CHAPTER 1

Introduction

1.1 STRUCTURAL DESIGN

The structural design of buildings, whether of structural steel or reinforced concrete, requires the determination of the overall proportions and dimensions of the supporting framework and the selection of the cross sections of individual members. In most cases the functional design, including the establishment of the number of stories and the floor plan, will have been done by an architect, and the structural engineer must work within the constraints imposed by this design. Ideally, the engineer and architect will collaborate throughout the design process to complete the project in an efficient manner. In effect, however, the design can be summed up as follows: The architect decides how the building should look; the engineer must make sure that it doesn't fall down. Although this distinction is an oversimplification, it affirms the first priority of the structural engineer: safety. Other important considerations include serviceability (how well the structure performs in terms of appearance and deflection) and economy. An economical structure requires an efficient use of materials and construction labor. Although this objective can usually be accomplished by a design that requires a minimum amount of material, savings can often be realized by using more material if it results in a simpler, more easily constructed project. In fact, materials account for a relatively small portion of the cost of a typical steel structure as compared with labor and other costs (Ruby and Matuska, 2009).

A good design requires the evaluation of several framing plans—that is, different arrangements of members and their connections. In other words, several alternative designs should be prepared and their costs compared. For each framing plan investigated, the individual components must be designed. To do so requires the structural analysis of the building frames and the computation of forces and bending moments in the individual members. Armed with this information, the structural designer can then select the appropriate cross section. Before any analysis, however, a decision must be made on the primary building material to be used; it will usually be reinforced concrete, structural steel, or both. Ideally, alternative designs should be prepared with each.

The emphasis in this book will be on the design of individual structural steel members and their connections. The structural engineer must select and evaluate the overall structural system in order to produce an efficient and economical design but cannot do so without a thorough understanding of the design of the components (the "building blocks") of the structure. Thus component design is the focus of this book.

Before discussing structural steel, we need to examine various types of structural members. Figure 1.1 shows a truss with vertical concentrated forces applied at the joints along the top chord. In keeping with the usual assumptions of truss analysis—pinned connections and loads applied only at the joints—each component of the truss will be a two-force member, subject to either axial compression or tension. For simply supported trusses loaded as shown—a typical loading condition—each of the top chord members will be in compression, and the bottom chord members will be in tension. The web members will either be in tension or compression, depending on their location and orientation and on the location of the loads.

Other types of members can be illustrated with the rigid frame of Figure 1.2a. The members of this frame are rigidly connected by welding and can be assumed to form a continuous structure. At the supports, the members are welded to a rectangular plate that is bolted to a concrete footing. Placing several of these frames in parallel and connecting them with additional members that are then covered with roofing material and walls produces a typical building system. Many important details have not been mentioned, but many small commercial buildings are constructed essentially in this manner. The design and analysis of each frame in the system begins with the idealization of the frame as a two-dimensional structure, as shown in Figure 1.2b. Because the frame has a plane of symmetry parallel to the page, we are able to treat the frame as two-dimensional and represent the frame members by their centerlines. (Although it is not shown in Figure 1.1, this same idealization is made with trusses, and the members are usually represented by their centerlines.) Note that the supports are represented as hinges (pins), not as fixed supports. If there is a possibility that the footing will undergo a slight rotation, or if the connection is flexible enough to allow a slight rotation, the support must be considered to be pinned. One assumption made in the usual methods of structural analysis is that deformations are very small, which means that only a slight rotation of the support is needed to qualify it as a pinned connection.

Once the geometry and support conditions of the idealized frame have been established, the loading must be determined. This determination usually involves apportioning a share of the total load to each frame. If the hypothetical structure under consideration is subjected to a uniformly distributed roof load, the portion carried by one frame will be a uniformly distributed line load measured in force per unit length, as shown in Figure 1.2b. Typical units would be kips per foot.

FIGURE 1.1

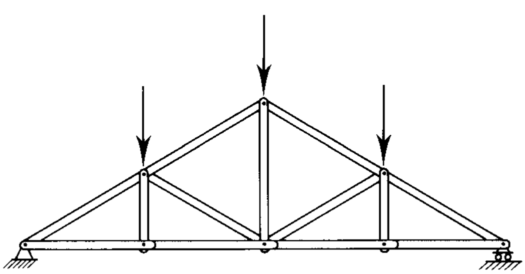
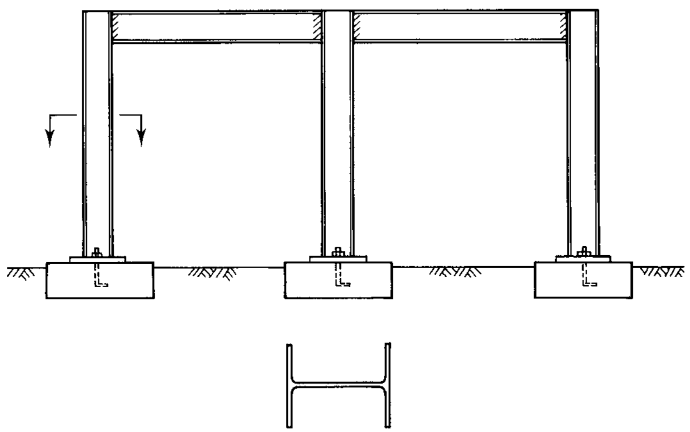
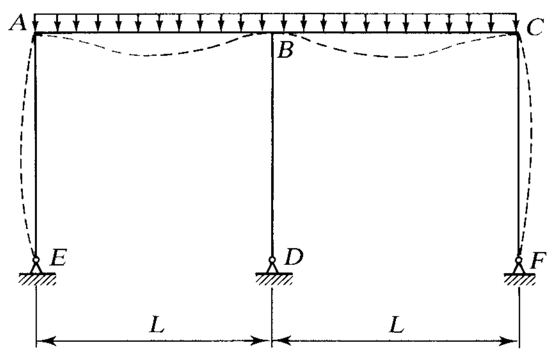


FIGURE 1.2



(a)



(b)

For the loading shown in Figure 1.2b, the frame will deform as indicated by the dashed line (drawn to a greatly exaggerated scale). The individual members of the frame can be classified according to the type of behavior represented by this deformed shape. The horizontal members *AB* and *BC* are subjected primarily to bending, or flexure, and are called *beams*. The vertical member *BD* is subjected to couples transferred from each beam, but for the symmetrical frame shown, they are equal and opposite, thereby canceling each other. Thus member *BD* is subjected only to axial compression arising from the vertical loads. In buildings, vertical compression members such as these are referred to as *columns*. The other two vertical members, *AE* and *CF*, must resist not only axial compression from the vertical loads but also a significant amount of bending. Such members are called *beam-columns*. In reality, all members, even those classified as beams or columns, will be subjected to both bending and axial load, but in many cases, the effects are minor and can be neglected.

In addition to the members described, this book covers the design of connections and the following special members: composite beams, composite columns, and plate girders.

1.2 LOADS

The forces that act on a structure are called *loads*. They belong to one of two broad categories: *dead load* and *live load*. Dead loads are those that are permanent, including the weight of the structure itself, which is sometimes called the *self-weight*. In addition to the weight of the structure, dead loads in a building include the weight of nonstructural components such as floor coverings, partitions, and suspended ceilings (with light fixtures, mechanical equipment, and plumbing). All of the loads mentioned thus far are forces resulting from gravity and are referred to as *gravity loads*. Live loads, which can also be gravity loads, are those that are not as permanent as dead loads. They may or may not be acting on the structure at any given time, and the location may not be fixed. Examples of live loads include furniture, equipment, and occupants of buildings. In general, the magnitude of a live load is not as well defined as that of a dead load, and it usually must be estimated. In many cases, a structural member must be investigated for various positions of a live load so that a potential failure condition is not overlooked.

If a live load is applied slowly and is not removed and reapplied an excessive number of times, the structure can be analyzed as if the load were static. If the load is applied suddenly, as would be the case when the structure supports a moving crane, the effects of impact must be accounted for. If the load is applied and removed many times over the life of the structure, fatigue stress becomes a problem, and its effects must be accounted for. Impact loading occurs in relatively few buildings, notably industrial buildings, and fatigue loading is rare, with thousands of load cycles over the life of the structure required before fatigue becomes a problem. For these reasons, all loading conditions in this book will be treated as static, and fatigue will not be considered.

Wind exerts a pressure or suction on the exterior surfaces of a building, and because of its transient nature, it properly belongs in the category of live loads. Because of the relative complexity of determining wind loads, however, wind is usually considered a separate category of loading. Because lateral loads are most detrimental to tall structures, wind loads are usually not as important for low buildings, but uplift on light roof systems can be critical. Although wind is present most of the time, wind loads of the magnitude considered in design are infrequent and are not considered to be fatigue loads.

Earthquake loads are another special category and need to be considered only in those geographic locations where there is a reasonable probability of occurrence. A structural analysis of the effects of an earthquake requires an analysis of the structure's response to the ground motion produced by the earthquake. Simpler methods are sometimes used in which the effects of the earthquake are simulated by a system of horizontal loads, similar to those resulting from wind pressure, acting at each floor level of the building.

Snow is another live load that is treated as a separate category. Adding to the uncertainty of this load is the complication of drift, which can cause much of the load to accumulate over a relatively small area.

Other types of live load are often treated as separate categories, such as hydrostatic pressure and soil pressure, but the cases we have enumerated are the ones ordinarily encountered in the design of structural steel building frames and their members.

1.3 BUILDING CODES

Buildings must be designed and constructed according to the provisions of a building code, which is a legal document containing requirements related to such things as structural safety, fire safety, plumbing, ventilation, and accessibility to the physically disabled. A building code has the force of law and is administered by a governmental entity such as a city, a county, or, for some large metropolitan areas, a consolidated government. Building codes do not give design procedures, but they do specify the design requirements and constraints that must be satisfied. Of particular importance to the structural engineer is the prescription of minimum live loads for buildings. Although the engineer is encouraged to investigate the actual loading conditions and attempt to determine realistic values, the structure must be able to support these specified minimum loads.

Although some large cities have their own building codes, many municipalities will modify a "model" building code to suit their particular needs and adopt it as modified. Model codes are written by various nonprofit organizations in a form that can be easily adopted by a governmental unit. Three national code organizations have developed model building codes: the *Uniform Building Code* (International Conference of Building Officials, 1999), the *Standard Building Code* (Southern Building Code Congress International, 1999), and the *BOCA National Building Code* (BOCA, 1999) (BOCA is an acronym for Building Officials and Code Administrators.) These codes have generally been used in different regions of the United States. The *Uniform Building Code* has been essentially the only one used west of the Mississippi, the *Standard Building Code* has been used in the southeastern states, and the *BOCA National Building Code* has been used in the northeastern part of the country.

A unified building code, the *International Building Code* (International Code Council, 2009), has been developed to eliminate some of the inconsistencies among the three national building codes. This was a joint effort by the three code organizations (ICBO, BOCA, and SBCCI). These organizations have merged into the International Code Council, and the new code has replaced the three regional codes.

Although it is not a building code, ASCE 7, *Minimum Design Loads for Buildings and Other Structures* (American Society of Civil Engineers, 2010) is similar in form to a building code. This standard provides load requirements in a format suitable for adoption as part of a code. The *International Building Code* incorporates much of ASCE 7 in its load provisions.

1.4 DESIGN SPECIFICATIONS

In contrast to building codes, design specifications give more specific guidance for the design of structural members and their connections. They present the guidelines and criteria that enable a structural engineer to achieve the objectives mandated by a building code. Design specifications represent what is considered to be good engineering practice based on the latest research. They are periodically revised and updated by the issuance of supplements or completely new editions. As with model building codes, design specifications are written in a legal format by nonprofit organizations. They have no legal standing on their own, but by presenting design criteria and limits in the form of legal mandates and prohibitions, they can easily be adopted, by reference, as part of a building code.

The specifications of most interest to the structural steel designer are those published by the following organizations.

1. **American Institute of Steel Construction (AISC):** This specification provides for the design of structural steel buildings and their connections. It is the one of primary concern in this book, and we discuss it in detail (AISC, 2010a).
2. **American Association of State Highway and Transportation Officials (AASHTO):** This specification covers the design of highway bridges and related structures. It provides for all structural materials normally used in bridges, including steel, reinforced concrete, and timber (AASHTO, 2010).
3. **American Railway Engineering and Maintenance-of-Way Association (AREMA):** The AREMA *Manual for Railway Engineering* covers the design of railway bridges and related structures (AREMA, 2010). This organization was formerly known as the American Railway Engineering Association (AREA).
4. **American Iron and Steel Institute (AISI):** This specification deals with cold-formed steel, which we discuss in Section 1.6 of this book (AISI, 2007).

1.5 STRUCTURAL STEEL

The earliest use of iron, the chief component of steel, was for small tools, in approximately 4000 B.C. (Murphy, 1957). This material was in the form of wrought iron, produced by heating ore in a charcoal fire. In the latter part of the eighteenth century and in the early nineteenth century, cast iron and wrought iron were used in various types of bridges. Steel, an alloy of primarily iron and carbon, with fewer impurities and less carbon than cast iron, was first used in heavy construction in the nineteenth century. With the advent of the Bessemer converter in 1855, steel began to displace wrought iron and cast iron in construction. In the United States, the first structural steel railroad bridge was the Eads bridge, constructed in 1874 in St. Louis, Missouri (Tall, 1964). In 1884, the first building with a steel frame was completed in Chicago.

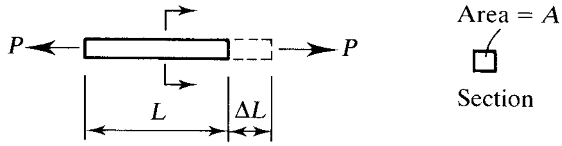
The characteristics of steel that are of the most interest to structural engineers can be examined by plotting the results of a tensile test. If a test specimen is subjected to an axial load *P*, as shown in Figure 1.3a, the stress and strain can be computed as follows:

$$f = \frac{P}{A} \quad \text{and} \quad \varepsilon = \frac{\Delta L}{L}$$

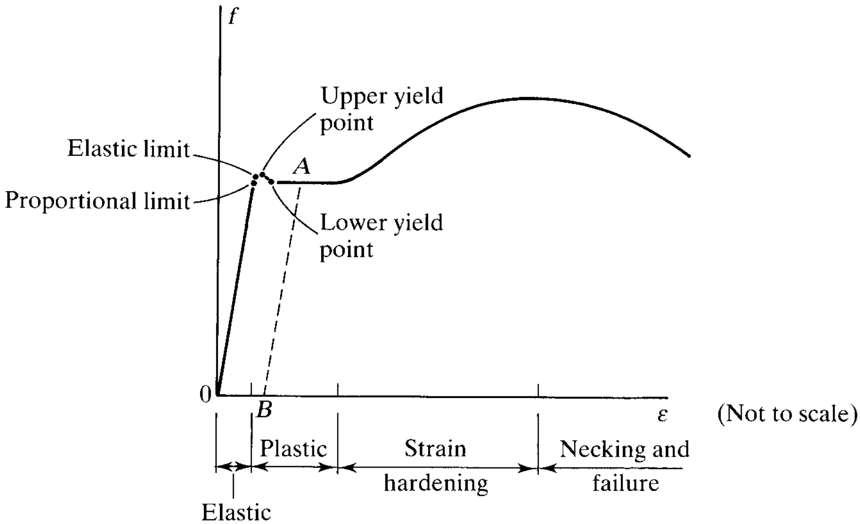
where

f = axial tensile stress
A = cross-sectional area
ε = axial strain
L = length of specimen
ΔL = change in length

FIGURE 1.3



(a)



(b)

If the load is increased in increments from zero to the point of fracture, and stress and strain are computed at each step, a stress–strain curve such as the one shown in Figure 1.3b can be plotted. This curve is typical of a class of steel known as *ductile*, or *mild*, *steel*. The relationship between stress and strain is linear up to the proportional limit; the material is said to follow *Hooke's law*. A peak value, the upper yield point, is quickly reached after that, followed by a leveling off at the lower yield point. The stress then remains constant, even though the strain continues to increase. At this stage of loading, the test specimen continues to elongate as long as the load is not removed, even though the load cannot be increased. This constant stress region is called the *yield plateau*, or *plastic range*. At a strain of approximately 12 times the strain at yield, strain hardening begins, and additional load (and stress) is required to cause additional elongation (and strain). A maximum value of stress is reached, after which the specimen begins to "neck down" as the stress decreases with increasing strain, and fracture occurs. Although the cross section is reduced during loading (the Poisson effect), the original cross-sectional area is used to compute all stresses. Stress computed in this way is known as *engineering stress*. If the original length is used to compute the strain, it is called *engineering strain*.

Steel exhibiting the behavior shown in Figure 1.3b is called *ductile* because of its ability to undergo large deformations before fracturing. Ductility can be measured by the elongation, defined as

$$e = \frac{L_f - L_0}{L_0} \times 100 \tag{1.1}$$

where

e = elongation (expressed as a percent)
L_f = length of the specimen at fracture
L_0 = original length

The elastic limit of the material is a stress that lies between the proportional limit and the upper yield point. Up to this stress, the specimen can be unloaded without permanent deformation; the unloading will be along the linear portion of the diagram, the same path followed during loading. This part of the stress–strain diagram is called the *elastic range*. Beyond the elastic limit, unloading will be along a straight line parallel to the initial linear part of the loading path, and there will be a permanent strain. For example, if the load is removed at point A in Figure 1.3b, the unloading will be along line AB, resulting in the permanent strain OB.

Figure 1.4 shows an idealized version of this stress–strain curve. The proportional limit, elastic limit, and the upper and lower yield points are all very close to one another and are treated as a single point called the *yield point*, defined by the stress F_y. The other point of interest to the structural engineer is the maximum value of stress that can be attained, called the *ultimate tensile strength*, F_u. The shape of this curve is typical of mild structural steels, which are different from one another primarily in the values of F_y and F_u. The ratio of stress to strain within the elastic range, denoted E and called *Young's modulus*, or *modulus of elasticity*, is the same for all structural steels and has a value of 29,000,000 psi (pounds per square inch) or 29,000 ksi (kips per square inch).

Figure 1.5 shows a typical stress–strain curve for high-strength steels, which are less ductile than the mild steels discussed thus far. Although there is a linear elastic portion and a distinct tensile strength, there is no well-defined yield point or yield plateau.

FIGURE 1.4

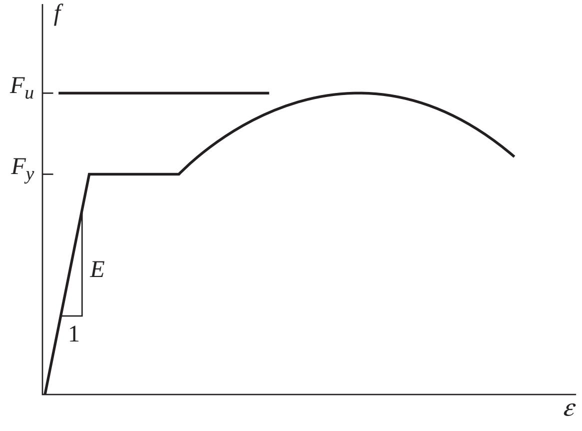
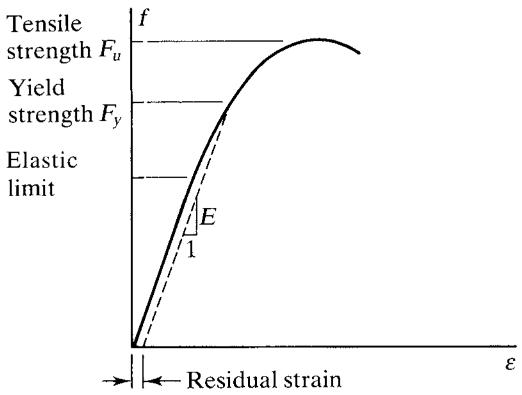


FIGURE 1.5



To use these higher-strength steels in a manner consistent with the use of ductile steels, some value of stress must be chosen as a value for F_y so that the same procedures and formulas can be used with all structural steels. Although there is no yield point, one needs to be defined. As previously shown, when a steel is stressed beyond its elastic limit and then unloaded, the path followed to zero stress will not be the original path from zero stress; it will be along a line having the slope of the linear portion of the path followed during loading—that is, a slope equal to E, the modulus of elasticity. Thus there will be a residual strain, or permanent set, after unloading. The yield stress for steel with a stress–strain curve of the type shown in Figure 1.5 is called the *yield strength* and is defined as the stress at the point of unloading that corresponds to a permanent strain of some arbitrarily defined amount. A strain of 0.002 is usually selected, and this method of determining the yield strength is called the *0.2% offset method.* As previously mentioned, the two properties usually needed in structural steel design are F_u and F_y, regardless of the shape of the stress–strain curve and regardless of how F_y was obtained. For this reason, the generic term *yield stress* is used, and it can mean either yield point or yield strength.

The various properties of structural steel, including strength and ductility, are determined by its chemical composition. Steel is an alloy, its principal component being iron. Another component of all structural steels, although in much smaller amounts, is carbon, which contributes to strength but reduces ductility. Other components of some grades of steel include copper, manganese, nickel, chromium, molybdenum, and silicon. Structural steels can be grouped according to their composition as follows.

1. **Plain carbon steels:** mostly iron and carbon, with less than 1% carbon.
2. **Low-alloy steels:** iron and carbon plus other components (usually less than 5%). The additional components are primarily for increasing strength, which is accomplished at the expense of a reduction in ductility.
3. **High-alloy or specialty steels:** similar in composition to the low-alloy steels but with a higher percentage of the components added to iron and carbon. These steels are higher in strength than the plain carbon steels and also have some special quality, such as resistance to corrosion.

Different grades of structural steel are identified by the designation assigned them by the American Society for Testing and Materials (ASTM). This organization

TABLE 1.1

Property	A36	A572 Gr. 50	A992
Yield point, min.	36 ksi	50 ksi	50 ksi
Tensile strength, min.	58 to 80 ksi	65 ksi	65 ksi
Yield to tensile ratio, max.	—	—	0.85
Elongation in 8 in., min.	20%	18%	18%

develops standards for defining materials in terms of their composition, properties, and performance, and it prescribes specific tests for measuring these attributes (ASTM, 2010a). One of the most commonly used structural steels is a mild steel designated as ASTM A36, or A36 for short. It has a stress–strain curve of the type shown in Figures 1.3b and 1.4 and has the following tensile properties.

Yield stress: $F_y = 36{,}000$ psi (36 ksi)

Tensile strength: $F_u = 58{,}000$ psi to 80,000 psi (58 ksi to 80 ksi)

A36 steel is classified as a plain carbon steel, and it has the following components (other than iron).

Carbon: 0.26% (maximum)

Phosphorous: 0.04% (maximum)

Sulfur: 0.05% (maximum)

These percentages are approximate, the exact values depending on the form of the finished steel product. A36 is a ductile steel, with an elongation as defined by Equation 1.1 of 20% based on an undeformed original length of 8 inches.

Steel producers who provide A36 steel must certify that it meets the ASTM standard. The values for yield stress and tensile strength shown are minimum requirements; they may be exceeded and usually are to a certain extent. The tensile strength is given as a range of values because for A36 steel, this property cannot be achieved to the same degree of precision as the yield stress.

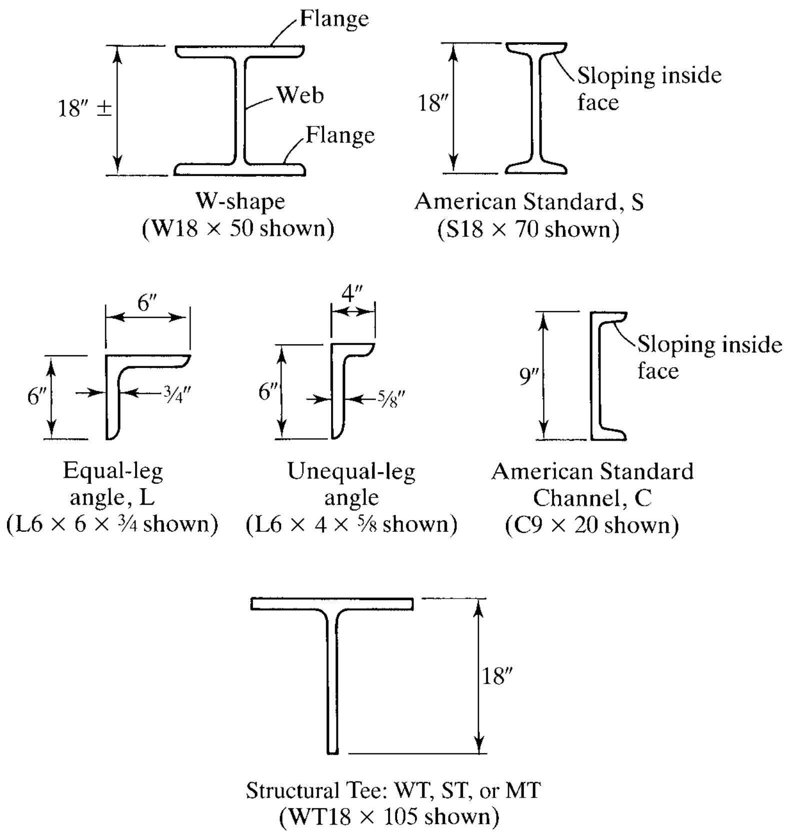
Other commonly used structural steels are ASTM A572 Grade 50 and ASTM A992. These two steels are very similar in both tensile properties and chemical composition, with a maximum carbon content of 0.23%. A comparison of the tensile properties of A36, A572 Grade 50, and A992 is given in Table 1.1.

1.6 STANDARD CROSS-SECTIONAL SHAPES

In the design process outlined earlier, one of the objectives—and the primary emphasis of this book—is the selection of the appropriate cross sections for the individual members of the structure being designed. Most often, this selection will entail choosing a standard cross-sectional shape that is widely available rather than requiring the fabrication of a shape with unique dimensions and properties. The selection of an "off-the-shelf" item will almost always be the most economical choice, even if it means using slightly more material. The largest category of standard shapes includes those produced by *hot-rolling*. In this manufacturing process, which takes place in a mill, molten steel is taken from an electric arc furnace and poured into a *continuous casting* system where the steel solidifies but is never allowed to cool completely. The hot steel passes through a series of rollers that squeeze the material into the desired cross-sectional shape. Rolling the steel while it is still hot allows it to be deformed with no resulting loss in ductility, as would be the case with cold-working. During the rolling process, the member increases in length and is cut to standard lengths, usually a maximum of 65 to 75 feet, which are subsequently cut (in a fabricating shop) to the lengths required for a particular structure.

Cross sections of some of the more commonly used hot-rolled shapes are shown in Figure 1.6. The dimensions and designations of the standard available shapes are defined in the ASTM standards (ASTM, 2010b). The *W-shape,* also called a *wide-flange shape,* consists of two parallel flanges separated by a single web. The orientation of these elements is such that the cross section has two axes of symmetry. A typical designation would be "W18 × 50," where W indicates the type of shape, 18 is the nominal depth parallel to the web, and 50 is the weight in pounds per foot of length. The nominal depth is the approximate depth expressed in whole inches. For some of the lighter shapes, it is equal to the depth to the nearest inch, but this is not a general rule for the W-shapes. All of the W-shapes of a given nominal size can be grouped into families that have the same depth from inside-of-flange to inside-of-flange but with different flange thicknesses.

FIGURE 1.6



The *American Standard,* or *S-shape,* is similar to the W-shape in having two parallel flanges, a single web, and two axes of symmetry. The difference is in the proportions: The flanges of the W are wider in relation to the web than are the flanges of the S. In addition, the outside and inside faces of the flanges of the W-shape are parallel, whereas the inside faces of the flanges of the S-shape slope with respect to the outside faces. An example of the designation of an S-shape is "S18 × 70," with the S indicating the type of shape, and the two numbers giving the depth in inches and the weight in pounds per foot. This shape was formerly called an *I-beam*.

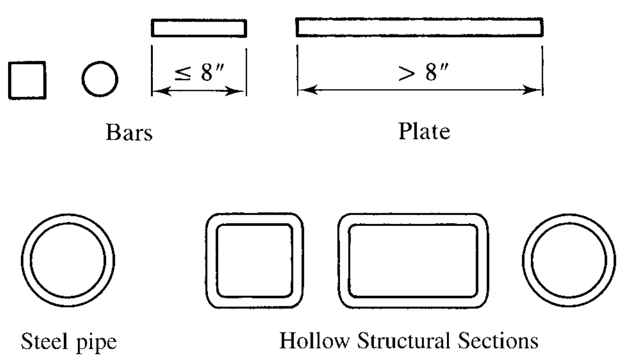
The angle shapes are available in either equal-leg or unequal-leg versions. A typical designation would be "L6 × 6 × ¾" or "L6 × 4 × ⅝." The three numbers are the lengths of each of the two legs as measured from the corner, or heel, to the toe at the other end of the leg, and the thickness, which is the same for both legs. In the case of the unequal-leg angle, the longer leg dimension is always given first. Although this designation provides all of the dimensions, it does not provide the weight per foot.

The *American Standard Channel,* or *C-shape,* has two flanges and a web, with only one axis of symmetry; it carries a designation such as "C9 × 20." This notation is similar to that for W- and S-shapes, with the first number giving the total depth in inches parallel to the web and the second number the weight in pounds per linear foot. For the channel, however, the depth is exact rather than nominal. The inside faces of the flanges are sloping, just as with the American Standard shape. Miscellaneous Channels—for example, the MC10 × 25—are similar to American Standard Channels.

The *Structural Tee* is produced by splitting an I-shaped member at middepth. This shape is sometimes referred to as a *split-tee*. The prefix of the designation is either WT, ST, or MT, depending on which shape is the "parent." For example, a WT18 × 105 has a nominal depth of 18 inches and a weight of 105 pounds per foot, and is cut from a W36 × 210. Similarly, an ST10 × 33 is cut from an S20 × 66, and an MT5 × 4 is cut from an M10 × 8. The "M" is for "miscellaneous." The M-shape has two parallel flanges and a web, but it does not fit exactly into either the W or S categories. The HP shape, used for bearing piles, has parallel flange surfaces, approximately the same width and depth, and equal flange and web thicknesses. HP-shapes are designated in the same manner as the W-shape; for example, HP14 × 117.

Other frequently used cross-sectional shapes are shown in Figure 1.7. *Bars* can have circular, square, or rectangular cross sections. If the width of a rectangular shape

FIGURE 1.7



is 8 inches or less, it is classified as a bar. If the width is more than 8 inches, the shape is classified as a *plate*. The usual designation for both is the abbreviation PL (for plate, even though it could actually be a bar) followed by the thickness in inches, the width in inches, and the length in feet and inches; for example, PL $\frac{3}{8} \times 5 \times 3'\text{-}2\frac{1}{2}''$. Although plates and bars are available in increments of $\frac{1}{16}$ inch, it is customary to specify dimensions to the nearest $\frac{1}{8}$ inch. Bars and plates are formed by hot-rolling.

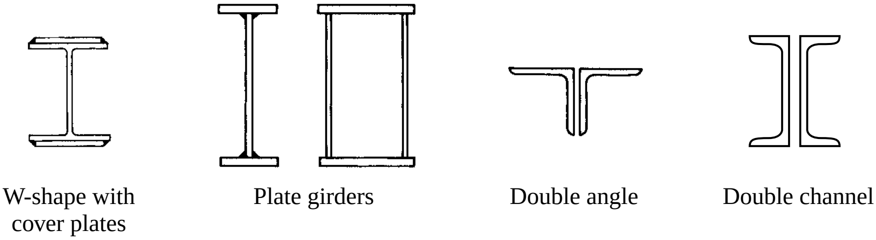
Also shown in Figure 1.7 are hollow shapes, which can be produced either by bending plate material into the desired shape and welding the seam or by hot-working to produce a seamless shape. The shapes are categorized as steel pipe, round HSS, and square and rectangular HSS. The designation HSS is for "Hollow Structural Sections."

Steel pipe is available as standard, extra-strong, or double-extra-strong, with designations such as Pipe 5 Std., Pipe 5 x-strong, or Pipe 5 xx-strong, where 5 is the nominal outer diameter in inches. The different strengths correspond to different wall thicknesses for the same outer diameter. For pipes whose thicknesses do not match those in the standard, extra-strong, or double-extra-strong categories, the designation is the outer diameter and wall thickness in inches, expressed to three decimal places; for example, Pipe 5.563×0.500.

Round HSS are designated by outer diameter and wall thickness, expressed to three decimal places; for example, HSS 8.625×0.250. Square and rectangular HSS are designated by nominal outside dimensions and wall thickness, expressed in rational numbers; for example, HSS $7 \times 5 \times \frac{3}{8}$. Most hollow structural sections available in the United States today are produced by cold-forming and welding (Sherman, 1997).

Other shapes are available, but those just described are the ones most frequently used. In most cases, one of these standard shapes will satisfy design requirements. If the requirements are especially severe, then a built-up section, such as one of those shown in Figure 1.8, may be needed. Sometimes a standard shape is augmented by additional cross-sectional elements, as when a cover plate is welded to one or both flanges of a W-shape. Building up sections is an effective way of strengthening an existing structure that is being rehabilitated or modified for some use other than the one for which it was designed. Sometimes a built-up shape must be used because none of the standard rolled shapes are large enough; that is, the cross section does not have enough area or moment of inertia. In such cases, plate girders can be used. These can be I-shaped sections, with two flanges and a web, or box sections, with two flanges and two webs. The components can be welded together and can be designed to have

FIGURE 1.8



exactly the properties needed. Built-up shapes can also be created by attaching two or more standard rolled shapes to each other. A widely used combination is a pair of angles placed back-to-back and connected at intervals along their length. This is called a *double-angle shape*. Another combination is the double-channel shape (either American Standard or Miscellaneous Channel). There are many other possibilities, some of which we illustrate throughout this book.

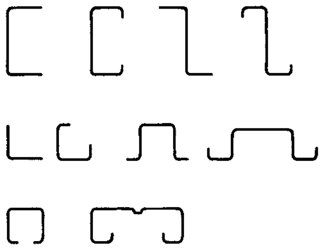
The most commonly used steels for rolled shapes and plate material are ASTM A36, A572, and A992. ASTM A36 is usually specified for angles, plates, S, M, and channel shapes; A572 Grade 50 for HP shapes; and A992 for W shapes. (These three steels were compared in Table 1.1 in Section 1.5.) Steel pipe is available in ASTM A53 Grade B only. ASTM A500 is usually specified for hollow structural sections (HSS). These recommendations are summarized in Table 1.2. Other steels can be used for these shapes, but the ones listed in Table 1.2 are the most common (Anderson and Carter, 2009).

Another category of steel products for structural applications is cold-formed steel. Structural shapes of this type are created by bending thin material such as sheet steel or plate into the desired shape without heating. Typical cross sections are shown in Figure 1.9. Only relatively thin material can be used, and the resulting shapes are suitable only for light applications. An advantage of this product is its versatility, since almost any conceivable cross-sectional shape can easily be formed. In addition, cold-working will increase the yield point of the steel, and under certain conditions it may be accounted for in design (AISI, 2007). This increase comes at the expense of a reduction in ductility, however. Because of the thinness of the cross-sectional elements, the problem of instability (discussed in Chapters 4 and 5) is a particularly important factor in the design of cold-formed steel structures.

TABLE 1.2

Shape	Preferred Steel
Angles	A36
Plates	A36
S, M, C, MC	A36
HP	A572 Grade 50
W	A992
Pipe	A53 Grade B (only choice)
HSS	A500 Grade B (round) or C (rectangular)

FIGURE 1.9



Problems

Note The following problems illustrate the concepts of stress and strain covered in Section 1.5. The materials cited in these problems are not necessarily steel.

1.5-1 A 20-foot-long W8 × 67 is suspended from one end. If the modulus of elasticity is 29,000 ksi, determine the following.

a. What is the maximum tensile stress?

b. What is the maximum normal strain?

1.5-2 The strain in member *AB* was measured to be 8.9×10^{-4}. If the member is an L3 × $2\frac{1}{2} \times \frac{1}{4}$ of A36 steel, determine the following.

a. What is the change in length in inches?

b. What is the force in the member?

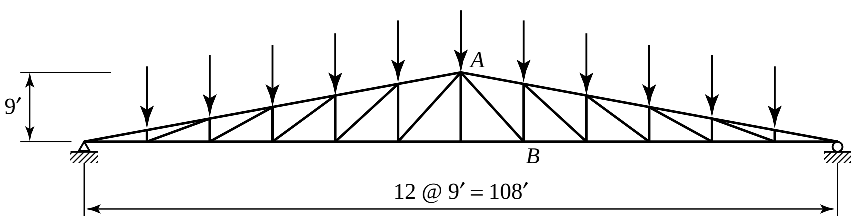


FIGURE P1.5-2

1.5-3 During a tensile test of a specimen of unknown material, an increase in length of 6.792×10^{-3} inches within the gage length was recorded at a load of 5000 lb. The specimen diameter was 0.5 inch and the gage length was 8 inches. (The gage length is the distance between two marks placed along the length of the specimen.)

a. Based on this one data point, what is the modulus of elasticity?

b. If the maximum load reached before fracture was 14,700 lb, what is the ultimate tensile stress?

1.5-4 A tensile test was conducted on a specimen with a diameter of 0.5 inch. A strain gage was bonded to the specimen so that the strain could be obtained directly. The following data were obtained:

Load (lb)	Strain (micro in./in.)
2,000	47
2,500	220
3,000	500
3,500	950
4,000	1,111
4,500	1,200
5,000	1,702

a. Create a table of stress and strain values.
b. Plot these data points, and draw a *best-fit* straight line through them.
c. What is the slope of this line? What does this value represent?

1.5-5 A tension test was conducted on a specimen with a circular cross section of diameter 0.5 inch and a gage length of 8 inches. (The gage length is the distance between two marks on the specimen. The deformation is measured within this length.) The stress and strain were computed from the test data and plotted. Two plots are shown here; the first one shows the entire test range, and the second shows a portion near the proportional limit.

a. Draw best-fit lines to obtain stress–strain curves.
b. Estimate the proportional limit.
c. Use the slope of the best-fit line to estimate the modulus of elasticity.
d. Estimate the 0.2% offset yield strength.
e. Estimate the ultimate stress.
f. If a load of 10 kips is applied and then removed, estimate the permanent deformation in inches.

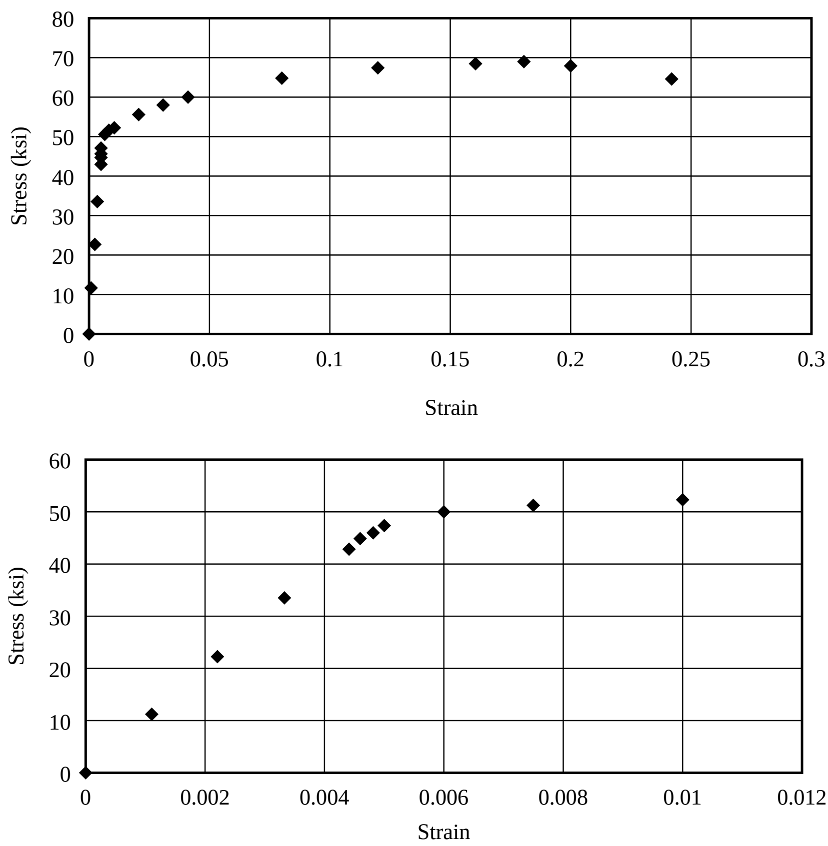


FIGURE P1.5-5

1.5-6 The data shown in the table were obtained from a tensile test of a metal specimen with a diameter of 0.500 inch and a gage length (the length over which the elongation is measured) of 2.00 inches. The specimen was not loaded to failure.

a. Generate a table of stress and strain values.

b. Plot these values and draw a best-fit line to obtain a stress–strain curve.

c. Use the slope of the best-fit line to estimate the modulus of elasticity.

d. Estimate the value of the proportional limit.

e. Use the 0.2% offset method to determine the yield strength.

Load (kips)	Elongation (in.)
0	0
1	0.0010
2	0.0014
2.5	0.0020
3.5	0.0024
5	0.0036
6	0.0044
7	0.0050
8	0.0060
9	0.0070
10	0.0080
11.5	0.0120
12	0.0180

1.5-7 The data shown in the table were obtained from a tensile test of a metal specimen with a rectangular cross section of 0.2 in.^{2} in area and a gage length (the length over which the elongation is measured) of 2.000 inches.

a. Generate a table of stress and strain values.

b. Plot these values and draw a best-fit line to obtain a stress–strain curve.

c. Determine the modulus of elasticity from the slope of the linear portion of the curve.

d. Estimate the value of the proportional limit.

e. Use the 0.2% offset method to determine the yield stress.

Load (kips)	Elongation $\times 10^3$ (in.)
0	0
0.5	0.160
1.0	0.352
1.5	0.706
2.0	1.012
2.5	1.434
3.0	1.712
3.5	1.986
4.0	2.286
4.5	2.612
5.0	2.938
5.5	3.274
6.0	3.632
6.5	3.976

Load (kips)	Elongation $\times 10^3$ (in.)
7.0	4.386
7.5	4.640
8.0	4.988
8.5	5.432
9.0	5.862
9.5	6.362
10.0	7.304
10.5	8.072
11.0	9.044
11.5	11.310
12.0	14.120
12.5	20.044
13	29.106

CHAPTER 2

Concepts in Structural Steel Design

2.1 DESIGN PHILOSOPHIES

As discussed earlier, the design of a structural member entails the selection of a cross section that will safely and economically resist the applied loads. Economy usually means minimum weight—that is, the minimum amount of steel. This amount corresponds to the cross section with the smallest weight per foot, which is the one with the smallest cross-sectional area. Although other considerations, such as ease of construction, may ultimately affect the choice of member size, the process begins with the selection of the lightest cross-sectional shape that will do the job. Having established this objective, the engineer must decide how to do it safely, which is where different approaches to design come into play. The fundamental requirement of structural design is that the required strength not exceed the available strength; that is,

$$\text{Required strength} \leq \text{available strength}$$

In *allowable strength design* (ASD), a member is selected that has cross-sectional properties such as area and moment of inertia that are large enough to prevent the maximum applied axial force, shear, or bending moment from exceeding an allowable, or permissible, value. This allowable value is obtained by dividing the nominal, or theoretical, strength by a factor of safety. This can be expressed as

$$\text{Required strength} \leq \text{allowable strength} \tag{2.1}$$

where

$$\text{Allowable strength} = \frac{\text{nominal strength}}{\text{safety factor}}$$

Strength can be an axial force strength (as in tension or compression members), a flexural strength (moment strength), or a shear strength.

If stresses are used instead of forces or moments, the relationship of Equation 2.1 becomes

$$\text{Maximum applied stress} \leq \text{allowable stress} \tag{2.2}$$

This approach is called *allowable stress design.* The allowable stress will be in the elastic range of the material (see Figure 1.3). This approach to design is also called *elastic design* or *working stress design.* Working stresses are those resulting from the working loads, which are the applied loads. Working loads are also known as *service* loads.

Plastic design is based on a consideration of failure conditions rather than working load conditions. A member is selected by using the criterion that the structure will fail at a load substantially higher than the working load. Failure in this context means either collapse or extremely large deformations. The term *plastic* is used because, at failure, parts of the member will be subjected to very large strainsÑ large enough to put the member into the plastic range (see Figure 1.3b). When the entire cross section becomes plastic at enough locations, •plastic hingesŽwill form at those locations, creating a *collapse mechanism.* As the actual loads will be less than the failure loads by a factor of safety known as the *load factor,* members designed this way are not unsafe, despite being designed based on what happens at failure. This design procedure is roughly as follows.

1. Multiply the working loads (service loads) by the load factor to obtain the failure loads.
2. Determine the cross-sectional properties needed to resist failure under these loads. (A member with these properties is said to have sufficient strength and would be at the verge of failure when subjected to the factored loads.)
3. Select the lightest cross-sectional shape that has these properties.

Members designed by plastic theory would reach the point of failure under the factored loads but are safe under actual working loads.

Load and resistance factor design (LRFD) is similar to plastic design in that strength, or the failure condition, is considered. Load factors are applied to the service loads, and a member is selected that will have enough strength to resist the factored loads. In addition, the theoretical strength of the member is reduced by the application of a resistance factor. The criterion that must be satisfied in the selection of a member is

$$\text{Factored load} \leq \text{factored strength} \tag{2.3}$$

In this expression, the factored load is actually the sum of all service loads to be resisted by the member, each multiplied by its own load factor. For example, dead loads will have load factors that are different from those for live loads. The factored strength is the theoretical strength multiplied by a resistance factor. Equation 2.3 can therefore be written as

$$\sum(\text{loads} \times \text{load factors}) \leq \text{resistance} \times \text{resistance factor} \tag{2.4}$$

The factored load is a failure load greater than the total actual service load, so the load factors are usually greater than unity. However, the factored strength is a reduced, usable strength, and the resistance factor is usually less than unity. The factored loads are the loads that bring the structure or member to its limit. In terms of safety, this *limit state* can be fracture, yielding, or buckling, and the factored resistance is the useful strength of the member, reduced from the theoretical value by the resistance factor. The limit state can also be one of serviceability, such as a maximum acceptable deflection.

2.2 AMERICAN INSTITUTE OF STEEL CONSTRUCTION SPECIFICATION

Because the emphasis of this book is on the design of structural steel building members and their connections, the Specification of the American Institute of Steel Construction is the design specification of most importance here. It is written and kept current by an AISC committee comprising structural engineering practitioners, educators, steel producers, and fabricators. New editions are published periodically, and supplements are issued when interim revisions are needed. Allowable stress design has been the primary method used for structural steel buildings since the first AISC Specification was issued in 1923, although plastic design was made part of the Specification in 1963. In 1986, AISC issued the first specification for load and resistance factor design along with a companion *Manual of Steel Construction.* The purpose of these two documents was to provide an alternative to allowable stress design, much as plastic design is an alternative. The current specification (AISC, 2010a) incorporates both LRFD and ASD.

The LRFD provisions are based on research reported in eight papers published in 1978 in the *Structural Journal of the American Society of Civil Engineers* (Ravindra and Galambos; Yura, Galambos, and Ravindra; Bjorhovde, Galambos, and Ravindra; Cooper, Galambos, and Ravindra; Hansell et al.; Fisher et al.; Ravindra, Cornell, and Galambos; Galambos and Ravindra, 1978).

Although load and resistance factor design was not introduced into the AISC Specification until 1986, it is not a recent concept; since 1974, it has been used in Canada, where it is known as *limit states design.* It is also the basis of most European building codes. In the United States, LRFD has been an accepted method of design for reinforced concrete for years and is the primary method authorized in the American Concrete Institute•s Building Code, where it is known as *strength design* (ACI, 2008). Current highway bridge design standards also use load and resistance factor design (AASHTO, 2010).

The AISC Specification is published as a stand-alone document, but it is also part of the *Steel Construction Manual,* which we discuss in the next section. Except for such specialized steel products as cold-formed steel, which is covered by a different specification (AISI, 2007), the AISC Specification is the standard by which virtually all structural steel buildings in this country are designed and constructed. Hence the student of structural steel design must have ready access to his document. The details of the Specification will be covered in the chapters that follow, but we discuss the overall organization here.

The Specification consists of three parts: the main body, the appendixes, and the Commentary. The body is alphabetically organized into Chapters A through N. Within each chapter, major headings are labeled with the chapter designation followed by a number. Furthermore, subdivisions are numerically labeled. For example, the types of structural steel authorized are listed in Chapter A, •General Provisions,Ž under Section A3, •Material,Ž and, under it, Section 1, •Structural Steel Materials.Ž The main body of the Specification is followed by appendixes 1..8. The Appendix section is followed by the Commentary, which gives background and elaboration on many of the provisions of the Specification. Its organizational scheme is the same as that of the Specification, so material applicable to a particular section can be easily located.

The Specification incorporates both U.S. customary and metric (SI) units. Where possible, equations and expressions are expressed in non-dimensional form by leaving quantities such as yield stress and modulus of elasticity in symbolic form, thereby avoiding giving units. When this is not possible, U.S. customary units are given, followed by SI units in parentheses. Although there is a strong move to metrication in the steel industry, most structural design in the United States is still done in U.S. customary units, and this textbook uses only U.S. customary units.

2.3 LOAD FACTORS, RESISTANCE FACTORS, AND LOAD COMBINATIONS FOR LRFD

Equation 2.4 can be written more precisely as

$$\Sigma \gamma_i Q_i \leq \phi R_n \tag{2.5}$$

where

Q_i = a load effect (a force or a moment)
γ_i = a load factor
R_n = the nominal resistance, or strength, of the component under consideration
ϕ = resistance factor

The factored resistance ϕR_n is called the *design strength*. The summation on the left side of Equation 2.5 is over the total number of load effects (including, but not limited to, dead load and live load), where each load effect can be associated with a different load factor. Not only can each load effect have a different load factor but also the value of the load factor for a particular load effect will depend on the combination of loads under consideration. Equation 2.5 can also be written in the form

$$R_u \leq \phi R_n \tag{2.6}$$

where

R_u = required strength = sum of factored load effects (forces or moments)

Section B2 of the AISC Specification says to use the load factors and load combinations prescribed by the governing building code. If the building code does not give them, then ASCE 7 (ASCE, 2010) should be used. The load factors and load combinations in this standard are based on extensive statistical studies and are prescribed by most building codes.

ASCE 7 presents the basic load combinations in the following form:

Combination 1: $1.4D$
Combination 2: $1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$
Combination 3: $1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (L \text{ or } 0.5W)$
Combination 4: $1.2D + 1.0W + L + 0.5(L_r \text{ or } S \text{ or } R)$

Combination 5: $1.2D + 1.0E + L + 0.2S$
Combination 6: $0.9D + 1.0W$
Combination 7: $0.9D + 1.0E$

where

D = dead load
L = live load due to occupancy
L_r = roof live load
S = snow load
R = rain or ice load*
W = wind load
E = earthquake (seismic load)

In combinations 3, 4, and 5, the load factor on L can be reduced to 0.5 if L is no greater than 100 pounds per square foot, except for garages or places of public assembly. In combinations with wind or earthquake loads, you should use a direction that produces the worst effects.

The ASCE 7 basic load combinations are also given in Part 2 of the AISC *Steel Construction Manual* (AISC 2011a), which will be discussed in Section 2.6 of this chapter. They are presented in a slightly different form as follows:

Combination 1: $1.4D$
Combination 2: $1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$
Combination 3: $1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (0.5L \text{ or } 0.5W)$
Combination 4: $1.2D + 1.0W + 0.5L + 0.5(L_r \text{ or } S \text{ or } R)$
Combination 5: $1.2D \pm 1.0E + 0.5L + 0.2S$
Combinations 6 and 7: $0.9D \pm (1.0W \text{ or } 1.0E)$

Here, the load factor on L in combinations 3, 4, and 5 is given as 0.5, which should be increased to 1.0 if L is greater than 100 pounds per square foot or for garages or places of public assembly. ASCE 7 combinations 6 and 7 arise from the expression shown by considering combination 6 to use $1.0W$ and combination 7 to use $1.0E$. In other words,

Combination 6: $0.9D \pm 1.0W$
Combination 7: $0.9D \pm 1.0E$

Combinations 6 and 7 account for the possibility of dead load and wind or earthquake load counteracting each other; for example, the net load effect could be the difference between $0.9D$ and $1.0W$ or between $0.9D$ and $1.0E$. (Wind or earthquake load may tend to overturn a structure, but the dead load will have a stabilizing effect.)

As previously mentioned, the load factor for a particular load effect is not the same in all load combinations. For example, in combination 2 the load factor for the live load L is 1.6, whereas in combination 3, it is 0.5. The reason is that the live load

*This load does not include *ponding*, a phenomenon that we discuss in Chapter 5.

is being taken as the dominant effect in combination 2, and one of the three effects, L_r, S, or R, will be dominant in combination 3. In each combination, one of the effects is considered to be at its •lifetime maximumŽ value and the others at their •arbitrary point in timeŽ values.

The resistance factor ϕ for each type of resistance is given by AISC in the Specification chapter dealing with that resistance, but in most cases, one of two values will be used: 0.90 for limit sates involving yielding or compression buckling and 0.75 for limit states involving rupture (fracture).

2.4 SAFETY FACTORS AND LOAD COMBINATIONS FOR ASD

For allowable strength design, the relationship between loads and strength (Equation 2.1) can be expressed as

$$R_a \le \frac{R_n}{\Omega} \tag{2.7}$$

where

R_a = required strength
R_n = nominal strength (same as for LRFD)
Ω = safety factor
R_n/Ω = allowable strength

The required strength R_a is the sum of the service loads or load effects. As with LRFD, specific combinations of loads must be considered. Load combinations for ASD are also given in ASCE 7. These combinations, as presented in the AISC *Steel Construction Manual* (AISC 2011a), are

Combination 1:	D
Combination 2:	$D + L$
Combination 3:	$D + (L_r \text{ or } S \text{ or } R)$
Combination 4:	$D + 0.75L + 0.75(L_r \text{ or } S \text{ or } R)$
Combination 5:	$D \pm (0.6W \text{ or } 0.7E)$
Combination 6a:	$D + 0.75L + 0.75(0.6W) + 0.75(L_r \text{ or } S \text{ or } R)$
Combination 6b:	$D + 0.75L \pm 0.75(0.7E) + 0.75S$
Combinations 7 and 8:	$0.6D \pm (0.6W \text{ or } 0.7E)$

The factors shown in these combinations are not load factors. The 0.75 factor in some of the combinations accounts for the unlikelihood that all loads in the combination will be at their lifetime maximum values simultaneously. The 0.7 factor applied to the seismic load effect E is used because ASCE 7 uses a strength approach (i.e., LRFD) for computing seismic loads, and the factor is an attempt to equalize the effect for ASD.

Corresponding to the two most common values of resistance factors in LRFD are the following values of the safety factor Ω in ASD: For limit states involving yielding or compression buckling, $\Omega = 1.67$.* For limit states involving rupture, $\Omega = 2.00$. The relationship between resistance factors and safety factors is given by

$$\Omega = \frac{1.5}{\phi} \tag{2.8}$$

For reasons that will be discussed later, this relationship will produce similar designs for LRFD and ASD, under certain loading conditions.

If both sides of Equation 2.7 are divided by area (in the case of axial load) or section modulus (in the case of bending moment), then the relationship becomes

$$f \leq F$$

where

f = applied stress
F = allowable stress

This formulation is called *allowable stress design.*

EXAMPLE 2.1

A column (compression member) in the upper story of a building is subject to the following loads:

Dead load:	109 kips compression
Floor live load:	46 kips compression
Roof live load:	19 kips compression
Snow:	20 kips compression

a. Determine the controlling load combination for LRFD and the corresponding factored load.
b. If the resistance factor ϕ is 0.90, what is the required *nominal* strength?
c. Determine the controlling load combination for ASD and the corresponding required service load strength.
d. If the safety factor Ω is 1.67, what is the required nominal strength based on the required service load strength?

SOLUTION Even though a load may not be acting directly on a member, it can still cause a load effect in the member. This is true of both snow and roof live load in this example. Although this building is subjected to wind, the resulting forces on the structure are resisted by members other than this particular column.

a. The controlling load combination is the one that produces the largest factored load. We evaluate each expression that involves dead load, D; live load resulting from occupancy, L; roof live load, L_r; and snow, S.

*The value of Ω is actually $1\frac{2}{3} = 5/3$ but has been rounded to 1.67 in the AISC specification.

Combination 1: $1.4D = 1.4(109) = 152.6$ kips

Combination 2: $1.2D + 1.6L + 0.5(L_r$ or S or $R)$. Because S is larger than L_r and $R = 0$, we need to evaluate this combination only once, using S.

$$1.2D + 1.6L + 0.5S = 1.2(109) + 1.6(46) + 0.5(20) = 214.4 \text{ kips}$$

Combination 3: $1.2D + 1.6(L_r$ or S or $R) + (0.5L$ or $0.5W)$. In this combination, we use S instead of L_r, and both R and W are zero.

$$1.2D + 1.6S + 0.5L = 1.2(109) + 1.6(20) + 0.5(46) = 185.8 \text{ kips}$$

Combination 4: $1.2D + 1.0W + 0.5L + 0.5(L_r$ or S or $R)$. This expression reduces to $1.2D + 0.5L + 0.5S$, and by inspection, we can see that it produces a smaller result than combination 3.

Combination 5: $1.2D \pm 1.0E + 0.5L + 0.2S$. As $E = 0$, this expression reduces to $1.2D + 0.5L + 0.2S$, which produces a smaller result than combination 4.

Combinations 6 and 7: $0.9D \pm (1.0W$ or $1.0E)$. These combinations do not apply in this example, because there are no wind or earthquake loads to counteract the dead load.

ANSWER Combination 2 controls, and the factored load is 214.4 kips.

b. If the factored load obtained in part (a) is substituted into the fundamental LRFD relationship, Equation 2.6, we obtain

$$R_u \le \phi R_n$$
$$214.4 \le 0.90R_n$$
$$R_n \ge 238 \text{ kips}$$

ANSWER The required nominal strength is 238 kips.

c. As with the combinations for LRFD, we will evaluate the expressions involving D, L, L_r, and S for ASD.

Combination 1: $D = 109$ kips. (Obviously this case will never control when live load is present.)

Combination 2: $D + L = 109 + 46 = 155$ kips

Combination 3: $D + (L_r$ or S or $R)$. Since S is larger than L_r, and $R = 0$, this combination reduces to $D + S = 109 + 20 = 129$ kips

Combination 4: $D + 0.75L + 0.75(L_r$ or S or $R)$. This expression reduces to $D + 0.75L + 0.75S = 109 + 0.75(46) + 0.75(20) = 158.5$ kips

Combination 5: $D \pm (0.6W$ or $0.7E)$. Because W and E are zero, this expression reduces to combination 1.

Combination 6a:	$D + 0.75L + 0.75(0.6W) + 0.75(L_r$ or S or $R)$. Because W and E are zero, this expression reduces to combination 4.
Combination 6b:	$D + 0.75L \pm 0.75(0.7E) + 0.75S$. This combination also gives the same result as combination 4.
Combinations 7 and 8:	$0.6D \pm (0.6W$ or $0.7E)$. These combinations do not apply in this example, because there are no wind or earthquake loads to counteract the dead load.

ANSWER Combination 4 controls, and the required service load strength is 158.5 kips.

d. From the ASD relationship, Equation 2.7,

$$R_a \leq \frac{R_n}{\Omega}$$

$$158.5 \leq \frac{R_n}{1.67}$$

$$R_n \geq 265 \text{ kips}$$

ANSWER The required nominal strength is 265 kips.

Example 2.1 illustrates that the controlling load combination for LRFD may not control for ASD.

When LRFD was introduced into the AISC Specification in 1986, the load factors were determined in such a way as to give the same results for LRFD and ASD when the loads consisted of dead load and a live load equal to three times the dead load. The resulting relationship between the resistance factor ϕ and the safety factor Ω, as expressed in Equation 2.8, can be derived as follows. Let R_n from Equations 2.6 and 2.7 be the same when $L = 3D$. That is,

$$\frac{R_u}{\phi} = R_a\Omega$$

$$\frac{1.2D + 1.6L}{\phi} = (D + L)\Omega$$

or

$$\frac{1.2D + 1.6(3D)}{\phi} = (D + 3D)\Omega$$

$$\Omega = \frac{1.5}{\phi}$$

CHAPTER 3

Tension Members

3.1 INTRODUCTION

Tension members are structural elements that are subjected to axial tensile forces. They are used in various types of structures and include truss members, bracing for buildings and bridges, cables in suspended roof systems, and cables in suspension and cable-stayed bridges. Any cross-sectional configuration may be used, because for any given material, the only determinant of the strength of a tension member is the cross-sectional area. Circular rods and rolled angle shapes are frequently used. Built-up shapes, either from plates, rolled shapes, or a combination of plates and rolled shapes, are sometimes used when large loads must be resisted. The most common built-up configuration is probably the double-angle section, shown in Figure 3.1, along with other typical cross sections. Because the use of this section is so widespread, tables of properties of various combinations of angles are included in the AISC *Steel Construction Manual*.

The stress in an axially loaded tension member is given by

$$f = \frac{P}{A}$$

where P is the magnitude of the load and A is the cross-sectional area (the area normal to the load). The stress as given by this equation is exact, provided that the cross section under consideration is not adjacent to the point of application of the load, where the distribution of stress is not uniform.

If the cross-sectional area of a tension member varies along its length, the stress is a function of the particular section under consideration. The presence of holes in a member will influence the stress at a cross section through the hole or holes. At these locations, the cross-sectional area will be reduced by an amount equal to the area removed by the holes. Tension members are frequently connected at their ends with bolts, as illustrated in Figure 3.2. The tension member shown, a $\frac{1}{2} \times 8$ plate, is connected to a *gusset plate*, which is a connection element whose purpose is to transfer the load from the member to a support or to another member. The area of the bar at section *a–a* is $(\frac{1}{2})(8) = 4$ in.^{2}, but the area at section *b–b* is only $4 - (2)(\frac{1}{2})(\frac{7}{8}) = 3.13$ in.^{2}

FIGURE 3.1

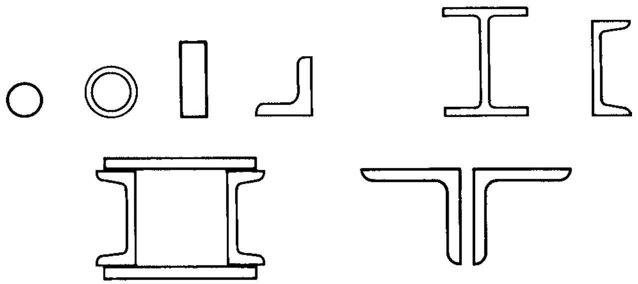
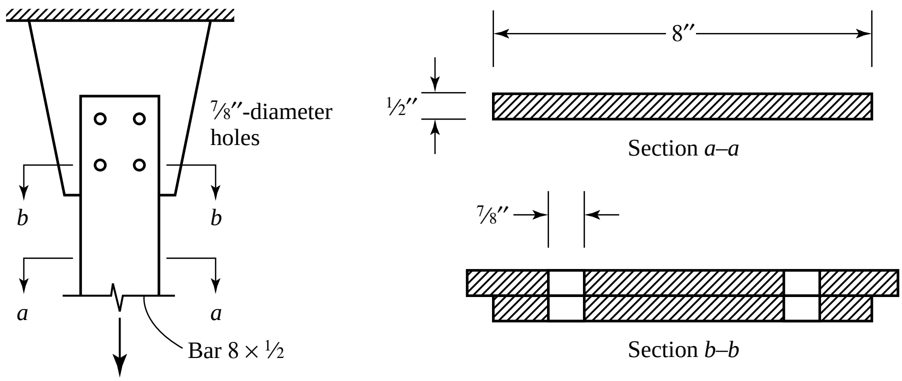


FIGURE 3.2



and will be more highly stressed. This reduced area is referred to as the *net area,* or *net section,* and the unreduced area is the *gross area.*

The typical design problem is to select a member with sufficient cross-sectional area to resist the loads. A closely related problem is that of analysis, or review, of a given member, where in the strength is computed and compared with the load. In general, analysis is a direct procedure, but design is an iterative process and may require some trial and error.

Tension members are covered in Chapter D of the Specification. Requirements that are common with other types of members are covered in Chapter B, "Design Requirements."

3.2 TENSILE STRENGTH

A tension member can fail by reaching one of two limit states: excessive deformation or fracture. To prevent excessive deformation, initiated by yielding, the load on the gross section must be small enough that the stress on the gross section is less than the yield stress F_y. To prevent fracture, the stress on the net section must be less than the tensile strength F_u. In each case, the stress P/A must be less than a limiting stress F or

$$\frac{P}{A} < F$$

Thus, the load P must be less than FA, or

$$P < FA$$

The *nominal* strength in yielding is

$$P_n = F_y A_g$$

and the nominal strength in fracture is

$$P_n = F_u A_e$$

where A_e is the *effective* net area, which may be equal to either the net area or, in some cases, a smaller area. We discuss effective net area in Section 3.3.

Although yielding will first occur on the net cross section, the deformation within the length of the connection will generally be smaller than the deformation in the remainder of the tension member. The reason is that the net section exists over a relatively small length of the member, and the total elongation is a product of the length and the strain (a function of the stress). Most of the member will have an unreduced cross section, so attainment of the yield stress on the gross area will result in larger total elongation. It is this larger deformation, not the first yield, that is the limit state.

LRFD: In load and resistance factor design, the factored tensile load is compared to the design strength. The design strength is the resistance factor times the nominal strength. Equation 2.6,

$$R_u = \phi R_n$$

can be written for tension members as

$$P_u \leq \phi_t P_n$$

where P_u is the governing combination of factored loads. The resistance factor ϕ_t is smaller for fracture than for yielding, reflecting the more serious nature of fracture.

For yielding, $\phi_t = 0.90$

For fracture, $\phi_t = 0.75$

Because there are two limit states, both of the following conditions must be satisfied:

$$P_u \leq 0.90 F_y A_g$$

$$P_u \leq 0.75 F_u A_e$$

The smaller of these is the design strength of the member.

ASD: In allowable strength design, the total service load is compared to the allowable strength (allowable load):

$$P_a \leq \frac{P_n}{\Omega_t}$$

where P_a is the required strength (applied load), and P_n/Ω_t is the allowable strength. The subscript “a” indicates that the required strength is for “allowable strength design,” but you can think of it as standing for “applied” load.

For yielding of the gross section, the safety factor Ω_t is 1.67, and the allowable load is

$$\frac{P_n}{\Omega_t} = \frac{F_y A_g}{1.67} = 0.6 F_y A_g$$

(The factor 0.6 appears to be a rounded value, but recall that 1.67 is a rounded value. If $\Omega_t = 5/3$ is used, the allowable load is exactly 0.6 $F_y A_g$.)

For fracture of the net section, the safety factor is 2.00 and the allowable load is

$$\frac{P_n}{\Omega_t} = \frac{F_u A_e}{2.00} = 0.5 F_u A_e$$

Alternatively, the service load stress can be compared to the allowable stress. This can be expressed as

$$f_t \leq F_t$$

where f_t is the applied stress and F_t is the allowable stress. For yielding of the gross section,

$$f_t = \frac{P_a}{A_g} \quad \text{and} \quad F_t = \frac{P_n/\Omega_t}{A_g} = \frac{0.6 F_y A_g}{A_g} = 0.6 F_y$$

For fracture of the net section,

$$f_t = \frac{P_a}{A_e} \quad \text{and} \quad F_t = \frac{P_n/\Omega_t}{A_e} = \frac{0.5 F_u A_e}{A_e} = 0.5 F_u$$

You can find values of F_y and F_u for various structural steels in Table 2-3 in the *Manual.* All of the steels that are available for various hot-rolled shapes are indicated by shaded areas. The black areas correspond to preferred materials, and the gray areas represent other steels that are available. Under the W heading, we see that A992 is the preferred material for W shapes, but other materials are available, usually at a higher cost. For some steels, there is more than one grade, with each grade having different values of F_y and F_u. In these cases, the grade must be specified along with the ASTM designation—for example, A572 Grade 50. For A242 steel, F_y and F_u depend on the thickness of the flange of the cross-sectional shape. This relationship is given in footnotes in the table. For example, to determine the properties of a W33 × 221 of ASTM A242 steel, first refer to the dimensions and properties table in Part 1 of the *Manual* and determine that the flange thickness t_f is equal to 1.28 inches. This matches the thickness range indicated in footnote*1*; therefore, $F_y = 50$ ksi and $F_u = 70$ ksi. Values of F_y and F_u for plates and bars are given in the *Manual* Table 2-4, and information on structural fasteners, including bolts and rods, can be found in Table 2-5.

The exact amount of area to be deducted from the gross area to account for the presence of bolt holes depends on the fabrication procedure. The usual practice is to drill or punch standard holes (i.e., not oversized) with a diameter 1⁄16 inch larger than the fastener diameter. To account for possible roughness around the edges of the hole, Section B4.3 of the AISC Specification (in the remainder of this book, references to the Specification will usually be in the form AISC B4.3) requires the addition of 1⁄16 inch to the actual hole diameter. This amounts to using an effective hole diameter 1⁄8 inch larger than the fastener diameter. In the case of slotted holes, 1⁄16 inch should be added to the actual *width* of the hole. You can find details related to standard, oversized, and slotted holes in AISC J3.2, “Size and Use of Holes” (in Chapter J, “Design of Connections”).

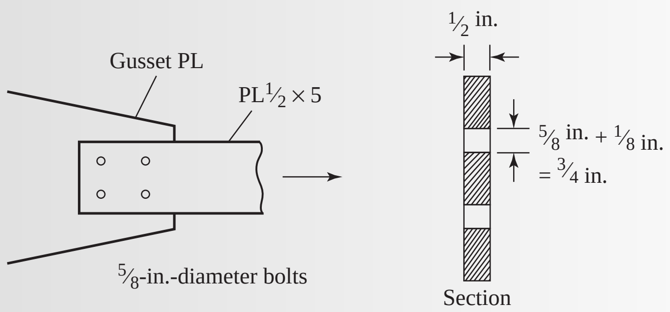
EXAMPLE 3.1

A ½ × 5 plate of A36 steel is used as a tension member. It is connected to a gusset plate with four ⅝-inch-diameter bolts as shown in Figure 3.3. Assume that the effective net area A_e equals the actual net area A_n (we cover computation of effective net area in Section 3.3).

a. What is the design strength for LRFD?

b. What is the allowable strength for ASD?

FIGURE 3.3



SOLUTION

For yielding of the gross section,

$$A_g = 5(1/2) = 2.5 \text{ in.}^2$$

and the nominal strength is

$$P_n = F_y A_g = 36(2.5) = 90.0 \text{ kips}$$

For fracture of the net section,

$$A_n = A_g - A_{holes} = 2.5 - (\tfrac{1}{2})(\tfrac{3}{4}) \times 2 \text{ holes}$$
$$= 2.5 - 0.75 = 1.75 \text{ in.}^2$$

$A_e = A_n = 1.75 \text{ in.}^2$ (This is true for this example, but A_e does not always equal A_n.)

The nominal strength is

$$P_n = F_u A_e = 58(1.75) = 101.5 \text{ kips}$$

a. The design strength based on yielding is

$$\phi_t P_n = 0.90(90) = 81.0 \text{ kips}$$

The design strength based on fracture is

$$\phi_t P_n = 0.75(101.5) = 76.1 \text{ kips}$$

ANSWER The design strength for LRFD is the smaller value: $\phi_t P_n = 76.1$ kips.

b. The allowable strength based on yielding is

$$\frac{P_n}{\Omega_t} = \frac{90}{1.67} = 53.9 \text{ kips}$$

The allowable strength based on fracture is

$$\frac{P_n}{\Omega_t} = \frac{101.5}{2.00} = 50.8 \text{ kips}$$

ANSWER The allowable service load is the smaller value = 50.8 kips.

Alternative Solution Using Allowable Stress: For yielding,

$$F_t = 0.6F_y = 0.6(36) = 21.6 \text{ ksi}$$

and the allowable load is

$$F_t A_g = 21.6(2.5) = 54.0 \text{ kips}$$

(The slight difference between this value and the one based on allowable strength is because the value of Ω in the allowable strength approach has been rounded from 5/3 to 1.67; the value based on the allowable stress is the more accurate one.) For fracture,

$$F_t = 0.5F_u = 0.5(58) = 29.0 \text{ ksi}$$

and the allowable load is

$$F_t A_e = 29.0(1.75) = 50.8 \text{ kips}$$

ANSWER The allowable service load is the smaller value = 50.8 kips.

Because of the relationship given by Equation 2.8, the allowable strength will always be equal to the design strength divided by 1.5. In this book, however, we will do the complete computation of allowable strength even when the design strength is available.

The effects of stress concentrations at holes appear to have been overlooked. In reality, stresses at holes can be as high as three times the average stress on the net section, and at fillets of rolled shapes they can be more than twice the average (McGuire, 1968). Because of the ductile nature of structural steel, the usual design practice is to neglect such localized overstress. After yielding begins at a point of stress concentration, additional stress is transferred to adjacent areas of the cross section. This stress redistribution is responsible for the “forgiving” nature of structural steel. Its ductility permits the initially yielded zone to deform without fracture as the stress on the remainder of the cross section continues to increase. Under certain conditions, however, steel may lose its ductility and stress concentrations can precipitate brittle fracture. These situations include fatigue loading and extremely low temperature.

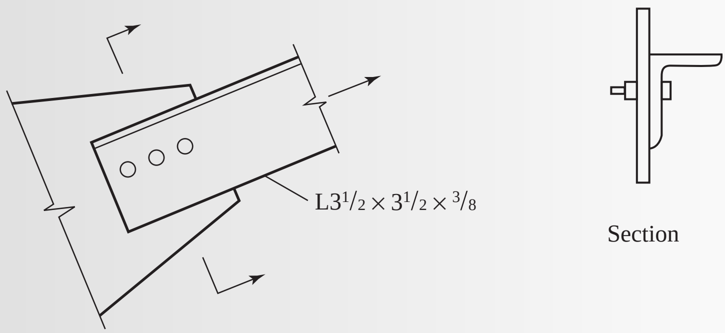
EXAMPLE 3.2

A single-angle tension member, an L3½ × 3½ × ⅜, is connected to a gusset plate with ⅞-inch-diameter bolts as shown in Figure 3.4. A36 steel is used. The service loads are 35 kips dead load and 15 kips live load. Investigate this member for compliance with the AISC Specification. Assume that the effective net area is 85% of the computed net area.

a. Use LRFD.

b. Use ASD.

FIGURE 3.4



SOLUTION First, compute the nominal strengths.

Gross section:

$$A_g = 2.50 \text{ in.}^2 \quad \text{(from Part 1 of the } \textit{Manual}\text{)}$$

$$P_n = F_y A_g = 36(2.50) = 90 \text{ kips}$$

Net section:

$$A_n = 2.50 - \left(\frac{3}{8}\right)\left(\frac{7}{8} + \frac{1}{8}\right) = 2.125 \text{ in.}^2$$

$$A_e = 0.85A_n = 0.85(2.125) = 1.806 \text{ in.}^2 \quad \text{(in } \textit{this} \text{ example)}$$

$$P_n = F_u A_e = 58(1.806) = 104.7 \text{ kips}$$

a. The design strength based on yielding is

$$\phi_t P_n = 0.90(90) = 81 \text{ kips}$$

The design strength based on fracture is

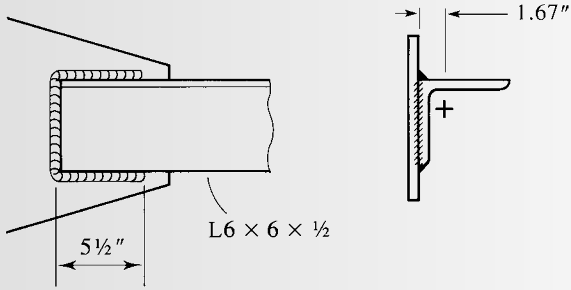
$$\phi_t P_n = 0.75(104.7) = 78.5 \text{ kips}$$

The design strength is the smaller value: $\phi_t P_n = 78.5$ kips

Factored load:

When only dead load and live load are present, the only load combinations with a chance of controlling are combinations 1 and 2.

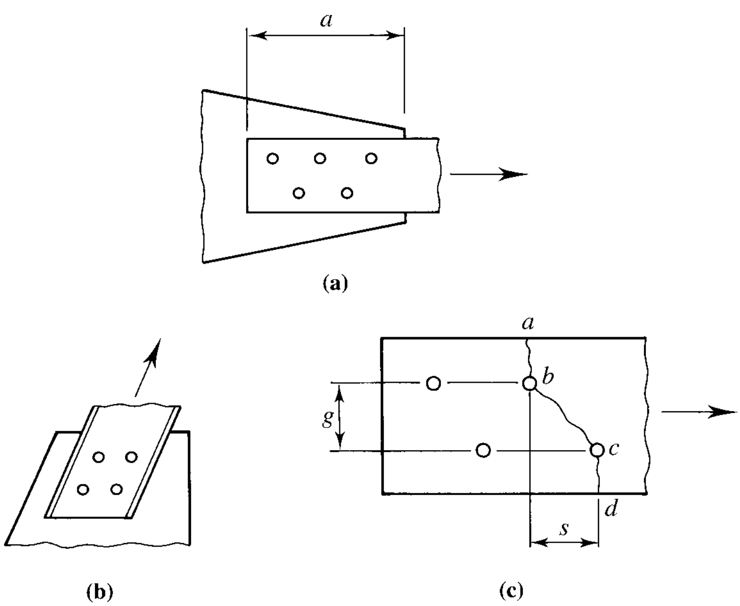
FIGURE 3.13



3.4 STAGGERED FASTENERS

If a tension member connection is made with bolts, the net area will be maximized if the fasteners are placed in a single line. Sometimes space limitations, such as a limit on dimension *a* in Figure 3.14a, necessitate using more than one line. If so, the reduction in cross-sectional area is minimized if the fasteners are arranged in a staggered pattern, as shown. Sometimes staggered fasteners are required by the geometry of a connection, such as the one shown in Figure 3.14b. In either case, any cross section passing through holes will pass through fewer holes than if the fasteners are not staggered.

FIGURE 3.14



If the amount of stagger is small enough, the influence of an offset hole may be felt by a nearby cross section, and fracture along an inclined path such as *abcd* in Figure 3.14c is possible. In such a case, the relationship $f = P/A$ does not apply, and stresses on the inclined portion *b–c* are a combination of tensile and shearing stresses. Several approximate methods have been proposed to account for the effects of staggered holes. Cochrane (1922) proposed that when deducting the area corresponding to a staggered hole, use a reduced diameter, given by

$$d' = d - \frac{s^2}{4g} \tag{3.2}$$

where d is the hole diameter, s is the stagger, or pitch, of the bolts (spacing in the direction of the load), and g is the gage (transverse spacing). This means that in a failure pattern consisting of both staggered and unstaggered holes, use d for holes at the end of a transverse line between holes ($s = 0$) and use d' for holes at the end of an inclined line between holes.

The AISC Specification, in Section B4.3b, uses this approach, but in a modified form. If the net area is treated as the product of a thickness times a net width, and the diameter from Equation 3.2 is used for all holes (since $d' = d$ when the stagger $s = 0$), the net width in a failure line consisting of both staggered and unstaggered holes is

$$\begin{aligned} w_n &= w_g - \Sigma d' \\ &= w_g - \Sigma\left(d - \frac{s^2}{4g}\right) \\ &= w_g - \Sigma d + \Sigma \frac{s^2}{4g} \end{aligned}$$

where w_n is the net width and w_g is the gross width. The second term is the sum of all hole diameters, and the third term is the sum of $s^2/4g$ for all inclined lines in the failure pattern.

When more than one failure pattern is conceivable, all possibilities should be investigated, and the one corresponding to the smallest load capacity should be used. Note that this method will not accommodate failure patterns with lines parallel to the applied load.

EXAMPLE 3.6

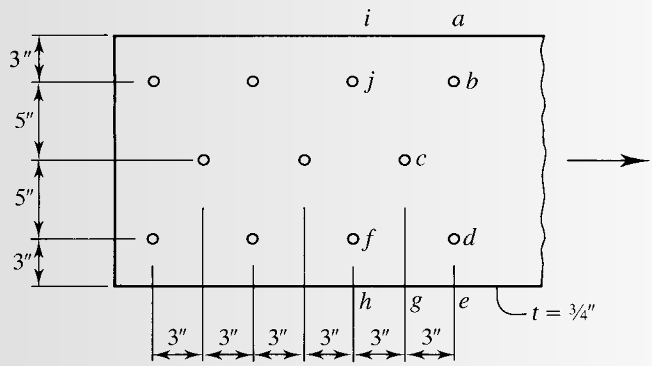
Compute the smallest net area for the plate shown in Figure 3.15. The holes are for 1-inch-diameter bolts.

SOLUTION

The effective hole diameter is 1 + ⅛ = 1⅛ in. For line *abde*,

$$w_n = 16 - 2(1.125) = 13.75 \text{ in.}$$

FIGURE 3.15



For line *abcde*,

$$w_n = 16 - 3(1.125) + \frac{2(3)^2}{4(5)} = 13.52 \text{ in.}$$

The second condition will give the smallest net area:

ANSWER $A_n = tw_n = 0.75(13.52) = 10.1 \text{ in.}^2$

Equation 3.2 can be used directly when staggered holes are present. In the computation of the net area for line *abcde* in Example 3.6,

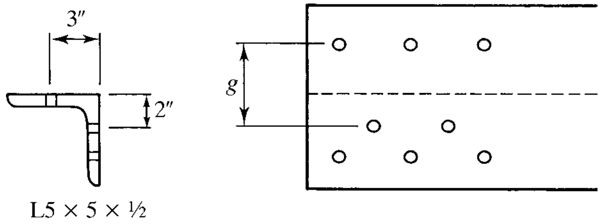
$$A_n = A_g - \sum t \times (d \text{ or } d')$$

$$= 0.75(16) - 0.75(1.125) - 0.75\left[1.125 - \frac{(3)^2}{4(5)}\right] \times 2 = 10.1 \text{ in.}^2$$

As each fastener resists an equal share of the load (an assumption used in the design of simple connections; see Chapter 7), different potential failure lines may be subjected to different loads. For example, line *abcde* in Figure 3.15 must resist the full load, whereas *ijfh* will be subjected to $8/11$ of the applied load. The reason is that $3/11$ of the load will have been transferred from the member before *ijfh* receives any load.

When lines of bolts are present in more than one element of the cross section of a rolled shape, and the bolts in these lines are staggered with respect to one another, the use of areas and Equation 3.2 is preferable to the net-width approach of the AISC Specification. If the shape is an angle, it can be visualized as a plate formed by "unfolding" the legs to more clearly identify the pitch and gage distances. AISC B4.3b specifies that any gage line crossing the heel of the angle be reduced by an amount that equals the angle thickness. Thus, the distance g in Figure 3.16, to be used in the $s^2/4g$ term, would be $3 + 2 - ½ = 4½$ inches.

FIGURE 3.16



EXAMPLE 3.7

An angle with staggered fasteners in each leg is shown in Figure 3.17. A36 steel is used, and holes are for ⅞-inch-diameter bolts.

a. Determine the design strength for LRFD.
b. Determine the allowable strength for ASD.

SOLUTION From the dimensions and properties tables, the gross area is $A_g = 6.80$ in.^{2}. The effective hole diameter is ⅞ + ⅛ = 1 in.

For line *abdf*, the net area is

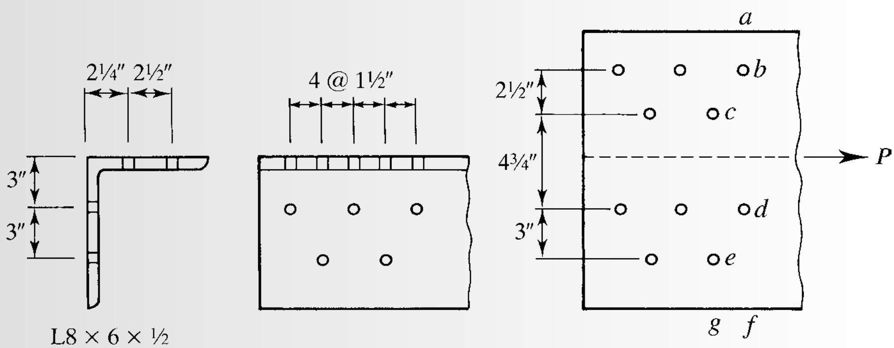
$$\begin{aligned} A_n &= A_g - \sum t_w \times (d \text{ or } d') \\ &= 6.80 - 0.5(1.0) \times 2 = 5.80 \text{ in.}^2 \end{aligned}$$

For line *abceg*,

$$A_n = 6.80 - 0.5(1.0) - 0.5\left[1.0 - \frac{(1.5)^2}{4(2.5)}\right] - 0.5(1.0) = 5.413 \text{ in.}^2$$

Because $\frac{1}{10}$ of the load has been transferred from the member by the fastener at *d*, this potential failure line must resist only $\frac{9}{10}$ of the load. Therefore, the net area

FIGURE 3.17



of 5.413 in.^{2} should be multiplied by $^{10}/_9$ to obtain a net area that can be compared with those lines that resist the full load. Use $A_n = 5.413(^{10}/_9) = 6.014$ in.^{2} For line *abcdeg*,

$$g_{cd} = 3 + 2.25 - 0.5 = 4.75 \text{ in.}$$

$$A_n = 6.80 - 0.5(1.0) - 0.5\left[1.0 - \frac{(1.5)^2}{4(2.5)}\right] - 0.5\left[1.0 - \frac{(1.5)^2}{4(4.75)}\right] - 0.5\left[1.0 - \frac{(1.5)^2}{4(3)}\right]$$

$$= 5.065 \text{ in.}^2$$

The last case controls; use

$$A_n = 5.065 \text{ in.}^2$$

Both legs of the angle are connected, so

$$A_e = A_n = 5.065 \text{ in.}^2$$

The nominal strength based on fracture is

$$P_n = F_u A_e = 58(5.065) = 293.8 \text{ kips}$$

The nominal strength based on yielding is

$$P_n = F_y A_g = 36(6.80) = 244.8 \text{ kips}$$

a. The design strength based on fracture is

$$\phi_t P_n = 0.75(293.8) = 220 \text{ kips}$$

The design strength based on yielding is

$$\phi_t P_n = 0.90(244.8) = 220 \text{ kips}$$

ANSWER Design strength = 220 kips.

b. For the limit state of fracture, the allowable stress is

$$F_t = 0.5F_u = 0.5(58) = 29.0 \text{ ksi}$$

and the allowable strength is

$$F_t A_e = 29.0(5.065) = 147 \text{ kips}$$

For yielding,

$$F_t = 0.6F_y = 0.6(36) = 21.6 \text{ ksi}$$

$$F_t A_g = 21.6(6.80) = 147 \text{ kips}$$

ANSWER Allowable strength = 147 kips.

EXAMPLE 3.8

Determine the smallest net area for the American Standard Channel shown in Figure 3.18. The holes are for ⅝-inch-diameter bolts.

SOLUTION

$$A_n = A_g - \sum t_w \times (d \text{ or } d')$$

$$d = \text{bolt diameter} + \frac{1}{8} = \frac{5}{8} + \frac{1}{8} = \frac{3}{4} \text{ in.}$$

Line *abe:*

$$A_n = A_g - t_w d = 3.82 - 0.437\left(\frac{3}{4}\right) = 3.49 \text{ in.}^2$$

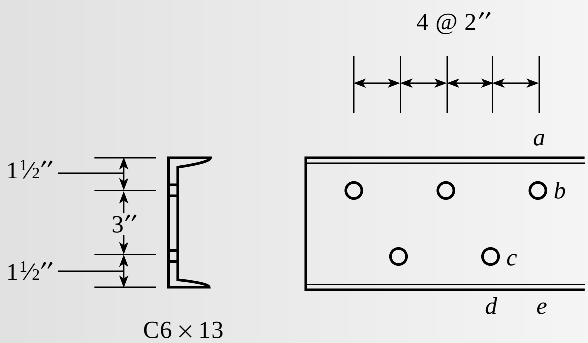
Line *abcd:*

$$A_n = A_g - t_w(d \text{ for hole at } b) - t_w(d' \text{ for hole at } c)$$

$$= 3.82 - 0.437\left(\frac{3}{4}\right) - 0.437\left[\frac{3}{4} - \frac{(2)^2}{4(3)}\right] = 3.31 \text{ in.}^2$$

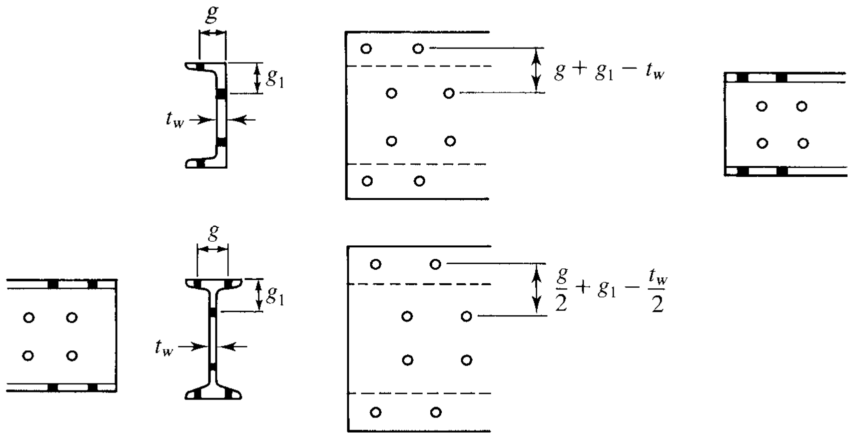
ANSWER Smallest net area = 3.31 in.^{2}

FIGURE 3.18



When staggered holes are present in shapes other than angles, and the holes are in different elements of the cross section, the shape can still be visualized as a plate, even if it is an I-shape. The AISC Specification furnishes no guidance for gage lines crossing a "fold" when the different elements have different thicknesses. A method for handling this case is illustrated in Figure 3.19. In Example 3.8, all of the holes are in one element of the cross section, so this difficulty does not arise. Example 3.9 illustrates the case of staggered holes in different elements of an S-shape.

FIGURE 3.19



EXAMPLE 3.9

Find the available strength of the S-shape shown in Figure 3.20. The holes are for ¾-inch-diameter bolts. Use A36 steel.

SOLUTION Compute the net area:

$$A_n = A_g - \sum t \times (d \text{ or } d')$$

$$\text{Effective hole diameter} = \frac{3}{4} + \frac{1}{8} = \frac{7}{8}$$

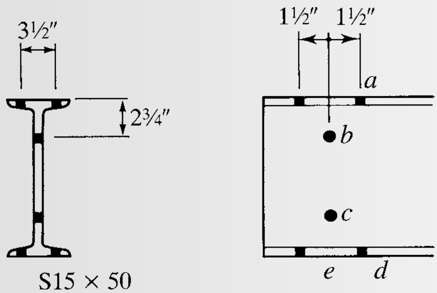
For line *ad*,

$$A_n = 14.7 - 4\left(\frac{7}{8}\right)(0.622) = 12.52 \text{ in.}^2$$

For line *abcd*, the gage distance for use in the $s^2/4g$ term is

$$\frac{g}{2} + g_1 - \frac{t_w}{2} = \frac{3.5}{2} + 2.75 - \frac{0.550}{2} = 4.225 \text{ in.}$$

FIGURE 3.20



Starting at a and treating the holes at b and d as the staggered holes gives

$$A_n = A_g - \sum t \times (d \text{ or } d')$$

$$= 14.7 - 2(0.622)\left(\frac{7}{8}\right) - (0.550)\left[\frac{7}{8} - \frac{(1.5)^2}{4(4.225)}\right]$$

$$-(0.550)\left(\frac{7}{8}\right) - 2(0.622)\left[\frac{7}{8} - \frac{(1.5)^2}{4(4.225)}\right] = 11.73 \text{ in.}^2$$

Line *abcd* controls. As all elements of the cross section are connected,

$$A_e = A_n = 11.73 \text{ in.}^2$$

For the net section, the nominal strength is

$$P_n = F_u A_e = 58(11.73) = 680.3 \text{ kips}$$

For the gross section,

$$P_n = F_y A_g = 36(14.7) = 529.2 \text{ kips}$$

LRFD SOLUTION The design strength based on fracture is

$$\phi_t P_n = 0.75(680.3) = 510 \text{ kips}$$

The design strength based on yielding is

$$\phi_t P_n = 0.90(529.2) = 476 \text{ kips}$$

Yielding of the gross section controls.

ANSWER Design strength = 476 kips.

ASD SOLUTION The allowable stress based on fracture is

$$F_t = 0.5F_u = 0.5(58) = 29.0 \text{ ksi}$$

and the corresponding allowable strength is $F_t A_e = 29.0(11.73) = 340$ kips.

The allowable stress based on yielding is

$$F_t = 0.6F_y = 0.6(36) = 21.6 \text{ ksi}$$

and the corresponding allowable strength is $F_t A_g = 21.6(14.7) = 318$ kips.

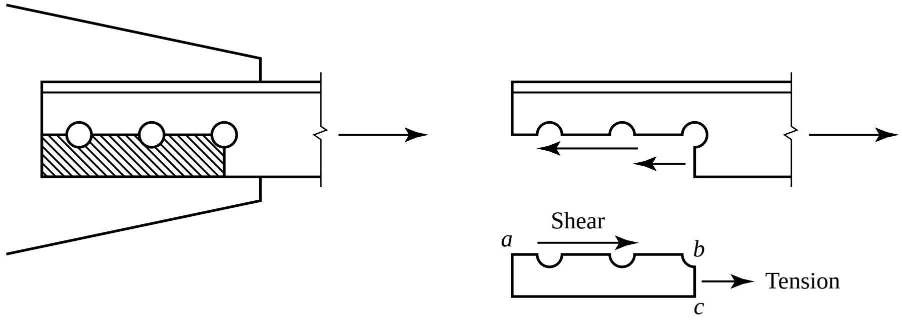
Yielding of the gross section controls.

ANSWER Allowable strength = 318 kips.

3.5 BLOCK SHEAR

For certain connection configurations, a segment or "block" of material at the end of the member can tear out. For example, the connection of the single-angle tension member shown in Figure 3.21 is susceptible to this phenomenon, called *block shear*.

FIGURE 3.21

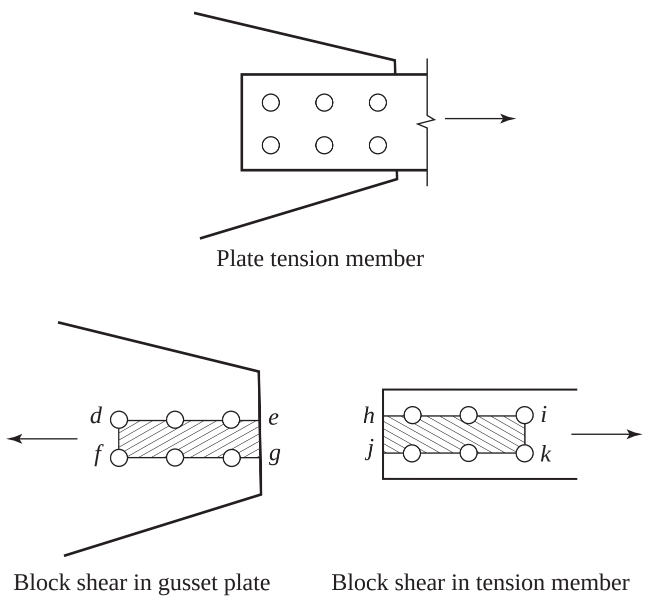


For the case illustrated, the shaded block would tend to fail by shear along the longitudinal section *ab* and by tension on the transverse section *bc*.

For certain arrangements of bolts, block shear can also occur in gusset plates. Figure 3.22 shows a plate tension member connected to a gusset plate. In this connection, block shear could occur in both the gusset plate and the tension member. For the gusset plate, tension failure would be along the transverse section *df*, and shear failure would occur on two longitudinal surfaces, *de* and *fg*. Block shear failure in the plate tension member would be tension on *ik* and shear on both *hi* and *jk*. This topic is not covered explicitly in AISC Chapter D ("Design of Members for Tension"), but the introductory user note directs you to Chapter J ("Design of Connections"), Section J4.3, "Block Shear Strength."

The model used in the AISC Specification assumes that failure occurs by rupture (fracture) on the shear area and rupture on the tension area. Both surfaces contribute to the total strength, and the resistance to block shear will be the sum of the strengths of the two surfaces. The shear rupture stress is taken as 60% of the tensile ultimate

FIGURE 3.22



stress, so the nominal strength in shear is $0.6F_uA_{nv}$ and the nominal strength in tension is F_uA_{nt},

where

A_{nv} = net area along the shear surface or surfaces
A_{nt} = net area along the tension surface

This gives a nominal strength of

$$R_n = 0.6F_uA_{nv} + F_uA_{nt} \tag{3.3}$$

The AISC Specification uses Equation 3.3 for angles and gusset plates, but for certain types of coped beam connections (to be covered in Chapter 5), the second term is reduced to account for nonuniform tensile stress. The tensile stress is nonuniform when some rotation of the block is required for failure to occur. For these cases,

$$R_n = 0.6F_uA_{nv} + 0.5F_uA_{nt} \tag{3.4}$$

The AISC Specification limits the $0.6F_uA_{nv}$ term to $0.6F_yA_{gv}$, where

$0.6F_y$ = shear yield stress
A_{gv} = gross area along the shear surface or surfaces

and gives one equation to cover all cases as follows:

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt} \le 0.6F_yA_{gv} + U_{bs}F_uA_{nt} \qquad \text{(AISC Equation J4-5)}$$

where U_{bs} = 1.0 when the tension stress is uniform (angles, gusset plates, and most coped beams) and U_{bs} = 0.5 when the tension stress is nonuniform. A nonuniform case is illustrated in the Commentary to the Specification.

For LRFD, the resistance factor ϕ is 0.75, and for ASD, the safety factor Ω is 2.00. Recall that these are the factors used for the fracture—or rupture—limit state, and block shear is a rupture limit state.

Although AISC Equation J4-5 is expressed in terms of bolted connections, block shear can also occur in welded connections, especially in gusset plates.

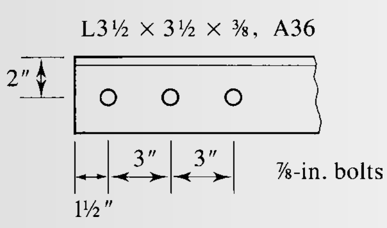
EXAMPLE 3.10

Compute the block shear strength of the tension member shown in Figure 3.23. The holes are for ⅞-inch-diameter bolts, and A36 steel is used.

a. Use LRFD.

b. Use ASD.

FIGURE 3.23



SOLUTION The shear areas are

$$A_{gv} = \frac{3}{8}(7.5) = 2.813 \text{ in.}^2$$

and, since there are 2.5 hole diameters,

$$A_{nv} = \frac{3}{8}\left[7.5 - 2.5\left(\frac{7}{8} + \frac{1}{8}\right)\right] = 1.875 \text{ in.}^2$$

The tension area is

$$A_{nt} = \frac{3}{8}\left[1.5 - 0.5\left(\frac{7}{8} + \frac{1}{8}\right)\right] = 0.3750 \text{ in.}^2$$

(The factor of 0.5 is used because there is one-half of a hole diameter in the tension section.)

Since the block shear will occur in an angle, $U_{bs} = 1.0$, and from AISC Equation J4-5,

$$\begin{aligned} R_n &= 0.6F_uA_{nv} + U_{bs}F_uA_{nt} \\ &= 0.6(58)(1.875) + 1.0(58)(0.3750) = 87.00 \text{ kips} \end{aligned}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(2.813) + 1.0(58)(0.3750) = 82.51 \text{ kips}$$

The nominal block shear strength is therefore 82.51 kips.

ANSWER a. The design strength for LRFD is $\phi R_n = 0.75(82.51) = 61.9$ kips.

b. The allowable strength for ASD is $\dfrac{R_n}{\Omega} = \dfrac{82.51}{2.00} = 41.3$ kips.

3.6 DESIGN OF TENSION MEMBERS

The design of a tension member involves finding a member with adequate gross and net areas. If the member has a bolted connection, the selection of a suitable cross section requires an accounting for the area lost because of holes. For a member with a rectangular cross section, the calculations are relatively straightforward. If a rolled shape is to be used, however, the area to be deducted cannot be predicted in advance because the member•s thickness at the location of the holes is not known.

A secondary consideration in the design of tension members is slenderness. If a structural member has a small cross section in relation to its length, it is said to be *slender*. A more precise measure is the slenderness ratio, L/r, where L is the member length and r is the minimum radius of gyration of the cross-sectional area. The minimum radius

SOLUTION The shear areas are

$$A_{gv} = \frac{3}{8}(7.5) = 2.813 \text{ in.}^2$$

and, since there are 2.5 hole diameters,

$$A_{nv} = \frac{3}{8}\left[7.5 - 2.5\left(\frac{7}{8} + \frac{1}{8}\right)\right] = 1.875 \text{ in.}^2$$

The tension area is

$$A_{nt} = \frac{3}{8}\left[1.5 - 0.5\left(\frac{7}{8} + \frac{1}{8}\right)\right] = 0.3750 \text{ in.}^2$$

(The factor of 0.5 is used because there is one-half of a hole diameter in the tension section.)

Since the block shear will occur in an angle, $U_{bs} = 1.0$, and from AISC Equation J4-5,

$$\begin{aligned} R_n &= 0.6F_uA_{nv} + U_{bs}F_uA_{nt} \\ &= 0.6(58)(1.875) + 1.0(58)(0.3750) = 87.00 \text{ kips} \end{aligned}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(2.813) + 1.0(58)(0.3750) = 82.51 \text{ kips}$$

The nominal block shear strength is therefore 82.51 kips.

ANSWER a. The design strength for LRFD is $\phi R_n = 0.75(82.51) = 61.9$ kips.

b. The allowable strength for ASD is $\frac{R_n}{\Omega} = \frac{82.51}{2.00} = 41.3$ kips.

3.6 DESIGN OF TENSION MEMBERS

The design of a tension member involves finding a member with adequate gross and net areas. If the member has a bolted connection, the selection of a suitable cross section requires an accounting for the area lost because of holes. For a member with a rectangular cross section, the calculations are relatively straightforward. If a rolled shape is to be used, however, the area to be deducted cannot be predicted in advance because the member•s thickness at the location of the holes is not known.

A secondary consideration in the design of tension members is slenderness. If a structural member has a small cross section in relation to its length, it is said to be *slender*. A more precise measure is the slenderness ratio, L/r, where L is the member length and r is the minimum radius of gyration of the cross-sectional area. The minimum radius of gyration is the one corresponding to the minor principal axis of the cross section. This value is tabulated for all rolled shapes in the properties tables in Part 1 of the *Manual*.

Although slenderness is critical to the strength of a compression member, it is inconsequential for a tension member. In many situations, however, it is good practice to limit the slenderness of tension members. If the axial load in a slender tension member is removed and small transverse loads are applied, undesirable vibrations or deflections might occur. These conditions could occur, for example, in a slack bracing rod subjected to wind loads. For this reason, the user note in AISC D1 suggests a maximum slenderness ratio of 300. It is only a recommended value because slenderness has no structural significance for tension members, and the limit may be exceeded when special circumstances warrant it. This limit does not apply to cables, and the user note explicitly excludes rods.

The central problem of all member design, including tension member design, is to find a cross section for which the required strength does not exceed the available strength. For tension members designed by LRFD, the requirement is

$$P_u \leq \phi_t P_n \quad \text{or} \quad \phi_t P_n \geq P_u$$

where P_u is the sum of the factored loads. To prevent yielding,

$$0.90 F_y A_g \geq P_u \quad \text{or} \quad A_g \geq \frac{P_u}{0.90 F_y}$$

To avoid fracture,

$$0.75 F_u A_e \geq P_u \quad \text{or} \quad A_e \geq \frac{P_u}{0.75 F_u}$$

For allowable strength design, if we use the allowable *stress* form, the requirement corresponding to yielding is

$$P_a \leq F_t A_g$$

and the required gross area is

$$A_g \geq \frac{P_a}{F_t} \quad \text{or} \quad A_g \geq \frac{P_a}{0.6 F_y}$$

For the limit state of fracture, the required effective area is

$$A_e \geq \frac{P_a}{F_t} \quad \text{or} \quad A_e \geq \frac{P_a}{0.5 F_u}$$

The slenderness ratio limitation will be satisfied if

$$r \geq \frac{L}{300}$$

where r is the minimum radius of gyration of the cross section and L is the member length.

EXAMPLE 3.11

A tension member with a length of 5 feet 9 inches must resist a service dead load of 18 kips and a service live load of 52 kips. Select a member with a rectangular cross section. Use A36 steel and assume a connection with one line of ⅞-inch-diameter bolts.

LRFD SOLUTION

$$P_u = 1.2D + 1.6L = 1.2(18) + 1.6(52) = 104.8 \text{ kips}$$

$$\text{Required } A_g = \frac{P_u}{\phi_t F_y} = \frac{P_u}{0.90 F_y} = \frac{104.8}{0.90(36)} = 3.235 \text{ in.}^2$$

$$\text{Required } A_e = \frac{P_u}{\phi_t F_u} = \frac{P_u}{0.75 F_u} = \frac{104.8}{0.75(58)} = 2.409 \text{ in.}^2$$

Try $t = 1$ in.

$$\text{Required } w_g = \frac{\text{required } A_g}{t} = \frac{3.235}{1} = 3.235 \text{ in.}$$

Try a $1 \times 3\frac{1}{2}$ cross section.

$$A_e = A_n = A_g - A_{hole}$$

$$= (1 \times 3.5) - \left(\frac{7}{8} + \frac{1}{8}\right)(1) = 2.5 \text{ in.}^2 > 2.409 \text{ in.}^2 \qquad \text{(OK)}$$

Check the slenderness ratio:

$$I_{min} = \frac{3.5(1)^3}{12} = 0.2917 \text{ in.}^4$$

$$A = 1(3.5) = 3.5 \text{ in.}^2$$

From $I = Ar^2$, we obtain

$$r_{min} = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{0.2917}{3.5}} = 0.2887 \text{ in.}^2$$

$$\text{Maximum } \frac{L}{r} = \frac{5.75(12)}{0.2887} = 239 < 300 \qquad \text{(OK)}$$

ANSWER

Use a PL $1 \times 3\frac{1}{2}$.

ASD SOLUTION

$$P_a = D + L = 18 + 52 = 70.0 \text{ kips}$$

For yielding, $F_t = 0.6F_y = 0.6(36) = 21.6$ ksi, and

$$\text{Required } A_g = \frac{P_a}{F_t} = \frac{70}{21.6} = 3.24 \text{ in.}^2$$

For fracture, $F_t = 0.5F_u = 0.5(58) = 29.0$ ksi, and

$$\text{Required } A_e = \frac{P_a}{F_t} = \frac{70}{29.0} = 2.414 \text{ in.}^2$$

(The rest of the design *procedure* is the same as for LRFD. The numerical results may be different)

Try $t = 1$ in.

$$\text{Required } w_g = \frac{\text{required } A_g}{t} = \frac{3.241}{1} = 3.241 \text{ in.}$$

Try a 1 × 3 ½ cross section.

$$A_e = A_n = A_g - A_{hole}$$
$$= (1 \times 3.5) - \left(\frac{7}{8} + \frac{1}{8}\right)(1) = 2.5 \text{ in.}^2 > 2.414 \text{ in.}^2 \qquad \text{(OK)}$$

Check the slenderness ratio:

$$I_{min} = \frac{3.5(1)^3}{12} = 0.2917 \text{ in.}^4$$
$$A = 1(3.5) = 3.5 \text{ in.}^2$$

From $I = Ar^2$, we obtain

$$r_{min} = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{0.2917}{3.5}} = 0.2887 \text{ in.}^2$$
$$\text{Maximum } \frac{L}{r} = \frac{5.75(12)}{0.2887} = 239 < 300 \qquad \text{(OK)}$$

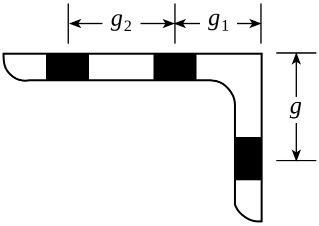
ANSWER Use a PL 1 × 3½.

Example 3.11 illustrates that once the required area has been determined, the procedure is the same for both LRFD and ASD. Note also that in this example, the required areas are virtually the same for LRFD and ASD. This is because the ratio of live load to dead load is approximately 3, and the two approaches will give the same results for this ratio.

The member in Example 3.11 is less than 8 inches wide and thus is classified as a bar rather than a plate. Bars should be specified to the nearest ¼ inch in width and to the nearest ⅛ inch in thickness (the precise classification system is given in Part 1 of the *Manual* under the heading "Plate Products"). It is common practice to use the PL (Plate) designation for both bars and plates.

If an angle shape is used as a tension member and the connection is made by bolting, there must be enough room for the bolts. Space will be a problem only when there are two lines of bolts in a leg. The usual fabrication practice is to punch or drill holes in standard locations in angle legs. These hole locations are given in Table 1-7A in Part 1 of the *Manual*. This table is located at the end of the dimensions and properties table for angles. Figure 3.24 presents this same information. Gage distance g applies when there is one line of bolts, and g_1 and g_2 apply when there are two lines. Figure 3.24 shows that an angle leg must be at least 5 inches long to accommodate two lines of bolts.

FIGURE 3.24



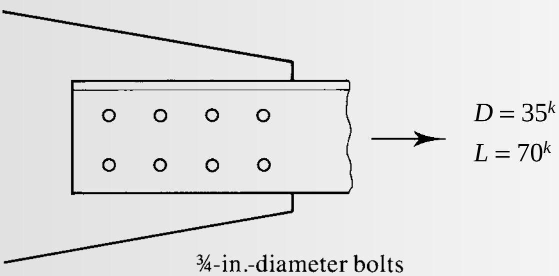
Usual Gages for Angles (inches)

Leg	8	7	6	5	4	3½	3	2½	2	1¾	1½	1⅜	1¼	1
g	4½	4	3½	3	2½	2	1¾	1⅜	1⅛	1	⅞	⅞	¾	⅝
g_1	3	2½	2¼	2										
g_2	3	3	2½	1¾										

EXAMPLE 3.12

Select an unequal-leg angle tension member 15 feet long to resist a service dead load of 35 kips and a service live load of 70 kips. Use A36 steel. The connection is shown in Figure 3.25.

FIGURE 3.25



LRFD SOLUTION

The factored load is

$$P_u = 1.2D + 1.6L = 1.2(35) + 1.6(70) = 154 \text{ kips}$$

$$\text{Required } A_g = \frac{P_u}{\phi_t F_y} = \frac{154}{0.90(36)} = 4.75 \text{ in.}^2$$

$$\text{Required } A_e = \frac{P_u}{\phi_t F_u} = \frac{154}{0.75(58)} = 3.54 \text{ in.}^2$$

The radius of gyration should be at least

$$\frac{L}{300} = \frac{15(12)}{300} = 0.6 \text{ in.}$$

To find the lightest shape that satisfies these criteria, we search the dimensions and properties table for the unequal-leg angle that has the smallest acceptable gross area and then check the effective net area. The radius of gyration can be checked by inspection. There are two lines of bolts, so the connected leg must be at least 5 inches long (see the usual gages for angles in Figure 3.24). Starting at either end of the table, we find that the shape with the smallest area that is at least equal to 4.75 in.^{2} is an L6 × 4 × ½ with an area of 4.75 in.^{2} and a minimum radius of gyration of 0.864 in.

Try L6 × 4 × ½.

$$A_n = A_g - A_{\text{holes}} = 4.75 - 2\left(\frac{3}{4} + \frac{1}{8}\right)\left(\frac{1}{2}\right) = 3.875 \text{ in.}^2$$

Because the length of the connection is not known, Equation 3.1 cannot be used to compute the shear lag factor U. Since there are four bolts in the direction of the load, we will use the alternative value of $U = 0.80$.

$$A_e = A_n U = 3.875(0.80) = 3.10 \text{ in.}^2 < 3.54 \text{ in.}^2 \qquad \text{(N.G.)*}$$

Try the next larger shape from the dimensions and properties tables.

Try L5 × 3½ × ⅝ ($A_g = 4.93$ in.^{2} and $r_{min} = 0.746$ in.)

$$A_n = A_g - A_{\text{holes}} = 4.93 - 2\left(\frac{3}{4} + \frac{1}{8}\right)\left(\frac{5}{8}\right) = 3.836 \text{ in.}^2$$

$$A_e = A_n U = 3.836(0.80) = 3.07 \text{ in.}^2 < 3.54 \text{ in.}^2 \qquad \text{(N.G.)}$$

(Note that this shape has slightly more gross area than that produced by the previous trial shape, but because of the greater leg thickness, slightly more area is deducted for the holes.) Passing over the next few heavier shapes,

Try L8 × 4 × ½ ($A_g = 5.80$ in.^{2} and $r_{min} = 0.863$ in.)

$$A_n = A_g - A_{\text{holes}} = 5.80 - 2\left(\frac{3}{4} + \frac{1}{8}\right)\left(\frac{1}{2}\right) = 4.925 \text{ in.}^2$$

$$A_e = A_n U = 4.925(0.80) = 3.94 \text{ in.}^2 > 3.54 \text{ in.}^2 \qquad \text{(OK)}$$

*The notation N.G. means "No Good."

ANSWER This shape satisfies all requirements, so use an L8 × 4 × ½.

ASD SOLUTION The total service load is

$$P_a = D + L = 35 + 70 = 105 \text{ kips}$$

$$\text{Required } A_g = \frac{P_a}{F_t} = \frac{P_a}{0.6F_y} = \frac{105}{0.6(36)} = 4.86 \text{ in.}^2$$

$$\text{Required } A_e = \frac{P_a}{0.5F_u} = \frac{105}{0.5(58)} = 3.62 \text{ in.}^2$$

$$\text{Required } r_{\text{min}} = \frac{L}{300} = \frac{15(12)}{300} = 0.6 \text{ in.}$$

Try L8 × 4 × ½ (A_g = 5.80 in.^{2} and r_{min} = 0.863 in.). For a shear lag factor U of 0.80,

$$A_n = A_g - A_{\text{holes}} = 5.80 - 2\left(\frac{3}{4} + \frac{1}{8}\right)\left(\frac{1}{2}\right) = 4.925 \text{ in.}^2$$

$$A_e = A_nU = 4.925(0.80) = 3.94 \text{ in.}^2 > 3.62 \text{ in.}^2 \qquad \text{(OK)}$$

ANSWER This shape satisfies all requirements, so use an L8 × 4 × ½.

The ASD solution in Example 3.12 is somewhat condensed, in that some of the discussion in the LRFD solution is not repeated and only the final trial is shown. All essential computations are included, however.

Tables for the Design of Tension Members

Part 5 of the *Manual* contains tables to assist in the design of tension members of various cross-sectional shapes, including Table 5-2 for angles. The use of these tables will be illustrated in the following example.

EXAMPLE 3.13

Design the tension member of Example 3.12 with the aid of the tables in Part 5 of the *Manual*.

LRFD SOLUTION From Example 3.12,

$$P_u = 154 \text{ kips}$$

$$r_{\text{min}} \geq 0.600 \text{ in.}$$

The tables for design of tension members give values of A_g and A_e for various shapes based on the assumption that $A_e = 0.75A_g$. In addition, the corresponding available strengths based on yielding and rupture (fracture) are given. All values available for angles are for A36 steel. Starting with the lighter shapes (the ones with the smaller gross area), we find that an L6 × 4 × ½, with $\phi_t P_n = 154$ kips based on the gross section and $\phi_t P_n = 155$ kips based on the net section, is a possibility. From the dimensions and properties tables in Part 1 of the *Manual*, $r_{min} = 0.864$ in. To check this selection, we must compute the actual net area. If we assume that $U = 0.80$,

$$A_n = A_g - A_{\text{holes}} = 4.75 - 2\left(\frac{3}{4} + \frac{1}{8}\right)\left(\frac{1}{2}\right) = 3.875 \text{ in.}^2$$

$$A_e = A_n U = 3.875(0.80) = 3.10 \text{ in.}^2$$

$$\phi_t P_n = \phi_t F_u A_e = 0.75(58)(3.10) = 135 \text{ kips} < 154 \text{ kips} \qquad \text{(N.G.)}$$

This shape did not work because the ratio of actual effective net area A_e to gross area A_g is not equal to 0.75. The ratio is closer to

$$\frac{3.10}{4.75} = 0.6526$$

This corresponds to a required $\phi_t P_n$ (based on rupture) of

$$\frac{0.75}{\text{actual ratio}} \times P_u = \frac{0.75}{0.6526}(154) = 177 \text{ kips}$$

Try an L8 × 4 × ½, with $\phi_t P_n = 188$ kips (based on yielding) and $\phi_t P_n = 189$ Kips (based on rupture strength, with $A_e = 0.75A_g = 4.31$ in.^{2}). From the dimensions and properties tables in Part 1 of the *Manual*, $r_{min} = 0.863$ in. The actual effective net area and rupture strength are computed as follows:

$$A_n = A_g - A_{\text{holes}} = 5.80 - 2\left(\frac{3}{4} + \frac{1}{8}\right)\left(\frac{1}{2}\right) = 4.925 \text{ in.}^2$$

$$A_e = A_n U = 4.925(0.80) = 3.94 \text{ in.}^2$$

$$\phi_t P_n = \phi_t F_u A_e = 0.75(58)(3.94) = 171 > 154 \text{ kips} \qquad \text{(OK)}$$

ANSWER Use an L8 × 4 × ½, connected through the 8-inch leg.

ASD SOLUTION From Example 3.12,

$$P_a = 105 \text{ kips}$$

$$\text{Required } r_{min} = 0.600 \text{ in.}$$

From *Manual* Table 5-2, try an L5 × 3½ × ⅝, with $P_n/\Omega_t = 106$ kips based on yielding of the gross section and $P_n/\Omega_t = 107$ kips based on rupture of the net section. From the dimensions and properties tables in Part 1 of the *Manual*, $r_{min} = 0.746$ in.

Using a shear lag factor U of 0.80, the actual effective net area is computed as follows:

$$A_n = A_g - A_{\text{holes}} = 4.93 - 2\left(\frac{3}{4} + \frac{1}{8}\right)\left(\frac{5}{8}\right) = 3.836 \text{ in.}^2$$
$$A_e = A_n U = 3.836(0.80) = 3.069 \text{ in.}^2$$

and the allowable strength based on rupture of the net section is

$$\frac{P_n}{\Omega_t} = \frac{F_u A_e}{\Omega_t} = \frac{58(3.069)}{2.00} = 89.0 \text{ kips} < 105 \text{ kips} \qquad \text{(N.G.)}$$

This shape did not work because the ratio of actual effective net area A_e to gross area A_g is not equal to 0.75. The ratio is closer to

$$\frac{3.069}{4.93} = 0.6225$$

This corresponds to a required P_n/Ω_t (based on rupture), for purposes of using Table 5-2, of

$$\frac{0.75}{0.6225}(105) = 127 \text{ kips}$$

Using this as a guide, try L6 × 4 × ⅝, with $P_n/\Omega_t = 126$ kips based on yielding of the gross section and $P_n/\Omega_t = 128$ kips based on rupture of the net section. From the dimensions and properties tables in Part 1 of the *Manual*, $r_{\min} = 0.859$ in.

$$A_n = A_g - A_{\text{holes}} = 5.86 - 2\left(\frac{3}{4} + \frac{1}{8}\right)\left(\frac{5}{8}\right) = 4.766 \text{ in.}^2$$
$$A_e = A_n U = 4.766(0.80) = 3.81 \text{ in.}^2$$
$$\frac{P_n}{\Omega_t} = \frac{F_u A_e}{\Omega_t} = \frac{58(3.81)}{2.00} = 111 \text{ kips} > 105 \text{ kips} \qquad \text{(OK)}$$

ANSWER Use an L6 × 4 × ⅝, connected through the 6-inch leg.

Note that if the effective net area must be computed, the tables do not save much effort. In addition, you must still refer to the dimensions and properties tables to find the radius of gyration. The tables for design do, however, provide all other information in a compact form, and the search may go more quickely.

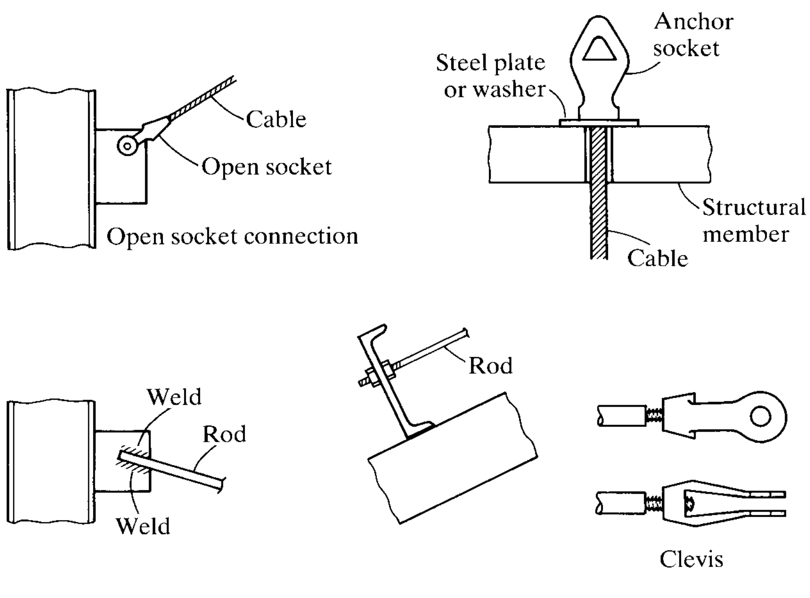
When structural shapes or plates are connected to form a built-up shape, they must be connected not only at the ends of the member but also at intervals along its length. A continuous connection is not required. This type of connection is called *stitching*, and the fasteners used are termed *stitch bolts*. The usual practice is to locate the points of stitching so that L/r for any component part does not exceed L/r for the built-up member. The user note in AISC D4 recommends that built-up shapes whose component parts are separated by intermittent fillers be connected at intervals such that the maximum L/r for any component does not exceed 300. Built-up shapes consisting of plates or a combination of plates and shapes are addressed in AISC Section J3.5 of Chapter J ("Design of Connections"). In general, the spacing of fasteners or welds should not exceed 24 times the thickness of the thinner plate, or 12 inches. If the member is of "weathering" steel subject to atmospheric corrosion, the maximum spacing is 14 times the thickness of the thinner part, or 7 inches.

3.7 THREADED RODS AND CABLES

When slenderness is not a consideration, rods with circular cross sections and cables are often used as tension members. The distinction between the two is that rods are solid and cables are made from individual strands wound together in ropelike fashion. Rods and cables are frequently used in suspended roof systems and as hangers or suspension members in bridges. Rods are also used in bracing systems; in some cases, they are pretensioned to prevent them from going slack when external loads are removed. Figure 3.26 illustrates typical rod and cable connection methods.

When the end of a rod is to be threaded, an upset end is sometimes used. This is an enlargement of the end in which the threads are to be cut. Threads reduce the cross-sectional area, and upsetting the end produces a larger gross area to start with. Standard upset ends with threads will actually have more net area in the threaded portion than in the unthreaded part. Upset ends are relatively expensive, however, and in most cases unnecessary.

FIGURE 3.26



The effective cross-sectional area in the threaded portion of a rod is called the *stress area* and is a function of the unthreaded diameter and the number of threads per inch. The ratio of stress area to nominal area varies but has a lower bound of approximately 0.75. The nominal tensile strength of the threaded rod can therefore be written as

$$P_n = A_s F_u = 0.75 A_b F_u \qquad (3.5)$$

where

A_s = stress area
A_b = nominal (unthreaded) area

The AISC Specification, in Chapter J, presents the nominal strength in a somewhat different form:

$$R_n = F_n A_b \qquad \text{(AISC Equation J3-1)}$$

where R_n is the nominal strength and F_n is given in Table J3.2 as $F_{nt} = 0.75F_u$. This associates the 0.75 factor with the ultimate tensile stress rather than the area, but the result is the same as that given by Equation 3.5.

For LRFD, the resistance factor ϕ is 0.75, so the strength relationship is

$$P_u \le \phi_t P_n \quad \text{or} \quad P_u \le 0.75(0.75 A_b F_u)$$

and the required area is

$$A_b = \frac{P_u}{0.75(0.75F_u)} \qquad (3.6)$$

For ASD, the safety factor Ω is 2.00, leading to the requirement

$$P_a \le \frac{P_n}{2.00} \quad \text{or} \quad P_a \le 0.5P_n$$

Using P_n from Equation 3.5, we get

$$P_a \le 0.5(0.75 A_b F_u)$$

If we divide both sides by the area A_b, we obtain the allowable stress

$$F_t = 0.5(0.75F_u) = 0.375F_u \qquad (3.7)$$

EXAMPLE 3.14

A threaded rod is to be used as a bracing member that must resist a service tensile load of 2 kips dead load and 6 kips live load. What size rod is required if A36 steel is used?

LRFD SOLUTION

The factored load is

$$P_u = 1.2(2) + 1.6(6) = 12 \text{ kips}$$

From Equation 3.6,

$$\text{Required area } = A_b = \frac{P_u}{0.75(0.75F_u)} = \frac{12}{0.75(0.75)(58)} = 0.3678 \text{ in.}^2$$

From $A_b = \dfrac{\pi d^2}{4}$,

$$\text{Required } d = \sqrt{\frac{4(0.3678)}{\pi}} = 0.684 \text{ in.}$$

ANSWER Use a ¾-inch-diameter threaded rod ($A_b = 0.442$ in.^{2}).

ASD SOLUTION The required strength is

$$P_a = D + L = 2 + 6 = 8 \text{ kips}$$

From Equation 3.7, the allowable tensile stress is

$$F_t = 0.375F_u = 0.375(58) = 21.75 \text{ ksi}$$

and the required area is

$$A_b = \frac{P_a}{F_t} = \frac{8}{21.75} = 0.3678 \text{ in.}^2$$

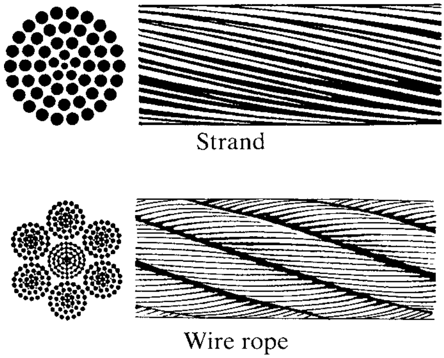
ANSWER Use a ¾-inch-diameter threaded rod ($A_b = 0.442$ in.^{2}).

To prevent damage during construction, rods should not be too slender. Although there is no specification requirement, a common practice is to use a minimum diameter of ⅝ inch.

Flexible cables, in the form of strands or wire rope, are used in applications where high strength is required and rigidity is unimportant. In addition to their use in bridges and cable roof systems, they are also used in hoists and derricks, as guy lines for towers, and as longitudinal bracing in metal building systems. The difference between strand and wire rope is illustrated in Figure 3.27. A strand consists of individual wires wound helically around a central core, and a wire rope is made of several strands laid helically around a core.

Selection of the correct cable for a given loading is usually based on both strength and deformation considerations. In addition to ordinary elastic elongation, an initial stretching is caused by seating or shifting of the individual wires, which results in a permanent stretch. For this reason, cables are often prestretched. Wire rope and strand are made from steels of much higher strength than structural steels and are not covered by the AISC Specification. The breaking strengths of various cables, as well as details of available fixtures for connections, can be obtained from manufacturers• literature.

FIGURE 3.27



3.8 TENSION MEMBERS IN ROOF TRUSSES

Many of the tension members that structural engineers design are components of trusses. For this reason, some general discussion of roof trusses is in order. A more comprehensive treatment of the subject is given by Lothars (1972).

When trusses are used in buildings, they usually function as the main supporting elements of roof systems where long spans are required. They are used when the cost and weight of a beam would be prohibitive. (A truss may be thought of as a deep beam with much of the web removed.) Roof trusses are often used in industrial or mill buildings, although construction of this type has largely given way to rigid frames. Typical roof construction with trusses supported by load-bearing walls is illustrated in Figure 3.28. In this type of construction, one end of the connection of the truss to the walls usually can be considered as pinned and the other as roller-supported. Thus the truss can be analyzed as an externally statically determinate structure. The supporting walls can be reinforced concrete, concrete block, brick, or a combination of these materials.

Roof trusses normally are spaced uniformly along the length of the building and are tied together by longitudinal beams called *purlins* and by x-bracing. The primary function of the purlins is to transfer loads to the top chord of the truss, but they can also act as part of the bracing system. Bracing is usually provided in the planes of both the top and bottom chords, but it is not required in every bay because lateral forces can be transferred from one braced bay to the other through the purlins.

Ideally, purlins are located at the truss joints so that the truss can be treated as a pin-connected structure loaded only at the joints. Sometimes, however, the roof deck cannot span the distance between joints, and intermediate purlins may be needed. In such cases, top chord members will be subjected to significant bending as well as axial compression and must be designed as beam–columns (Chapter 6).

Sag rods are tension members used to provide lateral support for the purlins. Most of the loads applied to the purlins are vertical, so there will be a component parallel to a sloping roof, which will cause the purlin to bend (sag) in that direction (Figure 3.29).

Sag rods can be located at the midpoint, the third points, or at more frequent intervals along the purlins, depending on the amount of support needed. The interval is a function of the truss spacing, the slope of the top chord, the resistance of the purlin

FIGURE 3.28

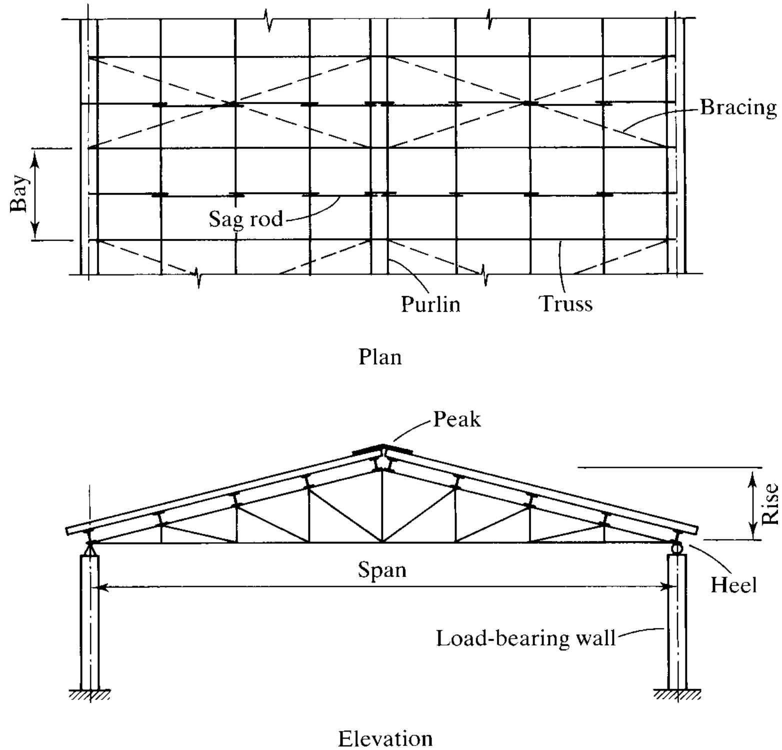
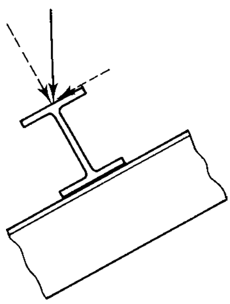


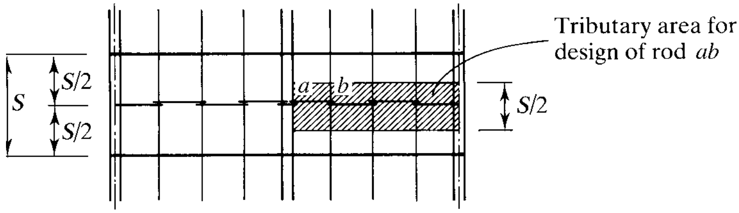
FIGURE 3.29



to this type of bending (most shapes used for purlins are very weak in this respect), and the amount of support furnished by the roofing. If a metal deck is used, it will usually be rigidly attached to the purlins, and sag rods may not be needed. Sometimes, however, the weight of the purlin itself is enough to cause problems, and sag rods may be needed to provide support during construction before the deck is in place.

If sag rods are used, they are designed to support the component of roof loads parallel to the roof. Each segment between purlins is assumed to support everything below it; thus the top rod is designed for the load on the roof area tributary to the rod, from the heel of the truss to the peak, as shown in Figure 3.30. Although the force will be different in each segment of rod, the usual practice is to use one size throughout.

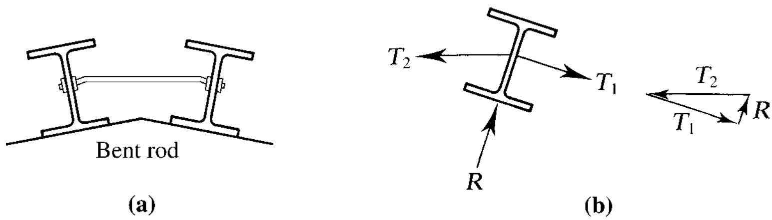
FIGURE 3.30



The extra amount of material in question is insignificant, and the use of the same size for each segment eliminates the possibility of a mix-up during construction.

A possible treatment at the peak or ridge is shown in Figure 3.31a. The tie rod between ridge purlins must resist the load from all of the sag rods on either side. The tensile force in this horizontal member has as one of its components the force in the upper sag-rod segment. A free-body diagram of one ridge purlin illustrates this effect, as shown in Figure 3.31b.

FIGURE 3.31



EXAMPLE 3.15

Fink trusses spaced at 20 feet on centers support W6 × 12 purlins, as shown in Figure 3.32a. The purlins are supported at their midpoints by sag rods. Use A36 steel and design the sag rods and the tie rod at the ridge for the following service loads.

Metal deck:	2 psf
Built-up roof:	5 psf
Snow:	18 psf of horizontal projection of the roof surface
Purlin weight:	12 pounds per foot (lb/ft) of length

SOLUTION Calculate loads.

Tributary width for each sag rod = 20/2 = 10 ft

Tributary area for deck and built-up roof = 10(46.6) = 466 ft^2

Dead load (deck and roof) = (2 + 5)(466) = 3262 lb

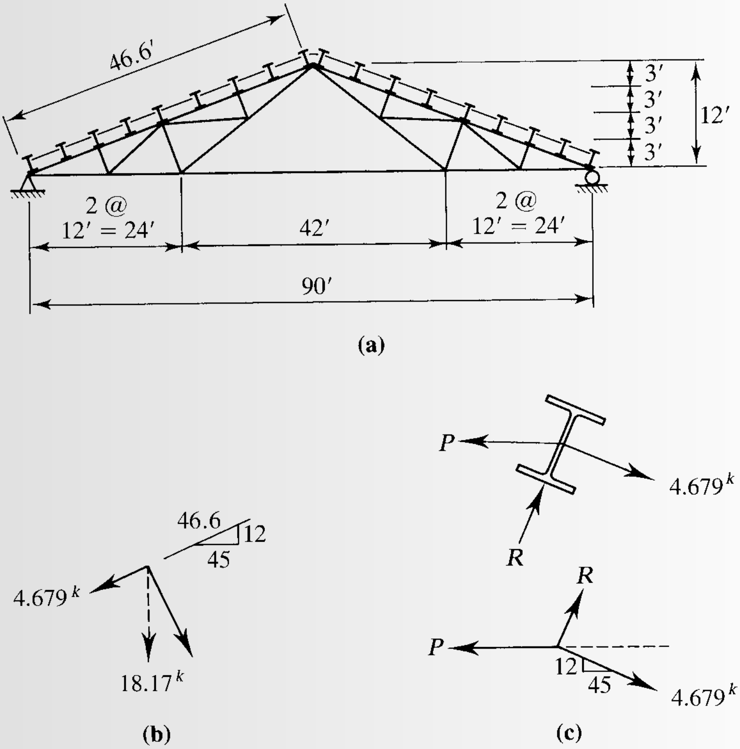
Total purlin weight = 12(10)(9) = 1080 lb

Total dead load = 3262 + 1080 = 4342 lb

Tributary area for snow load = 10(45) = 450 ft^2

Total snow load = 18(450) = 8100 lb

FIGURE 3.32



LRFD SOLUTION

Check load combinations.

Combination 2: $1.2D + 0.5S = 1.2(4342) + 0.5(8100) = 9260$ lb

Combination 3: $1.2D + 1.6S = 1.2(4342) + 1.6(8100) = 18{,}170$ lb

Combination 3 controls. (By inspection, the remaining combinations will not govern.)

For the component parallel to the roof (Figure 3.32b),

$$T = (18.17)\frac{12}{46.6} = 4.679 \text{ kips}$$

$$\text{Required } A_b = \frac{T}{\phi_t(0.75F_u)} = \frac{4.679}{0.75(0.75)(58)} = 0.1434 \text{ in.}^2$$

ANSWER

Use a ⅝-inch-diameter threaded rod ($A_b = 0.3068$ in.^{2}).

Tie rod at the ridge (Figure 3.32c):

$$P = (4.679)\frac{46.6}{45} = 4.845 \text{ kips}$$

$$\text{Required } A_b = \frac{4.845}{0.75(0.75)(58)} = 0.1485 \text{ in.}^2$$

ANSWER Use a ⅝-inch-diameter threaded rod ($A_b = 0.3068$ in.^{2}).

ASD SOLUTION By inspection, load combination 3 will control.

$$D + S = 4342 + 8100 = 12{,}440 \text{ lb}$$

The component parallel to the roof is

$$T = 12.44\left(\frac{12}{46.6}\right) = 3.203 \text{ kips}$$

The allowable tensile stress is $F_t = 0.375F_u = 0.375(58) = 21.75$ ksi.

$$\text{Required } A_b = \frac{T}{F_t} = \frac{3.203}{21.75} = 0.1473 \text{ in.}^2$$

ANSWER Use a ⅝-inch-diameter threaded rod ($A_b = 0.3068$ in.^{2}) for the sag rods.

Tie rod at the ridge:

$$P = 3.203\left(\frac{46.6}{45}\right) = 3.317 \text{ kips}$$

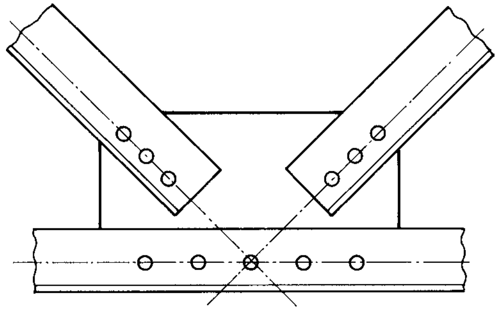
$$\text{Required } A_b = \frac{3.317}{21.75} = 0.1525 \text{ in.}^2$$

ANSWER Use a ⅝-inch-diameter threaded rod ($A_b = 0.3068$ in.^{2}) for the tie rod at the ridge.

For the usual truss geometry and loading, the bottom chord will be in tension and the top chord will be in compression. Some web members will be in tension and others will be in compression. When wind effects are included and consideration is given to different wind directions, the force in some web members may alternate between tension and compression. In this case, the affected member must be designed to function as both a tension member and a compression member.

In bolted trusses, double-angle sections are frequently used for both chord and web members. This design facilitates the connection of members meeting at a joint by permitting the use of a single gusset plate, as illustrated in Figure 3.33. When structural tee-shapes are used as chord members in welded trusses, the web angles can usually be welded to the stem of the tee. If the force in a web member is small, single angles can be used, although doing so eliminates the plane of symmetry from the truss and causes the web member to be eccentrically loaded. Chord members are usually fabricated as continuous pieces and spliced if necessary.

FIGURE 3.33



The fact that chord members are continuous and joints are bolted or welded would seem to invalidate the usual assumption that the truss is pin-connected. Joint rigidity does introduce some bending moment into the members, but it is usually small and considered to be a secondary effect. The usual practice is to ignore it. Bending caused by loads directly applied to members between the joints, however, must be taken into account. We consider this condition in Chapter 6, "Beam–Columns."

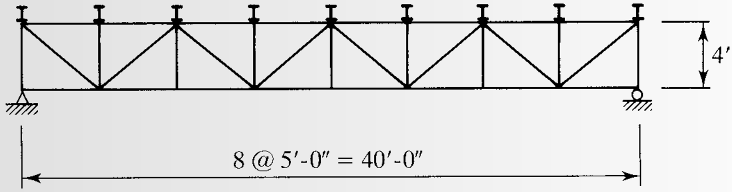
The *working lines* of the members in a properly detailed truss intersect at the *working point* at each joint. For a bolted truss, the bolt lines are the working lines, and in welded trusses the centroidal axes of the welds are the working lines. For truss analysis, member lengths are measured from working point to working point.

EXAMPLE 3.16

Select a structural tee for the bottom chord of the Warren roof truss shown in Figure 3.34. The trusses are welded and spaced at 20 feet. Assume that the bottom chord connection is made with 9-inch-long longitudinal welds at the flange. Use A992 steel and the following load data (wind is not considered in this example):

Purlins:	M8 × 6.5
Snow:	20 psf of horizontal projection
Metal deck:	2 psf
Roofing:	4 psf
Insulation:	3 psf

FIGURE 3.34



SOLUTION Calculate loads:

Snow = 20(40)(20) = 16,000 lb

Dead load (exclusive of purlins) =	Deck	2 psf
	Roof	4
	Insulation	3
	Total	9 psf

Total dead load = 9(40)(20) = 7200 lb

Total purlin weight = 6.5(20)(9) = 1170 lb

Estimate the truss weight as 10% of the other loads:

0.10(16,000 + 7200 + 1170) = 2437 lb

Loads at an interior joint are

$$D = \frac{7200}{8} + \frac{2437}{8} + 6.5(20) = 1335 \text{ lb}$$

$$S = \frac{16,000}{8} = 2000 \text{ lb}$$

At an exterior joint, the tributary roof area is half of that at an interior joint. The corresponding loads are

$$D = \frac{7200}{2(8)} + \frac{2437}{2(8)} + 6.5(20) = 732.3 \text{ lb}$$

$$S = \frac{16,000}{2(8)} = 1000 \text{ lb}$$

LRFD SOLUTION Load combination 3 will control:

$$P_u = 1.2D + 1.6S$$

At an interior joint,

$$P_u = 1.2(1.335) + 1.6(2.0) = 4.802 \text{ kips}$$

At an exterior joint,

$$P_u = 1.2(0.7323) + 1.6(1.0) = 2.479 \text{ kips}$$

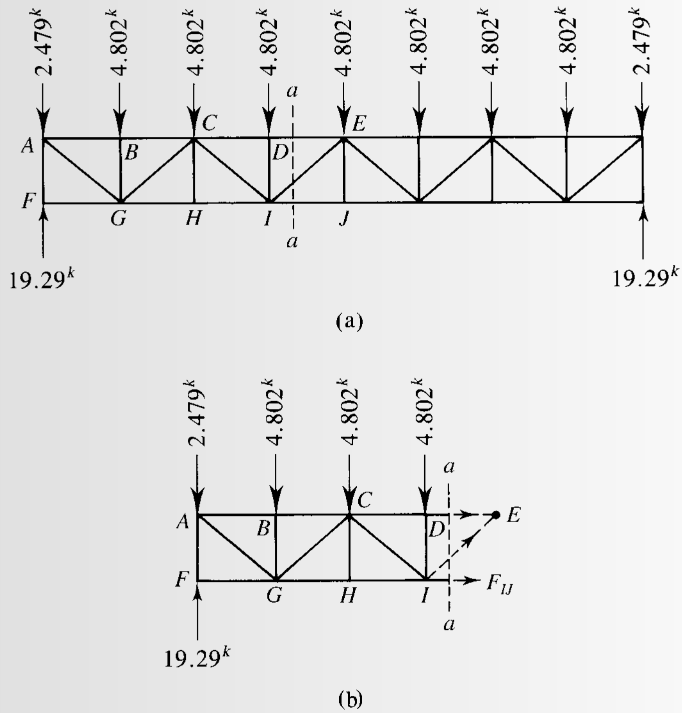
The loaded truss is shown in Figure 3.35a.

The bottom chord is designed by determining the force in each member of the bottom chord and selecting a cross section to resist the largest force. In this example, the force in member *IJ* will control. For the free body left of section *a–a* shown in Figure 3.35b,

$$\sum M_E = 19.29(20) - 2.479(20) - 4.802(15 + 10 + 5) - 4F_{IJ} = 0$$

$$F_{IJ} = 48.04 \text{ kips}$$

FIGURE 3.35



For the gross section,

$$\text{Required } A_g = \frac{F_{IJ}}{0.90F_y} = \frac{48.04}{0.90(50)} = 1.07 \text{ in.}^2$$

For the net section,

$$\text{Required } A_e = \frac{F_{IJ}}{0.75F_u} = \frac{48.04}{0.75(65)} = 0.985 \text{ in.}^2$$

Try an MT5 × 3.75:

$$A_g = 1.11 \text{ in.}^2 > 1.07 \text{ in.}^2 \quad \text{(OK)}$$

Compute the shear lag factor U from Equation 3.1.

$$U = 1 - \left(\frac{\bar{x}}{\ell}\right) = 1 - \left(\frac{1.51}{9}\right) = 0.8322$$

$$A_e = A_g U = 1.11(0.8322) = 0.924 \text{ in.}^2 < 0.985 \text{ in.}^2 \quad \text{(N.G.)}$$

Try an MT6 × 5:

$$A_g = 1.48 \text{ in.}^2 > 1.07 \text{ in.}^2 \quad \text{(OK)}$$

$$U = 1 - \left(\frac{\bar{x}}{\ell}\right) = 1 - \left(\frac{1.86}{9}\right) = 0.7933$$

$$A_e = A_g U = 1.48(0.7933) = 1.17 \text{ in.}^2 > 0.985 \text{ in.}^2 \quad \text{(OK)}$$

If we assume that the bottom chord is braced at the panel points,

$$\frac{L}{r} = \frac{5(12)}{0.594} = 101 < 300 \qquad \text{(OK)}$$

ANSWER Use an MT6 × 5.

ASD SOLUTION Load combination 3 will control. At an interior joint,

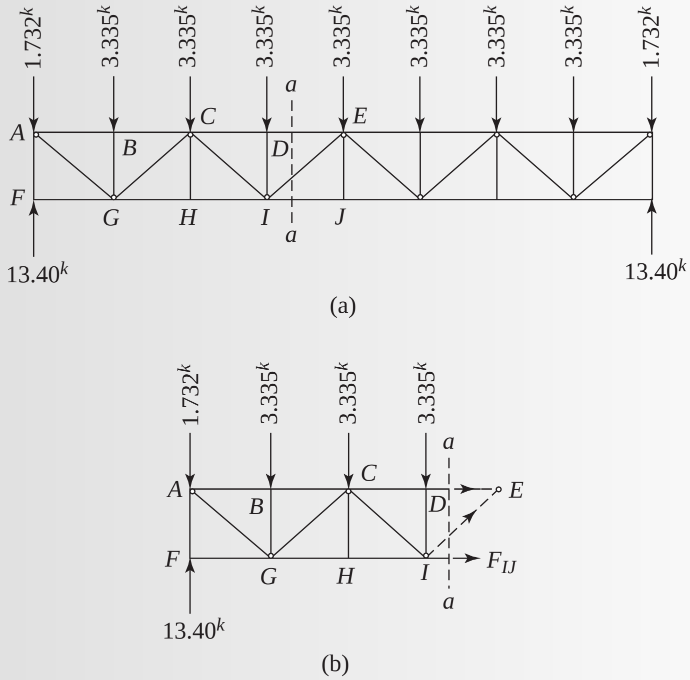
$$P_a = D + S = 1.335 + 2.0 = 3.335 \text{ kips}$$

At an exterior joint,

$$P_a = 0.7323 + 1.0 = 1.732 \text{ kips}$$

The loaded truss is shown in Figure 3.36a.

FIGURE 3.36



Member *IJ* is the bottom chord member with the largest force. For the free body shown in Figure 3.36b,

$$\sum M_E = 13.40(20) - 1.732(20) - 3.335(15 + 10 + 5) - 4F_{IJ} = 0$$

$$F_{IJ} = 33.33 \text{ kips}$$

For the gross section, $F_t = 0.6F_y = 0.6(36) = 21.6$ ksi

$$\text{Required } A_g = \frac{F_{IJ}}{F_t} = \frac{33.33}{21.6} = 1.54 \text{ in.}^2$$

For the net section, $F_t = 0.5F_u = 0.5(58) = 29.0$ ksi

$$\text{Required } A_e = \frac{F_{IJ}}{F_t} = \frac{33.33}{29.0} = 1.15 \text{ in.}^2$$

Try an MT6 × 5.4:

$$A_g = 1.59 \text{ in.}^2 > 1.54 \text{ in.}^2 \qquad \text{(OK)}$$

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{1.86}{9} = 0.7933$$

$$A_e = A_g U = 1.59(0.7933) = 1.26 \text{ in.}^2 > 1.15 \text{ in.}^2 \qquad \text{(OK)}$$

Assuming that the bottom chord is braced at the panel points, we get

$$\frac{L}{r} = \frac{5(12)}{0.566} = 106 < 300 \qquad \text{(OK)}$$

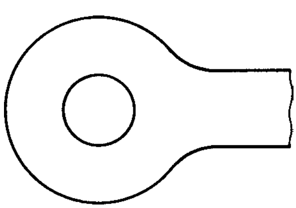
ANSWER Use an MT6 × 5.4.

3.9 PIN-CONNECTED MEMBERS

When a member is to be pin-connected, a hole is made in both the member and the parts to which it is connected and a pin is placed through the holes. This provides a connection that is as moment-free as can be fabricated. Tension members connected in this manner are subject to several types of failure, which are covered in AISC D5 and D6 and discussed in the following paragraphs.

The eyebar is a special type of pin-connected member in which the end containing the pin hole is enlarged, as shown in Figure 3.37. The design strength is based on yielding of the gross section. Detailed rules for proportioning eyebars are given in AISC D6 and are not repeated here. These requirements are based on experience and test programs for forged eyebars, but they are conservative when applied to eyebars thermally cut from plates (the present fabrication method). Eyebars were widely used in the past as single tension members in bridge trusses or were linked in chainlike fashion in suspension bridges. They are rarely used today.

FIGURE 3.37



Pin-connected members should be designed for the following limit states (see Figure 3.38).

1. **Tension** on the net effective area (Figure 3.38a):

   $\phi_t = 0.75,\ \Omega_t = 2.00, \qquad P_n = F_u(2tb_e)$ (AISC Equation D5-1)

2. **Shear** on the effective area (Figure 3.38b):

   $\phi_{sf} = 0.75,\ \Omega_{sf} = 2.00, \qquad P_n = 0.6F_uA_{sf}$ (AISC Equation D5-2)

3. **Bearing.** This requirement is given in Chapter J ("Connections, Joints, and Fasteners"), Section J7 (Figure 3.38c):

   $\phi = 0.75,\ \Omega = 2.00, \qquad P_n = 1.8F_yA_{pb}$ (AISC Equation J7-1)

4. **Tension** on the gross section:

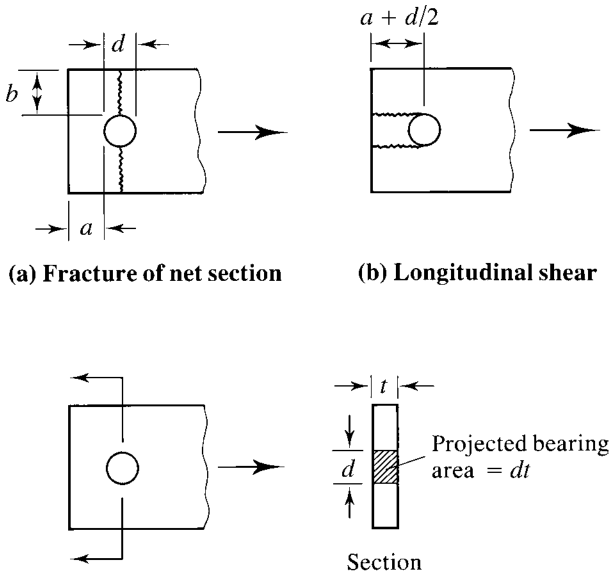
   $\phi_t = 0.90,\ \Omega_t = 1.67, \qquad P_n = F_yA_g$ (AISC Equation D2-1)

where

t = thickness of connected part
b_e = $2t + 0.63 \le b$
b = distance from edge of pin hole to edge of member, perpendicular to direction of force
A_{sf} = $2t(a + d/2)$
a = distance from edge of pin hole to edge of member, parallel to direction of force
d = pin diameter
A_{pb} = projected bearing area = dt

Additional requirements for the relative proportions of the pin and the member are covered in AISC D5.2

FIGURE 3.38



(a) Fracture of net section **(b) Longitudinal shear**

(c) Bearing

Problems

Tensile Strength

3.2-1 A PL ⅜ × 7 tension member is connected with three 1-inch-diameter bolts, as shown in Figure P3.2-1. The steel is A36. Assume that $A_e = A_n$ and compute the following.

a. The design strength for LRFD.

b. The allowable strength for ASD.

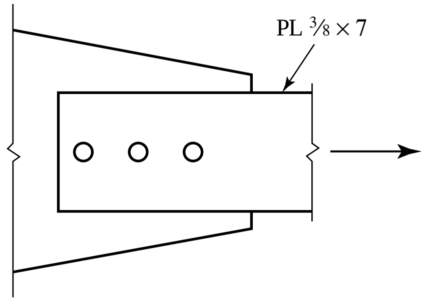


FIGURE P3.2-1

3.2-2 A PL ½ × 8 tension member is connected with six 1-inch-diameter bolts, as shown in Figure P3.2-2. The steel is ASTM A242. Assume that $A_e = A_n$ and compute the following.

a. The design strength for LRFD.

b. The allowable strength for ASD.

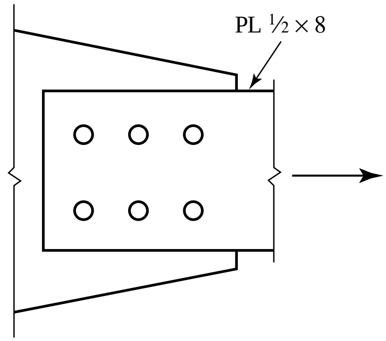


FIGURE P3.2-2

3.2-3 A C12 × 30 is connected with 1-in. diameter bolts in each flange, as shown in Figure P3.2-3. If $F_y = 50$ ksi, $F_u = 65$ ksi, and $A_e = 0.90A_n$, compute the following.

a. The design strength for LRFD.

b. The allowable strength for ASD.

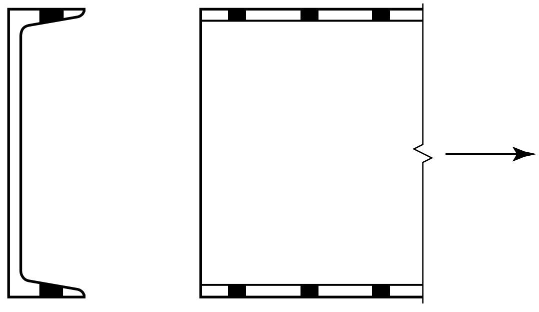


FIGURE P3.2-3

3.2-4 A PL $\frac{3}{8} \times 6$ tension member is welded to a gusset plate as shown in Figure P3.2-4. The steel is A36. Assume that $A_e = A_g$ and compute the following.

a. The design strength for LRFD.

b. The allowable strength for ASD.

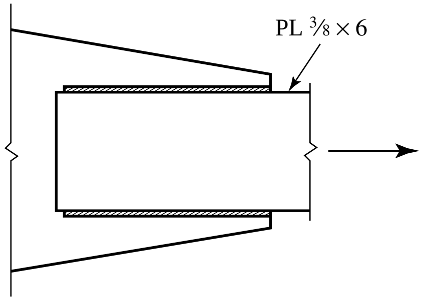


FIGURE P3.2-4

3.2-5 The tension member shown in Figure P3.2-5 is a PL $\frac{1}{2} \times 8$ of A36 steel. The member is connected to a gusset plate with-$1\frac{1}{8}$ inch-diameter bolts. It is subjected to the dead and live loads shown. Does this member have enough strength? Assume that $A_e = A_n$.

a. Use LRFD.

b. Use ASD.

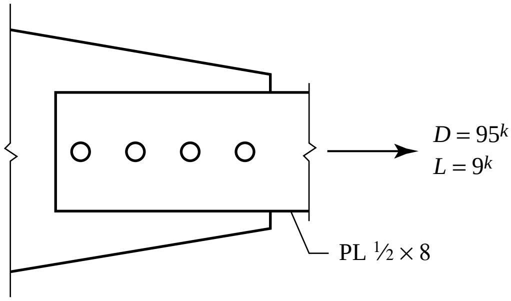


FIGURE P3.2-5

3.2-6 A double-angle tension member, 2L $3 \times 2 \times \frac{1}{4}$ LLBB, of A36 steel is subjected to a dead load of 12 kips and a live load of 36 kips. It is connected to a gusset plate with $\frac{3}{4}$-inch-diameter bolts through the long legs. Does this member have enough strength? Assume that $A_e = 0.85A_n$.

a. Use LRFD.

b. Use ASD.

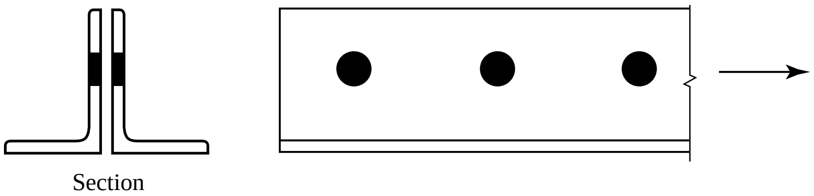


FIGURE P3.2-6

3.2-7 A C8 × 11.5 is connected to a gusset plate with $\frac{7}{8}$-inch-diameter bolts as shown in Figure P3.2-7. The steel is A572 Grade 50. If the member is subjected to dead load and live load only, what is the total service load capacity if the live-to-dead load ratio is 3? Assume that $A_e = 0.85A_n$.

a. Use LRFD.

b. Use ASD.

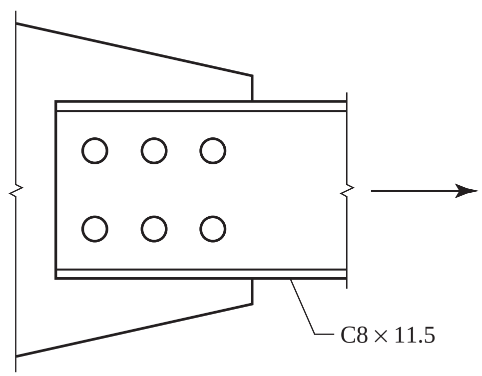


FIGURE P3.2-7

Effective area

3.3-1 Determine the effective area A_e for each case shown in Figure P3.3-1.

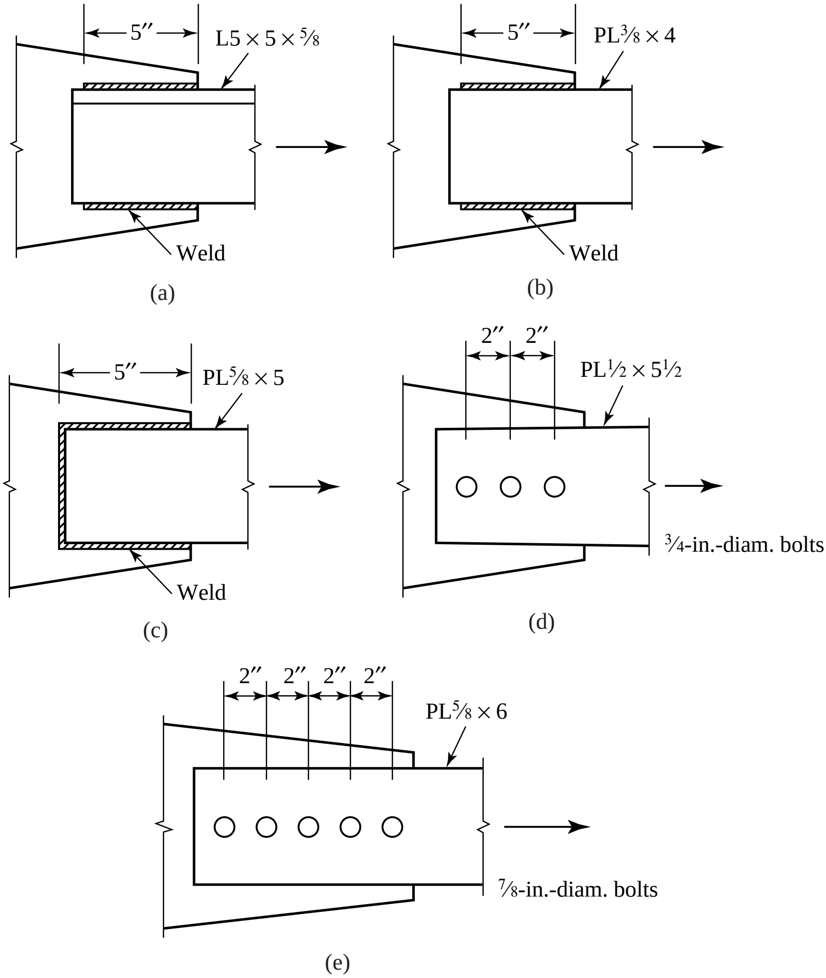


FIGURE P3.3-1

3.3-2 For the tension member shown, compute the following.

a. The tensile design strength.

b. The allowable tensile strength.

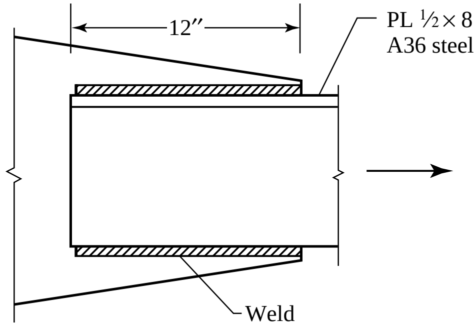


FIGURE P3.3-2

3.3-3 Determine the nominal tensile strength *based on the effective net area*.

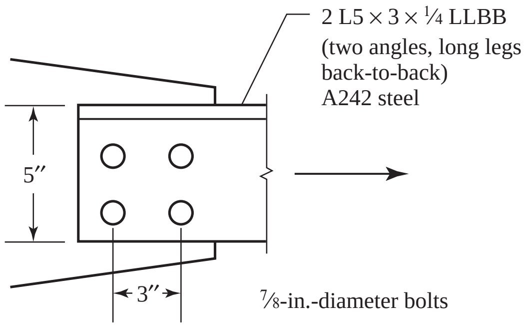


FIGURE P3.3-3

3.3-4 For the tension member shown, compute the following.

a. The tensile design strength.
b. The allowable tensile strength.

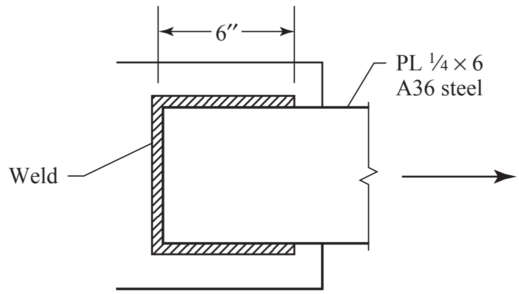


FIGURE P3.3-4

3.3-5 A W16 × 45 of A992 steel is connected to a plate at each flange as shown in Figure P3.3-5. Determine the nominal strength *based on the net section* as follows:

a. Use Equation 3.1 for the shear lag factor, U.
b. Use the alternative value of U from AISC Table D3.1.

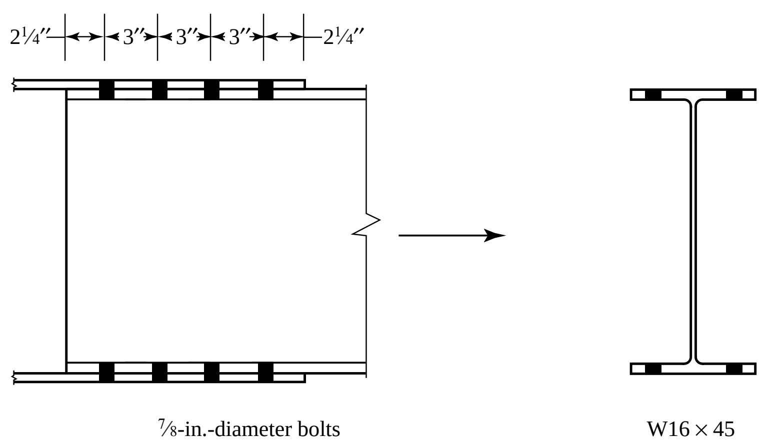


FIGURE P3.3-5

3.3-6 The tension member shown in Figure P3.3-6 is a C12 × 20.7 of A572 Grade 50 steel. Will it safely support a service dead load of 60 kips and a service live load of 125 kips? Use Equation 3.1 for U.

a. Use LRFD.

b. Use ASD.

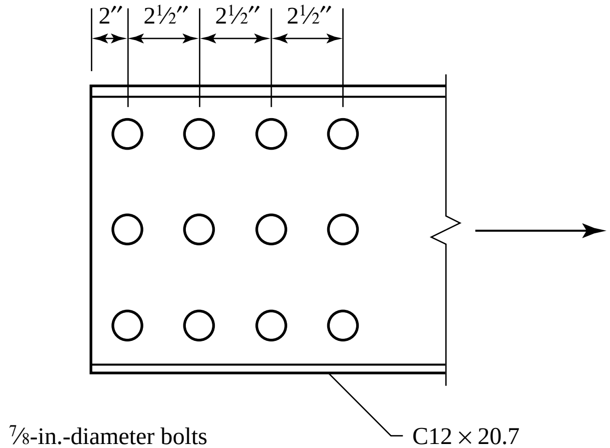


FIGURE P3.3-6

3.3-7 A double-angle tension member, 2L4 × 3 × ¼ LLBB, is connected with welds as shown in Figure P3.3-7. A36 steel is used.

a. Compute the available strength for LRFD.

b. Compute the available strength for ASD.

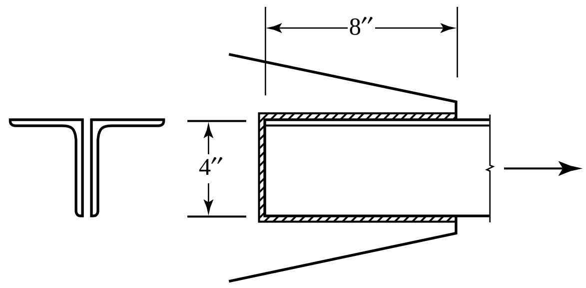


FIGURE P3.3-7

3.3-8 An L5 × 5 × ½ tension member of A242 steel is connected to a gusset plate with six ¾-inch-diameter bolts as shown in Figure P3.3-8. If the member is subject to dead load and live load only, what is the maximum total service load that can be applied if the ratio of live load to dead load is 2.0? Use the alternative value of U from AISC Table D3.1.

a. Use LRFD.

b. Use ASD.

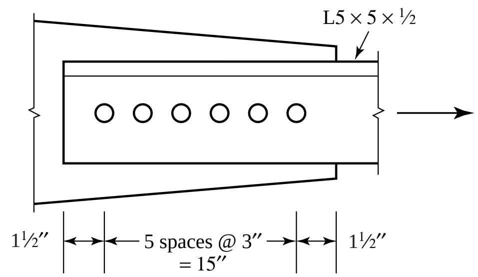


FIGURE P3.3-8

Staggered Fasteners

3.4-1 A36 steel is used for the tension member shown in Figure P3.4-1.

a. Determine the nominal strength based on the gross area.

b. Determine the nominal strength based on the net area.

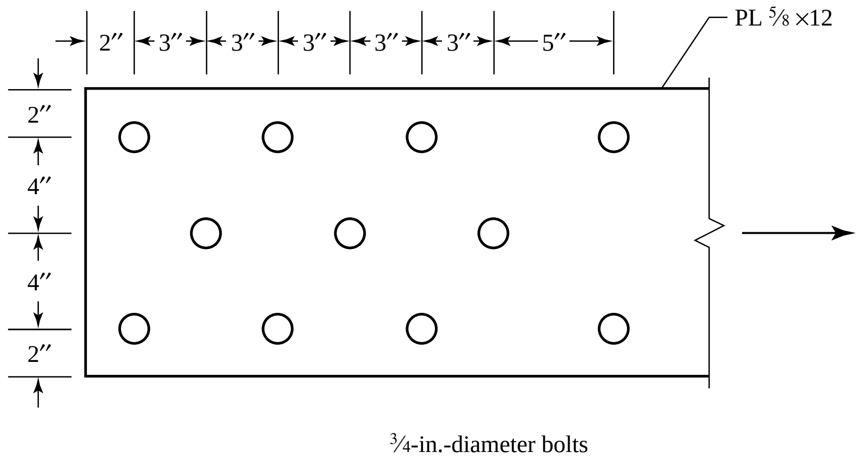


FIGURE P3.4-1

3.4-2 The tension member shown in Figure 3.4-2 is a PL ⅝ × 10, and the steel is A36. The bolts are ⅞-inch in diameter.

a. Determine the design strength for LRFD.

b. Determine the allowable strength for ASD.

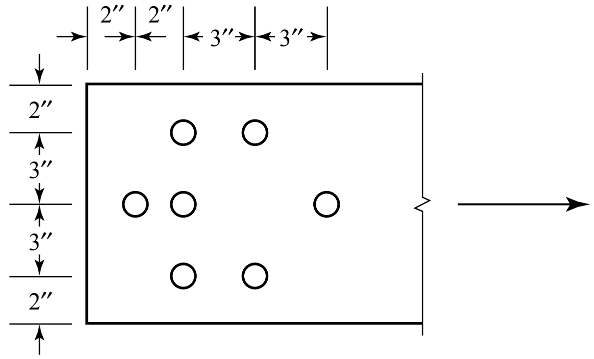


FIGURE P3.4-2

3.4-3 An MC 9 × 23.9 is connected with ¾-inch-diameter bolts as shown in Figure P3.4-3. A572 Grade 50 steel is used.

a. Determine the design strength.

b. Determine the allowable strength.

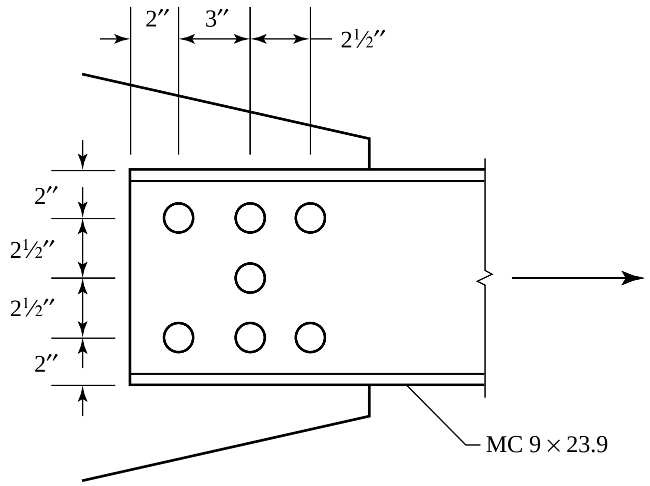


FIGURE P3.4-3

3.4-4 A992 steel is used for the tension member shown in Figure P3.4-4. The bolts are ¾ inch in diameter. The connection is to a ⅜-in.-thick gusset plate.

a. Determine the nominal strength based on the gross area.

b. Determine the nominal strength based on the effective net area.

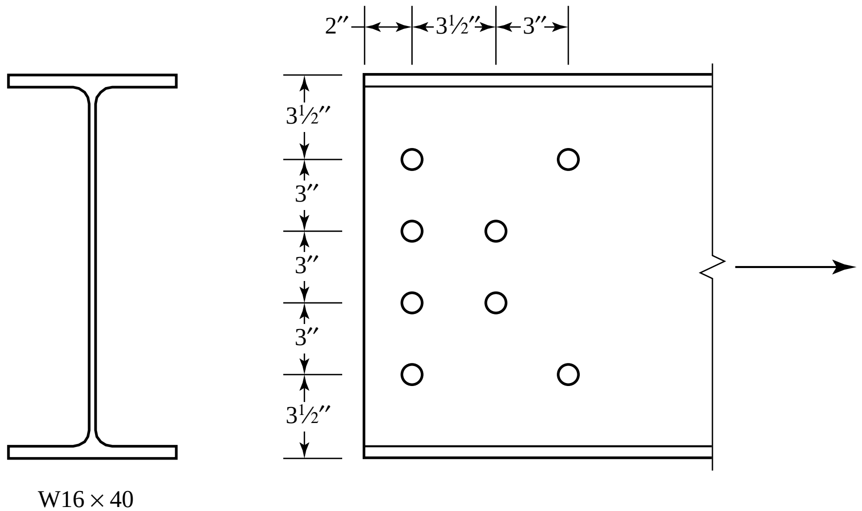


FIGURE P3.4-4

3.4-5 The tension member shown in Figure P3.4-5 is an L6 × 3½ × 5⁄16. The bolts are ¾ inch in diameter. If A36 steel is used, is the member adequate for a service dead load of 31 kips and a service live load of 31 kips?

a. Use LRFD.

b. Use ASD.

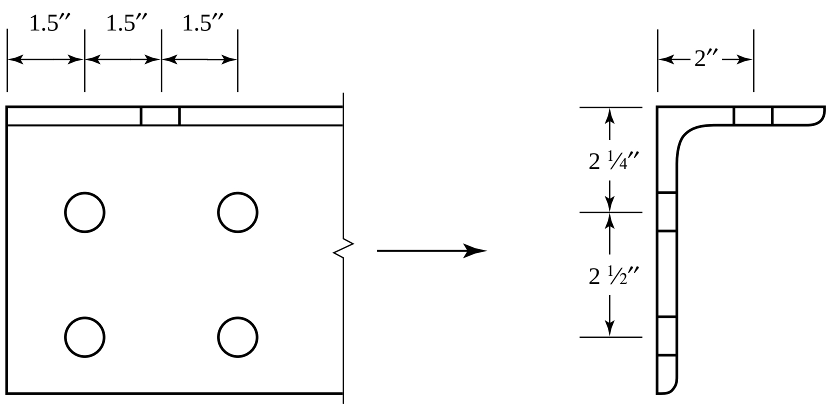


FIGURE P3.4-5

3.4-6 A double-channel shape, 2C10 × 20, of A572 Grade 50 steel is used for a built-up tension member as shown in Figure P3.4-6. The holes are for ½-inch-diameter bolts. Determine the total service load capacity if the live load is three times the dead load.

a. Use LRFD.

b. Use ASD.

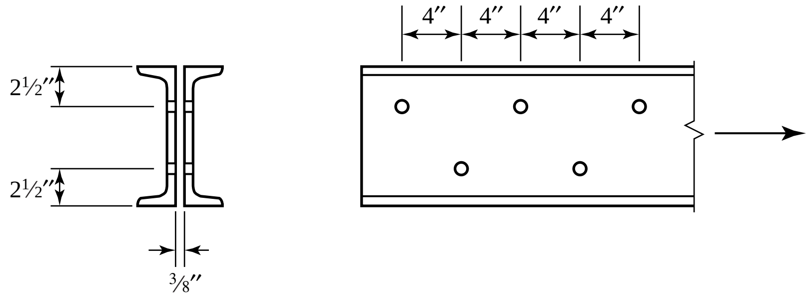


FIGURE P3.4-6

Block Shear

3.5-1 The tension member is a PL⅜ × 5½ of A242 steel. It is connected to a ⅜-in. thick gusset plate, also of A242 steel, with ¾-inch diameter bolts as shown in Figure P3.5-1. Determine the nominal block shear strength of the tension member.

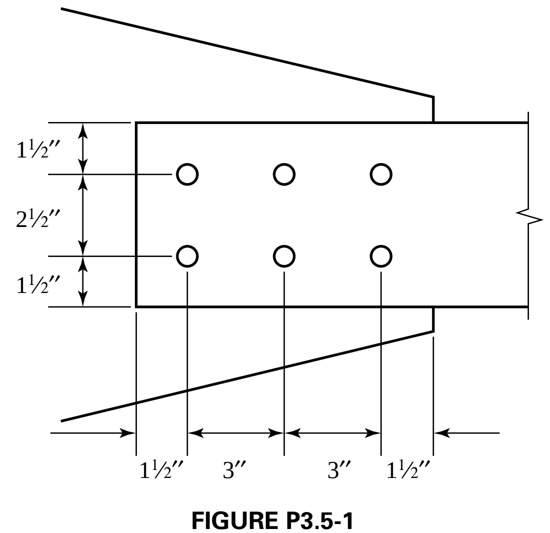


FIGURE P3.5-1

3.5-2 A square hollow structural section (HSS) is used as a tension member and is welded to a gusset plate of A36 steel as shown in Figure P3.5-2. Compute the nominal block shear strength of the gusset plate.

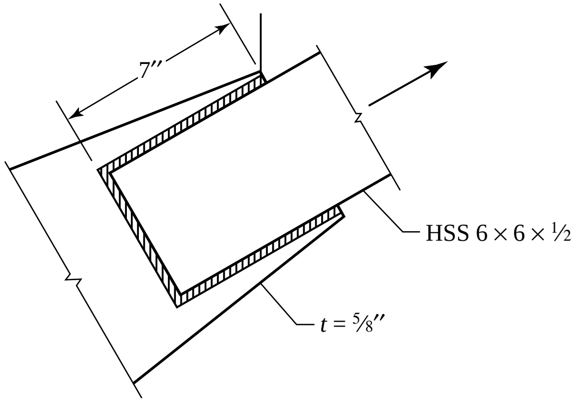


FIGURE P3.5-2

3.5-3 A WT8 × 13 of A992 steel is used as a tension member. The connection is with ⅞-in. diameter bolts as shown in Figure P3.5-3. Compute the nominal block shear strength.

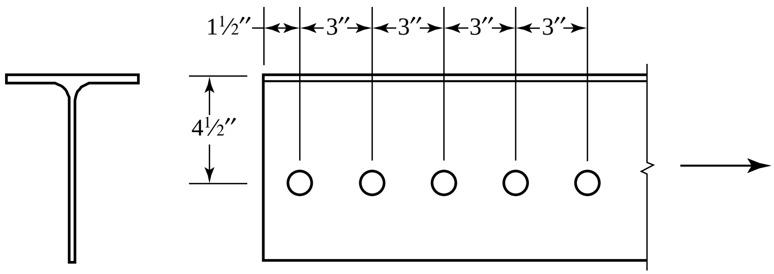


FIGURE P3.5-3

3.5-4 Compute the available block shear strength of the gusset plate.

a. Use LRFD.

b. Use ASD.

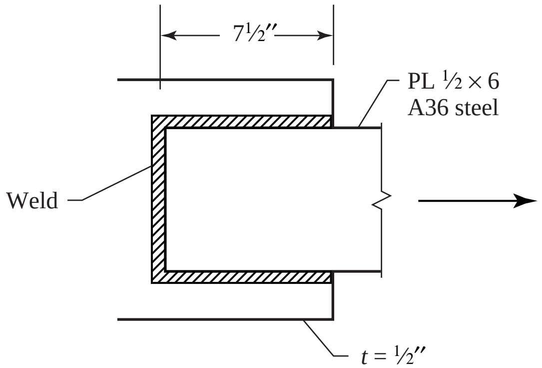


FIGURE P3.5-4

3.5-5 A C7 × 9.8 tension member is connected to a ⅜-in.-thick gusset plate as shown in Figure P3.5-5. Both the member and the gusset plate are A36 steel.

a. Compute the available block shear strength of the tension member for both LRFD and ASD.

b. Compute the available block shear strength of the gusset plate for both LRFD and ASD.

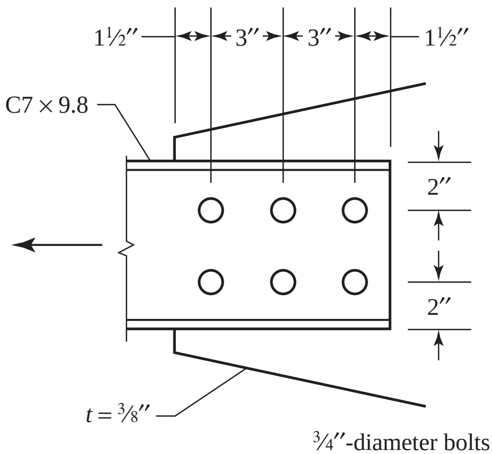


FIGURE P3.5-5

3.5-6 A double-channel shape, 2C8 × 18.75, is used as a tension member. The channels are bolted to a ⅜-inch gusset plate with ⅞-inch diameter bolts. The tension member is A572 Grade 50 steel and the gusset plate is A36. If LRFD is used, how much factored tensile load can be applied? Consider *all* limit states.

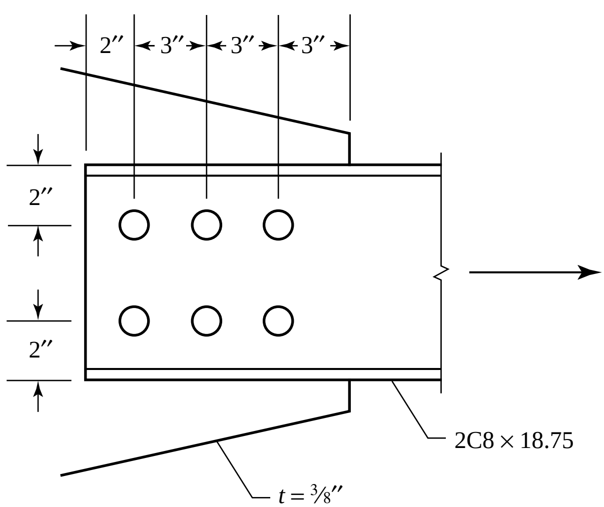


FIGURE P3.5-6

Design of Tension Members

3.6-1 Select a single-angle tension member of A36 steel to resist the following service loads: dead load = 50 kips, live load = 100 kips, and wind load = 45 kips. The member will be connected through one leg with 1-inch diameter bolts in two lines. There will be four bolts in each line. The member length is 20 feet.

a. Use LRFD.

b. Use ASD.

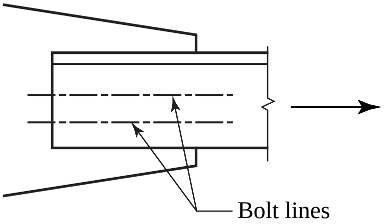


FIGURE P3.6-1

3.6-2 Use A36 steel and select a double-angle tension member to resist a service dead load of 20 kips and a service live load of 60 kips. Assume that the member will be connected to a ⅜-inch-thick gusset plate with a single line of five ⅞-inch diameter bolts. The member is 15 feet long.

a. Use LRFD.

b. Use ASD.

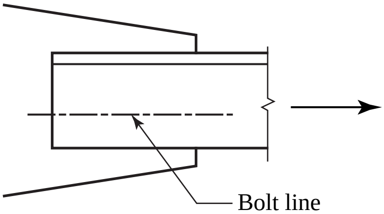


FIGURE P3.6-2

3.6-3 Select an ST shape to be used as a 20-ft-long tension member to resist the following service loads: dead load = 38 kips, live load = 115 kips, and snow load = 75 kips. The connection is through the flange with three ¾-inch diameter bolts in each line. Use A572 Grade 50 steel.

a. Use LRFD.

b. Use ASD.

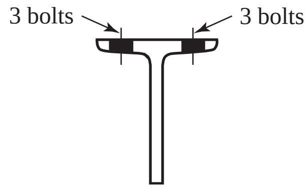


FIGURE P3.6-3

3.6-4 Select an S shape for the tension member shown in Figure P3.6-4. The member shown will be connected between two plates with eight ⅞-in. diameter bolts. The service dead load is 216 kips, the service live load is 25 kips, and the length is 22 ft. Use A36 steel.

a. Use LRFD.

b. Use ASD.

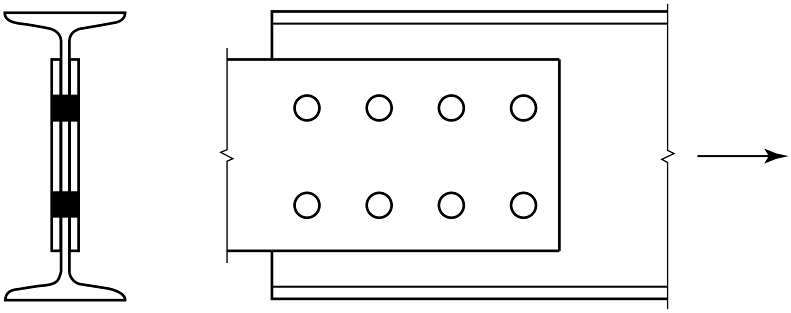


FIGURE P3.6-4

3.6-5 Choose a pipe to be used as a tension member to resist a service dead load of 10 kips and a service live load of 25 kips. The ends will be connected by welding completely around the circumference of the pipe. The length is 8 feet.

a. Use LRFD.

b. Use ASD.

3.6-6 Use LRFD and select an American Standard Channel shape for the following tensile loads: dead load = 54 kips, live load = 80 kips, and wind load = 75 kips. The connection will be with two 9-in.-long longitudinal welds. Use an estimated shear lag factor of $U = 0.85$. Once the member has been selected, compute the value of U with Equation 3.1 and revise the design if necessary. The length is 17.5 ft. Use $F_y = 50$ ksi and $F_u = 65$ ksi.

Threaded Rods and Cables

3.7-1 Select a threaded rod to resist a service dead load of 43 kips and a service live load of 4 kips. Use A36 steel.

a. Use LRFD.

b. Use ASD.

3.7-2 A W16 × 36 is supported by two tension rods *AB* and *CD*, as shown in Figure P3.7-2. The 30-kip load is a service live load. Use load and resistance factor design and select threaded rods of A36 steel for the following load cases.

a. The 30-kip load cannot move from the location shown.

b. The 30-kip load can be located anywhere between the two rods.

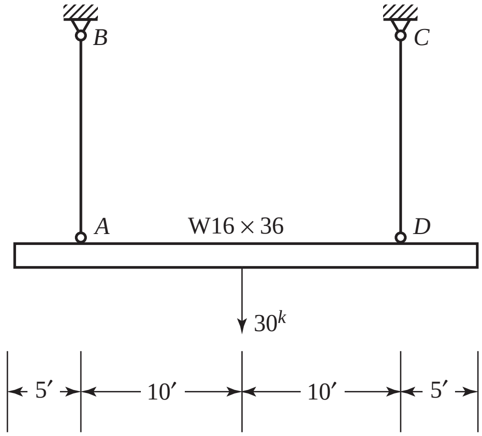


FIGURE P3.7-2

3.7-3 Same as problem 3.7-2, but use allowable *stress* design.

3.7-4 As shown in Figure P3.7-4, members *AC* and *BD* are used to brace the pin-connected structure against a horizontal wind load of 10 kips. Both of these members are assumed to be tension members and not resist any compression. For the load direction shown, member *AC* will resist the load in tension, and member *BD* will be unloaded. Select threaded rods of A36 steel for these members. Use load and resistance factor design.

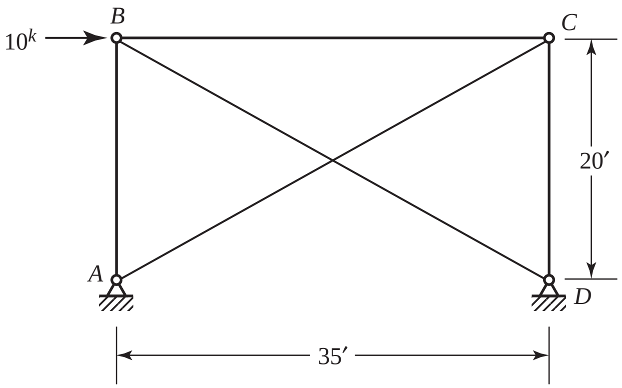


FIGURE P3.7-4

3.7-5 What size A36 threaded rod is required for member *AB*, as shown in Figure P3.7-5? The load is a service live load. (Neglect the weight of member *CB*.)

a. Use LRFD.

b. Use ASD.

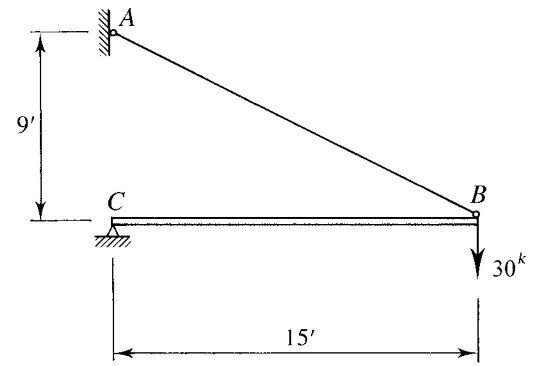


FIGURE P3.7-5

3.7-6 A pipe is supported at 12-foot intervals by a bent, threaded rod, as shown in Figure P3.7-6. If an 8-inch-diameter standard weight steel pipe full of water is used, what size A36 steel rod is required?

a. Use LRFD.

b. Use ASD.

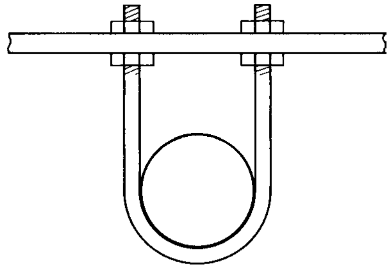


FIGURE P3.7-6

Tension Members in Roof Trusses

3.8-1 Use A992 steel and select a structural tee for the top chord of the welded roof truss shown in Figure P3.8-1. All connections are made with longitudinal plus transverse welds. Assume a connection length of 12 inches. The spacing of trusses in the roof system is 15 feet. Design for the following loads.

Snow: 20 psf of horizontal projection

Roofing: 12 psf

MC8 × 8.5 purlins

Truss weight: 1000 lb (estimated)

a. Use LRFD.
b. Use ASD.

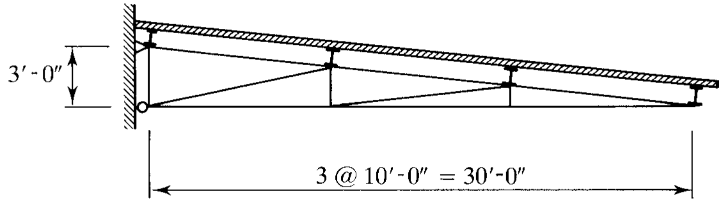


FIGURE P3.8-1

3.8-2 Use ASD and select single-angle shapes for the web tension members of the truss loaded as shown in Figure P3.8-2. The loads are service loads. All connections are with longitudinal welds. Use A36 steel and an estimated shear lag factor, U, of 0.85.

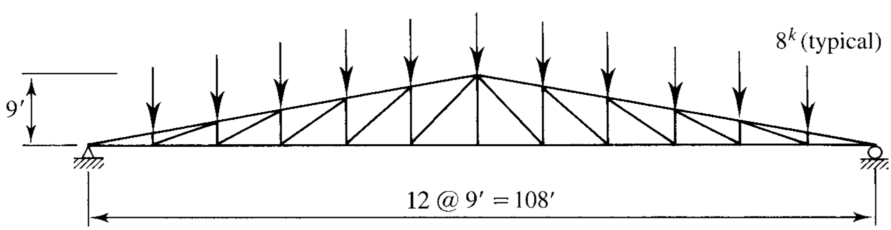


FIGURE P3.8-2

3.8-3 Compute the *factored* joint loads for the truss of Problem 3.8-2 for the following conditions.

Trusses spaced at 18 feet

Weight of roofing = 8 psf

Snow load = 20 psf of horizontal projection

W10 × 33 purlins located only at the joints

Total estimated truss weight = 5000 lb

3.8-4 Use LRFD and design the tension members of the roof truss shown in Figure P3.8-4. Use double-angle shapes throughout and assume ⅜-inch-thick gusset plates and welded connections. Assume a shear lag factor of $U = 0.80$. The trusses are spaced at 30 feet. Use A36 steel and design for the following loads.

Metal deck:	4 psf of roof surface
Built-up roof:	12 psf of roof surface
Purlins:	3 psf of roof surface (estimated)
Snow:	20 psf of horizontal projection
Truss weight:	5 psf of horizontal projection (estimated)

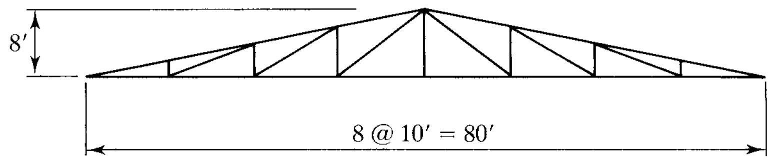


FIGURE P3.8-4

3.8-5 Use A36 steel and design sag rods for the truss of Problem 3.8-4. Assume that, once attached, the metal deck will provide lateral support for the purlins; therefore, the sag rods need to be designed for the purlin weight only.

a. Use LRFD.

b. Use ASD.

CHAPTER 4

Compression Members

4.1 INTRODUCTION

Compression members are structural elements that are subjected only to axial compressive forces; that is, the loads are applied along a longitudinal axis through the centroid of the member cross section, and the stress can be taken as $f = P/A$, where f is considered to be uniform over the entire cross section. This ideal state is never achieved in reality, however, because some eccentricity of the load is inevitable. Bending will result, but it usually can be regarded as secondary. As we shall see, the AISC Specification equations for compression member strength account for this accidental eccentricity.

The most common type of compression member occurring in buildings and bridges is the *column*, a vertical member whose primary function is to support vertical loads. In many instances, these members are also subjected to bending, and in these cases, the member is a *beam–column*. We cover this topic in Chapter 6. Compression members are also used in trusses and as components of bracing systems. Smaller compression members not classified as columns are sometimes referred to as *struts*.

In many small structures, column axial forces can be easily computed from the reactions of the beams that they support or computed directly from floor or roof loads. This is possible if the member connections do not transfer moment; in other words, if the column is not part of a rigid frame. For columns in rigid frames, there are calculable bending moments as well as axial forces, and a frame analysis is necessary. The AISC Specification provides for three methods of analysis to obtain the axial forces and bending moments in members of a rigid frame:

1. Direct analysis method
2. Effective length method
3. First-order analysis method

Except in very simple cases, computer software is used for the analysis. While the details of these three methods are beyond the scope of the present chapter, more will be said about them in Chapter 6 “Beam–Columns”. It is important to recognize, however, that these three methods are used to determine the *required* strengths of the members (axial loads and bending moments). The *available* strengths are computed by the methods of this chapter “Compression Members”, Chapter 5 “Beams”, and Chapter 6 “Beam–Columns”.

4.2 COLUMN THEORY

Consider the long, slender compression member shown in Figure 4.1a. If the axial load P is slowly applied, it will ultimately become large enough to cause the member to become unstable and assume the shape indicated by the dashed line. The member is said to have buckled, and the corresponding load is called the *critical buckling load*. If the member is stockier, as shown in Figure 4.1b, a larger load will be required to bring the member to the point of instability. For extremely stocky members, failure may occur by compressive yielding rather than buckling. Prior to failure, the compressive stress P/A will be uniform over the cross section at any point along the length, whether the failure is by yielding or by buckling. The load at which buckling occurs is a function of slenderness, and for very slender members this load could be quite small.

If the member is so slender (we give a precise definition of slenderness shortly) that the stress just before buckling is below the proportional limit—that is, the member is still elastic—the critical buckling load is given by

$$P_{cr} = \frac{\pi^2 EI}{L^2} \tag{4.1}$$

where E is the modulus of elasticity of the material, I is the moment of inertia of the cross-sectional area with respect to the minor principal axis, and L is the length of the member between points of support. For Equation 4.1 to be valid, the member must be elastic, and its ends must be free to rotate but not translate laterally. This end condition is satisfied by hinges or pins, as shown in Figure 4.2. This remarkable

FIGURE 4.1

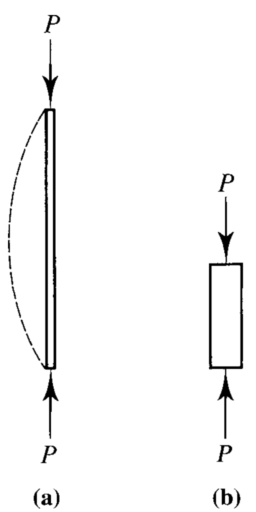
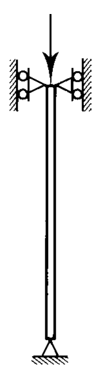


FIGURE 4.2



relationship was first formulated by Swiss mathematician Leonhard Euler and published in 1759. The critical load is sometimes referred to as the *Euler load* or the *Euler buckling load*. The validity of Equation 4.1 has been demonstrated convincingly by numerous tests. Its derivation is given here to illustrate the importance of the end conditions.

For convenience, in the following derivation, the member will be oriented with its longitudinal axis along the x-axis of the coordinate system given in Figure 4.3. The roller support is to be interpreted as restraining the member from translating either up or down. An axial compressive load is applied and gradually increased. If a temporary transverse load is applied so as to deflect the member into the shape indicated by the dashed line, the member will return to its original position when this temporary load is removed if the axial load is less than the critical buckling load. The critical buckling load, P_{cr}, is defined as the load that is just large enough to maintain the deflected shape when the temporary transverse load is removed.

The differential equation giving the deflected shape of an elastic member subjected to bending is

$$\frac{d^2y}{dx^2} = -\frac{M}{EI} \tag{4.2}$$

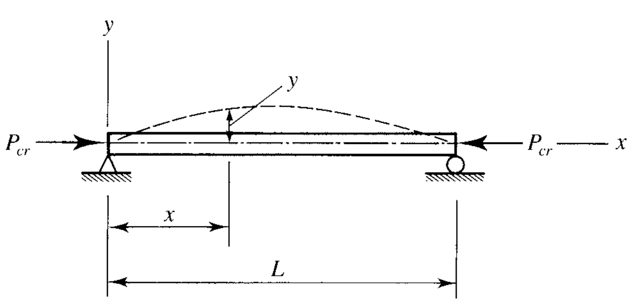
where x locates a point along the longitudinal axis of the member, y is the deflection of the axis at that point, and M is the bending moment at the point. E and I were previously defined, and here the moment of inertia I is with respect to the axis of bending (buckling). This equation was derived by Jacob Bernoulli and independently by Euler, who specialized it for the column buckling problem (Timoshenko, 1953). If we begin at the point of buckling, then from Figure 4.3 the bending moment is $P_{cr}y$. Equation 4.2 can then be written as

$$y'' + \frac{P_{cr}}{EI}y = 0$$

where the prime denotes differentiation with respect to x. This is a second-order, linear, ordinary differential equation with constant coefficients and has the solution

$$y = A\cos(cx) + B\sin(cx)$$

FIGURE 4.3



where

$$c = \sqrt{\frac{P_{cr}}{EI}}$$

and A and B are constants. These constants are evaluated by applying the following boundary conditions:

At $x = 0$, $y = 0$: $\quad 0 = A\cos(0) + B\sin(0) \qquad A = 0$

At $x = L$, $y = 0$: $\quad 0 = B\sin(cL)$

This last condition requires that $\sin(cL)$ be zero if B is not to be zero (the trivial solution, corresponding to $P = 0$). For $\sin(cL) = 0$,

$$cL = 0, \pi, 2\pi, 3\pi, \ldots = n\pi, \qquad n = 0, 1, 2, 3, \ldots$$

From

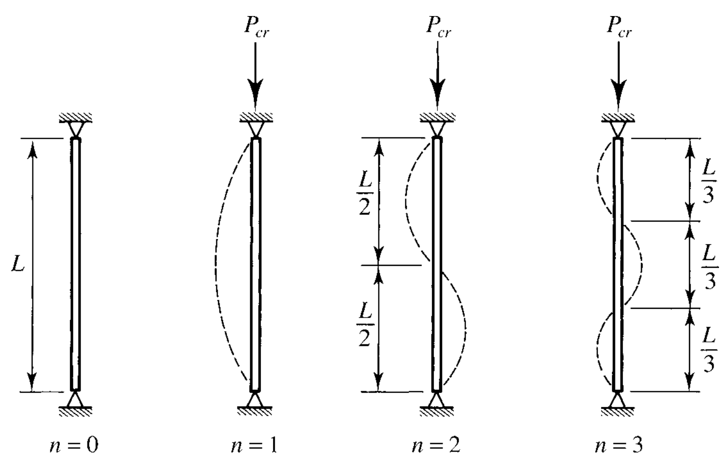
$$c = \sqrt{\frac{P_{cr}}{EI}}$$

we obtain

$$cL = \left(\sqrt{\frac{P_{cr}}{EI}}\right)L = n\pi, \quad \frac{P_{cr}}{EI}L^2 = n^2\pi^2 \quad \text{and} \quad P_{cr} = \frac{n^2\pi^2 EI}{L^2}$$

The various values of n correspond to different buckling modes; $n = 1$ represents the first mode, $n = 2$ the second, and so on. A value of zero gives the trivial case of no load. These buckling modes are illustrated in Figure 4.4. Values of n larger than 1 are not possible unless the compression member is physically restrained from deflecting at the points where the reversal of curvature would occur.

FIGURE 4.4



The solution to the differential equation is therefore

$$y = B \sin\left(\frac{n\pi x}{L}\right)$$

and the coefficient B is indeterminate. This result is a consequence of approximations made in formulating the differential equation; a linear representation of a nonlinear phenomenon was used.

For the usual case of a compression member with no supports between its ends, $n = 1$ and the Euler equation is written as

$$P_{cr} = \frac{\pi^2 EI}{L^2} \tag{4.3}$$

It is convenient to rewrite Equation 4.3 as

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 EAr^2}{L^2} = \frac{\pi^2 EA}{(L/r)^2}$$

where A is the cross-sectional area and r is the radius of gyration with respect to the axis of buckling. The ratio L/r is the slenderness ratio and is the measure of a member's slenderness, with large values corresponding to slender members.

If the critical load is divided by the cross-sectional area, the critical buckling stress is obtained:

$$F_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E}{(L/r)^2} \tag{4.4}$$

At this compressive stress, buckling will occur about the axis corresponding to r. Buckling will take place as soon as the load reaches the value given by Equation 4.3, and the column will become unstable about the principal axis corresponding to the largest slenderness ratio. This axis usually is the axis with the smaller moment of inertia (we examine exceptions to this condition later). Thus the minimum moment of inertia and radius of gyration of the cross section should ordinarily be used in Equations 4.3 and 4.4.

EXAMPLE 4.1

A W12 × 50 is used as a column to support an axial compressive load of 145 kips. The length is 20 feet, and the ends are pinned. Without regard to load or resistance factors, investigate this member for stability. (The grade of steel need not be known: The critical buckling load is a function of the modulus of elasticity, not the yield stress or ultimate tensile strength.)

SOLUTION For a W12 × 50,

$$\text{Minimum } r = r_y = 1.96 \text{ in.}$$

$$\text{Maximum } \frac{L}{r} = \frac{20(12)}{1.96} = 122.4$$

$$P_{cr} = \frac{\pi^2 EA}{(L/r)^2} = \frac{\pi^2(29{,}000)(14.6)}{(122.4)^2} = 278.9 \text{ kips}$$

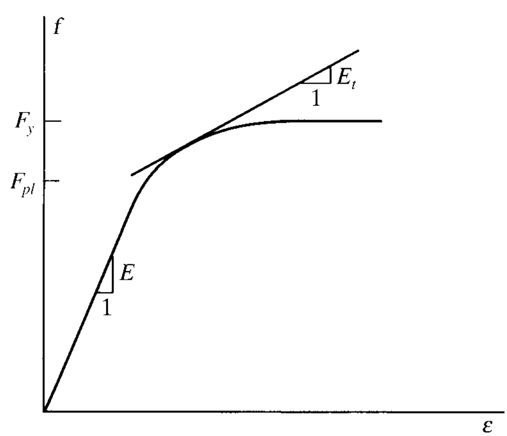
ANSWER Because the applied load of 145 kips is less than P_{cr}, the column remains stable and has an overall factor of safety against buckling of $278.9/145 = 1.92$.

Early researchers soon found that Euler's equation did not give reliable results for stocky, or less slender, compression members. The reason is that the small slenderness ratio for members of this type causes a large buckling stress (from Equation 4.4). If the stress at which buckling occurs is greater than the proportional limit of the material, the relation between stress and strain is not linear, and the modulus of elasticity E can no longer be used. (In Example 4.1, the stress at buckling is $P_{cr}/A = 278.9/14.6 = 19.10$ ksi, which is well below the proportional limit for any grade of structural steel.) This difficulty was initially resolved by Friedrich Engesser, who proposed in 1889 the use of a variable tangent modulus, E_t, in Equation 4.3. For a material with a stress–strain curve like the one shown in Figure 4.5, E is not a constant for stresses greater than the proportional limit F_{pl}. The tangent modulus E_t is defined as the slope of the tangent to the stress–strain curve for values of f between F_{pl} and F_y. If the compressive stress at buckling, P_{cr}/A, is in this region, it can be shown that

$$P_{cr} = \frac{\pi^2 E_t I}{L^2} \tag{4.5}$$

Equation 4.5 is identical to the Euler equation, except that E_t is substituted for E.

FIGURE 4.5



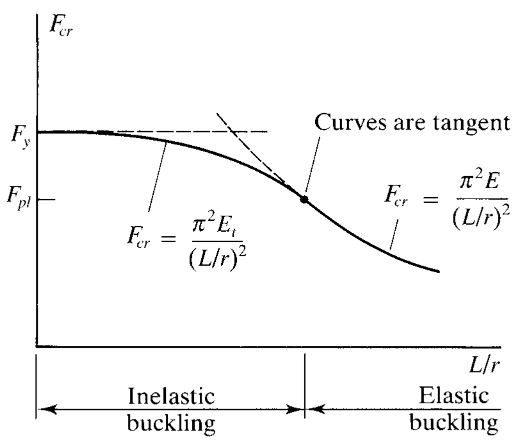
The stress–strain curve shown in Figure 4.5 is different from those shown earlier for ductile steel (in Figures 1.3 and 1.4) because it has a pronounced region of nonlinearity. This curve is typical of a compression test of a short length of W-shape called a *stub column,* rather than the result of a tensile test. The nonlinearity is primarily because of the presence of residual stresses in the W-shape. When a hot-rolled shape cools after rolling, all elements of the cross section do not cool at the same rate. The tips of the flanges, for example, cool faster than the junction of the flange and the web. This uneven cooling induces stresses that remain permanently. Other factors, such as welding and cold-bending to create curvature in a beam, can contribute to the residual stress, but the cooling process is its chief source.

Note that E_t is smaller than E and for the same L/r corresponds to a smaller critical load, P_{cr}. Because of the variability of E_t, computation of P_{cr} in the inelastic range by the use of Equation 4.5 is difficult. In general, a trial-and-error approach must be used, and a compressive stress–strain curve such as the one shown in Figure 4.5 must be used to determine E_t for trial values of P_{cr}. For this reason, most design specifications, including the AISC Specification, contain empirical formulas for inelastic columns.

Engesser's tangent modulus theory had its detractors, who pointed out several inconsistencies. Engesser was convinced by their arguments, and in 1895 he refined his theory to incorporate a reduced modulus, which has a value between E and E_t. Test results, however, always agreed more closely with the tangent modulus theory. Shanley (1947) resolved the apparent inconsistencies in the original theory, and today the tangent modulus formula, Equation 4.5, is accepted as the correct one for inelastic buckling. Although the load predicted by this equation is actually a lower bound on the true value of the critical load, the difference is slight (Bleich, 1952).

For any material, the critical buckling stress can be plotted as a function of slenderness, as shown in Figure 4.6. The tangent modulus curve is tangent to the Euler curve at the point corresponding to the proportional limit of the material. The composite curve, called a *column strength curve,* completely describes the strength of any column of a given material. Other than F_y, E, and E_t, which are properties of the material, the strength is a function only of the slenderness ratio.

FIGURE 4.6



Effective Length

Both the Euler and tangent modulus equations are based on the following assumptions:

1. The column is perfectly straight, with no initial crookedness.
2. The load is axial, with no eccentricity.
3. The column is pinned at both ends.

The first two conditions mean that there is no bending moment in the member before buckling. As mentioned previously, some accidental moment will be present, but in most cases it can be ignored. The requirement for pinned ends, however, is a serious limitation, and provisions must be made for other support conditions. The pinned-end condition requires that the member be restrained from lateral translation, but not rotation, at the ends. Constructing a frictionless pin connection is virtually impossible, so even this support condition can only be closely approximated at best. Obviously, all columns must be free to deform axially.

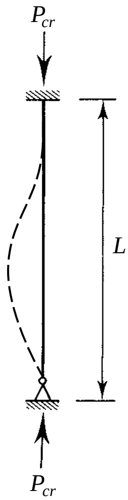
Other end conditions can be accounted for in the derivation of Equation 4.3. In general, the bending moment will be a function of x, resulting in a nonhomogeneous differential equation. The boundary conditions will be different from those in the original derivation, but the overall procedure will be the same. The form of the resulting equation for P_{cr} will also be the same. For example, consider a compression member pinned at one end and fixed against rotation and translation at the other, as shown in Figure 4.7. The Euler equation for this case, derived in the same manner as Equation 4.3, is

$$P_{cr} = \frac{2.05\pi^2 EI}{L^2}$$

or

$$P_{cr} = \frac{2.05\pi^2 EA}{(L/r)^2} = \frac{\pi^2 EA}{(0.70L/r)^2}$$

FIGURE 4.7



Thus this compression member has the same load capacity as a column that is pinned at both ends and is only 70% as long as the given column. Similar expressions can be found for columns with other end conditions.

The column buckling problem can also be formulated in terms of a fourth-order differential equation instead of Equation 4.2. This proves to be convenient when dealing with boundary conditions other than pinned ends.

For convenience, the equations for critical buckling load will be written as

$$P_{cr} = \frac{\pi^2 EA}{(KL/r)^2} \quad \text{or} \quad P_{cr} = \frac{\pi^2 E_t A}{(KL/r)^2} \qquad (4.6a/4.6b)$$

where KL is the *effective length*, and K is the *effective length factor*. The effective length factor for the fixed-pinned compression member is 0.70. For the most favorable condition of both ends fixed against rotation and translation, $K = 0.5$. Values of K for these and other cases can be determined with the aid of Table C-A-7.1 in the Commentary to AISC Specification Appendix 7. The three conditions mentioned thus far are included, as well as some for which end translation is possible. Two values of K are given: a theoretical value and a recommended design value to be used when the ideal end condition is approximated. Hence, unless a “fixed” end is perfectly fixed, the more conservative design values are to be used. Only under the most extraordinary circumstances would the use of the theoretical values be justified. Note, however, that the theoretical and recommended design values are the same for conditions (d) and (f) in Commentary Table C-A-7.1. The reason is that any deviation from a perfectly frictionless hinge or pin introduces rotational restraint and tends to reduce K. Therefore, use of the theoretical values in these two cases is conservative.

The use of the effective length KL in place of the actual length L in no way alters any of the relationships discussed so far. The column strength curve shown in Figure 4.6 is unchanged except for renaming the abscissa KL/r. The critical buckling stress corresponding to a given length, actual or effective, remains the same.

4.3 AISC REQUIREMENTS

The basic requirements for compression members are covered in Chapter E of the AISC Specification. The nominal compressive strength is

$$P_n = F_{cr} A_g \qquad \text{(AISC Equation E3-1)}$$

For LRFD,

$$P_u \leq \phi_c P_n$$

where

P_u = sum of the factored loads

ϕ_c = resistance factor for compression = 0.90

$\phi_c P_n$ = design compressive strength

For ASD,

$$P_a \le \frac{P_n}{\Omega_c}$$

where

P_a = sum of the service loads
Ω_c = safety factor for compression = 1.67
P_n/Ω_c = allowable compressive strength

If an allowable stress formulation is used,

$$f_a \le F_a$$

where

f_a = computed axial compressive stress = P_a/A_g
F_a = allowable axial compressive stress

$$= \frac{F_{cr}}{\Omega_c} = \frac{F_{cr}}{1.67} = 0.6F_{cr} \tag{4.7}$$

In order to present the AISC expressions for the critical stress F_{cr}, we first define the Euler load as

$$P_e = \frac{\pi^2 EA}{(KL/r)^2}$$

This is the critical buckling load according to the Euler equation. The Euler stress is

$$F_e = \frac{P_e}{A} = \frac{\pi^2 E}{(KL/r)^2} \qquad \text{(AISC Equation E3-4)}$$

With a slight modification, this expression will be used for the critical stress in the elastic range. To obtain the critical stress for elastic columns, the Euler stress is reduced as follows to account for the effects of initial crookedness:

$$F_{cr} = 0.877F_e \tag{4.8}$$

For inelastic columns, the tangent modulus equation, Equation 4.6b, is replaced by the exponential equation

$$F_{cr} = \left(0.658^{\frac{F_y}{F_e}}\right)F_y \tag{4.9}$$

With Equation 4.9, a direct solution for inelastic columns can be obtained, avoiding the trial-and-error approach inherent in the use of the tangent modulus equation. At the boundary between inelastic and elastic columns, Equations 4.8 and 4.9 give the same value of F_{cr}. This occurs when KL/r is approximately

$$4.71\sqrt{\frac{E}{F_y}}$$

To summarize,

$$\text{When } \frac{KL}{r} \le 4.71\sqrt{\frac{E}{F_y}}, \quad F_{cr} = (0.658^{F_y/F_e})F_y \tag{4.10}$$

$$\text{When } \frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}}, \quad F_{cr} = 0.877F_e \tag{4.11}$$

The AISC Specification provides for separating inelastic and elastic behavior based on either the value of KL/r (as in equations 4.10 and 4.11) or the value of the ratio F_y/F_e. The limiting value of F_y/F_e can be derived as follows. From AISC Equation E3-4,

$$\frac{KL}{r} = \sqrt{\frac{\pi^2 E}{F_e}}$$

For $\quad \dfrac{KL}{r} \le 4.71\sqrt{\dfrac{E}{F_y}},$

$$\sqrt{\frac{\pi^2 E}{F_e}} \le 4.71\sqrt{\frac{E}{F_y}}$$

$$\frac{F_y}{F_e} \le 2.25$$

The complete AISC Specification for compressive strength is as follows:

When $\dfrac{KL}{r} \le 4.71\sqrt{\dfrac{E}{F_y}}$ or $\dfrac{F_y}{F_e} \le 2.25$,

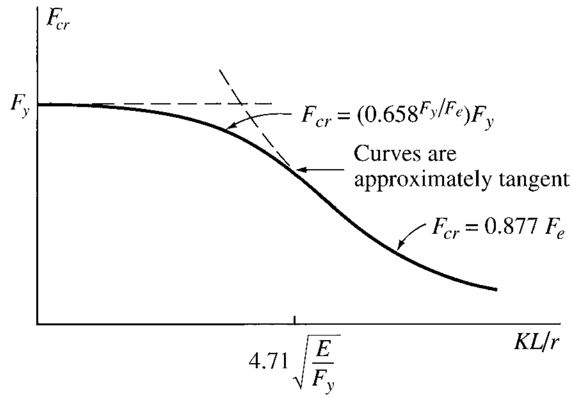
$$F_{cr} = (0.658^{F_y/F_e})F_y \qquad \text{(AISC Equation E3-2)}$$

When $\dfrac{KL}{r} > 4.71\sqrt{\dfrac{E}{F_y}}$ or $\dfrac{F_y}{F_e} > 2.25$,

$$F_{cr} = 0.877F_e \qquad \text{(AISC Equation E3-3)}$$

In this book, we will usually use the limit on KL/r, as expressed in Equations 4.10 and 4.11. These requirements are represented graphically in Figure 4.8.

FIGURE 4.8



AISC Equations E3-2 and E3-3 are a condensed version of five equations that cover five ranges of KL/r (Galambos, 1988). These equations are based on experimental and theoretical studies that account for the effects of residual stresses and an initial out-of-straightness of $L/1500$, where L is the member length. A complete derivation of these equations is given by Tide (2001).

Although AISC does not require an upper limit on the slenderness ratio KL/r, an upper limit of 200 is recommended (see user note in AISC E2). This is a practical upper limit, because compression members that are any more slender will have little strength and will not be economical.

EXAMPLE 4.2

A W14 × 74 of A992 steel has a length of 20 feet and pinned ends. Compute the design compressive strength for LRFD and the allowable compressive strength for ASD.

SOLUTION Slenderness ratio:

$$\text{Maximum } \frac{KL}{r} = \frac{KL}{r_y} = \frac{1.0(20 \times 12)}{2.48} = 96.77 < 200 \quad \text{(OK)}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29,000}{50}} = 113$$

Since 96.77 < 113, use AISC Equation E3-2.

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29,000)}{(96.77)^2} = 30.56 \text{ ksi}$$

$$F_{cr} = 0.658^{(F_y/F_e)} F_y = 0.658^{(50/30.56)}(50) = 25.21 \text{ ksi}$$

The nominal strength is

$$P_n = F_{cr}A_g = 25.21(21.8) = 549.6 \text{ kips}$$

LRFD SOLUTION The design strength is

$$\phi_c P_n = 0.90(549.6) = 495 \text{ kips}$$

ASD SOLUTION From Equation 4.7, the allowable stress is

$$F_a = 0.6F_{cr} = 0.6(25.21) = 15.13 \text{ ksi}$$

The allowable strength is

$$F_a A_g = 15.13(21.8) = 330 \text{ kips}$$

ANSWER Design compressive strength = 495 kips. Allowable compressive strength = 330 kips.

In Example 4.2, $r_y < r_x$, and there is excess strength in the x-direction. Square structural tubes (HSS) are efficient shapes for compression members because $r_y = r_x$ and the strength is the same for both axes. Hollow circular shapes are sometimes used as compression members for the same reason.

The mode of failure considered so far is referred to as *flexural* buckling, as the member is subjected to flexure, or bending, when it becomes unstable. For some cross-sectional configurations, the member will fail by twisting (torsional buckling) or by a combination of twisting and bending (flexural-torsional buckling). We consider these infrequent cases in Section 4.8.

4.4 LOCAL STABILITY

The strength corresponding to any *overall* buckling mode, however, such as flexural buckling, cannot be developed if the elements of the cross section are so thin that *local* buckling occurs. This type of instability is a localized buckling or wrinkling at an isolated location. If it occurs, the cross section is no longer fully effective, and the member has failed. I-shaped cross sections with thin flanges or webs are susceptible to this phenomenon, and their use should be avoided whenever possible. Otherwise, the compressive strength given by AISC Equations E3-2 and E3-3 must be reduced. The measure of this susceptibility is the width-to-thickness ratio of each cross-sectional element. Two types of elements must be considered: unstiffened elements, which are unsupported along one edge parallel to the direction of load, and stiffened elements, which are supported along both edges.

Limiting values of width-to-thickness ratios are given in AISC B4.1, “Classification of Sections for Local Buckling.” For compression members, shapes are classified as *slender* or *nonslender*. If a shape is slender, its strength limit state is local buckling, and the corresponding reduced strength must be computed. The width-to-thickness ratio is given the generic symbol λ. Depending on the particular cross-sectional element, λ for I shapes is either the ratio b/t or h/t_w, both of which are defined presently. If λ is greater than the specified limit (denoted λ_r), the shape is slender.

AISC Table B4.1a shows the upper limit, λ_r, for nonslender members of various cross-sectional shapes. If $\lambda \leq \lambda_r$, the shape is nonslender. Otherwise, the shape is slender. The table is divided into two parts: unstiffened elements and stiffened elements. (For beams, a shape can be *compact*, *noncompact*, or *slender*, and the limiting values of λ are given in AISC Table B4.1b. We cover beams in Chapter 5.) For I shapes, the projecting flange is considered to be an unstiffened element, and its width can be taken as half of the full nominal width. Using AISC notation gives

$$\lambda = \frac{b}{t} = \frac{b_f/2}{t_f} = \frac{b_f}{2t_f}$$

where b_f and t_f are the width and thickness of the flange. The upper limit is

$$\lambda_r = 0.56\sqrt{\frac{E}{F_y}}$$

The webs of I shapes are stiffened elements, and the stiffened width is the distance between the roots of the flanges. The width-to-thickness parameter is

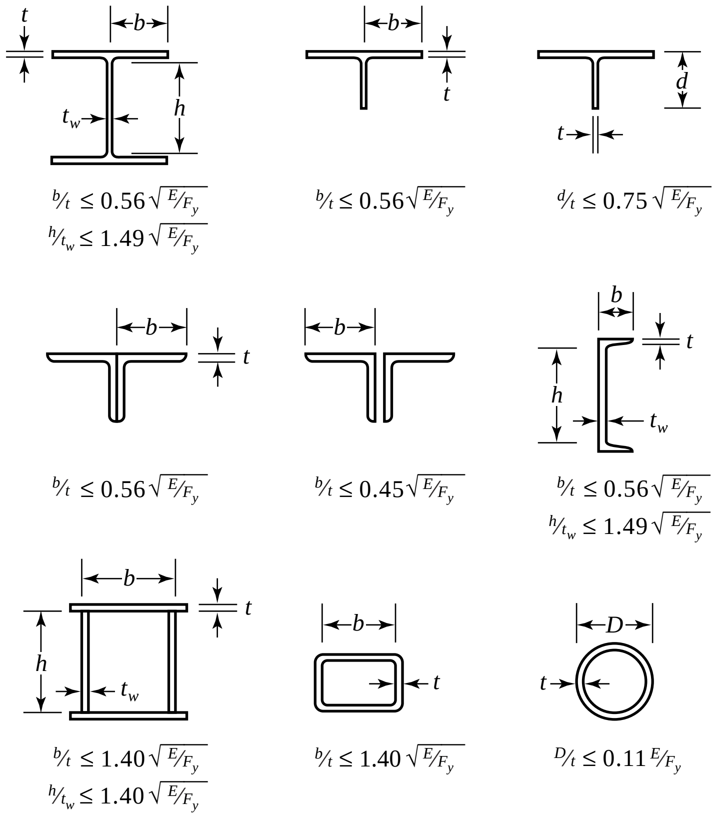
$$\lambda = \frac{h}{t_w}$$

where h is the distance between the roots of the flanges, and t_w is the web thickness. The upper limit is

$$\lambda_r = 1.49\sqrt{\frac{E}{F_y}}$$

Stiffened and unstiffened elements of various cross-sectional shapes are illustrated in Figure 4.9. The appropriate compression member limit, λ_r, from AISC B4.1 is given for each case.

FIGURE 4.9



EXAMPLE 4.3

Investigate the column of Example 4.2 for local stability.

SOLUTION

For a W14 × 74, $b_f = 10.1$ in., $t_f = 0.785$ in., and

$$\frac{b_f}{2t_f} = \frac{10.1}{2(0.785)} = 6.43$$

$$0.56\sqrt{\frac{E}{F_y}} = 0.56\sqrt{\frac{29,000}{50}} = 13.5 > 6.43 \qquad \text{(OK)}$$

$$\frac{h}{t_w} = \frac{d - 2k_{des}}{t_w} = \frac{14.2 - 2(1.38)}{0.450} = 25.4$$

where k_{des} is the *design* value of k. (Different manufacturers will produce this shape with different values of k. The *design* value is the smallest of these values. The *detailing* value is the largest.)

$$1.49\sqrt{\frac{E}{F_y}} = 1.49\sqrt{\frac{29{,}000}{50}} = 35.9 > 25.4 \qquad \text{(OK)}$$

ANSWER Local instability is not a problem.

In Example 4.3, the width-to-thickness ratios $b_f/2t_f$ and h/t_w were computed. This is not necessary, however, because these ratios are tabulated in the dimensions and properties table. In addition, shapes that are slender for compression are indicated with a footnote (footnote c).

It is permissible to use a cross-sectional shape that does not satisfy the width-to-thickness ratio requirements, but such a member may not be permitted to carry as large a load as one that does satisfy the requirements. In other words, the strength could be reduced because of local buckling. The overall procedure for making this investigation is as follows.

- If the width-to-thickness ratio λ is greater than λ_r, use the provisions of AISC E7 and compute a reduction factor Q.
- Compute KL/r and F_e as usual.
- If $\frac{KL}{r} \le 4.71\sqrt{\frac{E}{QF_y}}$ or $\frac{QF_y}{F_e} \le 2.25$,

$$F_{cr} = Q\left(0.658^{\frac{QF_y}{F_e}}\right)F_y \qquad \text{(AISC Equation E7-2)}$$

- If $\frac{KL}{r} > 4.71\sqrt{\frac{E}{QF_y}}$ or $\frac{QF_y}{F_e} > 2.25$,

$$F_{cr} = 0.877F_e \qquad \text{(AISC Equation E7-3)}$$

- The nominal strength is $P_n = F_{cr}A_g$ (AISC Equation E7-1)

The reduction factor Q is the product of two factors—Q_s for unstiffened elements and Q_a for stiffened elements. If the shape has no slender unstiffened elements, $Q_s = 1.0$. If the shape has no slender stiffened elements, $Q_a = 1.0$.

Many of the shapes commonly used as columns are not slender, and the reduction will not be needed. This includes most (but not all) W-shapes. However, a large number of hollow structural shapes (HSS), double angles, and tees have slender elements.

AISC Specification Section E7.1 gives the procedure for calculating Q_s for slender unstiffened elements. The procedure is straightforward, and involves comparing the width-to-thickness ratio with a limiting value and then computing Q_s from an expression that is a function of the width-to-thickness ratio, F_y, and E.

The computation of Q_a for slender stiffened elements is given in AISC E7.2 and is slightly more complicated than the procedure for unstiffened elements. The general procedure is as follows.

- Compute an effective area of the cross section. This requires a knowledge of the stress in the effective area, so iteration is required. The Specification allows a simplifying assumption, however, so iteration can be avoided.
- Compute $Q_a = A_e/A_g$, where A_e is the effective area, and A_g is the gross or unreduced area.

The details of the computation of Q_s and Q_a will not be given here but will be shown in the following example, which illustrates the procedure for an HSS.

EXAMPLE 4.4

Determine the axial compressive strength of an HSS 8 × 4 × ⅛ with an effective length of 15 feet with respect to each principal axis. Use $F_y = 46$ ksi.

SOLUTION

Compute the overall, or flexural, buckling strength.

$$\text{Maximum } \frac{KL}{r} = \frac{KL}{r_y} = \frac{15 \times 12}{1.71} = 105.3 < 200 \qquad \text{(OK)}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29,000}{46}} = 118$$

Since 105.3 < 118, use AISC Equation E3-2.

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29,000)}{(105.3)^2} = 25.81 \text{ ksi}$$

$$F_{cr} = 0.658^{(F_y/F_e)} F_y = 0.658^{(46/25.81)}(46) = 21.82 \text{ ksi}$$

The nominal strength is

$$P_n = F_{cr} A_g = 21.82(2.70) = 58.91 \text{ kips}$$

Check width-to-thickness ratios:

From the dimensions and properties table in the *Manual*, the width-to-thickness ratio for the larger overall dimension is

$$\frac{h}{t} = 66.0$$

The ratio for the smaller dimension is

$$\frac{b}{t} = 31.5$$

From AISC Table B4.1a, Case 6 (and Figure 4.9 in this book), the upper limit for nonslender elements is

$$1.40\sqrt{\frac{E}{F_y}} = 1.40\sqrt{\frac{29{,}000}{46}} = 35.15$$

Since $h/t > 1.40\sqrt{E/F_y}$, the larger dimension element is slender and the local buckling strength must be computed. (Although the limiting width-to-thickness ratio is labeled *b/t* in the table, that is a generic notation, and it applies to h/t as well.)

Because this cross-sectional element is a stiffened element, $Q_s = 1.0$, and Q_a must be computed from AISC Section E7.2. The shape is a rectangular section of uniform thickness, with

$$\frac{b}{t} \geq 1.40\sqrt{\frac{E}{f}},$$

So AISC E7.2 (b) applies, where

$$f = \frac{P_n}{A_e}$$

and A_e is the reduced effective area. The Specification user note for square and rectangular sections permits a value of $f = F_y$ to be used in lieu of determining f by iteration. From AISC Equation E7-18, the effective width of the slender element is

$$b_e = 1.92t\sqrt{\frac{E}{f}}\left[1 - \frac{0.38}{b/t}\sqrt{\frac{E}{f}}\right] \leq b \qquad \text{(AISC Equation E7-18)}$$

For the 8-inch side, using $f = F_y$ and the *design* thickness* from the dimensions and properties table,

$$b_e = 1.92(0.116)\sqrt{\frac{29{,}000}{46}}\left[1 - \frac{0.38}{(66.0)}\sqrt{\frac{29{,}000}{46}}\right] = 4.784 \text{ in.}$$

*The *design* thickness of an HSS is 0.93 times the *nominal* thickness (AISC B4.2). Using the design thickness in strength computations is a conservative way to account for tolerances in the manufacturing process.

From AISC B4.1(b) and the discussion in Part 1 of the *Manual*, the unreduced length of the 8-inch side between the corner radii can be taken as

$$b = 8 - 3t = 8 - 3(0.116) = 7.652 \text{ in.}$$

where the corner radius is taken as 1.5 times the design thickness.

The total loss in area is therefore

$$2(b - b_e)t = 2(7.652 - 4.784)(0.116) = 0.6654 \text{ in.}^2$$

and the reduced area is

$$A_e = 2.70 - 0.6654 = 2.035 \text{ in.}^2$$

The reduction factor is

$$Q_a = \frac{A_e}{A_g} = \frac{2.035}{2.70} = 0.7537$$

$$Q = Q_s Q_a = 1.0(0.7537) = 0.7537$$

Compute the local buckling strength:

$$4.71\sqrt{\frac{E}{QF_y}} = 4.71\sqrt{\frac{29{,}000}{0.7537(46)}} = 136.2$$

$$\frac{KL}{r} = 105.3 < 136.2 \qquad \therefore \text{ Use AISC Equation E7-2}$$

$$F_{cr} = Q\left(0.658^{\frac{QF_y}{F_e}}\right)F_y = 0.7537\left(0.658^{\frac{0.7537(46)}{25.81}}\right)46 = 19.76 \text{ ksi}$$

$$P_n = F_{cr}A_g = 19.76(2.70) = 53.35 \text{ kips}$$

Since this is less than the flexural buckling strength of 58.91 kips, local buckling controls.

LRFD SOLUTION

Design strength $= \phi_c P_n = 0.90(53.35) = 48.0$ kips

ASD SOLUTION

$$\text{Allowable strength} = \frac{P_n}{\Omega} = \frac{53.35}{1.67} = 32.0 \text{ kips}$$

(Allowable stress $= 0.6F_{cr} = 0.6(19.76) = 11.9$ ksi)

ALTERNATIVE SOLUTION WITH *f* DETERMINED BY ITERATION

As an initial trial value, use

$f = F_{cr} = 19.76$ ksi (the value obtained above after using an initial value of $f = F_y$)

$$b_e = 1.92(0.116)\sqrt{\frac{29{,}000}{19.76}}\left[1 - \frac{0.38}{(66.0)}\sqrt{\frac{29{,}000}{19.76}}\right] = 6.65 \text{ in.}$$

The total loss in area is

$$2(b - b_e)t = 2(7.652 - 6.65)(0.116) = 0.2325 \text{ in.}^2$$

and the reduced area is

$$A_e = 2.70 - 0.2325 = 2.468 \text{ in.}^2$$

The reduction factor is

$$Q_a = \frac{A_e}{A_g} = \frac{2.468}{2.70} = 0.9141$$

$$Q = Q_s Q_a = 1.0(0.9141) = 0.9141$$

Compute the local buckling strength.

$$4.71\sqrt{\frac{E}{QF_y}} = 4.71\sqrt{\frac{29{,}000}{0.9141(46)}} = 123.7$$

$$\frac{KL}{r} = 105.3 < 123.7 \qquad \therefore \text{ Use AISC Equation E7-2}$$

$$F_{cr} = Q\left(0.658^{\frac{QF_y}{F_e}}\right)F_y$$

$$= 0.9141\left(0.658^{\frac{0.9141(46)}{25.81}}\right)46 = 21.26 \text{ ksi} \neq 19.76 \text{ ksi (the assumed value)}$$

Try $f = 21.26$ ksi:

$$b_e = 1.92(0.116)\sqrt{\frac{29{,}000}{21.26}}\left[1 - \frac{0.38}{(66.0)}\sqrt{\frac{29{,}000}{21.26}}\right] = 6.477 \text{ in.}$$

The total loss in area is

$$2(b - b_e)t = 2(7.652 - 6.477)(0.116) = 0.2726 \text{ in.}^2$$

and the reduced area is

$$A_e = 2.70 - 0.2726 = 2.427 \text{ in.}^2$$

The reduction factor is

$$Q_a = \frac{A_e}{A_g} = \frac{2.427}{2.70} = 0.8989$$

$$Q = Q_s Q_a = 1.0(0.8989) = 0.8989$$

Compute the local buckling strength.

$$4.71\sqrt{\frac{E}{QF_y}} = 4.71\sqrt{\frac{29,000}{0.8989(46)}} = 124.7$$

$$\frac{KL}{r} = 105.3 < 124.7 \qquad \therefore \text{ Use AISC Equation E7-2}$$

$$F_{cr} = Q\left(0.658^{\frac{QF_y}{F_e}}\right)F_y = 0.8989\left(0.658^{\frac{0.8989(46)}{25.81}}\right)46$$

$$= 21.15 \text{ ksi} \neq 21.26 \text{ ksi}$$

Try $f = 21.15$ ksi:

$$b_e = 1.92(0.116)\sqrt{\frac{29,000}{21.15}}\left[1 - \frac{0.38}{(66.0)}\sqrt{\frac{29,000}{21.15}}\right] = 6.489 \text{ in.}$$

The total loss in area is

$$2(b - b_e)t = 2(7.652 - 6.489)(0.116) = 0.2698 \text{ in.}^2$$

and the reduced area is

$$A_e = 2.70 - 0.2698 = 2.430 \text{ in.}^2$$

The reduction factor is

$$Q_a = \frac{A_e}{A_g} = \frac{2.430}{2.70} = 0.9000$$

$$Q = Q_s Q_a = 1.0(0.9000) = 0.9000$$

Compute the local buckling strength.

$$4.71\sqrt{\frac{E}{QF_y}} = 4.71\sqrt{\frac{29,000}{0.9000(46)}} = 124.7$$

$$\frac{KL}{r} = 105.3 < 124.7 \qquad \therefore \text{ Use AISC Equation E7-2}$$

$$F_{cr} = Q\left(0.658^{\frac{QF_y}{F_e}}\right)F_y$$

$$= 0.9000\left(0.658^{\frac{0.9000(46)}{25.81}}\right)46 = 21.16 \text{ ksi} \approx 21.15 \text{ ksi (convergence)}$$

Recall that AISC Equation E7-18 for b_e applies when $b/t \geq 1.40\sqrt{E/f}$. In the present case,

$$1.40\sqrt{\frac{E}{f}} = 1.40\sqrt{\frac{29{,}000}{21.16}} = 51.8$$

Since 66 > 51.8, AISC Equation E7-18 does apply.

$$P_n = F_{cr}A_g = 21.16(2.70) = 57.13 \text{ kips} \quad \therefore \text{ Local buckling controls}$$

LRFD SOLUTION Design strength = $\phi_c P_n = 0.90(57.13) = 51.4$ kips

ASD SOLUTION Allowable strength $\dfrac{P_n}{\Omega} = \dfrac{57.13}{1.67} = 34.2$ kips

(Allowable stress = $0.6F_{cr} = 0.6(21.16) = 12.7$ ksi)

4.5 TABLES FOR COMPRESSION MEMBERS

The *Manual* contains many useful tables for analysis and design. For compression members whose strength is governed by flexural buckling (that is, not local buckling), Table 4-22 in Part 4 of the *Manual*, "Design of Compression Members," can be used. This table gives values of $\phi_c F_{cr}$ (for LRFD) and F_{cr}/Ω_c (for ASD) as a function of KL/r for various values of F_y. This table stops at the recommended upper limit of $KL/r = 200$. The available strength tables, however, are the most useful. These tables, which we will refer to as the "column load tables," give the available strengths of selected shapes, both $\phi_c P_n$ for LRFD and P_n/Ω_c for ASD, as a function of the effective length KL. These tables include values of KL up to those corresponding to $KL/r = 200$.

The use of the tables is illustrated in the following example.

EXAMPLE 4.5

Compute the available strength of the compression member of Example 4.2 with the aid of (a) Table 4-22 from Part 4 of the *Manual* and (b) the column load tables.

LRFD SOLUTION

a. From Example 4.2, $KL/r = 96.77$ and $F_y = 50$ ksi. Values of $\phi_c F_{cr}$ in Table 4-22 are given only for integer values of KL/r; for decimal values, KL/r may be rounded *up* or linear interpolation may be used. For uniformity, we use interpolation in this book for all tables unless otherwise indicated. For $KL/r = 96.77$ and $F_y = 50$ ksi,

$$\phi_c F_{cr} = 22.67 \text{ ksi}$$

$$\phi_c P_n = \phi_c F_{cr} A_g = 22.67(21.8) = 494 \text{ kips}$$

b. The column load tables in Part 4 of the *Manual* give the available strength for selected W-, HP-, single-angle, WT-, HSS, pipe, double-angle, and composite shapes. (We cover composite construction in Chapter 9.) The tabular values for the symmetrical shapes (W, HP, HSS and pipe) were calculated by using the minimum radius of gyration for each shape. From Example 4.2, $K = 1.0$, so

$$KL = 1.0(20) = 20 \text{ ft}$$

For a W14 × 74, $F_y = 50$ ksi and $KL = 20$ ft,

$$\phi_c P_n = 495 \text{ kips}$$

ASD SOLUTION

a. From Example 4.2, $KL/r = 96.77$ and $F_y = 50$ ksi. By interpolation, for $KL/r = 96.77$ and $F_y = 50$ ksi,

$$F_{cr}/\Omega_c = 15.07 \text{ ksi}$$

Note that this is the allowable stress, $F_a = 0.6F_{cr}$. Therefore, the allowable strength is

$$\frac{P_n}{\Omega_c} = F_a A_g = 15.07(21.8) = 329 \text{ kips}$$

b. From Example 4.2, $K = 1.0$, so

$$KL = 1.0(20) = 20 \text{ ft}$$

From the column load tables, for a W14 × 74 with $F_y = 50$ ksi and $KL = 20$ ft,

$$\frac{P_n}{\Omega_c} = 329 \text{ kips}$$

The values from Table 4-22 (*Manual*) are based on flexural buckling and AISC Equations E3-2 and E3-3. Thus, local stability is assumed, and width-thickness ratio limits must not be exceeded. Although some shapes in the column load tables exceed those limits (and they are identified with a "c" footnote), the tabulated strength has been computed according to the requirements of AISC Section E7, "Members with Slender Elements," and no further reduction is needed.

From a practical standpoint, if a compression member to be analyzed can be found in the column load tables, then these tables should be used. Otherwise, Table 4-22 can be used for the flexural buckling strength. If the member has slender elements, the local buckling strength must be computed using the provisions of AISC E7.

4.6 DESIGN

The selection of an economical rolled shape to resist a given compressive load is simple with the aid of the column load tables. Enter the table with the effective length and move horizontally until you find the desired available strength (or something slightly larger). In some cases, you must continue the search to be certain that you have found the lightest shape. Usually the category of shape (W, WT, etc.) will have been decided upon in advance. Often the overall nominal dimensions will also be known because of architectural or other requirements. As pointed out earlier, all tabulated values correspond to a slenderness ratio of 200 or less. The tabulated unsymmetrical shapes—the structural tees and the single and double angles—require special consideration and are covered in Section 4.8.

EXAMPLE 4.6

A compression member is subjected to service loads of 165 kips dead load and 535 kips live load. The member is 26 feet long and pinned at each end. Use A992 steel and select a W14 shape.

LRFD SOLUTION

Calculate the factored load:

$$P_u = 1.2D + 1.6L = 1.2(165) + 1.6(535) = 1054 \text{ kips}$$

$$\therefore \text{ Required design strength } \phi_c P_n = 1054 \text{ kips.}$$

From the column load tables for $KL = 1.0(26) = 26$ ft, a W14 × 145 has a design strength of 1230 kips.

ANSWER

Use a W14 × 145.

ASD SOLUTION

Calculate the total applied load:

$$P_a = D + L = 165 + 535 = 700 \text{ kips}$$

$$\therefore \text{ Required allowable strength } \frac{P_n}{\Omega_c} = 700 \text{ kips}$$

From the column load tables for $KL = 1.0(26) = 26$ ft, a W14 × 132 has an allowable strength of 702 kips.

ANSWER Use a W14 × 132.

EXAMPLE 4.7

Select the lightest W-shape that can resist a service dead load of 62.5 kips and a service live load of 125 kips. The effective length is 24 feet. Use ASTM A992 steel.

SOLUTION The appropriate strategy here is to find the lightest shape for each nominal depth in the column load tables and then choose the lightest overall.

LRFD SOLUTION The factored load is

$$P_u = 1.2D + 1.6L = 1.2(62.5) + 1.6(125) = 275 \text{ kips}$$

From the column load tables, the choices are as follows:

W8: There are no W8s with $\phi_c P_n \geq 275$ kips.

W10: W10 × 54, $\phi_c P_n = 282$ kips

W12: W12 × 58, $\phi_c P_n = 292$ kips

W14: W14 × 61, $\phi_c P_n = 293$ kips

Note that the strength is not proportional to the weight (which is a function of the cross-sectional area).

ANSWER Use a W10 × 54.

ASD SOLUTION The total applied load is

$$P_a = D + L = 62.5 + 125 = 188 \text{ kips}$$

From the column load tables, the choices are as follows:

W8: There are no W8s with $P_n/\Omega_c \geq 188$ kips.

W10: W10 × 54, $\dfrac{P_n}{\Omega_c} = 188$ kips

$$\text{W12:} \quad \text{W12} \times 58, \quad \frac{P_n}{\Omega_c} = 194 \text{ kips}$$

$$\text{W14:} \quad \text{W14} \times 61, \quad \frac{P_n}{\Omega_c} = 195 \text{ kips}$$

Note that the strength is not proportional to the weight (which is a function of the cross-sectional area).

ANSWER Use a W10 × 54.

For shapes not in the column load tables, a trial-and-error approach must be used. The general procedure is to assume a shape and then compute its strength. If the strength is too small (unsafe) or too large (uneconomical), another trial must be made. A systematic approach to making the trial selection is as follows:

1. Assume a value for the critical buckling stress F_{cr}. Examination of AISC Equations E3-2 and E3-3 shows that the theoretically maximum value of F_{cr} is the yield stress F_y.
2. Determine the required area. For LRFD,

$$\phi_c F_{cr} A_g \geq P_u$$

$$A_g \geq \frac{P_u}{\phi_c F_{cr}}$$

For ASD,

$$0.6F_{cr} \geq \frac{P_a}{A_g}$$

$$A_g \geq \frac{P_a}{0.6F_{cr}}$$

3. Select a shape that satisfies the area requirement.
4. Compute F_{cr} and the strength for the trial shape.
5. Revise if necessary. If the available strength is very close to the required value, the next tabulated size can be tried. Otherwise, repeat the entire procedure, using the value of F_{cr} found for the current trial shape as a value for Step 1.
6. Check local stability (check the width-to-thickness ratios). Revise if necessary.

EXAMPLE 4.8

Select a W18 shape of A992 steel that can resist a service dead load of 100 kips and a service live load of 300 kips. The effective length KL is 26 feet.

LRFD SOLUTION

$P_u = 1.2D + 1.6L = 1.2(100) + 1.6(300) = 600$ kips
Try $F_{cr} = 33$ ksi (an arbitrary choice of two-thirds F_y):

$$\text{Required } A_g = \frac{P_u}{\phi_c F_{cr}} = \frac{600}{0.90(33)} = 20.2 \text{ in.}^2$$

Try a W18 × 71:

$$A_g = 20.9 \text{ in.}^2 > 20.2 \text{ in.}^2 \qquad \text{(OK)}$$

$$\frac{KL}{r_{\min}} = \frac{26 \times 12}{1.70} = 183.5 < 200 \qquad \text{(OK)}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(183.5)^2} = 8.5 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113$$

Since $\frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}}$, AISC Equation E3-3 applies.

$$F_{cr} = 0.877F_e = 0.877(8.5) = 7.455 \text{ ksi}$$

$$\phi_c P_n = \phi_c F_{cr} A_g = 0.90(7.455)(20.9) = 140 \text{ kips} < 600 \text{ kips} \qquad \text{(N.G.)}$$

Because the initial estimate of F_{cr} was so far off, assume a value about halfway between 33 and 7.455 ksi. Try $F_{cr} = 20$ ksi.

$$\text{Required } A_g = \frac{P_u}{\phi_c F_{cr}} = \frac{600}{0.90(20)} = 33.3 \text{ in.}^2$$

Try a W18 × 119:

$$A_g = 35.1 \text{ in.}^2 > 33.3 \text{ in.}^2 \qquad \text{(OK)}$$

$$\frac{KL}{r_{\min}} = \frac{26 \times 12}{2.69} = 116.0 < 200 \qquad \text{(OK)}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(116.0)^2} = 21.27 \text{ ksi}$$

Since $\frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}} = 113$, AISC Equation E3-3 applies.

$$F_{cr} = 0.877F_e = 0.877(21.27) = 18.65 \text{ ksi}$$
$$\phi_c P_n = \phi_c F_{cr} A_g = 0.90(18.65)(35.1) = 589 \text{ kips} < 600 \text{ kips} \qquad \text{(N.G.)}$$

This is very close, so try the next larger size.

Try a W18 × 130:

$$A_g = 38.3 \text{ in.}^2$$
$$\frac{KL}{r_{min}} = \frac{26 \times 12}{2.70} = 115.6 < 200 \qquad \text{(OK)}$$
$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(115.6)^2} = 21.42 \text{ ksi}$$

Since $\frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}} = 113$, AISC Equation E3-3 applies.

$$F_{cr} = 0.877F_e = 0.877(21.42) = 18.79 \text{ ksi}$$
$$\phi_c P_n = \phi_c F_{cr} A_g = 0.90(18.79)(38.3) = 648 \text{ kips} > 600 \text{ kips} \qquad \text{(OK.)}$$

This shape is not slender (there is no footnote in the dimensions and properties table to indicate that it is), so local buckling does not have to be investigated.

ANSWER Use a W18 × 130.

ASD SOLUTION The ASD solution procedure is essentially the same as for LRFD, and the same trial values of F_{cr} will be used here.

$$P_a = D + L = 100 + 300 = 400 \text{ kips}$$

Try $F_{cr} = 33$ ksi (an arbitrary choice of two-thirds F_y):

$$\text{Required } A_g = \frac{P_a}{0.6F_{cr}} = \frac{400}{0.6(33)} = 20.2 \text{ in.}^2$$

Try a W18 × 71:

$$A_g = 20.9 \text{ in.}^2 > 20.2 \text{ in.}^2 \qquad \text{(OK)}$$
$$\frac{KL}{r_{min}} = \frac{26 \times 12}{1.70} = 183.5 < 200 \qquad \text{(OK)}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(183.5)^2} = 8.5 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113$$

Since $\dfrac{KL}{r} > 4.71\sqrt{\dfrac{E}{F_y}}$, AISC Equation E3-3 applies.

$$F_{cr} = 0.877F_e = 0.877(8.5) = 7.455 \text{ ksi}$$

$$\frac{P_n}{\Omega_c} = 0.6F_{cr}A_g = 0.6(7.455)(20.9) = 93.5 \text{ kips} < 400 \text{ kips} \qquad \text{(N.G.)}$$

Because the initial estimate of F_{cr} was so far off, assume a value about halfway between 33 and 7.455 ksi. Try $F_{cr} = 20$ ksi.

$$\text{Required } A_g = \frac{P_a}{0.6F_{cr}} = \frac{400}{0.6(20)} = 33.3 \text{ in.}^2$$

Try a W18 × 119:

$$A_g = 35.1 \text{ in.}^2 > 33.3 \text{ in.}^2 \qquad \text{(OK)}$$

$$\frac{KL}{r_{\min}} = \frac{26 \times 12}{2.69} = 116.0 < 200 \qquad \text{(OK)}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(116.0)^2} = 21.27 \text{ ksi}$$

Since $\dfrac{KL}{r} > 4.71\sqrt{\dfrac{E}{F_y}} = 113$, AISC Equation E3-3 applies.

$$F_{cr} = 0.877F_e = 0.877(21.27) = 18.65 \text{ ksi}$$

$$0.6F_{cr}A_g = 0.6(18.65)(35.1) = 393 \text{ kips} < 400 \text{ kips} \qquad \text{(N.G.)}$$

This is very close, so try the next larger size.

Try a W18 × 130:

$$A_g = 38.3 \text{ in.}^2$$

$$\frac{KL}{r_{\min}} = \frac{26 \times 12}{2.70} = 115.6 < 200 \qquad \text{(OK)}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2 (29,000)}{(115.6^2)} = 21.42 \text{ ksi}$$

Since $\dfrac{KL}{r} > 4.71\sqrt{\dfrac{E}{F_y}} = 113$, AISC Equation E3-3 applies.

$$F_{cr} = 0.877F_e = 0.877(21.42) = 18.79 \text{ ksi}$$

$$0.6F_{cr}A_g = 0.6(18.79)(38.3) = 432 \text{ kips} < 400 \text{ kips} \qquad \text{(OK)}$$

This shape is not slender (there is no footnote in the dimensions and properties table to indicate that it is), so local buckling does not have to be investigated.

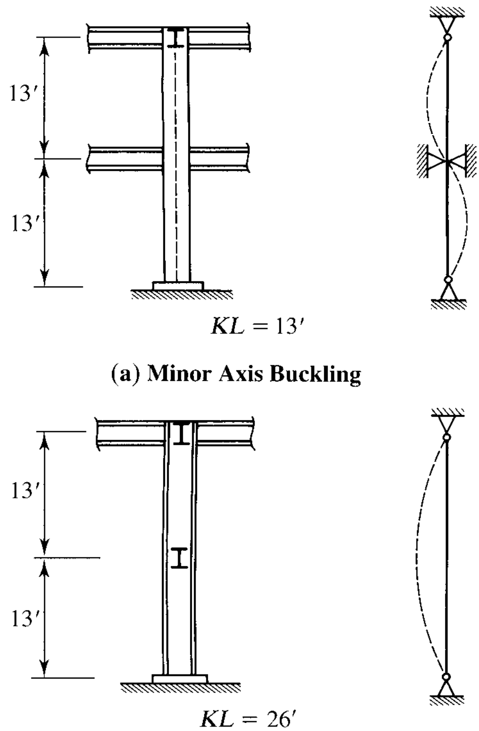
ANSWER Use a W18 × 130.

4.7 MORE ON EFFECTIVE LENGTH

We introduced the concept of effective length in Section 4.2, "Column Theory." All compression members are treated as pin-ended regardless of the actual end conditions but with an effective length *KL* that may differ from the actual length. With this modification, the load capacity of compression members is a function of only the slenderness ratio and modulus of elasticity. For a given material, the load capacity is a function of the slenderness ratio only.

If a compression member is supported differently with respect to each of its principal axes, the effective length will be different for the two directions. In Figure 4.10, a W-shape is used as a column and is braced by horizontal members in two perpendicular directions at the top. These members prevent translation of the column in all directions, but the connections, the details of which are not shown, permit small rotations to take place. Under these conditions, the member can be treated as

FIGURE 4.10



(a) Minor Axis Buckling

(b) Major Axis Buckling

pin-connected at the top. For the same reasons, the connection to the support at the bottom may also be treated as a pin connection. Generally speaking, a rigid, or fixed, condition is very difficult to achieve, and unless some special provisions are made, ordinary connections will usually closely approximate a hinge or pin connection. At midheight, the column is braced, but only in one direction.

Again, the connection prevents translation, but no restraint against rotation is furnished. This brace prevents translation perpendicular to the weak axis of the cross section but provides no restraint perpendicular to the strong axis. As shown schematically in Figure 4.10, if the member were to buckle about the major axis, the effective length would be 26 feet, whereas buckling about the minor axis would have to be in the second buckling mode, corresponding to an effective length of 13 feet. Because its strength decreases with increasing KL/r, a column will buckle in the direction corresponding to the largest slenderness ratio, so K_xL/r_x must be compared with K_yL/r_y. In Figure 4.10, the ratio $26(12)/r_x$ must be compared with $13(12)/r_y$ (where r_x and r_y are in inches), and the larger ratio would be used for the determination of the axial compressive strength.

EXAMPLE 4.9

A W12 × 58, 24 feet long, is pinned at both ends and braced in the weak direction at the third points, as shown in Figure 4.11. A992 steel is used. Determine the available compressive strength.

SOLUTION

$$\frac{K_xL}{r_x} = \frac{24(12)}{5.28} = 54.55$$

$$\frac{K_yL}{r_y} = \frac{8(12)}{2.51} = 38.25$$

K_xL/r_x, the larger value, controls.

LRFD SOLUTION

From Table 4-22 from Part 4 of the *Manual* and with $KL/r = 54.55$,

$$\phi_c F_{cr} = 36.24 \text{ ksi}$$

$$\phi_c P_n = \phi_c F_{cr} A_g = 36.24(17.0) = 616 \text{ kips}$$

ANSWER

Design strength = 616 kips.

ASD SOLUTION

From Table 4-22 with $KL/r = 54.55$,

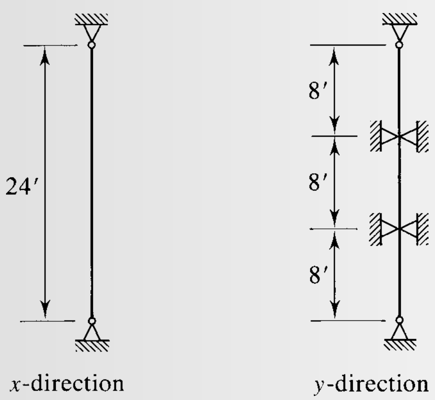
$$\frac{F_{cr}}{\Omega_c} = 24.09 \text{ ksi}$$

$$\frac{P_n}{\Omega_c} = \frac{F_{cr}}{\Omega_c} A_g = 24.09(17.0) = 410 \text{ kips}$$

ANSWER

Allowable strength = 410 kips.

FIGURE 4.11



The available strengths given in the column load tables are based on the effective length with respect to the y-axis. A procedure for using the tables with K_xL, however, can be developed by examining how the tabular values were obtained. Starting with a value of KL, the strength was obtained by a procedure similar to the following:

- KL was divided by r_y to obtain KL/r_y.
- F_{cr} was computed.
- The available strengths, $\phi_c P_n$ for LRFD and P_n/Ω_c for ASD, were computed.

Thus the tabulated strengths are based on the values of KL being equal to K_yL. If the capacity with respect to x-axis buckling is desired, the table can be entered with

$$KL = \frac{K_xL}{r_x/r_y}$$

and the tabulated load will be based on

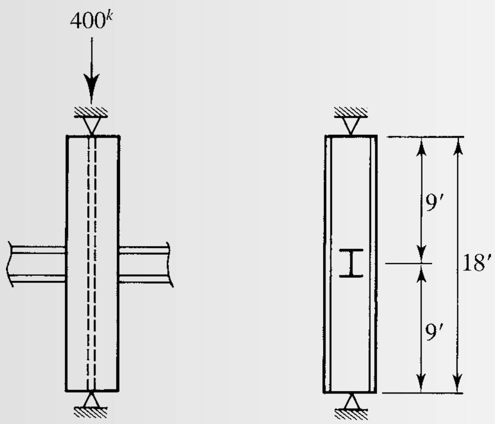
$$\frac{KL}{r_y} = \frac{K_xL/(r_x/r_y)}{r_y} = \frac{K_xL}{r_x}$$

The ratio r_x/r_y is given in the column load tables for each shape listed.

EXAMPLE 4.10

The compression member shown in Figure 4.12 is pinned at both ends and supported in the weak direction at midheight. A service load of 400 kips, with equal parts of dead and live load, must be supported. Use F_y = 50 ksi and select the lightest W-shape.

FIGURE 4.12



LRFD SOLUTION

Factored load = P_u = 1.2(200) + 1.6(200) = 560 kips

Assume that the weak direction controls and enter the column load tables with $KL = 9$ feet. Beginning with the smallest shapes, the first one found that will work is a W8 × 58 with a design strength of 634 kips.

Check the strong axis:

$$\frac{K_x L}{r_x/r_y} = \frac{18}{1.74} = 10.34 \text{ ft} > 9 \text{ ft}$$

$\therefore K_x L$ controls for this shape.

Enter the tables with $KL = 10.34$ feet. A W8 × 58 has an interpolated strength of

$$\phi_c P_n = 596 \text{ kips} > 560 \text{ kips} \quad \text{(OK)}$$

Next, investigate the W10 shapes. Try a W10 × 49 with a design strength of 568 kips.

Check the strong axis:

$$\frac{K_x L}{r_x/r_y} = \frac{18}{1.71} = 10.53 \text{ ft} > 9 \text{ ft}$$

$\therefore K_x L$ controls for this shape.

Enter the tables with $KL = 10.53$ feet. A W10 × 54 is the lightest W10, with an interpolated design strength of 594 kips.

Continue the search and investigate a W12 × 53 ($\phi_c P_n = 611$ kips for $KL = 9$ ft):

$$\frac{K_x L}{r_x/r_y} = \frac{18}{2.11} = 8.53 \text{ ft} < 9 \text{ ft}$$

$\therefore K_y L$ controls for this shape, and $\phi_c P_n = 611$ kips.

Determine the lightest W14. The lightest one with a possibility of working is a W14 × 61. It is heavier than the lightest one found so far, so it will not be considered.

ANSWER

Use a W12 × 53.

ASD SOLUTION

The required load capacity is $P = 400$ kips. Assume that the weak direction controls and enter the column load tables with $KL = 9$ feet. Beginning with the smallest shapes, the first one found that will work is a W8 × 58 with an allowable strength of 422 kips.

Check the strong axis:

$$\frac{K_x L}{r_x/r_y} = \frac{18}{1.74} = 10.34 \text{ ft} > 9 \text{ ft}$$

$\therefore K_x L$ controls for this shape.

Enter the tables with $KL = 10.34$ feet. A W8 × 58 has an interpolated strength of

$$\frac{P_n}{\Omega_c} = 397 \text{ kips} < 400 \text{ kips} \qquad \text{(N.G.)}$$

The next lightest W8 that will work is a W8 × 67.

$$\frac{K_x L}{r_x/r_y} = \frac{18}{1.75} = 10.29 \text{ ft} > 9 \text{ ft}$$

The interpolated allowable strength is

$$\frac{P_n}{\Omega_c} = 460 \text{ kips} > 400 \text{ kips} \qquad \text{(OK)}$$

Next, investigate the W10 shapes. Try a W10 × 60.

$$\frac{K_x L}{r_x/r_y} = \frac{18}{1.71} = 10.53 \text{ ft} > 9 \text{ ft}$$

The interpolated strength is

$$\frac{P_n}{\Omega_c} = 444 \text{ kips} > 400 \text{ kips} \qquad \text{(OK)}$$

Check the W12 shapes. Try a W12 × 53 ($P_n/\Omega_c = 407$ kips for $KL = 9$ ft):

$$\frac{K_x L}{r_x/r_y} = \frac{18}{2.11} = 8.53 \text{ ft} < 9 \text{ ft}$$

$\therefore K_y L$ controls for this shape, and $P_n/\Omega_c = 407$ kips.

Find the lightest W14. The lightest one with a possibility of working is a W14 × 61. Since it is heavier than the lightest one found so far, it will not be considered.

ANSWER Use a W12 × 53.

Whenever possible, the designer should provide extra support for the weak direction of a column. Otherwise, the member is inefficient: It has an excess of strength in one direction. When $K_x L$ and $K_y L$ are different, $K_y L$ will control unless r_x/r_y is smaller than $K_x L/K_y L$. When the two ratios are equal, the column has equal strength in both directions. For most of the W-shapes in the column load tables, r_x/r_y ranges between 1.6 and 1.8, but it is as high as 3.1 for some shapes.

EXAMPLE 4.11

The column shown in Figure 4.13 is subjected to a service dead load of 140 kips and a service live load of 420 kips. Use A992 steel and select a W-shape.

SOLUTION $K_xL = 20$ ft and maximum $K_yL = 8$ ft. The effective length K_xL will control whenever

$$\frac{K_xL}{r_x/r_y} > K_yL$$

or

$$r_x/r_y < \frac{K_xL}{K_yL}$$

In this example,

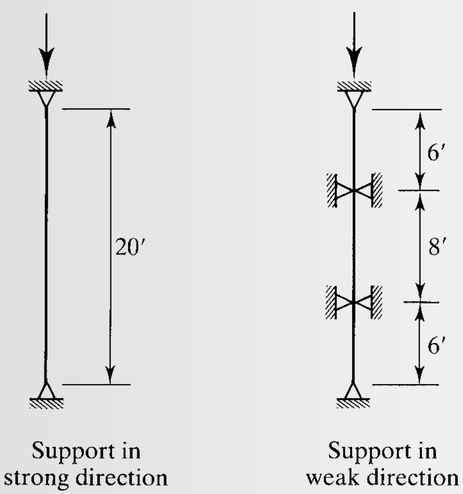
$$\frac{K_xL}{K_yL} = \frac{20}{8} = 2.5$$

so K_xL will control if $r_x/r_y < 2.5$. Since this is true for almost every shape in the column load tables, K_xL probably controls in this example.

Assume $r_x/r_y = 1.7$:

$$\frac{K_xL}{r_x/r_y} = \frac{20}{1.7} = 11.76 > K_yL$$

FIGURE 4.13



LRFD SOLUTION

$$P_u = 1.2D + 1.6L = 1.2(140) + 1.6(420) = 840 \text{ kips}$$

Enter the column load tables with KL = 12 feet. There are no W8 shapes with enough load capacity.

Try a W10 × 88 ($\phi_c P_n = 940$ kips):

$$\text{Actual } \frac{K_x L}{r_x/r_y} = \frac{20}{1.73} = 11.56 \text{ ft} < 12 \text{ ft}$$

$$\therefore \phi_c P_n > \text{required } 840 \text{ kips}$$

(By interpolation, $\phi_c P_n = 955$ kips.)

Check a W12 × 79:

$$\frac{K_x L}{r_x/r_y} = \frac{20}{1.75} = 11.43 \text{ ft.}$$

$$\phi_c P_n = 900 \text{ kips} > 840 \text{ kips} \qquad \text{(OK)}$$

Investigate W14 shapes. For r_x/r_y = 2.44 (the approximate ratio for all likely possibilities),

$$\frac{K_x L}{r_x/r_y} = \frac{20}{2.44} = 8.197 \text{ ft} > K_y L = 8 \text{ ft}$$

For KL = 9 ft, a W14 × 74, with a capacity of 854 kips, is the lightest W14-shape. Since 9 feet is a conservative approximation of the actual effective length, this shape is satisfactory.

ANSWER Use a W14 × 74 (lightest of the three possibilities).

ASD SOLUTION

$$P_a = D + L = 140 + 420 = 560 \text{ kips}$$

Enter the column load tables with KL = 12 feet. There are no W8 shapes with enough load capacity. Investigate a W10 × 88 (for KL = 12 ft, P_n/Ω_c = 625 kips):

$$\text{Actual } \frac{K_x L}{r_x/r_y} = \frac{20}{1.73} = 11.56 \text{ ft} < 12 \text{ ft}$$

$$\therefore \frac{P_n}{\Omega_c} > \text{required } 560 \text{ kips}$$

(By interpolation, $P_n/\Omega_c = 635$ kips.)

Check a W12 × 79:

$$\frac{K_x L}{r_x/r_y} = \frac{20}{1.75} = 11.43 \text{ ft} > K_y L = 8 \text{ ft}$$

$$\frac{P_n}{\Omega_c} = 599 \text{ kips } > 560 \text{ kips} \qquad \text{(OK)}$$

Investigate W14 shapes. Try a W14 × 74:

$$\frac{K_x L}{r_x/r_y} = \frac{20}{2.44} = 8.20 > K_y L = 8 \text{ ft}$$

For $KL = 8.20$ ft,

$$\frac{P_n}{\Omega_c} = 582 \text{ kips } > 560 \text{ kips} \qquad \text{(OK)}$$

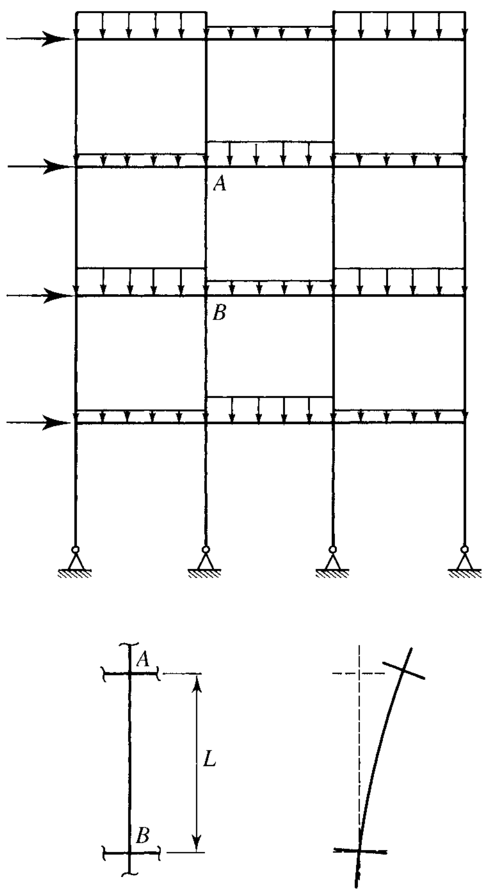
ANSWER Use a W14 × 74 (lightest of the three possibilities).

For isolated columns that are not part of a continuous frame, Table C-A-7.1 in the Commentary to Specification Appendix 7 will usually suffice. Consider, however, the rigid frame in Figure 4.14. The columns in this frame are not independent members but part of a continuous structure. Except for those in the lower story, the columns are restrained at both ends by their connection to beams and other columns. This frame is also unbraced, meaning that horizontal displacements of the frame are possible and all columns are subject to sidesway. If Table C-A-7.1 is used for this frame, the lower-story columns are best approximated by condition (f), and a value of $K = 2$ might be used. For a column such as AB, a value of $K = 1.2$, corresponding to condition (c), could be selected. A more rational procedure, however, will account for the degree of restraint provided by connecting members.

The rotational restraint provided by the beams, or girders, at the end of a column is a function of the rotational stiffnesses of the members intersecting at the joint. The rotational stiffness of a member is proportional to EI/L, where I is the moment of inertia of the cross section with respect to the axis of bending. Gaylord, Gaylord, and Stallmeyer (1992) show that the effective length factor K depends on the ratio of column stiffness to girder stiffness at each end of the member, which can be expressed as

$$G = \frac{\sum E_c I_c / L_c}{\sum E_g I_g / L_g} = \frac{\sum I_c / L_c}{\sum I_g / L_g} \qquad (4.12)$$

FIGURE 4.14



where

$\Sigma E_c I_c / L_c$ = sum of the stiffnesses of all columns at the end of the column under consideration.

$\Sigma E_g I_g / L_g$ = sum of the stiffnesses of all girders at the end of the column under consideration.

$E_c = E_g = E$, the modulus of elasticity of structural steel.

If a very slender column is connected to girders having large cross sections, the girders will effectively prevent rotation of the column. The ends of the column are approximately fixed, and K is relatively small. This condition corresponds to small values of G given by Equation 4.12. However, the ends of stiff columns connected to flexible beams can more freely rotate and approach the pinned condition, giving relatively large values of G and K.

The relationship between G and K has been quantified in the Jackson–Mooreland Alignment Charts (Johnston, 1976), which are reproduced in Figures C-A-7.1 and C-A-7.2 in the Commentary. To obtain a value of K from one of these nomograms, first calculate the value of G at each end of the column, letting one value be G_A and the other be G_B. Connect G_A and G_B with a straight line, and read the value of K on the middle scale. The effective length factor obtained in this manner is with respect to the axis of bending, which is the axis perpendicular to the plane of the frame. A separate analysis must be made for buckling about the other axis. Normally the beam-to-column connections in this direction will not transmit moment; sidesway is prevented by bracing; and K can be taken as 1.0.

EXAMPLE 4.12

The rigid frame shown in Figure 4.15 is unbraced. Each member is oriented so that its web is in the plane of the frame. Determine the effective length factor K_x for columns *AB* and *BC*.

SOLUTION

Column *AB:*

For joint *A*,

$$G = \frac{\sum I_c/L_c}{\sum I_g/L_g} = \frac{833/12 + 1070/12}{1350/20 + 1830/18} = \frac{158.6}{169.2} = 0.94$$

For joint *B*,

$$G = \frac{\sum I_c/L_c}{\sum I_g/L_g} = \frac{1070/12 + 1070/15}{169.2} = \frac{160.5}{169.2} = 0.95$$

ANSWER

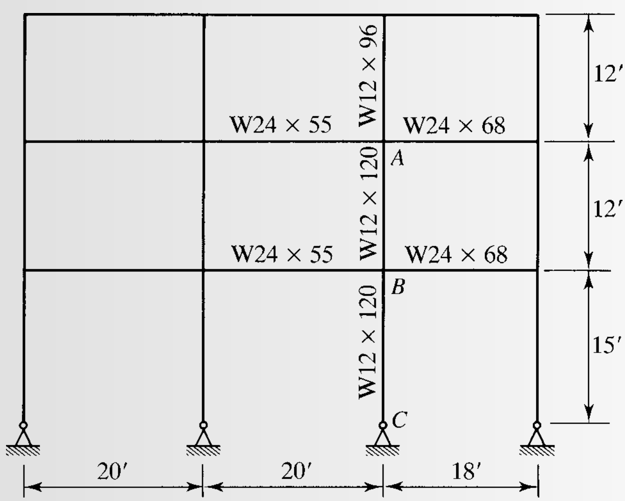
From the alignment chart for sidesway uninhibited (AISC Figure C-A-7.2), with $G_A = 0.94$ and $G_B = 0.95$, $K_x = 1.3$ for column *AB*.

Column *BC:*

For joint *B*, as before,

$G = 0.95$

FIGURE 4.15



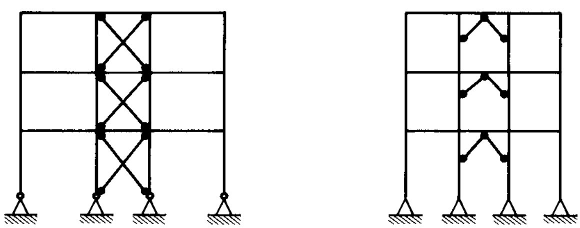
For joint C, a pin connection, the situation is analogous to that of a very stiff column attached to infinitely flexible girders—that is, girders of zero stiffness. The ratio of column stiffness to girder stiffness would therefore be infinite for a perfectly frictionless hinge. This end condition can only be approximated in practice, so the discussion accompanying the alignment chart recommends that G be taken as 10.0.

ANSWER From the alignment chart with $G_A = 0.95$ and $G_B = 10.0$, $K_x = 1.85$ for column BC.

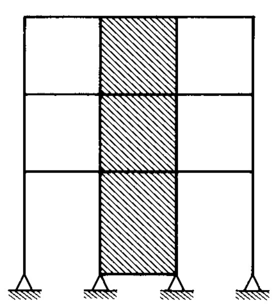
As pointed out in Example 4.12, for a pinned support, G should be taken as 10.0; for a fixed support, G should be taken as 1.0. The latter support condition corresponds to an infinitely stiff girder and a flexible column, corresponding to a theoretical value of $G = 0$. The discussion accompanying the alignment chart in the Commentary recommends a value of $G = 1.0$ because true fixity will rarely be achieved.

Unbraced frames are able to support lateral loads because of their moment-resisting joints. Often the frame is augmented by a bracing system of some sort; such frames are called *braced frames*. The additional resistance to lateral loads can take the form of diagonal bracing or rigid shear walls, as illustrated in Figure 4.16. In either case, the tendency for columns to sway is blocked within a given panel,

FIGURE 4.16



(a) Diagonal Bracing



(b) Shear Walls
(masonry, reinforced concrete,
or steel plate)

or bay, for the full height of the frame. This support system forms a cantilever structure that is resistant to horizontal displacements and also provides horizontal support for the other bays. Depending on the size of the structure, more than one bay may require bracing.

A frame must resist not only the tendency to sway under the action of lateral loads but also the tendency to buckle, or become unstable, under the action of vertical loads. Bracing to stabilize a structure against vertical loading is called *stability bracing*. Appendix 6 of the AISC Specification, "Stability Bracing for Columns and Beams," covers this type of bracing. Two categories are covered: *relative* and *nodal*. With relative bracing, a brace point is restrained relative to adjacent brace points. A relative brace is connected not only to the member to be braced but also to other members, as with diagonal bracing. With relative bracing, both the brace and other members contribute to stabilizing the member to be braced. Nodal bracing provides isolated support at specific locations on the member and is not relative to other brace points or other members. The provisions of AISC Appendix 6 give equations for the required strength and stiffness (resistance to deformation) of stability bracing. The provisions for columns are from the *Guide to Stability Design Criteria* (Galambos, 1998). The required strength and stiffness for stability can be added directly to the requirements for bracing to resist lateral loading. Stability bracing is discussed further in Chapter 5, "Beams," and Chapter 6, "Beam–Columns."

Columns that are members of braced rigid frames are prevented from sidesway and have some degree of rotational restraint at their ends. Thus they are in a category that lies somewhere between cases (a) and (d) in Table C-A-7.1 of the Commentary, and K is between 0.5 and 1.0. A value of 1.0 is therefore always conservative for members of braced frames and is the value prescribed by AISC Appendix 7.2.3(a) unless an analysis is made. Such an analysis can be made with the alignment chart for braced frames. Use of this nomogram would result in an effective length factor somewhat less than 1.0, and some savings could be realized.*

As with any design aid, the alignment charts should be used only under the conditions for which they were derived. These conditions are discussed in Section 7.2 of the Commentary to the Specification and are not enumerated here. Most of the conditions will usually be approximately satisfied; if they are not, the deviation will be on the conservative side. One condition that usually is not satisfied is the requirement that all behavior be elastic. If the slenderness ratio KL/r is less than $4.71\sqrt{E/F_y}$, the column will buckle inelastically, and the effective length factor obtained from the alignment chart will be overly conservative. A large number of columns are in this category. A convenient procedure for determining K for inelastic columns allows the alignment charts to be used (Yura, 1971; Disque, 1973; Geschwindner, 2010). To demonstrate the procedure, we begin with the critical buckling load for an inelastic

*If a frame is braced against sidesway, the beam-to-column connections need not be moment-resisting, and the bracing system could be designed to resist all sidesway tendency. If the connections are not moment-resisting, however, there will be no continuity between columns and girders, and the alignment chart cannot be used. For this type of braced frame, K_x should be taken as 1.0.

column given by Equation 4.6b. Dividing it by the cross-sectional area gives the buckling stress:

$$F_{cr} = \frac{\pi^2 E_t}{(KL/r)^2}$$

The rotational stiffness of a column in this state would be proportional to $E_t I_c / L_c$, and the appropriate value of G for use in the alignment chart is

$$G_{\text{inelastic}} = \frac{\sum E_t I_c / L_c}{\sum E I_g / L_g} = \frac{E_t}{E} G_{\text{elastic}}$$

Because E_t is less than E, $G_{\text{inelastic}}$ is less than G_{elastic}, and the effective length factor K will be reduced, resulting in a more economical design. To evaluate E_t/E, called the *stiffness reduction factor* (denoted by τ_b), consider the following relationship.

$$\frac{F_{cr\,(\text{inelastic})}}{F_{cr\,(\text{elastic})}} = \frac{\pi^2 E_t/(KL/r)^2}{\pi^2 E/(KL/r)^2} = \frac{E_t}{E} = \tau_b$$

From Galambos (1998), $F_{cr\,(\text{inelastic})}$ and $F_{cr\,(\text{elastic})}$ can be expressed as

$$F_{cr\,(\text{inelastic})} = \left(1 - \frac{\lambda^2}{4}\right) F_y \tag{4.13}$$

$$F_{cr\,(\text{elastic})} = \frac{F_y}{\lambda^2}$$

where

$$\lambda = \frac{KL}{r} \frac{1}{\pi} \sqrt{\frac{F_y}{E}}$$

Since

$$\tau_b = \frac{F_{cr\,(\text{inelastic})}}{F_{cr\,(\text{elastic})}}$$

then

$$F_{cr\,(\text{inelastic})} = \tau_b F_{cr\,(\text{elastic})} = \tau_b \left(\frac{F_y}{\lambda^2}\right)$$

$$\lambda^2 = \frac{\tau_b F_y}{F_{cr\,(\text{inelastic})}}$$

From Equation 4.13,

$$F_{cr\,(\text{inelastic})} = \left(1 - \frac{\lambda^2}{4}\right) F_y = \left(1 - \frac{\tau_b F_y}{4 F_{cr\,(\text{inelastic})}}\right) F_y$$

Using the notation $F_{cr} = F_{cr\,(\text{inelastic})}$ and solving for τ_b, we obtain

$$\tau_b = 4\left(\frac{F_{cr}}{F_y}\right)\left(1-\frac{F_{cr}}{F_y}\right)$$

This can be written in terms of forces as

$$\tau_b = 4\left(\frac{P_n}{P_y}\right)\left(1-\frac{P_n}{P_y}\right)$$

where

P_n = nominal compressive strength = $F_{cr}A_g$

P_y = compressive yield strength = $F_y A_g$

Substituting the required strength, αP_r, for the available strength, P_n, we have

$$\tau_b = 4\left(\frac{\alpha P_r}{P_y}\right)\left(1-\frac{\alpha P_r}{P_y}\right) \qquad \text{(AISC Equation C2-2b)}$$

where $\alpha = 1.0$ for LRFD and 1.6 for ASD. The required strength is computed at the factored load level, and the 1.6 factor is used to adjust the ASD service load level to a factored load level. The stiffness reduction factor, τ_b, is also used to adjust member stiffnesses for frame analysis. This is discussed in Chapter 6, "Beam–Columns."

EXAMPLE 4.13

A W10 × 54 of A992 steel is used as a column. It is subjected to a service dead load of 100 kips and a service live load of 200 kips. If the slenderness ratio makes this member an inelastic column, what is the stiffness reduction factor, τ_b?

LRFD SOLUTION

$$P_r = P_u = 1.2D + 1.6L = 1.2(100) + 1.6(200) = 440 \text{ kips}$$

$$P_y = F_y A_g = 50(15.8) = 790 \text{ kips}$$

From AISC Equation C2-2b,

$$\tau_b = 4\left(\frac{\alpha P_r}{P_y}\right)\left(1-\frac{\alpha P_r}{P_y}\right) = 4\left(\frac{1.0(440)}{790}\right)\left(1-\frac{1.0(440)}{790}\right) = 0.987$$

ANSWER $\tau_b = 0.987$.

ASD SOLUTION

$$P_r = P_a = D + L = 100 + 200 = 300 \text{ kips}$$

$$P_y = F_y A_g = 50(15.8) = 790 \text{ kips}$$

From AISC Equation C2-2b,

$$\tau_b = 4\left(\frac{\alpha P_r}{P_y}\right)\left(1-\frac{\alpha P_r}{P_y}\right) = 4\left(\frac{1.6(300)}{790}\right)\left(1-\frac{1.6(300)}{790}\right) = 0.954$$

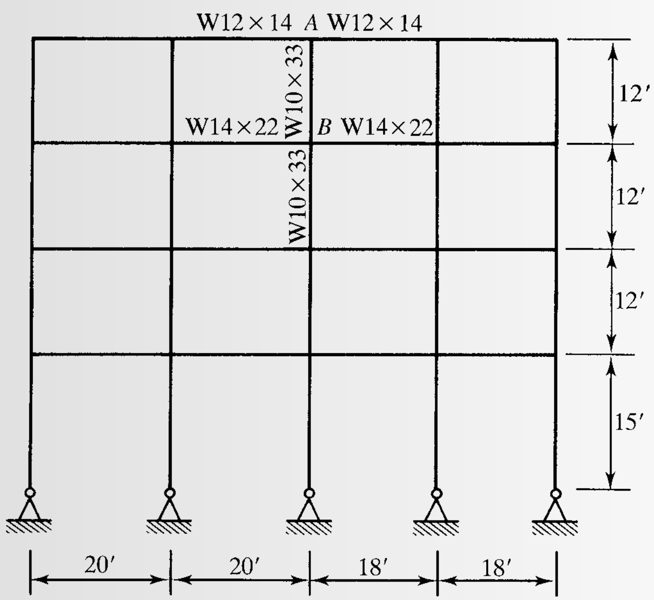
ANSWER $\tau_b = 0.954$.

If the end of a column is fixed ($G = 1.0$) or pinned ($G = 10.0$), the value of G at that end should not be multiplied by the stiffness reduction factor. Values of the stiffness reduction factor τ_b as a function of P_u/A_g and P_a/A_g are given in Table 4-21 in Part 4 of the *Manual*.

EXAMPLE 4.14

A rigid unbraced frame is shown in Figure 4.17. All members are oriented so that bending is about the strong axis. Lateral support is provided at each joint by simply connected bracing in the direction perpendicular to the frame. Determine the effective length factors with respect to each axis for member *AB*. The service dead load is 35.5 kips, and the service live load is 142 kips. A992 steel is used.

FIGURE 4.17



SOLUTION Compute elastic G factors:

For joint A,

$$\frac{\Sigma(I_c/L_c)}{\Sigma(I_g/L_g)} = \frac{171/12}{88.6/20+88.6/18} = \frac{14.25}{9.352} = 1.52$$

For joint B,

$$\frac{\Sigma(I_c/L_c)}{\Sigma(I_g/L_g)} = \frac{2(171/12)}{199/20+199/18} = \frac{28.5}{21.01} = 1.36$$

From the alignment chart for unbraced frames, $K_x = 1.45$, based on elastic behavior. Determine whether the column behavior is elastic or inelastic.

$$\frac{K_x L}{r_x} = \frac{1.45(12\times 12)}{4.19} = 49.83$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113$$

Since

$$\frac{K_x L}{r_x} < 4.71\sqrt{\frac{E}{F_y}}$$

behavior is inelastic, and the inelastic K factor can be used.

LRFD SOLUTION The factored load is

$$P_u = 1.2D + 1.6L = 1.2(35.5) + 1.6(142) = 269.8 \text{ kips}$$

Enter Table 4-21 in Part 4 of the *Manual* with

$$\frac{P_u}{A_g} = \frac{269.8}{9.71} = 27.79 \text{ ksi}$$

and obtain the stiffness reduction factor $\tau_b = 0.9877$ by interpolation.
For joint A,

$$G_{\text{inelastic}} = \tau_b \times G_{\text{elastic}} = 0.9877(1.52) = 1.50$$

For joint B,

$$G_{\text{inelastic}} = 0.9877(1.36) = 1.34$$

ANSWER From the alignment chart, $K_x = 1.43$. Because of the support conditions normal to the frame, K_y can be taken as 1.0.

ASD SOLUTION The applied load is

$$P_a = D + L = 35.5 + 142 = 177.5 \text{ kips}$$

Enter Table 4-21 in Part 4 of the *Manual* with

$$\frac{P_a}{A_g} = \frac{177.5}{9.71} = 18.28 \text{ ksi}$$

and obtain the stiffness reduction factor $\tau_b = 0.9703$ by interpolation.
For joint *A*,

$$G_{\text{inelastic}} = \tau_a \times G_{\text{elastic}} = 0.9703(1.52) = 1.47$$

For joint *B*,

$$G_{\text{inelastic}} = 0.9703(1.36) = 1.32$$

ANSWER From the alignment chart, $K_x = 1.43$. Because of the support conditions normal to the frame, K_y can be taken as 1.0.

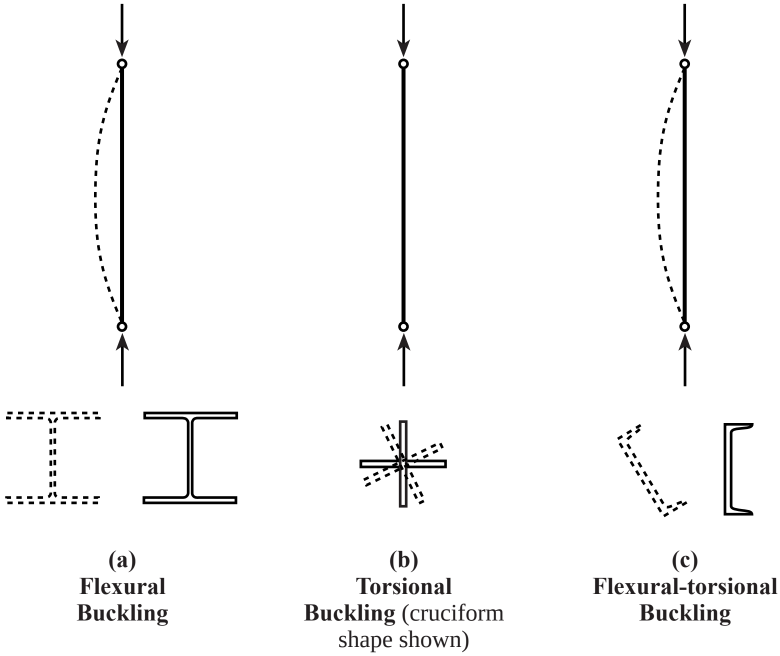
According to the AISC Specification, the effective length factor, *K*, should be determined by a "sidesway buckling analysis" (Chapters E, C, and Appendix 7). However, the use the alignment charts is acceptable (Nair, 2005).

4.8 TORSIONAL AND FLEXURAL-TORSIONAL BUCKLING

When an axially loaded compression member becomes unstable overall (that is, not locally unstable), it can buckle in one of three ways, as shown in Figure 4.18).

1. **Flexural buckling.** We have considered this type of buckling up to now. It is a deflection caused by bending, or flexure, about the axis corresponding to the largest slenderness ratio (Figure 4.18a). This is usually the minor principal axis—the one with the smallest radius of gyration. Compression members with any type of cross-sectional configuration can fail in this way.
2. **Torsional buckling.** This type of failure is caused by twisting about the longitudinal axis of the member. It can occur only with doubly symmetrical cross sections with very slender cross-sectional elements (Figure 4.18b). Standard hot-rolled shapes are not susceptible to torsional buckling, but members built up from thin plate elements may be and should be investigated. The cruciform shape shown is particularly vulnerable to this type of buckling. This shape can be fabricated from plates as shown in the figure, or built up from four angles placed back to back.
3. **Flexural-torsional buckling.** This type of failure is caused by a combination of flexural buckling and torsional buckling. The member bends and twists simultaneously (Figure 4.18c). This type of failure can occur only with unsymmetrical cross sections, both those with one axis of symmetry—such as channels, structural tees, double-angle shapes, and equal-leg single angles—and those with no axis of symmetry, such as unequal-leg single angles.

FIGURE 4.18



The AISC Specification requires an analysis of torsional or flexural-torsional buckling when appropriate. Section E4(a) of the Specification covers double-angle and tee-shaped members, and Section E4(b) provides a more general approach that can be used for other shapes. We discuss the general approach first. It is based on first determining a value of F_e, which then can be used with the flexural buckling equations, AISC Equations E3-2 and E3-3. The stress F_e can be defined as the elastic buckling stress corresponding to the controlling mode of failure, whether flexural, torsional, or flexural-torsional.

The equations for F_e given in AISC E4(b) are based on well-established theory given in *Theory of Elastic Stability* (Timoshenko and Gere, 1961). Except for some changes in notation, they are the same equations as those given in that work, with no simplifications. For doubly symmetrical shapes (torsional buckling),

$$F_e = \left[\frac{\pi^2 EC_w}{(K_z L)^2} + GJ\right]\frac{1}{I_x + I_y} \qquad \text{(AISC Equation E4-4)}$$

For singly symmetrical shapes (flexural-torsional buckling),

$$F_e = \frac{F_{ey} + F_{ez}}{2H}\left(1 - \sqrt{1 - \frac{4F_{ey}F_{ez}H}{(F_{ey} + F_{ez})^2}}\right) \qquad \text{(AISC Equation E4-5)}$$

where y is the axis of symmetry.

For shapes with *no* axis of symmetry (flexural-torsional buckling),

$$(F_e - F_{ex})(F_e - F_{ey})(F_e - F_{ez}) - F_e^2(F_e - F_{ey})\left(\frac{x_0}{\bar{r}_0}\right)^2 - F_e^2(F_e - F_{ex})\left(\frac{y_0}{\bar{r}_0}\right)^2 = 0 \qquad \text{(AISC Equation E4-6)}$$

This last equation is a cubic; F_e is the smallest root.

In the above equations, the z-axis is the longitudinal axis. The previously undefined terms in these three equations are defined as

C_w = warping constant (in.^{6})
K_z = effective length factor for *torsional* buckling, which is based on the amount of end restraint against twisting about the longitudinal axis
G = shear modulus (ksi) = 11,200 ksi for structural steel
J = torsional constant (equal to the polar moment of inertia only for circular cross sections) (in.^{4})

$$F_{ex} = \frac{\pi^2 E}{(K_x L/r_x)^2} \qquad \text{(AISC Equation E4-7)}$$

$$F_{ey} = \frac{\pi^2 E}{(K_y L/r_y)^2} \qquad \text{(AISC Equation E4-8)}$$

where y is the axis of symmetry for singly symmetrical shapes.

$$F_{ez} = \left(\frac{\pi^2 E C_w}{(K_z L)^2} + GJ\right)\frac{1}{A_g \bar{r}_0^2} \qquad \text{(AISC Equation E4-9)}$$

$$H = 1 - \frac{x_0^2 + y_0^2}{\bar{r}_0^2} \qquad \text{(AISC Equation E4-10)}$$

where z is the longitudinal axis and x_0, y_0 are the coordinates of the shear center of the cross section with respect to the centroid (in inches). The shear center is the point on the cross section through which a transverse load on a beam must pass if the member is to bend without twisting.

$$\bar{r}_0^2 = x_0^2 + y_0^2 + \frac{I_x + I_y}{A_g} \qquad \text{(AISC Equation E4-11)}$$

Values of the constants used in the equations for F_e can be found in the dimensions and properties tables in Part 1 of the *Manual*. Table 4.1 shows which constants are given for various types of shapes. Table 4.1 shows that the *Manual* does not give the constants $\bar{r}_0$ and H for tees, although they are given on the Companion CD. They are easily computed, however, if x_0 and y_0 are known. Since x_0 and y_0 are the coordinates of the

TABLE 4.1

Shape	Constants
W, M, S, HP, WT, MT, ST	J, C_w (In addition, the Manual Companion CD gives values of $\bar{r}_0$, and H for WT, MT, and ST shapes)
C	J, C_w, $\bar{r}_0$, H
MC, Angles	J, C_w, $\bar{r}_0$, (In addition, the Manual Companion CD gives values of H for MC and angle shapes.)
Double Angles	$\bar{r}_0$, H (J and C_w are double the values given for single angles.)

shear center with respect to the centroid of the cross section, the location of the shear center must be known. For a tee shape, it is located at the intersection of the centerlines of the flange and the stem. Example 4.15 illustrates the computation of $\bar{r}_0$ and H.

The need for a torsional buckling analysis of a doubly symmetrical shape will be rare. Similarly, shapes with no axis of symmetry are rarely used for compression members, and flexural-torsional buckling analysis of these types of members will seldom, if ever, need to be done. For these reasons, we limit further consideration to flexural-torsional buckling of shapes with one axis of symmetry. Furthermore, the most commonly used of these shapes, the double angle, is a built-up shape, and we postpone consideration of it until Section 4.9.

For singly symmetrical shapes, the flexural-torsional buckling stress F_e is found from AISC Equation E4-5. In this equation, y is defined as the axis of symmetry (regardless of the orientation of the member), and flexural-torsional buckling will take place only about this axis (flexural buckling about this axis will not occur). The x-axis (the axis of no symmetry) is subject only to flexural buckling. Therefore, for singly symmetrical shapes, there are two possibilities for the strength: either flexural-torsional buckling about the y-axis (the axis of symmetry) or flexural buckling about the x-axis (Timoshenko and Gere, 1961 and Zahn and Iwankiw, 1989). To determine which one controls, compute the strength corresponding to each axis and use the smaller value.

The procedure for flexural-torsional buckling analysis of double angles and tees given in AISC Section E4(a) is a modification of the procedure given in AISC E4(b). There is also some notational change: F_e becomes F_{cr}, F_{ey} becomes F_{cry}, and F_{ez} becomes F_{crz}.

To obtain F_{crz}, we can drop the first term of AISC Equation E4-11 to get

$$F_{crz} = \frac{GJ}{A_g \bar{r}_0^2} \qquad \text{(AISC Equation E4-3)}$$

This approximation is acceptable because for double angles and tees, the first term is negligible compared to the second term.

The nominal strength can then be computed as

$$P_n = F_{cr} A_g \qquad \text{(AISC Equation E4-1)}$$

where

$$F_{cr} = \left(\frac{F_{cry} + F_{crz}}{2H}\right)\left[1 - \sqrt{1 - \frac{4F_{cry}F_{crz}H}{(F_{cry} + F_{crz})^2}}\right] \quad \text{(AISC Equation E4-2)}$$

All other terms from Section E4(b) remain unchanged. This procedure, to be used with double angles and tees only, is more accurate than the procedure given in E4(b).

EXAMPLE 4.15

Compute the compressive strength of a WT12 × 81 of A992 steel. The effective length with respect to the *x*-axis is 25 feet 6 inches, the effective length with respect to the *y*-axis is 20 feet, and the effective length with respect to the *z*-axis is 20 feet.

SOLUTION

Because this shape is a nonslender WT, we use the approach of AISC E4(a). First, compute the flexural buckling strength for the *x*-axis (the axis of no symmetry):

$$\frac{K_x L}{r_x} = \frac{25.5 \times 12}{3.50} = 87.43$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(87.43)^2} = 37.44 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113$$

Since $\frac{KL}{r} < 4.71\sqrt{\frac{E}{F_y}}$, AISC Equation E3-2 applies:

$$F_{cr} = 0.658^{(F_y/F_e)} F_y = 0.658^{(50/37.44)}(50) = 28.59 \text{ ksi}$$

The nominal strength is

$$P_n = F_{cr} A_g = 28.59(23.9) = 683.3 \text{ kips}$$

Compute the flexural-torsional buckling strength about the *y*-axis (the axis of symmetry):

Compute F_{cry} using AISC E3:

$$\frac{K_y L}{r_y} = \frac{20 \times 12}{3.05} = 78.69$$

From AISC Equation E3-4,

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2 E}{(K_y L/r_y)^2} = \frac{\pi^2(29{,}000)}{(78.69)^2} = 46.22 \text{ ksi}$$

Since $K_y L/r_y < 4.71\sqrt{\dfrac{E}{F_y}} = 113,$

$$F_{cry} = 0.658^{(F_y/F_e)} F_y = 0.658^{(50/46.22)}(50) = 31.79 \text{ ksi}$$

Because the shear center of a tee is located at the intersection of the centerlines of the flange and the stem,

$$x_0 = 0$$

$$y_0 = \bar{y} - \frac{t_f}{2} = 2.70 - \frac{1.22}{2} = 2.090 \text{ in.}$$

$$\bar{r}_0^2 = x_0^2 + y_0^2 + \frac{I_x + I_y}{A_g} = 0 + (2.090)^2 + \frac{293 + 221}{23.9} = 25.87 \text{ in.}^2$$

$$H = 1 - \frac{x_0^2 + y_0^2}{\bar{r}_0^2} = 1 - \frac{0 + (2.090)^2}{25.87} = 0.8312$$

$$F_{crz} = \frac{GJ}{A_g \bar{r}_0^2} = \frac{11{,}200(9.22)}{23.9(25.87)} = 167.0 \text{ ksi}$$

$$F_{cry} + F_{crz} = 31.79 + 167.0 = 198.8 \text{ ksi}$$

$$F_{cr} = \left(\frac{F_{cry} + F_{crz}}{2H}\right)\left[1 - \sqrt{1 - \frac{4F_{cry}F_{crz}H}{(F_{cry} + F_{crz})^2}}\right]$$

$$= \frac{198.8}{2(0.8312)}\left[1 - \sqrt{1 - \frac{4(31.79)(167.0)(0.8312)}{(198.8)^2}}\right] = 30.63 \text{ ksi}$$

$$P_n = F_{cr} A_g = 30.63(23.9) = 732.1 \text{ kips}$$

The flexural buckling strength controls, and the nominal strength is 683.3 kips.

ANSWER For LRFD, the design strength is $\phi_c P_n = 0.90(683.3) = 615$ kips.

For ASD, the allowable stress is $F_a = 0.6F_{cr} = 0.6(28.59) = 17.15$ ksi, and the allowable strength is $F_a A_g = 17.15(23.9) = 410$ kips.

EXAMPLE 4.16

Compute the compressive strength of a C15 × 50 of A36 steel. The effective lengths with respect to the *x*, *y*, and *z* axes are each 13 feet.

SOLUTION AISC E4(b) must be used, because this shape is nonslender and is neither a double-angle shape nor a tee shape. Check the flexural buckling strength about the *y*-axis (this is the axis of no symmetry for a channel):

$$\frac{K_yL}{r_y} = \frac{13 \times 12}{0.865} = 180.3$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(180.3)^2} = 8.805 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{36}} = 133.7$$

Since $\dfrac{KL}{r} > 4.71\sqrt{\dfrac{E}{F_y}}$, AISC Equation E3-2 applies:

$$F_{cr} = 0.877F_e = 0.877(8.805) = 7.722 \text{ ksi}$$

The nominal strength is

$$P_n = F_{cr}A_g = 7.722(14.7) = 113.5 \text{ kips}$$

Compute the flexural-torsional buckling strength about the x-axis (this is the axis of symmetry for a channel):

$$\frac{K_xL}{r_x} = \frac{13 \times 12}{5.24} = 29.77$$

$$F_{ey} = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(29.77)^2} = 323.0 \text{ ksi}$$

$$F_{ez} = \left[\frac{\pi^2 EC_w}{(K_zL)^2} + GJ\right]\frac{1}{A\bar{r}_o^2}$$

$$= \left[\frac{\pi^2(29{,}000)(492)}{(13 \times 12)^2} + 11{,}200(2.65)\right]\frac{1}{14.7(5.49)^2} = 80.06 \text{ ksi } (\bar{r}_o \text{ is tabulated})$$

$$F_{ey} + F_{ez} = 323.0 + 80.06 = 403.1 \text{ ksi}$$

$$F_e = \left(\frac{F_{ey} + F_{ez}}{2H}\right)\left[1 - \sqrt{1 - \frac{4F_{ey}F_{ez}H}{(F_{ey} + F_{ez})^2}}\right]$$

$$= \frac{403.1}{2(0.937)}\left[1 - \sqrt{1 - \frac{4(323.0)(80.06)(0.937)}{(403.1)^2}}\right] = 78.46 \text{ ksi} \quad (H \text{ is tabulated})$$

Since

$$\frac{K_xL}{r_x} < 4.71\sqrt{\frac{E}{F_y}} = 133.7$$

use AISC Equation E3-2:

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(36/78.46)}(36) = 29.71 \text{ ksi}$$

The nominal strength is

$$P_n = F_{cr}A_g = 29.71(14.7) = 436.7 \text{ kips}$$

The flexural buckling strength controls, and the nominal strength is 113.5 kips.

ANSWER For LRFD, the design strength is $\phi_c P_n = 0.90(113.5) = 102$ kips.

For ASD, the allowable stress is $F_a = 0.6F_{cr} = 0.6(7.722) = 4.633$ ksi, and the allowable strength is $F_a A_g = 4.633(14.7) = 68.1$ kips.

The procedure used in Example 4.15, which is based on AISC Specification E4(a), should always be used for double angles and tees. In practice, however, the strength of most double angles and tees can be found in the column load tables. These tables give two sets of values of the available strength, one based on flexural buckling about the *x*-axis and one based on flexural-torsional buckling about the y axis. The flexural-torsional buckling strengths are based on the procedure of AISC E4(a).

Available compressive strength tables are also provided for single-angle members. The values of strength in these tables are not based on flexural-torsional buckling theory, but on the provisions of AISC E5.

When using the column load tables for unsymmetrical shapes, there is no need to account for slender compression elements, because that has already been done. If an analysis is being done for a member not in the column load tables, then any element slenderness must be accounted for.

4.9 BUILT-UP MEMBERS

If the cross-sectional properties of a built-up compression member are known, its analysis is the same as for any other compression member, provided the component parts of the cross section are properly connected. AISC E6 contains many details related to this connection, with separate requirements for members composed of two or more rolled shapes and for members composed of plates or a combination of plates and shapes. Before considering the connection problem, we will review the computation of cross-sectional properties of built-up shapes.

The design strength of a built-up compression member is a function of the slenderness ratio KL/r. Hence the principal axes and the corresponding radii of gyration about these axes must be determined. For homogeneous cross sections, the principal axes coincide with the centroidal axes. The procedure is illustrated in Example 4.17. The components of the cross section are assumed to be properly connected.

EXAMPLE 4.17

The column shown in Figure 4.19 is fabricated by welding a ⅜-inch by 4-inch cover plate to the flange of a W18 × 65. Steel with $F_y = 50$ ksi is used for both components. The effective length is 15 feet with respect to both axes. Assume that the components are connected in such a way that the member is fully effective and compute the strength based on flexural buckling.

FIGURE 4.19

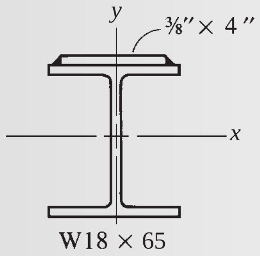


TABLE 4.2

Component	A	y	Ay
Plate	1.500	0.1875	0.2813
W	19.10	9.575	182.9
Σ	20.60		183.2

SOLUTION With the addition of the cover plate, the shape is slightly unsymmetrical, but the flexural-torsional effects will be negligible.

The vertical axis of symmetry is one of the principal axes, and its location need not be computed. The horizontal principal axis will be found by application of the *principle of moments:* The sum of moments of component areas about any axis (in this example, a horizontal axis along the top of the plate will be used) must equal the moment of the total area. We use Table 4.2 to keep track of the computations.

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{183.2}{20.60} = 8.893 \text{ in.}$$

With the location of the horizontal centroidal axis known, the moment of inertia with respect to this axis can be found by using the *parallel-axis theorem:*

$$I = \bar{I} + Ad^2$$

where

$\bar{I}$ = moment of inertia about the centroidal axis of a component area
A = area of the component
I = moment of inertia about an axis parallel to the centroidal axis of the component area
d = perpendicular distance between the two axes

The contributions from each component area are computed and summed to obtain the moment of inertia of the composite area. These computations are shown in Table 4.3, which is an expanded version of Table 4.2. The moment of inertia about the x-axis is

$$I_x = 1193 \text{ in.}^4$$

TABLE 4.3

Component	A	y	Ay	$\bar{I}$	d	$\bar{I} + Ad^2$
Plate	1.500	0.1875	0.2813	0.01758	8.706	113.7
W	19.10	9.575	182.9	1070	0.6820	1079
Σ	20.60		183.2			1193

For the vertical axis,

$$I_y = \frac{1}{12}\left(\frac{3}{8}\right)(4)^3 + 54.8 = 56.80 \text{ in.}^4$$

Since $I_y < I_x$, the y-axis controls.

$$r_{\min} = r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{56.80}{20.60}} = 1.661 \text{ in.}$$

$$\frac{KL}{r_{\min}} = \frac{15 \times 12}{1.661} = 108.4$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(108.4)^2} = 24.36 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113$$

Since $\dfrac{KL}{r} < 4.71\sqrt{\dfrac{E}{F_y}}$, use AISC Equation E3-2.

$$F_{cr} = 0.658^{(F_y/F_e)} F_y = 0.658^{(50/24.36)}(50) = 21.18 \text{ ksi}$$

The nominal strength is

$$P_n = F_{cr}A_g = 21.18(20.60) = 436.3 \text{ kips}$$

LRFD SOLUTION

The design strength is

$$\phi_c P_n = 0.90(436.3) = 393 \text{ kips}$$

ASD SOLUTION

From Equation 4.7, the allowable stress is

$$F_a = 0.6F_{cr} = 0.6(21.18) = 12.71 \text{ ksi}$$

The allowable strength is

$$F_a A_g = 12.71(20.60) = 262 \text{ kips}$$

ANSWER

Design compressive strength = 393 kips. Allowable compressive strength = 262 kips.

Connection Requirements for Built-Up Members Composed of Rolled Shapes

The most common built-up shape is one that is composed of rolled shapes, namely, the double-angle shape. This type of member will be used to illustrate the requirements for this category of built-up members. Figure 4.20 shows a truss compression member connected to gusset plates at each end. To maintain the back-to-back separation of the angles along the length, fillers (spacers) of the same thickness as the gusset plate are placed between the angles at equal intervals. The intervals must be small enough that the member functions as a unit. If the member buckles about the *x*-axis (flexural buckling), the connectors are not subjected to any calculated load, and the connection problem is simply one of maintaining the relative positions of the two components. To ensure that the built-up member acts as a unit, AISC E6.2 requires that the slenderness of an individual component be no greater than three-fourths of the slenderness of the built-up member; that is,

$$\frac{Ka}{r_i} \leq \frac{3}{4}\frac{KL}{r} \tag{4.14}$$

where

a = spacing of the connectors
r_i = smallest radius of gyration of the component
Ka/r_i = effective slenderness ratio of the component
KL/r = maximum slenderness ratio of the built-up member

If the member buckles about the axis of symmetry—that is, if it is subjected to flexural-torsional buckling about the *y*-axis—the connectors are subjected to shearing forces. This condition can be visualized by considering two planks used as a beam, as shown in Figure 4.21. If the planks are unconnected, they will slip along the surface of contact when loaded and will function as two separate beams. When connected by bolts (or any other fasteners, such as nails), the two planks will behave as a unit, and the resistance to slip will be provided by shear in the bolts. This behavior takes place

FIGURE 4.20

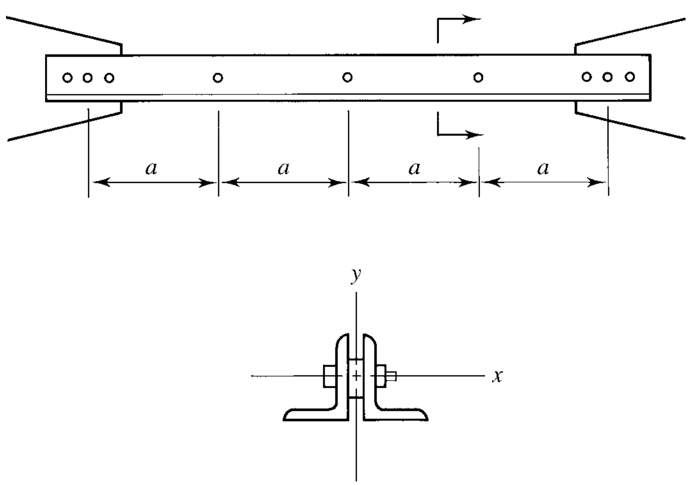
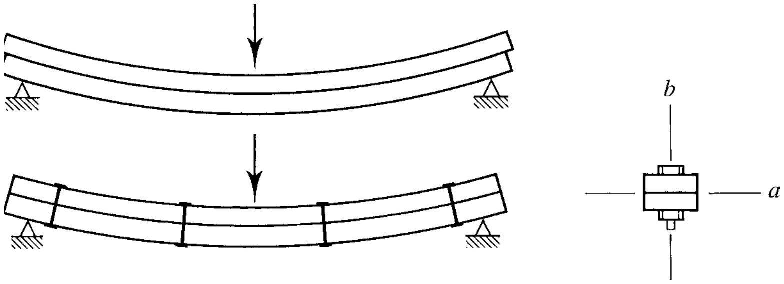


FIGURE 4.21



in the double-angle shape when bending about its y-axis. If the plank beam is oriented so that bending takes place about its other axis (the b-axis), then both planks bend in exactly the same manner, and there is no slippage and hence no shear. This behavior is analogous to bending about the x-axis of the double-angle shape. When the fasteners are subjected to shear, a modified slenderness ratio larger than the actual value may be required.

AISC E6 considers two categories of intermediate connectors: (1) snug-tight bolts and (2) welds or fully-tensioned bolts. We cover these connection methods in detail in Chapter 7, “Simple Connections.”

When the connectors are snug-tight bolts, the modified slenderness ratio is

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_0^2 + \left(\frac{a}{r_i}\right)^2} \qquad \text{(AISC Equation E6-1)}$$

where

$$\left(\frac{KL}{r}\right)_0 = \text{original unmodified slenderness ratio}$$

When the connectors are fully-tensioned bolts or welds, the modified slenderness ratio depends on the value of a/r_i:

When $a/r_i \leq 40$, the slenderness ratio is not modified; that is,

$$\left(\frac{KL}{r}\right)_m = \left(\frac{KL}{r}\right)_0 \qquad \text{(AISC Equation E6-2a)}$$

When $a/r_i > 40$,

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_0^2 + \left(\frac{K_i a}{r_i}\right)^2} \qquad \text{(AISC Equation E6-2b)}$$

where

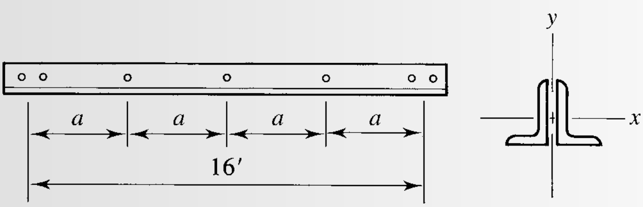
K_i = 0.5 for angles back-to-back
= 0.75 for channels back-to-back
= 0.86 for all other cases

The column load tables for double angles are based on the use of welds or fully tightened bolts. These tables show the number of intermediate connectors required for the given y-axis flexural-torsional buckling strength. The number of connectors needed for the x-axis flexural buckling strength must be determined from the requirement of Equation 4.14 that the slenderness of one angle between connectors must not exceed three-fourths of the overall slenderness of the double-angle shape.

EXAMPLE 4.18

Compute the available strength of the compression member shown in Figure 4.22. Two angles, 5 × 3 × ½, are oriented with the long legs back-to-back (2L5 × 3 × ½ LLBB) and separated by ⅜ inch. The effective length KL is 16 feet, and there are three fully tightened intermediate connectors. A36 steel is used.

FIGURE 4.22



SOLUTION Compute the flexural buckling strength for the x-axis:

$$\frac{K_x L}{r_x} = \frac{16(12)}{1.58} = 121.5$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2 (29{,}000)}{(121.5)^2} = 19.39 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{36}} = 134$$

Since $\frac{KL}{r} < 4.71\sqrt{\frac{E}{F_y}}$, use AISC Equation E3-2.

$$F_{cr} = 0.658^{(F_y/F_e)} F_y = 0.658^{(36/19.39)}(36) = 16.55 \text{ ksi}$$

The nominal strength is

$$P_n = F_{cr} A_g = 16.55(7.50) = 124.1 \text{ kips}$$

To determine the flexural-torsional buckling strength for the y-axis, use the modified slenderness ratio, which is based on the spacing of the connectors. The unmodified slenderness ratio is

$$\left(\frac{KL}{r}\right)_0 = \frac{KL}{r_y} = \frac{16(12)}{1.24} = 154.8$$

The spacing of the connectors is

$$a = \frac{16(12)}{4 \text{ spaces}} = 48 \text{ in.}$$

Then, from Equation 4.14,

$$\frac{Ka}{r_i} = \frac{Ka}{r_z} = \frac{48}{0.642} = 74.77 < 0.75(154.8) = 116.1 \quad \text{(OK)}$$

Compute the modified slenderness ratio, $(KL/r)_m$:

$$\frac{a}{r_i} = \frac{48}{0.642} = 74.77 > 40 \qquad \therefore \text{Use AISC Equation E6-2b}$$

$$\frac{K_i a}{r_i} = \frac{0.5(48)}{0.642} = 37.38$$

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_o^2 + \left(\frac{K_i a}{r_i}\right)^2} = \sqrt{(154.8)^2 + (37.38)^2} = 159.2$$

This value should be used in place of KL/r_y for the computation of F_{cry}:

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(159.2)^2} = 11.29 \text{ ksi}$$

Since $\dfrac{KL}{r} > 4.71\sqrt{\dfrac{E}{F_y}} = 134$,

$$F_{cry} = 0.877F_e = 0.877(11.29) = 9.901 \text{ ksi}$$

From AISC Equation E4-3,

$$F_{crz} = \frac{GJ}{A_g \bar{r}_o^2} = \frac{11{,}200(2 \times 0.322)}{7.50(2.51)^2} = 152.6 \text{ ksi}$$

$$F_{cry} + F_{crz} = 9.901 + 152.6 = 162.5 \text{ ksi}$$

$$F_{cr} = \left(\frac{F_{cry} + F_{crz}}{2H}\right)\left[1 - \sqrt{1 - \frac{4F_{cry}F_{crz}H}{(F_{cry} + F_{crz})^2}}\right]$$

$$= \frac{162.5}{2(0.646)}\left[1 - \sqrt{1 - \frac{4(9.832)(152.6)(0.646)}{(162.5)^2}}\right] = 9.599 \text{ ksi}$$

The nominal strength is

$$P_n = F_{cr}A_g = 9.599(7.50) = 71.99 \text{ kips}$$

Therefore the flexural-torsional buckling strength controls.

LRFD SOLUTION

The design strength is

$$\phi_c P_n = 0.90(71.99) = 64.8 \text{ kips}$$

ASD SOLUTION

From Equation 4.7, the allowable stress is

$$F_a = 0.6F_{cr} = 0.6(9.599) = 5.759 \text{ ksi}$$

The allowable strength is

$$F_a A_g = 5.759(7.50) = 43.2 \text{ kips}$$

ANSWER

Design compressive strength = 64.8 kips. Allowable compressive strength = 43.2 kips.

EXAMPLE 4.19

Design a 14-foot-long compression member to resist a service dead load of 12 kips and a service live load of 23 kips. Use a double-angle shape with the short legs back-to-back, separated by ⅜-inch. The member will be braced at midlength against buckling about the x-axis (the axis parallel to the long legs). Specify the number of intermediate connectors needed (the midlength brace will provide one such connector). Use A36 steel.

LRFD SOLUTION

The factored load is

$$P_u = 1.2D + 1.6L = 1.2(12) + 1.6(23) = 51.2 \text{ kips}$$

From the column load tables, select 2L 3½ × 3 × ¼ SLBB, weighing 10.8 lb/ft. The capacity of this shape is 53.2 kips, based on buckling about the y-axis with an effective length of 14 feet. (The strength corresponding to flexural buckling about the x-axis is 63.1 kips, based on an effective length of $^{14}/_{2} = 7$ feet.) Note that this shape is a slender-element cross section, but this is taken into account in the tabular values.

Bending about the y-axis subjects the fasteners to shear, so a sufficient number of fasteners must be provided to account for this action. The table reveals that three intermediate connectors are required. (This number also satisfies Equation 4.14.)

ANSWER Use 2L 3½ × 3 × ¼ SLBB with three intermediate connectors within the 14-foot length.

ASD SOLUTION The total load is

$$P_a = D + L = 12 + 23 = 35 \text{ kips}$$

From the column load tables, select 2L 3¼ × 3 × ¼ SLBB, weighing 10.8 lb/ft. The capacity is 35.4 kips, based on buckling about the y axis, with an effective length of 14 feet. (The strength corresponding to flexural buckling about the x axis is 42.0 kips, based on an effective length of $^{14}/_{2} = 7$ feet.) Note that this shape is a slender-element section, but this is taken into account in the tabular values.

Bending about the y axis subjects the fasteners to shear, so a sufficient number of fasteners must be provided to account for this action. The table reveals that three intermediate connectors are required. (This number also satisfies Equation 4.14.)

ANSWER Use 2L 3½ × 3 × ¼ SLBB with three intermediate connectors within the 14-foot length.

Connection Requirements for Built-Up Members Composed of Plates or Both Plates and Shapes

When a built-up member consists of two or more rolled shapes separated by a substantial distance, plates must be used to connect the shapes. AISC E6 contains many details regarding the connection requirements and the proportioning of the plates. Additional connection requirements are given for other built-up compression members composed of plates or plates and shapes.

Problems

AISC Requirements

4.3-1 Use AISC Equation E3-2 or E3-3 and determine the nominal axial compressive strength for the following cases:

a. $L = 15$ ft

b. $L = 20$ ft

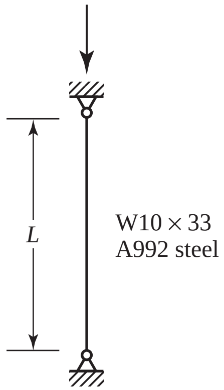


FIGURE P4.3-1

4.3-2 Compute the nominal axial compressive strength of the member shown in Figure P4.3-2. Use AISC Equation E3-2 or E3-3.

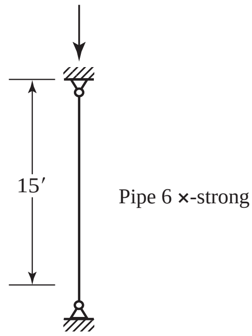


FIGURE P4.3-2

4.3-3 Compute the nominal compressive strength of the member shown in Figure P4.3-3. Use AISC Equation E3-2 or E3-3.

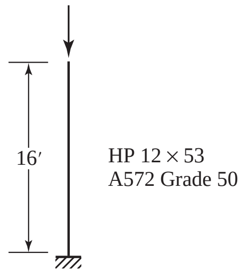


FIGURE P4.3-3

4.3-4 Determine the available strength of the compression member shown in Figure P4.3-4, in each of the following ways:

a. Use AISC Equation E3-2 or E3-3. Compute both the design strength for LRFD and the allowable strength for ASD.
b. Use Table 4-22 from Part 4 of the *Manual.* Compute both the design strength for LRFD and the allowable strength for ASD.

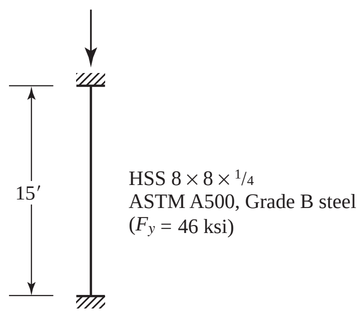


FIGURE P4.3-4

4.3-5 A W18 × 119 is used as a compression member with one end fixed and the other end fixed against rotation but free to translate. The length is 12 feet. If A992 steel is used, what is the available compressive strength?

a. Use AISC Equation E3-2 or E3-3. Compute both the design strength for LRFD and the allowable strength for ASD.
b. Use Table 4-22 from Part 4 of the *Manual.* Compute both the design strength for LRFD and the allowable strength for ASD.

4.3-6 Does the column shown in Figure P4.3-6 have enough available strength to support the given service loads?

a. Use LRFD.
b. Use ASD.

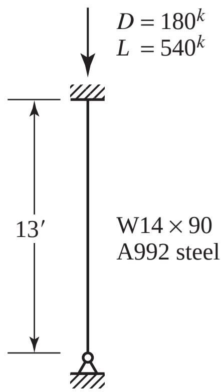


FIGURE P4.3-6

4.3-7 Determine whether the compression member shown in Figure P4.3-7 is adequate to support the given service loads.

a. Use LRFD.

b. Use ASD.

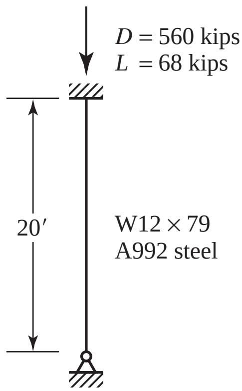


FIGURE P4.3-7

4.3-8 Determine the maximum axial compressive service load that can be supported if the live load is twice as large as the dead load. Use AISC Equation E3-2 or E3-3.

a. Use LRFD.

b. Use ASD.

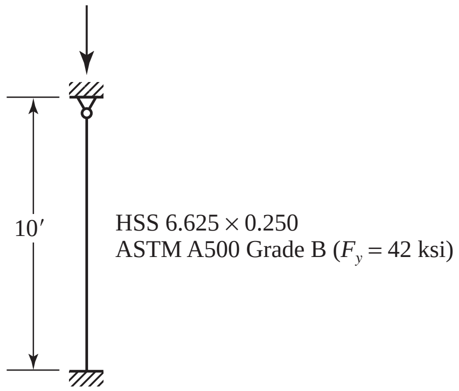


FIGURE P4.3-8

Local Stability

4.4-1 An HSS10 × 8 × 3⁄16 is used as a compression member with one end pinned and the other end fixed against rotation but free to translate. The length is 12 feet. Compute the nominal compressive strength for A500 Grade B steel ($F_y = 46$ ksi). *Note that this is a slender-element compression member, and the equations of AISC Section E7 must be used.*

4.4-2 A W21 × 101 is used as a compression member with one end fixed and the other end free. The length is 10 feet. What is the nominal compressive strength if $F_y = 50$ ksi? *Note that this is a slender-element compression member, and the equations of AISC Section E7 must be used.*

Design

4.6-1
a. Select a W12 of A992 steel. Use the column load tables.
   1. Use LRFD.
   2. Use ASD.

b. Select a W18 of A992 steel. Use the trial-and-error approach covered in Section 4.6.
   1. Use LRFD.
   2. Use ASD.

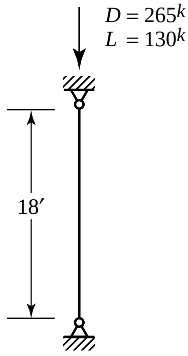


FIGURE P4.6-1

4.6-2 A 15-foot long column is pinned at the bottom and fixed against rotation but free to translate at the top. It must support a service dead load of 100 kips and a service live load of 100 kips.

a. Select a W12 of A992 steel. Use the column load tables.
   1. Use LRFD.
   2. Use ASD.

b. Select a W16 of A992 steel. Use the trial-and-error approach covered in Section 4.6.
   1. Use LRFD.
   2. Use ASD.

4.6-3 Select a square HSS ($F_y = 46$ ksi).

a. Use LRFD.

b. Use ASD.

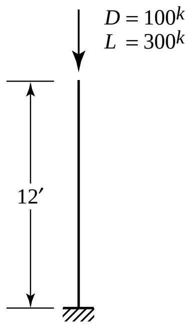


FIGURE P4.6-3

4.6-4 Select a steel pipe. Specify whether your selection is Standard, Extra-Strong, or Double-Extra Strong.

a. Use LRFD.

b. Use ASD.

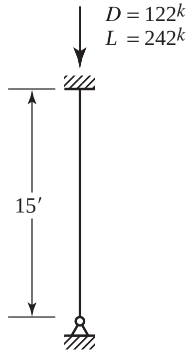


FIGURE P4.6-4

4.6-5 Select an HP-shape for the conditions of Problem 4.6-4. Use $F_y = 50$ ksi.
a. Use LRFD.
b. Use ASD.

4.6-6 Select a rectangular (not square) HSS for the conditions of Problem 4.6-3.
a. Use LRFD.
b. Use ASD.

4.6-7 For the conditions shown in Figure P4.6-7, use LRFD and do the following.
a. Select a W10 of A992 steel.
b. Select a square HSS.
c. Select a rectangular HSS.
d. Select a round HSS.

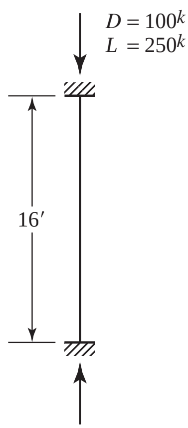


FIGURE P4.6-7

4.6-8 Same as Problem 4.6-7, but use ASD.

4.6-9 For the conditions shown in Figure P4.6-7, use LRFD and select the lightest W21 shape of A992 steel. Do not exclude slender shapes from consideration.

Effective Length

4.7-1 A W18 × 97 with $F_y = 60$ ksi is used as a compression member. The length is 13 feet. Compute the nominal strength for $K_x = 2.2$ and $K_y = 1.0$.

4.7-2 An HSS 10 × 6 × 5⁄16 with $F_y = 46$ ksi is used as a column. The length is 16 feet. Both ends are pinned, and there is support against weak axis buckling at a point 6 feet from the top. Determine

a. the design strength for LRFD.

b. the allowable *stress* for ASD.

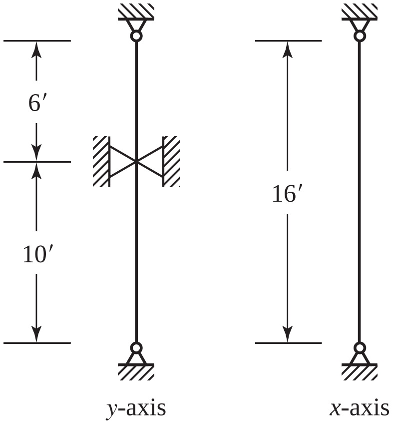


FIGURE P4.7-2

4.7-3 A W12 × 65 of A572 Grade 60 steel is used as a compression member. It is 26 feet long, pinned at each end, and has additional support in the weak direction at a point 12 feet from the top. Can this member resist a service dead load of 180 kips and a service live load of 320 kips?

a. Use LRFD.

b. Use ASD.

4.7-4 Use A992 steel and select a W12 shape for an axially loaded column to meet the following specifications: The length is 24 feet, both ends are pinned, and there is bracing in the weak direction at a point 10 feet from the top. The service dead load is 142 kips, and the service live load is 356 kips.

a. Use LRFD.

b. Use ASD.

4.7-5 Use A992 steel and select a W shape.

a. Use LRFD.

b. Use ASD.

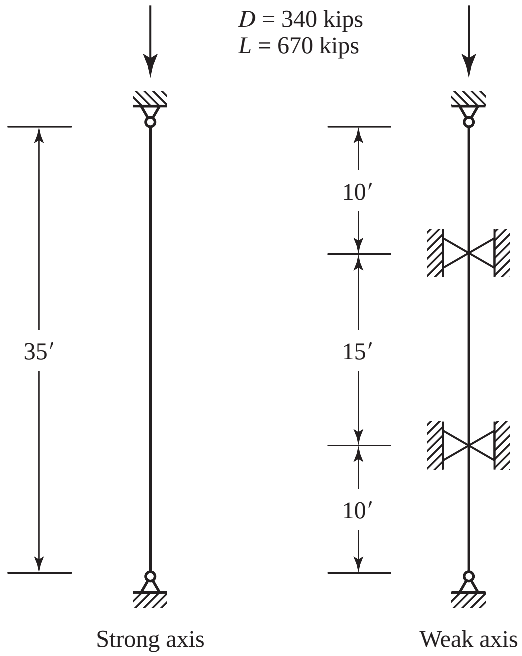


FIGURE P4.7-5

4.7-6 Select a square HSS for use as a 15-foot-long compression member that must resist a service dead load of 35 kips and a service live load of 80 kips. The member will be pinned at each end, with additional support in the weak direction at midheight. Use A500 Grade B steel ($F_y = 46$ ksi).

a. Use LRFD.

b. Use ASD.

4.7-7 Select the best rectangular (not square) HSS for a column to support a service dead load of 30 kips and a service live load of 90 kips. The member is 22 feet long and is pinned at the ends. It is supported in the weak direction at a point 12 feet from the top. Use $F_y = 46$ ksi.

a. Use LRFD.

b. Use ASD.

4.7-8 The frame shown in Figure P4.7-8 is unbraced, and bending is about the x-axis of the members. All beams are W16 × 40, and all columns are W12 × 58.

a. Determine the effective length factor K_x for column AB. Do not consider the stiffness reduction factor.

b. Determine the effective length factor K_x for column BC. Do not consider the stiffness reduction factor.

c. If $F_y = 50$ ksi, is the stiffness reduction factor applicable to these columns?

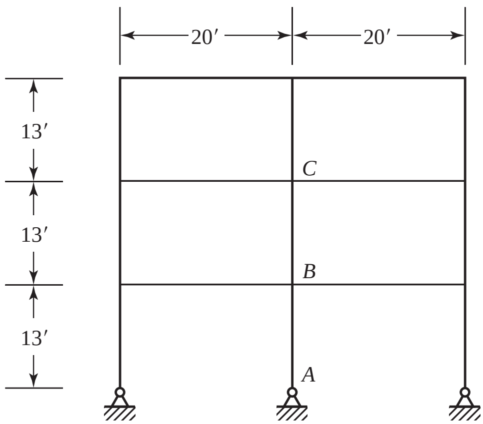


FIGURE P4.7-8

4.7-9 The given frame is unbraced, and bending is about the x axis of each member. The axial dead load supported by column AB is 155 kips, and the axial live load is 460 kips. $F_y = 50$ ksi. Determine K_x for member AB. Use the stiffness reduction factor if applicable.

a. Use LRFD.

b. Use ASD.

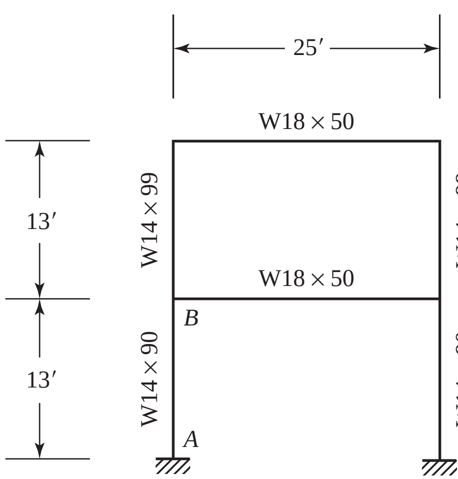


FIGURE P4.7-9

4.7-10 The rigid frame shown in Figure P4.7-10 is unbraced. The members are oriented so that bending is about the strong axis. Support conditions in the direction perpendicular to the plane of the frame are such that $K_y = 1.0$. The beams are W16 × 57, and the columns are W10 × 100. A992 steel is used. The axial compressive dead load is 90 kips, and the axial compressive live load is 110 kips.

a. Determine the axial compressive design strength of column *AB*. Use the stiffness reduction factor if applicable.
b. Determine the allowable axial compressive strength of column *AB*. Use the stiffness reduction factor if applicable.

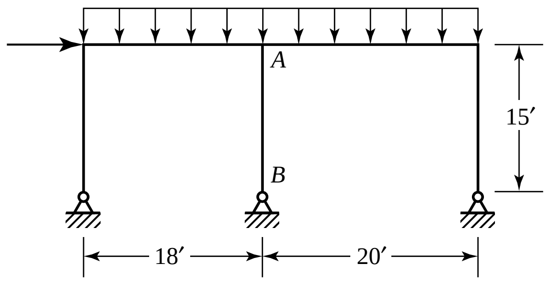


FIGURE P4.7-10

4.7-11 The frame shown in Figure P4.7-11 is unbraced against sidesway. Relative moments of inertia of the members have been assumed for preliminary design purposes. Use the alignment chart and determine K_x for members *AB*, *BC*, *DE*, and *EF*.

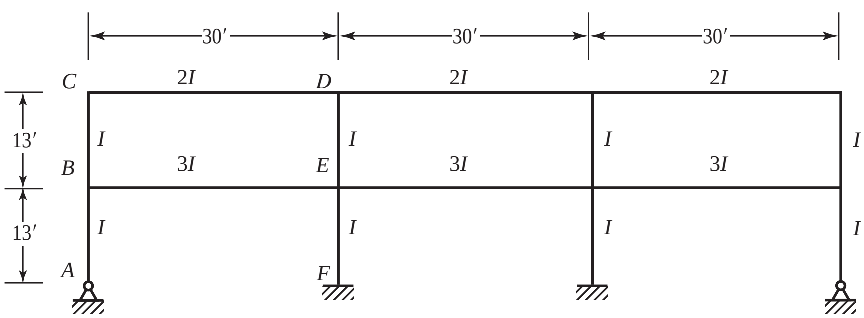


FIGURE P4.7-11

4.7-12 An unbraced frame is shown in Figure P4.7-12. Use LRFD and the alignment chart to check the adequacy of the following columns for $F_y = 50$ ksi. Use the stiffness reduction factor if applicable. Use $K_y = 1.0$.

a. Column *AB*, $P_u = 750$ kips.
b. Column *MN*, $P_u = 1000$ kips.
c. Column *BC*, $P_u = 600$ kips.
d. Column *LM*, $P_u = 1200$ kips.
e. Column *FG*, $P_u = 240$ kips.
f. Column *HI*, $P_u = 480$ kips.

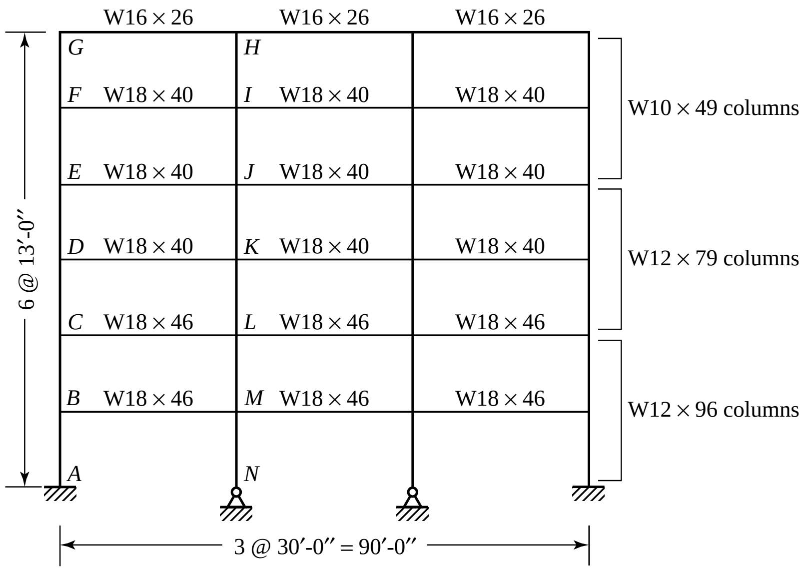


FIGURE P4.7-12

4.7-13 The rigid frame shown in Figure P4.7-13 is unbraced in the plane of the frame. In the direction perpendicular to the frame, the frame is braced at the joints. The connections at these points of bracing are simple (moment-free) connections. Roof girders are W14 × 26, and floor girders are W16 × 40. Member *BC* is a W12 × 50. Use A992 steel and select a W-shape for *AB*. Assume that the controlling load combination causes no moment in *AB*. The service dead load is 48 kips and the service live load is 72 kips. Use LRFD.

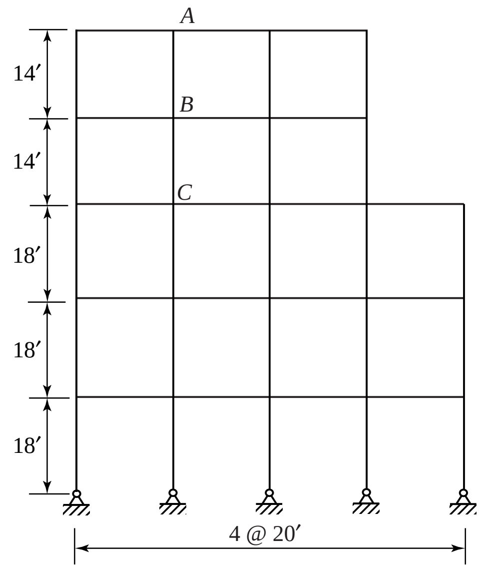


FIGURE P4.7-13

Torsional and Flexural-Torsional Buckling

4.8-1 Use A992 steel and compute the nominal compressive strength of a WT10.5 × 66 with an effective length of 16 feet with respect to each axis. Use the AISC Specification equations. Do not use the column load tables.

4.8-2 Use A36 steel and compute the nominal strength of the column shown in Figure P4.8-2. The member ends are fixed in all directions (x, y, and z).

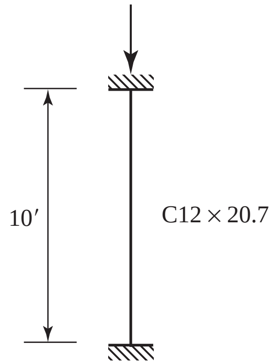


FIGURE P4.8-2

4.8-3 Select a WT section for the compression member shown in Figure P4.8-3. The load is the total service load, with a live-to-dead load ratio of 2:1. Use $F_y = 50$ ksi.

a. Use LRFD.

b. Use ASD.

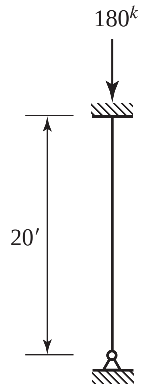


FIGURE P4.8-3

4.8-4 Select an American Standard Channel for the compression member shown in Figure P4.8-4. Use A572 Grade 50 steel. The member ends are fixed in all directions (x, y, and z).

a. Use LRFD.

b. Use ASD.

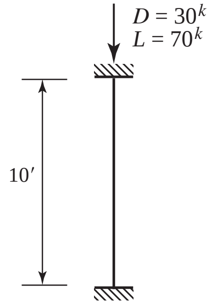


FIGURE P4.8-4

Built-Up Members

4.9-1 Verify the value of r_y given in Part 1 of the *Manual* for the double-angle shape 2L4 × 3½ × ¼ SLBB. The angles will be connected to a ⅜ -inch-thick gusset plate.

4.9-2 Verify the values of y_2, r_x, and r_y given in Part 1 of the *Manual* for the combination shape consisting of an S12 × 31.8 with a C8 × 11.5 cap channel.

4.9-3 A column is built up from four 5 × 5 × ¾ angle shapes as shown in Figure P4.9-3. The plates are not continuous but are spaced at intervals along the column length and function to maintain the separation of the angles. They do not contribute to the cross-sectional properties. Compute r_x and r_y.

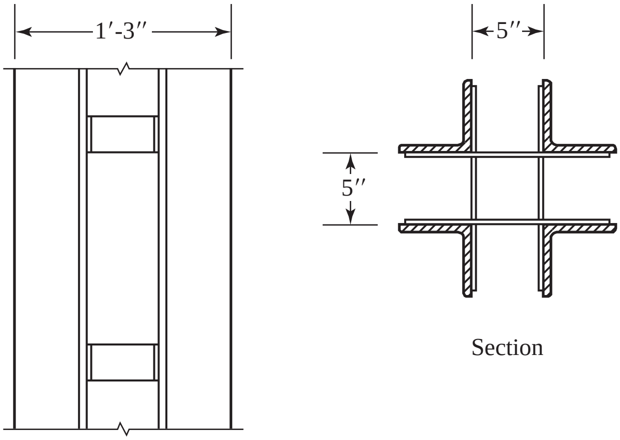


FIGURE P4.9-3

4.9-4 An unsymmetrical compression member consists of a ½ × 12 top flange, a ½ × 6 bottom flange, and a 5⁄16 × 16 web (the shape is symmetrical about an axis parallel to the web depth). Compute the radius of gyration about each of the principal axes.

4.9-5 Compute the nominal axial compressive strength based on flexural buckling (no torsional or flexural-torsional buckling). Assume that the cross-sectional elements are connected such that the built-up shape is fully effective. ASTM A242 steel is used.

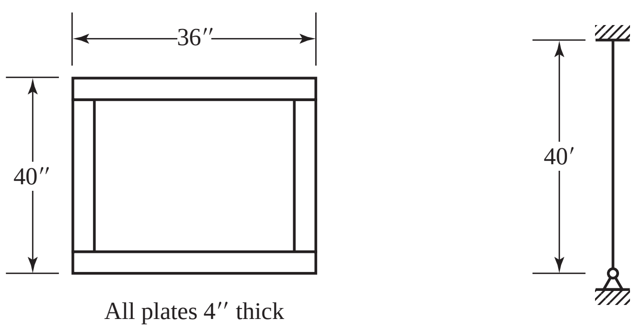


FIGURE P4.9-5

4.9-6 Compute the axial compressive design strength based on flexural buckling (no torsional or flexural-torsional buckling). Assume that the cross-sectional elements are connected such that the built-up shape is fully effective.

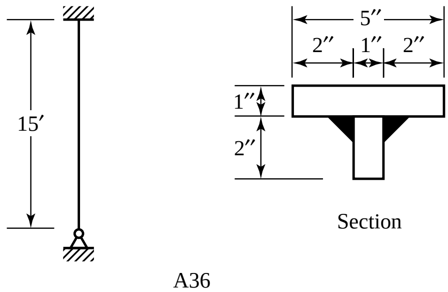


FIGURE P4.9-6

4.9-7 A compression member is made up of two channels, 2C5 × 9, placed back-to-back and separated by 3⁄8 inch (for connection to a 3⁄8-inch-thick gusset plate). The two components are connected along their length in such a way as to maintain the 3⁄8-inch separation. The effective length with respect to each axis is 14 feet, and A242 Grade 50 steel is used.

a. Verify the value of r_y given in the properties table in the *Manual*.
b. Neglect flexural-torsional buckling and compute the allowable axial compressive strength.

4.9-8 In order to reinforce a column in an existing structure, two channels are welded to the column as shown in Figure P4.9-8. $F_y = 50$ ksi for both the column and the channels. The effective length with respect to each axis is 16 feet. What is the available axial compressive strength? What is the percent increase in strength?

a. Use LRFD.
b. Use ASD.

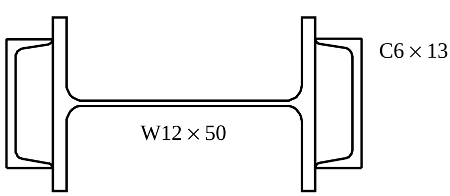


FIGURE P4.9-8

4.9-9 A compression member is built up from a W14 × 90 and a W10 × 49, both of A992 steel.

a. Compute r_x and r_y for the built-up shape.

b. Neglect flexural-torsional buckling and compute the available strength for $K_xL = K_yL = 30$ feet.

   i. Use LRFD.

   ii. Use ASD.

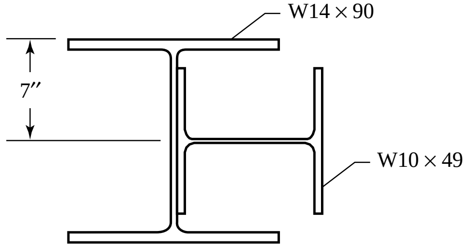


FIGURE P4.9-9

4.9-10 Compute the design strength for LRFD and the allowable strength for ASD for the following double-angle shape: 2L8 × 4 × ¾, long legs ⅜-in. back-to-back, $F_y = 36$ ksi; KL is 20 feet for all axes, and there are two intermediate connectors. Use the procedure of AISC Section E4(a). Do not use the column load tables. Compare the flexural and the flexural-torsional buckling strengths.

4.9-11 For the conditions shown in Figure P4.9-11, select a double-angle section (⅜-in. gusset plate connection). Use A36 steel. Specify the number of intermediate connectors.

a. Use LRFD.

b. Use ASD.

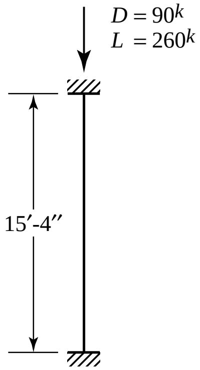


FIGURE P4.9-11

4.9-12 Use ASD and select a WT section for the compression member shown in Figure P4.9-12. The load shown is the total service load, consisting of dead and live loads. Use A992 steel.

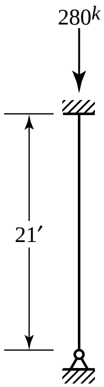


FIGURE P4.9-12

4.9-13 Use ASD and select a double-angle shape for the top chord of the truss of Problem 3.8-2. Use $K_x = K_y = 1.0$. Assume ⅜-inch gusset plates, and use A36 steel.

CHAPTER 5

Beams

5.1 INTRODUCTION

Beams are structural members that support transverse loads and are therefore subjected primarily to flexure, or bending. If a substantial amount of axial load is also present, the member is referred to as a *beam–column* (beam–columns are considered in Chapter 6). Although some degree of axial load will be present in any structural member, in many practical situations this effect is negligible and the member can be treated as a beam. Beams are usually thought of as being oriented horizontally and subjected to vertical loads, but that is not necessarily the case. A structural member is considered to be a beam if it is loaded so as to cause bending.

Commonly used cross-sectional shapes include the W, S, and M shapes. Channel shapes are sometimes used, as are beams built up from plates, in the form of I or box shapes. For reasons to be discussed later, doubly symmetric shapes such as the standard rolled W, M, and S shapes are the most efficient.

Coverage of beams in the AISC Specification is spread over two chapters: Chapter F, "Design of Members for Flexure," and Chapter G, "Design of Members for Shear." Several categories of beams are covered in the Specification; in this book, we cover the most common cases in the present chapter, and we cover a special case, plate girders, in Chapter 10.

Figure 5.1 shows two types of beam cross sections; a hot-rolled doubly-symmetric I shape and a welded doubly-symmetric built-up I shape. The hot-rolled I shape is the one most commonly used for beams. Welded shapes usually fall into the category classified as plate girders.

For flexure (shear will be covered later), the required and available strengths are moments. For load and resistance factor design (LRFD), Equation 2.6 can be written as

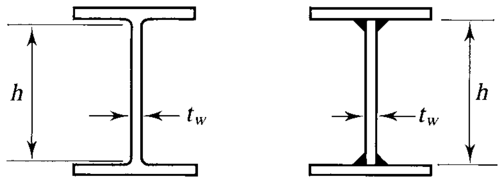
$$M_u \le \phi_b M_n \qquad (5.1)$$

where

M_u = required moment strength = maximum moment caused by the controlling load combination from ASCE 7

ϕ_b = resistance factor for bending (flexure) = 0.90

FIGURE 5.1



M_n = nominal moment strength

The right-hand side of Equation 5.1 is the design strength, sometimes called the *design moment.*

For allowable strength design (ASD), Equation 2.7 can be written as

$$M_a \leq \frac{M_n}{\Omega_b} \tag{5.2}$$

where

M_a = required moment strength = maximum moment corresponding to the controlling load combination from ASCE 7

Ω_b = safety factor for bending = 1.67

Equation 5.2 can also be written as

$$M_a \leq \frac{M_n}{1.67} = 0.6M_n$$

Dividing both sides by the elastic section modulus S (which will be reviewed in the next section), we get an equation for *allowable stress design:*

$$\frac{M_a}{S} \leq \frac{0.6M_n}{S}$$

or

$$f_b \leq F_b$$

where

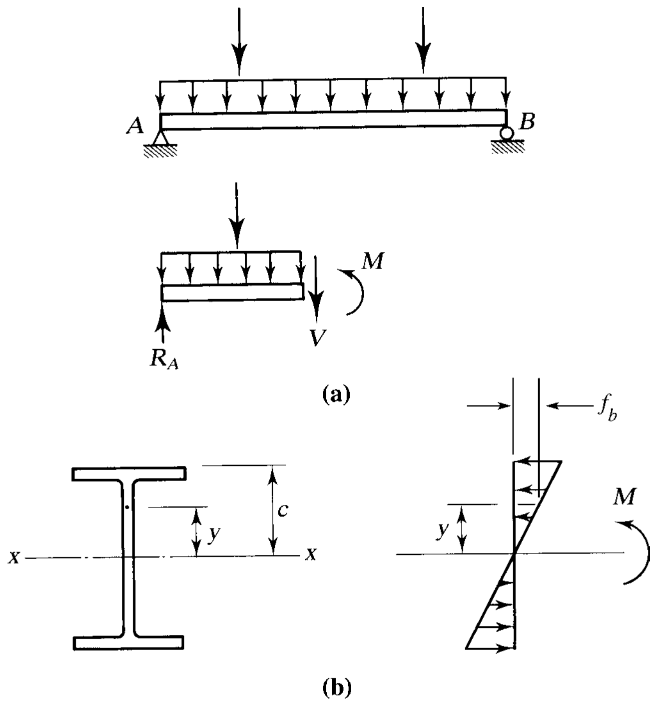
f_b = maximum computed bending stress

F_b = allowable bending stress

5.2 BENDING STRESS AND THE PLASTIC MOMENT

To be able to determine the nominal moment strength M_n, we must first examine the behavior of beams throughout the full range of loading, from very small loads to the point of collapse. Consider the beam shown in Figure 5.2a, which is oriented so that bending is about the major principal axis (for an I shape, it will be the x–x axis). For a linear elastic material and small deformations, the distribution of bending stress will be as shown in Figure 5.2b, with the stress assumed to be uniform across the width

FIGURE 5.2



of the beam. (Shear is considered separately in Section 5.8.) From elementary mechanics of materials, the stress at any point can be found from the flexure formula:

$$f_b = \frac{My}{I_x} \tag{5.3}$$

where M is the bending moment at the cross section under consideration, y is the perpendicular distance from the neutral plane to the point of interest, and I_x is the moment of inertia of the area of the cross section with respect to the neutral axis. For a homogeneous material, the neutral axis coincides with the centroidal axis. Equation 5.3 is based on the assumption of a linear distribution of strains from top to bottom, which in turn is based on the assumption that cross sections that are plane before bending remain plane after bending. In addition, the beam cross section must have a vertical axis of symmetry, and the loads must be in the longitudinal plane containing this axis. Beams that do not satisfy these criteria are considered in Section 5.15. The maximum stress will occur at the extreme fiber, where y is maximum. Thus there are two maxima: maximum compressive stress in the top fiber and maximum tensile stress in the bottom fiber. If the neutral axis is an axis of symmetry, these two stresses will be equal in magnitude. For maximum stress, Equation 5.3 takes the form:

$$f_{max} = \frac{Mc}{I_x} = \frac{M}{I_x/c} = \frac{M}{S_x} \tag{5.4}$$

where c is the perpendicular distance from the neutral axis to the extreme fiber, and S_x is the elastic section modulus of the cross section. For any cross-sectional shape, the section modulus will be a constant. For an unsymmetrical cross section, S_x will have two values: one for the top extreme fiber and one for the bottom. Values of S_x for standard rolled shapes are tabulated in the dimensions and properties tables in the *Manual*.

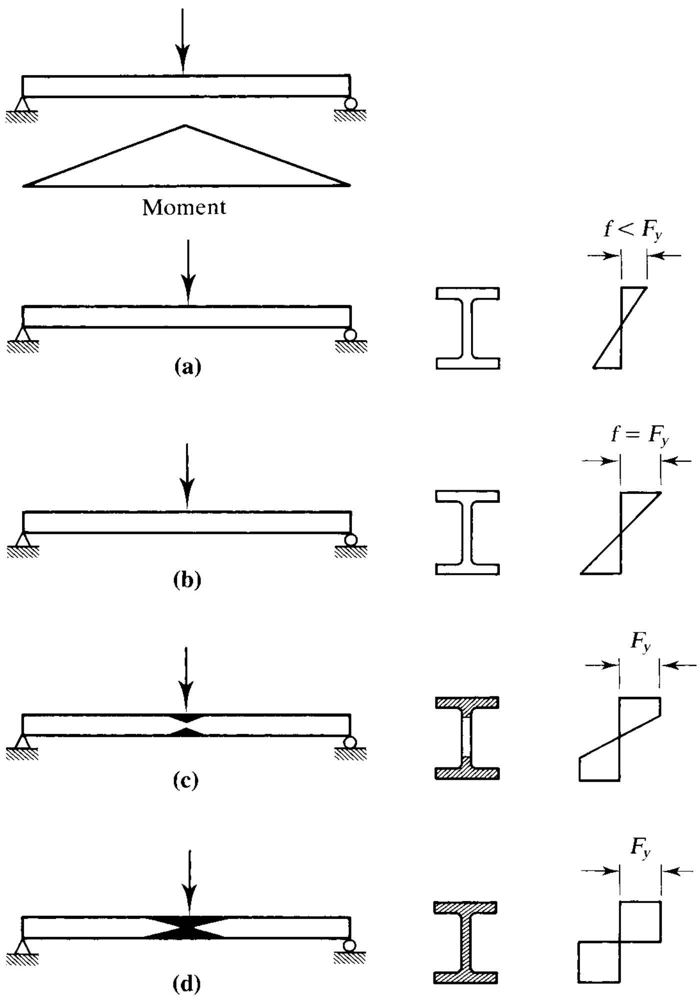
Equations 5.3 and 5.4 are valid as long as the loads are small enough that the material remains within its linear elastic range. For structural steel, this means that the stress f_{max} must not exceed F_y and that the bending moment must not exceed

$$M_y = F_y S_x$$

where M_y is the bending moment that brings the beam to the point of yielding.

In Figure 5.3, a simply supported beam with a concentrated load at midspan is shown at successive stages of loading. Once yielding begins, the distribution of stress

FIGURE 5.3



on the cross section will no longer be linear, and yielding will progress from the extreme fiber toward the neutral axis. At the same time, the yielded region will extend longitudinally from the center of the beam as the bending moment reaches M_y at more locations. These yielded regions are indicated by the dark areas in Figure 5.3c and d. In Figure 5.3b, yielding has just begun. In Figure 5.3c, the yielding has progressed into the web, and in Figure 5.3d the entire cross section has yielded. The additional moment required to bring the beam from stage b to stage d is 10 to 20% of the yield moment, M_y, for W shapes. When stage d has been reached, any further increase in the load will cause collapse, since all elements of the cross section have reached the yield plateau of the stress–strain curve and unrestricted plastic flow will occur. A *plastic hinge* is said to have formed at the center of the beam, and this hinge along with the actual hinges at the ends of the beam constitute an unstable mechanism. During plastic collapse, the mechanism motion will be as shown in Figure 5.4. Structural analysis based on a consideration of collapse mechanisms is called *plastic analysis*. An introduction to plastic analysis and design is presented in the Appendix of this book.

The plastic moment capacity, which is the moment required to form the plastic hinge, can easily be computed from a consideration of the corresponding stress distribution. In Figure 5.5, the compressive and tensile stress resultants are shown, where A_c is the cross-sectional area subjected to compression, and A_t is the area in tension. These are the areas above and below the plastic neutral axis, which is not necessarily the same as the elastic neutral axis. From equilibrium of forces,

$$C = T$$
$$A_c F_y = A_t F_y$$
$$A_c = A_t$$

Thus the plastic neutral axis divides the cross section into two equal areas. For shapes that are symmetrical about the axis of bending, the elastic and plastic neutral axes are the same. The plastic moment, M_p, is the resisting couple formed by the two equal and opposite forces, or

$$M_p = F_y(A_c)a = F_y(A_t)a = F_y\left(\frac{A}{2}\right)a = F_y Z$$

FIGURE 5.4

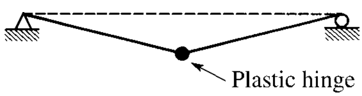
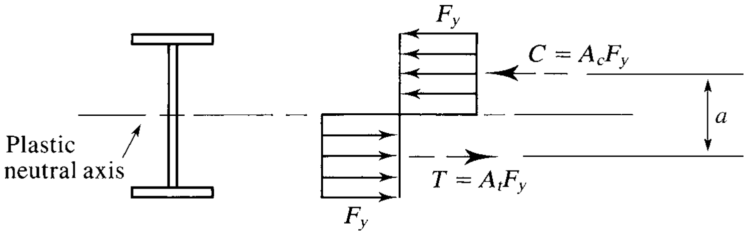


FIGURE 5.5



where

A = total cross-sectional area

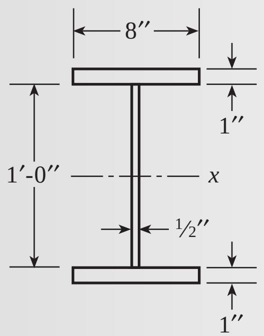
a = distance between the centroids of the two half-areas

$Z = \left(\frac{A}{2}\right)a$ = plastic section modulus

EXAMPLE 5.1

For the built-up shape shown in Figure 5.6, determine (a) the elastic section modulus S and the yield moment M_y and (b) the plastic section modulus Z and the plastic moment M_p. Bending is about the x-axis, and the steel is A572 Grade 50.

FIGURE 5.6



SOLUTION

a. Because of symmetry, the elastic neutral axis (the x-axis) is located at mid-depth of the cross section (the location of the centroid). The moment of inertia of the cross section can be found by using the parallel axis theorem, and the results of the calculations are summarized in Table 5.1.

TABLE 5.1

Component	$\bar{I}$	A	d	$\bar{I} + Ad^2$
Flange	0.6667	8	6.5	338.7
Flange	0.6667	8	6.5	338.7
Web	72	—	—	72.0
Sum				749.4

The elastic section modulus is

$$S = \frac{I}{c} = \frac{749.4}{1 + (12/2)} = \frac{749.4}{7} = 107 \text{ in.}^3$$

and the yield moment is

$$M_y = F_yS = 50(107) = 5350 \text{ in.-kips} = 446 \text{ ft-kips}$$

ANSWER $S = 107$ in.^{3} and $M_y = 446$ ft-kips.

b. Because this shape is symmetrical about the x-axis, this axis divides the cross section into equal areas and is therefore the plastic neutral axis. The centroid of the top half-area can be found by the principle of moments. Taking moments about the x-axis (the neutral axis of the entire cross section) and tabulating the computations in Table 5.2, we get

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{61}{11} = 5.545 \text{ in.}$$

TABLE 5.2

Component	A	y	Ay
Flange	8	6.5	52
Web	3	3	9
Sum	11		61

FIGURE 5.7

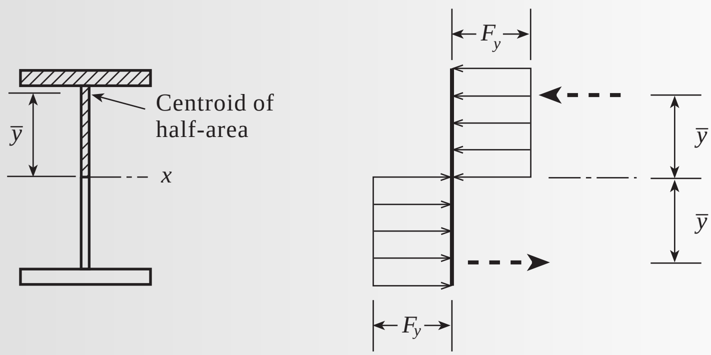


Figure 5.7 shows that the moment arm of the internal resisting couple is

$$a = 2\bar{y} = 2(5.545) = 11.09 \text{ in.}$$

and that the plastic section modulus is

$$\left(\frac{A}{2}\right)a = 11(11.09) = 122 \text{ in.}^3$$

The plastic moment is

$$M_p = F_y Z = 50(122) = 6100 \text{ in.-kips} = 508 \text{ ft-kips}$$

ANSWER $Z = 122$ in.^{3} and $M_p = 508$ ft-kips.

EXAMPLE 5.2

Compute the plastic moment, M_p, for a W10 × 60 of A992 steel.

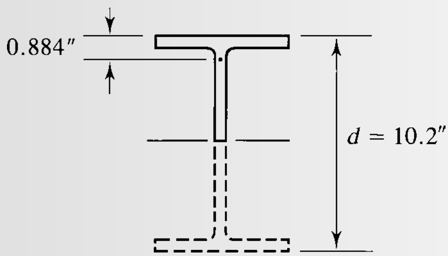
SOLUTION From the dimensions and properties tables in Part 1 of the *Manual*,

$$A = 17.7 \text{ in.}^2$$

$$\frac{A}{2} = \frac{17.7}{2} = 8.85 \text{ in.}^2$$

The centroid of the half-area can be found in the tables for WT shapes, which are cut from W shapes. The relevant shape here is the WT5 × 30, and the distance from the outside face of the flange to the centroid is 0.884 inch, as shown in Figure 5.8.

FIGURE 5.8



$$a = d - 2(0.884) = 10.2 - 2(0.884) = 8.432 \text{ in.}$$

$$Z = \left(\frac{A}{2}\right)a = 8.85(8.432) = 74.62 \text{ in.}^3$$

This result, when rounded to three significant figures, is the same as the value given in the dimensions and properties tables.

ANSWER $M_p = F_y Z = 50(74.62) = 3731$ in.-kips = 311 ft-kips.

5.3 STABILITY

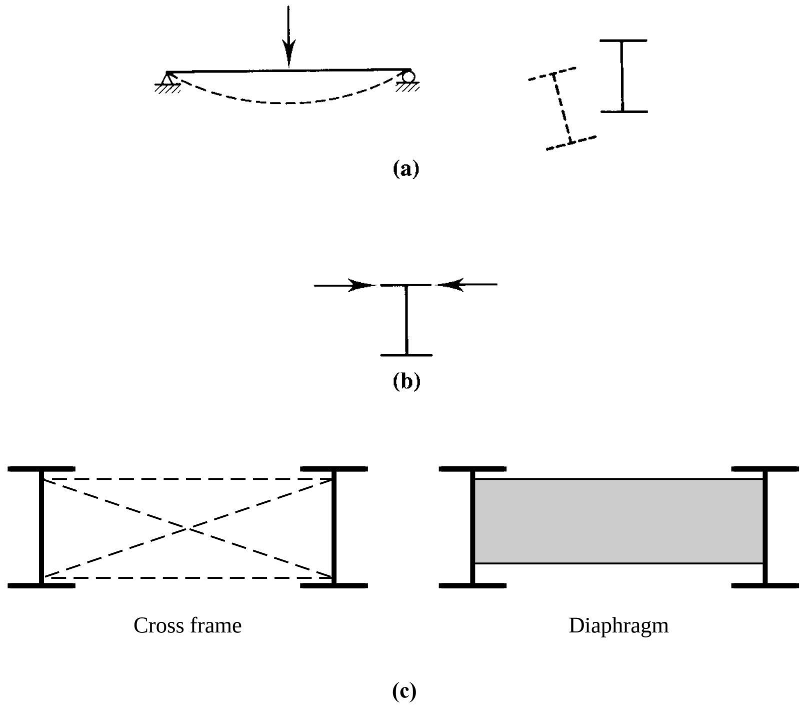
If a beam can be counted on to remain stable up to the fully plastic condition, the nominal moment strength can be taken as the plastic moment capacity; that is,

$$M_n = M_p$$

Otherwise, M_n will be less than M_p.

As with a compression member, instability can be in an overall sense or it can be local. Overall buckling is illustrated in Figure 5.9a. When a beam bends, the compression region (above the neutral axis) is analogous to a column, and in a manner similar to a column, it will buckle if the member is slender enough. Unlike a column,

FIGURE 5.9

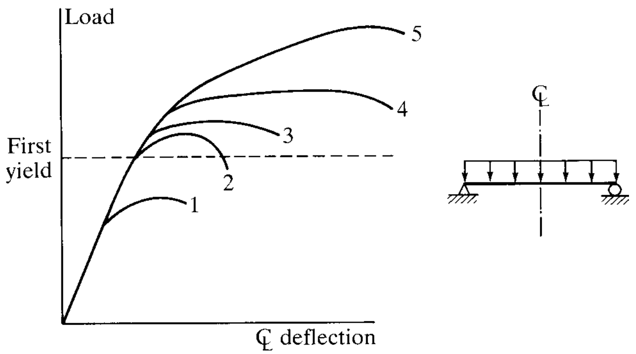


however, the compression portion of the cross section is restrained by the tension portion, and the outward deflection (flexural buckling) is accompanied by twisting (torsion). This form of instability is called *lateral-torsional buckling* (LTB). Lateral-torsional buckling can be prevented by bracing the beam against twisting at sufficiently close intervals. This can be accomplished with either of two types of stability bracing: lateral bracing, illustrated schematically in Figure 5.9b, and torsional bracing, represented in Figure 5.9c. Lateral bracing, which prevents lateral translation, should be applied as close to the compression flange as possible. Torsional bracing prevents twist directly; it can be either nodal or continuous, and it can take the form of either cross frames or diaphragms. The nodal and relative categories were defined in Chapter 4, "Compression Members." Appendix 6 of the AISC Specification gives the strength and stiffness requirements for beam bracing. These provisions are based on the work of Yura (2001). As we shall see, the moment strength depends in part on the unbraced length, which is the distance between points of bracing.

Whether the beam can sustain a moment large enough to bring it to the fully plastic condition also depends on whether the cross-sectional integrity is maintained. This integrity will be lost if one of the compression elements of the cross section buckles. This type of buckling can be either compression flange buckling, called *flange local buckling* (FLB), or buckling of the compression part of the web, called *web local buckling* (WLB). As discussed in Chapter 4, "Compression Members," whether either type of local buckling occurs will depend on the width-to-thickness ratios of the compression elements of the cross section.

Figure 5.10 further illustrates the effects of local and lateral-torsional buckling. Five separate beams are represented on this graph of load versus central deflection.

FIGURE 5.10



Curve 1 is the load-deflection curve of a beam that becomes unstable (in any way) and loses its load-carrying capacity before first yield (see Figure 5.3b) is attained. Curves 2 and 3 correspond to beams that can be loaded past first yield but not far enough for the formation of a plastic hinge and the resulting plastic collapse. If plastic collapse can be reached, the load-deflection curve will have the appearance of either curve 4 or curve 5. Curve 4 is for the case of uniform moment over the full length of the beam, and curve 5 is for a beam with a variable bending moment (moment gradient). Safe designs can be achieved with beams corresponding to any of these curves, but curves 1 and 2 represent inefficient use of material.

5.4 CLASSIFICATION OF SHAPES

AISC classifies cross-sectional shapes as compact, noncompact, or slender, depending on the values of the width-to-thickness ratios. For I shapes, the ratio for the projecting flange (an *unstiffened* element) is $b_f/2t_f$, and the ratio for the web (a *stiffened* element) is h/t_w. The classification of shapes is found in Section B4 of the Specification, "Member Properties," in Table B4.1b (Table B4.1a is for compression members). It can be summarized as follows. Let

λ = width-to-thickness ratio
λ_p = upper limit for compact category
λ_r = upper limit for noncompact category

Then

if $\lambda \leq \lambda_p$ and the flange is continuously connected to the web, the shape is compact;
if $\lambda_p < \lambda \leq \lambda_r$, the shape is noncompact; and
if $\lambda > \lambda_r$, the shape is slender.

The category is based on the worst width-to-thickness ratio of the cross section. For example, if the web is compact and the flange is noncompact, the shape is classified as noncompact. Table 5.3 has been extracted from AISC Table B4.1b and is specialized for hot-rolled I-shaped cross sections.

Table 5.3 also applies to channels, except that λ for the flange is b_f/t_f.

TABLE 5.3 Width-to-Thickness Parameters*

Element	λ	λ_p	λ_r
Flange	$\dfrac{b_f}{2t_f}$	$0.38\sqrt{\dfrac{E}{F_y}}$	$1.0\sqrt{\dfrac{E}{F_y}}$
Web	$\dfrac{h}{t_w}$	$3.76\sqrt{\dfrac{E}{F_y}}$	$5.70\sqrt{\dfrac{E}{F_y}}$

*For hot-rolled I shapes in flexure.

5.5 BENDING STRENGTH OF COMPACT SHAPES

A beam can fail by reaching M_p and becoming fully plastic, or it can fail by

1. lateral-torsional buckling (LTB), either elastically or inelastically;
2. flange local buckling (FLB), elastically or inelastically; or
3. web local buckling (WLB), elastically or inelastically.

If the maximum bending stress is less than the proportional limit when buckling occurs, the failure is said to be *elastic*. Otherwise, it is *inelastic*. (See the related discussion in Section 4.2, "Column Theory.")

For convenience, we first categorize beams as compact, noncompact, or slender, and then determine the moment resistance based on the degree of lateral support. The discussion in this section applies to two types of beams: (1) hot-rolled I shapes bent about the strong axis and loaded in the plane of the weak axis, and (2) channels bent about the strong axis and either loaded through the shear center or restrained against twisting. (The shear center is the point on the cross section through which a transverse load must pass if the beam is to bend without twisting.) Emphasis will be on I shapes. C-shapes are different only in that the width-to-thickness ratio of the flange is b_f/t_f rather than $b_f/2t_f$.

We begin with *compact shapes*, defined as those whose webs are continuously connected to the flanges and that satisfy the following width-to-thickness ratio requirements for the flange and the web:

$$\frac{b_f}{2t_f} \le 0.38\sqrt{\frac{E}{F_y}} \quad \text{and} \quad \frac{h}{t_w} \le 3.76\sqrt{\frac{E}{F_y}}$$

The web criterion is met by all standard I and C shapes listed in the *Manual* for $F_y \le 65$ ksi; therefore, in most cases only the flange ratio needs to be checked (note that built-up welded I shapes can have noncompact or slender webs). Most shapes will also satisfy the flange requirement and will therefore be classified as compact. The noncompact shapes are identified in the dimensions and properties table with a footnote (footnote f). Note that compression members have different criteria than flexural members, so a shape could be compact for flexure but slender for compression. As discussed in Chapter 4, shapes with slender compression elements are identified with a footnote (footnote c). If the beam is compact and has continuous lateral support, or if the unbraced length is very short, the nominal moment strength, M_n, is the full plastic moment capacity of the shape, M_p. For members with inadequate lateral support, the moment resistance is limited by the lateral-torsional buckling strength, either inelastic or elastic.

The first category, laterally supported compact beams, is quite common and is the simplest case. For a doubly-symmetric, compact I- or C-shaped section bent about its major axis, AISC F2.1 gives the nominal strength as

$$M_n = M_p \qquad \text{(AISC Equation F2-1)}$$

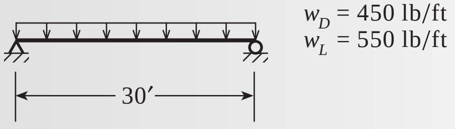
where

$$M_p = F_y Z_x$$

EXAMPLE 5.3

The beam shown in Figure 5.11 is a W16 × 31 of A992 steel. It supports a reinforced concrete floor slab that provides continuous lateral support of the compression flange. The service dead load is 450 lb/ft. This load is superimposed on the beam; it does not include the weight of the beam itself. The service live load is 550 lb/ft. Does this beam have adequate moment strength?

FIGURE 5.11



SOLUTION

First, determine the nominal flexural strength. Check for compactness.

$$\frac{b_f}{2t_f} = 6.28 \qquad \text{(from Part 1 of the } \textit{Manual}\text{)}$$

$$0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29{,}000}{50}} = 9.15 > 6.28 \quad \therefore \text{ The flange is compact.}$$

$$\frac{h}{t_w} < 3.76\sqrt{\frac{E}{F_y}} \qquad \therefore \text{ The web is compact.}$$

(The web is compact for all shapes in the *Manual* for $F_y \leq 65$ ksi.)

This shape can also be identified as compact because there is no footnote in the dimensions and properties tables to indicate otherwise. Because the beam is compact and laterally supported, the nominal flexural strength is

$$M_n = M_p = F_y Z_x = 50(54.0) = 2700 \text{ in.-kips} = 225.0 \text{ ft kips.}$$

Compute the maximum bending moment. The total service dead load, including the weight of the beam, is

$$w_D = 450 + 31 = 481 \text{ lb/ft}$$

For a simply supported, uniformly loaded beam, the maximum bending moment occurs at midspan and is equal to

$$M_{max} = \frac{1}{8} wL^2$$

where w is the load in units of force per unit length, and L is the span length. Then

$$M_D = \frac{1}{8} w_D L^2 = \frac{1}{8}(0.481)(30)^2 = 54.11 \text{ ft-kips}$$

$$M_L = \frac{1}{8}(0.550)(30)^2 = 61.88 \text{ ft-kips}$$

LRFD SOLUTION

The dead load is less than 8 times the live load, so load combination 2 controls:

$$M_u = 1.2M_D + 1.6M_L = 1.2(54.11) + 1.6(61.88) = 164 \text{ ft-kips.}$$

Alternatively, the loads can be factored at the outset:

$$w_u = 1.2w_D + 1.6w_L = 1.2(0.481) + 1.6(0.550) = 1.457 \text{ kips/ft}$$

$$M_u = \frac{1}{8} w_u L^2 = \frac{1}{8}(1.457)(30)^2 = 164 \text{ ft-kips}$$

The design strength is

$$\phi_b M_n = 0.90(225.0) = 203 \text{ ft-kips} > 164 \text{ ft-kips} \qquad \text{(OK)}$$

ANSWER

The design moment is greater than the factored-load moment, so the W16 × 31 is satisfactory.

ASD SOLUTION

ASD load combination 2 controls.

$$M_a = M_D + M_L = 54.11 + 61.88 = 116.0 \text{ ft-kips}$$

Alternatively, the loads can be added before the moment is computed:

$$w_a = w_D + w_L = 0.481 + 0.550 = 1.031 \text{ kips/ft}$$

$$M_a = \frac{1}{8} w_a L^2 = \frac{1}{8}(1.031)(30)^2 = 116.0 \text{ ft-kips}$$

The allowable moment is

$$\frac{M_n}{\Omega_b} = \frac{M_n}{1.67} = 0.6M_n = 0.6(225.0) = 135 \text{ ft-kips} > 116 \text{ ft-kips} \qquad \text{(OK)}$$

Allowable stress solution:

The applied stress is

$$f_b = \frac{M_a}{S_x} = \frac{116.0(12)}{47.2} = 29.5 \text{ ksi}$$

The allowable stress is

$$F_b = \frac{0.6M_n}{S_x} = \frac{0.6(225.0)(12)}{47.2} = 34.3 \text{ ksi}$$

Since $f_b < F_b$, the beam has enough strength.

ANSWER The W16 × 31 is satisfactory.

The allowable stress solution can be simplified if a slight approximation is made. The allowable stress can be written as

$$F_b = \frac{0.6M_n}{S_x} = \frac{0.6F_yZ_x}{S_x}$$

If an average value of $Z_x/S_x = 1.1$ is used (this is conservative),

$$F_b = 0.6F_y(1.1) = 0.66F_y$$

If this value is used in Example 5.3,

$$F_b = 0.66(50) = 33.0 \text{ ksi}$$

which is conservative by about 4%. Thus, for compact, laterally-supported beams, the allowable stress can be conservatively taken as $0.66F_y$. (This value of allowable stress has been used in AISC allowable stress design specifications since 1963.)

We can formulate an allowable stress approach that requires no approximation if we use the plastic section modulus instead of the elastic section modulus. From

$$\frac{M_n}{\Omega_b} \geq M_a$$

and

$$\frac{M_n}{\Omega_b} = \frac{F_yZ_y}{1.67} = 0.6F_yZ_x$$

The required plastic section modulus is

$$Z_x \geq \frac{M_a}{0.6F_y}$$

Thus, if the bending stress is based on the plastic section modulus Z_x,

$$f_b = \frac{M_a}{Z_x} \quad \text{and} \quad F_b = 0.6F_y$$

This approach is useful when designing compact, laterally-supported beams.

The moment strength of compact shapes is a function of the unbraced length, L_b, defined as the distance between points of lateral support, or bracing. In this book, we indicate points of lateral support with an "×," as shown in Figure 5.12. The relationship between the nominal strength, M_n, and the unbraced length is shown in Figure 5.13. If the unbraced length is no greater than L_p, to be defined presently, the beam is considered to have full lateral support, and $M_n = M_p$. If L_b is greater than L_p but less than or equal to the parameter L_r, the strength is based on inelastic LTB. If L_b is greater than L_r, the strength is based on elastic LTB.

The equation for the theoretical elastic lateral-torsional buckling strength can be found in *Theory of Elastic Stability* (Timoshenko and Gere, 1961). With some notational changes, the nominal moment strength is

$$M_n = F_{cr}S_x$$

FIGURE 5.12

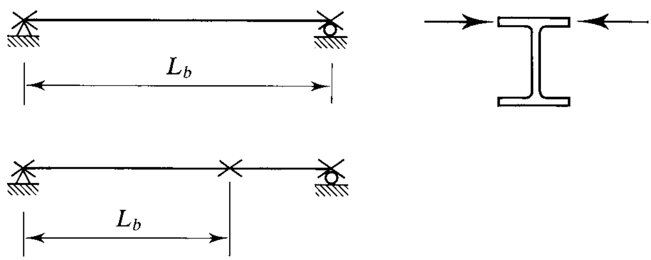
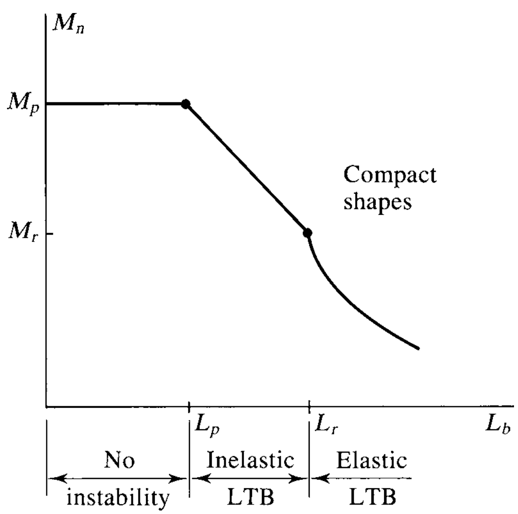


FIGURE 5.13



where F_{cr} is the elastic buckling stress and is given by

$$F_{cr} = \frac{\pi}{L_b S_x}\sqrt{EI_y GJ + \left(\frac{\pi E}{L_b}\right)^2 I_y C_w}, \text{ ksi} \tag{5.5}$$

where

L_b = unbraced length (in.)
I_y = moment of inertia about the weak axis of the cross section (in.^{4})
G = shear modulus of structural steel = 11,200 ksi
J = torsional constant (in.^{4})
C_w = warping constant (in.^{6})

(The constants G, J, and C_w were defined in Chapter 4 in the discussion of torsional and lateral-torsional buckling of columns.)

Equation 5.5 is valid as long as the bending moment within the unbraced length is uniform (nonuniform moment is accounted for with a factor C_b, which is explained later). The AISC Specification gives a different, but equivalent, form for the elastic buckling stress F_{cr}. AISC gives the nominal moment strength as

$$M_n = F_{cr} S_x \le M_p \qquad \text{(AISC Equation F2-3)}$$

where

$$F_{cr} = \frac{C_b \pi^2 E}{(L_b/r_{ts})^2}\sqrt{1 + 0.078\frac{Jc}{S_x h_0}\left(\frac{L_b}{r_{ts}}\right)^2} \qquad \text{(AISC Equation F2-4)}$$

and

C_b = factor to account for nonuniform bending within the unbraced length L_b. This factor will be covered following Example 5.4.

$$r_{ts}^2 = \frac{\sqrt{I_y C_w}}{S_x} \qquad \text{(AISC Equation F2-7)}$$

c = 1.0 for doubly-symmetric I shapes (AISC Equation F2-8a)

$$= \frac{h_0}{2}\sqrt{\frac{I_y}{C_w}} \quad \text{for channels} \qquad \text{(AISC Equation F2-8b)}$$

h_0 = distance between flange centroids = $d - t_f$

If the moment when lateral-torsional buckling occurs is greater than the moment corresponding to first yield, the strength is based on inelastic behavior. The moment corresponding to first yield is

$$M_r = 0.7F_y S_x$$

where the yield stress has been reduced by 30% to account for the effect of residual stress. As shown in Figure 5.13, the boundary between elastic and inelastic behavior will be for an unbraced length of L_r, which is the value of L_b obtained from AISC Equation F2-4 when F_{cr} is set equal to $0.7F_y$ with $C_b = 1.0$. The following equation results:

$$L_r = 1.95 r_{ts} \frac{E}{0.7F_y} \sqrt{\frac{Jc}{S_x h_0} + \sqrt{\left(\frac{Jc}{S_x h_0}\right)^2 + 6.76\left(\frac{0.7F_y}{E}\right)^2}} \quad \text{(AISC Equation F2-6)}$$

As with columns, inelastic buckling of beams is more complicated than elastic buckling, and empirical formulas are often used. The following equation is used by AISC:

$$M_n = C_b\left[M_p - (M_p - 0.7F_y S_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \quad \text{(AISC Equation F2-2)}$$

where the $0.7F_y S_x$ term is the yield moment adjusted for residual stress, and

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}} \quad \text{(AISC Equation F2-5)}$$

Summary of Nominal Flexural Strength

The nominal bending strength for compact I and C-shaped sections can be summarized as follows:

For $L_b \le L_p$,

$$M_n = M_p \quad \text{(AISC Equation F2-1)}$$

For $L_p < L_b \le L_r$,

$$M_n = C_b\left[M_p - (M_p - 0.7F_y S_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \quad \text{(AISC Equation F2-2)}$$

For $L_b > L_r$,

$$M_n = F_{cr} S_x \le M_p \quad \text{(AISC Equation F2-3)}$$

where

$$F_{cr} = \frac{C_b \pi^2 E}{(L_b/r_{ts})^2} \sqrt{1 + 0.078 \frac{Jc}{S_x h_0}\left(\frac{L_b}{r_{ts}}\right)^2} \quad \text{(AISC Equation F2-4)}$$

EXAMPLE 5.4

Determine the flexural strength of a W14 × 68 of A242 steel subject to

a. Continuous lateral support.
b. An unbraced length of 20 ft with $C_b = 1.0$.
c. An unbraced length of 30 ft with $C_b = 1.0$.

SOLUTION To determine the yield stress of a W14 × 68 of A242 steel, we refer to Table 2-3 in Part 2 of the *Manual*. The yield stress is a function of the flange thickness, which for this shape is 0.720 in. This corresponds to footnote 1, so a W14 × 68 is available in A242 steel with a yield stress F_y of 50 ksi. Next, determine whether this shape is compact, noncompact, or slender:

$$\frac{b_f}{2t_f} = 6.97 \quad \text{(from Part 1 of the } \textit{Manual}\text{)}$$

$$0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29{,}000}{50}} = 9.15 > 6.97 \qquad \therefore \text{ The flange is compact.}$$

The web is compact for all shapes in the *Manual* for $F_y \le 65$ ksi; therefore, a W14 × 68 is compact for $F_y = 50$ ksi. (This determination could also be made by observing that there is no footnote in the dimensions and properties tables to indicate that the shape is not compact.)

a. Because the beam is compact and laterally supported, the nominal flexural strength is

$$M_n = M_p = F_y Z_x = 50(115) = 5750 \text{ in.-kips} = 479.2 \text{ ft-kips}$$

LRFD SOLUTION The design strength is

$$\phi_b M_n = 0.90(479.2) = 431 \text{ ft-kips}$$

ASD SOLUTION The allowable moment strength is

$$\frac{M_n}{\Omega_b} = \frac{M_n}{1.67} = 0.6M_n = 0.6(479.2) = 288 \text{ ft-kips}$$

b. $L_b = 20$ ft and $C_b = 1.0$. First, determine L_p and L_r:

$$L_p = 1.76r_y\sqrt{\frac{E}{F_y}} = 1.76(2.46)\sqrt{\frac{29{,}000}{50}} = 104.3 \text{ in.} = 8.692 \text{ ft}$$

The following terms will be needed in the computation of L_r:

$$r_{ts}^2 = \frac{\sqrt{I_y C_w}}{S_x} = \frac{\sqrt{121(5380)}}{103} = 7.833 \text{ in.}^2$$

$$r_{ts} = \sqrt{7.833} = 2.799 \text{ in.}$$

(r_{ts} can also be found in the dimensions and properties tables. For a W14 × 68, it is given as 2.80 in.)

$$h_o = d - t_f = 14.0 - 0.720 = 13.28 \text{ in.}$$

(h_o can also be found in the dimensions and properties tables. For a W14 × 68, it is given as 13.3 in.)
For a doubly-symmetric I shape, $c = 1.0$. From AISC Equation F2-6,

$$L_r = 1.95 r_{ts} \frac{E}{0.7F_y} \sqrt{\frac{Jc}{S_x h_o} + \sqrt{\left(\frac{Jc}{S_x h_o}\right)^2 + 6.76\left(\frac{0.7F_y}{E}\right)^2}}$$

$$\frac{Jc}{S_x h_o} = \frac{3.01(1.0)}{103(13.28)} = 0.002201$$

$$L_r = 1.95(2.799)\frac{29,000}{0.7(50)}\sqrt{0.002201 + \sqrt{(0.002201)^2 + 6.76\left[\frac{0.7(50)}{29,000}\right]^2}}$$

$$= 351.3 \text{ in.} = 29.28 \text{ ft}$$

Since $L_p < L_b < L_r$,

$$M_n = C_b\left[M_p - (M_p - 0.7F_y S_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p$$

$$= 1.0\left[5,750 - (5,750 - 0.7 \times 50 \times 103)\left(\frac{20 - 8.692}{29.28 - 8.692}\right)\right]$$

$$= 4572 \text{ in.-kips} = 381.0 \text{ ft-kips} < M_p = 479.2 \text{ ft.-kips}$$

LRFD SOLUTION

The design strength is

$$\phi_b M_n = 0.90(381.0) = 343 \text{ ft-kips}$$

ASD SOLUTION

The allowable moment strength is

$$\frac{M_n}{\Omega_b} = 0.6M_n = 0.6(381.0) = 229 \text{ ft-kips}$$

c. $L_b = 30$ ft and $C_b = 1.0$

$L_b > L_r = 29.28$ ft, so elastic lateral-torsional buckling controls.

From AISC Equation F2-4,

$$F_{cr} = \frac{C_b \pi^2 E}{(L_b/r_{ts})^2}\sqrt{1 + 0.078\frac{Jc}{S_x h_o}\left(\frac{L_b}{r_{ts}}\right)^2}$$

$$= \frac{1.0\pi^2(29,000)}{\left(\dfrac{30\times 12}{2.799}\right)^2}\sqrt{1 + 0.078\frac{3.01(1.0)}{103(13.28)}\left(\frac{30\times 12}{2.799}\right)^2} = 33.90 \text{ ksi}$$

From AISC Equation F2-3,

$$M_n = F_{cr}S_x = 33.90(103) = 3492 \text{ in.-kips} = 291.0 \text{ ft-kips} < M_p = 479.2 \text{ ft-kips}$$

LRFD SOLUTION

$$\phi_b M_n = 0.90(291.0) = 262 \text{ ft-kips}$$

ASD SOLUTION

$$M_n/\Omega_b = 0.6M_n = 0.6(291.0) = 175 \text{ ft-kips}$$

If the moment within the unbraced length L_b is uniform (constant), there is no *moment gradient* and $C_b = 1.0$. If there is a moment gradient, the value of C_b is given by

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C} \qquad \text{(AISC Equation F1-1)}$$

where

M_{max} = absolute value of the maximum moment within the unbraced length (including the end points of the unbraced length)

M_A = absolute value of the moment at the quarter point of the unbraced length

M_B = absolute value of the moment at the midpoint of the unbraced length

M_C = absolute value of the moment at the three-quarter point of the unbraced length

AISC Equation F1-1 is valid for doubly-symmetric members and for singly-symmetric members in single curvature.

When the bending moment is uniform, the value of C_b is

$$C_b = \frac{12.5M}{2.5M + 3M + 4M + 3M} = 1.0$$

EXAMPLE 5.5

Determine C_b for a uniformly loaded, simply supported W shape with lateral support at its ends only.

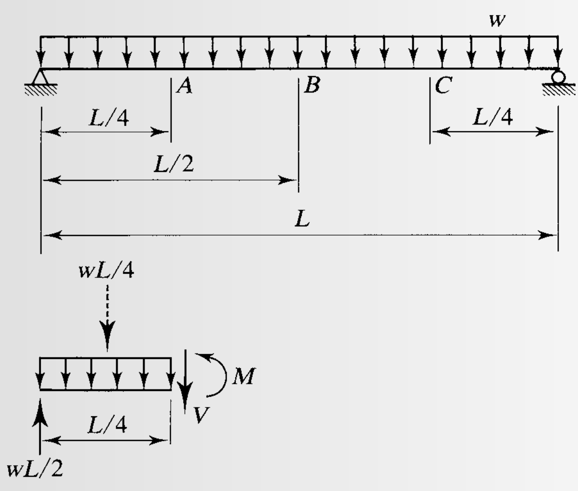
SOLUTION Because of symmetry, the maximum moment is at midspan, so

$$M_{\max} = M_B = \frac{1}{8}wL^2$$

Also because of symmetry, the moment at the quarter point equals the moment at the three-quarter point. From Figure 5.14,

$$M_A = M_C = \frac{wL}{2}\left(\frac{L}{4}\right) - \frac{wL}{4}\left(\frac{L}{8}\right) = \frac{wL^2}{8} - \frac{wL^2}{3} = \frac{3}{2\,32}wL^2$$

FIGURE 5.14



Since this is a W shape (doubly symmetric), AISC Equation (F1-1) is applicable.

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5\left(\frac{1}{8}\right)}{2.5\left(\frac{1}{8}\right) + 3\left(\frac{3}{32}\right) + 4\left(\frac{1}{8}\right) + 3\left(\frac{3}{32}\right)} = 1.14$$

ANSWER $C_b = 1.14$.

Figure 5.15 shows the value of C_b for several common cases of loading and lateral support. Values of C_b for other cases can be found in Part 3 of the *Manual*, "Design of Flexural Members."

For unbraced cantilever beams, AISC specifies a value of C_b of 1.0. A value of 1.0 is always conservative, regardless of beam configuration or loading, but in some cases it may be excessively conservative.

The effect of C_b on the nominal strength is illustrated in Figure 5.16. Although the strength is directly proportional to C_b, this graph clearly shows the importance of observing the upper limit of M_p, regardless of which equation is used for M_n.

Part 3 of the *Steel Construction Manual*, "Design of Flexural Members," contains several useful tables and charts for the analysis and design of beams. For example, Table 3-2, "W Shapes, Selection by Z_x," (hereafter referred to as the "Z_x table"), lists shapes commonly used as beams, arranged in order of available flexural strength—both $\phi_b M_{px}$ and M_{px}/Ω_b. Other useful constants that are tabulated include L_p and L_r (which is particularly tedious to compute). These two constants can

FIGURE 5.15

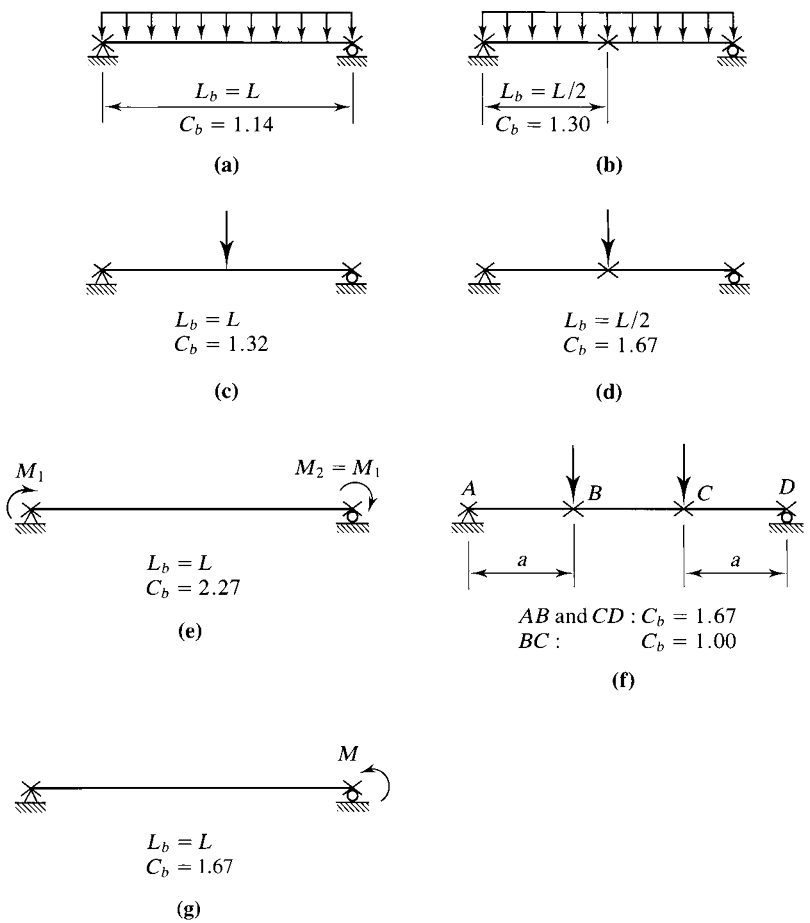
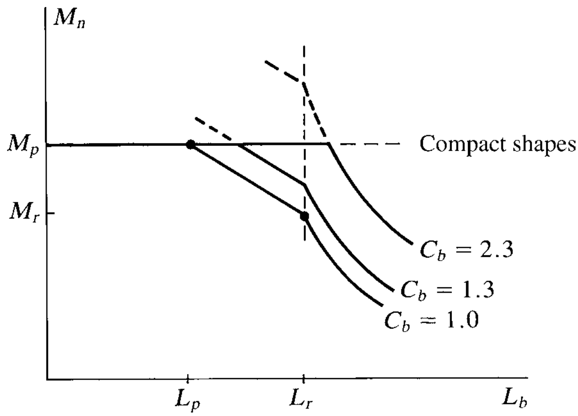


FIGURE 5.16



also be found in several other tables in Part 3 of the *Manual.* We cover additional design aids in other sections of this chapter.

5.6 BENDING STRENGTH OF NONCOMPACT SHAPES

As previously noted, most standard W, M, S, and C shapes are compact. A few are noncompact because of the flange width-to-thickness ratio, but none are slender.

In general, a noncompact beam may fail by lateral-torsional buckling, flange local buckling, or web local buckling. Any of these types of failure can be in either the elastic range or the inelastic range. The strength corresponding to each of these three limit states must be computed, and the smallest value will control.

From AISC F3, for flange local buckling, if $\lambda_p < \lambda \le \lambda_r$, the flange is noncompact, buckling will be inelastic, and

$$M_n = M_p - (M_p - 0.7F_yS_x)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right) \qquad \text{(AISC Equation F3-1)}$$

where

$$\lambda = \frac{b_f}{2t_f}$$

$$\lambda_p = 0.38\sqrt{\frac{E}{F_y}}$$

$$\lambda_r = 1.0\sqrt{\frac{E}{F_y}}$$

The webs of all hot-rolled shapes in the *Manual* are compact, so the noncompact shapes are subject only to the limit states of lateral-torsional buckling and flange local buckling. Built-up welded shapes, however, can have noncompact or slender webs as well as noncompact or slender flanges. These cases are covered in AISC Sections F4 and F5. Built-up shapes, including plate girders, are covered in Chapter 10 of this textbook.

EXAMPLE 5.6

A simply supported beam with a span length of 45 feet is laterally supported at its ends and is subjected to the following service loads:

Dead load = 400 lb/ft (including the weight of the beam)

Live load = 1000 lb/ft

If $F_y = 50$ ksi, is a W14 × 90 adequate?

SOLUTION Determine whether the shape is compact, noncompact, or slender:

$$\lambda = \frac{b_f}{2t_f} = 10.2$$

$$\lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29{,}000}{50}} = 9.15$$

$$\lambda_r = 1.0\sqrt{\frac{E}{F_y}} = 1.0\sqrt{\frac{29{,}000}{50}} = 24.1$$

Since $\lambda_p < \lambda < \lambda_r$, this shape is noncompact. Check the capacity based on the limit state of flange local buckling:

$$M_p = F_y Z_x = 50(157) = 7850 \text{ in.-kips}$$

$$M_n = M_p - (M_p - 0.7F_y S_x)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)$$

$$= 7850 - (7850 - 0.7 \times 50 \times 143)\left(\frac{10.2 - 9.15}{24.1 - 9.15}\right)$$

$$= 7650 \text{ in.-kips} = 637.5 \text{ ft-kips}$$

Check the capacity based on the limit state of lateral-torsional buckling. From the Z_x table,

$L_p = 15.1$ ft and $L_r = 42.5$ ft

$L_b = 45 \text{ ft} > L_r$ ∴ Failure is by *elastic* LTB.

From Part 1 of the *Manual*,

$$I_y = 362 \text{ in.}^4$$
$$r_{ts} = 4.11 \text{ in.}$$
$$h_o = 13.3 \text{ in.}$$
$$J = 4.06 \text{ in.}^4$$
$$C_w = 16{,}000 \text{ in.}^6$$

For a uniformly loaded, simply supported beam with lateral support at the ends,

$$C_b = 1.14 \qquad \text{(Fig. 5.15a)}$$

For a doubly-symmetric I shape, $c = 1.0$. AISC Equation F2-4 gives

$$F_{cr} = \frac{C_b \pi^2 E}{(L_b/r_{ts})^2}\sqrt{1 + 0.078\frac{Jc}{S_x h_o}\left(\frac{L_b}{r_{ts}}\right)^2}$$
$$= \frac{1.14\pi^2(29{,}000)}{\left(\frac{45\times 12}{4.11}\right)^2}\sqrt{1 + 0.078\frac{4.06(1.0)}{143(13.3)}\left(\frac{45\times 12}{4.11}\right)^2} = 37.20 \text{ ksi}$$

From AISC Equation F2-3,

$$M_n = F_{cr}S_x = 37.20(143) = 5320 \text{ in.-kips} < M_p = 7850 \text{ in.-kips}$$

This is smaller than the nominal strength based on flange local buckling, so lateral-torsional buckling controls.

LRFD SOLUTION

The design strength is

$$\phi_b M_n = 0.90(5320) = 4788 \text{ in.-kips} = 399 \text{ ft-kips}$$

The factored load and moment are

$$w_u = 1.2w_D + 1.6w_L = 1.2(0.400) + 1.6(1.000) = 2.080 \text{ kips/ft}$$
$$M_u = \frac{1}{8}w_u L^2 = \frac{1}{8}(2.080)(45)^2 = 527 \text{ ft-kips} > 399 \text{ ft-kips} \quad \text{(N.G.)}$$

ANSWER

Since $M_u > \phi_b M_n$, the beam does not have adequate moment strength.

ASD SOLUTION

The allowable stress is

$$F_b = 0.6F_{cr} = 0.6(37.20) = 22.3 \text{ ksi}$$

The applied bending moment is

$$M_a = \frac{1}{8}w_a L^2 = \frac{1}{8}(0.400 + 1.000)(45)^2 = 354.4 \text{ ft-kips}$$

and the applied stress is

$$f_b = \frac{M_a}{S_x} = \frac{354.4(12)}{143} = 29.7 \text{ ksi} > 22.3 \text{ ksi} \quad \text{(N.G.)}$$

ANSWER Since $f_b > F_b$, the beam does not have adequate moment strength.

Noncompact shapes are identified in the Z_x table by an "*f*" footnote (this same identification is used in the dimensions and properties tables). Noncompact shapes are also treated differently in the Z_x table in the following way. The tabulated value of L_p is the value of unbraced length at which the nominal strength based on inelastic lateral-torsional buckling equals the nominal strength based on flange local buckling, that is, the maximum unbraced length for which the nominal strength can be taken as the strength based on flange local buckling. (Recall that L_p for compact shapes is the maximum unbraced length for which the nominal strength can be taken as the plastic moment.) For the shape in Example 5.6, equate the nominal strength based on FLB to the strength based on inelastic LTB (AISC Equation F2-2), with $C_b = 1.0$:

$$M_n = M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right) \tag{5.6}$$

The value of L_r was given in Example 5.6 and is unchanged. The value of L_p, however, must be computed from AISC Equation F2-5:

$$L_p = 1.76r_y\sqrt{\frac{E}{F_y}} = 1.76(3.70)\sqrt{\frac{29{,}000}{50}} = 156.8 \text{ in.} = 13.07 \text{ ft}$$

Returning to Equation 5.6, we obtain

$$7650 = 7850 - (7850 - 0.7 \times 50 \times 143)\left(\frac{L_b - 13.07}{42.6 - 13.07}\right)$$

$$L_b = 15.2 \text{ ft}$$

This is the value tabulated as L_p for a W14 × 90 with $F_y = 50$ ksi. Note that

$$L_p = 1.76r_y\sqrt{\frac{E}{F_y}}$$

could still be used for noncompact shapes. If doing so resulted in the equation for inelastic LTB being used when L_b was not really large enough, the strength based on FLB would control anyway.

In addition to the different meaning of L_p for noncompact shapes in the Z_x table, the available strength values, $\phi_b M_{px}$ and M_p/Ω_b, are based on flange local buckling rather than the plastic moment.

5.7 SUMMARY OF MOMENT STRENGTH

The procedure for computation of nominal moment strength for I and C-shaped sections bent about the *x* axis will now be summarized. All terms in the following equations have been previously defined, and AISC equation numbers will not be shown. This summary is for compact and noncompact shapes (noncompact flanges) only (no slender shapes).

1. Determine whether the shape is compact.
2. If the shape is compact, check for lateral-torsional buckling as follows.

   If $L_b \le L_p$, there is no LTB, and $M_n = M_p$

   If $L_p < L_b \le L_r$, there is inelastic LTB, and

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p$$

   If $L_b > L_r$, there is elastic LTB, and

$$M_n = F_{cr}S_x \le M_p$$

where

$$F_{cr} = \frac{C_b\pi^2E}{(L_b/r_{ts})^2}\sqrt{1 + 0.078\frac{Jc}{S_xh_0}\left(\frac{L_b}{r_{ts}}\right)^2}$$

3. If the shape is noncompact because of the flange, the nominal strength will be the smaller of the strengths corresponding to flange local buckling and lateral-torsional buckling.

   a. Flange local buckling:

      If $\lambda \le \lambda_p$, there is no FLB

      If $\lambda_p < \lambda \le \lambda_r$, the flange is noncompact, and

$$M_n = M_p - (M_p - 0.7F_yS_x)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)$$

   b. Lateral-torsional buckling:

      If $L_b \le L_p$, there is no LTB

      If $L_p < L_b \le L_r$, there is inelastic LTB, and

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p$$

If $L_b > L_r$, there is elastic LTB, and

$$M_n = F_{cr} S_x \leq M_p$$

where

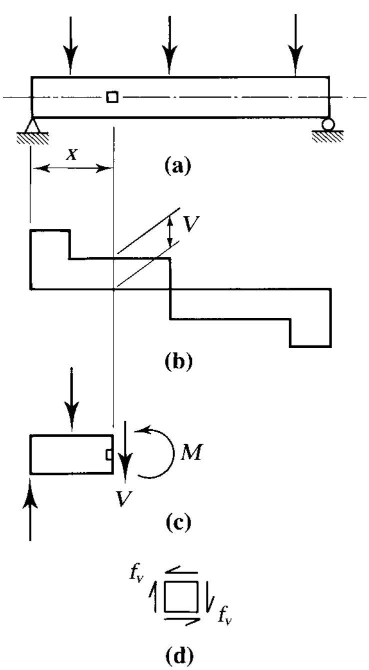
$$F_{cr} = \frac{C_b \pi^2 E}{(L_b / r_{ts})^2} \sqrt{1 + 0.078 \frac{Jc}{S_x h_0} \left(\frac{L_b}{r_{ts}} \right)^2}$$

5.8 SHEAR STRENGTH

Beam shear strength is covered in Chapter G of the AISC Specification, "Design of Members for Shear." Both hot-rolled shapes and welded built-up shapes are covered. We discuss hot-rolled shapes in the present chapter of this book and built-up shapes in Chapter 10, "Plate Girders." The AISC provisions for hot-rolled shapes are covered in Section G2.1.

Before covering the AISC provisions for shear strength, we will first review some basic concepts from mechanics of materials. Consider the simple beam of Figure 5.17. At a distance x from the left end and at the neutral axis of the cross section, the state of stress is as shown in Figure 5.17d. Because this element is located at the neutral

FIGURE 5.17



axis, it is not subjected to flexural stress. From elementary mechanics of materials, the shearing stress is

$$f_v = \frac{VQ}{Ib} \tag{5.7}$$

where

f_v = vertical and horizontal shearing stress at the point of interest
V = vertical shear force at the section under consideration
Q = first moment, about the neutral axis, of the area of the cross section between the point of interest and the top or bottom of the cross section
I = moment of inertia about the neutral axis
b = width of the cross section at the point of interest

Equation 5.7 is based on the assumption that the stress is constant across the width b, and it is therefore accurate only for small values of b. For a rectangular cross section of depth d and width b, the error for $d/b = 2$ is approximately 3%. For $d/b = 1$, the error is 12% and for $d/b = \frac{1}{4}$, it is 100% (Higdon, Ohlsen, and Stiles, 1960). For this reason, Equation 5.7 cannot be applied to the flange of a W-shape in the same manner as for the web.

Figure 5.18 shows the shearing stress distribution for a W shape. Superimposed on the actual distribution is the average stress in the web, V/A_w, which does not differ much from the maximum web stress. Clearly, the web will completely yield long before the flanges begin to yield. Because of this, yielding of the web represents one of the shear limit states. Taking the shear yield stress as 60% of the tensile yield stress, we can write the equation for the stress in the web at failure as

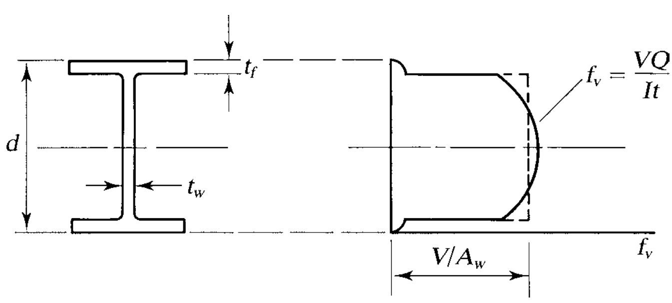
$$f_v = \frac{V_n}{A_w} = 0.6F_y$$

where A_w = area of the web. The nominal strength corresponding to this limit state is therefore

$$V_n = 0.6F_yA_w \tag{5.8}$$

and will be the nominal strength in shear provided that there is no shear buckling of the web. Whether that occurs will depend on h/t_w, the width-to-thickness ratio of the

FIGURE 5.18



web. If this ratio is too large—that is, if the web is too slender—the web can buckle in shear, either inelastically or elastically.

AISC Specification Requirements for Shear

For LRFD, the relationship between required and available strength is

$$V_u \leq \phi_v V_n$$

where
V_u = maximum shear based on the controlling combination of factored loads
ϕ_v = resistance factor for shear

For ASD, the relationship is

$$V_a \leq \frac{V_n}{\Omega_v}$$

where
V_a = maximum shear based on the controlling combination of service loads
Ω_v = safety factor for shear

As we will see, the values of the resistance factor and safety factor will depend on the web width-to-thickness ratio.

Section G2.1 of the AISC Specification covers both beams with stiffened webs and beams with unstiffened webs. In most cases, hot-rolled beams will not have stiffeners, and we will defer treatment of stiffened webs until Chapter 10. The basic strength equation is

$$V_n = 0.6F_y A_w C_v \qquad \text{(AISC Equation G2-1)}$$

where
A_w = area of the web ≈ dt_w
d = overall depth of the beam
C_v = ratio of critical web stress to shear yield stress

The value of C_v depends on whether the limit state is web yielding, web inelastic buckling, or web elastic buckling.

Case 1: For hot-rolled I shapes with

$$\frac{h}{t_w} \leq 2.24\sqrt{\frac{E}{F_y}}$$

The limit state is shear yielding, and

$$C_v = 1.0 \qquad \text{(AISC Equation G2-2)}$$
$$\phi_v = 1.00$$
$$\Omega_v = 1.50$$

Most W shapes with $F_y \leq 50$ ksi fall into this category (see User Note in AISC G2.1[a]).

Case 2: For all other doubly and singly symmetric shapes,

$\phi_v = 0.90$
$\Omega_v = 1.67$

and C_v is determined as follows:

For $\dfrac{h}{t_w} \le 1.10\sqrt{\dfrac{k_v E}{F_y}}$, there is no web instability, and

$$C_v = 1.0 \qquad \text{(AISC Equation G2-3)}$$

(This corresponds to Equation 5.8 for shear yielding.)

For $1.10\sqrt{\dfrac{k_v E}{F_y}} < \dfrac{h}{t_w} \le 1.37\sqrt{\dfrac{k_v E}{F_y}}$, inelastic web buckling can occur, and

$$C_v = \frac{1.10\sqrt{\dfrac{k_v E}{F_y}}}{h/t_w} \qquad \text{(AISC Equation G2-4)}$$

For $\dfrac{h}{t_w} > 1.37\sqrt{\dfrac{k_v E}{F_y}}$, the limit state is elastic web buckling, and

$$C_v = \frac{1.51 k_v E}{(h/t_w)^2 F_y} \qquad \text{(AISC Equation G2-5)}$$

where

$k_v = 5$

This value of k_v is for unstiffened webs with $h/t_w < 260$. Although section G2.1 of the Specification does not give $h/t_w = 260$ as an upper limit, no value of k_v is given when $h/t_w \ge 260$. In addition, AISC F13.2, “Proportioning Limits for I-Shaped Members,” states that h/t_w in unstiffened girders shall not exceed 260.

AISC Equation G2-5 is based on elastic stability theory, and AISC Equation G2-4 is an empirical equation for the inelastic region, providing a transition between the limit states of web yielding and elastic web buckling.

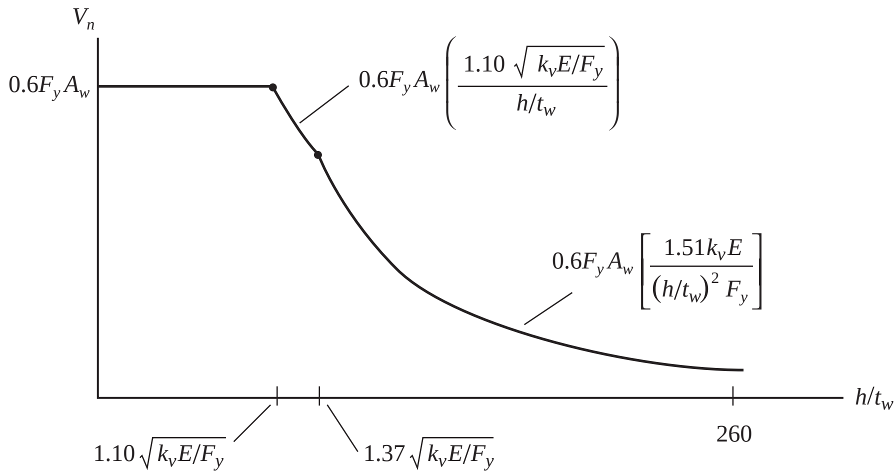
The relationship between shear strength and the web width-to-thickness ratio is analogous to that between flexural strength and the width-to-thickness ratio (for FLB) and between flexural strength and the unbraced length (for LTB). This relationship is illustrated in Figure 5.19.

Allowable Stress Formulation

The allowable strength relation

$$V_a \le \frac{V_n}{\Omega_v}$$

FIGURE 5.19



can also be written in terms of stress as

$$f_v \le F_v$$

where

$$f_v = \frac{V_a}{A_w} = \text{applied shear stress}$$

$$F_v = \frac{V_n/\Omega_v}{A_w} = \frac{0.6F_yA_wC_v/\Omega_v}{A_w} = \text{allowable shear stress}$$

For the most common case of hot-rolled I shapes with $h/t_w \le 2.24\sqrt{E/F_y}$,

$$F_v = \frac{0.6F_yA_w(1.0)/1.5\ 0}{A_w} = 0.4F_y$$

Shear is rarely a problem in rolled steel beams; the usual practice is to design a beam for flexure and then to check it for shear.

EXAMPLE 5.7

Check the beam in Example 5.6 for shear.

SOLUTION

From the dimensions and properties tables in Part 1 of the *Manual*, the web width-to-thickness ratio of a W14 × 90 is

$$\frac{h}{t_w} = 25.9$$

and the web area is $A_w = dt_w = 14.0(0.440) = 6.160 \text{ in.}^2$

$$2.24\sqrt{\frac{E}{F_y}} = 2.24\sqrt{\frac{29{,}000}{50}} = 54.0$$

Since

$$\frac{h}{t_w} < 2.24\sqrt{\frac{E}{F_y}}$$

the strength is governed by shear yielding of the web and $C_v = 1.0$. (As pointed out in the Specification User Note, this will be the case for most W shapes with $F_y \leq 50$ ksi.) The nominal shear strength is

$$V_n = 0.6F_yA_wC_v = 0.6(50)(6.160)(1.0) = 184.8 \text{ kips}$$

LRFD SOLUTION

Determine the resistance factor ϕ_v.

$$\text{Since } \frac{h}{t_w} < 2.24\sqrt{\frac{E}{F_y}},$$

$$\phi_v = 1.00$$

and the design shear strength is

$$\phi_v V_n = 1.00(184.8) = 185 \text{ kips}$$

From Example 5.6, $w_u = 2.080$ kips/ft and $L = 45$ ft. For a simply supported, uniformly loaded beam, the maximum shear occurs at the support and is equal to the reaction.

$$V_u = \frac{w_u L}{2} = \frac{2.080(45)}{2} = 46.8 \text{ kips} < 185 \text{ kips} \quad \text{(OK)}$$

ASD SOLUTION

Determine the safety factor Ω_v.

$$\text{Since } \frac{h}{t_w} < 2.24\sqrt{\frac{E}{F_y}},$$

$$\Omega_v = 1.50$$

and the allowable shear strength is

$$\frac{V_n}{\Omega_v} = \frac{184.8}{1.50} = 123 \text{ kips}$$

From Example 5.6, the total service load is

$$w_a = w_D + w_L = 0.400 + 1.000 = 1.4 \text{ kips/ft}$$

The maximum shear is

$$V_a = \frac{w_a L}{2} = \frac{1.4(45)}{2} = 31.5 \text{ kips} < 123 \text{ kips} \quad \text{(OK)}$$

Alternately, a solution in terms of stress can be done. Since shear yielding controls ($C_v = 1.0$) and $\Omega_v = 1.50$, the allowable shear stress is

$$F_v = 0.4F_y = 0.4(50) = 20 \text{ ksi}$$

The required shear strength (stress) is

$$f_a = \frac{V_a}{A_w} = \frac{31.5}{6.160} = 5.11 \text{ ksi} < 20 \text{ ksi} \quad \text{(OK)}$$

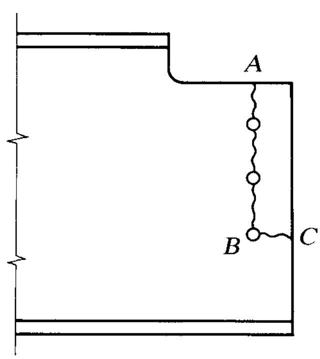
ANSWER The required shear strength is less than the available shear strength, so the beam is satisfactory.

Values of $\phi_v V_n$ and V_n/Ω_v are given in several tables in Part 3 of the *Manual*, including the Z_x table, so computation of shear strength is unnecessary for hot-rolled shapes.

Block Shear

Block shear, which was considered earlier in conjunction with tension member connections, can occur in certain types of beam connections. To facilitate the connection of beams to other beams so that the top flanges are at the same elevation, a short length of the top flange of one of the beams may be cut away, or *coped*. If a coped beam is connected with bolts as in Figure 5.20, segment *ABC* will tend to tear out.

FIGURE 5.20



The applied load in this case will be the vertical beam reaction, so shear will occur along line *AB* and there will be tension along *BC*. Thus the block shear strength will be a limiting value of the reaction.

We covered the computation of block shear strength in Chapter 3, but we will review it here. Failure is assumed to occur by rupture (fracture) on the shear area (subject to an upper limit) and rupture on the tension area. AISC J4.3, "Block Shear Strength," gives the following equation for block shear strength:

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt} \leq 0.6F_yA_{gv} + U_{bs}F_uA_{nt} \qquad \text{(AISC Equation J4-5)}$$

where

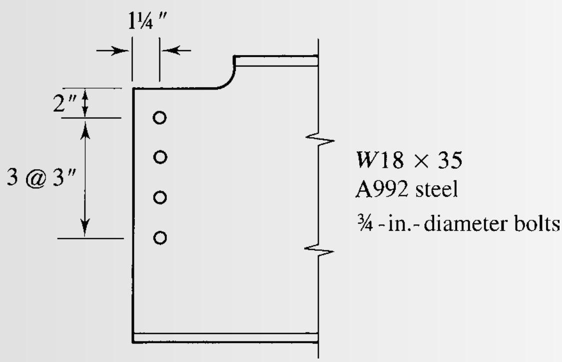
A_{gv} = gross area in shear (in Figure 5.20, length *AB* times the web thickness)
A_{nv} = net area along the shear surface or surfaces
A_{nt} = net area along the tension surface (in Figure 5.20, along *BC*)
U_{bs} = 1.0 when the tensile stress is uniform (for most coped beams)
= 0.5 when the tension stress is not uniform (coped beams with two lines of bolts or with nonstandard distance from bolts to end of beam) (Ricles and Yura, 1983)

For LRFD, $\phi = 0.75$. For ASD, $\Omega = 2.00$.

EXAMPLE 5.8

Determine the maximum reaction, based on block shear, that can be resisted by the beam shown in Figure 5.21.

FIGURE 5.21



SOLUTION The effective hole diameter is $\frac{3}{4} + \frac{1}{8} = \frac{7}{8}$ in. The shear areas are

$$A_{gv} = t_w(2+3+3+3) = 0.300(11) = 3.300 \text{ in.}^2$$

$$A_{nv} = 0.300\left[11 - 3.5\left(\frac{7}{8}\right)\right] = 2.381 \text{ in.}^2$$

The net tension area is

$$A_{nt} = 0.300\left[1.25 - \frac{1}{2}\left(\frac{7}{8}\right)\right] = 0.2438 \text{ in.}^2$$

Since the block shear will occur in a coped beam with a single line of bolts, $U_{bs} = 1.0$. From AISC Equation J4-5,

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt} = 0.6(65)(2.381) + 1.0(65)(0.2438) = 108.7 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(65)(3.300) + 1.0(65)(0.2438) = 144.5 \text{ kips}$$

The nominal block shear strength is therefore 108.7 kips.

LRFD SOLUTION The maximum factored load reaction is the design strength: $\phi R_n = 0.75(108.7) = 81.5$ kips

ASD SOLUTION The maximum service load reaction is the allowable strength: $\frac{R_n}{\Omega} = \frac{108.7}{2.00} = 54.4$ kips

5.9 DEFLECTION

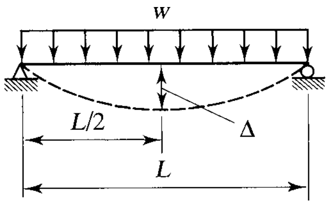
In addition to being safe, a structure must be *serviceable*. A serviceable structure is one that performs satisfactorily, not causing any discomfort or perceptions of unsafety for the occupants or users of the structure. For a beam, being serviceable usually means that the deformations, primarily the vertical sag, or deflection, must be limited. Excessive deflection is usually an indication of a very flexible beam, which can lead to problems with vibrations. The deflection itself can cause problems if elements attached to the beam can be damaged by small distortions. In addition, users of the structure may view large deflections negatively and wrongly assume that the structure is unsafe.

For the common case of a simply supported, uniformly loaded beam such as that in Figure 5.22, the maximum vertical deflection is

$$\Delta = \frac{5}{384}\frac{wL^4}{EI}$$

Deflection formulas for a variety of beams and loading conditions can be found in Part 3, “Design of Flexural Members,” of the *Manual*. For more unusual situations,

FIGURE 5.22



$$\Delta = \frac{5}{384}\frac{wL^4}{EI}$$

standard analytical methods such as the method of virtual work may be used. Deflection is a serviceability limit state, not one of strength, so deflections should always be computed with *service* loads.

The appropriate limit for the maximum deflection depends on the function of the beam and the likelihood of damage resulting from the deflection. The AISC Specification furnishes little guidance other than a statement in Chapter L, "Design for Serviceability," that deflections should not be excessive. There is, however, a more detailed discussion in the Commentary to Chapter L. Appropriate limits for deflection can usually be found from the governing building code, expressed as a fraction of the span length L, such as $L/360$. Sometimes a numerical limit, such as 1 inch, is appropriate. The limits given in the International Building Code (ICC, 2009) are typical. Table 5.4 shows some of the deflection limits given by that code.

The limits shown in Table 5.4 for deflection due to dead load plus live load do not apply to steel beams, because the dead load deflection is usually compensated for by some means, such as *cambering*. Camber is a curvature in the opposite direction of the dead load deflection curve and can be accomplished by bending the beam, with or without heat. When the dead load is applied to the cambered beam, the curvature is removed, and the beam becomes level. Therefore, only the live load deflection is of concern in the completed structure. Dead load deflection can also be accounted for by pouring a variable depth slab with a level top surface, the variable depth being a consequence of the deflection of the beam (this is referred to as *ponding* of the concrete). Detailed coverage of control of dead load deflection is given in an AISC seminar series (AISC, 1997a) and several papers (Ruddy, 1986; Ricker, 1989; and Larson and Huzzard, 1990).

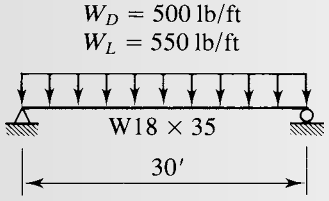
TABLE 5.4
Deflection Limits

Type of member	Max. live load defl.	Max. dead + live load defl.	Max. snow or wind load defl.
Roof beam:			
Supporting plaster ceiling	$L/360$	$L/240$	$L/360$
Supporting nonplaster ceiling	$L/240$	$L/180$	$L/240$
Not supporting a ceiling	$L/180$	$L/120$	$L/180$
Floor beam	$L/360$	$L/240$	—

EXAMPLE 5.9

Compute the dead load and live load deflections for the beam shown in Figure 5.23. If the maximum permissible *live* load deflection is $L/360$, is the beam satisfactory?

FIGURE 5.23



SOLUTION It is more convenient to express the deflection in inches than in feet, so units of inches are used in the deflection formula. The dead load deflection is

$$\Delta_D = \frac{5}{384}\frac{w_D L^4}{EI} = \frac{5}{384}\frac{(0.500/12)(30 \times 12)^4}{29,000(510)} = 0.616 \text{ in.}$$

The live load deflection is

$$\Delta_L = \frac{5}{384}\frac{w_L L^4}{EI} = \frac{5}{384}\frac{(0.550/12)(30 \times 12)^4}{29,000(510)} = 0.678 \text{ in.}$$

The maximum permissible live load deflection is

$$\frac{L}{360} = \frac{30(12)}{360} = 1.0 \text{ in.} > 0.678 \text{ in.} \quad \text{(OK)}$$

ANSWER The beam satisfies the deflection criterion.

Ponding is one deflection problem that does affect the safety of a structure. It is a potential hazard for flat roof systems that can trap rainwater. If drains become clogged during a storm, the weight of the water will cause the roof to deflect, thus providing a reservoir for still more water. If this process proceeds unabated, collapse can occur. The AISC specification requires that the roof system have sufficient stiffness to prevent ponding, and it prescribes limits on stiffness parameters in Appendix 2, “Design for Ponding.”

5.10 DESIGN

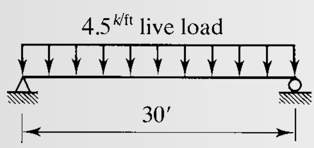
Beam design entails the selection of a cross-sectional shape that will have enough strength and that will meet serviceability requirements. As far as strength is concerned, flexure is almost always more critical than shear, so the usual practice is to design for flexure and then check shear. The design process can be outlined as follows.

1. Compute the required moment strength (i.e., the factored load moment M_u for LRFD or the unfactored moment M_a for ASD). The weight of the beam is part of the dead load but is unknown at this point. A value may be assumed and verified after a shape is selected, or the weight may be ignored initially and checked after a shape has been selected. Because the beam weight is usually a small part of the total load, if it is ignored at the beginning of a design problem, the selected shape will usually be satisfactory when the moment is recomputed.
2. Select a shape that satisfies this strength requirement. This can be done in one of two ways.
   a. Assume a shape, compute the available strength, and compare it with the required strength. Revise if necessary. The trial shape can be easily selected in only a limited number of situations (as in Example 5.10).
   b. Use the beam design charts in Part 3 of the *Manual*. This method is preferred, and we explain it following Example 5.10.
3. Check the shear strength.
4. Check the deflection.

EXAMPLE 5.10

Select a standard hot-rolled shape of A992 steel for the beam shown in Figure 5.24. The beam has continuous lateral support and must support a uniform service live load of 4.5 kips/ft. The maximum permissible live load deflection is $L/240$.

FIGURE 5.24



LRFD SOLUTION

Ignore the beam weight initially then check for its effect after a selection is made.

$$w_u = 1.2w_D + 1.6w_L = 1.2(0) + 1.6(4.5) = 7.2 \text{ kips/ft}$$

$$\text{Required moment strength } M_u = \frac{1}{8}w_u L^2 = \frac{1}{8}(7.2)(30)^2 = 810.0 \text{ ft-kips}$$

$$= \text{required } \phi_b M_n$$

Assume that the shape will be compact. For a compact shape with full lateral support,

$$M_n = M_p = F_y Z_x$$

From $\phi_b M_n \geq M_u$,

$$\phi_b F_y Z_x \geq M_u$$

$$Z_x \geq \frac{M_u}{\phi_b F_y} = \frac{810.0(12)}{0.90(50)} = 216 \text{ in.}^3$$

The Z_x table lists hot-rolled shapes normally used as beams in order of decreasing plastic section modulus. Furthermore, they are grouped so that the shape at the top of each group (in bold type) is the lightest one that has enough section modulus to satisfy a required section modulus that falls within the group. In this example, the shape that comes closest to meeting the section modulus requirement is a W21 × 93, with $Z_x = 221$ in.^{3}, but the lightest one is a W24 × 84, with $Z_x = 224$ in.^{3}. Because section modulus is not directly proportional to area, it is possible to have more section modulus with less area, and hence less weight.

Try a W24 × 84. This shape is compact, as assumed (noncompact shapes are marked as such in the table); therefore $M_n = M_p$, as assumed.

Account for the beam weight.

$$w_u = 1.2w_D + 1.6w_L = 1.2(0.084) + 1.6(4.5) = 7.301 \text{ kips/ft}$$

$$\text{Required moment strength} = M_u = \frac{1}{8} w_u L^2 = \frac{1}{8}(7.301)(30)^2 = 821.4 \text{ ft-kips}$$

The required section modulus is

$$Z_x = \frac{M_u}{\phi_b F_y} = \frac{821.4(12)}{0.90(50)} = 219 \text{ in.}^3 < 224 \text{ in.}^3 \quad \text{(OK)}$$

In lieu of basing the search on the required section modulus, the design strength $\phi_b M_p$ could be used, because it is directly proportional to Z_x and is also tabulated. Next, check the shear:

$$V_u = \frac{w_u L}{2} = \frac{7.301(30)}{2} = 110 \text{ kips}$$

From the Z_x table,

$$\phi_v V_n = 340 \text{ kips} > 110 \text{ kips} \qquad \text{(OK)}$$

Finally, check the deflection. The maximum permissible live load deflection is $L/240 = (30 \times 12)/240 = 1.5$ inch.

$$\Delta_L = \frac{5}{384}\frac{w_L L^4}{EI_x} = \frac{5}{384}\frac{(4.5/12)(30 \times 12)^4}{29{,}000(2370)} = 1.19 \text{ in.} < 1.5 \text{ in.} \qquad \text{(OK)}$$

ANSWER Use a W24 × 84.

ASD SOLUTION Ignore the beam weight initially, then check for its effect after a selection is made.

$$w_a = w_D + w_L = 0 + 4.5 = 4.5 \text{ kips/ft}$$

$$\text{Required moment strength} = M_a = \frac{1}{8} w_a L^2 = \frac{1}{8}(4.5)(30)^2 = 506.3 \text{ ft-kips}$$

$$= \text{required } \frac{M_n}{\Omega_b}$$

Assume that the shape will be compact. For a compact shape with full lateral support,

$$M_n = M_p = F_y Z_x$$

From $\frac{M_p}{\Omega_b} \geq M_a$,

$$\frac{F_y Z_x}{\Omega_b} \geq M_a$$

$$Z_x \geq \frac{\Omega_b M_a}{F_y} = \frac{1.67(506.3 \times 12)}{50} = 203 \text{ in.}^3$$

The Z_x table lists hot-rolled shapes normally used as beams in order of decreasing plastic section modulus. They are arranged in groups, with the lightest shape in each group at the top of that group. For the current case, the shape with a section modulus closest to 203 in.^{3} is a W18 × 97, but the lightest shape with sufficient section modulus is a W24 × 84, with $Z_x = 224$ in.^{3}

Try a W24 × 84. This shape is compact, as assumed (if it were noncompact, there would be a footnote in the Z_x table). Therefore, $M_n = M_p$ as assumed. Account for the beam weight:

$$w_a = w_D + w_L = 0.084 + 4.5 = 4.584 \text{ kips/ft}$$

$$M_a = \frac{1}{8} w_a L^2 = \frac{1}{8}(4.584)(30)^2 = 515.7 \text{ ft-kips}$$

The required plastic section modulus is

$$Z_x = \frac{\Omega_b M_a}{F_y} = \frac{1.67(515.7 \times 12)}{50} = 207 \text{ in.}^3 < 224 \text{ in.}^3 \quad \text{(OK)}$$

Instead of searching for the required section modulus, the search could be based on the required value of M_p/Ω_b, which is also tabulated. Because M_p/Ω_b is proportional to Z_x, the results will be the same.

Another approach is to use the allowable stress for compact laterally supported shapes. From Section 5.5 of this book, with the flexural stress based on the plastic section modulus,

$$F_b = 0.6F_y = 0.6(50) = 30.0 \text{ ksi}$$

and the required section modulus (before the beam weight is included) is

$$Z_x = \frac{M_a}{F_b} = \frac{506.3 \times 12}{30} = 203 \text{ in.}^3$$

Next, check the shear. The required shear strength is

$$V_a = \frac{w_a L}{2} = \frac{4.584(30)}{2} = 68.8 \text{ kips}$$

From the Z_x table, the available shear strength is

$$\frac{V_n}{\Omega_v} = 227 \text{ kips} > 68.8 \text{ kips} \quad \text{(OK)}$$

Check deflection. The maximum permissible live load deflection is

$$\frac{L}{240} = \frac{30 \times 12}{240} = 1.5 \text{ in.}$$

$$\Delta_L = \frac{5}{384} \frac{w_L L^4}{EI_x} = \frac{5}{384} \frac{(4.5/12)(30 \times 12)^4}{29{,}000(2370)} = 1.19 \text{ in.} < 1.5 \text{ in.} \quad \text{(OK)}$$

ANSWER Use a W24 × 84.

In Example 5.10, it was first assumed that a compact shape would be used, and then the assumption was verified. However, if the search is made based on available strength ($\phi_b M_p$ or M_p/Ω_b) rather than section modulus, it is irrelevant whether the shape is compact or noncompact. This is because for noncompact shapes, the tablulated values of $\phi_b M_p$ and M_p/Ω_b are based on flange local buckling and not the plastic moment (see Section 5.6). This means that for laterally supported beams, the Z_x table can be used for design without regard to whether the shape is compact or noncompact.

Beam Design Charts

Many graphs, charts, and tables are available for the practicing engineer, and these aids can greatly simplify the design process. For the sake of efficiency, they are widely used in design offices, but you should approach their use with caution and not allow basic principles to become obscured. It is not our purpose to describe in this book all available design aids in detail, but some are worthy of note, particularly the curves of moment strength versus unbraced length given in Part 3 of the *Manual*.

These curves will be described with reference to Figure 5.25, which shows a graph of nominal moment strength as a function of unbraced length L_b for a particular compact shape. Such a graph can be constructed for any cross-sectional shape and specific values of F_y and C_b by using the appropriate equations for moment strength.

The design charts in the *Manual* comprise a family of graphs similar to the one shown in Figure 5.25. Two sets of curves are available, one for W shapes with F_y = 50 ksi and one for C and MC shapes with F_y = 36 ksi. Each graph gives the flexural strength of a standard hot-rolled shape. Instead of giving the nominal strength M_n, however, both the allowable moment strength M_n/Ω_b and the design moment strength $\phi_b M_n$ are given. Two scales are shown on the vertical axis—one for M_n/Ω_b and one for $\phi_b M_n$. All curves were generated with $C_b = 1.0$. For other values of C_b, simply multiply the moment from the chart by C_b. However, the strength can never exceed the value represented by the horizontal line at the left side of the graph. For a compact shape, this represents the strength corresponding to yielding (reaching the plastic moment M_p). If the curve is for a noncompact shape, the horizontal line represents the flange local buckling strength.

Use of the charts is illustrated in Figure 5.26, where two such curves are shown. Any point on this graph, such as the intersection of the two dashed lines, represents

FIGURE 5.25

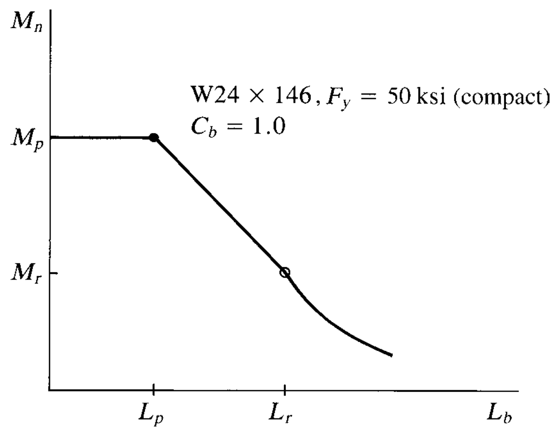
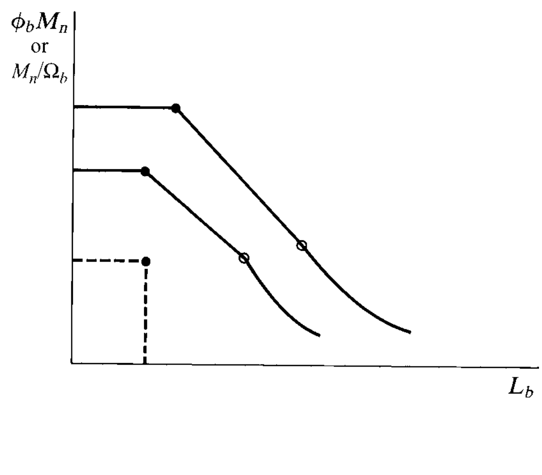


FIGURE 5.26



an available moment strength and an unbraced length. If the moment is a required moment capacity, then any curve above the point corresponds to a beam with a larger moment capacity. Any curve to the right is for a beam with exactly the required moment capacity, although for a larger unbraced length. In a design problem, therefore, if the charts are entered with a given unbraced length and a required strength, curves above and to the right of the point correspond to acceptable beams. If a dashed portion of a curve is encountered, then a curve for a lighter shape lies above or to the right of the dashed curve. Points on the curves corresponding to L_p are indicated by a solid circle; L_r is represented by an open circle.

In the LRFD solution of Example 5.10, the required design strength was 810.0 ft-kips, and there was continuous lateral support. For continuous lateral support, L_b can be taken as zero. From the charts, the first solid curve above the 810.0 ft-kip mark is for a W24 × 84, the same as selected in Example 5.10. Although $L_b = 0$ is not on this particular chart, the smallest value of L_b shown is less than L_p for all shapes on that page.

The beam curve shown in Figure 5.25 is for a compact shape, so the value of M_n for sufficiently small values of L_b is M_p. As discussed in Section 5.6, if the shape is noncompact, the maximum value of M_n will be based on flange local buckling. The maximum unbraced length for which this condition is true will be different from the value of L_p obtained with AISC Equation F2-5. The moment strength of noncompact shapes is illustrated graphically in Figure 5.27, where the maximum nominal strength is denoted M_p', and the maximum unbraced length for which this strength is valid is denoted L_p'.

Although the charts for compact and noncompact shapes are similar in appearance, M_p and L_p are used for compact shapes, whereas M_p' and L_p' are used for noncompact shapes. (This notation is not used in the charts or in any of the other design aids in the *Manual*.) Whether a shape is compact or noncompact is irrelevant to the *use* of the charts.

FIGURE 5.27

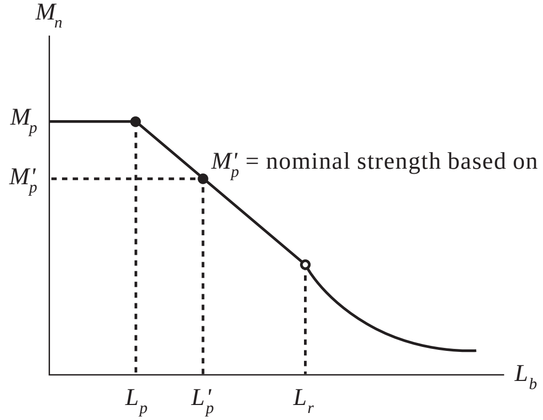
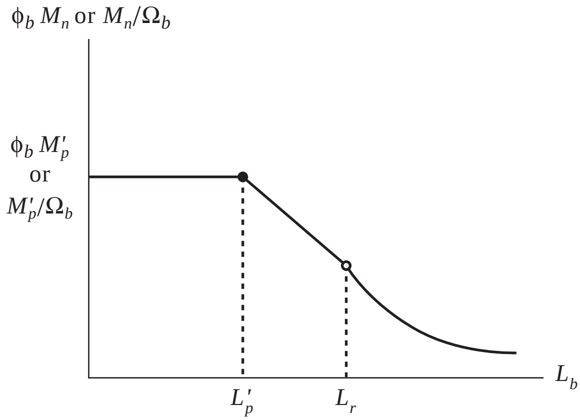


FIGURE 5.27 (*continued*)

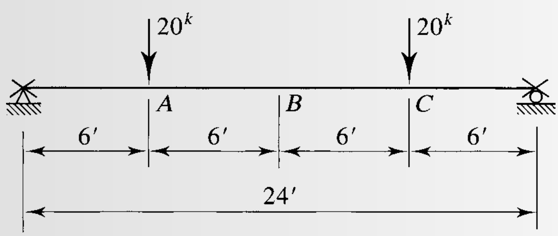


Form of the strength curve in the charts

EXAMPLE 5.11

The beam shown in Figure 5.28 must support two concentrated *live* loads of 20 kips each at the quarter points. The maximum live load deflection must not exceed $L/240$. Lateral support is provided at the ends of the beam. Use A992 steel and select a W shape.

FIGURE 5.28



SOLUTION

If the weight of the beam is neglected, the central half of the beam is subjected to a uniform moment, and

$$M_A = M_B = M_C = M_{max}, \quad \therefore \; C_b = 1.0$$

Even if the weight is included, it will be negligible compared to the concentrated loads, and C_b can still be taken as 1.0, permitting the charts to be used without modification.

LRFD SOLUTION

Temporarily ignoring the beam weight, the factored-load moment is

$$M_u = 6(1.6 \times 20) = 192 \text{ ft-kips}$$

From the charts, with $L_b = 24$ ft, **try W12 × 53:**

$$\phi_b M_n = 209 \text{ ft-kips} > 192 \text{ ft-kips} \qquad \text{(OK)}$$

Now, we account for the beam weight:

$$M_u = 192 + \frac{1}{8}(1.2 \times 0.053)(24)^2 = 197 \text{ ft-kips} < 209 \text{ ft-kips} \quad \text{(OK)}$$

The shear is

$$V_u = 1.6(20) + \frac{1.2(0.053)(24)}{2} = 32.8 \text{ ft-kips}$$

From the Z_x table (or the uniform load table),

$$\phi_v V_n = 125 \text{ kips} > 32.8 \text{ kips} \qquad \text{(OK)}$$

The maximum permissible live load deflection is

$$\frac{L}{240} = \frac{24(12)}{240} = 1.20 \text{ in.}$$

From Table 3-23, "Shears, Moments, and Deflections," in Part 3 of the *Manual*, the maximum deflection (at midspan) for two equal and symmetrically placed loads is

$$\Delta = \frac{Pa}{24EI}(3L^2 - 4a^2)$$

where
P = magnitude of concentrated load
a = distance from support to load
L = span length

$$\Delta = \frac{20(6 \times 12)}{24EI}[3(24 \times 12)^2 - 4(6 \times 12)^2] = \frac{13.69 \times 10^6}{EI}$$

$$= \frac{13.69 \times 10^6}{29{,}000(425)} = 1.11 \text{ in.} < 1.20 \text{ in.} \quad \text{(OK)}$$

ANSWER Use a W12 × 53.

ASD SOLUTION The required flexural strength (not including the beam weight) is

$$M_a = 6(20) = 120 \text{ ft-kips}$$

From the charts, with $L_b = 24$ ft, **try W12 × 53.**

$$\frac{M_n}{\Omega_b} = 139 \text{ ft-kips} > 120 \text{ ft-kips} \quad \text{(OK)}$$

Account for the beam weight:

$$M_a = 6(20) + \frac{1}{8}(0.053)(24)^2 = 124 \text{ ft-kips} < 139 \text{ ft-kips} \quad \text{(OK)}$$

The required shear strength is

$$V_a = 20 + \frac{0.053(24)}{2} = 20.6 \text{ kips}$$

From the Z_x table (or the uniform load table),

$$\frac{V_n}{\Omega_v} = 83.2 \text{ kips} > 20.6 \text{ kips} \quad \text{(OK)}$$

Since deflections are computed with service loads, the deflection check is the same for both LRFD and ASD. From the LRFD solution,

$$\Delta = 1.11 \text{ in.} < 1.20 \text{ in.} \qquad \text{(OK)}$$

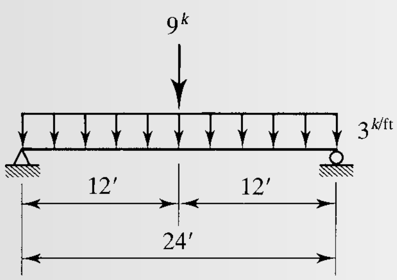
ANSWER Use a W12 × 53.

Although the charts are based on $C_b = 1.0$, they can easily be used for design when C_b is not 1.0; simply divide the required strength by C_b before entering the charts. We illustrate this technique in Example 5.12.

EXAMPLE 5.12

Use A992 steel and select a rolled shape for the beam in Figure 5.29. The concentrated load is a service live load, and the uniform load is 30% dead load and 70% live load. Lateral bracing is provided at the ends and at midspan. There is no restriction on deflection.

FIGURE 5.29



SOLUTION Neglect the beam weight and check it later.

$$w_D = 0.30(3) = 0.9 \text{ kips/ft}$$
$$w_L = 0.70(3) = 2.1 \text{ kips/ft}$$

LRFD SOLUTION

$$w_u = 1.2(0.9) + 1.6(2.1) = 4.44 \text{ kips/ft}$$
$$P_u = 1.6(9) = 14.4 \text{ kips}$$

The factored loads and reactions are shown in Figure 5.30. Next, determine the moments required for the computation of C_b. The bending moment at a distance x from the left end is

$$M = 60.48x - 4.44x\left(\frac{x}{2}\right) = 60.48x - 2.22x^2 \quad (\text{for } x \le 12 \text{ ft})$$

For $x = 3$ ft, $M_A = 60.48(3) - 2.22(3)^2 = 161.5$ ft-kips

For $x = 6$ ft, $M_B = 60.48(6) - 2.22(6)^2 = 283.0$ ft-kips

For $x = 9$ ft, $M_C = 60.48(9) - 2.22(9)^2 = 364.5$ ft-kips

For $x = 12$ ft, $M_{max} = M_u = 60.48(12) - 2.22(12)^2 = 406.1$ ft-kips

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C}$$
$$= \frac{12.5(406.1)}{2.5(406.1) + 3(161.5) + 4(283.0) + 3(364.5)} = 1.36$$

Enter the charts with an unbraced length $L_b = 12$ ft and a bending moment of

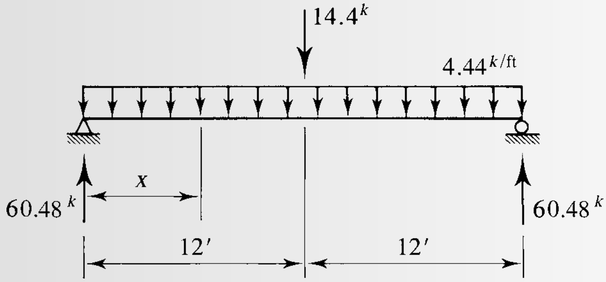
$$\frac{M_u}{C_b} = \frac{406.1}{1.36} = 299 \text{ ft-kips}$$

Try W21 × 48:

$$\phi_b M_n = 311 \text{ ft-kips} \qquad (\text{for } C_b = 1)$$

Since $C_b = 1.36$, the actual design strength is $1.36(311) = 423$ ft-kips. But the design strength cannot exceed $\phi_b M_p$, which is only 398 ft-kips (obtained from the chart),

FIGURE 5.30



so the actual design strength must be taken as

$$\phi_b M_n = \phi_b M_p = 398 \text{ ft-kips} < M_u = 406.1 \text{ ft-kips} \qquad \text{(N.G.)}$$

For the next trial shape, move up in the charts to the next solid curve and **try W18 × 55**. For L_b = 12 ft, the design strength from the chart is 335 ft-kips for $C_b = 1$. The strength for $C_b = 1.36$ is

$$\phi_b M_n = 1.36(335) = 456 \text{ ft-kips} > \phi_b M_p = 420 \text{ ft-kips}$$

$$\therefore \quad \phi_b M_n = \phi_b M_p = 420 \text{ ft-kips} > M_u = 406.1 \text{ ft-kips} \qquad \text{(OK)}$$

Check the beam weight.

$$M_u = 406.1 + \frac{1}{8}(1.2 \times 0.055)(24)^2 = 411 \text{ ft-kips} < 420 \text{ ft-kips} \qquad \text{(OK)}$$

The maximum shear is

$$V_u = 60.48 + \frac{1.2(0.055)}{2}(24) = 61.3 \text{ kips}$$

From the Z_x tables,

$$\phi_v V_n = 212 \text{ kips} > 61.3 \text{ kips} \qquad \text{(OK)}$$

ANSWER Use a W18 × 55.

ASD SOLUTION The applied loads are

$$w_a = 3 \text{ kips/ft} \quad \text{and} \quad P_a = 9 \text{ kips}$$

The left-end reaction is

$$\frac{w_a L + P_a}{2} = \frac{3(24) + 9}{2} = 40.5 \text{ kips}$$

and the bending moment at a distance x from the left end is

$$M = 40.5x - 3x\left(\frac{x}{2}\right) = 40.5x - 1.5x^2 \quad (\text{for } x \le 12 \text{ ft})$$

Compute the moments required for the computation of C_b:

For x = 3 ft, $M_A = 40.5(3) - 1.5(3)^2 = 108.0$ ft-kips
For x = 6 ft, $M_B = 40.5(6) - 1.5(6)^2 = 189.0$ ft-kips
For x = 9 ft, $M_C = 40.5(9) - 1.5(9)^2 = 243.0$ ft-kips
For x = 12 ft, $M_{max} = 40.5(12) - 1.5(12)^2 = 270.0$ ft-kip

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(270)}{2.5(270) + 3(108) + 4(189) + 3(243)} = 1.36$$

Enter the charts with an unbraced length $L_b = 12$ ft and a bending moment of

$$\frac{M_a}{C_b} = \frac{270}{1.36} = 199 \text{ ft-kips}$$

Try W21 × 48. For $C_b = 1$,

$$M_n/\Omega_b = 207 \text{ ft-kips}$$

For $C_b = 1.36$, the actual allowable strength is 1.36(207) = 282 ft-kips, but the strength cannot exceed M_p/Ω_n, which is only 265 ft-kips (this can be obtained from the chart), so the actual allowable strength must be taken as

$$\frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 265 \text{ ft-kips} < M_a = 270 \text{ ft-kips} \quad \text{(N.G.)}$$

Move up in the charts to the next solid curve and **try W18 × 55**. For $L_b = 12$ ft, the allowable strength for $C_b = 1$ is 223 ft-kips. The strength for $C_b = 1.36$ is

$$\frac{M_n}{\Omega_b} = 1.36(223) = 303 \text{ ft-kips} > \frac{M_p}{\Omega_b} = 280 \text{ ft-kips}$$

$$\therefore \quad \frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 280 \text{ ft-kips} > M_a = 270 \text{ ft-kips} \quad \text{(OK)}$$

Account for the beam weight.

$$M_a = 270 + \frac{1}{8}(0.055)(24)^2 = 274 \text{ ft-kips} < 280 \text{ ft-kips} \quad \text{(OK)}$$

The maximum shear is

$$V_a = \frac{9 + 3.055(24)}{2} = 41.2 \text{ kips}$$

From the Z_x table (or the uniform load table)

$$\frac{V_n}{\Omega_v} = 141 \text{ kips} > 41.2 \text{ kips} \quad \text{(OK)}$$

ANSWER Use a W18 × 55.

In Example 5.12, the value of C_b is the same (to three significant figures) for both the factored and the unfactored moments. The two computed values will always be nearly the same, and for this reason, it makes no practical difference which moments are used.

If deflection requirements control the design of a beam, a minimum required moment of inertia is computed, and the lightest shape having this value is sought. This task is greatly simplified by the moment of inertia selection tables in Part 3 of the *Manual*. We illustrate the use of these tables in Example 5.13, following a discussion of the design procedure for a beam in a typical floor or roof system.

5.11 FLOOR AND ROOF FRAMING SYSTEMS

When a distributed load acts on an area such as a floor in a building, certain portions of that load are supported by various components of the floor system. The actual distribution is difficult to determine, but it can be approximated quite easily. The basic idea is that of *tributary areas*. In the same way that tributaries flow into a river and contribute to the volume of water in it, the loads on certain areas of a structural surface "flow" into a structural component. The concept of tributary areas was first discussed in Section 3.8 in the coverage of tension members in roof trusses.

Figure 5.31 shows a typical floor framing plan for a multistory building. Part (a) of the figure shows one of the rigid frames comprising the building. Part (b) shows what would be seen if a horizontal section were cut through the building above one of the floors and the lower portion viewed from above. The gridwork thus exposed consists of the column cross sections (in this case, wide-flange structural steel shapes), girders connecting the columns in the east-west direction, and intermediate floor beams such as *EF* spanning between the girders. Girders are defined as beams that support other beams, although sometimes the term is applied to large beams in general. The floor beams, which fill in the panels defined by the columns, are sometimes called *filler beams*. The columns and girders along any of the east-west lines make up an individual frame. The frames are connected by the beams in the north-south direction, completing the framework for the building. There may also be secondary components, such as bracing for stability, that are not shown in Figure 5.31.

Figure 5.31(c) shows a typical bay of the floor framing system. When columns are placed in a rectangular grid, the region between four columns is called a *bay*. The bay size, such as 30 ft $\times$ 40 ft, is a measure of the geometry of a building. Figure 5.31(d) is a cross section of this bay, showing the floor beams as wide-flange steel shapes supporting a reinforced concrete floor slab.

The overall objective of a structure is to transmit loads to the foundation. As far as floor loads are concerned, this transmission of loads is accomplished as follows:

1. Floor loads, both live and dead, are supported by the floor slab.
2. The weight of the slab, along with the loads it supports, is supported by the floor beams.
3. The floor beams transmit their loads, including their own weight, to the girders.
4. The girders and their loads are supported by the columns.
5. The column loads are supported by the columns of the story below. The column loads accumulate from the top story to the foundation.

(The route taken by the loads from one part of the structure to another is sometimes called the *load path*.) This is a fairly accurate representation of what happens, but it is not exact. For example, part of the slab and its load will be supported directly by the girders, but most of it will be carried by the floor beams.

FIGURE 5.31

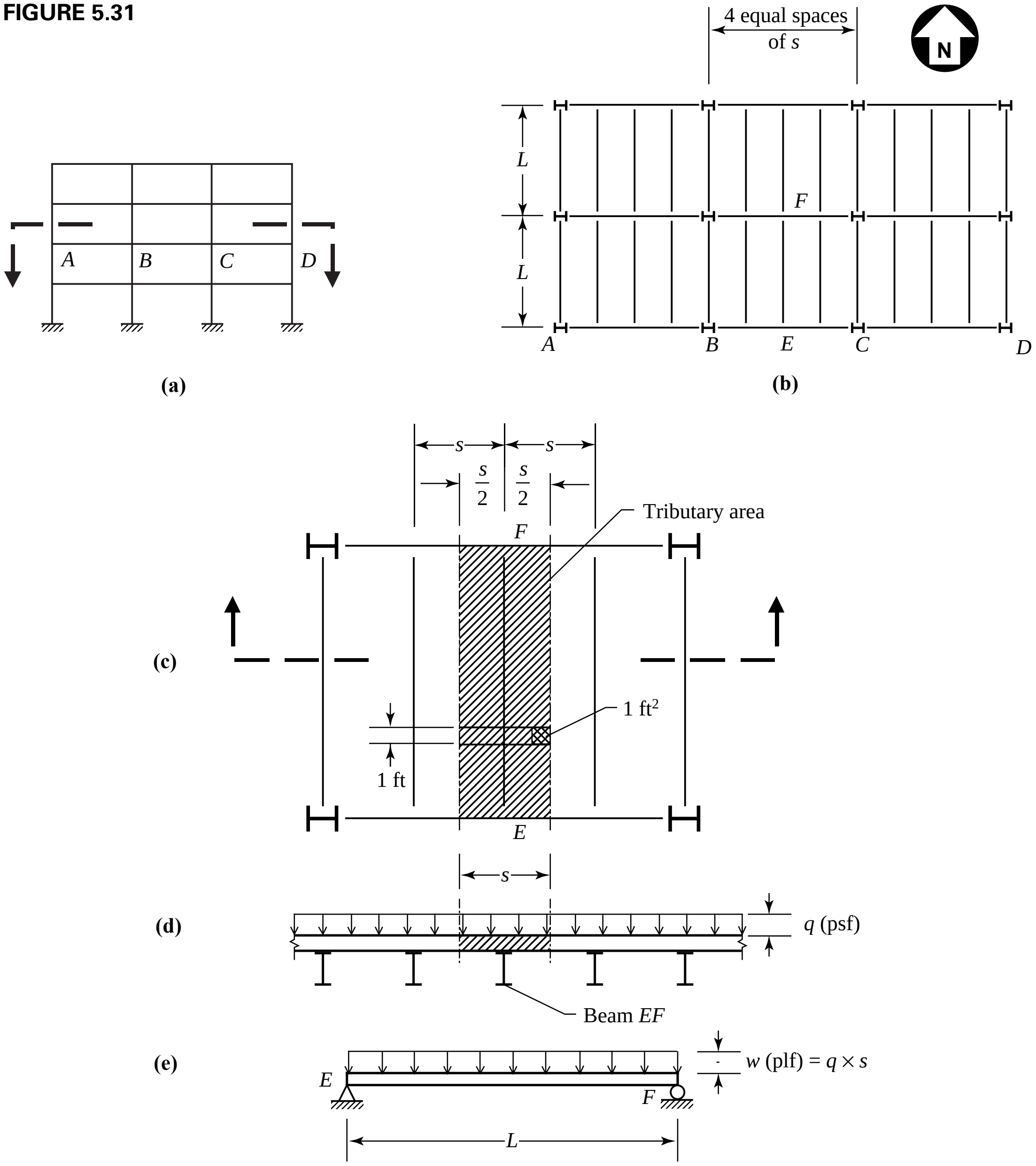


Figure 5.31(c) shows a shaded area around floor beam *EF*. This is the tributary area for this member, and it consists of half the floor between beam *EF* and the adjacent beam on each side. Thus, the total width of floor being supported is equal to the beam spacing *s* if the spacing is uniform. If the load on the floor is uniformly distributed, we can express the uniform load on beam *EF* as a force per unit length (for example, pounds per linear foot [plf]) by multiplying the floor load in force per unit area (for example, pounds per

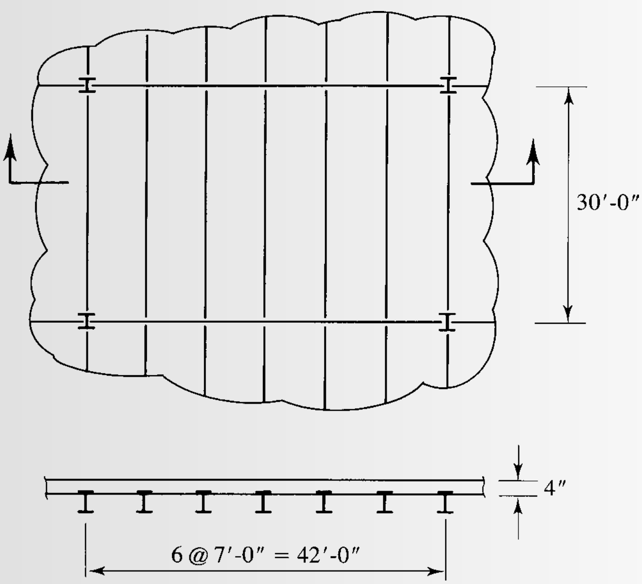
square foot [psf]) by the tributary width *s*. Figure 5.31(e) shows the final beam model (for the usual floor framing connections, the beams can be treated as simply supported).

For convenience, the weight of a reinforced concrete floor slab is usually expressed in pounds per square foot of floor surface. This way, the slab weight can be combined with other loads similarly expressed. If the floor consists of a metal deck and concrete fill, the combined weight can usually be obtained from the deck manufacturer's literature. If the floor is a slab of uniform thickness, the weight can be calculated as follows. Normal-weight concrete weighs approximately 145 pounds per cubic foot. If 5 pcf is added to account for the reinforcing steel, the total weight is 150 pcf. The volume of slab contained in one square foot of floor is 1 ft^2 × the slab thickness t. For a thickness expressed in inches, the slab weight is therefore $(t/12)(150)$ psf. For lightweight concrete, a unit weight of 115 pounds per cubic foot can be used in lieu of more specific data.

EXAMPLE 5.13

Part of a floor framing system is shown in Figure 5.32. A 4-inch-thick reinforced concrete floor slab of normal-weight concrete is supported by floor beams spaced at 7 feet. The floor beams are supported by girders, which in turn are supported by the columns. In addition to the weight of the structure, loads consist of a uniform live load of 80 psf and moveable partitions, to be accounted for by using a uniformly distributed load of 20 pounds per square foot of floor surface. The maximum live load deflection must not exceed 1/360 of the span length. Use A992 steel

FIGURE 5.32



and design the floor beams. Assume that the slab provides continuous lateral support of the floor beams.

SOLUTION The slab weight is

$$w_{slab} = \frac{t}{12}(150) = \frac{4}{12}(150) = 50 \text{ psf}$$

Assume that each beam supports a 7-ft width (tributary width) of floor.

Slab: 50(7) = 350 lb/ft

Partitions: 20(7) = 140 lb/ft

Live load: 80(7) = 560 lb/ft

The beam weight will be accounted for once a trial selection has been made.

Since the partitions are moveable, they will be treated as live load. This is consistent with the provisions of the International Building Code (ICC, 2009). The dead and live loads are, therefore,

$$w_D = 0.350 \text{ lb/ft (excluding the beam weight)}$$
$$w_L = 0.560 + 0.140 = 0.700 \text{ lb/ft}$$

LRFD SOLUTION The total factored load is

$$w_u = 1.2w_D + 1.6w_L = 1.2(0.350) + 1.6(0.700) = 1.540 \text{ kips/ft}$$

The typical floor-beam connection will provide virtually no moment restraint, and the beams can be treated as simply supported. Hence,

$$M_u = \frac{1}{8}w_uL^2 = \frac{1}{8}(1.540)(30)^2 = 173 \text{ ft-kips}$$

Since the beams have continuous lateral support, the Z_x table can be used.

Try a W14 × 30:

$$\phi_b M_n = 177 \text{ ft-kips} > 173 \text{ ft-kips} \qquad \text{(OK)}$$

Check the beam weight.

$$M_u = 173 + \frac{1}{8}(1.2 \times 0.030)(30)^2 = 177 \text{ ft-kips} \qquad \text{(OK)}$$

The maximum shear is

$$V_u \approx \frac{1.540(30)}{2} = 23.1 \text{ kips}$$

From the Z_x table,

$$\phi_v V_n = 112 \text{ kips} > 23.1 \text{ kips} \qquad \text{(OK)}$$

The maximum permissible deflection is

$$\frac{L}{360} = \frac{30(12)}{360} = 1.0 \text{ in.}$$

$$\Delta_L = \frac{5}{384}\frac{w_L L^4}{EI} = \frac{5}{384}\frac{(0.700/12)(30\times 12)^4}{29{,}000(291)} = 1.51 \text{ in.} > 1.0 \text{ in.} \qquad \text{(N.G.)}$$

Solving the deflection equation for the required moment of inertia yields

$$I_{\text{required}} = \frac{5w_L L^4}{384E\Delta_{\text{required}}} = \frac{5(0.700/12)(30\times 12)^4}{384(29{,}000)(1.0)} = 440 \text{ in.}^4$$

Part 3 of the *Manual* contains selection tables for both I_x and I_y. These tables are organized in the same way as the Z_x table, so selection of the lightest shape with sufficient moment of inertia is simple. From the I_x table, **try a W18 × 35:**

$$I_x = 510 \text{ in.}^4 > 440 \text{ in.}^4 \qquad \text{(OK)}$$

$$\phi_b M_n = 249 \text{ ft-kips} > 177 \text{ ft-kips} \qquad \text{(OK)}$$

$$\phi_v V_n = 159 \text{ kips} > 23.1 \text{ kips} \qquad \text{(OK)}$$

ANSWER Use a W18 × 35.

ASD SOLUTION Account for the beam weight after a selection has been made.

$$w_a = w_D + w_L = 0.350 + 0.700 = 1.05 \text{ kips/ft}$$

If we treat the beam connection as a simple support, the required moment strength is

$$M_a = \frac{1}{8}w_a L^2 = \frac{1}{8}(1.05)(30)^2 = 118 \text{ ft-kips} = \text{required } \frac{M_n}{\Omega_b}$$

For a beam with full lateral support, the Z_x table can be used.

Try a W16 × 31:

$$\frac{M_n}{\Omega_b} = 135 \text{ ft-kips} > 118 \text{ ft-kips} \qquad \text{(OK)}$$

(A W14 × 30 has an allowable moment strength of exactly 118 ft-kips, but the beam weight has not yet been accounted for.)

Account for the beam weight:

$$M_a = 118 + \frac{1}{8}(0.031)(30)^2 = 122 \text{ ft-kips} < 135 \text{ ft-kips} \qquad \text{(OK)}$$

The required shear strength is

$$V_a = \frac{w_a L}{2} = \frac{(1.05 + 0.031)(30)}{2} = 16.2 \text{ kips}$$

From the Z_x table, the available shear strength is

$$\frac{V_n}{\Omega_v} = 87.5 \text{ kips} > 16.2 \text{ kips} \quad \text{(OK)}$$

Check deflection. The maximum permissible live load deflection is

$$\frac{L}{360} = \frac{30 \times 12}{360} = 1.0 \text{ in.}$$

$$\Delta_L = \frac{5}{384}\frac{w_L L^4}{EI_x} = \frac{5}{384}\frac{(0.700/12)(30 \times 12)^4}{29{,}000(375)} = 1.17 \text{ in.} > 1.0 \text{ in.} \quad \text{(N.G.)}$$

Solve the deflection equation for the required moment of inertia:

$$I_{\text{required}} = \frac{5 w_L L^4}{384 E \Delta_{\text{required}}} = \frac{5(0.700/12)(30 \times 12)^4}{384(29{,}000)(1.0)} = 440 \text{ in.}^4$$

Part 3 of the *Manual* contains selection tables for both I_x and I_y. From the I_x table, try a W18 × 35:

$$I_x = 510 \text{ in.}^4 > 440 \text{ in.}^2 \quad \text{(OK)}$$

$$\frac{M_n}{\Omega_b} = 166 \text{ ft-kips} > 122 \text{ ft-kips} \quad \text{(OK)}$$

$$\frac{V_n}{\Omega_v} = 106 \text{ kips} > 16.2 \text{ kips} \quad \text{(OK)}$$

ANSWER Use a W18 × 35.

Note that in Example 5.13, the design was controlled by serviceability rather than strength. This is not unusual, but the recommended sequence in beam design is still to select a shape for moment and then check shear and deflection. Although there is no limit on the dead load deflection in this example, this deflection may be needed if the beam is to be cambered.

$$\Delta_D = \frac{5}{384}\frac{w_{\text{slab+beam}} L^4}{EI} = \frac{5}{384}\frac{[(0.350 + 0.035)/12](30 \times 12)^4}{29{,}000(510)} = 0.474 \text{ in.}$$

5.12 HOLES IN BEAMS

If beam connections are made with bolts, holes will be punched or drilled in the beam web or flanges. In addition, relatively large holes are sometimes cut in beam webs to provide space for utilities such as electrical conduits and ventilation ducts. Ideally, holes should be placed in the web only at sections of low shear, and holes should be made in the flanges at points of low bending moment. That will not always be possible, so the effect of the holes must be accounted for.

For relatively small holes such as those for bolts, the effect will be small, particularly for flexure, for two reasons. First, the reduction in the cross section is usually small. Second, adjacent cross sections are not reduced, and the change in cross section is actually more of a minor discontinuity than a "weak link."

Holes in a beam flange are of concern for the tension flange only, since bolts in the compression flange will transmit the load through the bolts. This is the same rationale that is used for compression members, where the net area is not considered. The AISC Specification requires that bolt holes in beam flanges be accounted for when the nominal tensile rupture strength (fracture strength) of the flange is less than the nominal tensile yield strength—that is, when

$$F_u A_{fn} < F_y A_{fg} \tag{5.9}$$

where

A_{fn} = net tension flange area
A_{fg} = gross tension flange area

If $F_y/F_u > 0.8$, the Specification requires that the right hand side of Equation 5.9 be increased by 10%. Equation 5.9 can be written more generally as follows:

$$F_u A_{fn} < Y_t F_y A_{fg} \tag{5.10}$$

where

$Y_t = 1.0$ for $F_y/F_u \le 0.8$
$= 1.1$ for $F_y/F_u > 0.8$

Note that, for A992 steel, the preferred steel for W shapes, the *maximum* value of F_y/F_u is 0.85. This means that unless more information is available, use $Y_t = 1.1$.
If the condition of Equation 5.10 exists—that is, if

$$F_u A_{fn} < Y_t F_y A_{fg}$$

then AISC F13.1 requires that the nominal flexural strength be limited by the condition of flexural rupture. This limit state corresponds to a flexural stress of

$$f_b = \frac{M_n}{S_x(A_{fn}/A_{fg})} = F_u \tag{5.11}$$

where $S_x(A_{fn}/A_{fg})$ can be considered to be a "net" elastic section modulus. The relationship of Equation 5.11 corresponds to a nominal flexural strength of

$$M_n = \frac{F_u A_{fn}}{A_{fg}} S_x$$

The AISC requirement for holes in beam flanges can be summarized as follows: If

$F_u A_{fn} < Y_t F_y A_{fg}$

The nominal flexural strength cannot exceed

$$M_n = \frac{F_u A_{fn}}{A_{fg}} S_x \qquad \text{(AISC Equation F13-1)}$$

where

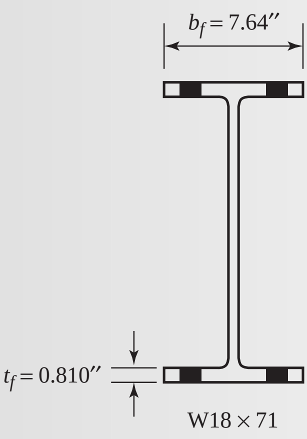
$Y_t = 1.0$ for $F_y/F_u \le 0.8$
$= 1.1$ for $F_y/F_u > 0.8$

The constant Y_t should be taken as 1.1 for A992 steel or if the maximum value of F_y/F_u is not known.

EXAMPLE 5.14

The shape shown in Figure 5.33 is a W18 × 71 with holes in each flange for 1-inch-diameter bolts. The steel is A992. Compute the nominal flexural strength for an unbraced length of 10 feet. Use $C_b = 1.0$.

FIGURE 5.33



SOLUTION To determine the nominal flexural strength M_n, all applicable limit states must be checked. From the Z_x table, a W18 × 71 is seen to be a compact shape (no footnote to indicate otherwise). Also from the Z_x table, $L_p = 6.00$ ft and $L_r = 19.6$ ft. Therefore, for an unbraced length $L_b = 10$ ft,

$$L_p < L_b < L_r$$

and the beam is subject to inelastic lateral-torsional buckling. The nominal strength for this limit state is given by

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \qquad \text{(AISC Equation F2-2)}$$

where

$$M_p = F_yZ_x = 50(146) = 7300 \text{ in.-kips}$$

$$M_n = 1.0\left[7300 - (7300 - 0.7\times 50\times 127)\left(\frac{10-6}{19.6-6}\right)\right] = 6460 \text{ in.-kips}$$

Check to see if the flange holes need to be accounted for. The gross area of one flange is

$$A_{fg} = t_fb_f = 0.810(7.64) = 6.188 \text{ in.}^2$$

The effective hole diameter is

$$d_h = 1 + \frac{1}{8} = 1\frac{1}{8} \text{ in.}$$

and the net flange area is

$$A_{fn} = A_{fg} - t_f\sum d_h = 6.188 - 0.810(2\times 1.125) = 4.366 \text{ in.}^2$$

$$F_uA_{fn} = 65(4.366) = 283.8 \text{ kips}$$

Determine Y_t. For A992 steel, the maximum F_y/F_u ratio is 0.85. Since this is greater than 0.8, use $Y_t = 1.1$.

$$Y_tF_yA_{fg} = 1.1(50)(6.188) = 340.3 \text{ kips}$$

Since $F_uA_{fn} < Y_tF_yA_{fg}$, the holes must be accounted for. From AISC Equation F13-1,

$$M_n = \frac{F_uA_{fn}}{A_{fg}}S_x = \frac{283.8}{6.188}(127) = 5825 \text{ in.-kips}$$

This value is less than the LTB value of 6460 in.-kips, so it controls.

ANSWER $M_n = 5825$ in.-kips $= 485$ ft-kips.

Beams with large holes in their webs will require special treatment and are beyond the scope of this book. *Design of Steel and Composite Beams with Web Openings* is a useful guide to this topic (Darwin, 1990).

5.13 OPEN-WEB STEEL JOISTS

Open-web steel joists are prefabricated trusses of the type shown in Figure 5.34. Many of the smaller ones use a continuous circular bar to form the web members and are commonly called *bar* joists. They are used in floor and roof systems in a wide variety of structures. For a given span, an open-web joist will be lighter in weight than a rolled shape, and the absence of a solid web allows for the easy passage of duct work and electrical conduits. Depending on the span length, open-web joists may be more economical than rolled shapes, although there are no general guidelines for making this determination.

Open-web joists are available in standard depths and load capacities from various manufacturers. Some open-web joists are designed to function as floor or roof joists, and others are designed to function as girders, supporting the concentrated reactions from joists. The AISC Specification does not cover open-web steel joists; a separate organization, the Steel Joist Institute (SJI), exists for this purpose. All aspects of steel joist usage, including their design and manufacture, are addressed in the publication *Standard Specifications, Load Tables, and Weight Tables for Steel Joists and Joist Girders* (SJI, 2005).

An open-web steel joist can be selected with the aid of the standard load tables (SJI, 2005). These tables give load capacities in pounds per foot of length for various standard joists. Tables are available for both LRFD and ASD, in either U.S. Customary units or metric units. One of the LRFD tables is reproduced in Figure 5.35. For each combination of span and joist, a pair of load values is given. The top number is the total load capacity in pounds per foot. The bottom number is the live load per foot that will produce a deflection of 1/360 of the span length. For span lengths in the shaded areas, special bridging (interconnection of joists) is required. The ASD tables use the same format, but the loads are unfactored. The first number in the designation is the nominal depth in inches. The table also gives the approximate weight in pounds per foot of length. Steel fabricators who furnish open-web steel joists must certify that a particular joist of a given designation, such as a 10K1 of span length 20 feet, will have a safe load capacity of at least the value given in the table. Different manufacturers'10K1 joists may have

FIGURE 5.34

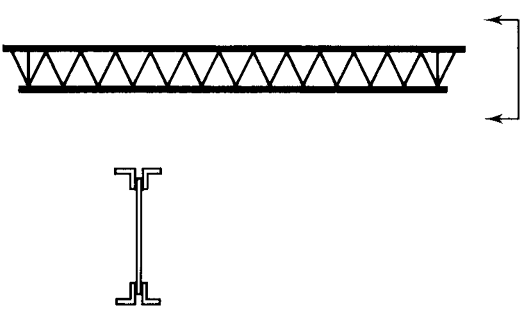


FIGURE 5.35

LRFD

STANDARD LOAD TABLE FOR OPEN WEB STEEL JOISTS, K-SERIES
Based on a 50 ksi Maximum Yield Strength - Loads Shown in Pounds per Linear Foot (plf)

Joist Designation	8K1*	10K1	12K1	12K3	12K5	14K1	14K3	14K4	14K6	16K2	16K3	16K4	16K5	16K6	16K7	16K9
Depth (in.)	8	8	8	8	8	14	14	14	14	16	16	16	16	16	16	16
Approx. Wt (lbs./ft.)	5.1	5.0	5.0	5.7	7.1	5.2	6.0	6.7	7.7	5.5	6.3	7.0	7.5	8.1	8.6	10.0
Span (ft.) ↓ 8	825 550															
9	825 550															
10	825 480	825 550														
11	798 377	825 542														
12	666 288	825 455	825 550	825 550	825 550											
13	565 225	718 363	825 510	825 510	825 510											
14	486 179	618 289	750 425	825 463	825 463	825 550	825 550	825 550	825 550							
15	421 145	537 234	651 344	814 428	825 434	766 475	825 507	825 507	825 507							
16	369 119	469 192	570 282	714 351	825 396	672 390	825 467	825 467	825 467	825 550	825 550	825 550	825 550	825 550	825 550	825 550
17		415 159	504 234	630 291	825 366	592 324	742 404	825 443	825 443	768 488	825 526	825 526	825 526	825 526	825 526	825 526
18		369 134	448 197	561 245	760 317	528 272	661 339	795 397	825 408	684 409	762 456	825 490	825 490	825 490	825 490	825 490
19		331 113	402 167	502 207	681 269	472 230	592 287	712 336	825 383	612 347	682 386	820 452	825 455	825 455	825 455	825 455
20		298 97	361 142	453 177	613 230	426 197	534 246	642 287	787 347	552 297	615 330	739 386	825 426	825 426	825 426	825 426
21			327 123	409 153	555 198	385 170	483 212	582 248	712 299	499 255	556 285	670 333	754 373	822 405	822 406	822 406
22			298 106	373 132	505 172	351 147	439 184	529 215	648 259	454 222	505 247	609 289	687 323	747 351	825 385	825 385
23			271 93	340 116	462 150	321 128	402 160	483 188	592 226	415 194	462 216	556 252	627 282	682 307	760 339	825 363
24			249 81	312 101	423 132	294 113	367 141	442 165	543 199	381 170	424 189	510 221	576 248	627 269	697 298	825 346
25						270 100	339 124	408 145	501 175	351 150	390 167	469 195	529 219	576 238	642 263	771 311
26						249 88	313 110	376 129	462 156	324 133	360 148	433 173	489 194	532 211	592 233	711 276
27						231 79	289 98	349 115	427 139	300 119	334 132	402 155	453 173	493 188	549 208	658 246
28						214 70	270 88	324 103	397 124	279 106	310 118	373 138	421 155	459 168	510 186	612 220
29										259 95	289 106	348 124	391 139	427 151	475 167	570 198
30										241 86	270 96	324 112	366 126	399 137	444 151	532 178
31										226 78	252 87	304 101	342 114	373 124	415 137	498 161
32										213 71	237 79	285 92	321 103	349 112	388 124	466 147

Source: *Standard Specifications, Load Tables, and Weight Tables for Steel Joists and Joist Girders*. Myrtle Beach, S.C.: Steel Joist Institute, 2005. Reprinted with permission.

*8K1 is no longer made as of 2010.

different member cross sections, but they all must have a nominal depth of 10 inches and, for a span length of 20 feet, a factored load capacity of at least 361 pounds per foot.

The open-web steel joists that are designed to function as floor or roof joists (in contrast to girders) are available as open-web steel joists (K-series, both standard and KCS), longspan steel joists (LH-series), and deep longspan steel joists (DLH-series). Standard load tables are given for each of these categories. The higher you move up the series, the greater the available span lengths and load-carrying capacities become. At the lower end, an 8K1 is available with a span length of 8 feet and a factored load capacity of 825 pounds per foot, whereas a 72DLH19 can support a load of 745 pounds per foot on a span of 144 feet.

With the exception of the KCS joists, all open-web steel joists are designed as simply supported trusses with uniformly distributed loads on the top chord. This loading subjects the top chord to bending as well as axial compression, so the top chord is designed as a beam–column (see Chapter 6). To ensure stability of the top chord, the floor or roof deck must be attached in such a way that continuous lateral support is provided.

KCS joists are designed to support both concentrated loads and distributed loads (including nonuniform distributions). To select a KCS joist, the engineer must compute a maximum moment and shear in the joist and enter the KCS tables with these values. (The KCS joists are designed to resist a uniform moment and a constant shear.) If concentrated loads must be supported by an LH or a DLH joist, a special analysis should be requested from the manufacturer.

Both top and bottom chord members of K-series joists must be made of steel with a yield stress of 50 ksi, and the web members may have a yield stress of either 36 ksi or 50 ksi. All members of LH- and DLH-series joists can be made with steel of any yield stress between 36 ksi and 50 ksi inclusive. The load capacity of K-series joists must be verified by the manufacturer by testing. No testing program is required for LH- or DLH-series joists.

Joist girders are designed to support open-web steel joists. For a given span, the engineer determines the number of joist spaces, then from the joist girder weight tables selects a depth of girder. The joist girder is designated by specifying its depth, the number of joist spaces, the load at each loaded top-chord panel point of the joist girder, and a letter to indicate whether the load is factored ("F") or unfactored ("K"). For example, using LRFD and U.S. Customary units, a 52G9N10.5F is 52 inches deep, provides for 9 equal joist spaces on the top chord, and will support 10.5 kips of factored load at each joist location. The joist girder weight tables give the weight in pounds per linear foot for the specified joist girder for a specific span length.

EXAMPLE 5.15

Use the load table given in Figure 5.35 to select an open-web steel joist for the following floor system and loads:

Joist spacing = 3 ft 0 in.

Span length = 20 ft 0 in.

The loads are

3-in. floor slab

Other dead load: 20 psf

Live load: 50 psf

The live load deflection must not exceed $L/360$.

SOLUTION For the dead loads of

$$\text{Slab:}\quad 150\left(\frac{3}{12}\right) = 37.5 \text{ psf}$$

Other dead load: = 20 psf
Joist weight: = 3 psf (est.)
Total: = 60.5 psf

$$w_D = 60.5(3) = 181.5 \text{ lb/ft}$$

For the live load of 50 psf,

$$w_L = 50(3) = 150 \text{ lb/ft}$$

The factored load is

$$w_u = 1.2w_D + 1.6w_L = 1.2\,(181.5) + 1.6(150) = 458 \text{ lb/ft}$$

Figure 5.35 indicates that the following joists satisfy the load requirement: a 12K5, weighing approximately 7.1 lb/ft; a 14K3, weighing approximately 6.0 lb/ft; and a 16K2, weighing approximately 5.5 lb/ft. No restriction was placed on the depth, so we choose the lightest joist, a 16K2.

To limit the live load deflection to $L/360$, the live load must not exceed

$$297 \text{ lb/ft} > 150 \text{ lb/ft} \qquad \text{(OK)}$$

ANSWER Use a 16K2.

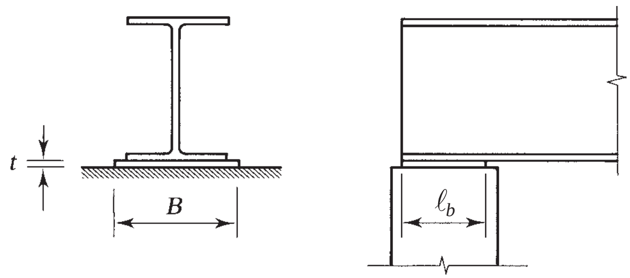
The standard load tables also include a K-series economy table, which facilitates the selection of the lightest joist for a given load.

5.14 BEAM BEARING PLATES AND COLUMN BASE PLATES

The design procedure for column base plates is similar to that for beam bearing plates, and for that reason we consider them together. In addition, the determination of the thickness of a column base plate requires consideration of flexure, so it logically belongs in this chapter rather than in Chapter 4. In both cases, the function of the plate is to distribute a concentrated load to the supporting material.

Two types of beam bearing plates are considered: one that transmits the beam reaction to a support such as a concrete wall and one that transmits a load to the top flange of a beam. Consider first the beam support shown in Figure 5.36. Although many beams are

FIGURE 5.36



connected to columns or other beams, the type of support shown here is occasionally used, particularly at bridge abutments. The design of the bearing plate consists of three steps.

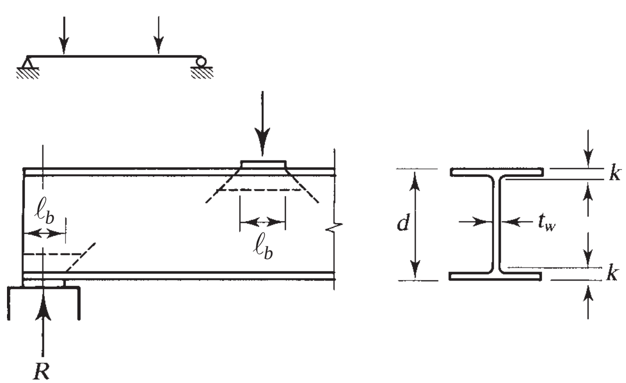
1. Determine dimension ℓ_b so that web yielding and web crippling are prevented.
2. Determine dimension B so that the area $B \times \ell_b$ is sufficient to prevent the supporting material (usually concrete) from being crushed in bearing.
3. Determine the thickness t so that the plate has sufficient bending strength.

Web yielding, web crippling, and concrete bearing strength are addressed by AISC in Chapter J, "Design of Connections."

Web Yielding

Web yielding is the compressive crushing of a beam web caused by the application of a compressive force to the flange directly above or below the web. This force could be an end reaction from a support of the type shown in Figure 5.36, or it could be a load delivered to the top flange by a column or another beam. Yielding occurs when the compressive stress on a horizontal section through the web reaches the yield point. When the load is transmitted through a plate, web yielding is assumed to take place on the nearest section of width t_w. In a rolled shape, this section will be at the toe of the fillet, a distance k from the outside face of the flange (this dimension is tabulated in the dimensions and properties tables in the *Manual*). If the load is assumed to distribute itself at a slope of 1 : 2.5, as shown in Figure 5.37, the area at the support

FIGURE 5.37



subject to yielding is $t_w(2.5k + \ell_b)$. Multiplying this area by the yield stress gives the nominal strength for web yielding at the support:

$$R_n = F_y t_w(2.5k + \ell_b) \qquad \text{(AISC Equation J10-3)}$$

The bearing length ℓ_b at the support should not be less than k.

At the interior load, the length of the section subject to yielding is

$$2(2.5k) + \ell_b = 5k + \ell_b$$

and the nominal strength is

$$R_n = F_y t_w(5k + \ell_b) \qquad \text{(AISC Equation J10-2)}$$

For LRFD, the design strength is ϕR_n, where $\phi = 1.0$.
For ASD, the allowable strength is R_n/Ω, where $\Omega = 1.50$.

Web Crippling

Web crippling is buckling of the web caused by the compressive force delivered through the flange. For an interior load, the nominal strength for web crippling is

$$R_n = 0.80t_w^2\left[1+3\left(\frac{\ell_b}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_y t_f}{t_w}} \qquad \text{(AISC Equation J10-4)}$$

For a load at or near the support (no greater than half the beam depth from the end), the nominal strength is

$$R_n = 0.40t_w^2\left[1+3\left(\frac{\ell_b}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_y t_f}{t_w}} \quad \text{for } \frac{\ell_b}{d} \le 0.2 \qquad \text{(AISC Equation J10-5a)}$$

or

$$R_n = 0.40t_w^2\left[1+\left(\frac{4\ell_b}{d}-0.2\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_y t_f}{t_w}} \quad \text{for } \frac{\ell_b}{d} > 0.2$$

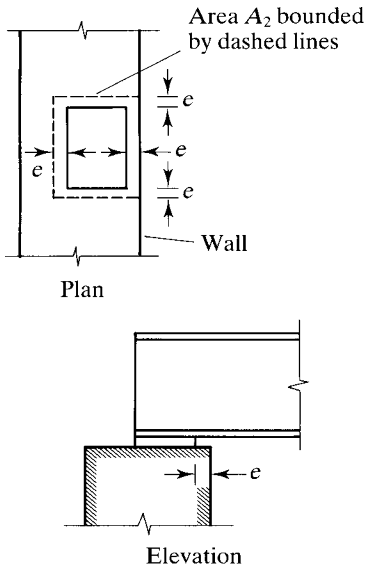
(AISC Equation J10-5b)

The resistance factor for this limit state is $\phi = 0.75$. The safety factor is $\Omega = 2.00$.

Concrete Bearing Strength

The material used for a beam support can be concrete, brick, or some other material, but it usually will be concrete. This material must resist the bearing load applied by the steel plate. The nominal bearing strength specified in AISC J8 is the same as that given in the American Concrete Institute's Building Code (ACI, 2008) and may be

FIGURE 5.38



used if no other building code requirements are in effect. If the plate covers the full area of the support, the nominal strength is

$$P_p = 0.85 f_c' A_1 \qquad \text{(AISC Equation J8-1)}$$

If the plate does not cover the full area of the support,

$$P_p = 0.85 f_c' A_1 \sqrt{\frac{A_2}{A_1}} \leq 1.7 f_c' A_1 \qquad \text{(AISC Equation J8-2)}$$

where

f_c' = 28-day compressive strength of the concrete
A_1 = bearing area
A_2 = full area of the support

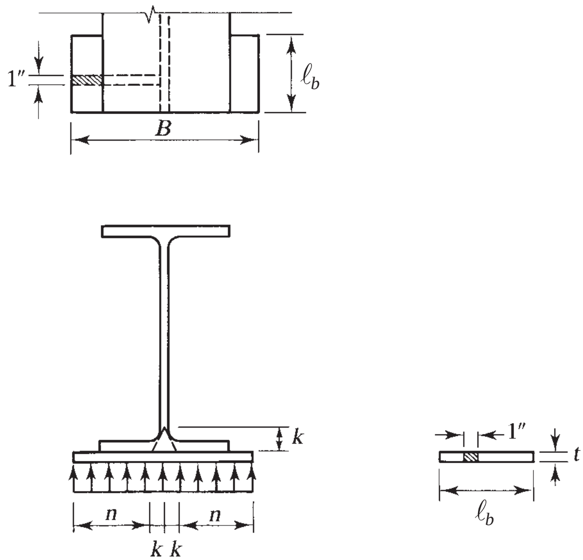
If area A_2 is not concentric with A_1, then A_2 should be taken as the largest concentric area that is geometrically similar to A_1, as illustrated in Figure 5.38.

For LRFD, the design bearing strength is $\phi_c P_p$, where $\phi_c = 0.65$. For ASD, the allowable bearing strength is P_p/Ω_c, where $\Omega_c = 2.31$.

Plate Thickness

Once the length and width of the plate have been determined, the average bearing pressure is treated as a uniform load on the bottom of the plate, which is assumed to be supported at the top over a central width of $2k$ and length ℓ_b, as shown in Figure 5.39. The plate is then considered to bend about an axis parallel to the beam span. Thus the plate is treated as a cantilever of span length $n = (B - 2k)/2$ and a width of ℓ_b.

FIGURE 5.39



For convenience, a 1-inch width is considered, with a uniform load in pounds per linear inch numerically equal to the bearing pressure in pounds per square inch.

From Figure 5.39, the maximum bending moment in the plate is

$$M = \frac{R}{B\ell_b} \times n \times \frac{n}{2} = \frac{Rn^2}{2B\ell_b}$$

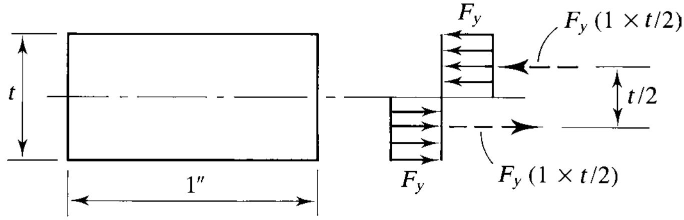
where R is the beam reaction and $R/B\ell_b$ is the average bearing pressure between the plate and the concrete. For a rectangular cross section bent about the minor axis, the nominal moment strength M_n is equal to the plastic moment capacity M_p. As illustrated in Figure 5.40, for a rectangular cross section of unit width and depth t, the plastic moment is

$$M_p = F_y\left(1 \times \frac{t}{2}\right)\left(\frac{t}{2}\right) = F_y\frac{t^2}{4}$$

For LRFD, Since the design strength must at least equal the factored-load moment,

$$\phi_b M_p \geq M_u$$

FIGURE 5.40



$$0.90F_y\frac{t^2}{4} \geq \frac{R_u n^2}{2B\ell_b}$$

$$t \geq \sqrt{\frac{2R_u n^2}{0.90B\ell_b F_y}} \tag{5.12}$$

or

$$t \geq \sqrt{\frac{2.22R_u n^2}{B\ell_b F_y}} \tag{5.13}$$

where R_u is the factored-load beam reaction.

For ASD, the allowable flexural strength must at least equal the applied moment:

$$\frac{M_p}{\Omega_b} \geq M_a$$

$$\frac{F_y t^2/4}{1.67} \geq \frac{R_a n^2}{2B\ell_b}$$

$$t \geq \sqrt{\frac{3.34R_a n^2}{B\ell_b F_y}} \tag{5.14}$$

where R_a is the service-load beam reaction.

EXAMPLE 5.16

Design a bearing plate to distribute the reaction of a W21 × 68 with a span length of 15 feet 10 inches center-to-center of supports. The total service load, including the beam weight, is 9 kips/ft, with equal parts dead and live load. The beam is to be supported on reinforced concrete walls with $f'_c = 3500$ psi. For the beam, $F_y = 50$ ksi, and $F_y = 36$ ksi for the plate.

LRFD SOLUTION

The factored load is

$$w_u = 1.2w_D + 1.6w_L = 1.2(4.5) + 1.6(4.5) = 12.60 \text{ kips/ft}$$

and the reaction is

$$R_u = \frac{w_u L}{2} = \frac{12.60(15.83)}{2} = 99.73 \text{ kips}$$

Determine the length of bearing ℓ_b required to prevent web yielding. From AISC Equation J10-3, the nominal strength for this limit state is

$$R_n = F_y t_w(2.5k + \ell_b)$$

For $\phi R_n \geq R_u$,

$$1.0(50)(0.430)[2.5(1.19) + \ell_b] \geq 99.73$$

resulting in the requirement

$$\ell_b \geq 1.66 \text{ in.}$$

(Note that two values of k are given in the dimensions and properties tables: a decimal value, called the *design* dimension, and a fractional value, called the *detailing* dimension. We always use the design dimension in calculations.)

Use AISC Equation J10-5 to determine the value of ℓ_b required to prevent web crippling. Assume $\ell_b/d > 0.2$ and try the second form of the equation, J10-5(b). For $\phi R_n \geq R_u$,

$$\phi(0.40)t_w^2\left[1+\left(\frac{4\ell_b}{d}-0.2\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_y t_f}{t_w}} \geq R_u$$

$$0.75(0.40)(0.430)^2\left[1+\left(\frac{4\ell_b}{21.1}-0.2\right)\left(\frac{0.430}{0.685}\right)^{1.5}\right]\sqrt{\frac{29,000(50)(0.685)}{0.430}} \geq 99.73$$

This results in the requirement

$$\ell_b \geq 3.00 \text{ in.}$$

Check the assumption:

$$\frac{\ell_b}{d} = \frac{3.00}{21.1} = 0.14 < 0.2 \quad \text{(N.G.)}$$

For $\ell_b/d \leq 0.2$,

$$\phi(0.40)t_w^2\left[1+3\left(\frac{\ell_b}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_y t_f}{t_w}} \geq R_u \qquad \text{[AISC Equation J10-5(a)]}$$

$$0.75(0.40)(0.430)^2\left[1+3\left(\frac{\ell_b}{21.1}\right)\left(\frac{0.430}{0.685}\right)^{1.5}\right]\sqrt{\frac{29,000(50)(0.685)}{0.430}} \geq 99.73$$

resulting in the requirement

$$\ell_b \geq 2.59 \text{ in.}$$

and

$$\frac{\ell_b}{d} = \frac{2.59}{21.1} = 0.12 < 0.2 \quad \text{(OK)}$$

Try ℓ_b = 6 in. Determine dimension B from a consideration of bearing strength. If we conservatively assume that the full area of the support is used, the required plate area A_1 can be found as follows:

$$\phi_c P_p \geq R_u$$

From AISC Equation J8-1, $P_p = 0.85 f_c' A_1$. Then

$$\phi_c(0.85 f_c' A_1) \geq R_u$$
$$0.65(0.85)(3.5)A_1 \geq 99.73$$
$$A_1 \geq 51.57 \text{ in.}^2$$

The minimum value of dimension B is

$$B = \frac{A_1}{\ell_b} = \frac{51.57}{6} = 8.60 \text{ in.}$$

The flange width of a W21 × 68 is 8.27 inches, making the plate slightly wider than the flange, which is desirable. Rounding up, **try B = 10 in.**
Compute the required plate thickness:

$$n = \frac{B - 2k}{2} = \frac{10 - 2(1.19)}{2} = 3.810 \text{ in.}$$

From Equation 5.13,

$$t = \sqrt{\frac{2.22 R_u n^2}{B \ell_b F_y}} = \sqrt{\frac{2.22(99.73)(3.810)^2}{10(6)(36)}} = 1.22 \text{ in.}$$

ANSWER Use a PL 1¼ × 6 × 10.

ASD SOLUTION

$$w_a = w_D + w_L = 9 \text{ kips/ft}$$

$$R_a = \frac{w_a L}{2} = \frac{9(15.83)}{2} = 71.24 \text{ kips}$$

Determine the length of bearing ℓ_b required to prevent web yielding. From AISC Equation J10-3, the nominal strength is

$$R_n = F_y t_w (2.5k + \ell_b)$$

For $\frac{R_n}{\Omega} \geq R_a$,

$$\frac{50(0.430)[2.5(1.19) + \ell_b]}{1.50} \geq 71.24$$

$$\ell_b \geq 2.00 \text{ in.}$$

Determine the value of ℓ_b required to prevent web crippling. Assume $\ell_b/d \le 0.2$ and use AISC Equation J10-5a:

$$\frac{R_n}{\Omega} = \frac{1}{\Omega}(0.40)t_w^2\left[1+3\left(\frac{\ell_b}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_y t_f}{t_w}} \ge R_a$$

$$\frac{1}{2.00}(0.40)(0.430)^2\left[1+3\left(\frac{\ell_b}{21.1}\right)\left(\frac{0.430}{0.685}\right)^{1.5}\right]\sqrt{\frac{29{,}000(50)(0.685)}{0.430}} \ge 71.24$$

$$\ell_b \ge 3.78 \text{ in.}$$

$$\frac{\ell_b}{d} = \frac{3.78}{21.1} = 0.179 < 0.2 \quad \text{(OK)}$$

Try $\ell_b = 6$ in. Conservatively assume that the full area of the support is used and determine B from a consideration of bearing strength. Using AISC Equation J8-1, we obtain

$$\frac{P_p}{\Omega_c} = \frac{0.85 f_c' A_1}{\Omega_c} \ge R_a$$

$$\frac{0.85(3.5)A_1}{2.31} \ge 71.24$$

$$A_1 \ge 55.32 \text{ in.}^2$$

The minimum value of dimension B is

$$B = \frac{A_1}{\ell_b} = \frac{55.32}{6} = 9.22 \text{ in.}$$

Try $B = 10$ in.:

$$n = \frac{B-2k}{2} = \frac{10-2(1.19)}{2} = 3.810 \text{ in.}$$

From Equation 5.14,

$$t \ge \sqrt{\frac{3.34 R_a n^2}{B\ell_b F_y}} = \sqrt{\frac{3.34(71.24)(3.810)^2}{10(6)(36)}} = 1.27 \text{ in.}$$

ANSWER Use a PL $1\frac{1}{2} \times 6 \times 10$.

If the beam is not laterally braced at the load point (in such a way as to prevent relative lateral displacement between the loaded compression flange and the tension flange), the Specification requires that *web sidesway buckling* be investigated (AISC J10.4). When loads are applied to both flanges, *web compression buckling* must be checked (AISC J10.5).