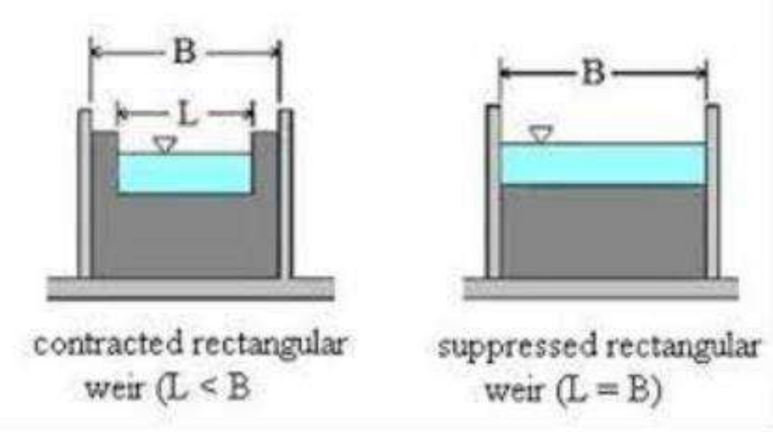
CLASSIFICATION OF NOTCHES AND WEIRS

The notches are classified as :

1. According to the shape of the opening :
   (*a*) Rectangular notch,
   (*b*) Triangular notch,
   (*c*) Trapezoidal notch, and
   (*d*) Stepped notch.



2. According to the effect of the sides on the nappe :
   (*a*) Notch with end contraction.
   (*b*) Notch without end contraction or suppressed notch.



CLASSIFICATION OF NOTCHES AND WEIRS

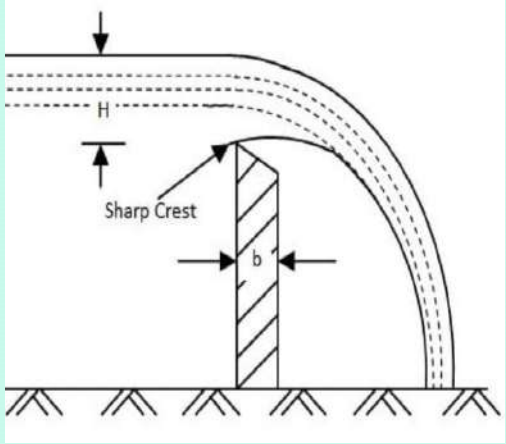
Weirs are classified according to the shape of the opening, the shape of the crest, the effect of the sides on the nappe and nature of discharge. The following are important classifications.

(*a*) According to the shape of the opening :

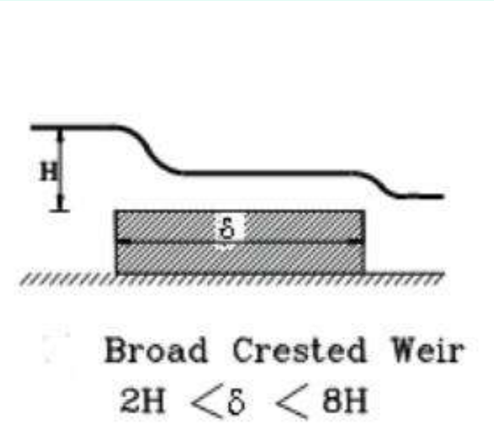
(*i*) Rectangular weir,
(*ii*) Triangular weir, and
(*iii*) Trapezoidal weir (Cipolletti weir)

(*b*) According to the shape of the crest :

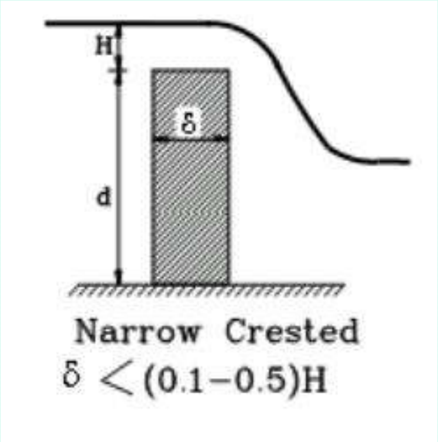
(*i*) Sharp-crested weir,
(*ii*) Broad-crested weir,
(*iii*) Narrow-crested weir, and
(*iv*) Ogee-shaped weir.



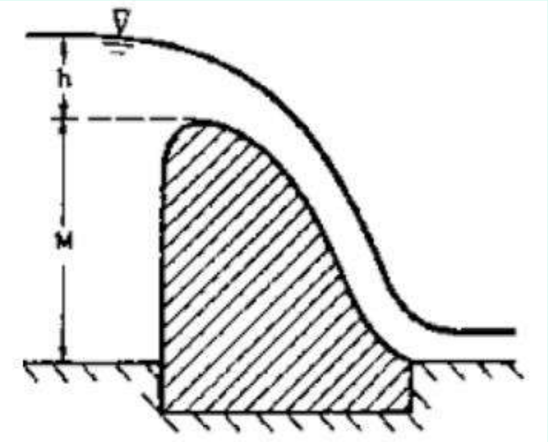
(*i*)



(*ii*)



(*iii*)

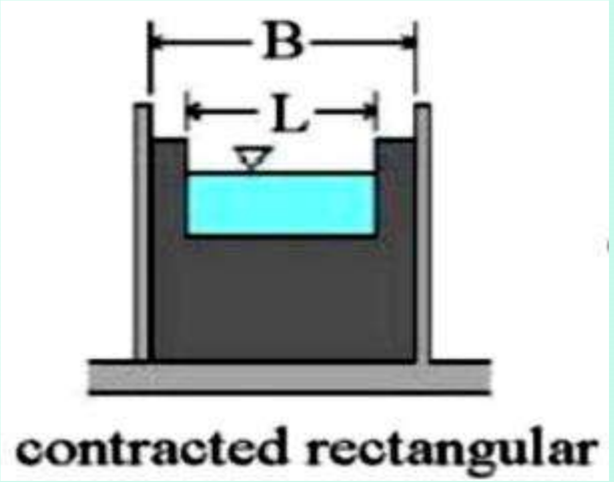


(*iv*)

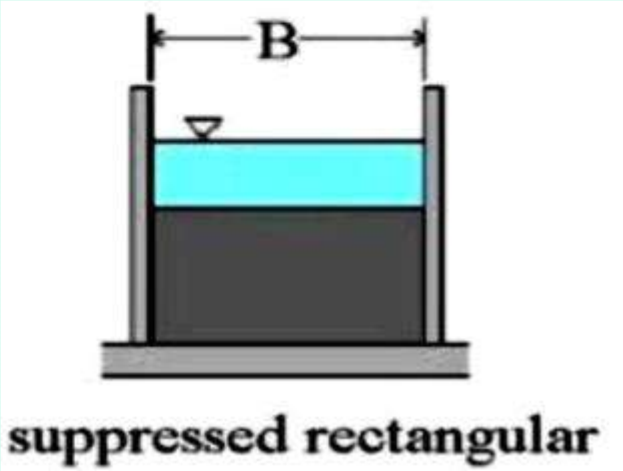
CLASSIFICATION OF NOTCHES AND WEIRS

(*c*) According to the effect of sides on the emerging nappe :

(*i*) Weir with end contraction, and (*ii*) Weir without end contraction.



(i)



(ii)

DISCHARGE OVER A RECTANGULAR NOTCH OR WEIR

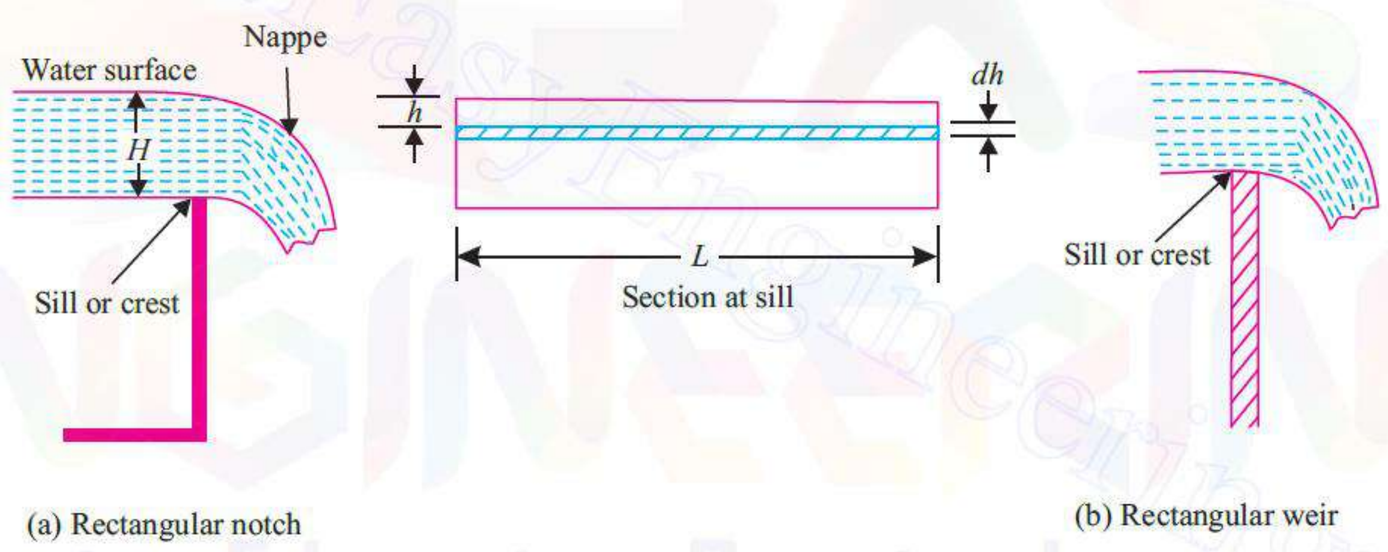


Fig. 9.1. Rectangular notch and weir.

H = Height of water above sill of the notch,

L = Length of notch or weir, and

C_d = Co-efficient of discharge.

Let us consider a horizontal strip of water of thickness dh at a depth h from the water level as shown in Fig. 9·1.

$$\text{Area of strip} = L \times dh$$

Theoretical velocity of water flowing through strip

$$= \sqrt{2gh}$$

The discharge through the strip,

$$dQ = C_d \times \text{area of strip} \times \text{theoretical velocity}$$

$$= C_d \times L \times dh \times \sqrt{2gh} \qquad ...(i)$$

DISCHARGE OVER A RECTANGULAR NOTCH OR WEIR

The total discharge, over the whole notch, may be found out by integrating the above equation within the limits 0 and H.

$$Q = \int_0^H C_d \times L \times \sqrt{2gh} \times dh$$

$$= C_d \times L \times \sqrt{2g} \int_0^H (h)^{1/2}\, dh$$

$$= C_d \times L \times \sqrt{2g} \left[\frac{h^{1/2+1}}{\frac{1}{2}+1} \right]_0^H$$

$$= C_d \times L \times \sqrt{2g} \left[\frac{h^{3/2}}{3/2} \right]_0^H$$

$$= 2/3 \times C_d \times L \times \sqrt{2g}\,(H)^{3/2}$$

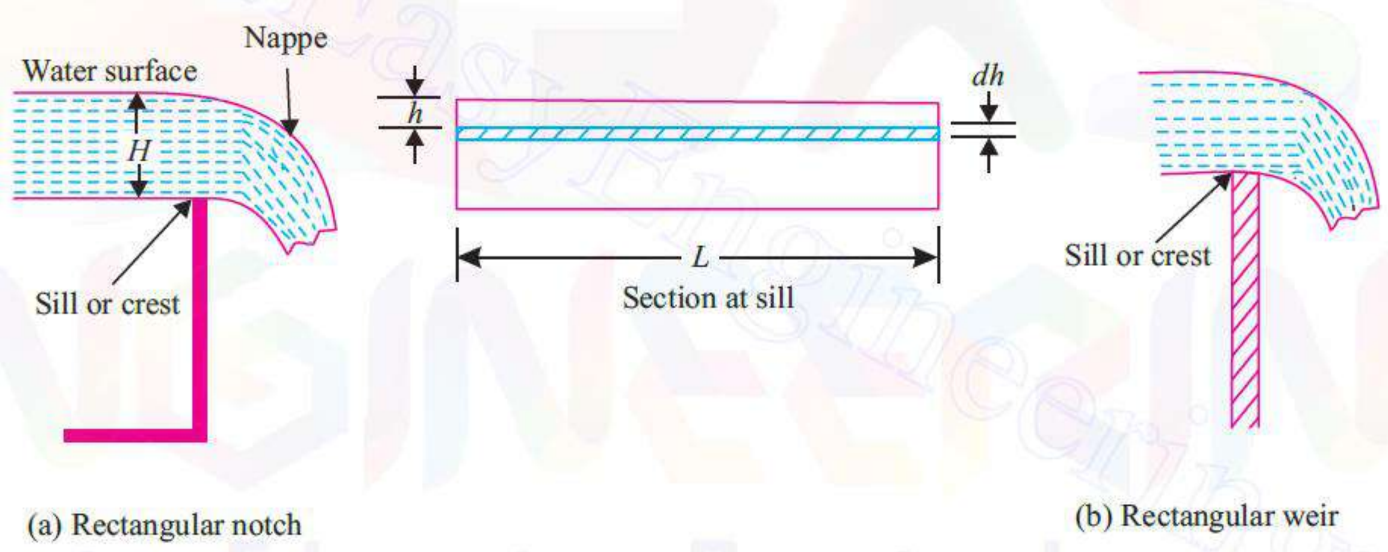


Fig. 9.1. Rectangular notch and weir.

$$Q = \frac{2}{3} C_d \,.\, L \sqrt{2g}\,(H)^{3/2} \qquad \ldots(9\cdot1)$$

Note. The expression for discharge over a rectangular notch or weir is *same*.

DISCHARGE OVER A RECTANGULAR NOTCH OR WEIR

Bernoulli's equation along the streamline

Pressure head at point 1 = $H - z_1$

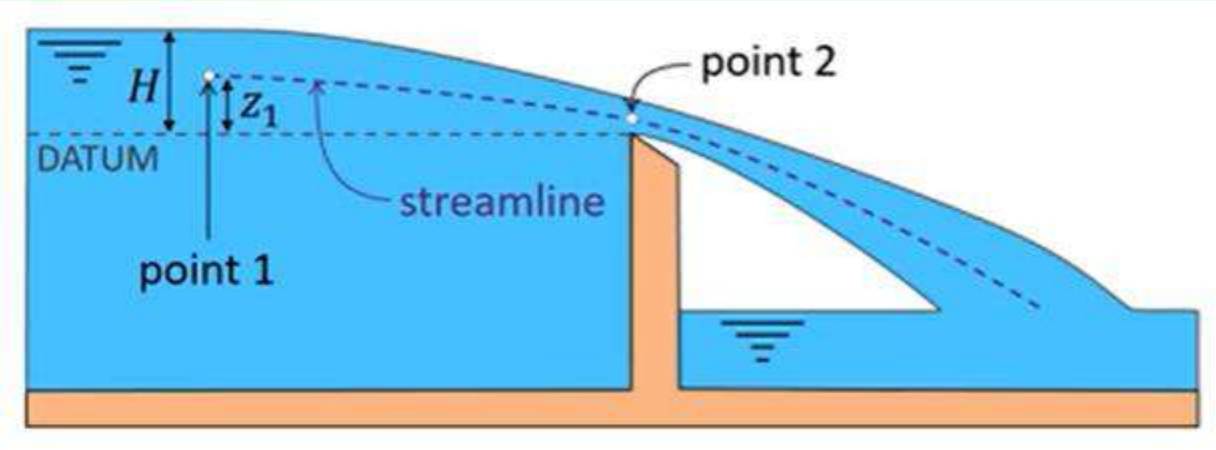
Pressure head at point 2 = 0

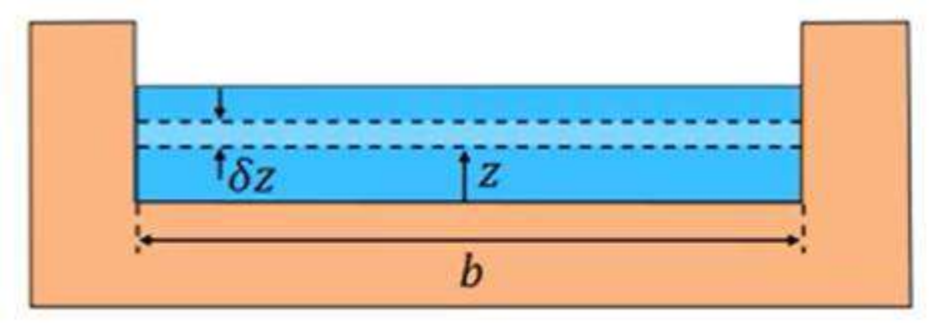
$$\not{z_1} + (H - \not{z_1}) + \frac{u_1^2}{2g} = z_2 + 0 + \frac{u_2^2}{2g}$$

$$u_2 = \sqrt{2g(H - z_2) + u_1^2}$$

$$\delta Q = b\delta z\sqrt{2g(H - z) + u_1^2}$$

$$Q = b\sqrt{2g}\int_0^H \left(H - z + \frac{u_1^2}{2g}\right)^{1/2} dz$$





$$Q = \frac{2}{3}b\sqrt{2g}\left\{\left(H + \frac{u_1^2}{2g}\right)^{3/2} - \left(\frac{u_1^2}{2g}\right)^{3/2}\right\}$$

Q_{ideal}

$$Q = \frac{2}{3}c_d b\sqrt{2g}\left\{\left(H + \frac{u_1^2}{2g}\right)^{3/2} - \left(\frac{u_1^2}{2g}\right)^{3/2}\right\}$$

if u_1 is very small $\quad Q = \frac{2}{3}c_d b\sqrt{2g}H^{3/2}$

Note. The expression for discharge over a rectangular notch or weir is *same*.

Source: https://www.youtube.com/watch?v=gxJWAUqGX9w

DISCHARGE OVER A RECTANGULAR NOTCH OR WEIR

Example. 9.1. *A rectangular notch 2·0 m wide has a constant head of 500 mm. Find the discharge over the notch, if co-efficient of discharge for the notch is 0·62.*

Solution. Length of the notch, $L = 2{\cdot}0$ m

Head over notch, $H = 500$ mm $= 0{\cdot}5$ m

Co-efficient of discharge, $C_d = 0{\cdot}62$

Discharge, Q:

Using the relation,

$$Q = \frac{2}{3} C_d \, . \, L\sqrt{2g} \, (H)^{3/2}$$

$$= \frac{2}{3} \times 0\cdot 62 \times 2\cdot 0 \times \sqrt{2 \times 9\cdot 81} \times (0\cdot 5)^{3/2}$$

$$= \mathbf{1{\cdot}294 \ m^3/s} \ \textbf{(Ans.)}$$

DISCHARGE OVER A TRIANGULAR NOTCH OR WEIR

Refer to Fig. 9·2. *A* triangular notch is also called a *V*-notch.

Let, H = Head of water above the apex of the notch,

θ = Angle of the notch, and

C_d = Co-efficient of discharge.

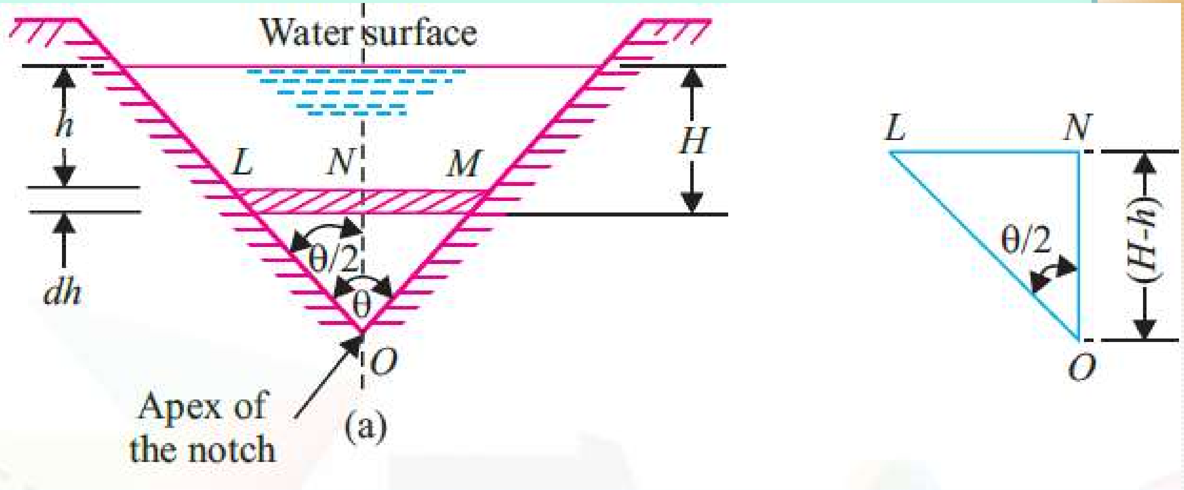


Fig. 9.2. The triangular notch.

$$Q = \frac{8}{15} C_d \sqrt{2g} \tan \frac{\theta}{2} H^{5/2} \quad \ldots(9\cdot2)$$

For a *right angled V-notch*, if $C_d = 0\cdot6$,

$$\left(\theta = 90°, \therefore \tan \frac{\theta}{2} = 1\right)$$

Then,
$$Q = \frac{8}{15} \times 0\cdot6 \sqrt{2 \times 9\cdot81} \times 1 \times H^{5/2}$$
$$= 1\cdot417\, H^{5/2} \quad \ldots(9\cdot3)$$

DISCHARGE OVER A TRIANGULAR NOTCH OR WEIR

Example 9.4. *During an experiment in a laboratory, 0·05 m^3 of water flowing over a right-angled notch was collected in one minute. If the head of the sill is 50 mm calculate the co-efficient of discharge of the notch.*

Solution. Discharge, $Q = 0{\cdot}05\ m^3/min = 0{\cdot}000833\ m^3/s$

Angle of notch, $\theta = 90°$

Head of the sill, $H = 50\ mm = 0{\cdot}05\ m$

Co-efficient of discharge, C_d:

Using the relation:

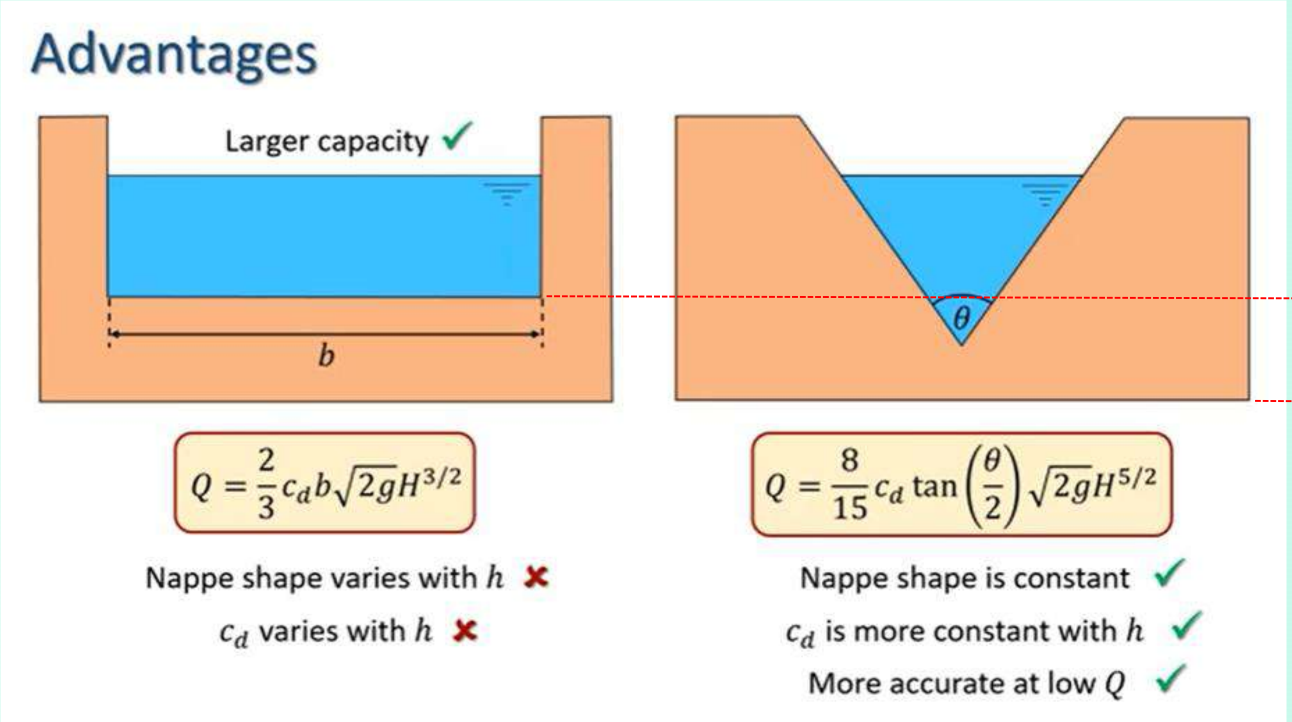
$$Q = \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2} \times H^{5/2}$$

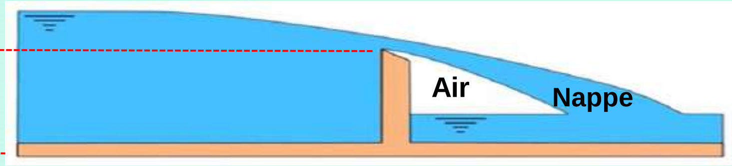
$$0 \cdot 000833 = \frac{8}{15} \times C_d \times \sqrt{2 \times 9 \cdot 81} \times \tan\left(\frac{90°}{2}\right) \times (0 \cdot 05)^{5/2}$$

$$= \frac{8}{15} \times C_d \times 4 \cdot 429 \times 1 \times 0 \cdot 000559 = 0 \cdot 00132\ C_d$$

$$C_d = \frac{0 \cdot 000833}{0 \cdot 00132} = \mathbf{0 \cdot 63}$$

Comparison of Rectangular and triangle weirs





Sectional side of the rectangular weir

Source: https://www.youtube.com/watch?v=gxJWAUqGX9w

EFFECT ON DISCHARGE OVER A NOTCH OR WEIR DUE TO ERROR IN THE MEASUREMENT OF HEAD

$$\frac{dQ}{Q} = \frac{K \times 3/2 \times H^{1/2}\, dH}{KH^{3/2}} = \frac{3}{2}\frac{dH}{H} \qquad \ldots(9{\cdot}5)$$

Eqn. (9·5) shows that an *error of 1% in measuring H will produce* 1·5 % *error in discharge over a rectangular notch or weir.*

$$\frac{dQ}{Q} = \frac{K \times 5/2 \times H^{3/2}\, dH}{K \times H^{5/2}} = \frac{5}{2}\frac{dH}{H} \qquad \ldots(9{\cdot}6)$$

Eqn. (9·6) shows that an error or *1% in measuring H will produce 2·5% error in discharge over a triangular notch or weir.*

Note: Equations (9.5) and (9.6) are not required.

DISCHARGE OVER A TRAPEZOIDAL NOTCH OR WEIR

Fig. 9·3 shows a trapezoidal notch or weir which is a combination of a rectangular and a triangular notch or weir. As such the discharge over such a notch or weir will be the *sum of the discharges over the rectangular and triangular notches or weirs.*

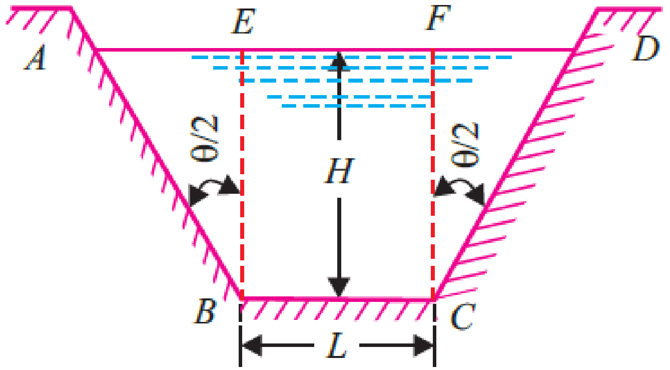


Fig. 9.3 The trapezoidal notch.

Let, H = Height of water over the notch,

L = Length of the rectangular portion (or crest) of the notch.,

C_{d1} = Co-efficient of discharge for the rectangular portion, and

C_{d2} = Co-efficient of discharge for the triangular portion.

The discharge through the rectangular portion $BCFE$ is given by (Eqn. 9·1),

$$Q_1 = \frac{2}{3} C_{d1} L \sqrt{2g}\, H^{3/2}$$

The discharge through two triangular notches ABE and FCD is equal to the discharge through a single triangular notch of angle θ and is given by [Eqn. 9.2],

$$Q_2 = \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2} \times H^{5/2}$$

∴ Discharge through trapezoidal notch or weir $ABCDA$,

$$Q = Q_1 + Q_2$$

$$= \frac{2}{3} C_{d1} L \sqrt{2g}\, H^{3/2} + \frac{8}{15} C_{d2} \sqrt{2g} \tan\frac{\theta}{2} \times H^{5/2} \qquad ...(9·4)$$

DISCHARGE OVER A STEPPED NOTCH

A **stepped notch** is *a combination of rectangular notches* as shown in Fig. 9·5. The discharge through a stepped notch is equal to the sum of the discharges through the different rectangular notches.

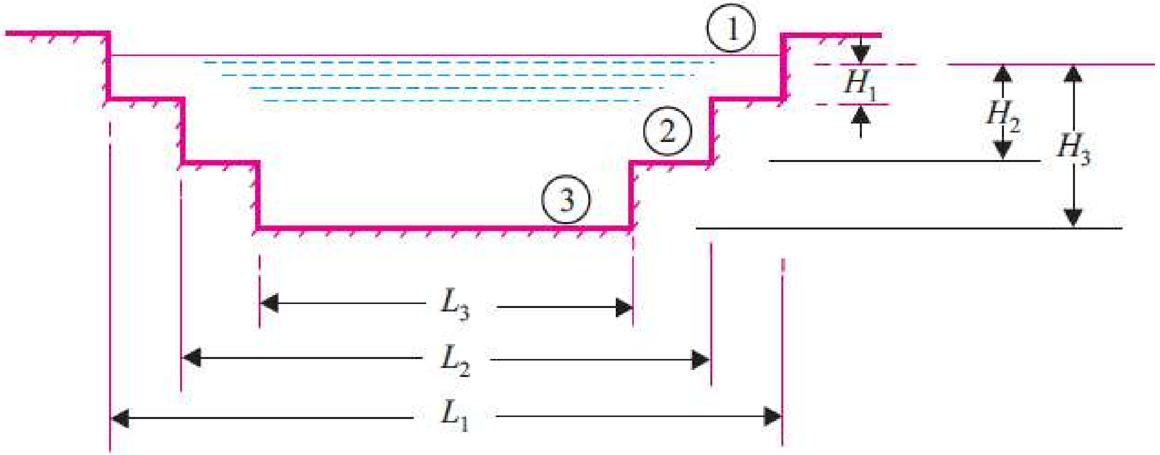


Fig. 9.5. The stepped notch.

H_1 = Height of water above sill of notch 1,
L_1 = Length of notch 1,
H_2, L_2 = Corresponding values for notch 2,
H_3, L_3 = Corresponding values for notch 3, and
C_d = Co-efficient of discharge for all notches.

The discharge over the notch 1,

$$Q_1 = \frac{2}{3} C_d . L_1 \sqrt{2g}\ H_1^{3/2}$$

Similarly, discharge over the notch 2,

$$Q_2 = \frac{2}{3} C_d . L_2 \sqrt{2g} \left[H_2^{3/2} - H_1^{3/2}\right]$$

and, discharge over the notch 3,

$$Q_3 = \frac{2}{3} C_d \cdot L_3 \sqrt{2g} \left[H_3^{3/2} - H_2^{3/2}\right]$$

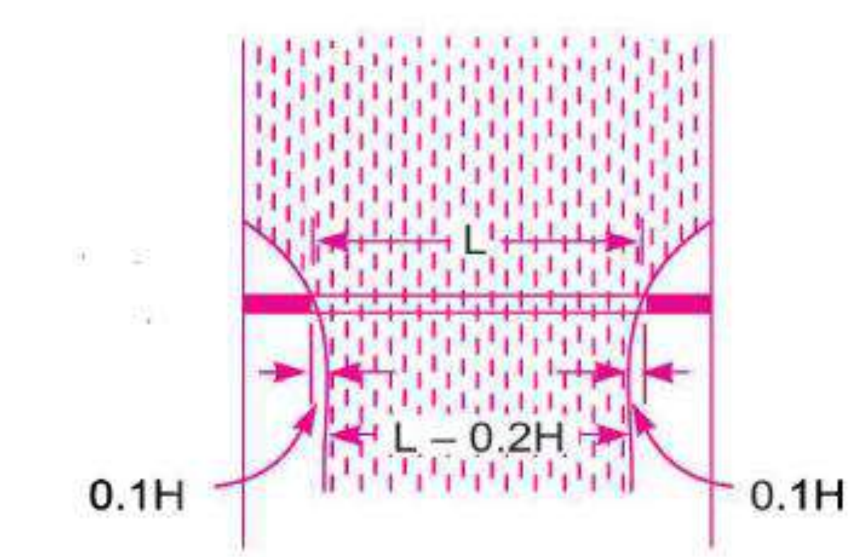
$\therefore$ Total discharge, $Q = Q_1 + Q_2 + Q_3$

Francis's Formula:

- The end contraction decreases the effective length of the crest of weir and hence decreases the discharge.
- Each end contraction reduces the crest length by $0{\cdot}1\ H$, where H is the head over the weir.

For a rectangular weir there are *two end contractions only* and hence *effective length* (L – 0.1nH)

$$= L - 0{\cdot}1 \times 2 \times H = L - 0{\cdot}2\ H$$



and discharge,

$$Q = \frac{2}{3} \times C_d \times (L - 0{\cdot}2\ H) \times \sqrt{2g}\ H^{3/2} \qquad \text{...(9·8)}$$

If there are n end contractions, we may write the empirical formula proposed by Francis as:

$$Q = \frac{2}{3} \times C_d \times (L - 0{\cdot}1\ nH) \times \sqrt{2g}\ H^{3/2} \qquad \text{... 9·8 }(a)$$

Example 9.12. *A 30 metres long weir is divided into 10 equal bays by vertical posts, each 0·6 m wide. Using Francis's formula, calculate the discharge over the weir under an effective head of 1 metre.*

Solution. Length of the weir = 30 m

Number of bays = 10

∴ Number of vertical posts = 10 – 1 = 9

Width of each post = 0·6 m

∴ Effective length, $L = 30 - 9 \times 0{\cdot}6 = 24{\cdot}6$ m

Number of end contractions, $n = 2 \times 10 = 20$

(*one bay has two end contractions*)

Head of water, H = 1 m

Discharge, *Q*:

Using Francis's formula,

$$Q = 1{\cdot}84\,(L - 0{\cdot}1 n H)\, H^{3/2}$$

$$= 1{\cdot}84\,(24{\cdot}6 - 0{\cdot}1 \times 20 \times 1) \times (1)^{3/2}$$

$$= \mathbf{41{\cdot}58\ m^3/s}$$ **(Ans.)**

CIPPOLETTI WEIR OR NOTCH

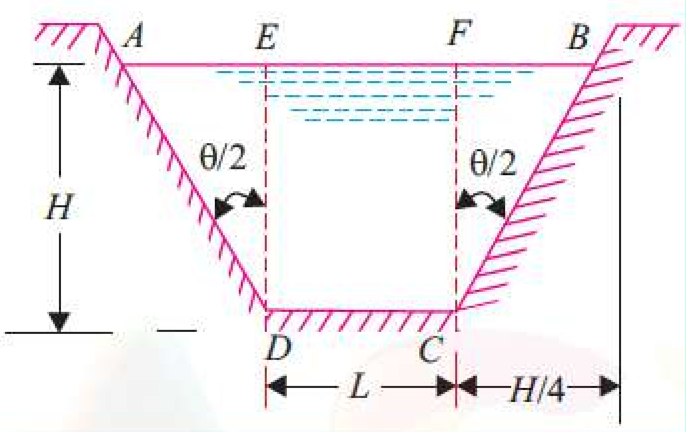


Fig. 9.8 Cippoletti weir.

Cippoletti weir or notch is a special type of trapezoidal weir having side slopes of 1 horizontal to 4 vertical.

Discharge over Cipolletti weir

= Discharge over rectangular weir without end contraction at same base length

$$Q = \frac{2}{3} C_d L \sqrt{2g} \times H^{3/2} \quad ...(9 \cdot 15)$$

DISCHARGE OVER A BROAD CRESTED WEIR

H = height of water above the crest
L = length of the crest

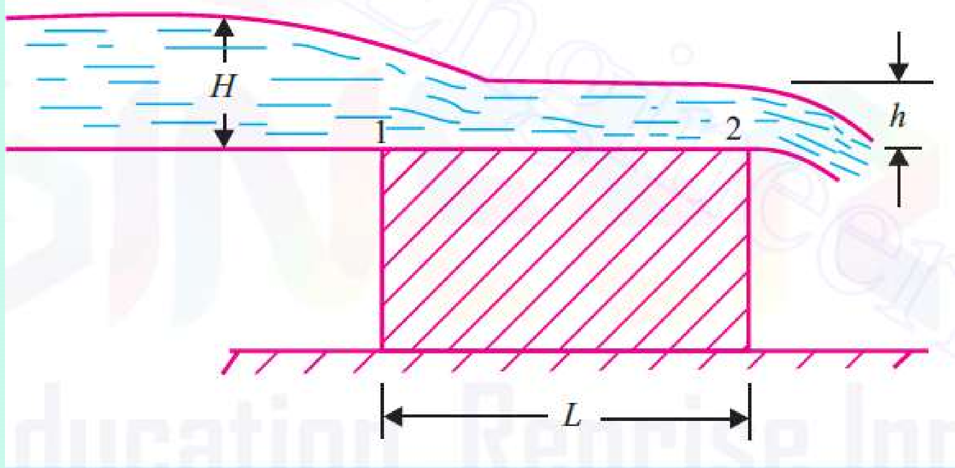


Fig. 9.9. Broad-crested weir.

If $2L > H$, the weir is called broad-crested weir

$$Q = \frac{2}{3\sqrt{3}} C_d \times L \times \sqrt{2g} \times H^{3/2}$$

$$Q = 1{\cdot}705 \times C_d \times L \times H^{3/2} \quad \text{...(9·17)}$$

DISCHARGE OVER A NARROW-CRESTED WEIR

In case of a narrow-crested weir, $2L < H$. This weir is similar to a rectangular weir or notch and hence, Q is given by:

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2} \quad \text{...(9·18)}$$

DISCHARGE OVER AN OGEE WEIR

In the Fig. 9·10 is shown an Ogee weir, in which the crest of the weir rises upto maximum height of 1·115 H and then falls as shown (where, H = height of water above inlet of the weir). The discharge over an Ogee weir is the same as that of a rectangular weir and is given by:

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2} \qquad \ldots(9\cdot18)$$

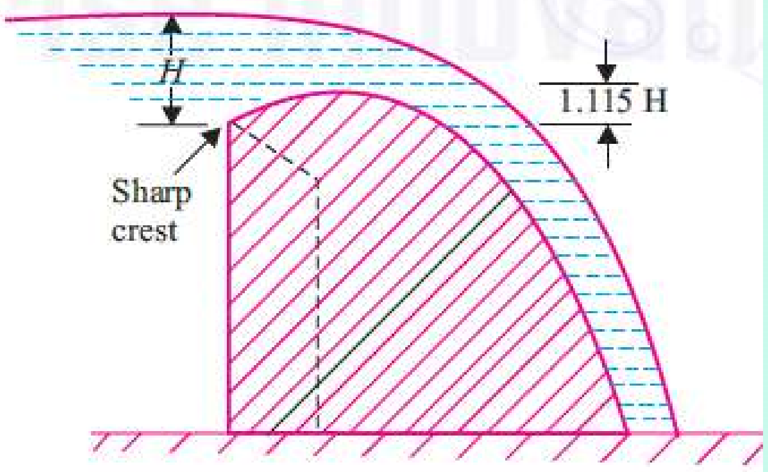


Fig. 9.10. An Ogee weir.

DISCHARGE OVER SUBMERGED OR DROWNED WEIR

A *weir is said to be* **submerged** *or* **drowned weir** *if the water level on its downstream side is above its crest*. Such a weir is shown in Fig. 9·11. The total discharge over the weir is obtained by dividing the weir into *two parts*. *The portion between upstream and downstream water surfaces may be treated as* **free weir** and portion between downstream water surface and crest as a **drowned weir.**

H = Height of water on the upstream side of the weir, and

h = Height of water on the downstream side of the weir.

Q_1 = Discharge over upper portion

$$= \frac{2}{3} . C_{d1} . L . \sqrt{2g}\ (H - h)^{3/2}$$

Q_2 = Discharge through drowned portion

$= C_{d2} \times$ area of flow $\times$ velocity of flow

$$= C_{d2} . L . h . \sqrt{2g\ (H - h)}$$

where, C_{d1} and C_{d2} are the respective discharge co-efficients.

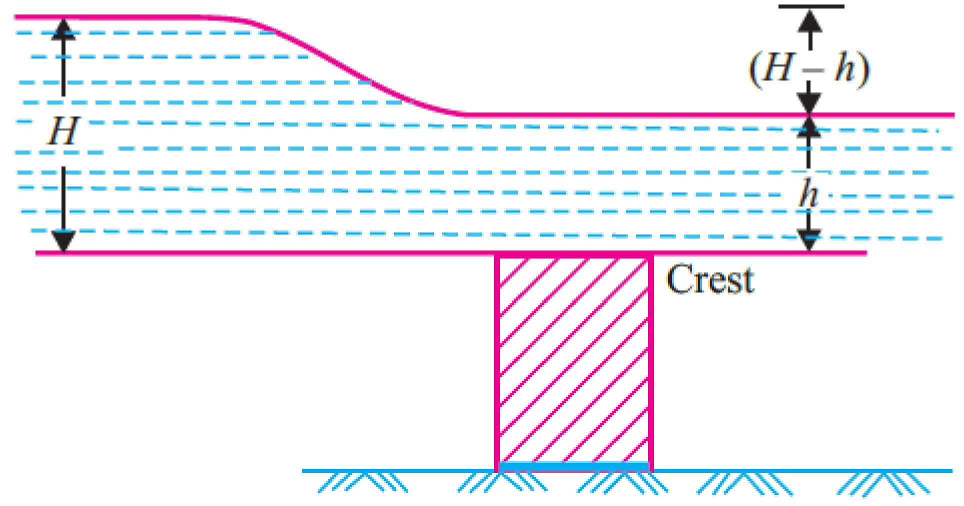


Fig. 9.11. Submerged weir.

$\therefore$ Total discharge, $Q = Q_1 + Q_2$

$$= \frac{2}{3} . C_{d1} . L . \sqrt{2g}\ (H - h)^{3/2} + C_{d2} . L . h . \sqrt{2g\ (H - h)} \quad ...(9·19)$$

DISCHARGE OVER SUBMERGED OR DROWNED WEIR

Example 9.17. *In a submerged weir of 2·5 m length the heights of water on the upstream and downstream sides are 0·2 m and 0·1 m respectively. Find the discharge over the weir if discharge co-efficients for free and drowned portions are 0·62 and 0·8 respectively.*

Solution. Length of weir, L = 25 m

Height of water on upstream side, H = 0.2 m

Height of water on downstream side, h = 0.1 m

C_{d1} = 0.62

C_{d2} = 0.8

Discharge over the weir, Q:

Total discharge, Q = Q_1 (discharge through free portion) + Q_2 (discharge through the drowned portion)

$$= \frac{2}{3} C_{d1} \times L \times \sqrt{2g}\,(H-h)^{3/2} + C_{d2} \times L \times h \times \sqrt{2g\,(H-h)} \quad \text{...[Eqn. (9·19)]}$$

$$= \frac{2}{3} \times 0\cdot62 \times 2\cdot5 \times \sqrt{2 \times 9\cdot81}\,(0\cdot2 - 0\cdot1)^{3/2}$$

$$+ 0\cdot8 \times 2\cdot5 \times 0\cdot1 \times \sqrt{2 \times 9\cdot81\,(0\cdot2 - 0\cdot1)}$$

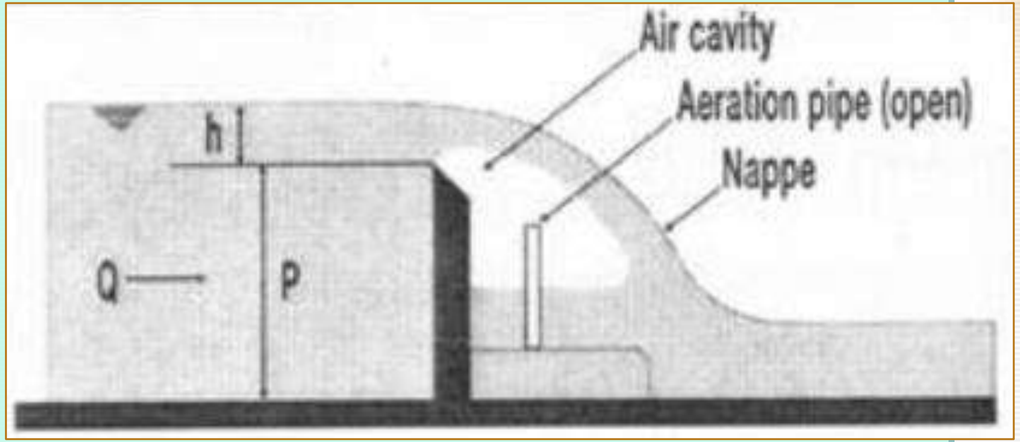
$= 0\cdot1447 + 0\cdot2801 =$ **0·4248 m³/s** **(Ans.)**

Determining the coefficients of discharge (Cd)

For a rectangular sharp crested weir: $Q=\frac{2}{3}Cd\,b\,(2g)^{\frac{1}{2}}\,h^{\frac{3}{2}}$

where:

Q	= Volume flowrate	(m^3/s)
	= Volume/time (using volumetric tank)	
Cd	= Coefficient of discharge	(Dimensionless)
B	= Breadth of weir	(m)
h	= Head above crest of weir (upstream)	(m)
g	= Gravitational constant	(9.81 m/s^2)
P	= Height of weir crest above bed	(m)



When the rectangular weir extends across the whole width of the channel it is called a suppressed weir and the Rehbock formula can be applied to determine Cd as follows:

$$Cd = 0.602 + 0.083 \times \frac{h}{P}$$