1

Fluid Mechanics

Dimensions & Units

1st Dimensions

Mass	Length	Time	Force
M	L	T	F

Types of systems

i- M – L – T system

ii- F – L – T system

2nd Units

System / Quantity	Mass	Length	Time	Force
Standard International (S.I)	kg	m	sec	N
French System (c.g.s.)	gm	cm	sec	dyne
British (English)	slug	ft	sec	lb
Kilogram weight system	kg	m	sec	kg_w

1- Length (l)

1 ft = 12 inch

inch = 2.54 cm

e.g.

1 ft = 12 * 2.54

= 30.48 cm

1 ft = 0.3048 m

yard = 3 ft

mile = 1760 yard

mile = 1760*3*0.3048

=1609 m

m = 100 cm

' → feet, " → inch

$$1\ m = \frac{1}{0.3048}\ ft$$

= 3.28 ft

2- Mass (m)

$\boxed{1\ slug = 14.59\ Kg}$, $1\ ton = 1000\ Kg$, $1Kg = 1000\ gm$

3- Volume (∀)

$1\ m^3 = 1000\ litre = 10^6\ cm^3$, $1\ gallon = 3.785\ litre$

4- Velocity (V)

$V = \frac{length}{time} = LT^{-1}$ (m/sec) or (ft/sec)

5- Acceleration (a)

$a = \frac{Velocity}{time} = \frac{dV}{dt} = LT^{-2}$

6- Gravitational accelaration (g)

$\boxed{g = 9.81\ m/sec^2}$ $\boxed{g = 32.2\ ft/sec^2}$

7- Force (F)

$F = mass * accelaration = \boxed{MLT^{-2}}$

$N = Kg.m/sec^2$

$dyne = gm.cm/sec^2$

$lb = slug.ft/sec^2$

pound (lb)

$\boxed{1\ lb = 4.44\ N}$

$1\ N = 10^5\ dyne$

$1\ Kip = 1000\ lb$

$1\ Kg_w = 9.81\ N$

$1\ gm_w = 981\ dyne$

$\boxed{1\ Kg_w = 2.205\ lb}$

8 - Density (ρ)

$$\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{m}{\forall} = ML^{-3}$$

$$1\ gm/cm^3 = 1000\ Kg/m^3 = 1.94\ slug/ft^3$$

Density of water

System	SI	C.g.S.	English
ρ_w	$1000\ Kg/m^3$	$1\ gm/cm^3$	$1.94\ slug/ft^3$

9 - Specific Weight (γ)

$$\gamma = \frac{\text{Weight}}{\text{Volume}} = \frac{W}{\forall} = FL^{-3} = ML^{-2}T^{-2}$$

($FL^{-3} \rightarrow MLT^{-2}$)

$$\boxed{\gamma = \rho g}$$

Specific weight of water

System	SI	C.g.S.	English
γ_w	$9810\ N/m^3$	$981\ dyne/cm^3$	$62.4\ lb/ft^3$

10 - Specific Volume $= \frac{1}{\rho} = M^{-1}L^3$

11 - Specific gravity (S.G.) = Relative density (r.d.)

$$S.G = r.d = \frac{\rho_{liquid}}{\rho_{water}} = \frac{\gamma_{liquid}}{\gamma_{water}}$$ (no Units)

e.g. Mercury

S.G of Hg = 13.6

$\gamma_{H.g} = 13.6\ \gamma_{water}$

$13.6 * 9810$ SI

$13.6 * 62.4$ English

12 - Pressure (P) = Stress (τ)

$$P = \tau = \frac{\text{Force}}{\text{Area}} = \rho g h = F L^{-2} = M L^{-1} T^{-2}$$

Pa (Pascal) = N/m^2

Psi = pounds per square inch ($lb/inch^2$)

Psf = pounds per square feet (lb/ft^2)

e.g// Convert P = 1 Psi → Psf

$\because$ 1 ft = 12 inch

$$\therefore P = 1 \frac{lb}{inch^2} * (12)^2 \frac{inch^2}{ft^2} = 144\ lb/ft^2$$

1 Psi = 144 Psf

13 - Discharge (Q) التصرف

$$Q = \frac{\text{Volume}}{\text{time}} = \frac{\forall}{t} = \text{Velocity} * \text{Area} = V.A = L^3 T^{-1}$$

$1\ m^3/sec = 10^6\ cm^3/sec$

كمية الحركة

14 - Momentum = mass * Velocity = Force * time

15 - Energy (E) = Work = Torque (T) = Moment

Work = Force * distance = F.L = ML^2T^{-2}

Joule = N.m

عدد اللفات

16 - No of revolutions (N) = (n) Speed of ROtation

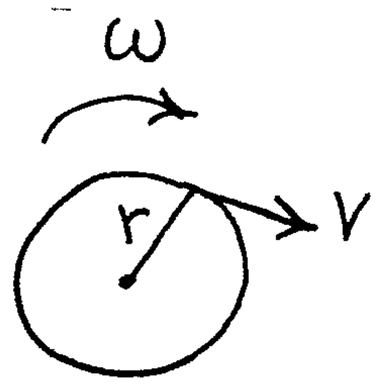
N = no. of revolutions/minute (r.p.m.)

n = no. of revolutions/second (r.p.s)

17 - Angular Velocity (ω) السرعة الزاوية

$$\omega = \frac{\theta}{t} = \frac{2\pi N}{60} = 2\pi n \quad (rad/sec)$$

$$V = \omega r \quad (m/sec) \text{ or } (ft/sec)$$



18 - Power (P)

$$P = \text{Force} * \text{Velocity} = FLT^{-1} = ML^2T^{-3}$$

$$\text{Watt} = N.m/sec$$

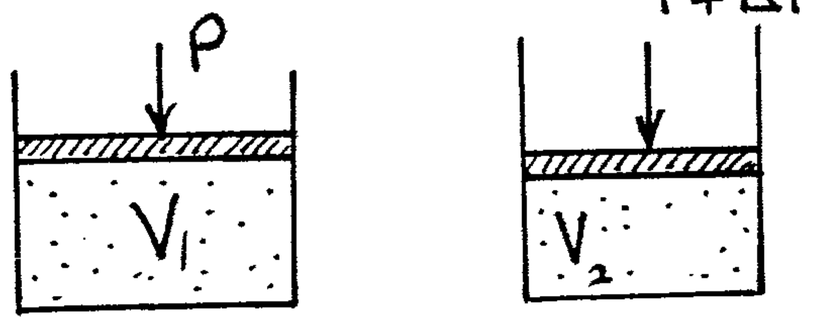
$$\text{Power} = T.\omega = \frac{N.m.rad}{sec}$$

T: Torque, ω: angular velocity

HP = Horsepower = Watt / 735

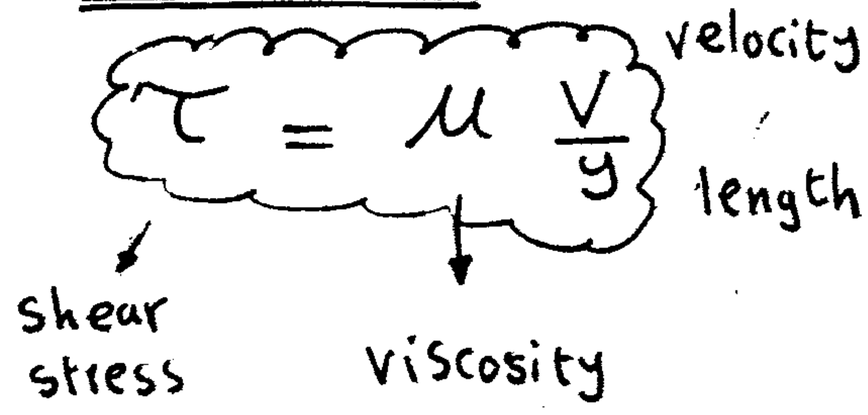
19- Bulk modulus (K)

$$K = \frac{\Delta P}{-\frac{\Delta V}{V}} = \frac{\text{Stress}}{\text{Strain}}$$



Viscosity اللزوجة (مقاومة السائل للإحتكاك)

$$\tau = \mu \frac{V}{y}$$



20 - Coeff of Viscosity = Dynamic Viscosity = Absolute Viscosity (μ ميو)

$$\tau = \mu \frac{V}{y}$$

$$\frac{MLT^{-2}}{L^2} = \mu \frac{LT^{-1}}{L} \Rightarrow \mu = \boxed{ML^{-1}T^{-1}}$$

$$FL^{-2} = \mu \frac{LT^{-1}}{L} \Rightarrow \mu = \boxed{\frac{FT}{L^2}}$$

$$\mu = \tau \frac{y}{V} = \frac{Kg}{m.sec} = \frac{N}{m^2}.sec = Pa.S$$

$$= \frac{gm}{cm.sec} = \frac{dyne.sec}{cm^2} = Poise$$

$$= \frac{slug}{ft.sec} = \frac{lb.sec}{ft^2}$$

$ML^{-1}T^{-1}$ $\qquad$ $FL^{-2}T$

$$\boxed{Poise = 0.1\ Pa.S}$$

$\mu_{water} \simeq 0.001 Pa.S$

$\simeq 0.01 poise$

21- Kinematic Viscosity (ν) نيو

$$\boxed{\nu = \frac{\mu}{\rho}} = \frac{ML^{-1}T^{-1}}{ML^{-3}} = \boxed{L^2T^{-1}}$$

$\nu = (cm^2/sec) = stoke$

$= (m^2/sec)$ or (ft^2/sec)

$$\boxed{Stoke = 10^{-4}\ m^2/sec}$$

$\nu_{water} = 10^{-6}\ m^2/sec = 10^{-2}$ stoke

22- Surface tension (σ)

$$\sigma = \frac{Work}{Area} = \frac{F.L}{L^2} = FL^{-1} = MT^{-2}$$

$\sigma = N/m = Kg/sec^2$ $\qquad$ $\sigma_{water} = 0.07\ N/m$

$P = \frac{4\sigma}{d}$, $h = \frac{4\sigma\cos\theta}{\gamma d}$

pressure, diameter, height

Example Find the dimension of the following relation

$$\frac{V}{\sqrt{K/\rho}}$$

$V = LT^{-1}$

$K = \frac{\text{stress}}{\text{Strain}} = \frac{MLT^{-2}/L^2}{M^0L^0T^0} = ML^{-1}T^{-2}$

$\therefore \frac{V}{\sqrt{K/\rho}} = \frac{LT^{-1}}{\sqrt{\frac{ML^{-1}T^{-2}}{ML^{-3}}}} = M^0L^0T^0$ (Dimensionless)

Convert $\gamma = 1000\ Kg_w/m^3 \longrightarrow lb/ft^3$

$1\ Kg_w = 2.205\ lb$, $1\ ft = 0.3048\ m$

$$\gamma = 1000\ \frac{Kg_w}{m^3} * 2.205\ \frac{lb}{Kg_w} * (0.3048)^3\ \frac{m^3}{ft^3} = 62.4\ \frac{lb}{ft^3}$$

Convert $g = 9.81\ m/sec^2 \longrightarrow ft/sec^2$

$$g = 9.81\ \frac{m}{sec^2} * (0.3048)\frac{ft}{m} = 32.2\ \frac{ft}{sec^2}$$

Convert $P = 7\ Kg_w/cm^2 \longrightarrow N/m^2$

$1\ Kg_w = 9.81\ N$

$$P = 7\ \frac{Kg_w}{cm^2} * 9.81\ \frac{N}{Kg_w} * (100)^2\ \frac{cm^2}{m^2} = 686700\ N/m^2$$

$\therefore$ $\boxed{Kg_w/cm^2 = 9.81 \times 10^4\ N/m^2}$

[1] Gasoline

(a) $\gamma = 710 \ \frac{dyne}{cm^3}$ Convert to $\frac{N}{m^3}$, $\frac{lb}{ft^3}$

$$\gamma = 710 \ \frac{dyne}{cm^3} * \frac{1 \ N}{10^5 \ dyne} * \frac{(100)^3 \ cm^3}{1^3 \ m^3} = 7100 \ \frac{N}{m^3}$$

$$\gamma = 7100 \ \frac{N}{m^3} * \frac{1 \ lb}{4.44 \ N} * \frac{(0.3048)^3 \ m^3}{1^3 \ ft^3} = 45.3 \ \frac{lb}{ft^3}$$

(b) $\mu = 4640.84$ poise Convert to Pa.S , $\frac{lb}{ft^2}.s$

$$\mu = 4640.84 \ Poise * \frac{0.1 \ Pa.S}{1 \ Poise} = 464.084 \ Pa.s \ \left(\frac{N}{m^2}.s\right)$$

$$\mu = 464.084 \ \frac{N}{m^2}.s * \frac{1 \ lb}{4.44 \ N} * \frac{(0.3048)^2 \ m^2}{1^2 \ ft^2} = 9.71 \ \frac{lb}{ft^2}.s$$

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[3] (a) $\frac{Vy}{\nu} = \frac{LT^{-1}.L}{L^2T^{-1}} = M^0L^0T^0$

(b) $\frac{\rho Vy}{\mu} = \frac{ML^{-3}.LT^{-1}.L}{ML^{-1}T^{-1}} = M^0L^0T^0$

(c) $\frac{V}{\sqrt{gy}} = \frac{LT^{-1}}{\sqrt{LT^{-2}.L}} = \frac{LT^{-1}}{\sqrt{L^2T^{-2}}} = \frac{LT^{-1}}{LT^{-1}} = M^0L^0T^0$

(d) $\frac{\rho}{\rho V^2}$

* $F = MLT^{-2}$

$$\rho = FL^{-2} = ML^{-1}T^{-2}$$

$$\frac{\rho}{\rho V^2} = \frac{ML^{-1}T^{-2}}{ML^{-3}(LT^{-1})^2} = \frac{L^2T^{-2}}{L^2T^{-2}} = M^0L^0T^0$$

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[4] $h = FLV^2/gd$

$$F = \frac{hgd}{LV^2} = \frac{L \cdot LT^{-2} \cdot L}{L(LT^{-1})^2} = \frac{L^2T^{-2}}{L^2T^{-2}} = M^0L^0T^0$$

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[5] (a) $\frac{V^2}{g} = \frac{L^2T^{-2}}{LT^{-2}} = L$ (ft), (m)

(b) $\frac{\rho}{\gamma} = \frac{FL^{-2}}{FL^{-3}} = L$ (ft), (m)

(c) $\rho V^2 = ML^{-3} \cdot L^2T^{-2} = ML^{-1}T^{-2} = FL^{-2}$

$\because F = MLT^{-2} \Rightarrow M = FL^{-1}T^2$

(lb/ft²), (N/m²)

(d) $\gamma y = FL^{-3} \cdot L = FL^{-2}$ (lb/ft²), ($N/m^2 = Pa$)

(e) $\frac{d\rho}{dx} = \frac{FL^{-2}}{L} = FL^{-3}$ (lb/ft³), (N/m³)

(f) $\frac{\tau}{y} = \frac{FL^{-2}}{L} = FL^{-3}$ (lb/ft³), (N/m³)

(g) $\rho QV = \frac{M}{L^3} \cdot \frac{L^3}{T} \cdot \frac{L}{T} = MLT^{-2} = F$ (lb), (N)

(h) $\gamma QL = \frac{F}{L^3} \cdot \frac{L^3}{T} \cdot L = FLT^{-1}$ (lb.ft/s), (N.m/s)

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6 $W_{full} = 1.5\ KN = 1.5 \times 10^3\ N$ $\quad W_{empty} = 119\ N$

$\forall = 179$ litres $= 0.179\ m^3$

$\gamma = ?$, $\rho = ?$, S.G. $= ?$

Sol

$W_{oil} = 1.5 * 10^3 - 119 = 1381\ N$

$\gamma_{oil} = \frac{W_{oil}}{\forall_{oil}} = \frac{1381}{0.179} = \underline{\underline{7715\ N/m^3}}$

$\rho_{oil} = \frac{\gamma_{oil}}{g} = \frac{7715}{9.81} = \underline{\underline{786.4\ Kg/m^3}}$

$S.G. = \frac{\rho_{oil}}{\rho_{water}} = \frac{786.4}{1000} = \underline{\underline{0.7864}}$

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7 (a) S.G. $= 0.82$ $\quad \rho_{fluid} = ?$

$\rho_{fluid} = S.G. * \rho_w = 0.82 * 1000 = \underline{\underline{820\ Kg/m^3}}$

(b) $m = 250\ Kg$ $\quad a = 12\ m/s$ $\quad F = ?$

$F = m * a = 250 * 12 = \underline{\underline{3000\ N}}$

(c)

$\rho_{oil} = 850\ kg/m^3 \quad \mu = 6 * 10^{-3}\ kg/ms$

$r.d = ? \quad , \quad \nu = ?$

Sol

$$r.d. = \frac{\rho_{oil}}{\rho_{water}} = \frac{850}{1000} = \underline{\underline{0.85}}$$

$$\nu = \frac{\mu}{\rho} = \frac{6 * 10^{-3}}{850} = \underline{\underline{7.06 * 10^{-6}\ m^2/s}}$$

(d)

$S.g. = 0.92 \qquad \mu = 0.085$ Poise $\left(\frac{dyne}{cm^2}.sec\right)$

$\mu_{British} = ? \qquad \nu_{stokes} = ?$

Sol

$1\ lb = 4.44\ N = 4.44 \times 10^5\ dyne$

$$\mu = 0.085 \frac{dyne}{cm^2}.s * \frac{1\ lb * (30.48)^2\ cm^2}{4.44 * 10^5\ dyne * 1^2\ ft^2} = \underline{\underline{1.77 \times 10^{-4}}} \frac{lb}{ft^2}.s$$

$$\rho = S.g. * \rho_{water} = 0.92 * 1\ gm/cm^3 = 0.92\ gm/cm^3$$

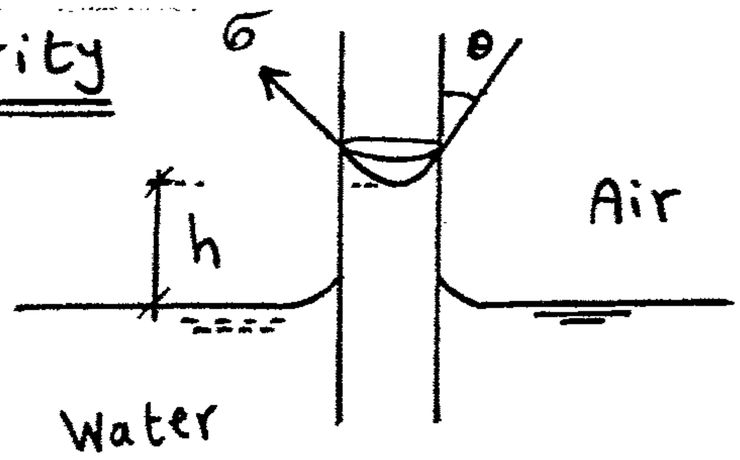
$$\nu = \frac{\mu}{\rho} = \frac{0.085}{0.92} = \underline{\underline{0.0924\ cm^2/s \quad (stokes)}}$$

	Quantity	Eqn	F-L-T	M-L-T	S.I.	C.G.S.	English	Other
1	Length	L	L	L	m	cm	ft (')	inch (")
2	Mass	m	$F L^{-1} T^{2}$	M	Kg	gm	slug	
3	Volume	$V = L^3$	L^3	L^3	m^3	cm^3 (cc)	ft^3	gallon
4	Velocity (Speed)	$v = \frac{L}{t}$	$L T^{-1}$	$L T^{-1}$	m/sec	cm/sec	ft/sec	
5	Acceleration	$a = \frac{L}{t^2}$	$L T^{-2}$	$L T^{-2}$	m/sec^2	cm/sec^2	ft/sec^2	
6	Gravity	g	$L T^{-2}$	$L T^{-2}$	m/sec^2	cm/sec^2	ft/sec^2	
7	Force	F = m.a	F	$M L T^{-2}$	$Kg.m/sec^2$ N	$gm.cm/sec^2$ dyne	$slug.ft/sec^2$ lb	
8	Density	$\rho = \frac{m}{V}$	$F L^{-4} T^{2}$	$M L^{-3}$	Kg/m^3	gm/cm^3	$slug/ft^3$	
9	Specific Weight	$\gamma = \frac{W}{V} = \rho g$	$F L^{-3}$	$M L^{-2} T^{-2}$	N/m^3	$dyne/cm^3$	lb/ft^3	
10	Specific Volume	$\frac{1}{\rho}$	$F^{-1} L^{4} T^{-2}$	$M^{-1} L^{3}$	m^3/Kg	cm^3/gm	$ft^3/slug$	
11	Specific Gravity	$S.G. = \frac{\rho_{liquid}}{\rho_{water}}$	- - -	- - -	- - -	- - -	- - -	- - -
12	Pressure (Stress)	$p = \frac{F}{A} = \rho g h$	$F L^{-2}$	$M L^{-1} T^{-2}$	N/m^2 Pa	$dyne/cm^2$	lb/ft^2 (psf) lb/in^2 (psi)	Bar, Torr atmosphere (mm Hg) (m water)

	Quantity	Eqn	F-L-T	M-L-T	S.I.	C.G.S.	English	Other
13	Discharge	$Q = A.v$	$L^3 T^{-1}$	$L^3 T^{-1}$	m^3/sec (cum ecs)	cm^3/sec	ft^3/sec (cfs)	
14	Momentum	$M = m.v = F.t$	F T	M L T^{-1}	Kg.m/sec N.sec	gm.cm/sec dyne.sec	slug.ft/sec lb.sec	
15	Energy (Work) Torque (Moment)	$W = F.L$	F L	M L^2 T^{-2}	N.m Joule	dyne.cm erg	lb.ft	
16	Rotation (N, n)	$n = \frac{\omega}{2\pi}$	T^{-1}	T^{-1}	r.p.m. r.p.s.	r.p.m. r.p.s.	r.p.m. r.p.s.	
17	Angular Velocity	$\omega = \frac{v}{r}$	T^{-1}	T^{-1}	rad/sec	rad/sec	rad/sec	
18	Power	$P = F.v$	F L T^{-1}	M L^2 T^{-3}	N.m/sec Watt	dyne.cm/sec	lb.ft/sec	Horse
19	Bulk Modulus	$K = \frac{\Delta p}{-\frac{\Delta V}{V}}$	F L^{-2}	M L^{-1} T^2	N/m^2	$dyne/cm^2$	lb/ft^2	
20	Coeff. of Viscosity (Dynamic Viscosity)	$\mu = \tau.\frac{y}{v}$	F L^{-2} T	M L^{-1} T^{-1}	$N.sec/m^2$ Kg/m/sec Pa.sec	$dyne.sec/cm^2$ gm/cm/sec Poise	$lb.sec/ft^2$ slug/ft/sec	
21	Kinematic Viscosity	$\nu = \frac{\mu}{\rho}$	$L^2 T^{-1}$	$L^2 T^{-1}$	m^2/sec	cm^2/sec Stoke	ft^2/sec	
22	Surface Tension	$\delta = \frac{\text{Work}}{\text{Area}}$	F L^{-1}	M T^{-2}	Kg/sec^2 N/m	gm/sec^2 dyne/cm	$slug/sec^2$ lb/ft	

2 Physical properties of Fluids

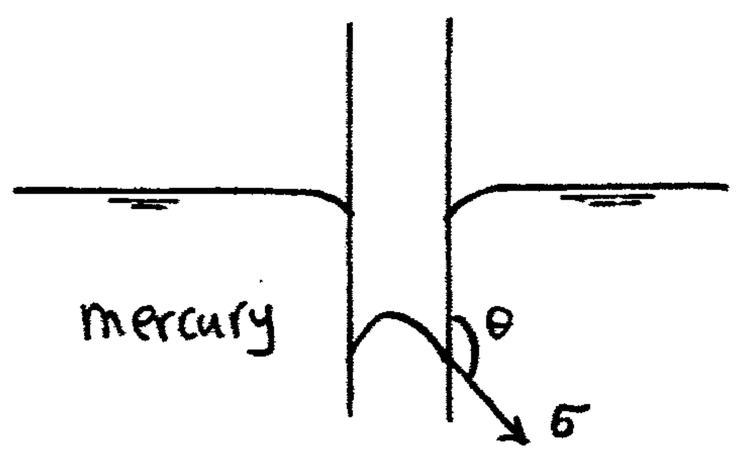
Capillarity



adhesion > Cohesion

تماسك بين الجزيئات التصاق مع الجدار

⇒ rise (wetting)



adhesion < Cohesion

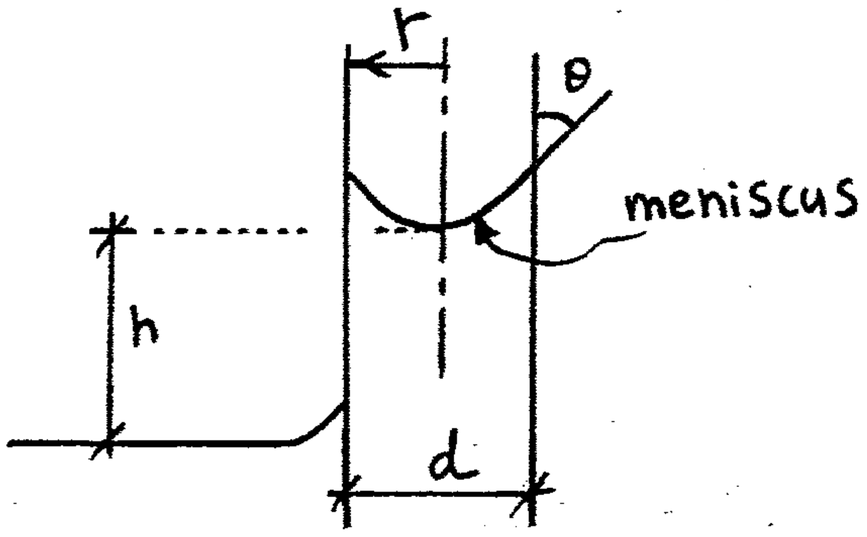
⇒ fall (no wetting)

∵ adhesion between glass and water > Cohesion of water

∴ The liquid column is forced up until its weight is balanced by the Force

$$\sigma (\pi d) \cos\theta = \frac{\pi d^2}{4} * h * \gamma$$

$$\boxed{h = \frac{4\sigma \cos\theta}{\gamma d}}$$



N.B

Water, clean glass θ = Zero

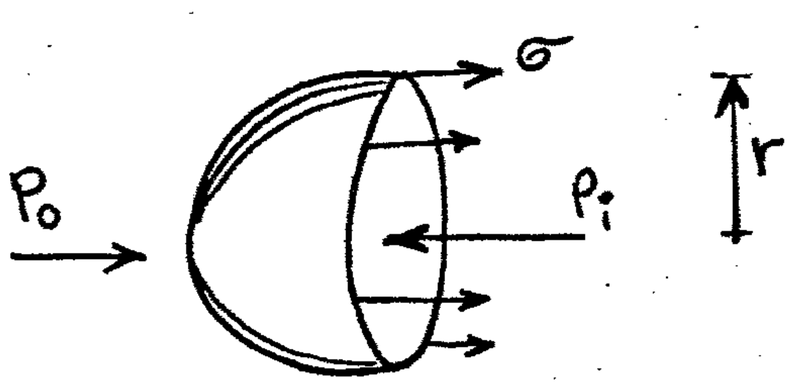
Water droplet

$$(P_i - P_o)\pi r^2 = \sigma (2\pi r)$$

$$P_i - P_o = \frac{2\sigma}{r}$$

or

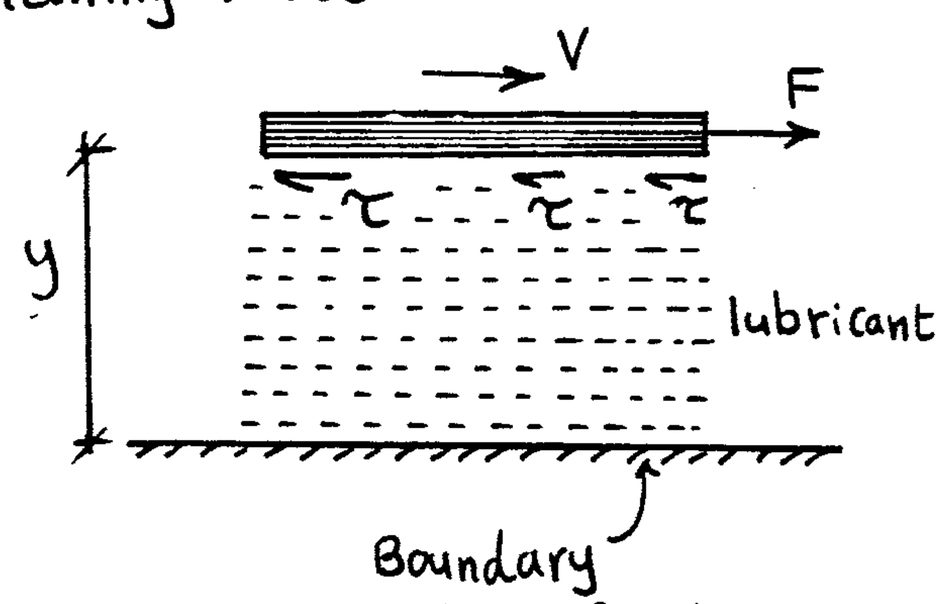
$$\boxed{\Delta P = \frac{4\sigma}{d}}$$

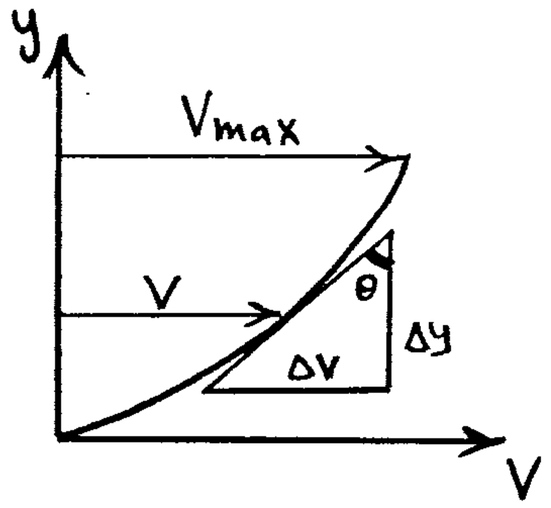


* Soap bubble $\Delta P = \frac{8\sigma}{d}$

Viscosity اللزوجه

It is a property of a fluid which determines its resistance to flow and shearing Force





Velocity distribution

$F \propto V$

$F \propto \frac{1}{y}$

$F \propto A$

$$\frac{F}{A} \propto \frac{V}{y}$$

$$\tau = \mu \frac{dV}{dy}$$

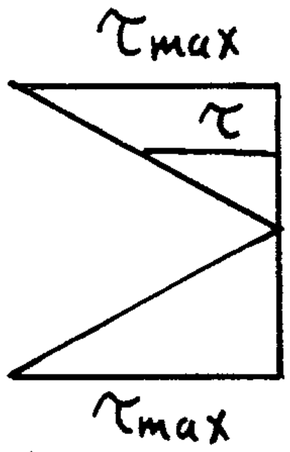
Shear stress — Viscosity — Velocity gradient

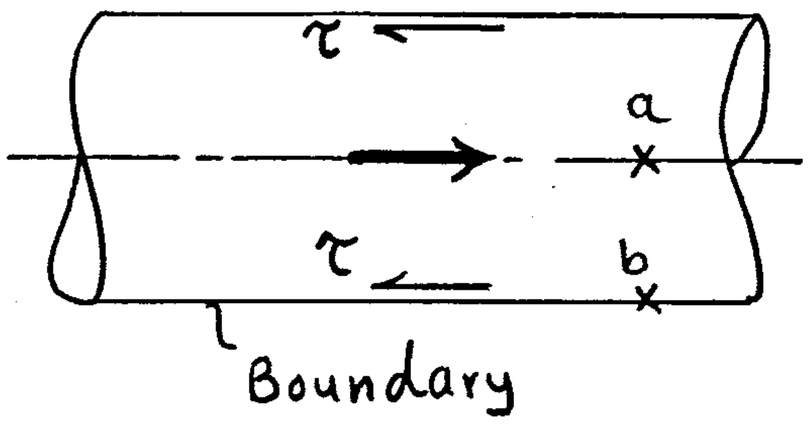
Newton's eqn of Viscosity

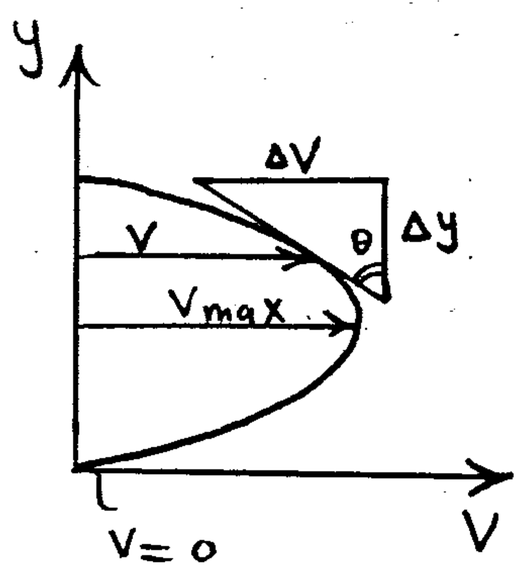
$$\tan\theta = \frac{dV}{dy}$$

$$F = \mu A \frac{dV}{dy}$$

Friction force







Shear Stress distribution

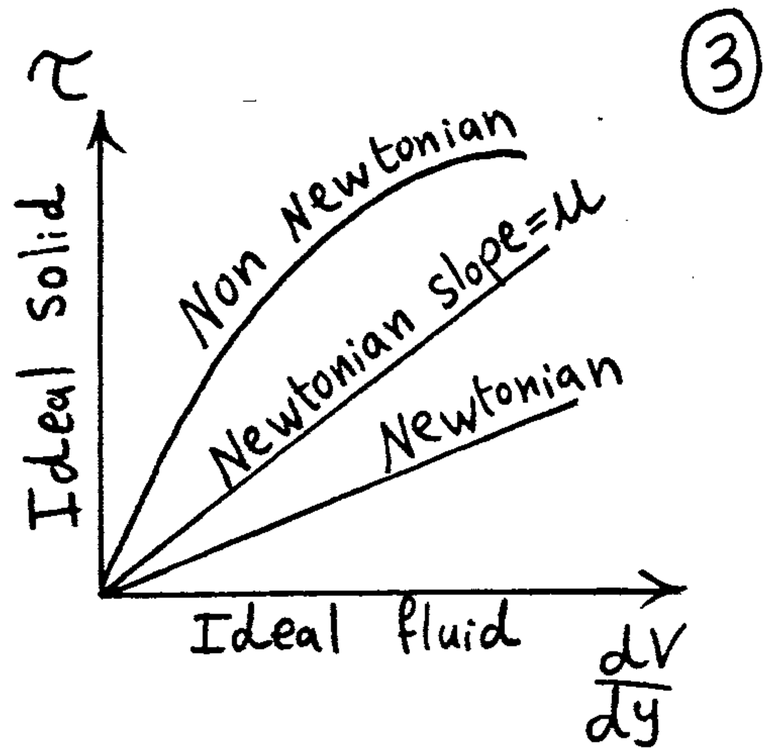
Velocity distribution

at point a $V = V_{max}$ $\frac{dV}{dy} = 0$ $\tau = 0$

at point b $V = 0$ $\frac{dV}{dy} = \tan\theta_{max}$ $\tau = \tau_{max}$

Newtonian Fluids e.g. oil, air, water

* They are fluids with Constant Viscosity
* Shear stress is linearly dependant on velocity gradient



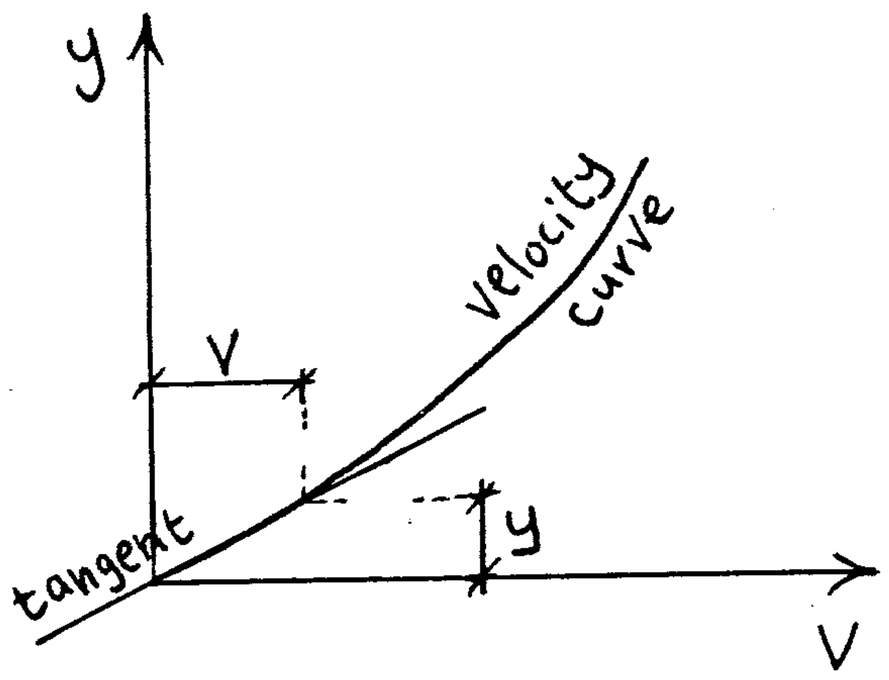
Non-Newtonian Fluids e.g. Ink, Paint, blood

If y is very Small (معظم السوائل)

Velocity distribution ≃ Straight line

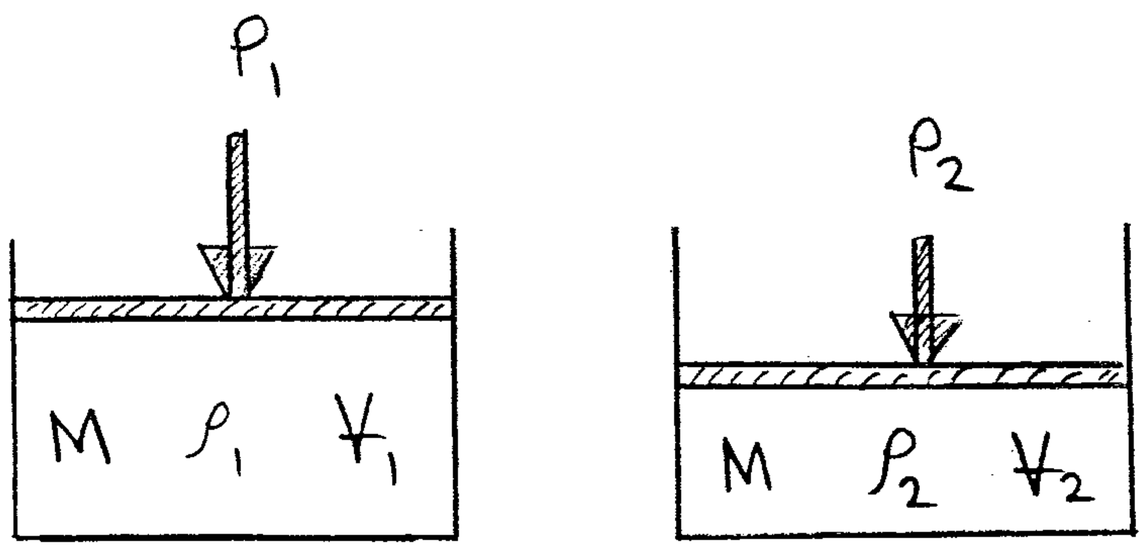
$$\frac{dV}{dy} \simeq \frac{V}{y}$$

$$\boxed{\tau = \mu \frac{V}{y}}$$



Bulks modulus

$$\boxed{K = \frac{\Delta P}{-\frac{\Delta \forall}{\forall}}}$$



As the mass is Constant ⇒ with the increase in Pressure the Volume decreases (−ve) and the density increases (+ve)

$M = \rho \forall$ (الكتلة ثابتة) بتفاضل هذه المعادلة

$$0 = \rho d\forall + \forall d\rho$$

$$\frac{d\rho}{\rho} = -\frac{d\forall}{\forall} \Rightarrow \boxed{K = \frac{\Delta P}{+\frac{\Delta \rho}{\rho}}}$$

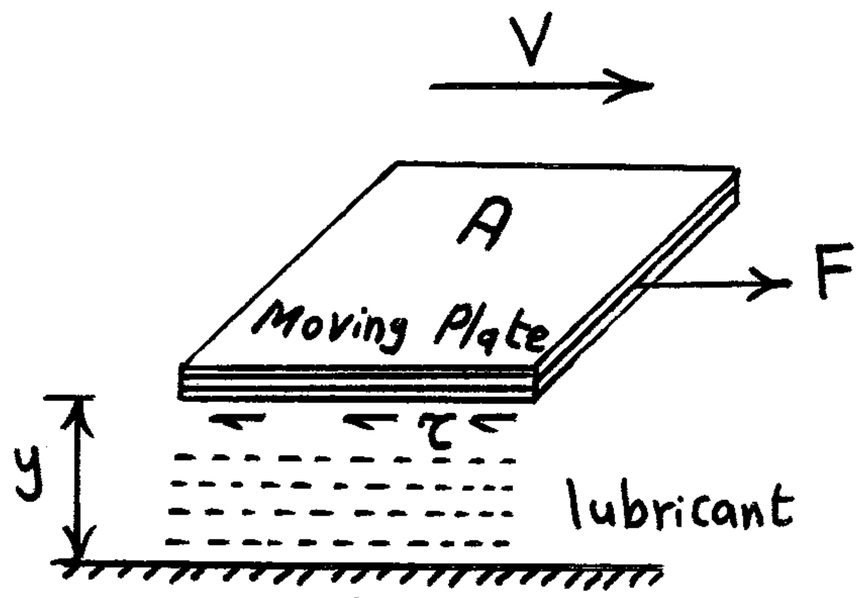
Applications

1- Plate moving with uniform velocity

a- against a horizontal Plane

Resistance (Friction force) = $\tau . A$

$$\boxed{F = \mu \frac{V}{y} A}$$

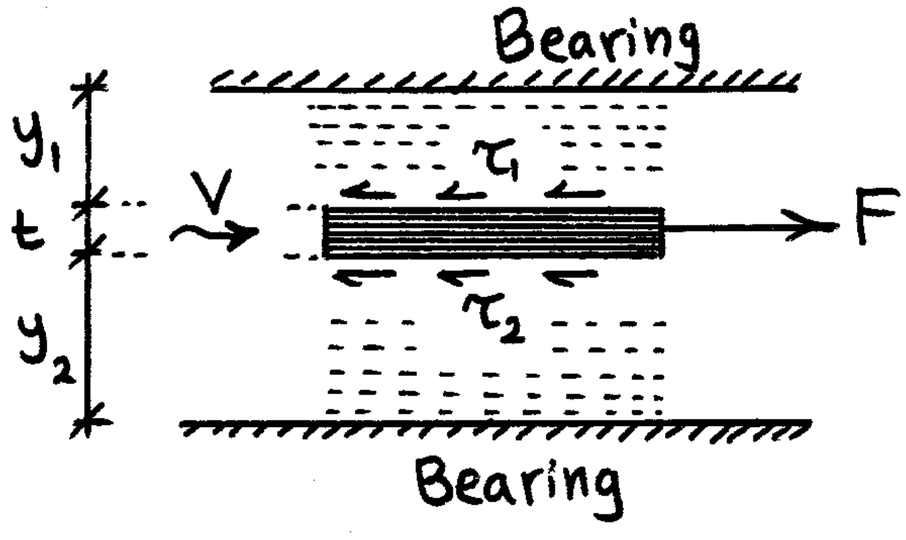


b- between 2 Planes

$$\boxed{F = \tau_1 A + \tau_2 A}$$

$$\tau_1 = \mu \frac{V}{y_1}$$

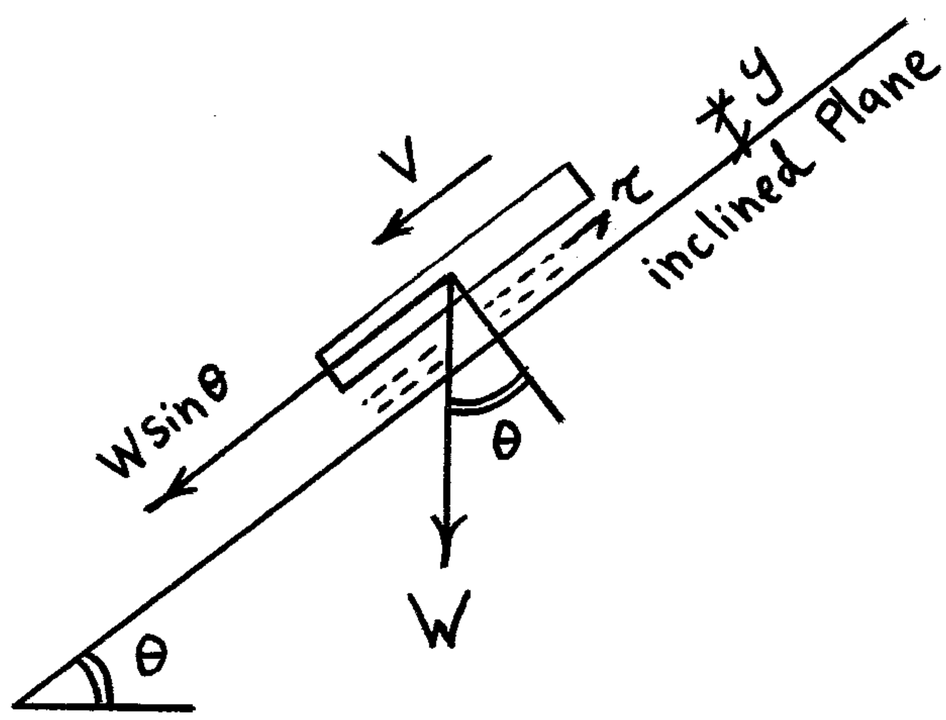
$$\tau_2 = \mu \frac{V}{y_2}$$



C - against an inclined Plane

at uniform Velocity $\Rightarrow (\Sigma F = 0)$

$$\boxed{W \sin\theta = \mu \frac{V}{y} A}$$



2- Cylinder (اسطوانة) moving with uniform Velocity

$$\text{Resistance} = \tau . A$$

$A = 2\pi r L$ مساحة خارجية لإسطوانة

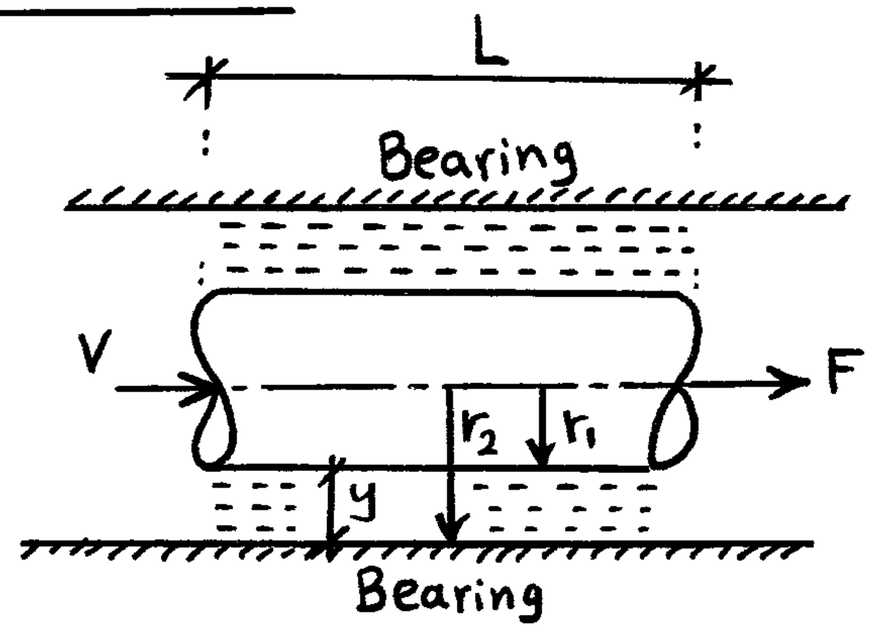
$$y = r_2 - r_1$$

a- Inner Cylinder moving horizontally

$$F = \mu \frac{V}{y} A$$

$$A = 2\pi r_1 L$$

$$\boxed{F = \mu \frac{V}{r_2 - r_1} 2\pi r_1 L}$$

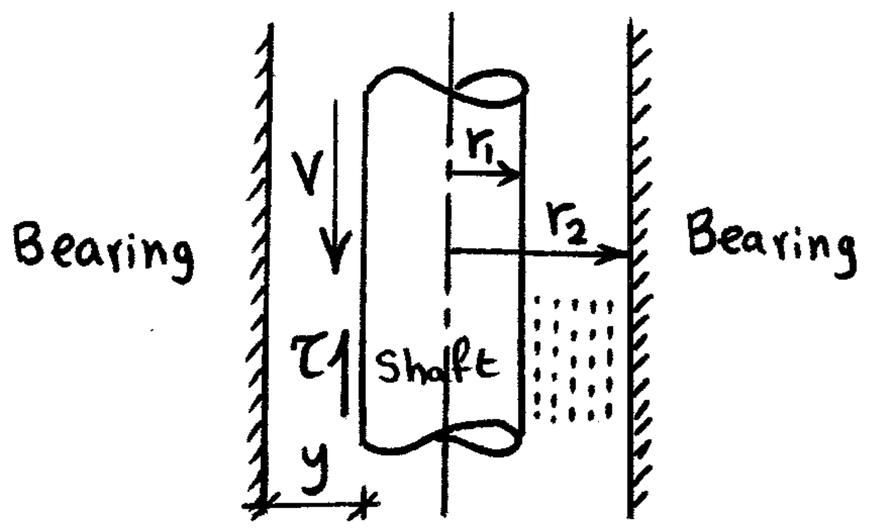


b- Inner Cylinder moving Vertically under gravity

$$W = \mu \frac{V}{y} A$$

$$A = 2\pi r_1 L$$

$$\boxed{W = \mu \frac{V}{r_2 - r_1} 2\pi r_1 L}$$



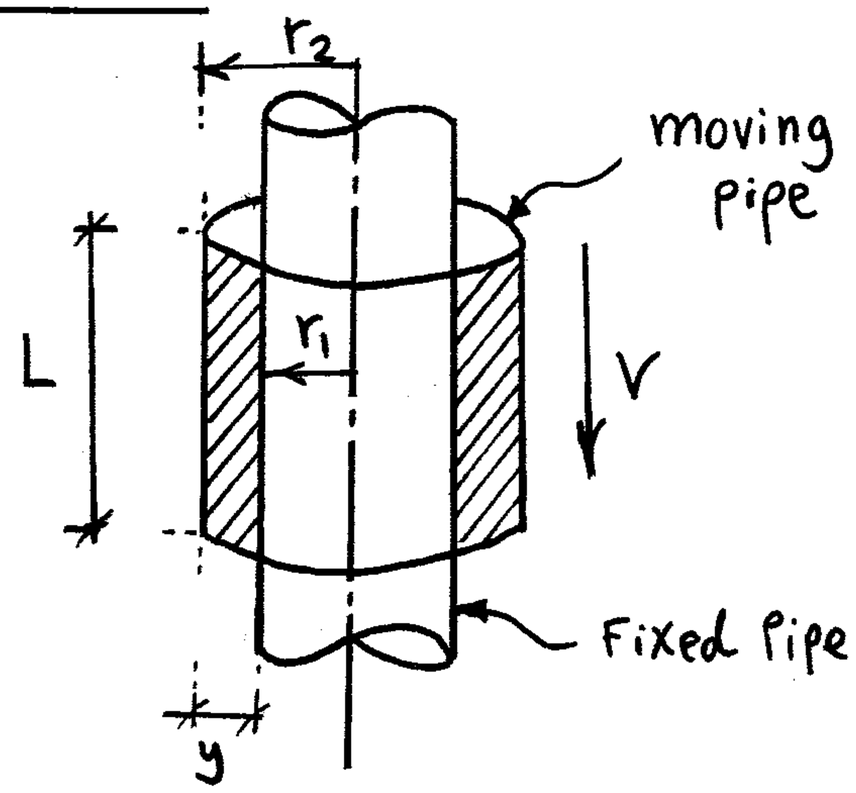
C- Outer moving and the inner fixed

$$W = \mu \frac{V}{y} A$$

$$A = 2\pi r_2 L$$

$$y = r_2 - r_1$$

$$\boxed{W = \mu \frac{V}{r_2 - r_1} 2\pi r_2 L}$$



More Applications on viscosity

Power & Torque needed to overcome friction for rotating objects

A- Constant shear stress بدون تكامل

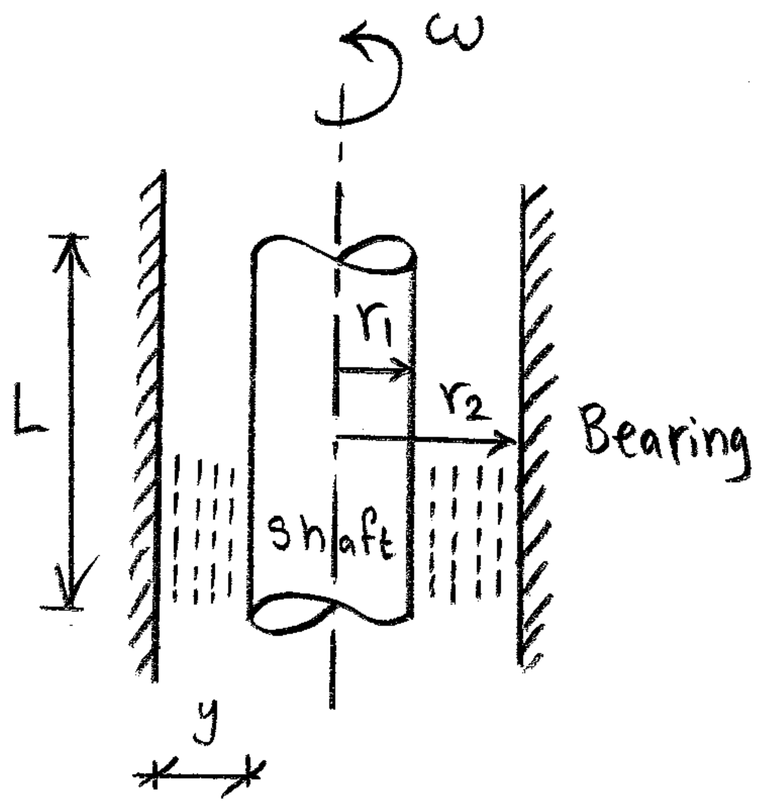
Power = Torque * ω

Torque = F * r

Force = τ * A

Cylinder rotating about its axis

1-Inner cylinder rotating, outer at rest



1- $\tau = \mu \frac{V}{y}$

$y = r_2 - r_1$

$V = \omega \underline{r_1}$

$\therefore \tau = \mu \frac{\omega r_1}{r_2 - r_1}$

2- $F = \tau * A$

$A = 2\pi \underline{r_1} L$

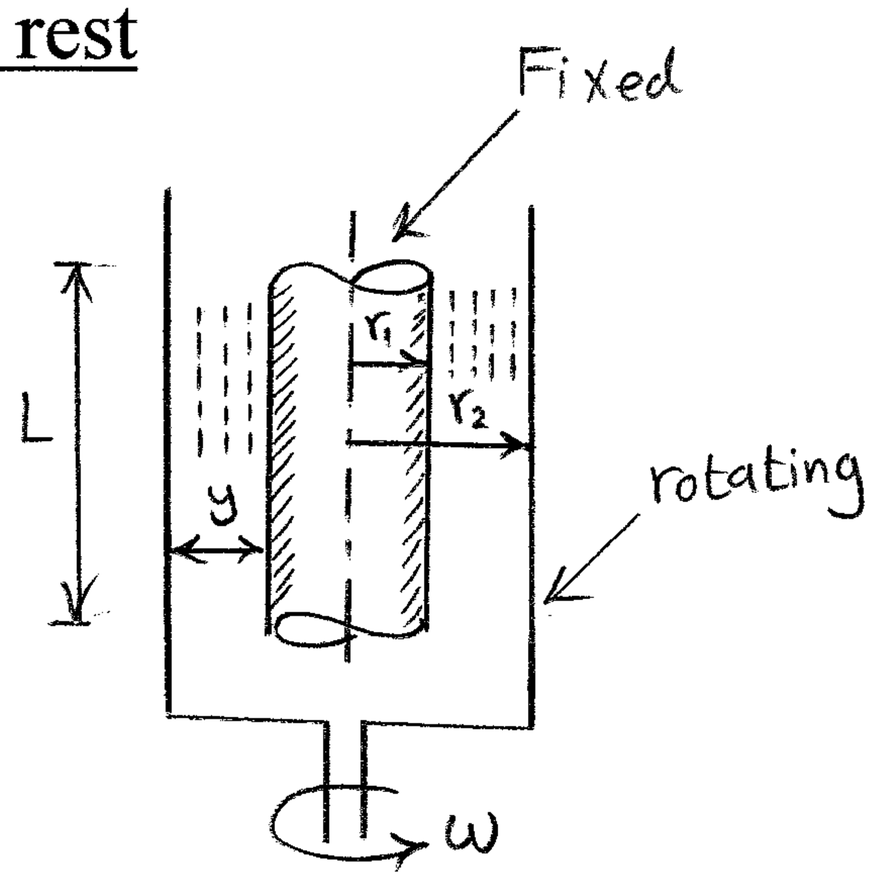
$F = \mu \frac{\omega r_1^2}{r_2 - r_1} \cdot 2\pi L$

3- $T = F * \underline{r_1}$

$T = \mu \frac{\omega r_1^3}{r_2 - r_1} 2\pi L$

4- $P = T * \omega = \mu \frac{\omega^2 r_1^3}{r_2 - r_1} 2\pi L$

2-Outer cylinder rotating, inner at rest



1- $\tau = \mu \frac{V}{y}$

$y = r_2 - r_1$

$V = \omega \underline{\underline{r_2}}$

$\therefore \tau = \mu \frac{\omega r_2}{r_2 - r_1}$

2- $F = \tau . A$

$A = 2\pi \underline{\underline{r_2}} L$

$\therefore F = \mu \frac{\omega r_2^2}{r_2 - r_1} . 2\pi L$

3- $T = F . \underline{\underline{r_2}}$

$T = \mu \frac{\omega r_2^3}{r_2 - r_1} . 2\pi L$

4- $P = T . \omega$

$P = \mu \frac{\omega^2 r_2^3}{r_2 - r_1} . 2\pi L$

B- Shear stress variable with radius محتاج تكامل

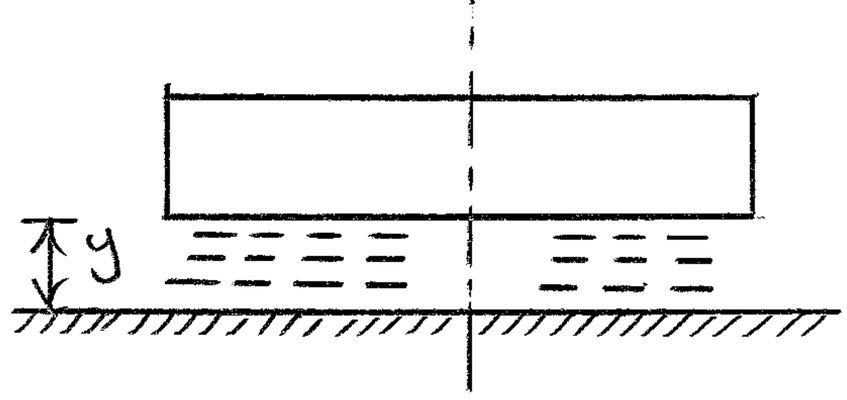
- كلما زاد نصف القطر كلما زادت السرعة كلما زاد الاحتكاك

$$P = T * \omega$$

$$T = \int dF * r$$

$$dF = \tau * dA$$

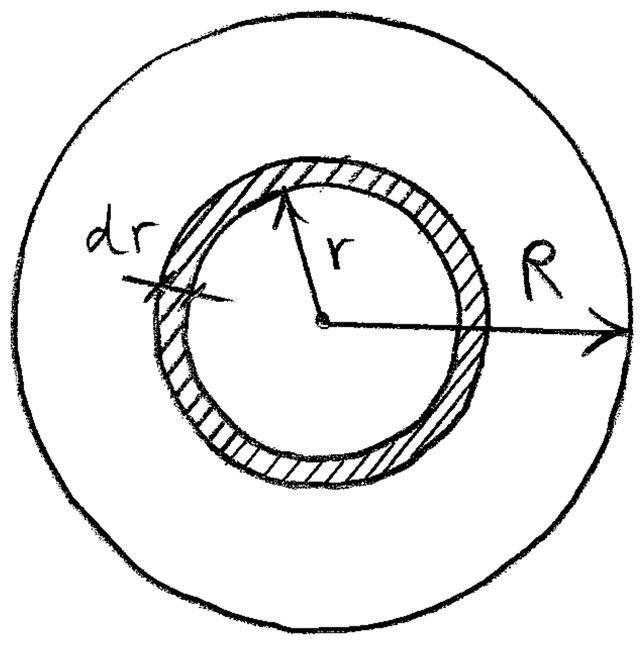
1- Disk rotating about its axis



1- $\tau = \mu \frac{V}{y}$

$V = \omega r$

$\tau = \mu \frac{\omega r}{y}$



2- $dF = \tau . dA$

$dA = 2\pi r . dr$

$dF = 2\pi \mu \frac{\omega r^2}{y} . dr$

3- $\int dT = \int dF . r$ (R, 0)

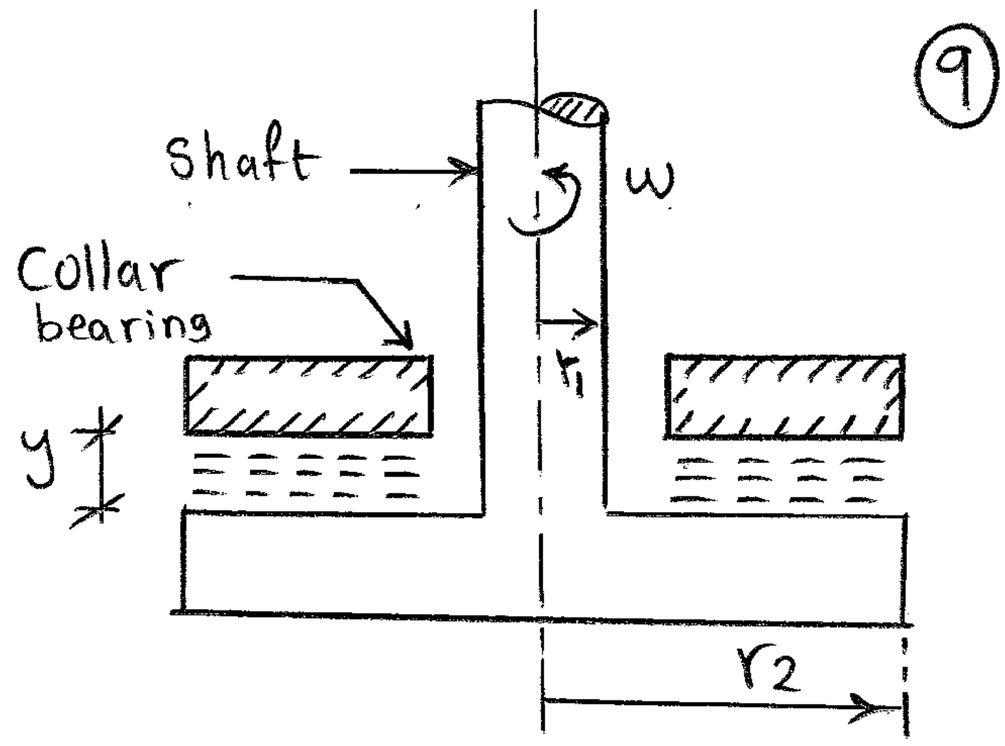
$= 2\pi \mu \frac{\omega}{y} \int_0^R r^3 dr$

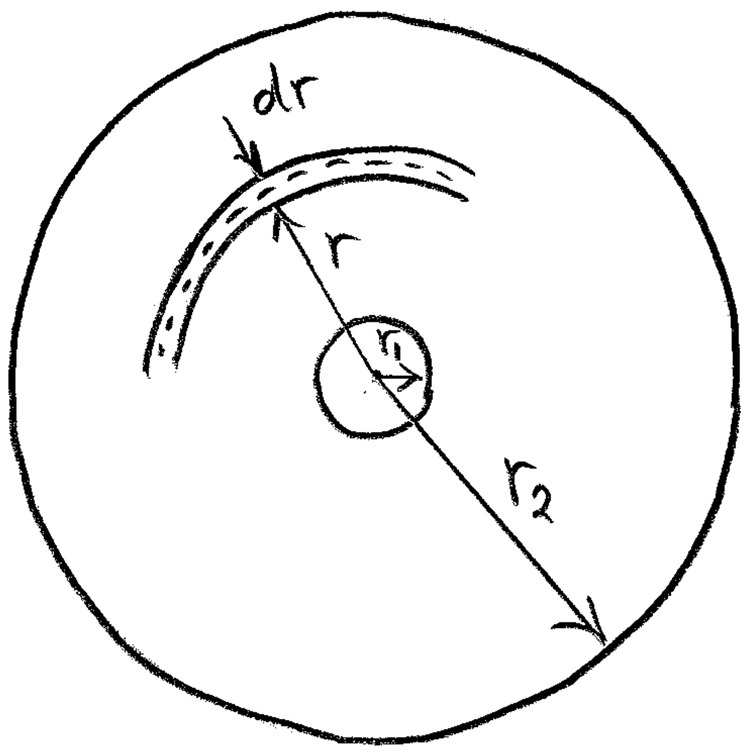
$T = 2\pi \mu \frac{\omega}{y} \frac{R^4}{4}$

4- $P = T . \omega$

$P = \frac{\pi \mu \omega^2 R^4}{2y}$

2- Collar Bearing





1- $\tau = \mu \frac{V}{y}$

$V = wr$

$\tau = \mu \frac{wr}{y}$

2- $dF = \tau . dA$

$dA = 2\pi r . dr$

$dF = 2\pi\mu \frac{wr^2}{y} . dr$

3- $T = \int_{r_1}^{r_2} dF . r$

$= 2\pi\mu \frac{w}{y} \int_{r_1}^{r_2} r^3 dr$

$= 2\pi\mu \frac{w}{y} \left(\frac{r_2^4 - r_1^4}{4} \right)$

4- $P = T . \omega$

$= \frac{\pi\mu\omega^2}{2y} . (r_2^4 - r_1^4)$

القوانين المستنتجة لا تحفظ

إنما يتم استنتاجها أثناء حل المسائل

المساحة المعرضة للإحتكاك تقع بين الدائرتين

ونظراً لأن السرعة متغيرة على المساحة

نأخذ مساحة شريحة معرضة لسرعة ثابتة ثم نجرى عليها التكامل

What is the diameter of a spherical water drop if the inside pressure is 15 N/m^2 and the surface tension is 0.074 N/m.

Water drop $P_i = 15\ N/m^2$ $\sigma = 0.074\ N/m$ $d = ?$

Solution

$$\Delta P = \frac{4\sigma}{d}$$

$$15 = \frac{4\,(0.074)}{d} \Rightarrow d = 0.0197\ m = \underline{\underline{1.97\ cm}}$$

10

The pressure within a bubble of soapy water of 0.05 cm diameter is 5.75 gm/cm^2 greater than that of the atmosphere. Calculate the surface tension in the soapy water in S.I. units.

Soap bubble $d = 0.05\ cm$ $\Delta p = 5.75\ gm/cm^2$

Surface tension $\sigma = ?$ in SI

Sol

$$\Delta P = 5.75\ gm/cm^2 = 5.75 * 10^{-3}\ Kg_w/cm^2$$

$$\because 1\ Kg_w/cm^2 = 9.81 * 10^4\ N/m^2$$

$$\Delta P = \frac{5.75}{1000} * 9.81 * 10^4 = 564.075\ N/m^2$$

$$\sigma = \frac{\Delta P . d}{8} = \frac{564.075 * 0.05 * 10^{-2}}{8} = \underline{\underline{0.035\ N/m}}$$

9 **Calculate** the capillary effect in millimeters in a glass tube of 4 mm diam., when immersed in (i) water and (ii) in mercury. The temperature of liquid is 20° C and the values of surface tension of water and mercury at this temperature in contact with air are 0.0075 kg/m and 0.052 kg/m respectively. The contact angle for water = 0 and for mercury = 130°. (11)

Water $\sigma = 0.0075 \ kg_w/m$, $\alpha = 0$

Mercury $\sigma = 0.052 \ kg_w/m$, $\alpha = 130^\circ$

$d = 4\,mm$ * $S.g_{Hg} = 13.6$ تحفظ

Solution

1st Water

$$h = \frac{4\sigma \cos\alpha}{\gamma d} = \frac{4 * 0.0075 \ \frac{kg_w}{m} * 9.81 \frac{N}{kg_w} * \cos 0}{9810 \ \frac{N}{m^3} * 4 * 10^{-3}}$$

$$= 7.5 * 10^{-3} \ m = \underline{7.5\,mm} \quad (rise)$$

2nd Mercury

$$h = \frac{4\sigma \cos\alpha}{\gamma d} = \frac{4 * 0.052 * 9.81 * \cos 130}{13.6 * 9810 * 4 * 10^{-3}}$$

$$= -2.46 * 10^{-3} \ m = \underline{-2.46\,mm} \quad (depression)$$

---

8 To **what** height will water rise in a glass tube if its diameter is ($\sigma = 0.072$ N/m):

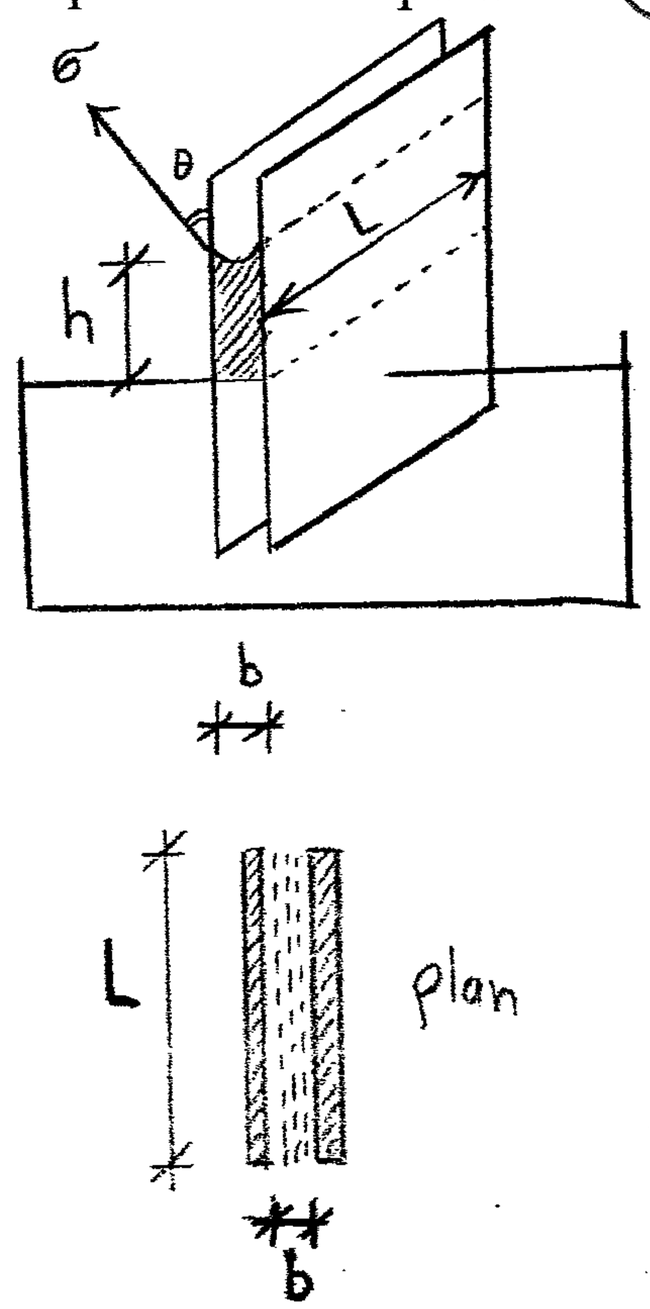
a) 1.50 cm b) 2.0 mm

assume α (angle of Contact) $= 0^\circ$

$$h_1 = \frac{4\sigma \cos\alpha}{\gamma d} = \frac{4(0.072) * 1}{9810\,(0.015)} = \underline{1.96\ mm}$$

$$h_2 = \frac{4\,(0.072)}{9810 * 2 * 10^{-3}} = \underline{14.68\ mm}$$

Determine the capillary rise of water between two parallel glass plates 5 mm apart. (12)



$b = 5\ mm$ $\quad \sigma_{water} = 0.074\ N/m$

$h = ?$

Sol

Weight = Surface tension force

$\gamma h L b = \sigma \cos\theta * 2L$

$$h = \frac{2\sigma \cos\theta}{\gamma b}$$

for water & glass $\Rightarrow \theta = 0$

$$h = \frac{2\,(0.074)}{9810\,(5*10^{-3})} = 3*10^{-3} = \underline{\underline{3\ mm}}$$

---

11 A liquid compressed in a cylinder has a volume of 1 liter at a pressure of 100 N/cm^2 and a volume of 995 cm^3 at a pressure of 200 N/cm^2. What is its bulk modulus of compressibility.

$\forall_1 = 1\ litre = 1000\ cm^3$ $\quad P_1 = 100\ N/cm^2$

$\forall_2 = 995\ cm^3$ $\quad P_2 = 200\ N/cm^2$

$K = ?$

Sol

$\Delta \forall = \forall_2 - \forall_1 = 995 - 1000 = -5\ cm^3$

$\Delta P = P_2 - P_1 = 200 - 100 = 100\ N/cm^2$

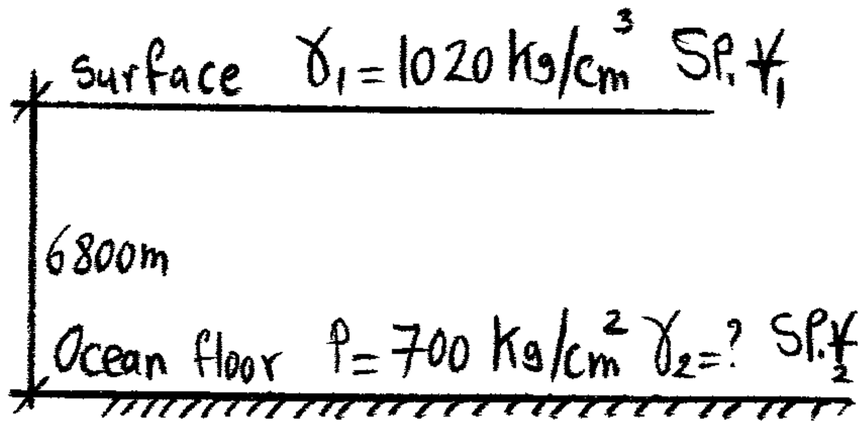
$$K = \frac{\Delta P}{-\frac{\Delta V}{V}} = \frac{100}{-\left(\frac{-5}{100}\right)} = \underline{\underline{20\,000\ N/cm^2}}$$

12 At a depth of 6.8 km in the ocean, the pressure is 700 kg/cm^2 above the atmospheric pressure. Assuming specific weight of water at the surface equal to 1020 kg/m^3 and the average bulk modulus of elasticity equals 2.5x10^4 kg/cm^2. CALCULATE the specific weight at that depth. Also, CALCULATE the change in specific volume from the surface to 6.8 km depth. (13)

$K = 2.5 * 10^4 \ kg/cm^2 \qquad \Delta Sp.\forall = ? \qquad \gamma_{ocean\ floor} = ?$

Solution

$$K = \frac{\Delta P}{\frac{\Delta \rho}{\rho}} = \frac{\Delta P}{\frac{\Delta \gamma}{\gamma}}$$



$$2.5 * 10^4 \ \frac{kg_w}{cm^2} = \frac{700 \ \frac{kg_w}{cm^2}}{\frac{\Delta \gamma}{1020 \ \frac{kg}{cm^3}}} \Rightarrow \Delta\gamma = 28.56 \ \frac{kg_w}{cm^3}$$

$$\gamma_2 = \gamma_1 + \Delta\gamma = 1020 + 28.56 = 1048.56 \ \frac{kg_w}{cm^3}$$

(ocean floor)

$$\Delta Sp.\forall = Sp.\forall_1 - Sp.\forall_2$$

$$= \frac{1}{\gamma_1} - \frac{1}{\gamma_2}$$

$$= \frac{1}{1020} - \frac{1}{1048.56} = 2.67 * 10^{-5} \ cm^3/kg$$

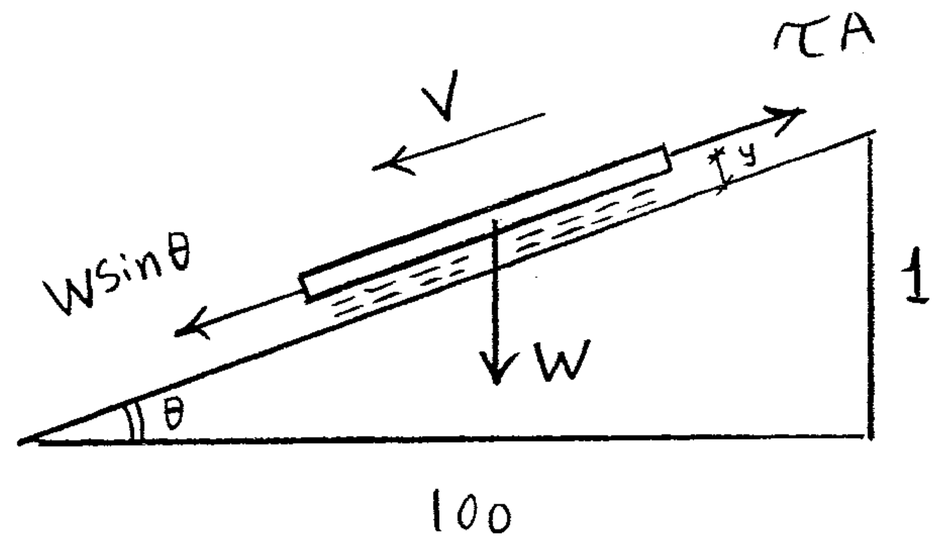
(14)

The space between a square smooth flat plate (50 x 50) cm^2, and a smooth inclined plane (1:100) is filled with an oil film (S.G. = 0.9) of 0.01 cm thickness. Determine the kinematic viscosity in stokes if the plate is 2.3 kg.

$A = 50 * 50 = 2500\ cm^2$

$y = 0.01\ cm \qquad V = 9\ cm/s$

$m = 2.3\ kg = 2300\ gm$



$\tan\theta = \frac{1}{100} \Rightarrow \theta = 0.573$

$S.g = 0.9 \qquad \nu = ?$

Sol

$W \sin\theta = \tau A$

$mg \sin\theta = \mu \frac{V}{y} A$

$2300\,(981) \sin 0.573 = \mu \frac{9}{0.01} 2500 \Rightarrow \mu = 0.01$ poise

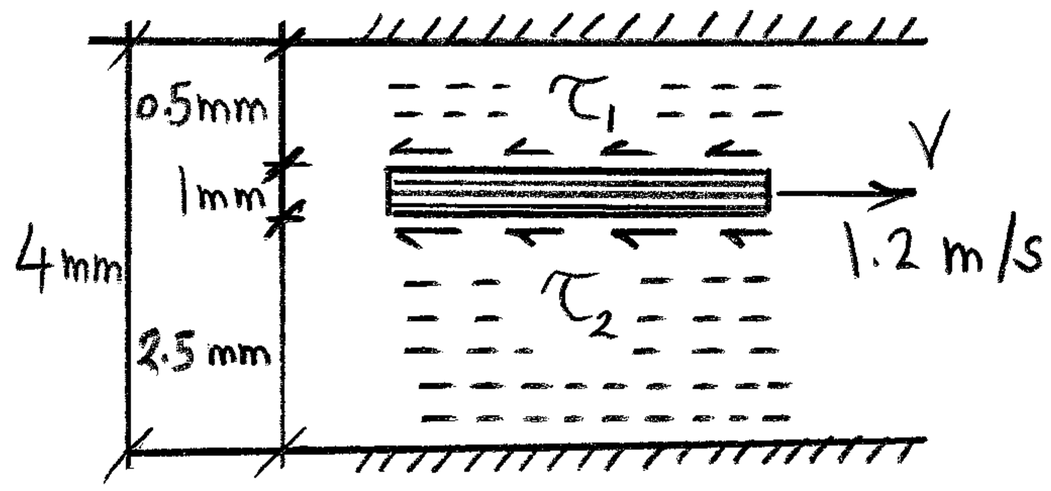
$\nu = \frac{\mu}{\rho} = \frac{0.01}{0.9 * 1} = \underline{\underline{0.011\ \text{Stokes}}}$

---

17 $A = 2m * 3m$

$\mu = 0.07$ Poise $(dyne.s/cm^2)$

$= 0.007\ Pa.s\ (N.s/m^2)$



Solution

$\tau = \tau_1 + \tau_2$

$= \mu \frac{V}{y_1} + \mu \frac{V}{y_2}$

$= 0.007 * 1.2 \left(\frac{1}{0.5 * 10^{-3}} + \frac{1}{2.5 * 10^{-3}} \right) = 20.16\ N/m^2$

$F = \tau A = 20.16 * (2 * 3) = \underline{\underline{121\ N}}$

* Pa.S = 10 Poise

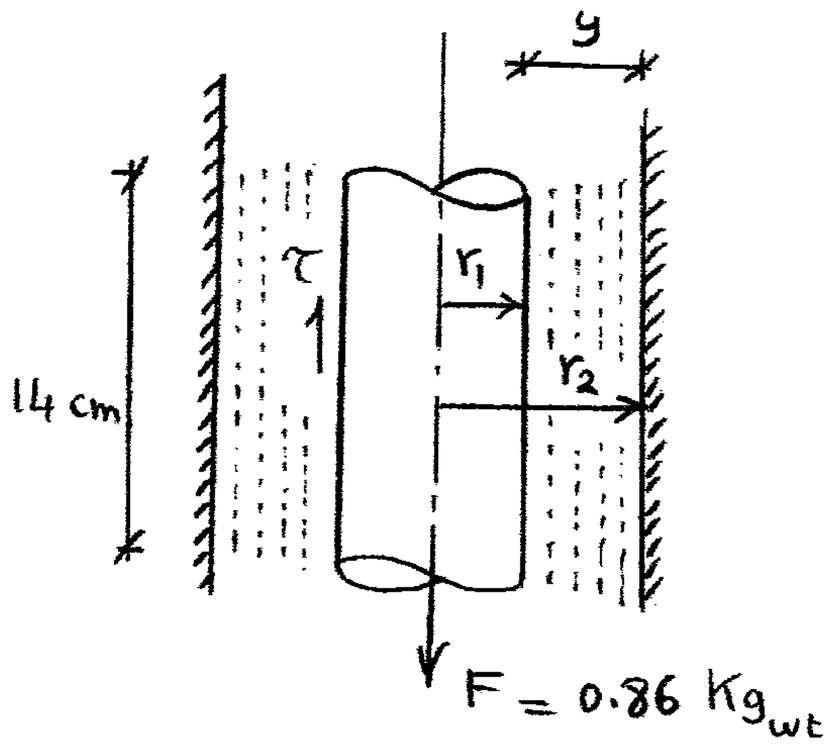
14 A piston 11.96 cm diameter and 1 cm long works in a cylinder 12 cm diameter. A lubricating oil which fills the space between them has a viscosity 0.65 poise. **Calculate** the speed at which the piston will move through the cylinder when an axial load of 1.0 kg is applied. Neglect the inertia of the piston. (15)

$\mu = 0.65$ poise $(dyne.s/cm^2)$

$F = 0.86\ kg_w \times 1000 = 860\ gm_w$

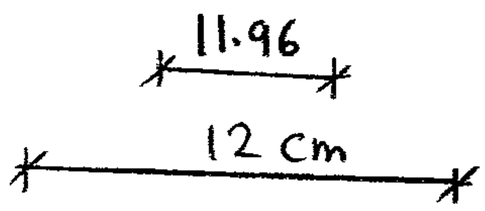
$= 860\ gm_w \times 981 = 8.43 \times 10^5$ dyne

$V = ?$



Sol

$y = r_2 - r_1 = \frac{12 - 11.96}{2} = 0.02$ cm



$A = 2\pi \underline{\underline{r_1}} L = 2\pi * \frac{11.96}{2} * 14 = 526\ cm^2$

$F = \mu \frac{V}{y} A$

$8.43 * 10^5 = 0.65 * \frac{V}{0.02} * 526 \Rightarrow V = \underline{\underline{49.3\ cm/s}}$

A piece of pipe 30 cm long weighting 1.5 kg and having internal diameter of 5.125 cm is slipped over a vertical shaft 5.0 cm in diameter and allowed to fall under its own weight Calculate the maximum velocity attained by the falling pipe if a film of oil having viscosity equals 0.5 lb.s/ft^2 is maintained between the pipe and the shaft.

$W = 1.5\ kg_w$ $\mu = 0.5\ lb.s/ft^2$

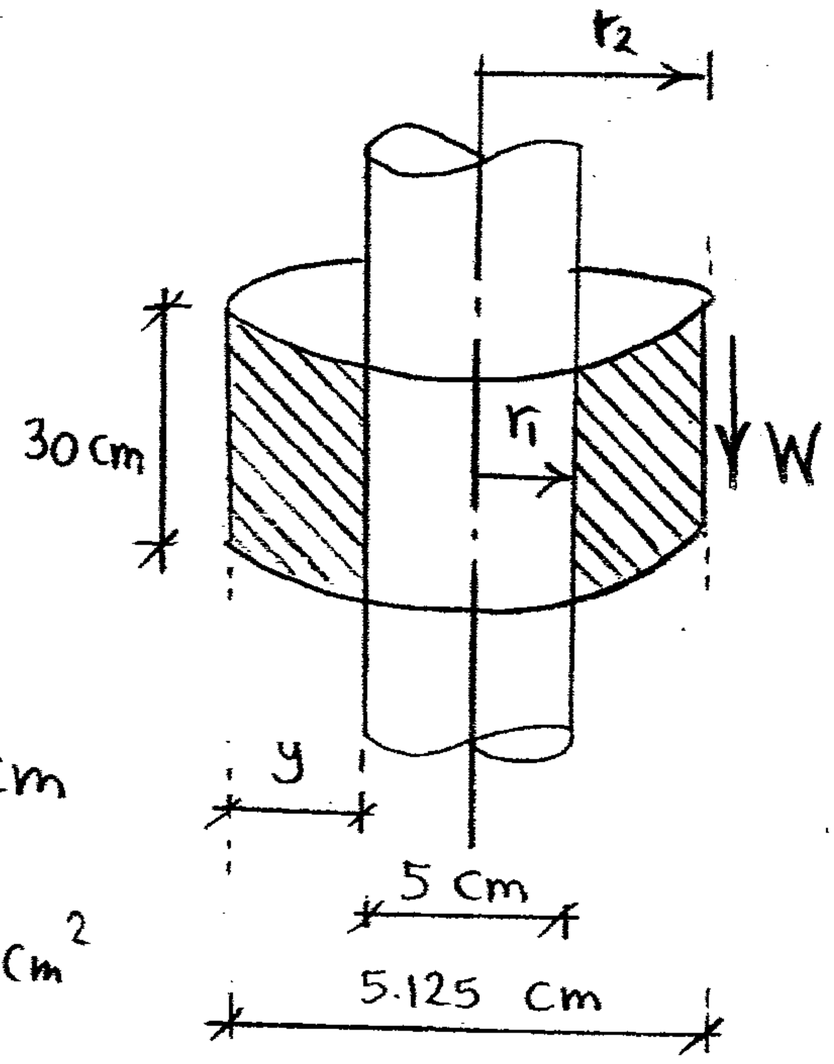
$V = ?$

Sol

$W = \tau . A = \mu \frac{V}{y} A$

$y = r_2 - r_1 = \frac{5.125 - 5}{2} = 0.0625$ cm

$A = 2\pi \underline{\underline{r_2}} L = 2\pi \left(\frac{5.125}{2}\right) 30 = 483\ cm^2$



$$\mu = 0.5 \frac{lb}{ft^2}.s * \frac{4.44\ N * 1\ ft^2}{1\ lb * (0.3048)^2 m^2} = 23.89 \frac{N}{m^2}.s \ (Pa.s)$$

$$W = \mu \frac{V}{y} . A$$

$$1.5 * 9.81 = 23.89 \frac{V}{0.0625 * 10^{-2}} * 483 * 10^{-4}$$

$$\Rightarrow V = 7.97 * 10^{-3}\ m/s \simeq \underline{0.8\ cm/s}$$

---

16 A cylinder of 0.12 m radius rotates concentrically inside of a fixed cylinder of 0.122 m radius. Both cylinders are 0.30 m long. **Determine** the viscosity of the liquid which fills the space between the cylinders if a torque of 1 N.m is required to maintain an angular velocity of 2 rad/s.

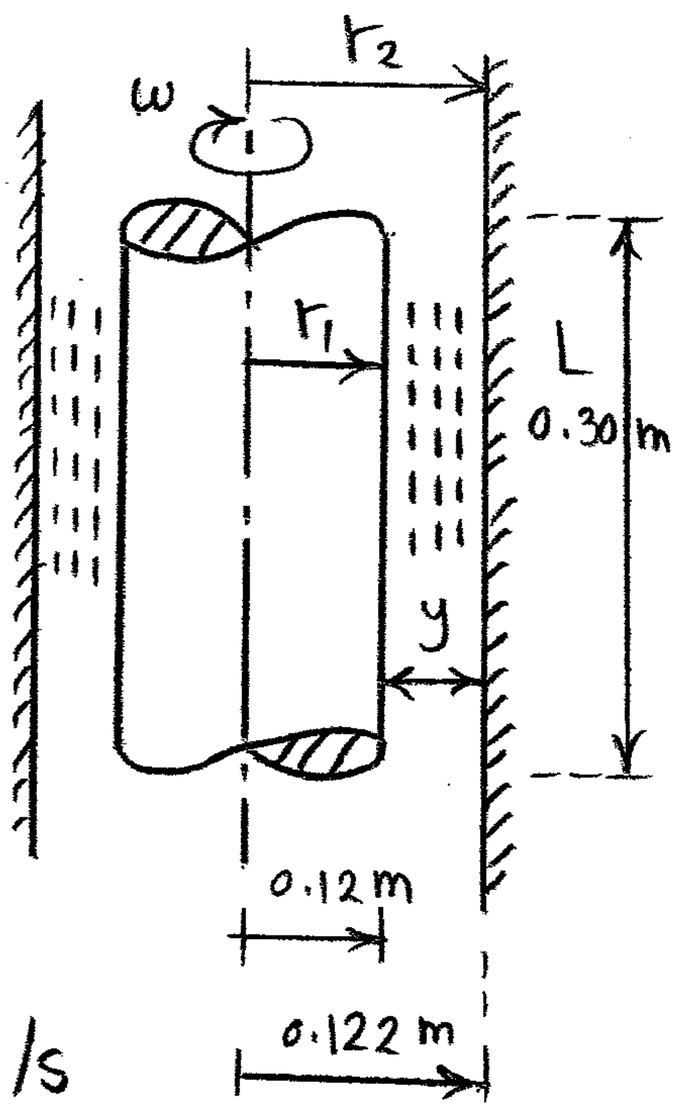
$T = 1\ N.m$, $\omega = 2\ rad/s$

$\mu = ?$

Solution

$$F = \frac{T}{r} = \frac{1}{0.12} = 8.33\ N$$

$$F = \tau . A = \mu \frac{V}{y} A$$



$$y = r_2 - r_1 = 0.122 - 0.12 = 0.002 m$$

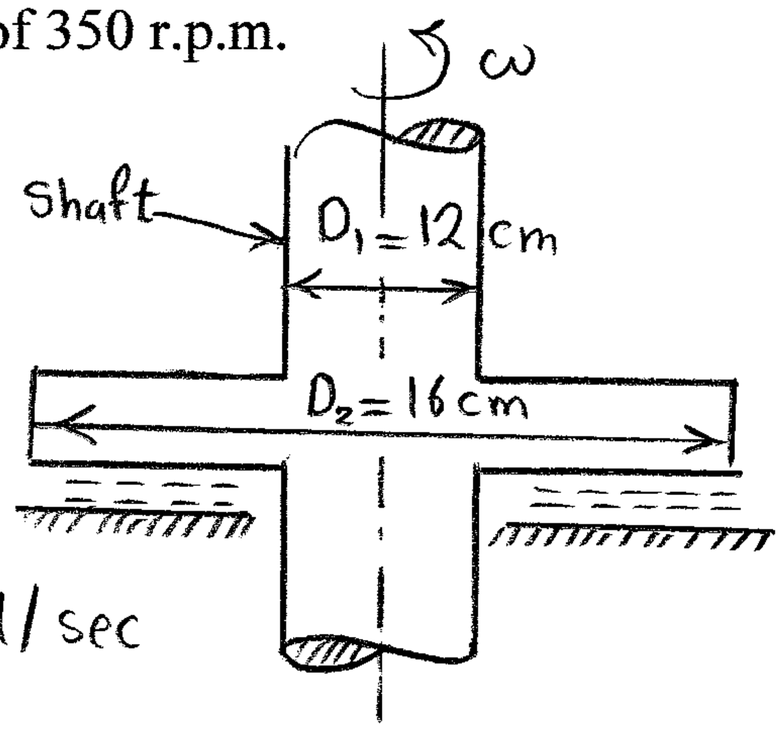
$$V = \omega r_1 = 2(0.12) = 0.24\ m/s$$

$$A = 2\pi r_1 L = 2\pi (0.12)\, 0.3 = 0.226\ m^2$$

$$F = \mu \frac{V}{y} A$$

$$8.33 = \mu \frac{0.24}{0.002} . 0.226 \qquad \Rightarrow \mu = \underline{0.307\ Pa.s}$$

15 The thrust of a shaft is taken by a collar bearing provided with a forced lubrication system. The lubrication system maintains a film of oil of uniform thickness between the surface of the collar and the bearing. The external and internal diameters of collar are 16 and 12 cms. respectively. The thickness of oil film is 0.02 cms. and coefficient of viscosity is 0.91 poise. **Find** the horse-power lost in over coming friction when the shaft is rotated at a speed of 350 r.p.m. (17)



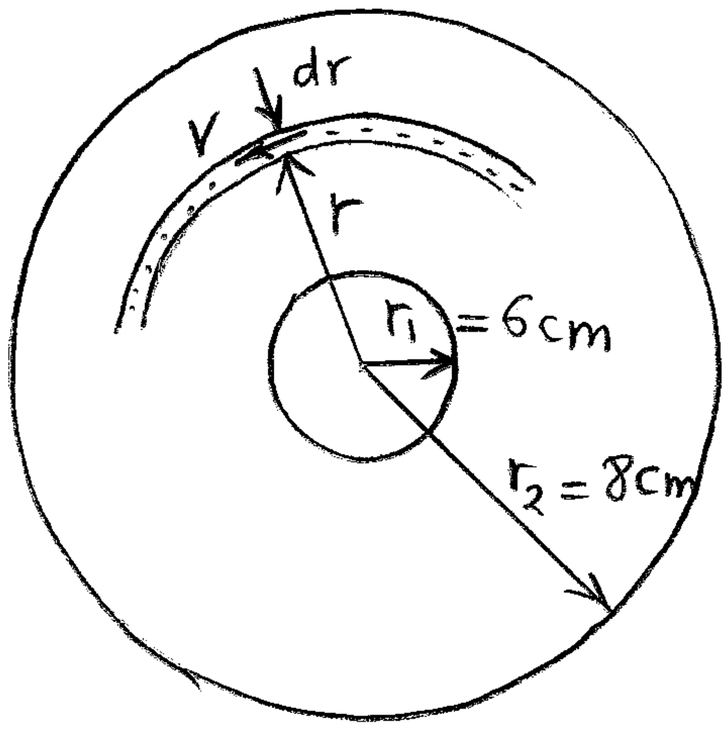
$\mu = 0.91$ poise $= 0.091$ Pa.s

$N = 350$ rev/min $\quad HP = ?$

Solution

$$\omega = \frac{2\pi N}{60} = \frac{2\pi(350)}{60} = 36.65 \text{ rad/sec}$$

$$\tau = \mu \frac{V}{y} = \mu \frac{\omega r}{y}$$



$$dF = \tau . dA$$

$$= \mu \frac{\omega r}{y} . 2\pi r \, dr$$

$$= 2\pi \mu \frac{\omega}{y} r^2 \, dr$$

$$T = \int dF . r$$

$$= 2\pi \mu \frac{\omega}{y} \int_{r_1}^{r_2} r^3 \, dr$$

$$* \int_{r_1}^{r_2} r^3 dr = \frac{r_2^4 - r_1^4}{4}$$

$$= 2\pi (0.091) \frac{36.65}{0.02 * 10^{-2}} \frac{(0.08)^4 - (0.06)^4}{4} = 0.733 \text{ N.m}$$

$$P = T . \omega = 0.733 (36.65) = 26.88 \text{ watt}$$

$$HP = \frac{P}{735} = \frac{26.88}{735} = \underline{0.0366 \text{ horse power}}$$

The velocity distribution in a fluid over a plate is given by the expression ($v = 0.3\,y - y^2$), where v is the velocity in m/sec at a distance y meters from the plate. Determine the shear stress at y = 0 and y = 0.1 m. Take $\mu = 8.8$ poise. (18)

$V = 0.3y - y^2$ $\quad \tau_1 = ?$ at $y = 0$

$\mu = 8.8$ Poise $= 0.88$ Pa.s $\quad \tau_2 = ?$ at $y = 0.1$

Solution

$$\frac{dV}{dy} = 0.3 - 2y$$

$$\tau = \mu \frac{dV}{dy} = 0.88\,(0.3 - 2y)$$

$$\tau_{1\,y=0} = 0.88\,(0.3 - 0) = \underline{0.264\ N/m^2}$$

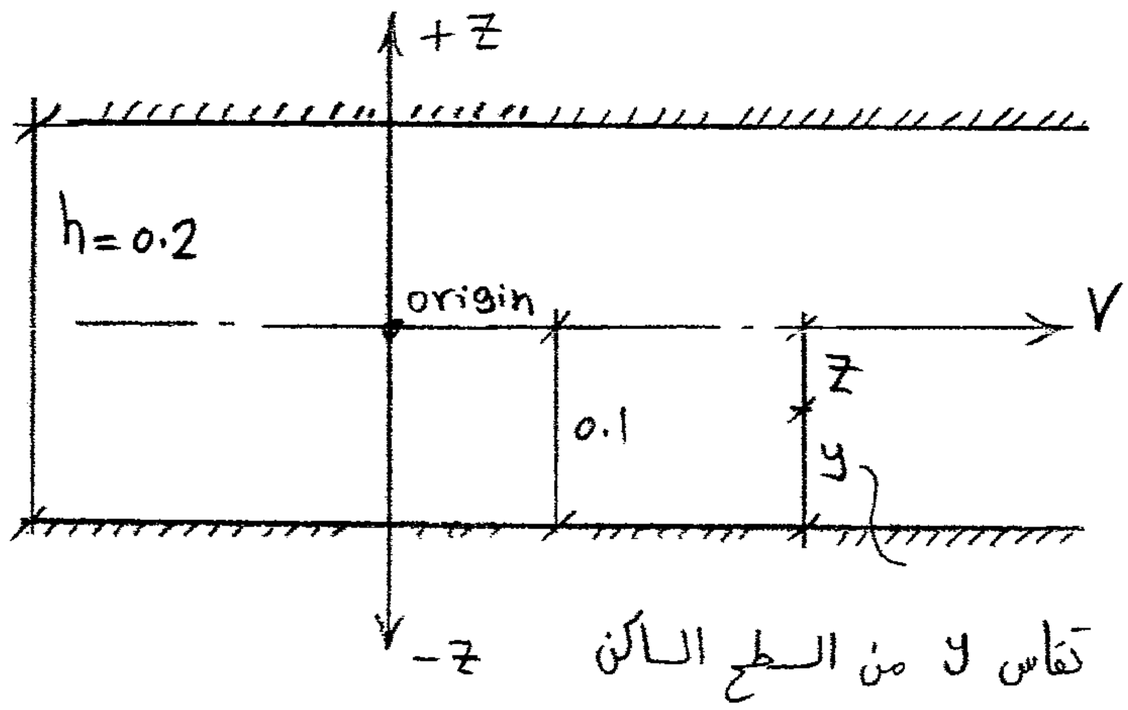
$$\tau_{2\,y=0.1} = 0.88\,(0.3 - 2(0.1)) = \underline{0.088\ N/m^2}$$

18

$$\frac{V}{u_{max}} = 1 - \left(\frac{2z}{h}\right)^2$$

$u_{max} = 30$ cm/s

$\mu = 0.01$ poise , $h = 0.2$ cm



تقاس y من السطح السفلي

Draw Shear Stress distribution , Velocity distribution , Shear at boundary

Sol

1- Eqn for Shear Stress distribution

$$\tau = \mu \frac{dV}{dy}$$

$$V = u_{max} - 4u_{max}\frac{z^2}{h^2}$$

$$V = U_{max} - 4U_{max}\frac{Z^2}{h^2}$$

$$\frac{dV}{dz} = \frac{-8U_{max}Z}{h^2}$$

* نلاحظ أنه إذا تم مساواة $\frac{dV}{dz}$ بصفر

فإن أقصى سرعة تتحقق عند $z=0$ أى فى المنتصف

$$\because Z = 0.1 - y$$

$$\therefore dz = -dy$$

$$\frac{dV}{dy} = \frac{8U_{max}Z}{h^2}$$

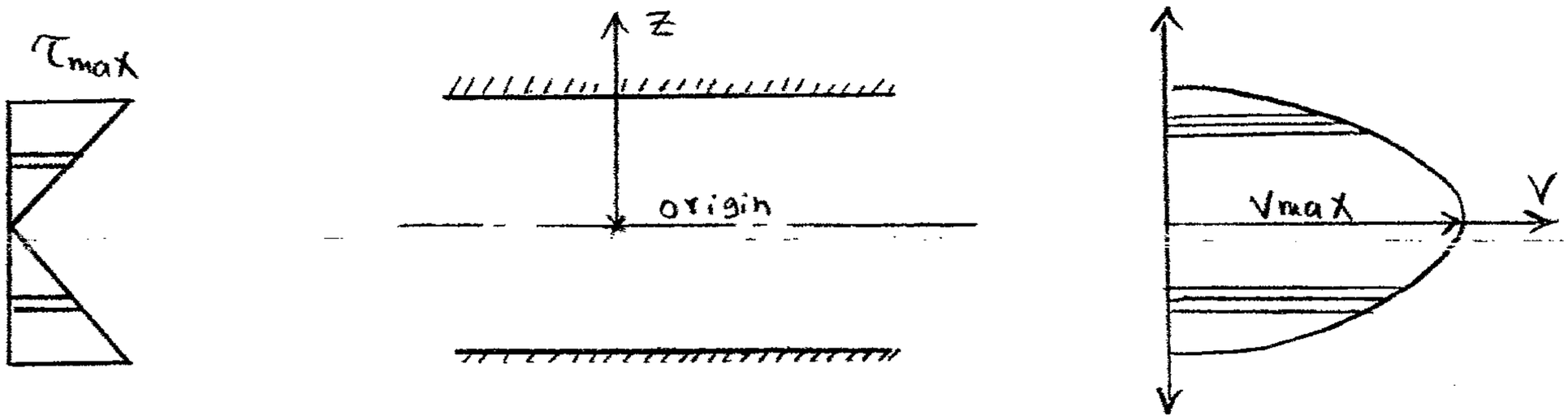
من معادلة τ نلاحظ أن $\tau=0$ عند $z=0$ (فى المنتصف)

$$\tau = \mu\frac{dV}{dy} = \frac{8\mu U_{max}Z}{h^2} = \frac{8*0.01*30*Z}{0.2} = 60Z$$

2- Eqn for velocity distribution

$$V = U_{max} - \frac{4U_{max}Z^2}{h^2} = 30 - \frac{4(30)Z^2}{(0.2)^2} = 30 - 3000Z^2$$

Z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
τ	0	0.6	1.2	1.8	2.4	3.0	3.6	4.2	4.8	5.4	6
V	30	29.7	28.8	27.3	25.2	22.5	19.2	15.3	10.8	5.7	0



Shear stress distribution

Velocity distribution

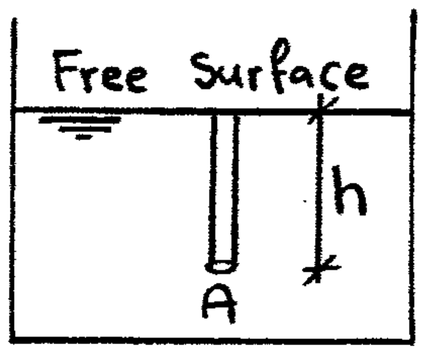
3- Shear stress at the boundarys

$$\tau = 60Z = 60(0.1) = 6 \text{ dyne/cm}^2$$

3

Hydrostatic Pressure

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{\rho g h A}{A} = \gamma h$$



$$\boxed{P = \gamma H}$$

Presure — Specific Weight — Pressure head

Pascal's hydrostatic equation

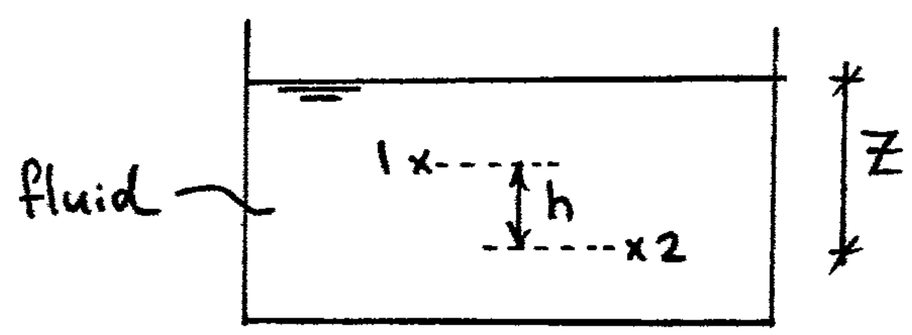
units ⇒ $N/m^2 = Pa$ $\quad lb/ft^2 = Psf$ $\quad lb/inch^2 = Psi$

e.g.

$P_2 > P_1$

$P_2 = P_1 + \gamma_f h$

$\because$ S.G. $= \frac{\gamma_f}{\gamma_w}$

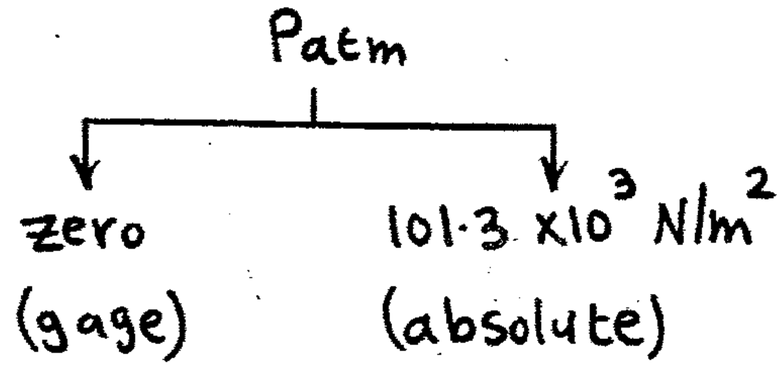


$$\boxed{P_2 = P_1 + \text{S.G. } \gamma_w h}$$

$$\boxed{P_1 = P_2 - \text{S.G. } \gamma_w h}$$

$P_2 = \gamma_f Z$ (gage)

$P_2 = P_{atm} + \gamma_f Z$ (absolute)



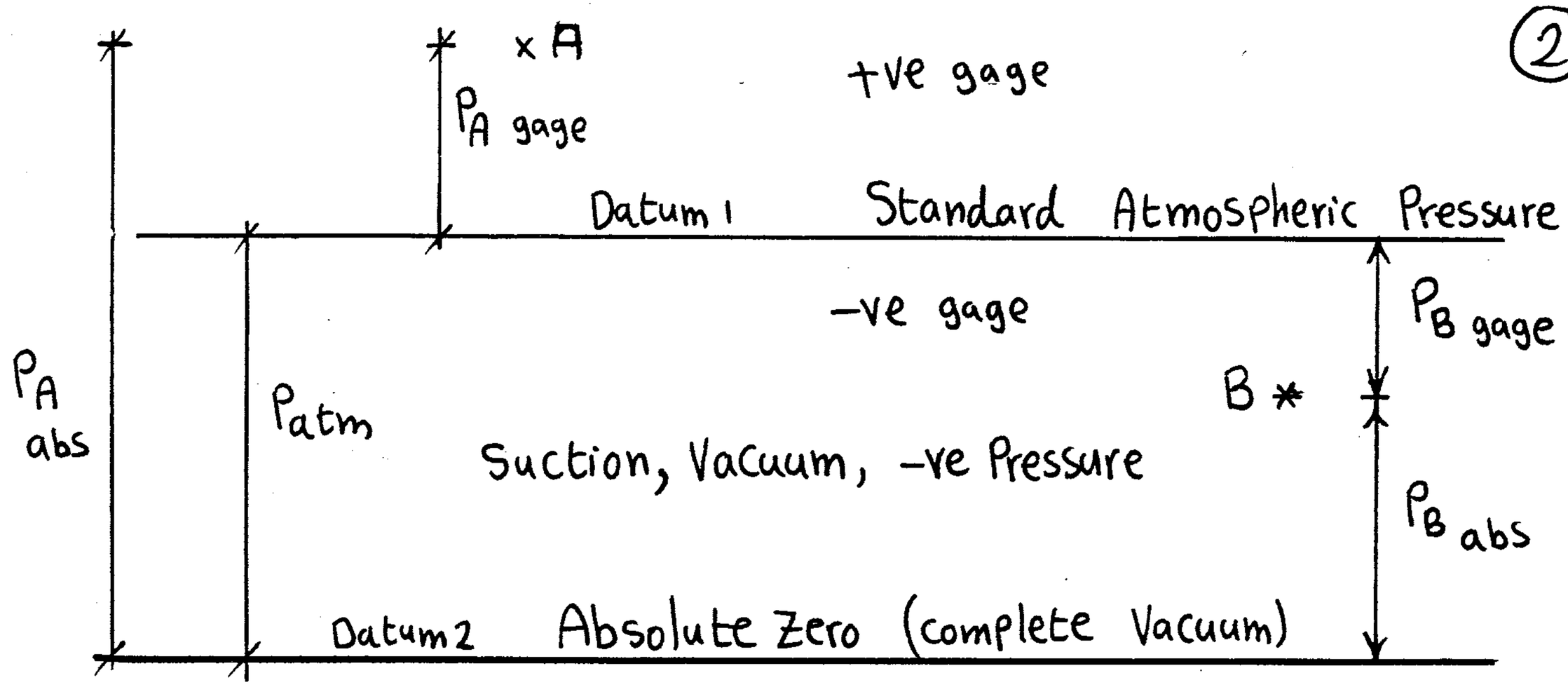
Gauge Pressure

It is the pressure measured by an instrument, in which the atmospheric pressure is taken as a datum

Absolute Pressure

It is the Sum of the atmospheric and gauge Pressure

$$\boxed{P_{abs} = P_{atm} + P_{gage}}$$



e.g.//

$P_A = 20$ KPa (gage)

$P_A = 101.3 + 20 = 121.3$ KPa (absolute)

$P_B = -40$ KN/m² } (gauge)
$= 40$ KN/m² vacuum
$= 40$ KN/m² suction

$P_B = 101.3 - 40 = 61.3$ KN/m² a (absolute)

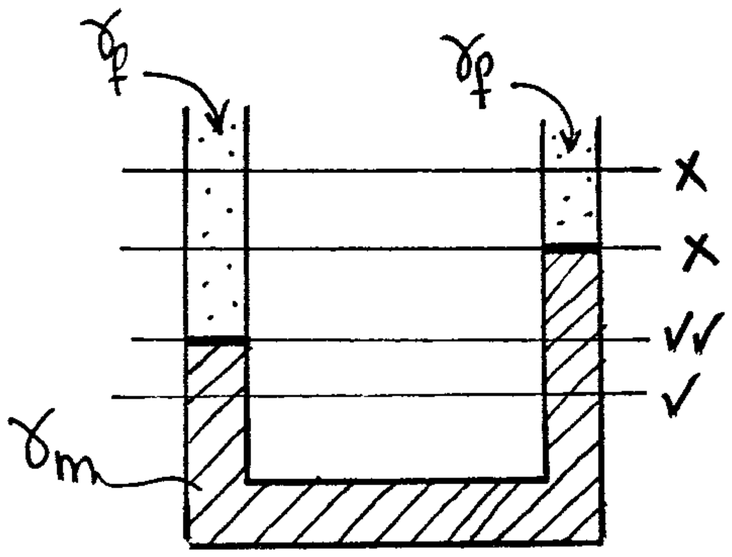
شروط تساوى الضغط عند نقطتين (المستوى الفاصل)

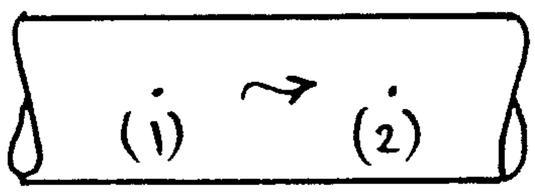
١- أنه تقع النقطتين فى نفس السائل

٢- السائل ساكن

٣- السائل متصل

٤- المستوى أفقى





$P_1 \neq P_2$

السائل غير متصل

$P_1 \neq P_2$

السائل غير ساكن

Standard Values of P_{atm}

P_{atm} = 0.76 m Hg = $0.76\ \gamma_{Hg}$ = $0.76 \times 13.6 \times 9810$

= 10.33 m Water = $10.33\ \gamma_w$ = $10.33 \times 1 \times 9810$

= 101.3×10^3 N/m^2 (Pa) $\simeq$ 1×10^5 N/m^2 = 1 bar

= 14.7 Psi a = 14.7×144 lb/ft^2 (Psf)

= 1 atmosphere = 34 ft water

= Zero (gauge) = 1.03 kg_w/cm^2

Pressure measurements

1- Barometer

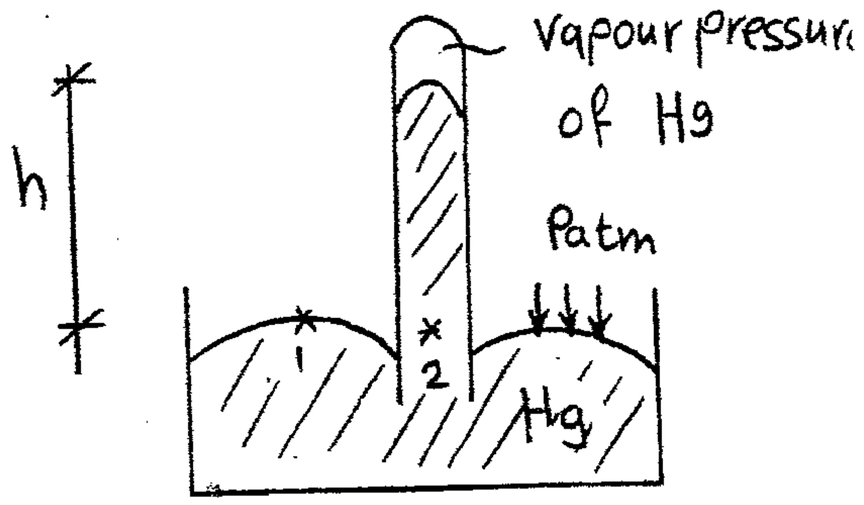
The barometer measures the atmospheric pressure at its location in absolute units

P_{atm} = Barometric pressure = local atmospheric pressure

P_{atm} = Standard value = 101.3×10^3 Pa لو غير معطى

a- Mercury Barometer

* It measures the atmospheric Pressure in absolute units

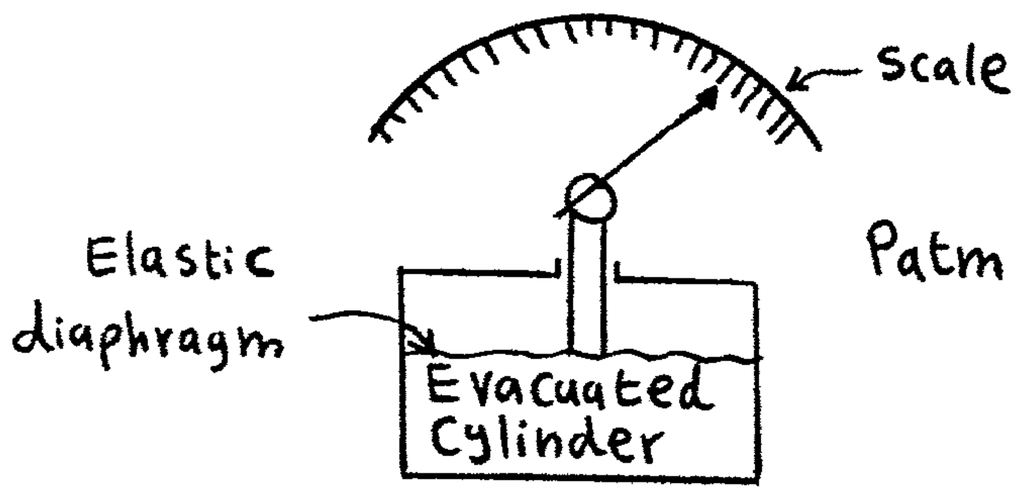


* When a tube filled with Mercury is inverted in a reservoir filled with Mercury, the Mercury drops until its height is balanced by the atmospheric Pressure

$$P_{atm} = \gamma_{H.g}\, h$$

b- Aneroid Barometer

It measures the diffrence between the atmospheric Pressure and an evacuated cylinder



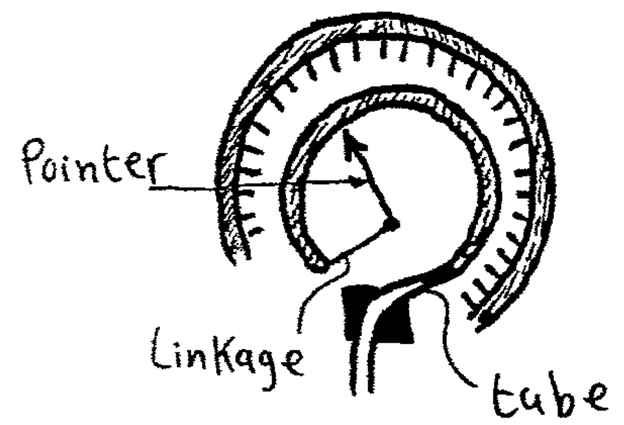
2- Pressure gauges

Bourdon gauge

It measures the Pressure relative to the Pressure Surrounding the gauge

$$P_{gauge} = P_{in} - P_{out}$$

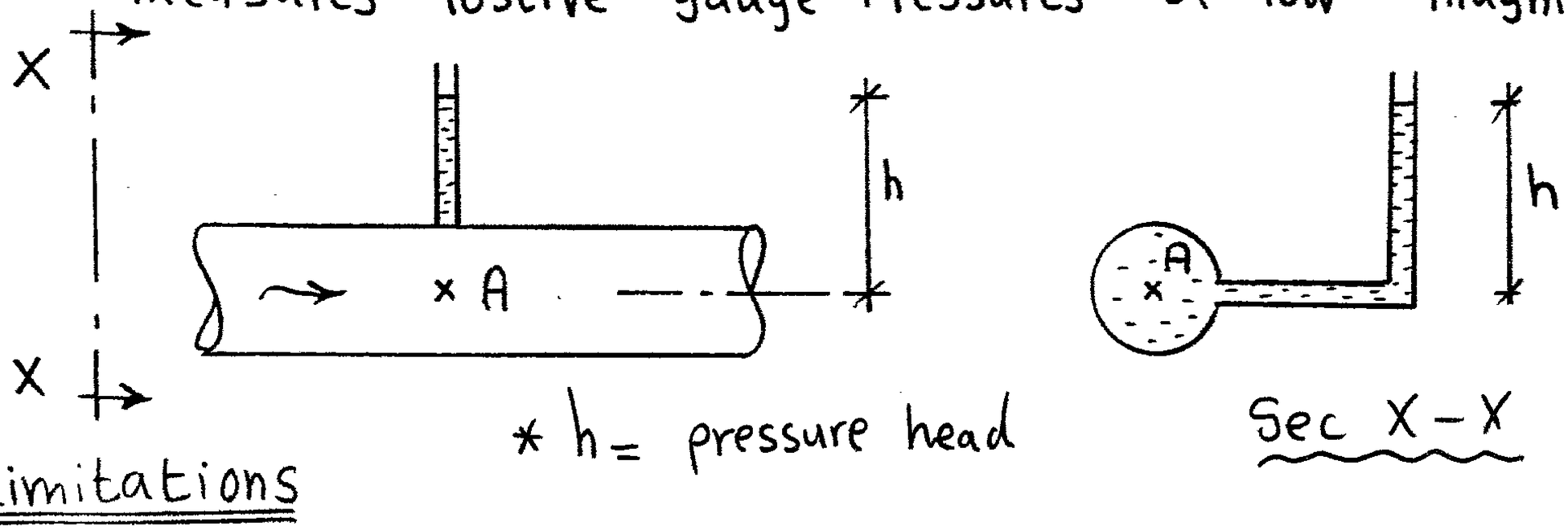
If $P_{in} = P_{out} \Rightarrow$ Reading = 0



3- Piezometer

لائل واحد

It measures Postive gauge Pressures of low magnitudes

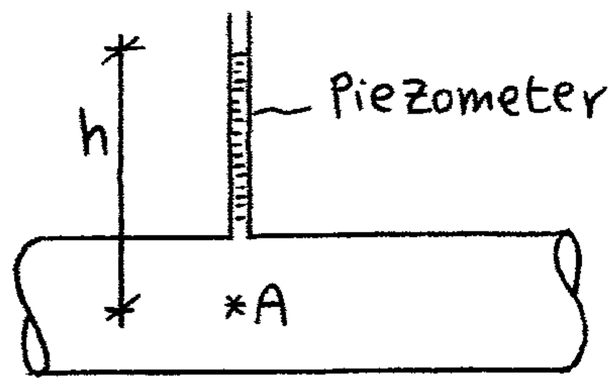


Limitations

a- Piezometers does not Work for negative Pressures

b- It is impractical to mesaure large Pressure
(we need a very long tube)

Pressure head



Pressure head is the height of a column of fluid that will produce the given intensity of Pressure

$h = \frac{P}{\gamma}$ = Pressure head

When a Piezometer is inserted in a tube the height of which the fluid rises is the Pressure head.

4- Manometers استخدام سائلين

It measures fluid pressures by using different fluids which may be heavier or lighter than the fluid concerned

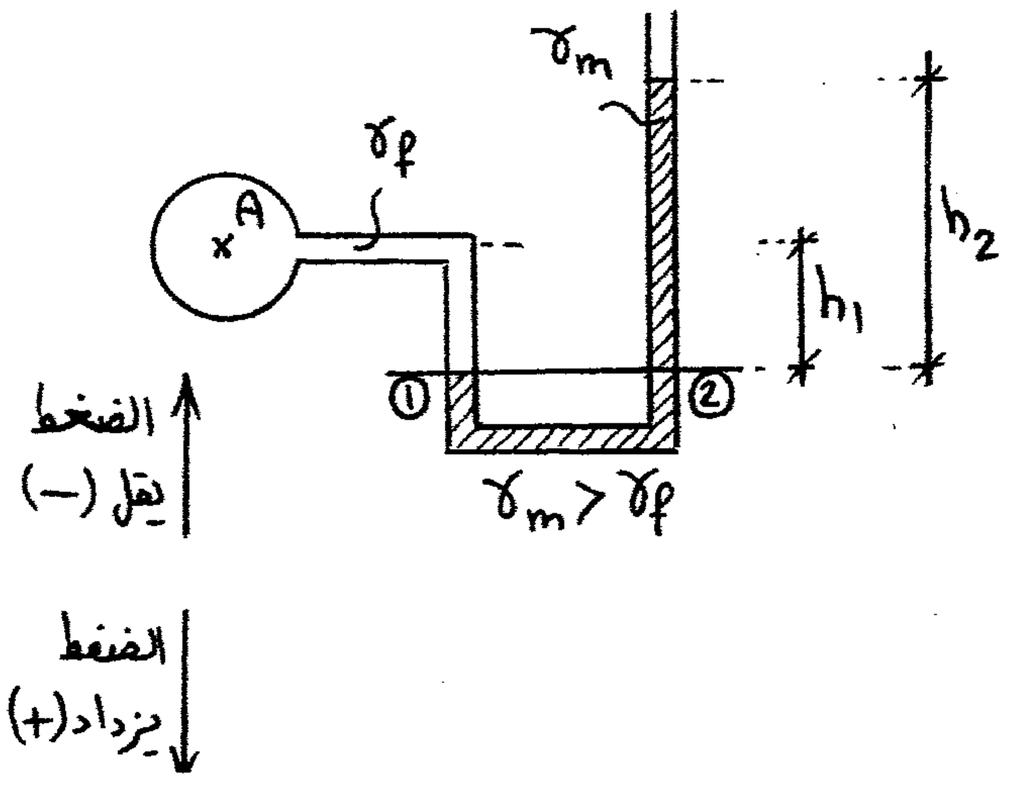
a- Simple manometer

$$P_1 = P_2$$

$$P_1 = P_A + \gamma_f h_1$$

$$P_2 = P_{atm} + \gamma_m h_2$$

$$\Rightarrow P_A = P_{atm} + \gamma_m h_2 - \gamma_f h_1$$



Differential manometers are used

- When only the difference between two pressures are desired

U-tube manometer is used

- When there is a big pressure difference
- A heavy liquid such as mercury is used

Inverted U-tube manometer is used

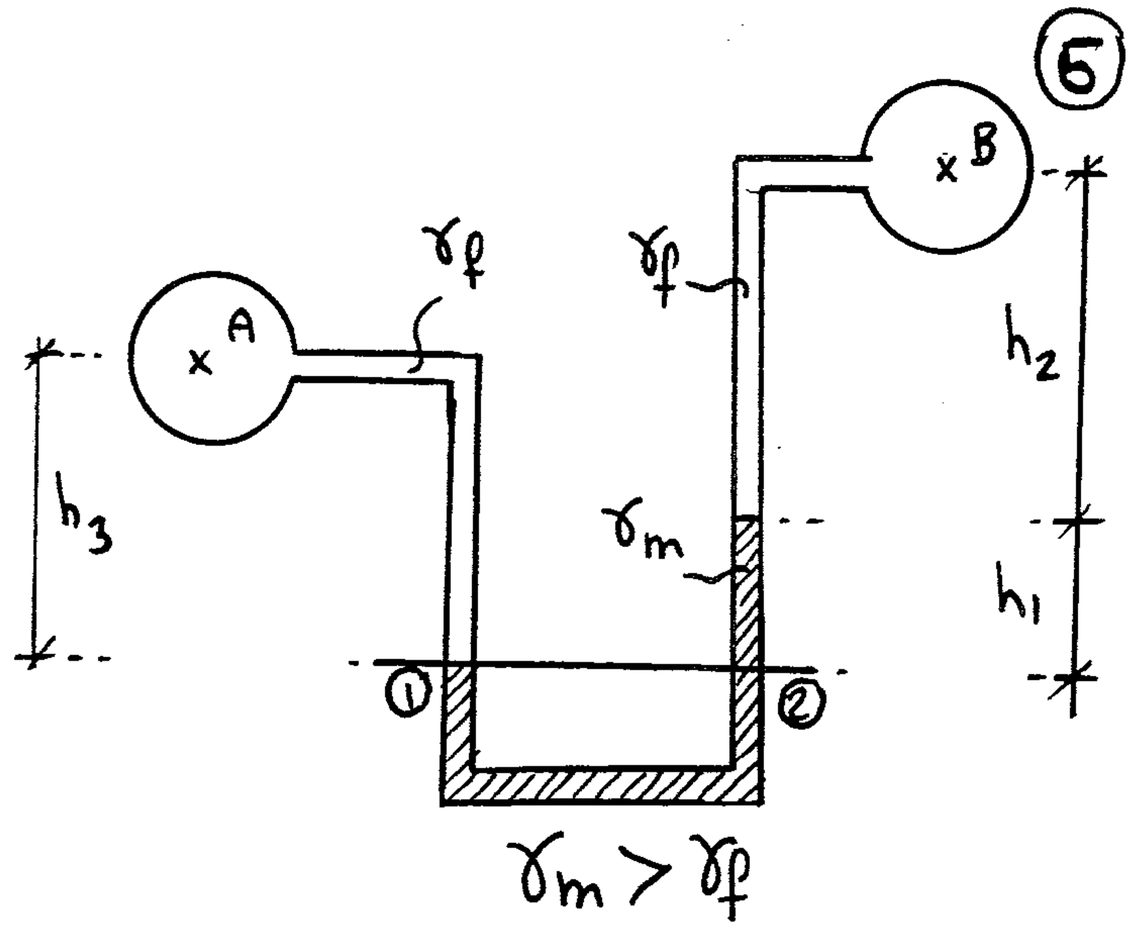
- When there is a small pressure difference
- A light liquid such as oil is used.

b- Differential manometer

$$P_1 = P_2$$

$$P_1 = P_A + \gamma_f h_3$$

$$P_2 = P_B + \gamma_f h_2 + \gamma_m h_1$$

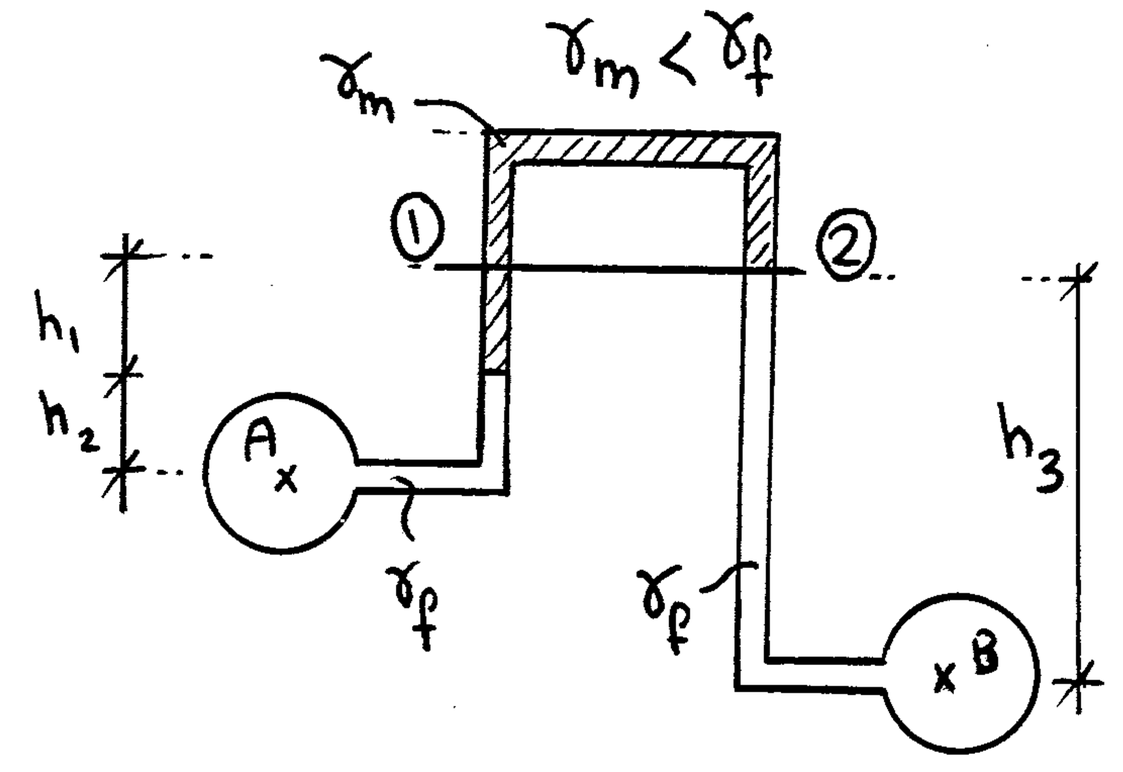


c- Inverted U tube Manometer

$$P_1 = P_2$$

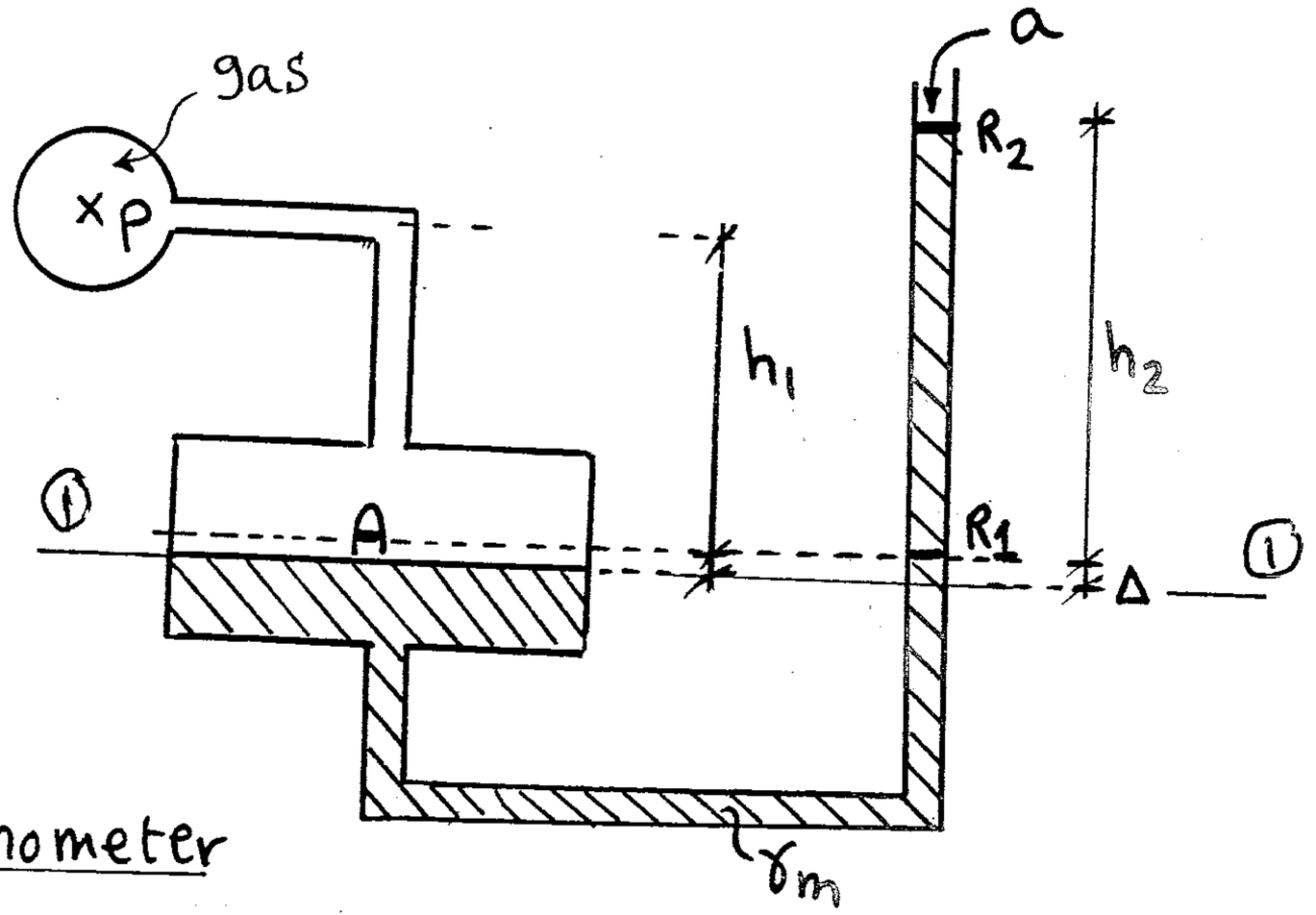
$$P_1 = P_A - \gamma_f h_2 - \gamma_m h_1$$

$$P_2 = P_B - \gamma_f h_3$$



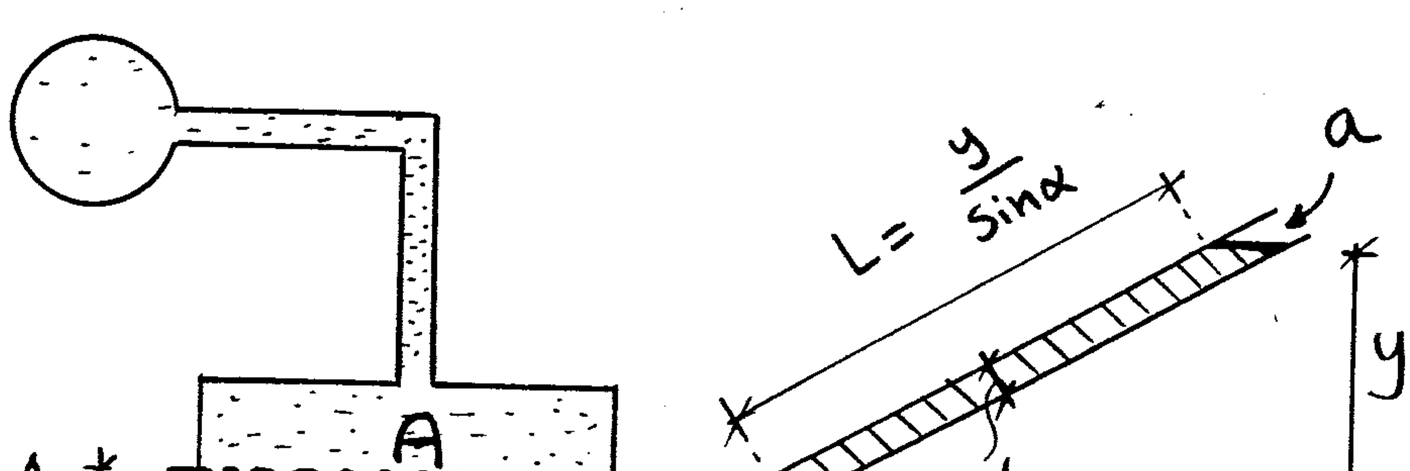
d- Micromanometer

i- Vertical tube Manometer



$$A\Delta = h_2 a$$

ii- Inclined tube manometer



$$A\Delta = La$$

$$a = \frac{\pi d^2}{4}$$

Pascal's law

The intensity of Pressure at any Point in a fluid at rest, is the Same in all directions

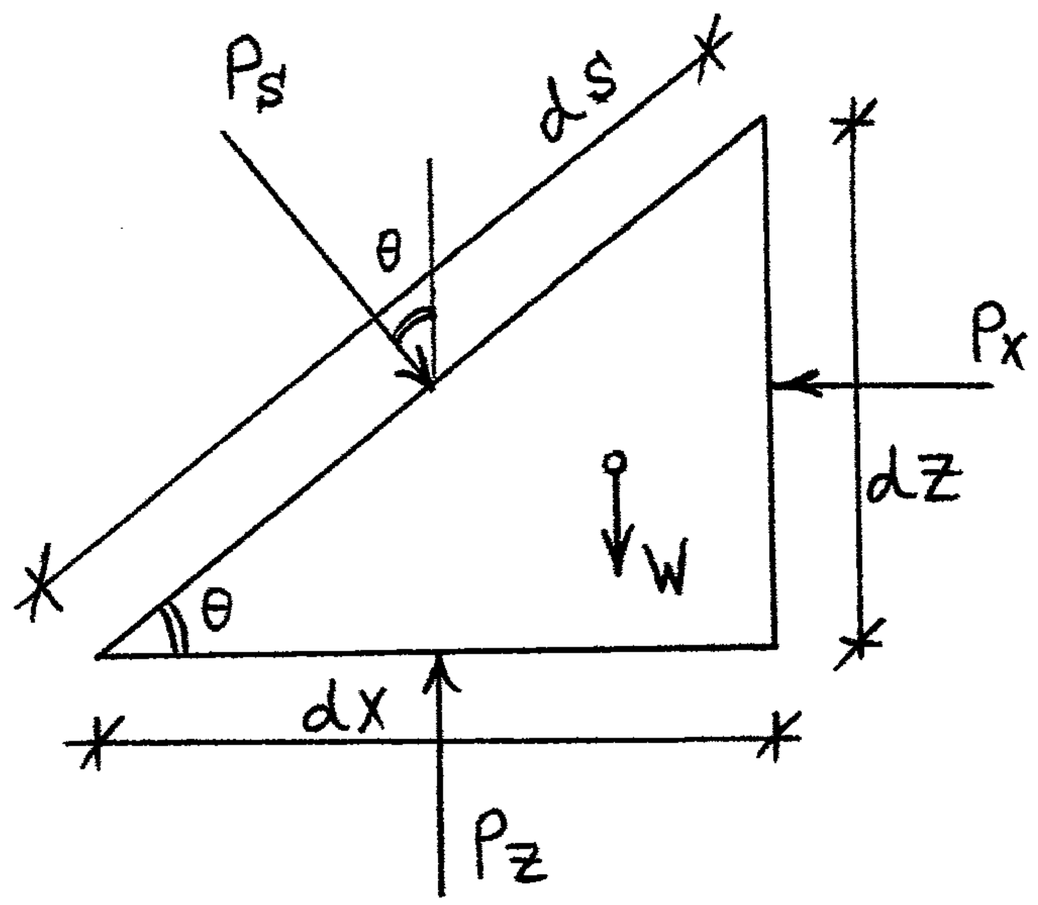
Consider a triangular prism of Very small size

$$\sum F_x = 0$$

$$P_s . dS \sin\theta = P_x . dz$$

$$P_s . \cancel{dS} \frac{dz}{\cancel{dS}} = P_x . \cancel{dz}$$

$$\therefore \boxed{P_s = P_x}$$



$$\sum F_z = 0$$

$$P_s . dS \cos\theta + W = P_z . dx$$

$$P_s . \cancel{dS} . \frac{dx}{\cancel{dS}} + \frac{1}{2} \cancel{dx} dz \gamma = P_z . \cancel{dx}$$

$$P_s + \frac{1}{2} dz \gamma = P_z$$

$\hookrightarrow dz \simeq 0$

$$\sin\theta = \frac{dz}{dS}$$

$$\cos\theta = \frac{dx}{dS}$$

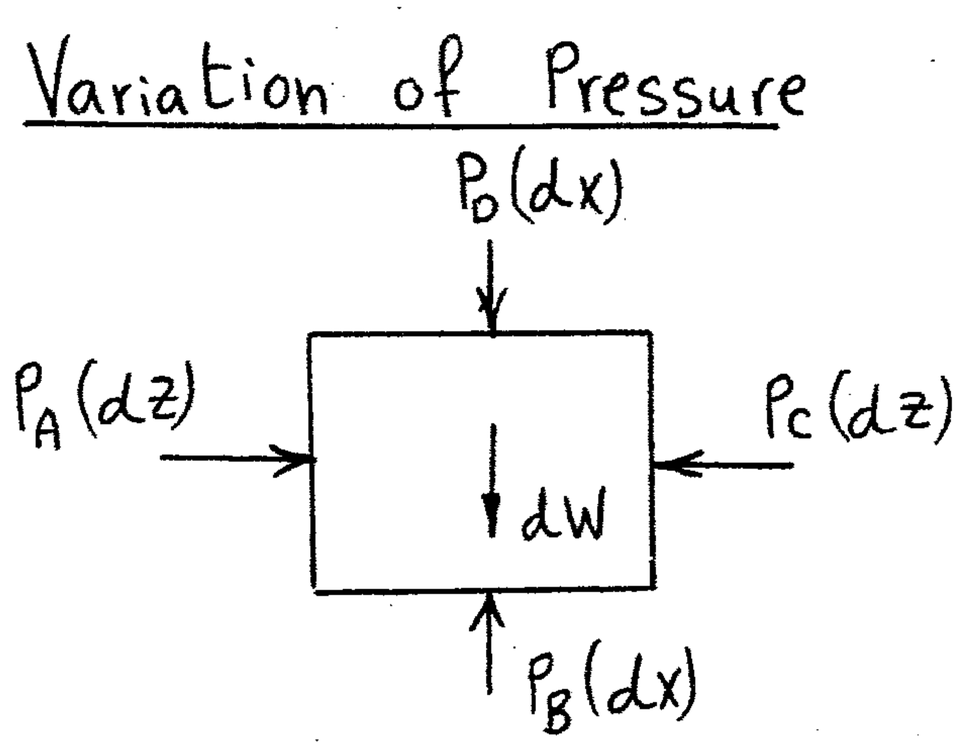
$$\boxed{P_s = P_z}$$

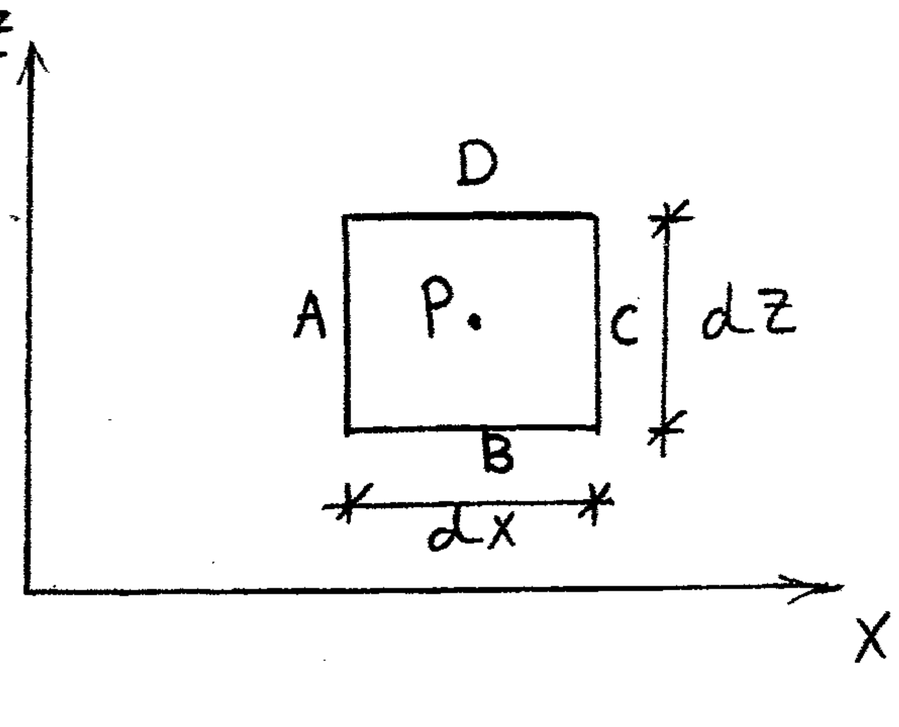
$$\boxed{P_x = P_z = P_s}$$

Pascal's law

Intensity of Pressure means rate of change of Pressure in a certain direction $\left(\frac{dP}{dx}, \frac{\partial P}{\partial z}\right)$

Variation of Pressure





Consider a fluid element of Size $dx\,dz$ and unit length

Let the Static pressure at the Center of the element = P

$$\sum F_x = P_A(dz) - P_C(dz) = 0 \quad \ldots\ldots (1)$$

$$\sum F_z = P_B(dx) - P_D(dx) - dW = 0 \quad \ldots\ldots (2)$$

$$\because P_A = P - \frac{\partial P}{\partial x} \cdot \frac{dx}{2}, \qquad P_C = P + \frac{\partial P}{\partial x} \cdot \frac{dx}{2}$$

$$P_B = P - \frac{\partial P}{\partial z} \cdot \frac{dz}{2} \qquad P_D = P + \frac{\partial P}{\partial z} \cdot \frac{dz}{2}$$

$$dW = \gamma (dz)(dx)$$

From (1)

$$\left(P - \frac{\partial P}{\partial x}\,\frac{dx}{2}\right)(dz) - \left(P + \frac{\partial P}{\partial x}\,\frac{dx}{2}\right)(dz) = 0$$

$$\boxed{\frac{\partial P}{\partial x} = 0}$$

$\therefore$ Pressure does not vary in horizontal direction

From ②

$$\left(P - \frac{\partial P}{\partial z}\cdot\frac{dz}{2}\right)(dx) - \left(P + \frac{\partial P}{\partial z}\cdot\frac{dz}{2}\right)(dx) - \gamma(dz)(dx) = 0 \qquad (9)$$

$$-\frac{\partial P}{\partial z}\,dz - \gamma\,dz = 0$$

$$\boxed{\frac{\partial P}{\partial z} = -\gamma}$$

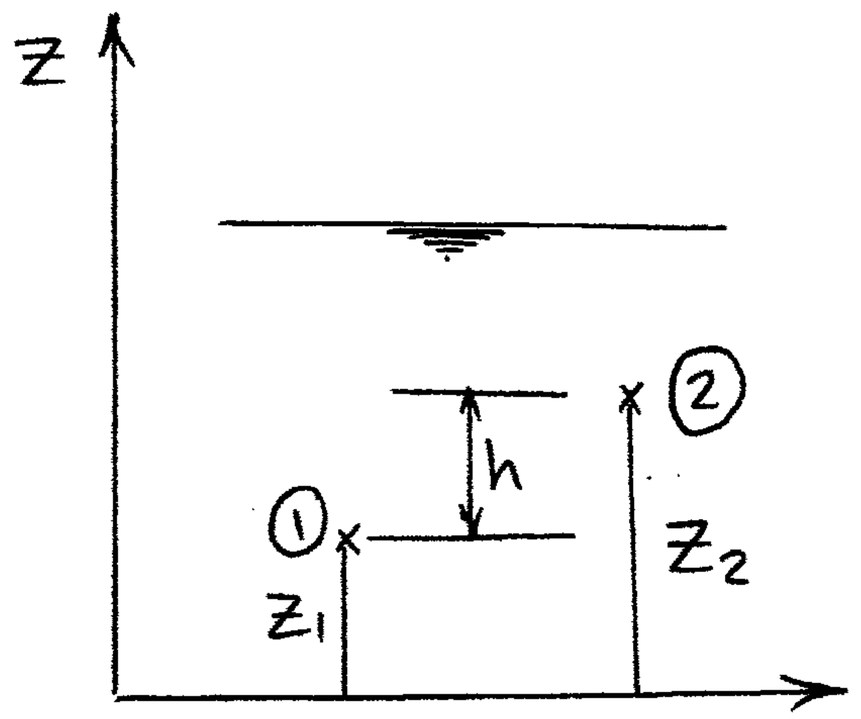
Pressure Varies in Vertical direction

For two points ①, ②

$$\int_{P_1}^{P_2} dP = -\gamma \int_{z_1}^{z_2} dz$$

$$\therefore P_2 - P_1 = -\gamma(z_2 - z_1) = -\gamma h$$

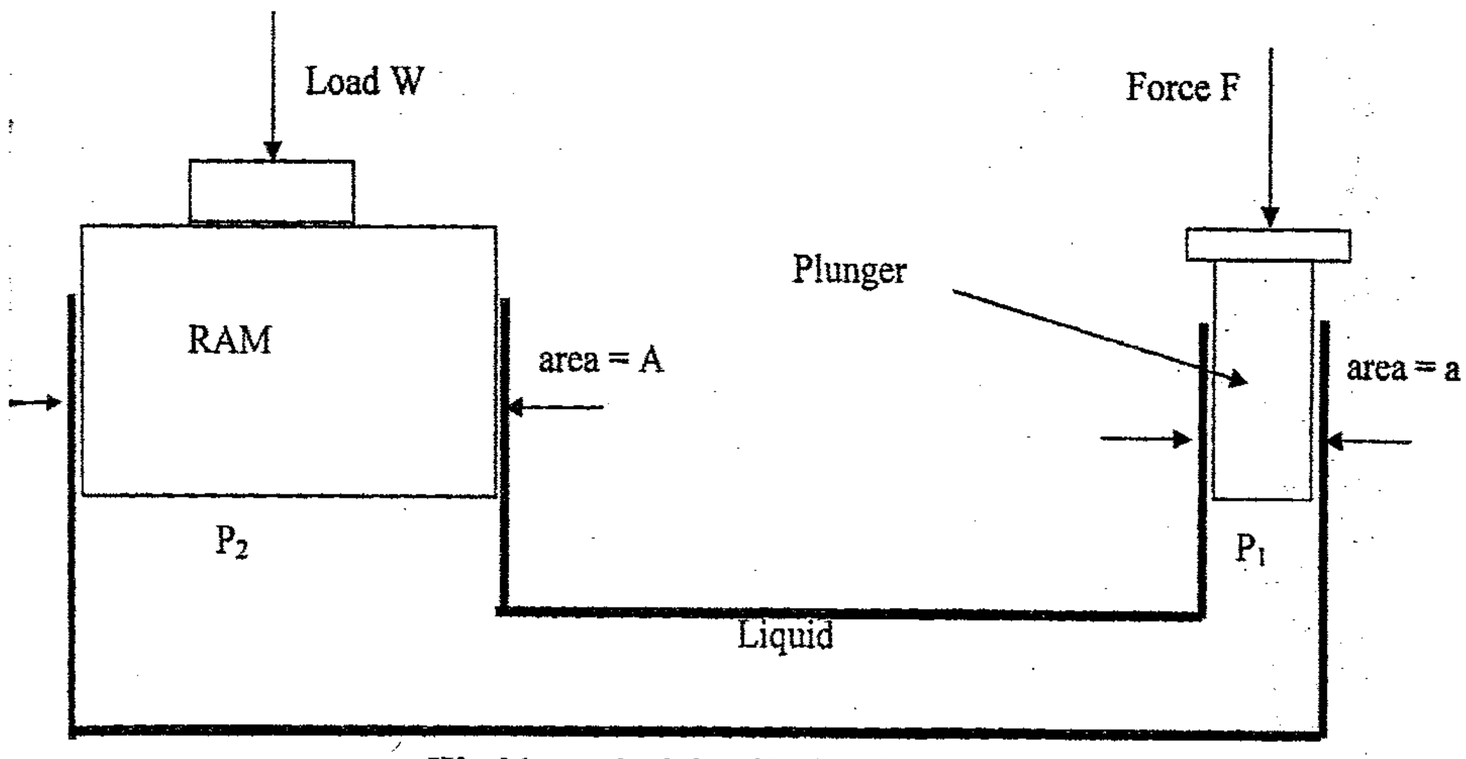
$$\therefore \boxed{P_1 = P_2 + \gamma h}$$



X

Pascal's application

1- hydraulic press
2- hydraulic jack
3- hydraulic lift
4- hydraulic crane



Working principle of hydraulic press

By applying a small force F on the plunger a larger load can be lifted by the ram

1b Convert $P = 1.03\ kg_w/cm^2$

first $P = 1.03 \frac{kg_w}{cm^2} * 9.81 * 10^4 = 101\,043\ N/m^2$

i) meters of water

$P = \gamma_w h$

$* S.g_{Hg} = 13.6$ ليه

$101043 = 9810\ h \Rightarrow h = \underline{10.3\ m}$

iii) cm Hg

$P = \gamma_{Hg}\ h_{Hg}$

$101043 = 13.6 * 9810\ h_{Hg} \Rightarrow h_{Hg} = 0.757\ m = \underline{75.7\ cm}$

1c Convert pressure = 12 m water

i) Pressure in lb/in^2

$h = \frac{12\ m}{0.3048} = 39.37\ ft$

$P = \gamma_w h_w = 62.4 * 39.37 = 2456\ lb/ft^2$

$P = \frac{2456}{144} = \underline{17.06\ lb/in^2}$ $* 1\ lb/in^2 = 144\ lb/ft^2$

ii) cm Hg

$P = \gamma_w h_w = \gamma_{Hg} h_{Hg}$

$h_{Hg} = \frac{\gamma_w h_w}{\gamma_{Hg}} = \frac{\gamma_w (12)}{13.6\ \gamma_w} = 0.882\ m = \underline{88.2\ cm}$

If $P_{atm} = 101\ KN/m^2$

i) $P_{gage} = 1.59 \times 10^6\ N/m^2$ $\qquad$ $P_{abs} = ?$

$$P_{abs} = P_{atm} + P_{gage}$$

$$= 101 * 10^3 + 1.59 * 10^6 = \underline{\underline{1.691 * 10^6\ N/m^2}}$$

ii) $P_{abs} = 0$ $\qquad$ $P_{gage} = ?$

$$P_{abs} = P_{atm} + P_{gage}$$

$$0 = 101 * 10^3 + P_{gage} \Rightarrow P_{gage} = \underline{\underline{-101 * 10^3\ N/m^2}}$$

iV) $P_{atm} = ?$ bars

$$P_{atm} = \frac{101 \times 10^3}{10^5} = \underline{\underline{1.01\ bar}} \qquad * \ 1\ bar = 10^5\ Pa$$

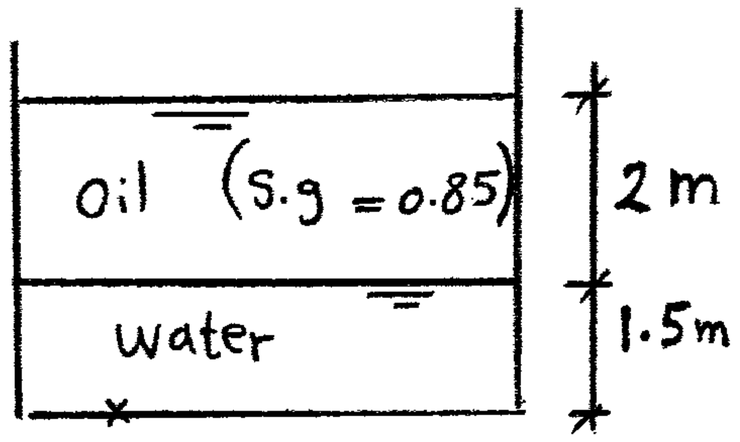
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[2] $P_1 = ?$ (P_a)

$$P_1 = \gamma_{oil} h_{oil} + \gamma_w h_w$$

$$= (0.85 * 9810) * 2 + 9810 * 1.5$$

$$= \underline{\underline{31\ 392\ N/m^2\ (P_a)}}$$



---

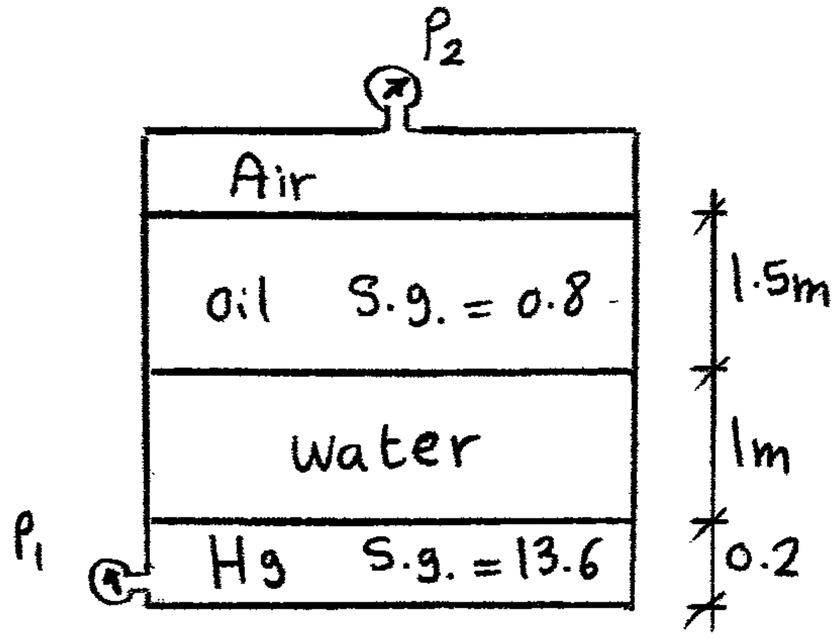
[4] $P_1 = 60\ kPa\ (abs)$, $P_2 = ?$

$$P_1 = P_{air} + \gamma_{oil} h_{oil} + \gamma_w h_w + \gamma_{Hg} h_{Hg}$$

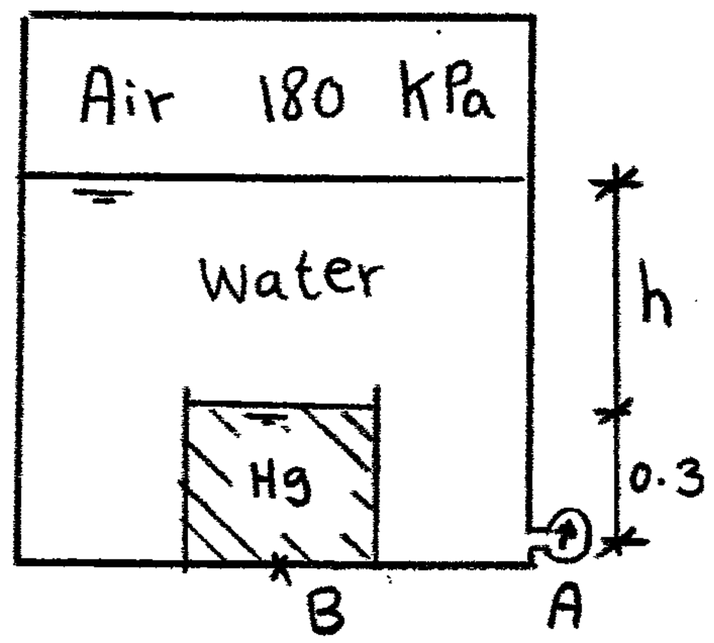
$$P_{air} = 60 * 10^3 - 0.8\ (9810)\ 1.5$$

$$- 9810(1) - 13.6\ (9810)\ 0.2$$

$$= 11\ 735\ Pa = \underline{\underline{11.73\ kPa\ (abs)}}$$



[9] $P_B = 300\ kPa$, $h = ?$, $P_A = ?\ kPa\ abs?$



Solution

$$P_B = P_{air} + \gamma_w h_w + \gamma_{Hg} h_{Hg}$$

$$300*10^3 = 180*10^3 + 9810\,h + 13.6(9810)\,0.3$$

$$\Rightarrow h = \underline{\underline{8.15\ m}} = 815.3\ cm$$

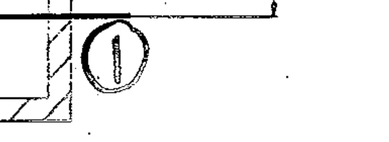
$$P_A = P_{air} + \gamma_w (h + 0.3)$$

$$= 180*10^3 + 9810\,(8.15 + 0.3) = 262\,920\ Pa$$

$$P_{A_{abs}} = 262\,920 + 1013*10^3 = 364\,220\ Pa = \underline{\underline{364.2\ kPa}}$$

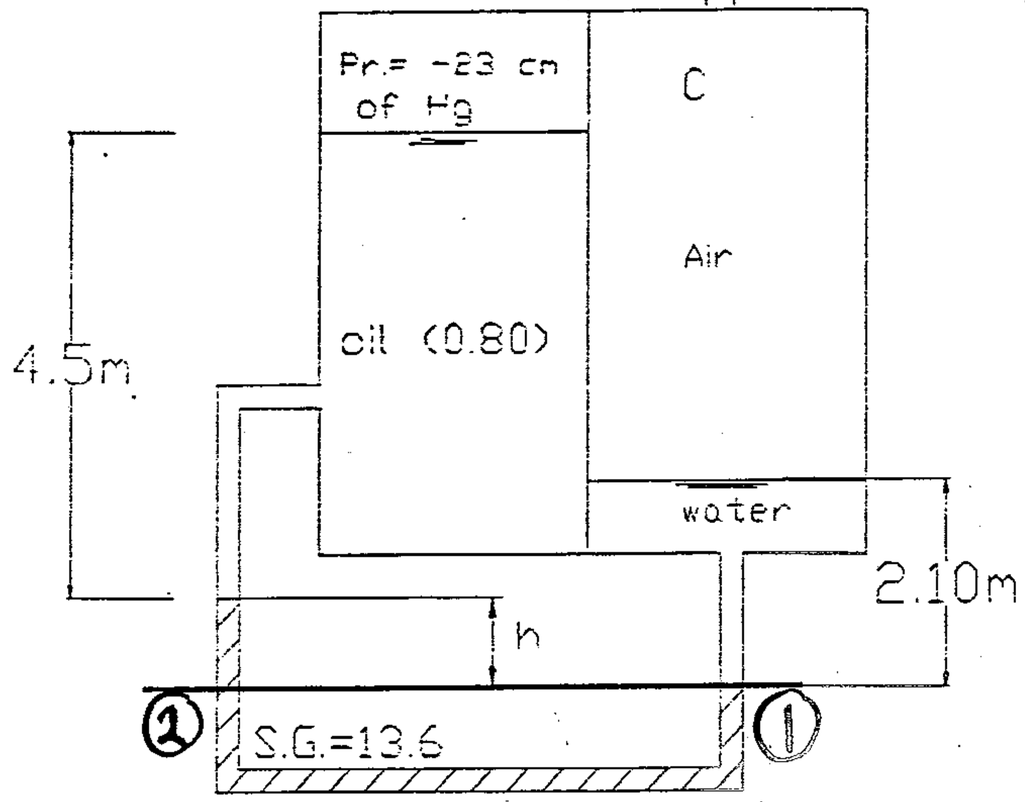
---

[15] $h = ?$



$$P_{gage} = 0.2\,\frac{kg}{cm^2} * 9.81\,\frac{N}{kg} * \frac{(100)^2 cm^2}{1\ m}$$

$$= 19.62\ kPa$$



$$P_{Hg} = -23\ cm\ Hg = S.g\,\gamma_w h$$

$$= 13.6(9810)(-0.23)$$

$$P_1 = P_2$$

$$P_1 = P_{gage} + \gamma_w h_w$$

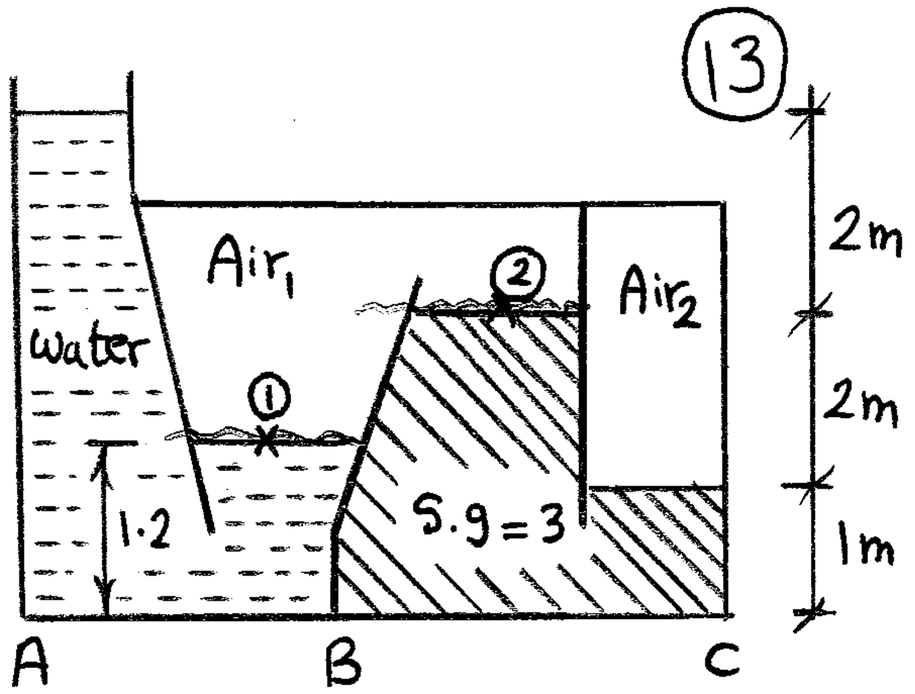
$$= 19.62*10^3 + 9810(2.10) \rightarrow (1)$$

$$P_2 = P_{Hg} + \gamma_{oil} h_{oil} + \gamma_{Hg} h_{Hg}$$

$$= 13.6(9810)(-0.23) + 0.8(9810)\,4.5 + 13.6(9810)\,h \rightarrow (2)$$

from (1), (2) $\Rightarrow$ $h = 0.266$

8 Where is the max pressure



Solution

$P_{AB} = \gamma_w h_w$

$= 9810(5) = \underline{49\,050\ N/m^2}$

$P_1 = P_{AB} - 9810(1.2) = 37\,278\ N/m^2$

$P_2 = P_1 - \gamma_{air}(1.8)$ ($\gamma_{air} \simeq 0$) * $\gamma_{air} = 0$

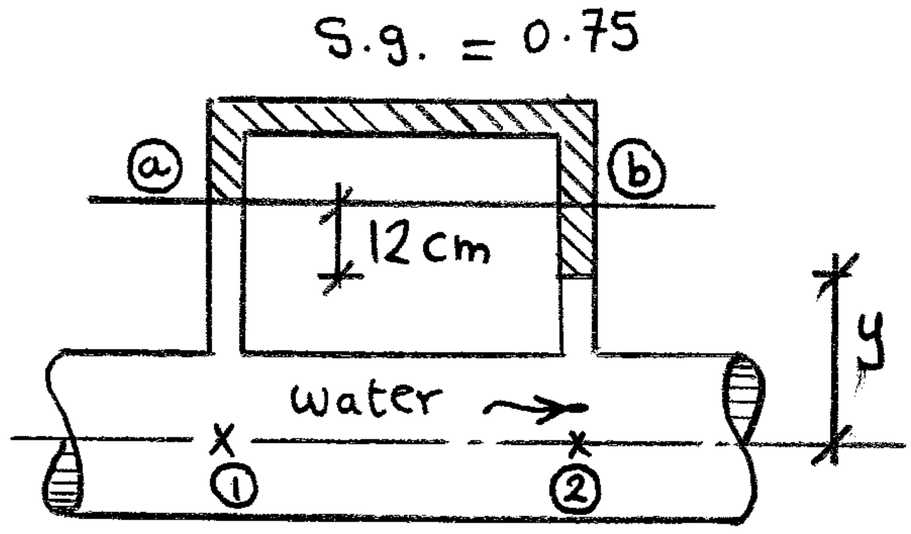
$\therefore P_2 = P_1 = 37\,278$

$P_{BC} = P_2 + (3 * 9810) * 3m = \underline{125\,568\ N/m^2}$

$P_{BC} > P_{AB} \Rightarrow$ max pressure will occur at BC

---

7 $P_1 - P_2 = ?$ bars



$\therefore P_a = P_b$

$P_a = P_1 - \gamma_w y - \gamma_w(0.12) \rightarrow ①$

$P_b = P_2 - \gamma_w y - 0.75\gamma_w(0.12) \rightarrow ②$

from ①, ②

$P_1 - \cancel{\gamma_w y} - \gamma_w(0.12) = P_2 - \cancel{\gamma_w y} - 0.75\gamma_w(0.12)$

$P_1 - P_2 = 9810(0.12) - 0.75(9810)\,0.12 = 294.3\ Pa$

$= \frac{294.3}{10^5} = 2.943 * 10^{-3}$ bars $\simeq 2.94$ mBar

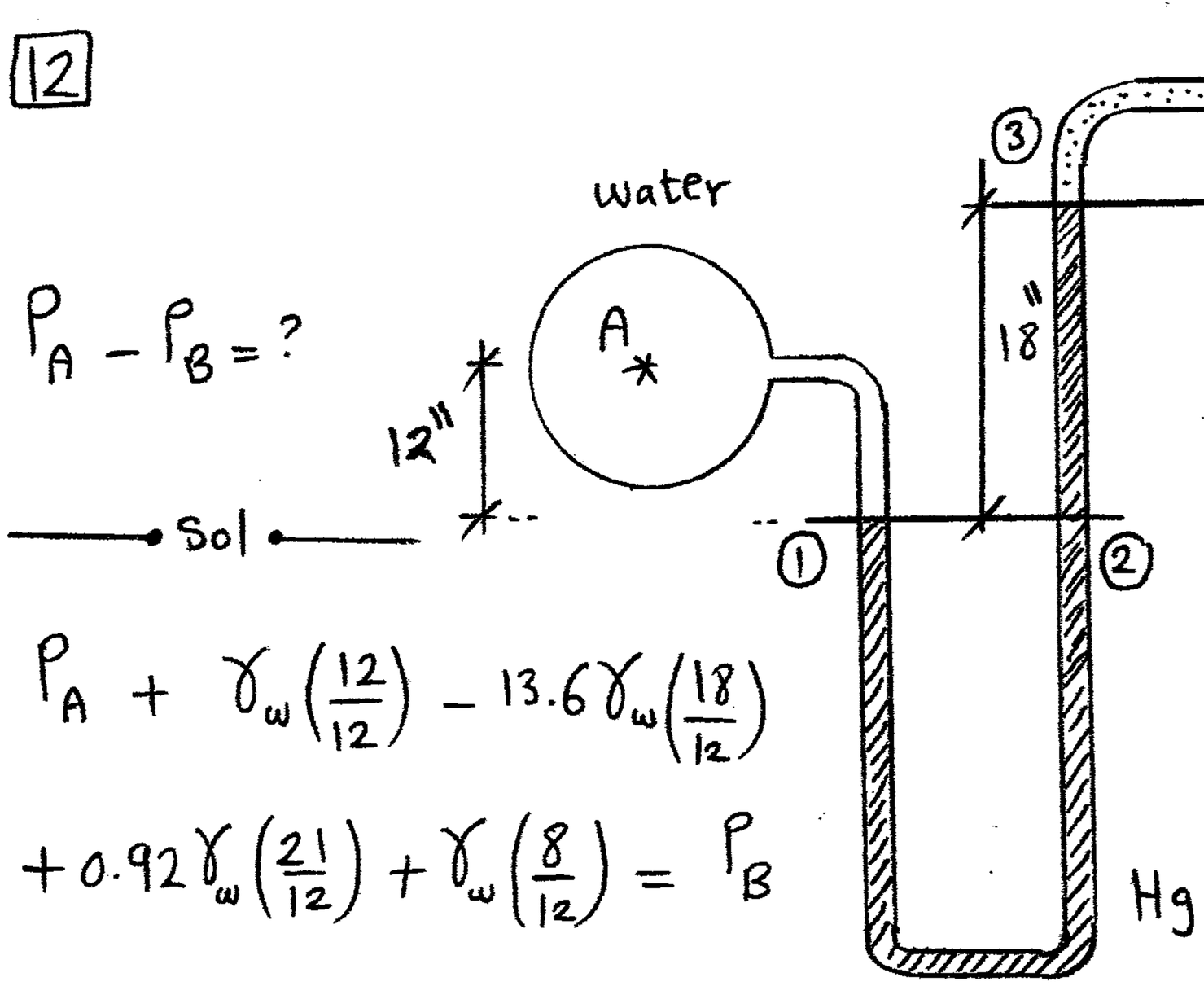
Another solution

$P_1 - \gamma_w y - 0.12\gamma_w + 0.75(0.12\gamma_w) + \gamma_w y = P_2$

$\Rightarrow P_1 - P_2 = \checkmark$ (+ve ↓, ↑ −ve)

12

$P_A - P_B = ?$



$$P_A + \gamma_w\left(\frac{12}{12}\right) - 13.6\gamma_w\left(\frac{18}{12}\right) + 0.92\gamma_w\left(\frac{21}{12}\right) + \gamma_w\left(\frac{8}{12}\right) = P_B$$

$$P_A - P_B = 17.12\,\gamma_w = 17.12\,(62.4) = \underline{1068.5\ lb/ft^2}$$

If oil was exchanged by air

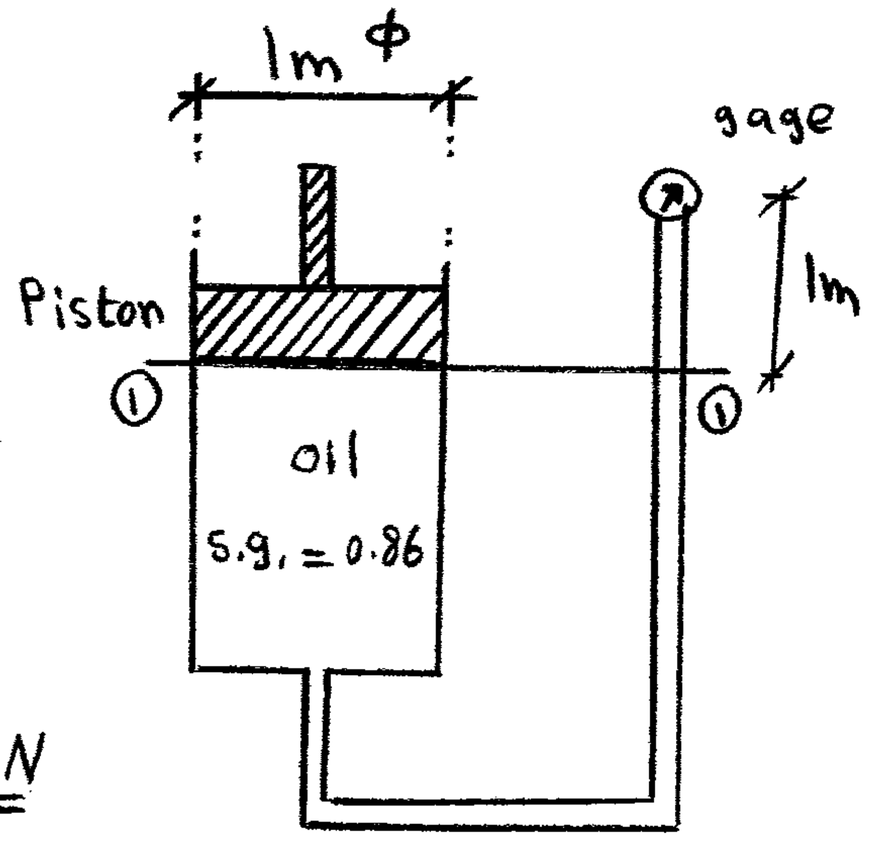
$$P_A + \gamma_w\left(\frac{12}{12}\right) - 13.6\gamma_w\left(\frac{18}{12}\right) + \underbrace{\gamma_{air}}_{\simeq 0}\left(\frac{21}{12}\right) + \gamma_w\left(\frac{8}{12}\right) = P_B$$

$$\therefore\ P_A - P_B = \underline{1169\ lb/ft^2}$$

For the configuration shown, **Calculate** the weight of the piston if the gage pressure is 70 KPa .

15

$P_{gage} = 70\ KPa \qquad W_{piston} = ?$



Sol

$$P_1 = 70 * 10^3 + 0.86(9810)\ 1$$

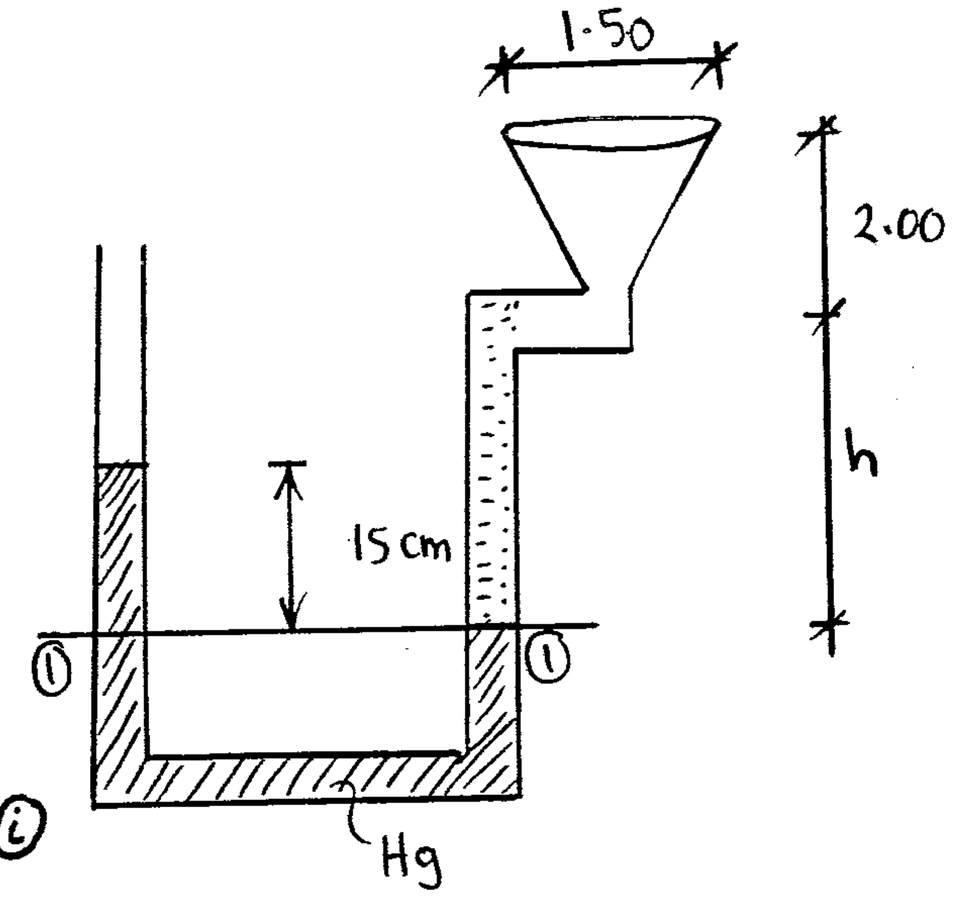
$$= 78436\ N/m^2$$

$$W = P.A = 78436\ \frac{\pi (1)^2}{4} = \underline{61603\ N}$$

16

A conical vessel is connected to a U tube manometer as shown in the figure. The reading of the manometer given in the figure shows when the vessel is empty. **Find** the reading of the manometer when the vessel is completely filled with water.

15



a- When the vessel is empty

$P_1 = \gamma_{Hg} h$

$= 13.6 * 9810 * 0.15 = 20012$ Pa → (i)

$P_1 = \gamma_w h$

$= 9810 h$ → (ii)

from (i), (ii)

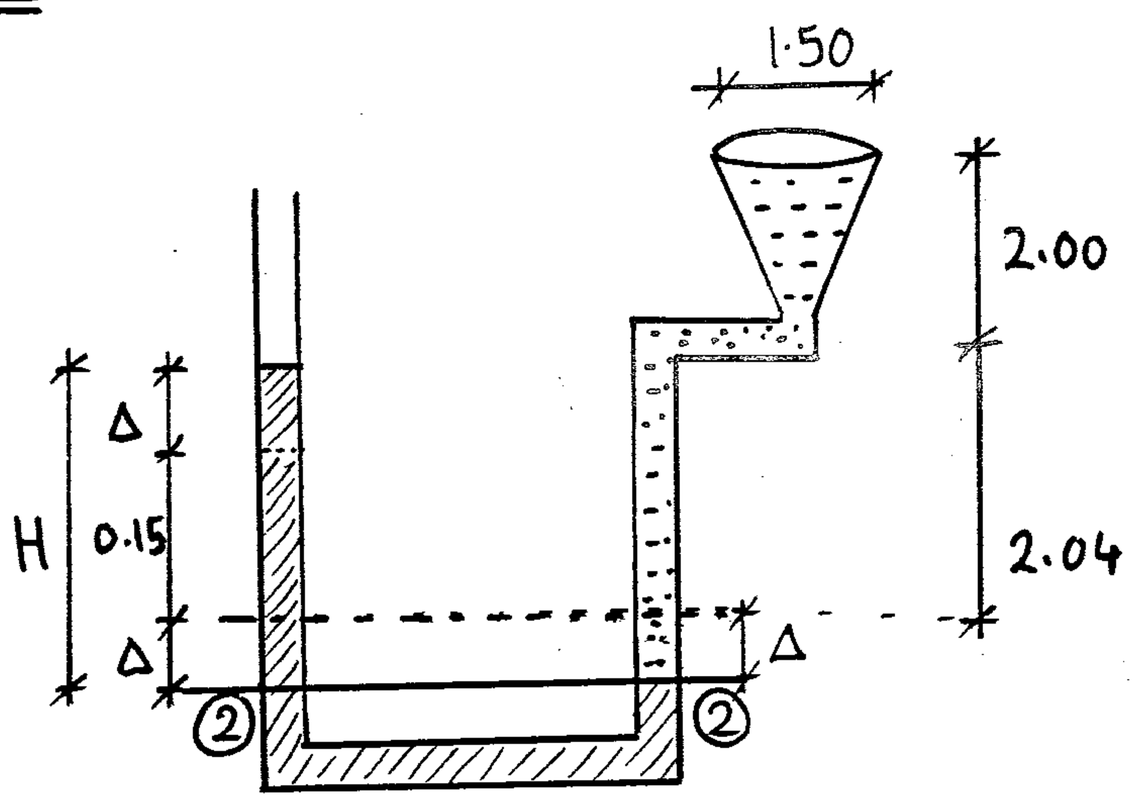
$9810 h = 20012 \Rightarrow h = \underline{2.04 \text{ m}}$

b- When the vessel is full

$P_2 = \gamma H$

$= \gamma_{H.g} (2\Delta + 0.15)$ → (i)

$P_2 = \gamma_w (2 + 2.04 + \Delta)$ → (ii)



from (i), (ii)

$13.6 \gamma_w (2\Delta + 0.15) = \gamma_w (4.04 + \Delta)$

$26.2 \Delta = 2$

$\Delta = 0.076$ m

Deflection = $H = 2\Delta + 0.15 = 0.30$ m = $\underline{30 \text{ cm}}$

14 $A = 50\ a$

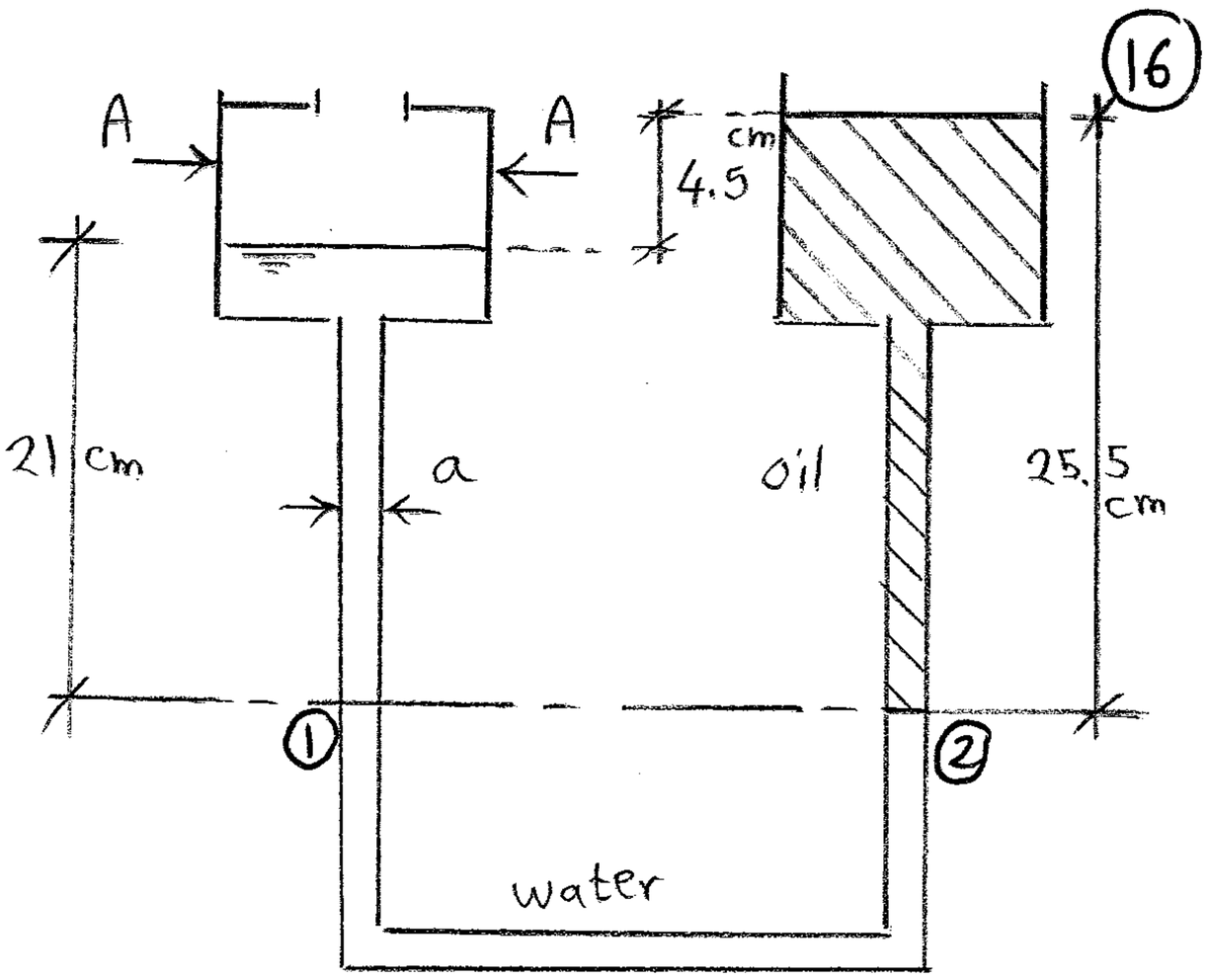
Find P for a diffrence 1"

a- Before increasing pressure

$$P_1 = P_2$$

$$\gamma_w (21) = S.g.\ \gamma_w (25.5)$$

$$S.g. = 0.823$$



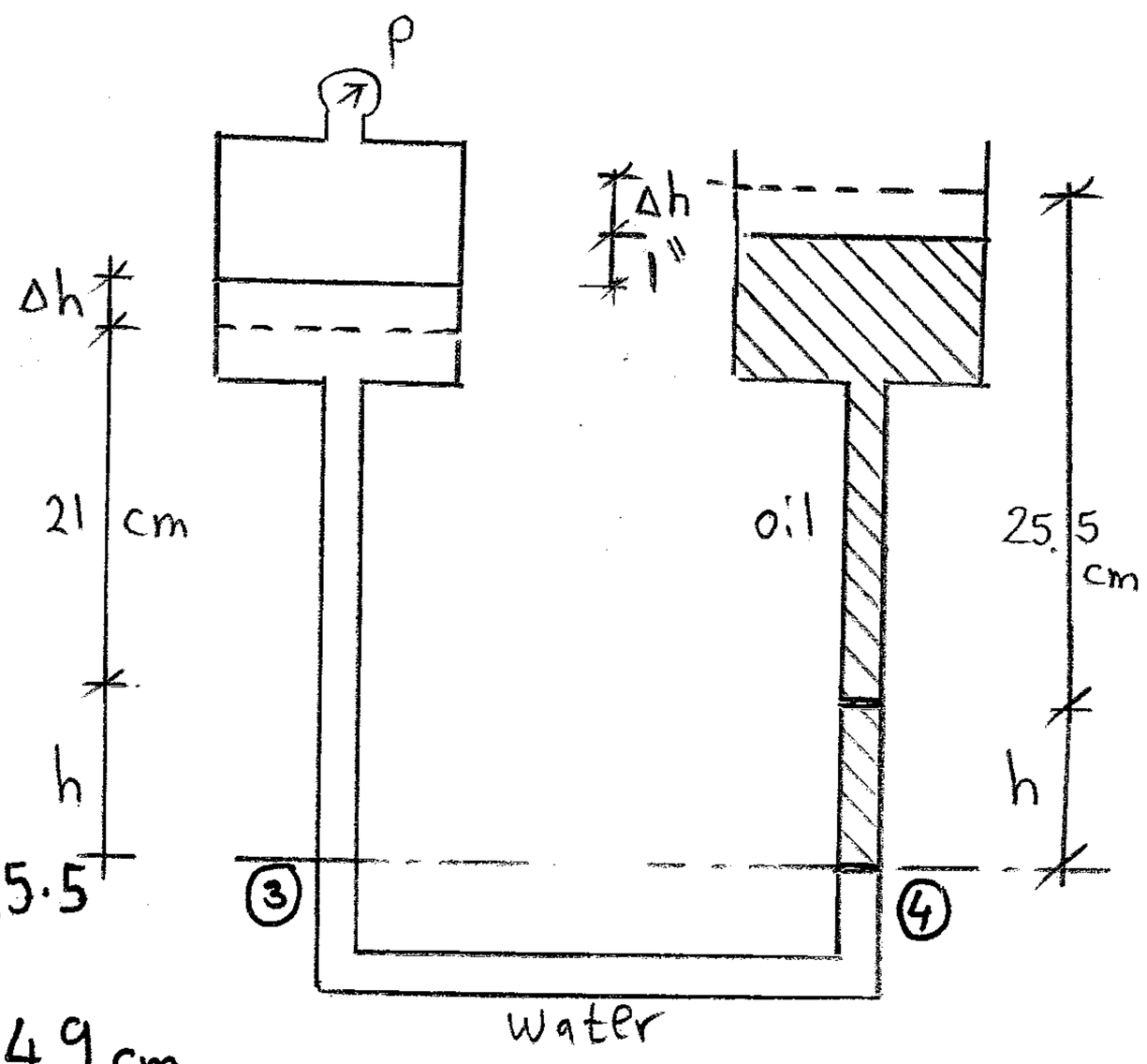
b- After increasing pressure

$$Volume_{①} = Volume_{②}$$

$$A(\Delta h) = a\,h$$

$$50\,a\,(\Delta h) = a\,h$$

$$\boxed{h = 50\ \Delta h}$$



$\therefore h + 21 + \Delta h + 2.54 + \Delta h = h + 25.5$

$\therefore \Delta h = 0.98\ cm \Rightarrow h = 49\ cm$

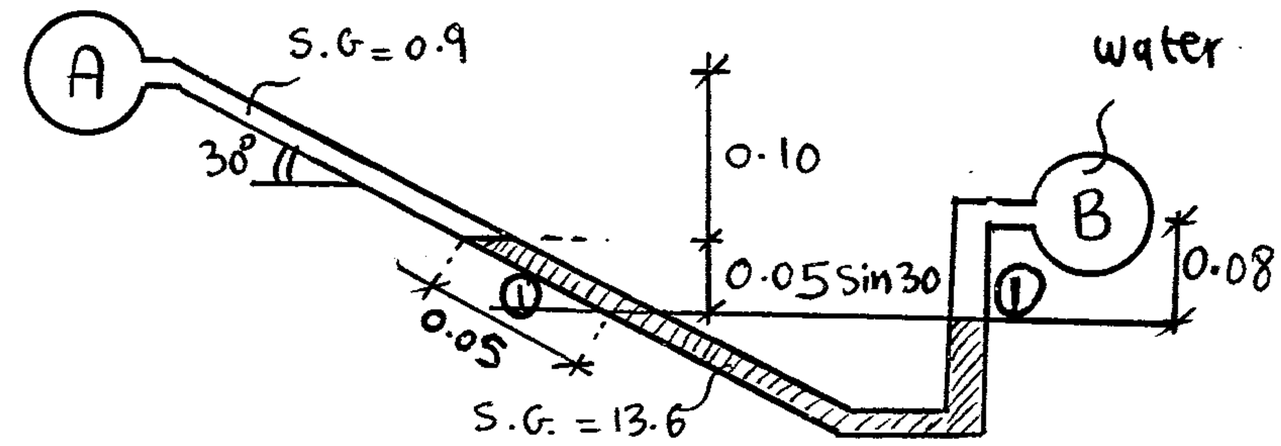
$$P_3 = P_4$$

$$P + \gamma_w (h + 21 + \Delta h) = 0.823\ \gamma_w (h + 25.5)$$

$$P + 981\,(49 + 21 + 0.98) = 0.823\,(981)(49 + 25.5)$$

$$P = -9482.8\ \frac{dyne}{cm^2} = -9.5\ Kdyne/cm^2$$

Determine the differential reading along the inclined leg of the mercury manometer, if the pressure in pipe A is decreased 12 kPa and the pressure in pipe B remains unchanged. The fluid in A has a specific gravity of 0.90 and the fluid in B is water. (17)

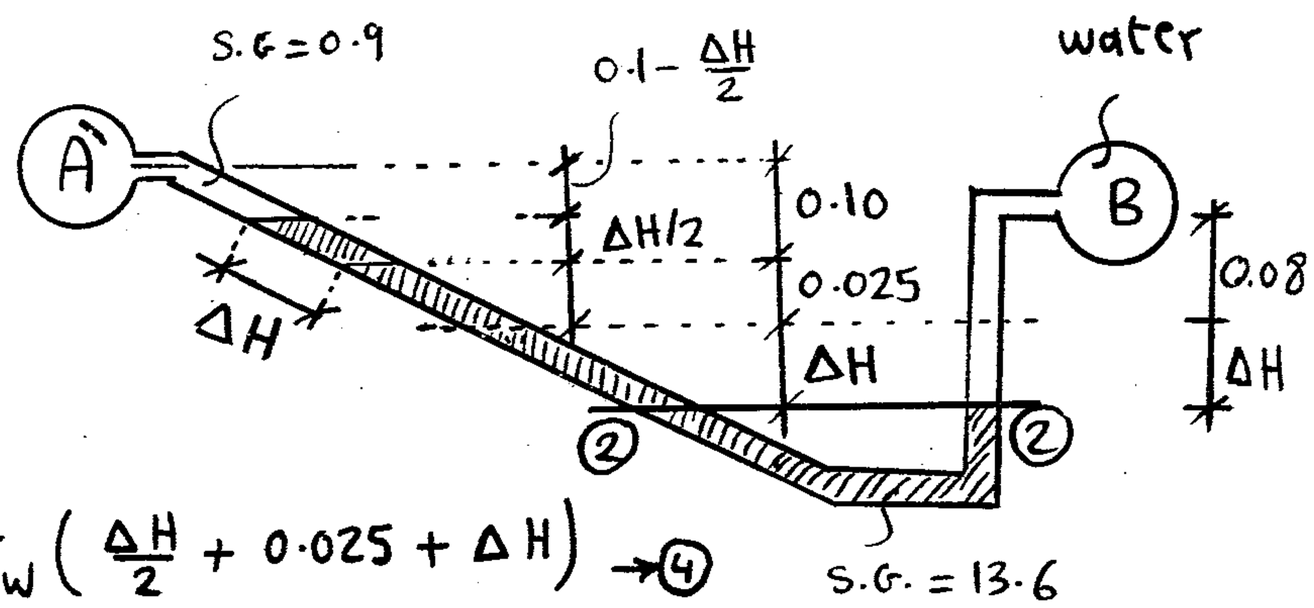


(a)

$$P_1 = P_A + 0.10(0.9)\gamma_w + 0.05\sin 30\,(13.6)\gamma_w$$

$$P_1 = P_B + 0.08\,\gamma_w$$

$$P_A + 0.35\,\gamma_w = P_B \rightarrow (1)$$



(b)

$$P_2 = P_B + (0.08 + \Delta H)\gamma_w \rightarrow (2)$$

$$P_A' = P_A - 12 \times 10^3 \rightarrow (3)$$

$$P_2 = P_A' + \left(0.1 - \frac{\Delta H}{2}\right) 0.9\,\gamma_w + 13.6\,\gamma_w \left(\frac{\Delta H}{2} + 0.025 + \Delta H\right) \rightarrow (4)$$

From (1), (2), (3), (4)

$$P_2 = \cancel{P_A} - 12\times10^3 + \left(0.1 - \frac{\Delta H}{2}\right)0.9\gamma_w + 13.6\gamma_w\left(\frac{\Delta H}{2} + 0.025 + \Delta H\right) = \cancel{P_A} + 0.35\gamma_w + (0.08 + \Delta H)\gamma_w$$

$$\gamma_w\left(13.6\left(3\frac{\Delta H}{2}\right) - 0.9\,\frac{\Delta H}{2} - \Delta H\right) = 12\times10^3 + \gamma_w\left(0.35 + 0.08 - 13.6(0.025) - 0.9(0.1)\right)$$

$$\gamma_w\,(18.95\,\Delta H) = 12\times10^3$$

$$\Delta H = \frac{12\times10^3}{9810 * 18.95} = 0.0645\ m$$

Total VI deflection $= \Delta H + 0.025 + \frac{\Delta H}{2} = 0.1218\ m$

$= \underline{\underline{121.8\ mm}}$

Rest of sheet 2

[3] $P = 50$ kPa

$h = ?$ m waters , m alchol $(S.g = 0.75)$

m mercury $(S.g = 13.6)$

Sol//

$$P = S.g.\ \gamma_w\ h$$

$$h = \frac{P}{S.g\ \gamma_w}$$

$$h_w = \frac{50 * 10^3}{1 * 9810} = \underline{5.1 \text{ m}}$$

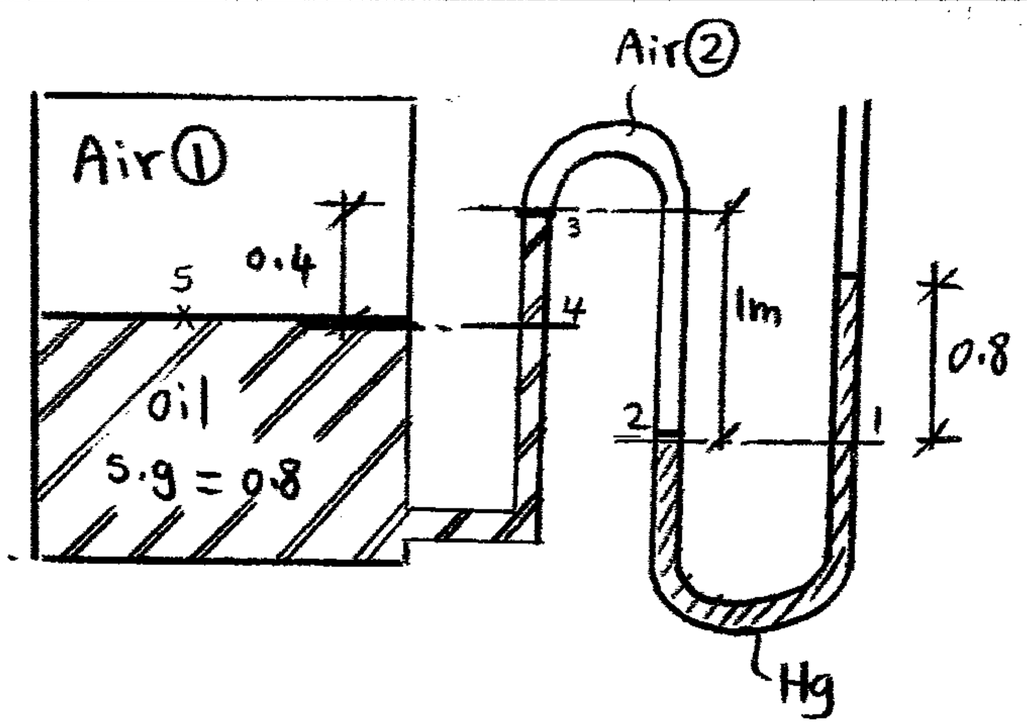
$$h_{alchol} = \frac{50 * 10^3}{0.75 * 9810} = \underline{6.8 \text{ m}}$$

$$h_{mercury} = \frac{50 * 10^3}{13.6 * 9810} = \underline{0.37 \text{ m}}$$

---

[13] $P_{Air\ ①} = ?$

Sol//



$$P_1 = \gamma_{Hg}\ (0.8)$$

$$P_2 = P_1$$

$$P_3 = P_2$$

$$P_4 = P_3 + \gamma_{oil}\ (0.4)$$

$$P_{air\ ①} = P_5 = P_4$$

+ve ↓ ↑ -ve

Another solution

$$0 + \gamma_{Hg}\ (0.8) + \gamma_{oil}\ (0.4) = P_{air\ ①}$$

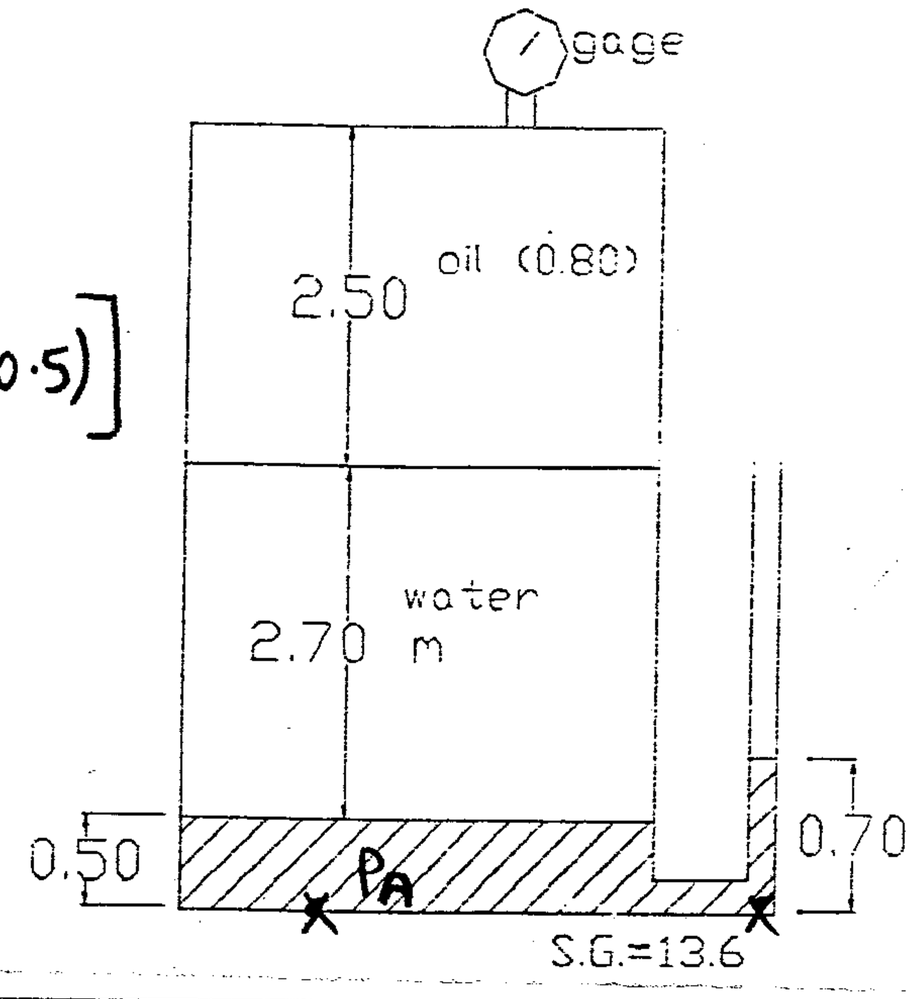
$$\therefore P_{air\ ①} = 13.6\,(9810)\ 0.8 + 0.8\,(9810)\ 0.4 = \underline{109872 \text{ Pa}}$$

10 $P_g = ?$

$P_A = 13.6\,(0.7)(9810)$

$= P_g + 9810\left[0.8(2.50) + 1(2.7) + 13.6(0.5)\right]$

$P_g = -\underline{19423.8}$ Pa

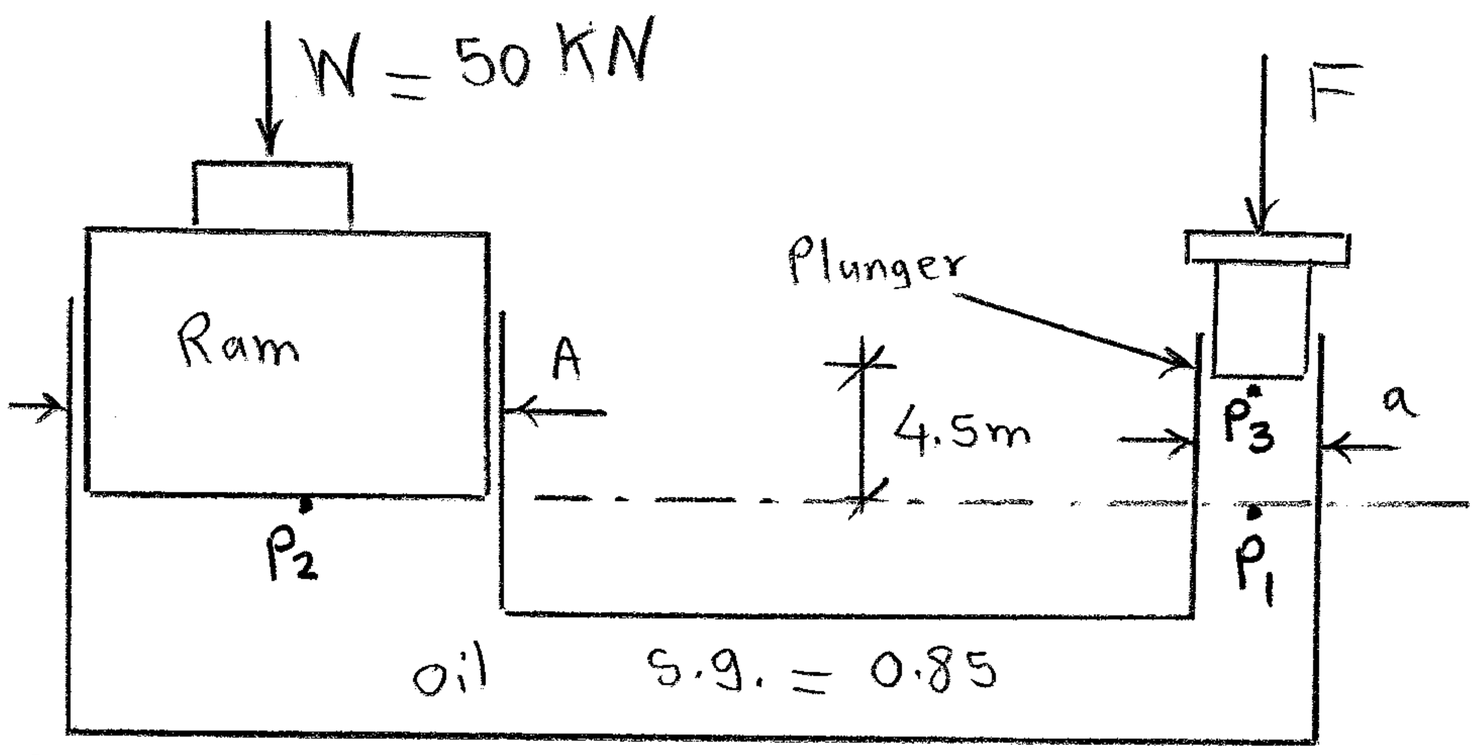


Sheet 1 no. 13

$a = 0.004\ m^2$

$A = 0.4\ m^2$

$F = ?$



Solution

$$P_2 = \frac{W}{A} = \frac{50 * 10^3}{0.4} = 125\,000 \text{ Pa}$$

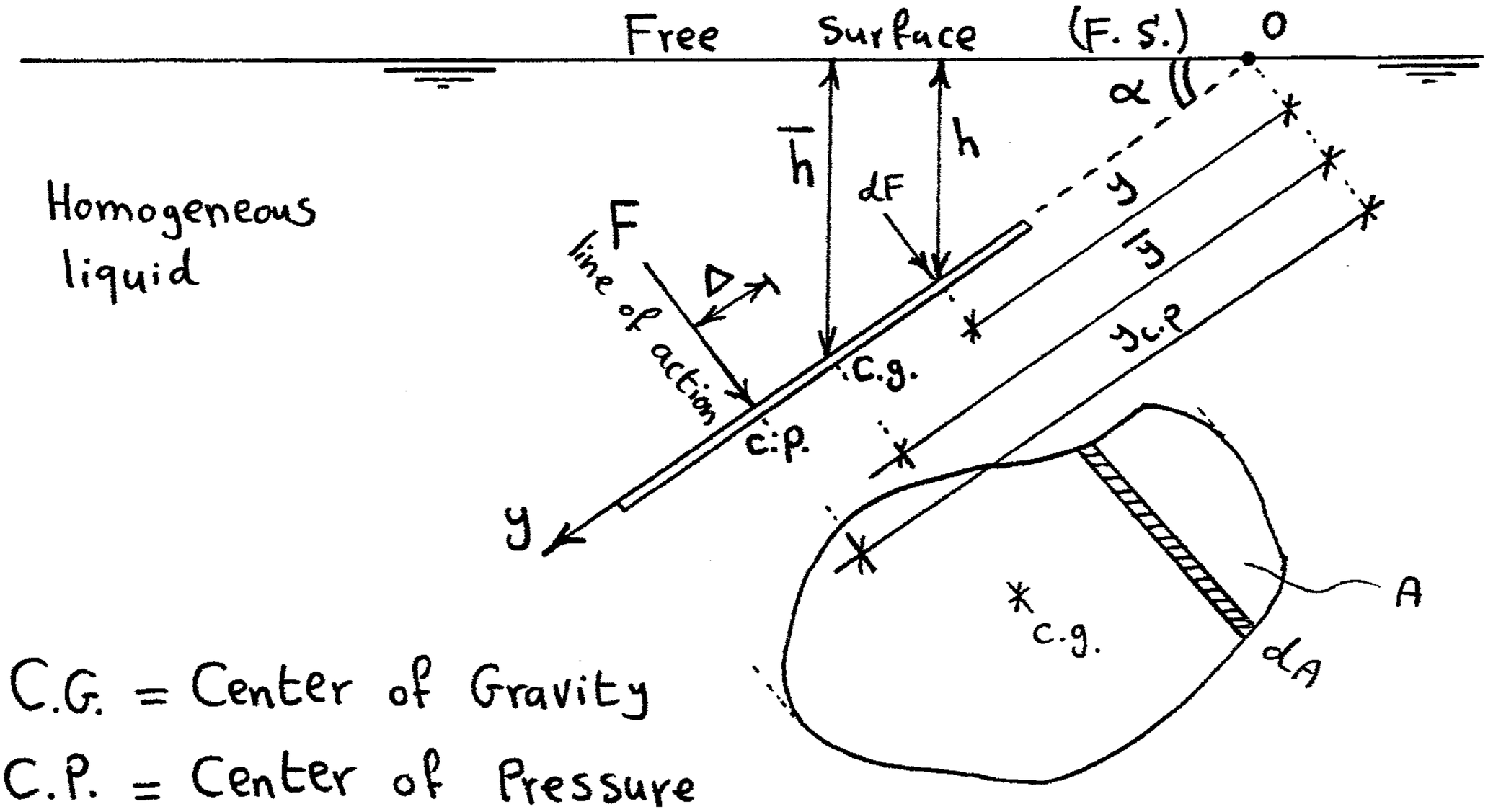
$$P_1 = P_3 + \gamma_{oil}\, h_{oil} \qquad \because P_1 = P_2$$

$$P_2 = \frac{F}{a} + S.g\,\gamma_w\, h_{oil}$$

$$125000 = \frac{F}{0.004} + 0.85\,(9810)(4.5)$$

$$\Rightarrow F = \underline{650\ N}$$

4 Forces on Plane Surfaces



Homogeneous liquid

C.G. = Center of Gravity

C.P. = Center of Pressure

A = Area of immersed surface ($\perp$ to the page)

$\bar{h}$ = Vl distance from C.G. to the free Surface (F.S.)

$\bar{y}$ = inclined distance from C.G. to the free Surface (F.S.)

α = زاوية ميل السطح على الأفقى

To determine the resultant hydrostatic force

$$dF = P.dA = \gamma h\, dA \qquad \sin\alpha = \frac{h}{y}$$

$$= \gamma y \sin\alpha\, dA \qquad \Rightarrow h = y\sin\alpha$$

$$\therefore \int dF = \gamma \sin\alpha \int_{o}^{A} y\, dA$$

$$\because \int y\, dA = A\bar{y} \quad \text{((First moment of Area about point o))}$$

$$\therefore F = \gamma \sin\alpha\, A\bar{y}$$

$$\boxed{F = \gamma A \bar{h}} \qquad \boxed{\bar{y} = \frac{\bar{h}}{\sin\alpha}}$$

To determine the line of action

The moment dM due to the force about O is

$$dM = dF\,y$$

$$= (\gamma\, y \sin\alpha\, dA)\, y$$

$$\int dM = \gamma \sin\alpha \int_0^A y^2\, dA$$

$$\because \int dM = F.\, y_{c.p}$$

$$\because \int_0^A y^2\, dA = I_o \qquad \text{((Second moment of Area about point o))}$$

$$\therefore F\, y_{c.p} = \gamma \sin\alpha\, I_o$$

$$\gamma\, \bar{y} \sin\alpha\, A\, y_{c.p} = \gamma \sin\alpha\, I_o$$

$$y_{c.p} = \frac{I_o}{A\bar{y}}$$

$$\because I_o = I_{c.g} + A\bar{y}^2$$

$$y_{c.p} = \frac{I_{c.g} + A\bar{y}^2}{A\bar{y}}$$

$$y_{c.p} = \frac{I_{c.g}}{A\bar{y}} + \bar{y}$$

Δ

$$\Delta = \frac{I_{c.g}}{A\bar{y}}$$

⇐ مكان تأثير الـResultant (F)
دائماً أسفل الـ c.g بمسافه Δ

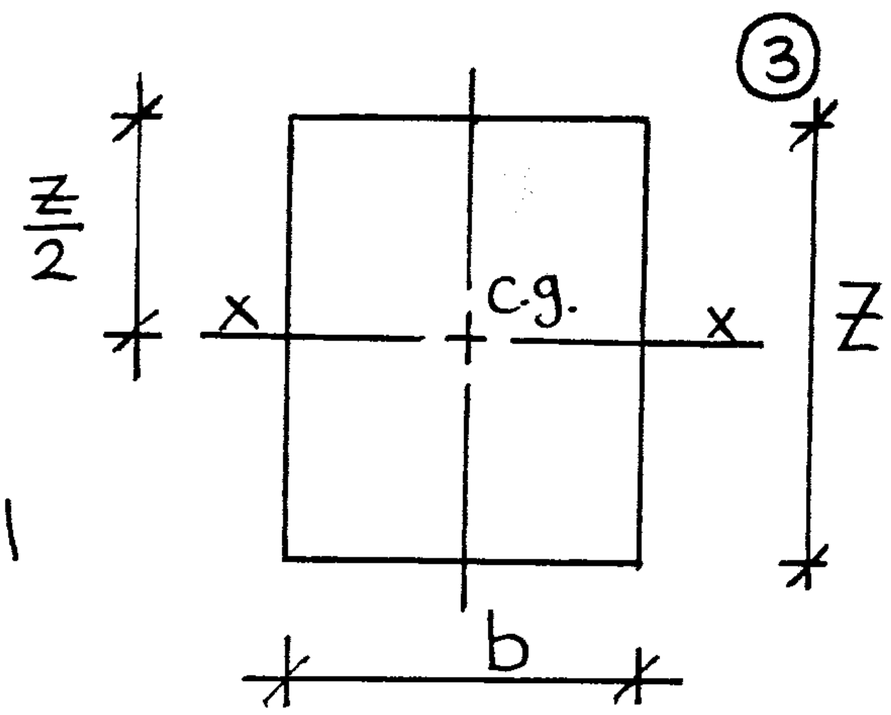
Properties of Area

1- Rectangle

$$A = bz$$

$$I_{xx} = \frac{bz^3}{12}$$

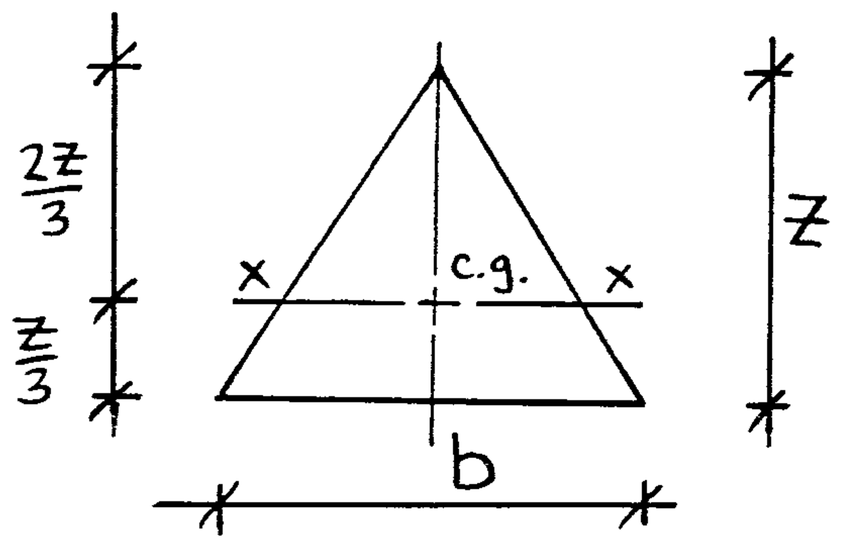
المختفى * الظاهر^3 / ١٢



2- Triangle

$$A = \frac{1}{2} bz$$

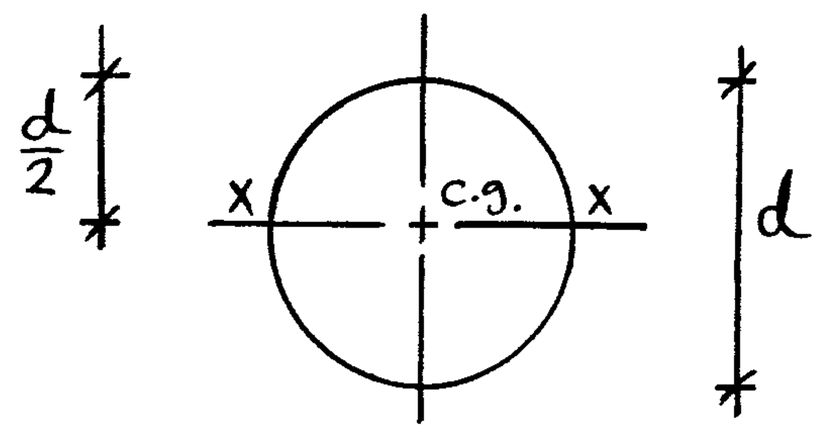
$$I_{xx} = \frac{bz^3}{36}$$



3- Circle

$$A = \frac{\pi d^2}{4}$$

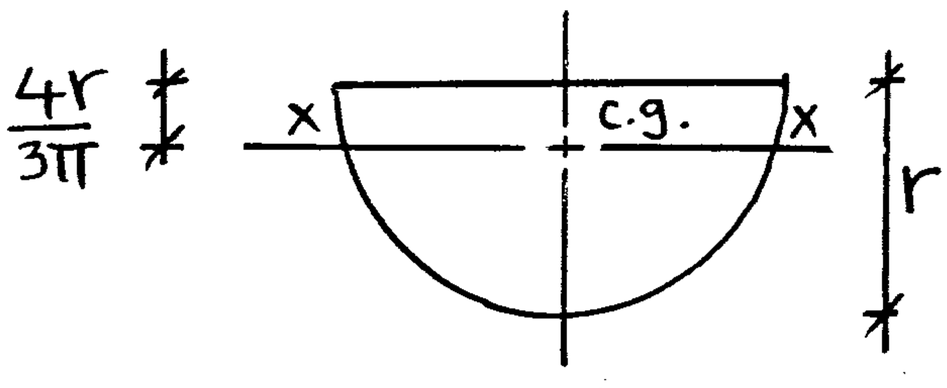
$$I_{xx} = \frac{\pi d^4}{64}$$



4- Semicircle

$$A = \frac{\pi d^2}{8}$$

$$I_{xx} = \frac{\pi d^4}{128} - \frac{d^4}{18\pi} = 0.11 r^4$$



e.g

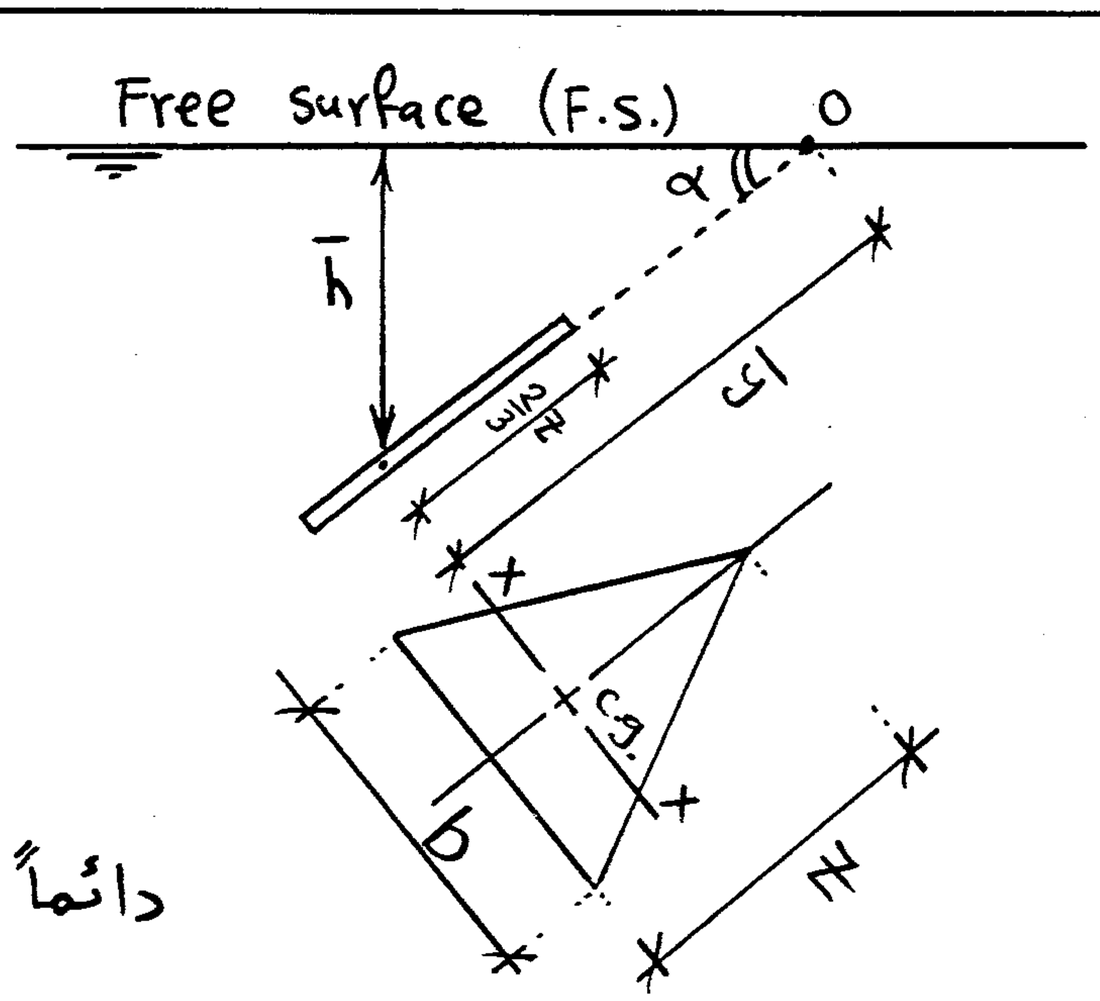
Calculate

① $A = \frac{1}{2} bz$

② $I_{xx} = \frac{bz^3}{36}$

③ $\bar{h}$ = ✓

④ $\bar{y} = \frac{\bar{h}}{\sin\alpha}$



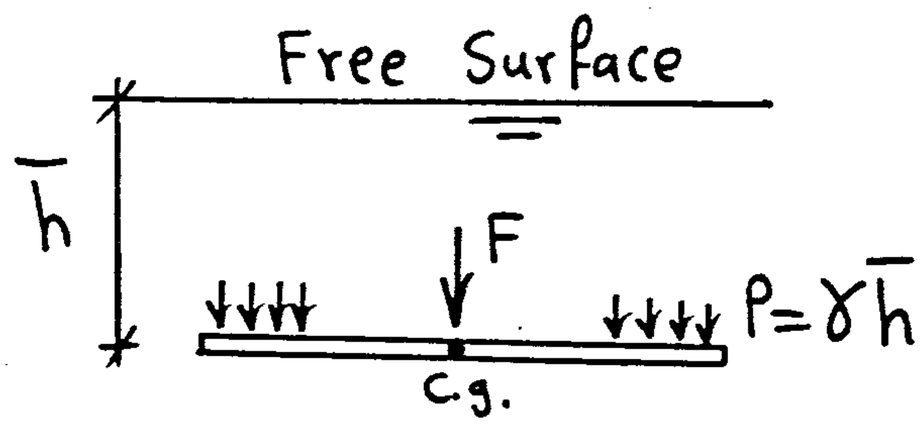
دائماً التكعيب هو البعد المرئ (z)

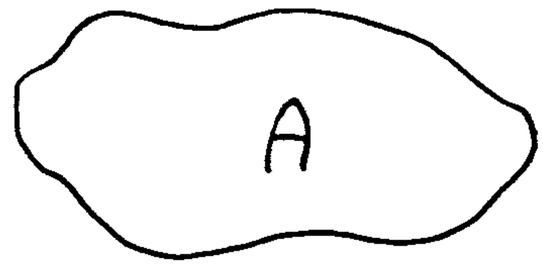
Special Cases

1- $\alpha = 0$

$$\bar{y} = h/\sin\alpha = \infty$$

$$\Delta = \frac{I_{c.g}}{A\bar{y}} = \frac{I_{c.g}}{\infty} = 0$$





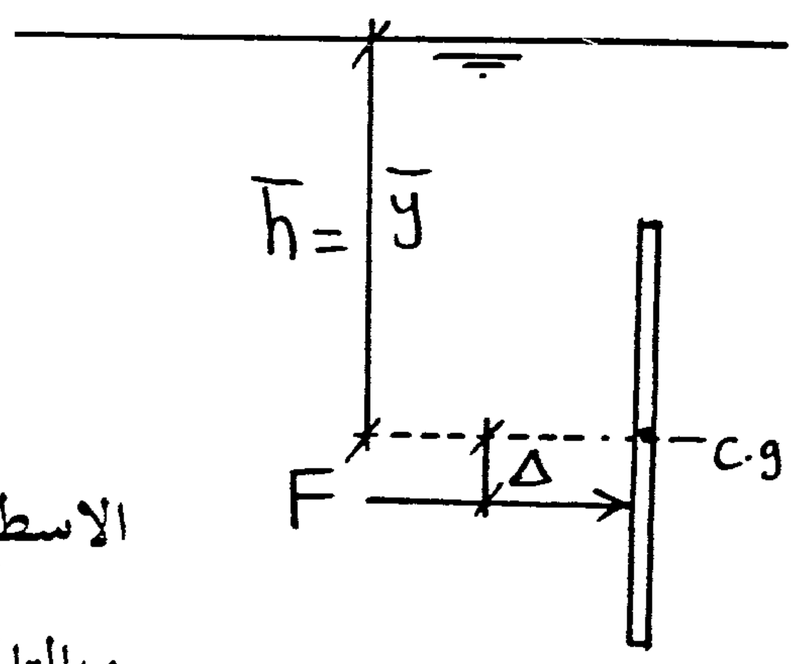
الاسطح المستوية الأفقية يكون عليها ضغط منتظم

وبالتالى تؤثر القوة F فى ال C.g. لهذا السطح

2- $\alpha = 90^\circ$

$$\bar{y} = \bar{h}$$

$$\Delta = \frac{I_{c.g}}{A\bar{h}}$$



الاسطح الرأسية أو المائلة يكون عليها ضغط غير منتظم

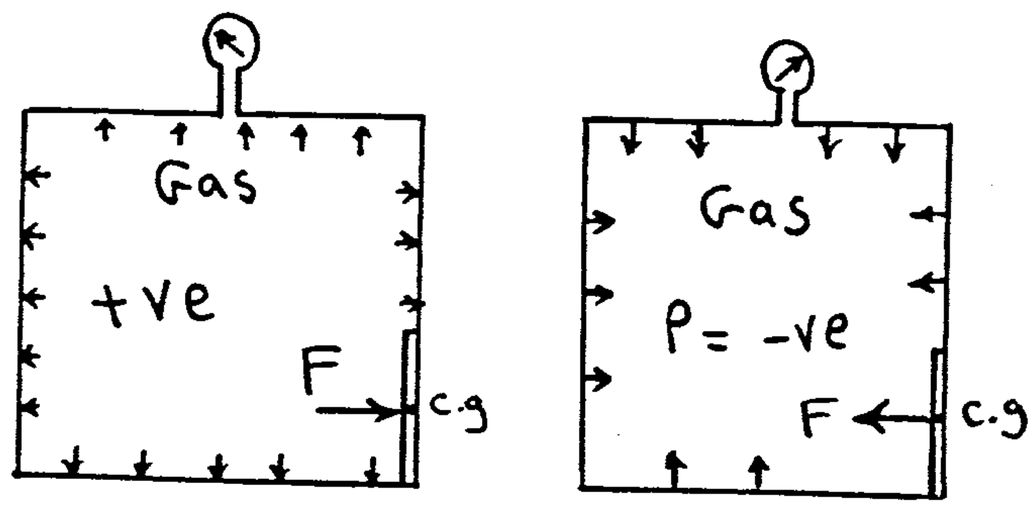
وبالتالى تؤثر القوة F أسفل ال C.g. بمسافة Δ

3- Gas

ضغط الغاز موزع بانتظام على جدران الخزان

وبالتالى تؤثر القوة F فى ال C.g. للبوابة

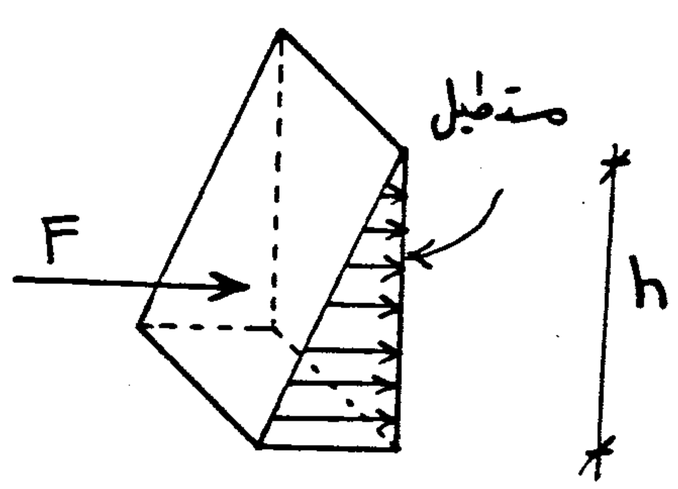
$$F = P.A_{gate}$$



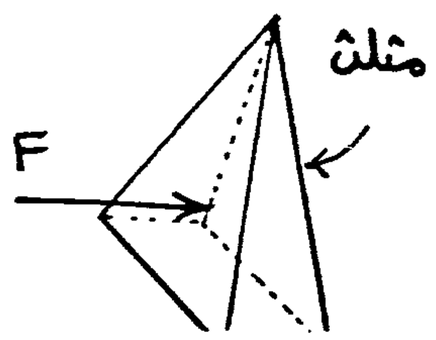
Pressure prism منشور الضغط

تؤثر القوة F فى مركز ثقل المنشور أى فى ثلث الارتفاع من القاعدة

أو (أسفل مركز الثقل فى حالة المستطيلات بمسافة $\Delta = \frac{h}{6}$)



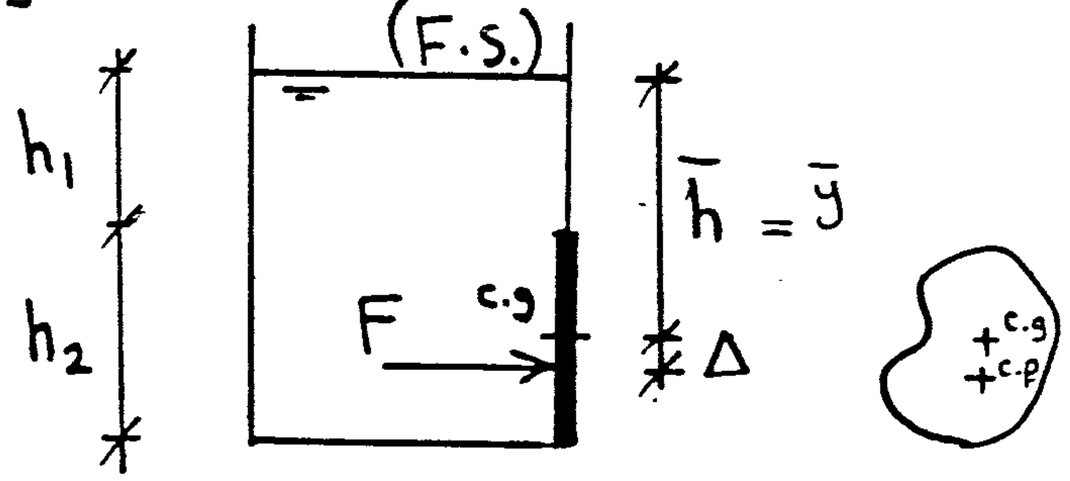
تؤثر القوة F فى مركز ثقل الهرم أو أسفل مركز ثقل المثلث بمسافة Δ



1- The Equation (for any surface)

$$F = \gamma A \bar{h}$$

$$\Delta = \frac{I_{c.g}}{A \bar{y}}$$



Closed tank with 2 fluids

∵ البوابة كلها مغمورة فى السائل فإن أى ضغط فوق السائل γ_2 سوف يؤثر بانتظام على البوابة

أى يؤثر بقوة فى ال C.g. للبوابة

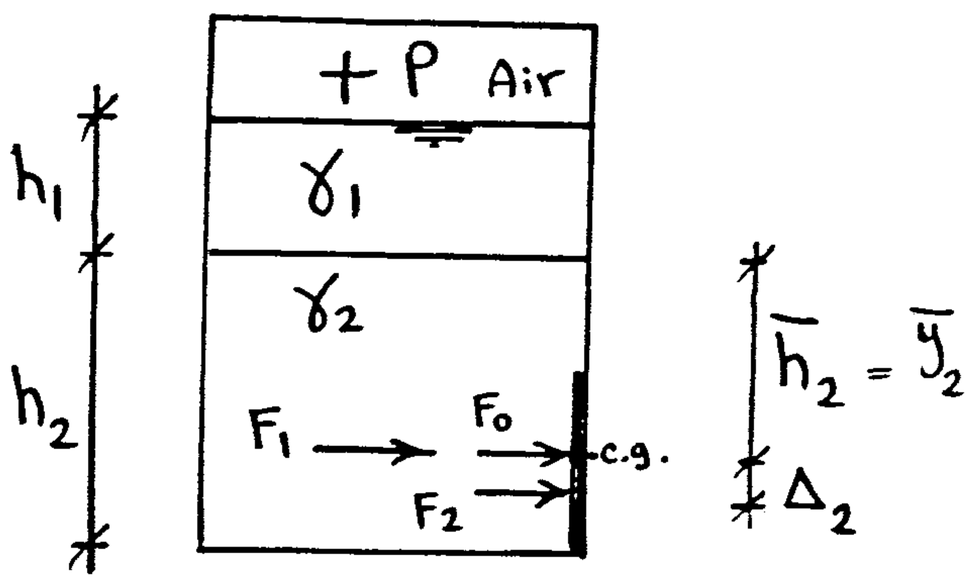
$$F_o = P.A$$

$$F_1 = \gamma_1 h_1 A$$

$$F_2 = \gamma_2 A \bar{h_2}$$

$$F_{total} = F_o + F_1 + F_2$$

$$\Delta_2 = \frac{I_{c.g}}{A \bar{y_2}}$$



لإيجاد مكان المحصلة F_{total} نأخذ العزم حول إحدى القوى

عزم القوى حول نقطة = عزم المحصلة حول نفس النقطة

$$F_2 \Delta_2 = F_{total} Z \quad \Rightarrow \quad \text{get } Z$$

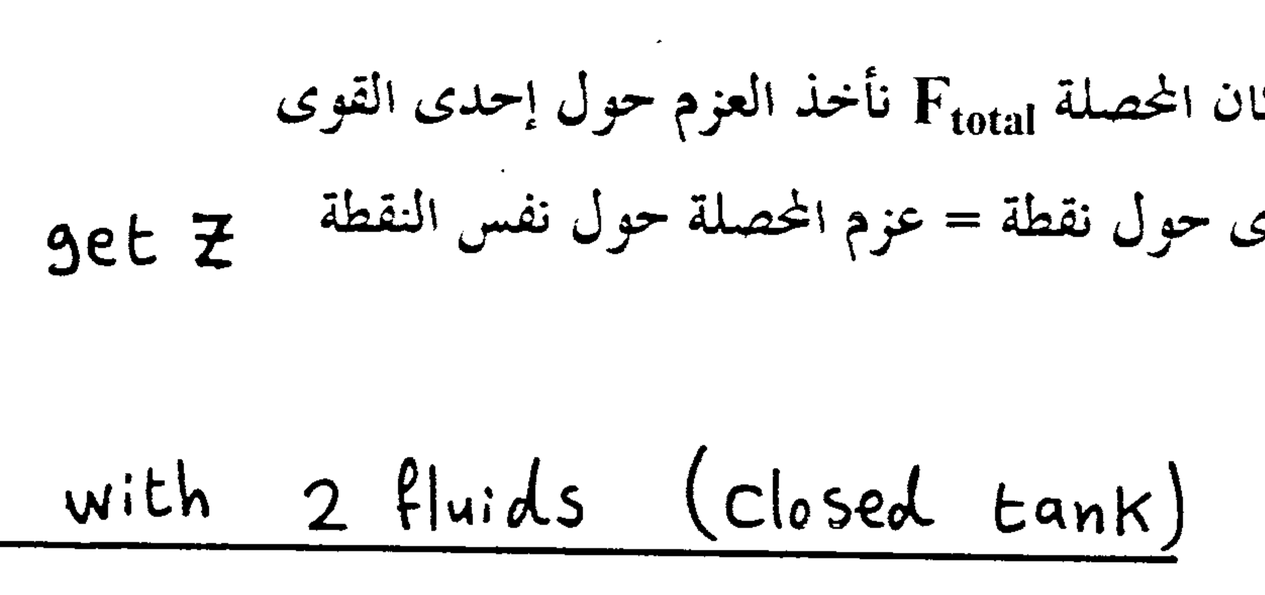
Inclined Surface with 2 fluids (closed tank)

$$F_1 = (\gamma_1 h_1 + P) A$$

$$F_2 = \gamma_2 A \bar{h_2}$$

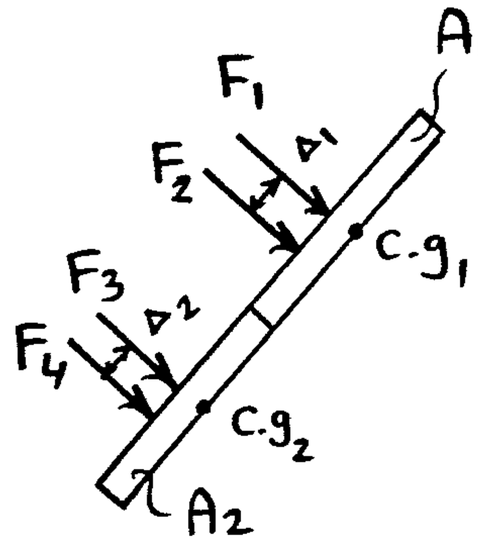
$$\bar{y_2} = \frac{\bar{h_2}}{\sin\alpha}$$

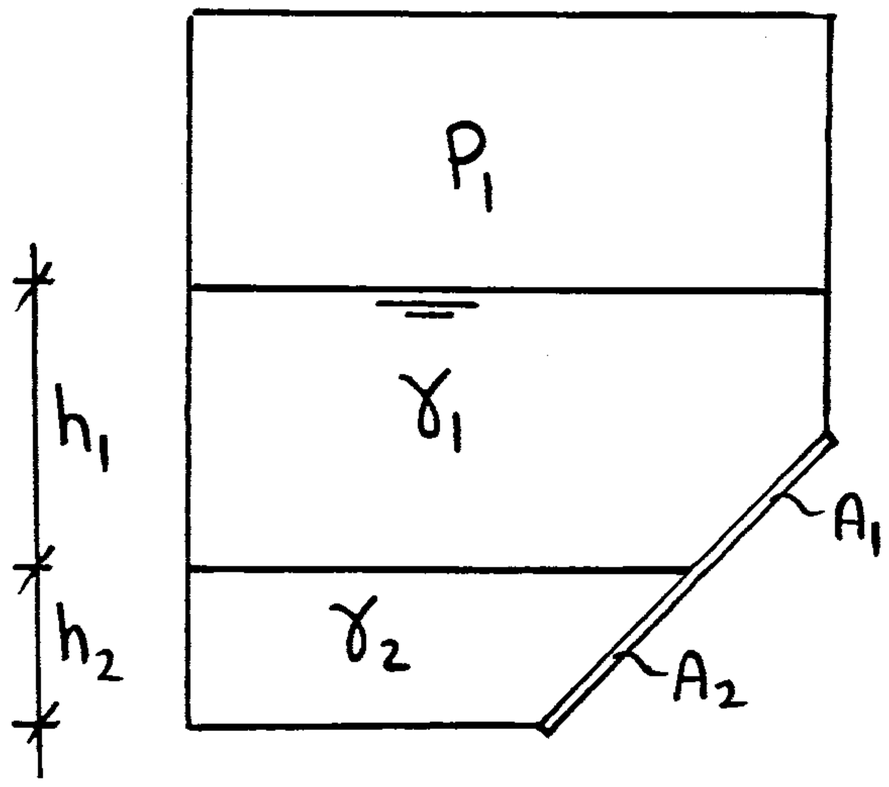
$$\Delta_2 = \frac{I_{c.g}}{A \bar{y_2}}$$



Special Case

Gate subjected to 2 fluids





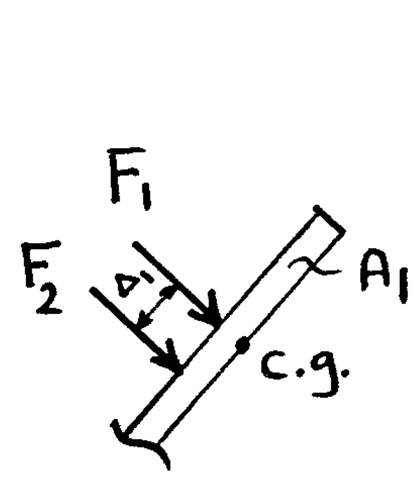
$$F_1 = P_1 A_1$$

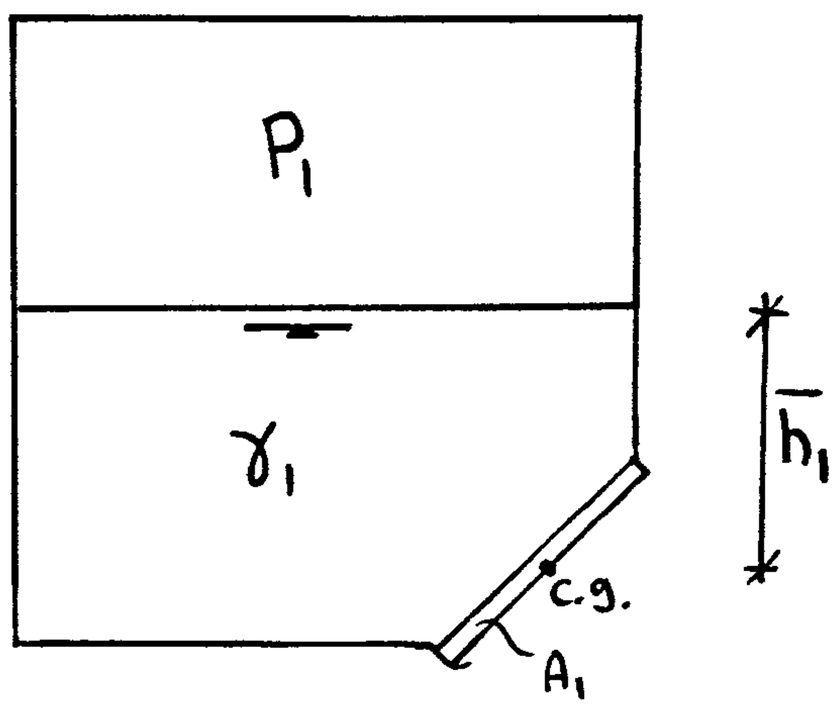
$$F_2 = \gamma_1 A_1 \overline{h_1}$$

$$\Delta = \frac{I_{c.g_1}}{A_1 \overline{y_1}}$$

$$\overline{y_1} = \frac{\overline{h_1}}{\sin\alpha}$$

نقسم المسألة إلى جزئين





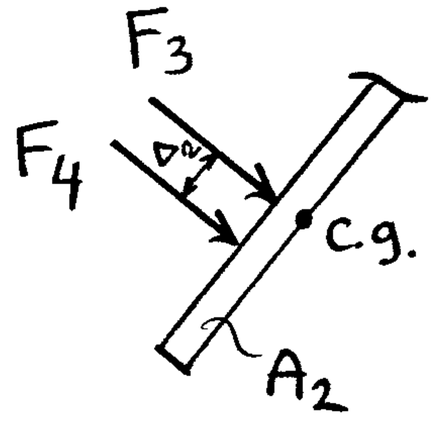
$$P_2 = P_1 + \gamma h_1$$

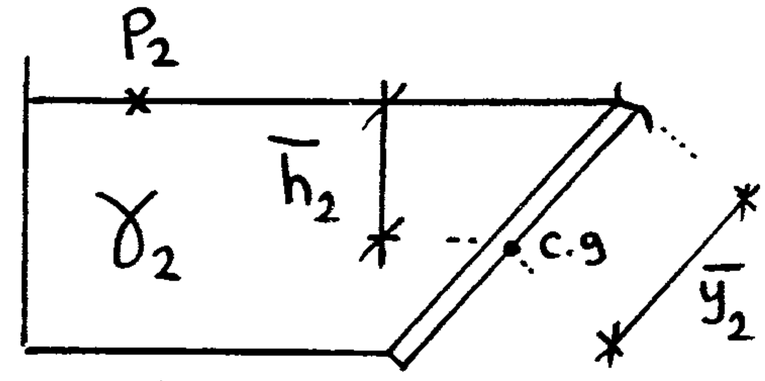
$$F_3 = P_2 A_2$$

$$F_4 = \gamma_2 A_2 \overline{h_2}$$

$$\Delta = \frac{I_{c.g_2}}{A_2 \overline{y_2}}$$

$$\overline{y_2} = \frac{\overline{h_2}}{\sin\alpha}$$





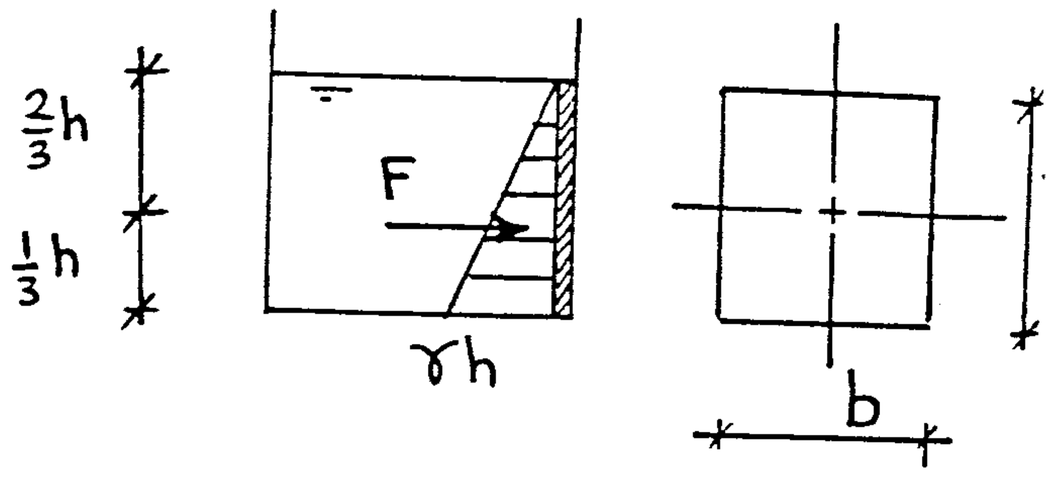
2- Pressure distribution

(For rectangular Surfaces only)

طريقة توزيع الضغط

$$F = \frac{1}{2}\gamma h \; h \; b$$

$$= \frac{1}{2}\gamma h^2 b$$



طريقة المعادلات

$$F = \gamma A \bar{h}$$

$$A = bh$$

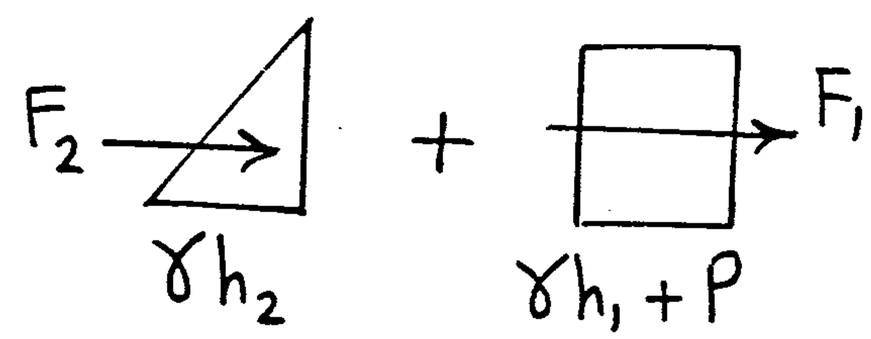
$$\bar{h} = \frac{h}{2}$$

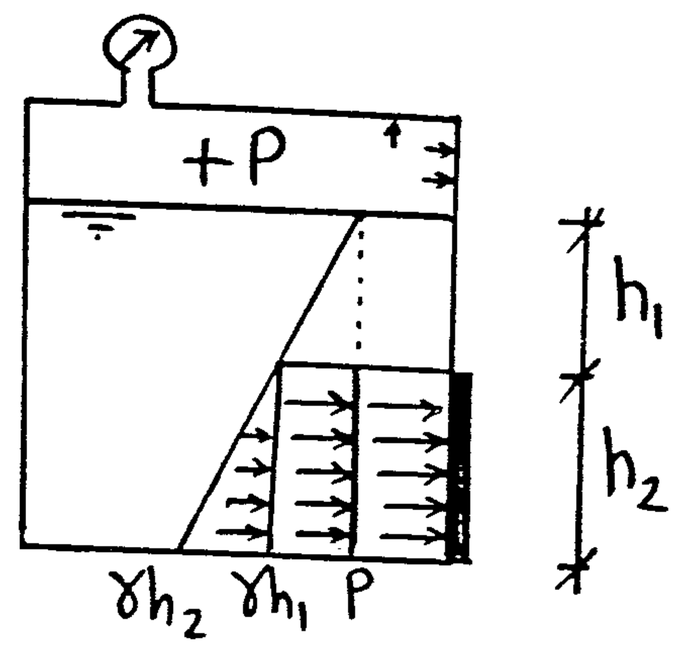
$$F = \gamma \, bh \, \frac{h}{2} = \frac{1}{2}\gamma h^2 b$$

Closed tank with +ve pressure

$$F_1 = (\gamma h_1 + P) h_2 b$$

$$F_2 = \frac{1}{2}(\gamma h_2) h_2 b$$

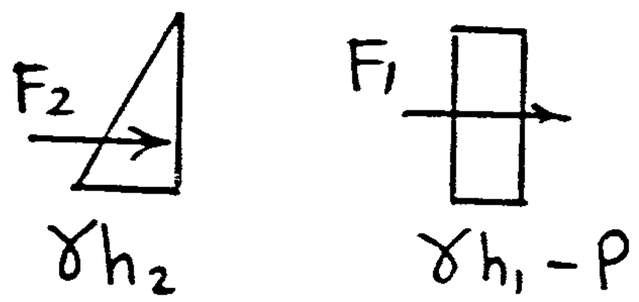


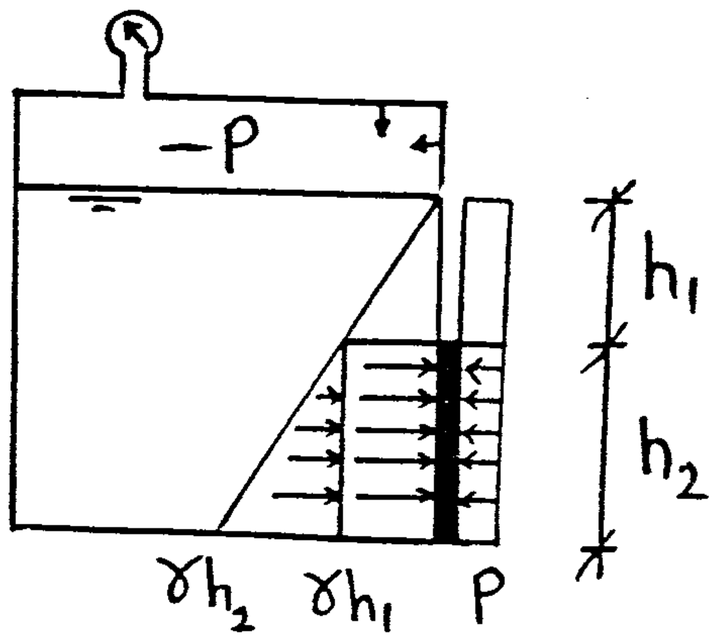


Closed tank with -ve pressure

$$F_1 = (\gamma h_1 - P) h_2 b$$

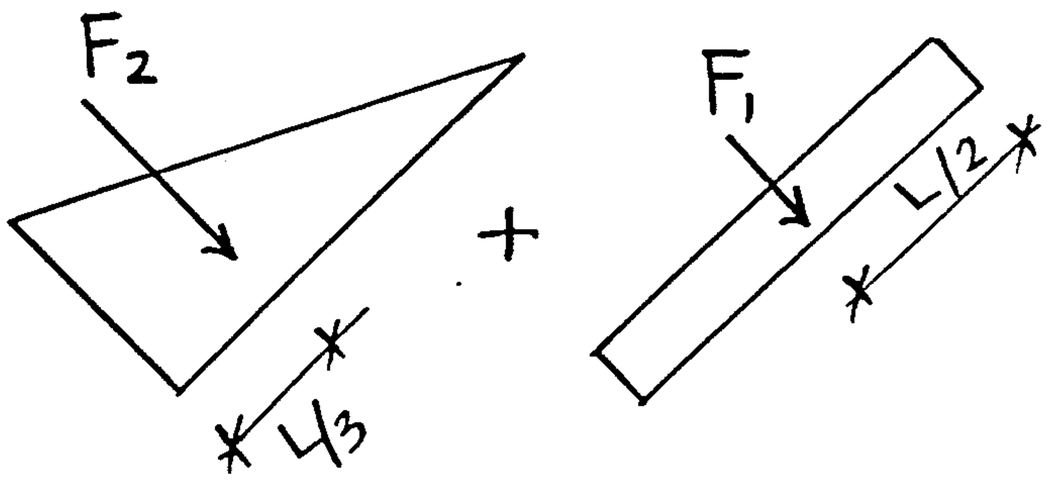
$$F_2 = \frac{1}{2}(\gamma h_2) h_2 b$$

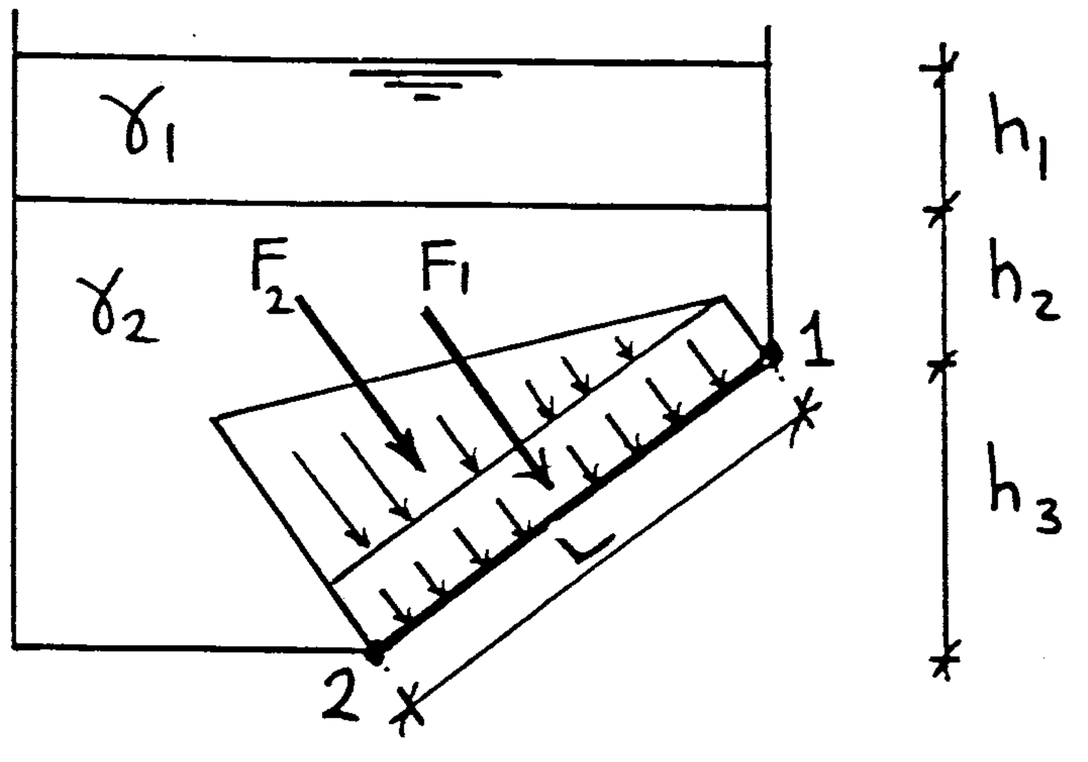




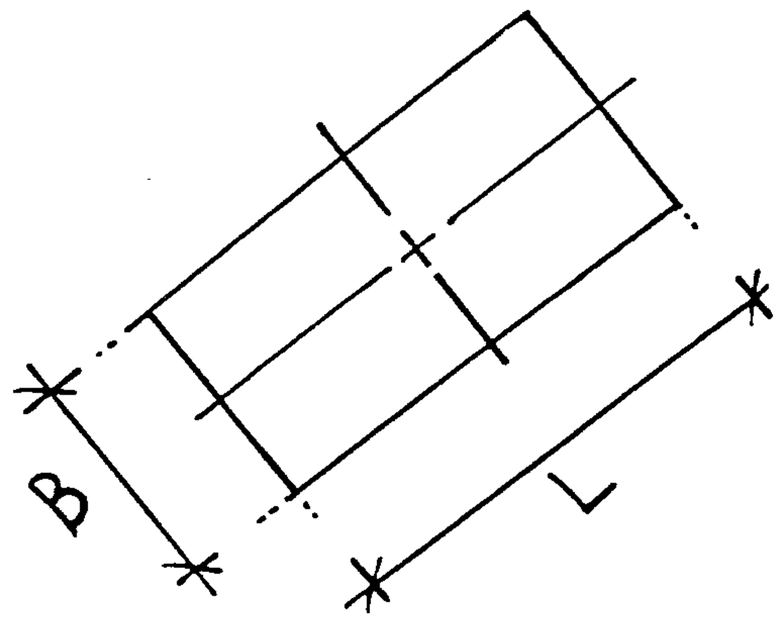
Inclined surface with 2 fluids

$$P_1 = \gamma_1 h_1 + \gamma_2 h_2$$
$$P_2 = P_1 + \gamma_2 h_3$$



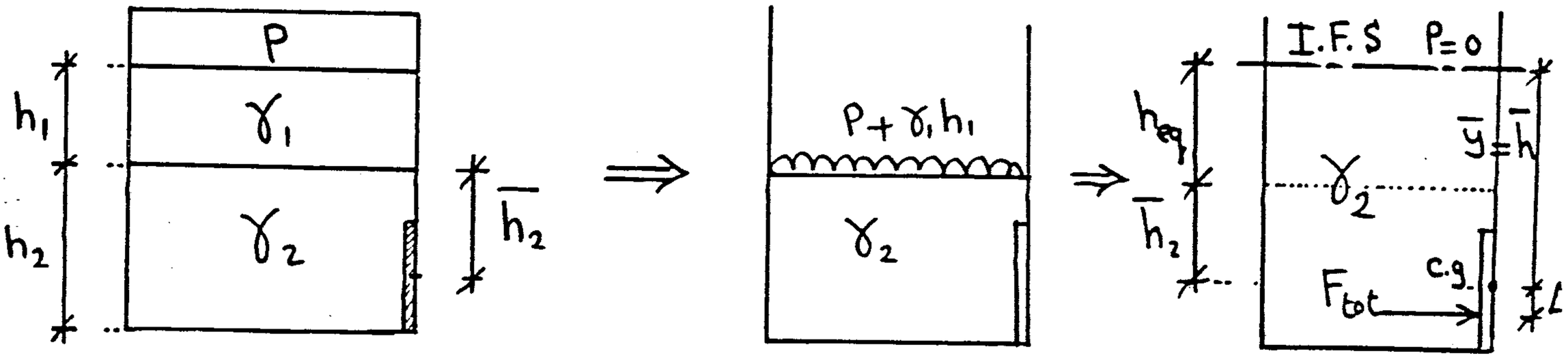


$$F_1 = P_1 A = = (\gamma_1 h_1 + \gamma_2 h_2) L B$$
$$F_2 = \frac{(P_2 - P_1)}{2} A = \frac{1}{2} \gamma_2 h_3 L B$$



3-Imaginary Free Surface (I.F.S.)

فكرة هذه الطريقة هى محاولة إيجاد Free Surface للبوابة المغمورة فى السائل



يتم تحويل الضغط الناتج عن السائل $\gamma_1 h_1$ والضغط P إلى ارتفاع مكافئ من السائل γ_2

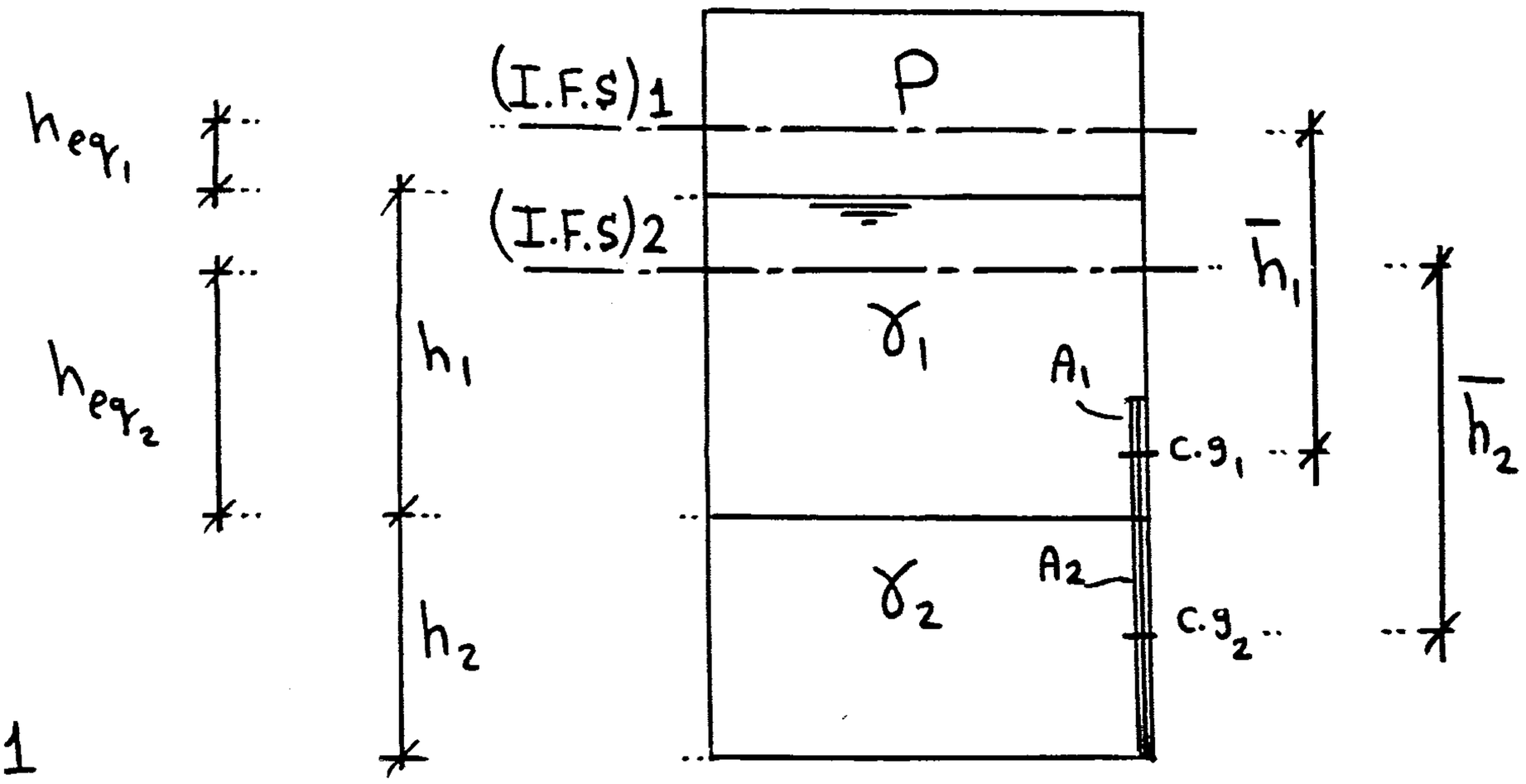
$$h_{eq} = \frac{P + \gamma_1 h_1}{\gamma_2}$$

$$F_{total} = \gamma_2 A \bar{h}$$

$$\Delta = \frac{I_{c.g}}{A \bar{y}}$$

$\bar{y}$ ٢ ١

Gate Subjected to 2 fluids



نتعامل مع كل مساحة على حدة

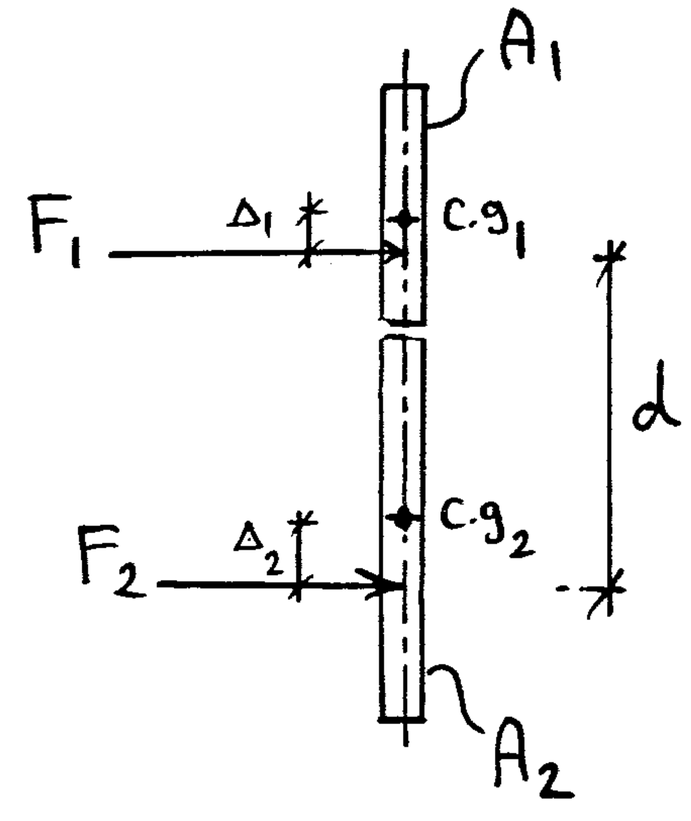
for Area 1

$$h_{eq} = \frac{P}{\gamma_1} \Rightarrow \text{get } \bar{h}_1$$

$$F_1 = \gamma A_1 \bar{h}_1$$

$$\Delta_1 = \frac{I_{c.g_1}}{A_1 \bar{y}_1}$$

$$\bar{y} = \bar{h}_1$$



for Area 2

$$h_{eq_2} = \frac{P + \gamma_1 h_1}{\gamma_2} \Rightarrow \text{get } \bar{h}_2$$

$$F_2 = \gamma A_2 \bar{h}_2$$

$$\Delta_2 = \frac{I_{c.g}}{A_2 \bar{y}_2}$$

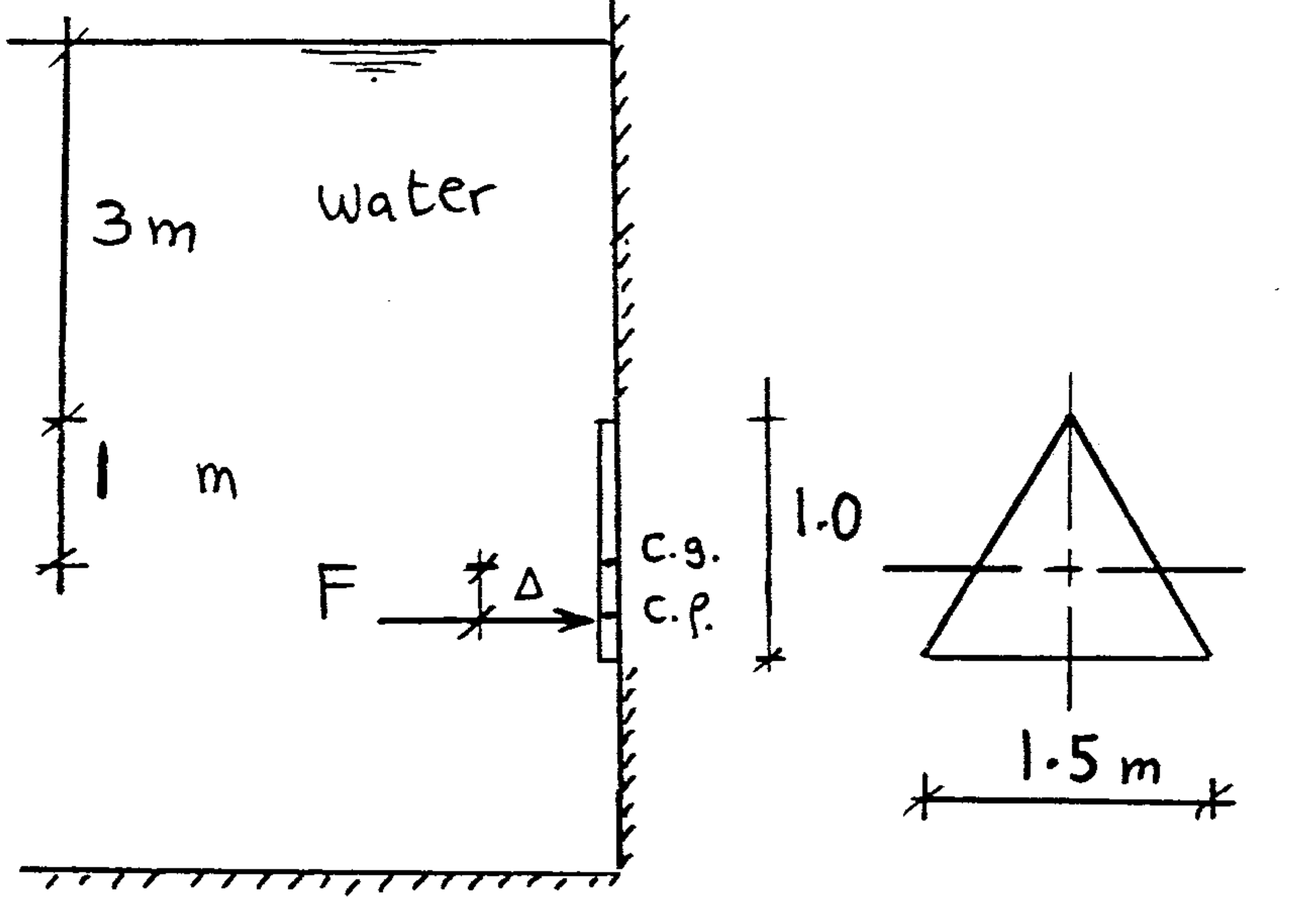
$$F = F_1 + F_2$$

$$F_2 d = F Z \Rightarrow \text{get } Z \quad \text{(line of action)}$$

2 Triangular

F = ? C.P. = ?

Sol



$F = \gamma A \bar{h}$

$\gamma_w = 9810 \ N/m^3$

$A = \frac{1}{2}(1.5)(1.0) = 0.75 m^2$

$\bar{h} = 3 + \frac{2}{3}(1.0) = 3.67 \ m$

$F = 9810\,(0.75)\,3.67 = \underline{\underline{26977 \ N}}$

$\Delta = \frac{I_{c.g.}}{A \bar{y}}$

$I_{c.g.} = \frac{\text{المختفى} * \text{المرئى}^3}{36}$

$I_{c.g.} = \frac{bZ^3}{36} = \frac{1.5\,(1.0)^3}{36} = 0.0416 \ m^4$

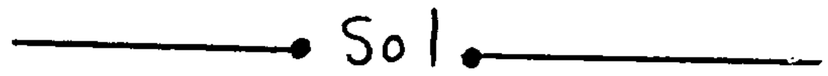
$\bar{y} = \frac{\bar{h}}{\sin\alpha} = \bar{h} = 3.67$

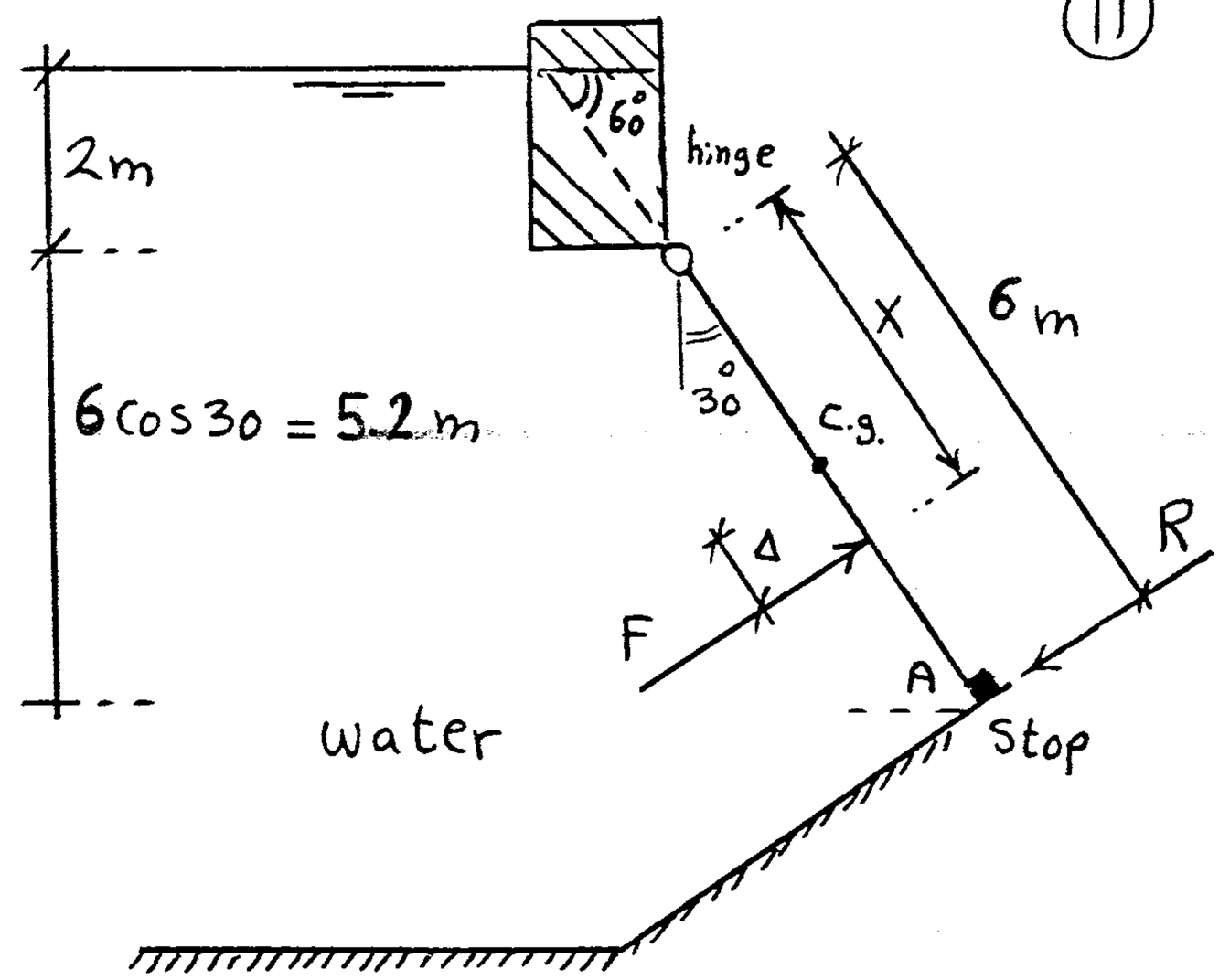
$\Delta = \frac{I_{c.g.}}{A\bar{y}} = \frac{0.04166}{(0.75)(3.67)} = 0.01514$

$C.P. = \bar{h} + \Delta = 3.67 + 0.01514 = \underline{\underline{3.682 \ m}}$

3 Rectangular

$B = 5\ m \quad R = ?$





$$F = \gamma A \bar{h}$$

$$\gamma = 9810\ N/m^3$$

$$A = 6 * 5 = 30\ m^2$$

$$\bar{h} = 2 + \frac{5.2}{2} = 4.6\ m$$

$$\therefore F = 9810\,(30)\ 4.6 = \underline{\underline{1\,353\,780\ N}}$$

$$\Delta = \frac{I_{c.g.}}{A \bar{y}}$$

$$I_{c.g.} = \frac{5(6)^3}{12} = 90\ m^4$$

$$\bar{y} = \frac{\bar{h}}{\sin\alpha} = \frac{4.6}{\sin 60^\circ} = 5.311\ m$$

$$\Delta = \frac{I_{c.g.}}{A \bar{y}} = \frac{90}{30(5.311)} = 0.5648$$

$$X = 3 + 0.5648 = 3.5648$$

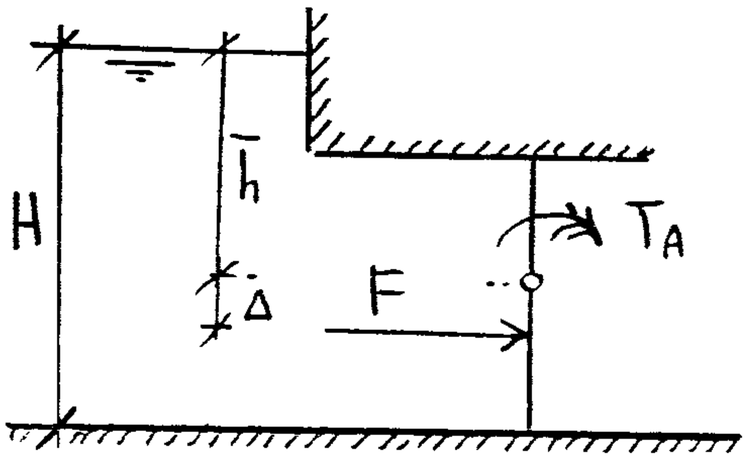
$$\overset{\curvearrowright}{M}_{hinge} = 0$$

$$6R = 1\,353\,780\ (3.5648)$$

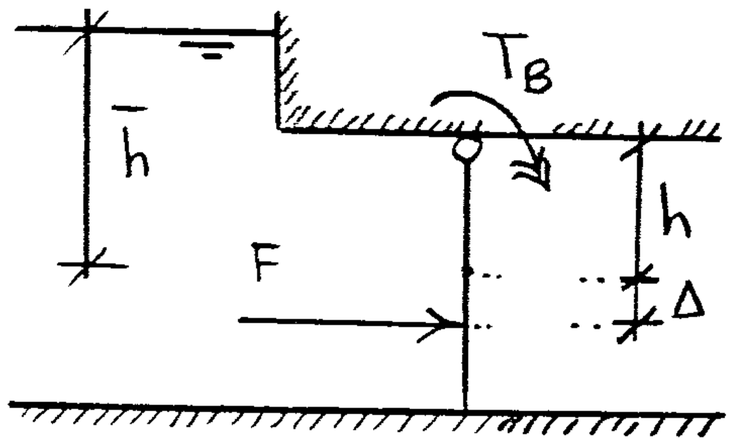
$$\Rightarrow R = \underline{\underline{804\,325\ N}}$$

4

Gate A



Gate B



a)

$$F = \gamma A \bar{h} \qquad \Delta = \frac{I_{c.g}}{A\bar{h}}$$

$$T_A = F.\Delta = \gamma A \bar{h} * \frac{I_{cg}}{A\bar{h}} = \gamma I_{cg}$$

T_A increases with H (X)

---

b) $T_B = F(h+\Delta) = \gamma A \bar{h}\left(h + \frac{I_{cg}}{A\bar{h}}\right) = \gamma\left(A\underline{\bar{h}}h + I_{cg}\right)$

T_B increases with H (✓)

---

c) T_A does not change with H (✓)

d) T_B does not change with H (X)

---

e) The hydrostatic force on gate A increases as H increases

$F = \gamma A \underline{\bar{h}} \qquad F \propto \bar{h} \Rightarrow F \propto H$ (✓)

---

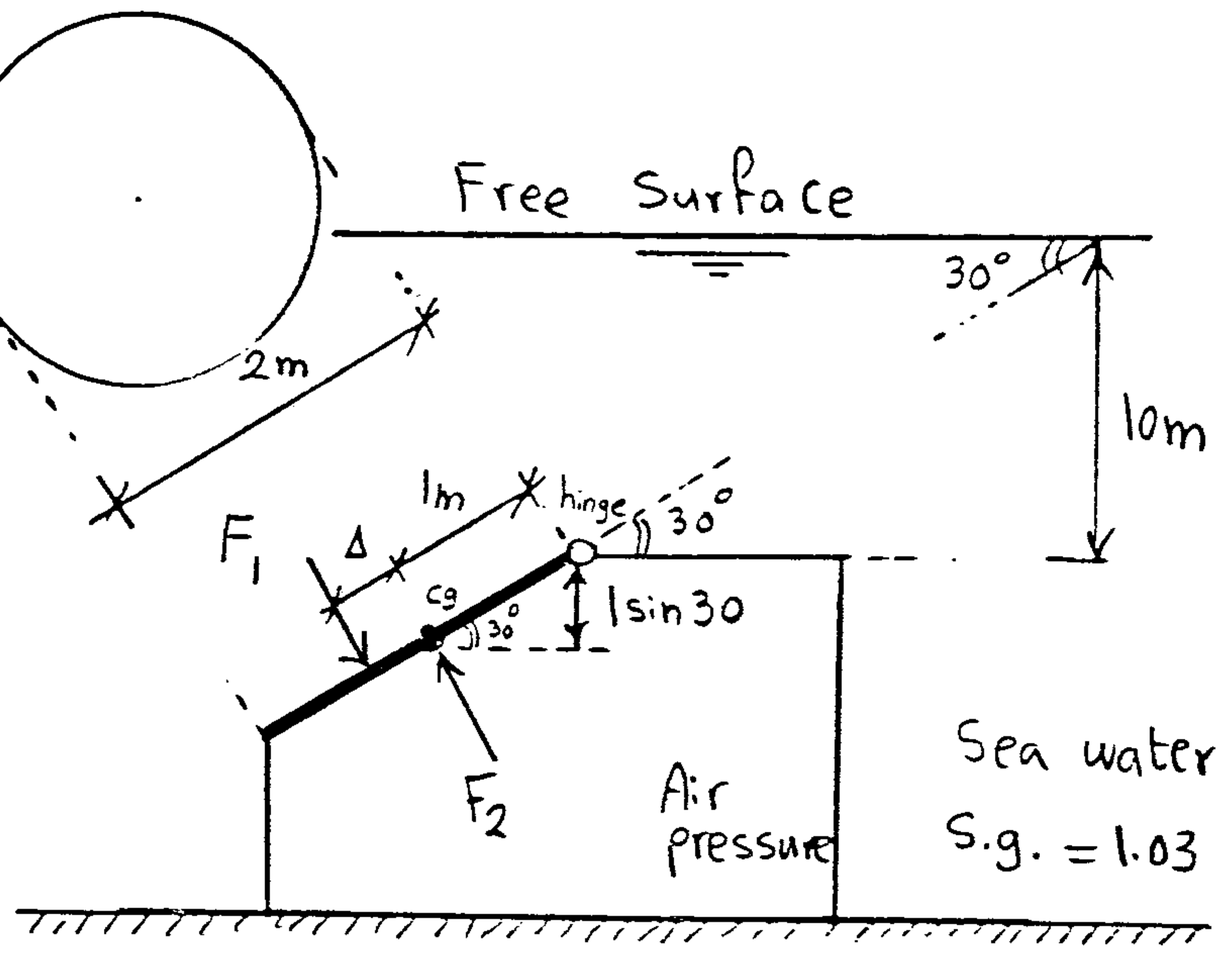
f) The distance between C.P. and C.g. is constant as H increases

$$\Delta = \frac{I_{cg}}{A\frac{\bar{h}}{\sin\alpha}} \Rightarrow \Delta \propto \frac{1}{\bar{h}}$$ (X)

---

g) T_A must increase as H increases (X)

7 A 2m diameter gate is located in an inclined wall and hinged on one edge. Determine the minimum air pressure within the container to open the gate



Air pressure = ?

Sol

$$F_1 = \gamma_{Sea} A \bar{h}$$

$$A = \pi \frac{(2)^2}{4} = 3.14\ m^2$$

$$\bar{h} = 10 + 1\sin 30 = 10.5\ m$$

$$F = 1.03\,(9810) * 3.14 * 10.5 = \underline{333139\ N}$$

$$\Delta = \frac{I_{cg}}{A\bar{y}}$$

$$I_{cg} = \frac{\pi d^4}{64} = \frac{\pi (2)^4}{64} = 0.785$$

$$\bar{y} = \frac{\bar{h}}{\sin\alpha} = \frac{10.5}{\sin 30} = 21\ m$$

$$\Delta = \frac{I_{cg}}{A\bar{y}} = \frac{0.785}{3.14\ (21)} = 0.012\ m$$

$$\circlearrowright M_{hinge} = 0$$

$$F_2\,(1) = F_1\,(1+\Delta)$$

$$F_2 = 333139\,(1 + 0.012) = 337105\ N$$

$$\text{Air Pressure} = \frac{F_2}{A} = \frac{337105}{3.14} = \underline{107303\ N/m^2}$$

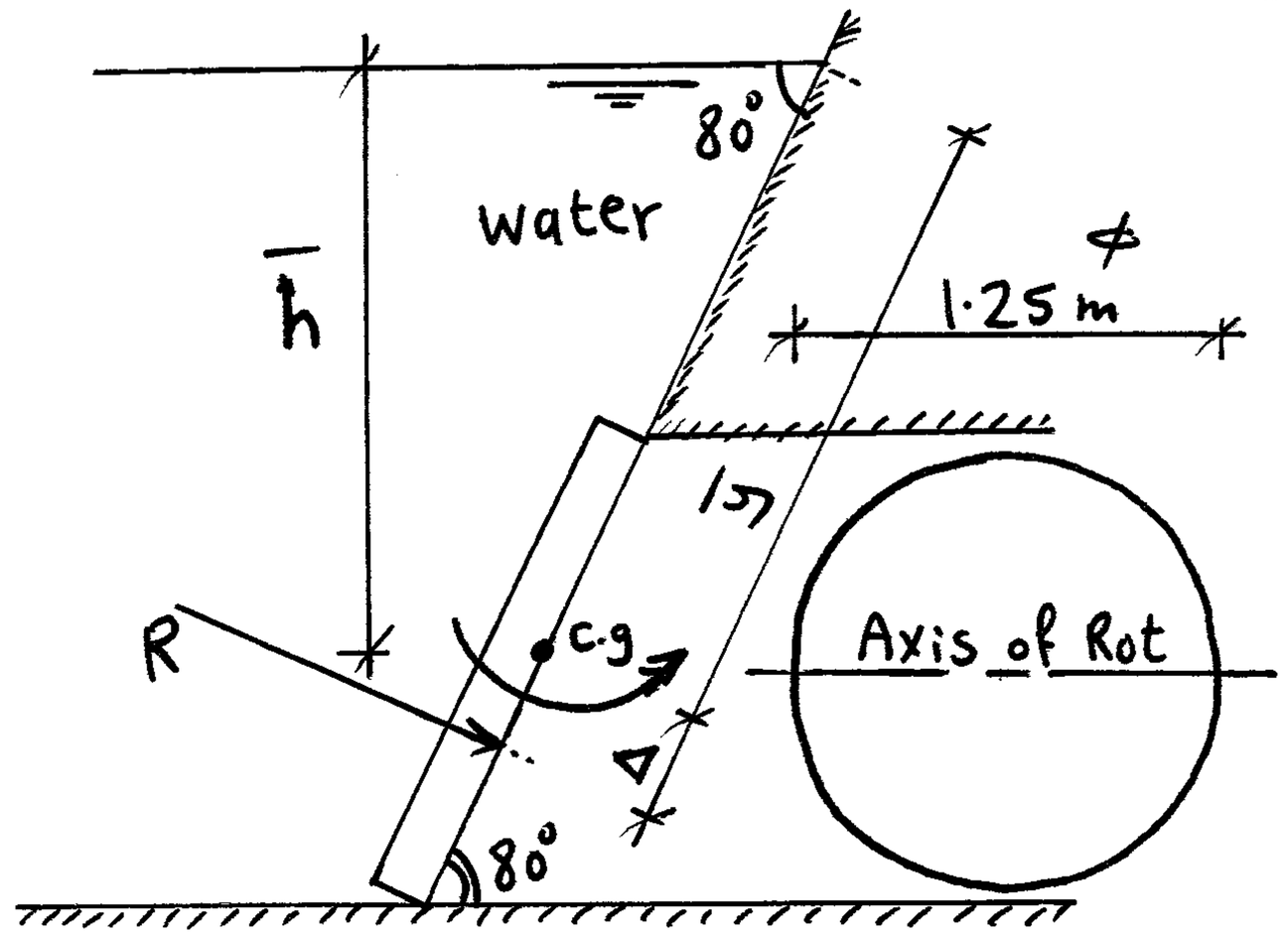
5

Turning Moment

$$M = R\Delta$$

$$R = \gamma A \bar{h}$$

$$\Delta = \frac{I_{c.g}}{A\bar{y}}$$



$$M = \gamma A \bar{h} \quad \frac{I_{c.g}}{A\bar{y}}$$

$$\therefore \bar{y} = \frac{\bar{h}}{\sin\alpha}$$

$$M = \gamma \bar{h} \quad \frac{I_{c.g}}{\bar{h}} \times \sin\alpha$$

$$M = \gamma I_{c.g} \sin\alpha$$

$$= \gamma \frac{\pi d^4}{64} \sin\alpha$$

$$= 9810 \frac{\pi (1.25)^4}{64} \sin 80^\circ = \underline{\underline{1157 \text{ N.m}}}$$

5

$B = 2 \text{ m} \qquad \forall_{conc} = ?$

$S.g._{conc} = 2.42$

Sol

$$F = \gamma A \bar{h}$$

$\gamma_w = 9810 \text{ N/m}^3$

$A = 6 * 2 = 12 \text{ m}^2$

$\bar{h} = \frac{6}{2} = 3$

$F = 9810\ (12)\ 3 = 353\,160 \text{ N}$

$$\Delta = \frac{I_{c.g}}{A \bar{y}}$$

$I_{c.g.} = \frac{2\ (6)^3}{12} = 36$

$\Delta = \frac{I_{c.g.}}{A \bar{y}} = \frac{36}{12\ (3)} = 1$

$\sum M\circlearrowright_{hinge} = 0$

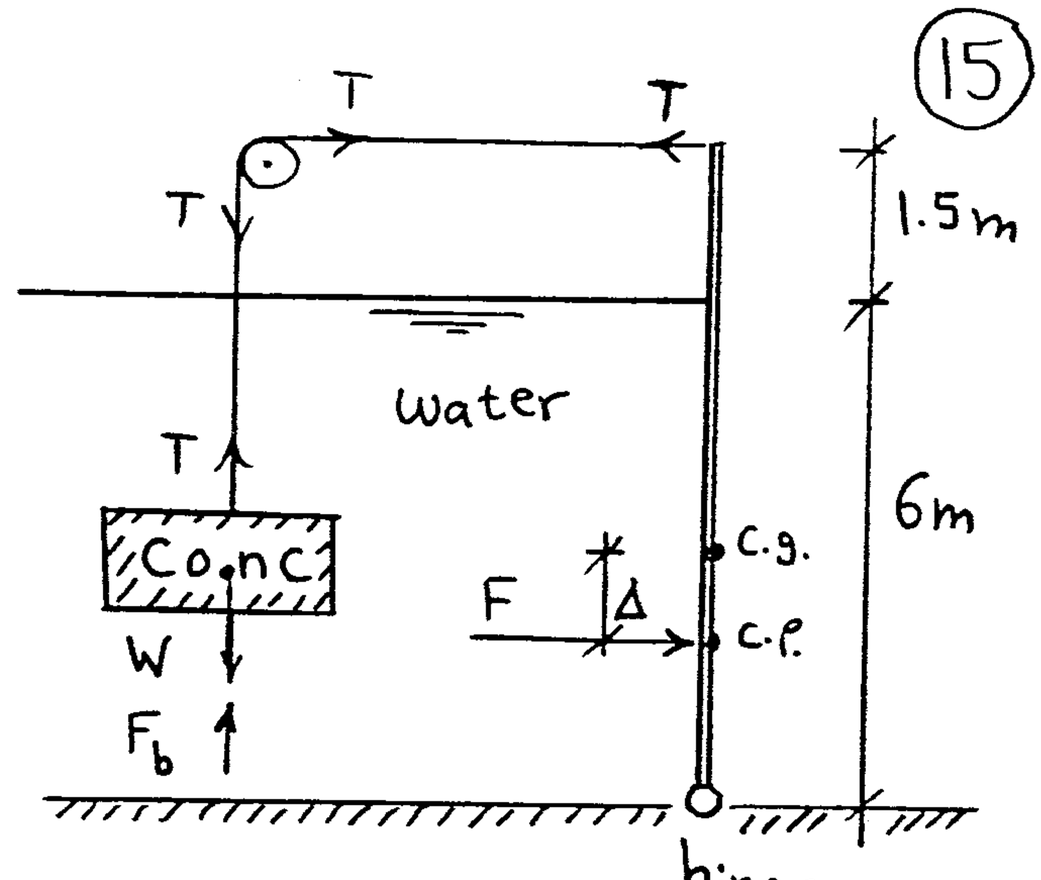
$7.5\ T = 353\,160\ (2) \Rightarrow T = 94\,176 \text{ N}$

∴ When a body is immersed in water, a bouyant force F_b acts upwards which equals to the weight of liquid displaced by the body (Archimedes)

$\therefore T + F_b = W_{conc}$

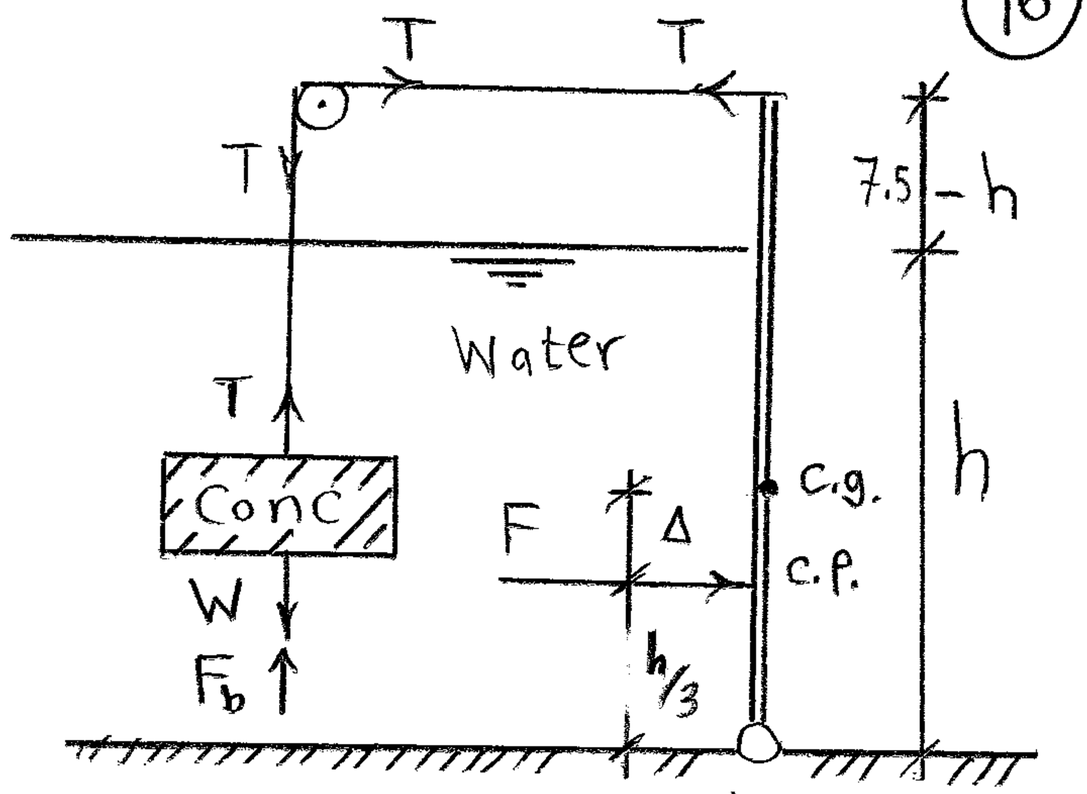
$94\,176 + \gamma_w \forall = \gamma_{conc} \forall$

$94\,176 + 9810 \forall = 2.42(9810) \forall \Rightarrow \forall = \underline{6.76 \text{ m}^3}$



If $\forall$ is reduced 30 %

$h = ?$



sol//

$$\forall_2 = \frac{70}{100} * \forall_1 = 0.7(6.76)$$

$$= \underline{\underline{4.73\ m^3}}$$

$$T + F_b = W_{conc}$$

$$T + \gamma_w \forall = \gamma_{conc} \forall$$

$$T + 9810(4.73) = 2.42(9810)(4.73) \quad \Rightarrow T = 65\,890\ N$$

$$F = \gamma A \bar{h}$$

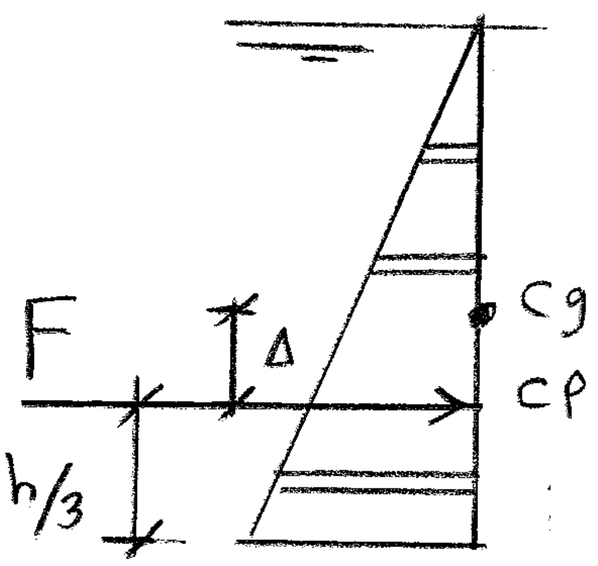
$\gamma_w = 9810$

$A = h * 2 = 2h$

$\bar{h} = \frac{h}{2}$

$$F = 9810\,(2h)\,\frac{h}{2} = 9810\,h^2$$

$$\Delta = \frac{I_{cg}}{A\bar{y}} = \frac{h}{6} \quad \text{(rectangle)}$$



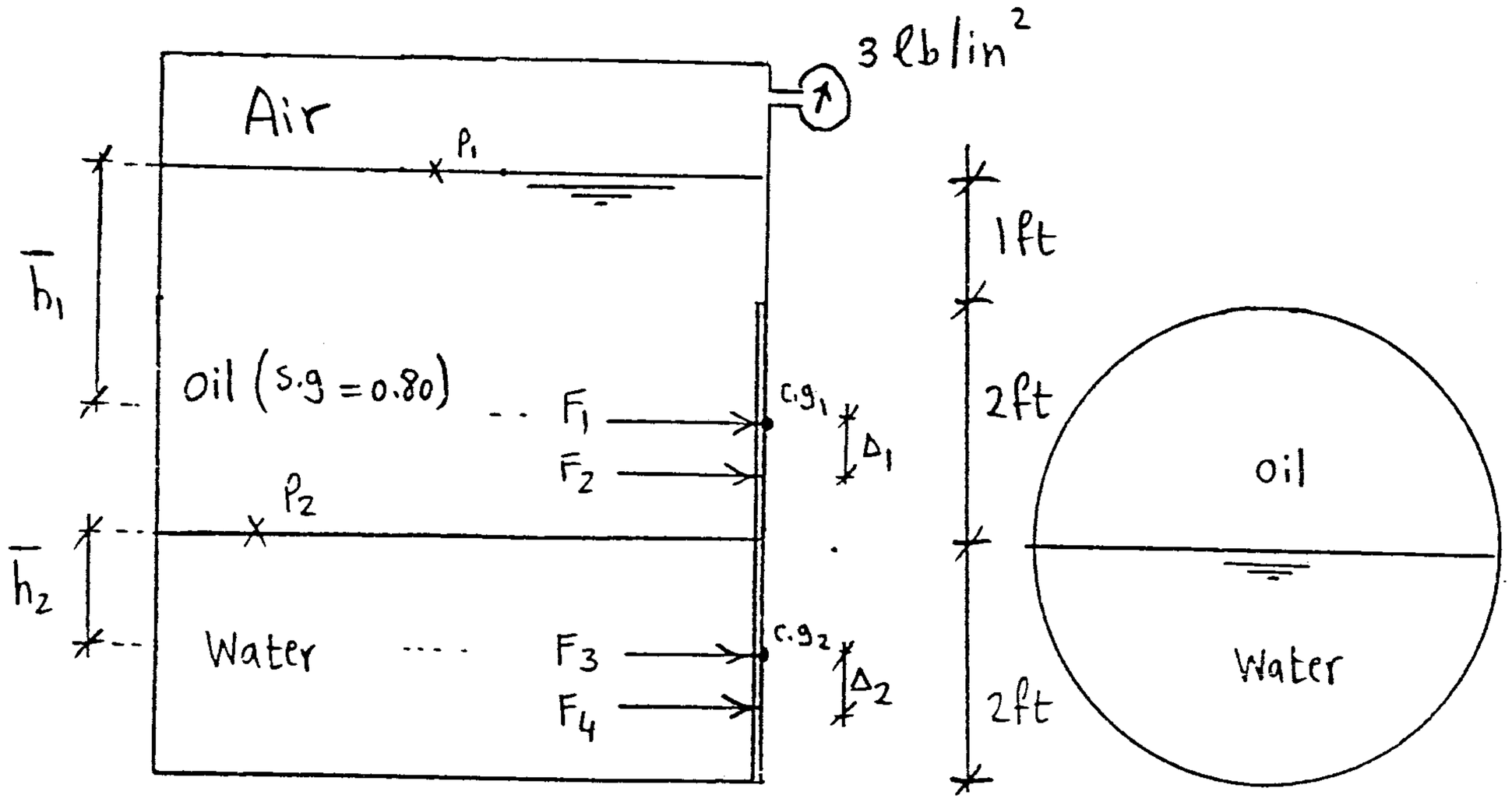
$$\sum M_{hinge} = 0$$

$$7.5\ T = F\left(\frac{h}{3}\right)$$

$$7.5(65890) = 9810\ h^2\ \frac{h}{3}$$

$$\Rightarrow h = \underline{\underline{5.33\ m}}$$

Find the force exerted by fluids on the circular gate as shown in figure and the location of the center of pressure. (17)



1st Solution (Equation method)

$$P_1 = 3\,Psi = 3 * 144 = 432\ psf$$

$$A_1 = \frac{\pi r^2}{2} = \frac{\pi (2)^2}{2} = 6.28\ ft^2$$

$$F_1 = P_1 A_1 = 432\,(6.28) = \underline{2714\ lb}$$

$$\bar{h}_1 = 1 + 2 - \frac{4(2)}{3\pi} = 2.15\ ft$$

$$F_2 = \gamma A_1 \bar{h}_1 = 0.8\,(62.4)\,6.28\,(2.15) = \underline{674\ lb}$$

$$I_{c.g.} = \frac{\pi d^4}{128} - \frac{d^4}{18\pi} = \frac{\pi (4)^4}{128} - \frac{(4)^4}{18\pi} = 1.76\ ft^4$$

$$\bar{y}_1 = \bar{h}_1 = 2.15$$

$$\Delta_1 = \frac{I_{c.g.}}{A \bar{y}_1} = \frac{1.76}{6.28\,(2.15)} = \underline{0.13}$$

$$P_2 = 432 + 0.8\,(62.4)\,3 = 581.7\ Psf$$

$$F_3 = P_2 A_2 = 581.7\,(6.28) = \underline{3653\ lb}$$

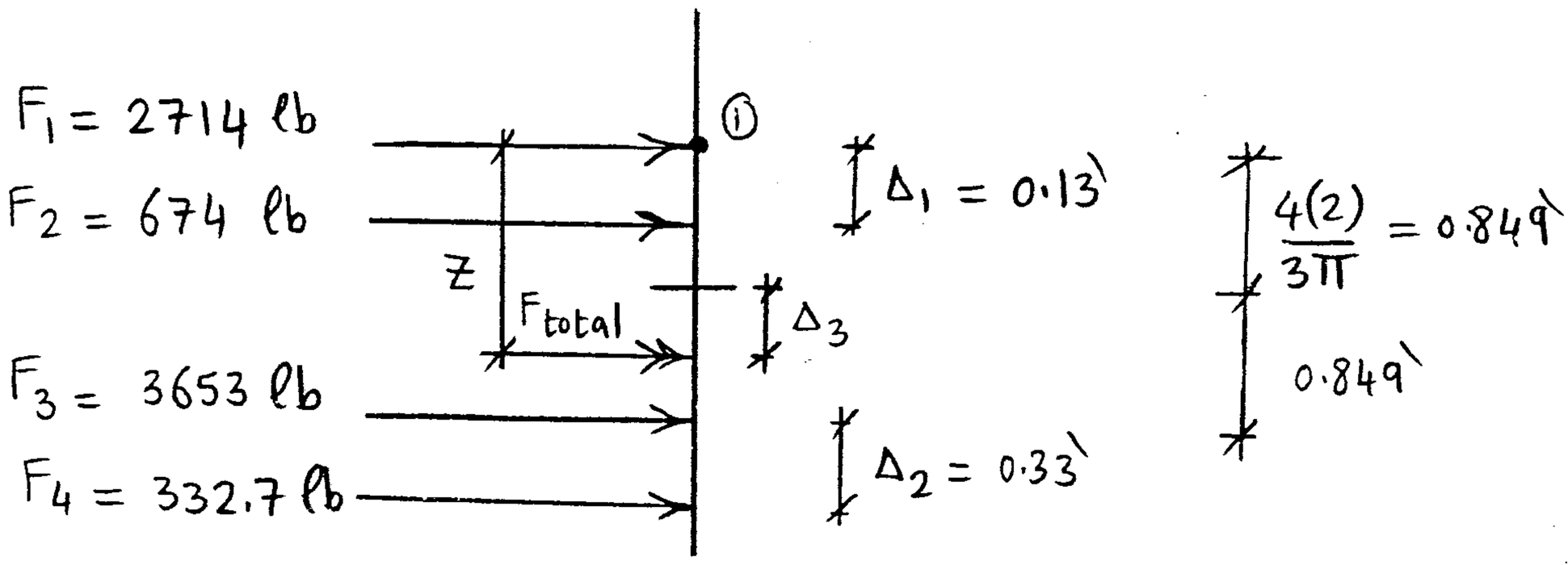
$$\bar{h}_2 = \frac{4r}{3\pi} = \frac{4(2)}{3\pi} = 0.849$$

$$F_4 = \gamma_2 A_2 \bar{h}_2 = 62.4\,(6.28)\,0.849 = \underline{\underline{332.7\ lb}}$$

$$I_{c.g.} = 1.76$$

$$\bar{y}_2 = \bar{h}_2 = 0.849$$

$$\Delta_2 = \frac{I_{c.g.}}{A_2 \bar{y}_2} = \frac{1.76}{6.28\,(0.849)} = \underline{\underline{0.33\ ft}}$$



$$F_{total} = 2714 + 674 + 3653 + 332.7 = \boxed{7373.7\ lb}$$

$$\Sigma M_{①}$$

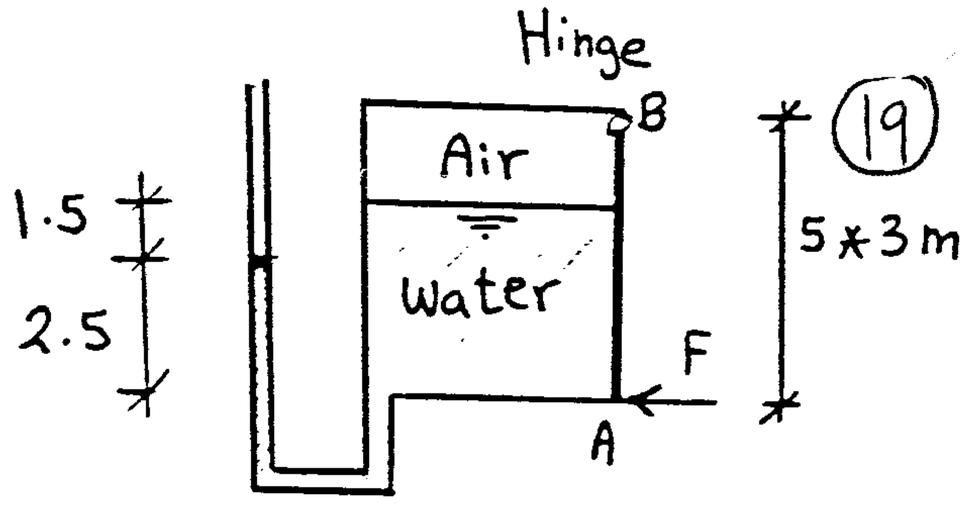
$$7373.7(z) = 674\,(0.13) + 3653\,(0.849 * 2) + 332.7\,(2.028)$$

$$\Rightarrow z = 0.944$$

location of C.P.

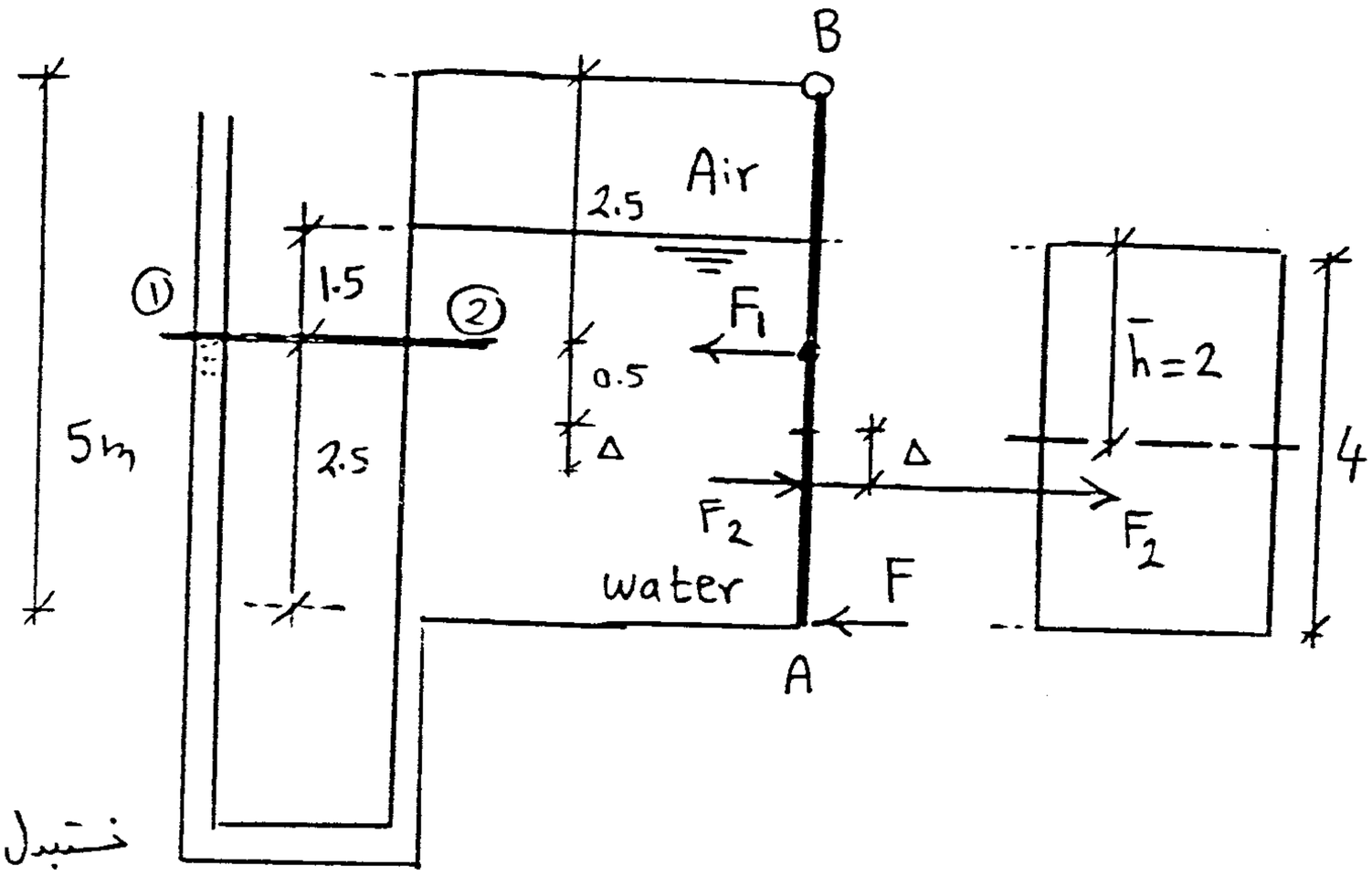
$$\Delta_3 = 0.944 - 0.849 = \boxed{0.095}$$

Find the Force F at A to hold the gate closed.



$B = 3m \qquad F = ?$

Solution



$$P_1 = P_2$$

$$0 = P_{air} + \gamma_w (1.5)$$

$$\therefore P_{air} = -9810\,(1.5)$$

$$= \underline{\underline{-14715\ N/m^2}}$$

F_1 ستبدل ضغط الهواء بقوة في الـ c.g

$$F_1 = P_{air} A_{gate} = 14715\,(5*3) = \underline{\underline{220\,725\ N}}$$

$$F_2 = \gamma A \bar{h}$$

$$= 9810\,[4*3]\,2 = \underline{\underline{235\,440\ N}}$$

$$\Delta = \frac{I_{cg}}{A\bar{y}} = \frac{\frac{3(4)^3}{12}}{(3*4)*2} = \frac{2}{3}$$

$$\sum M_{hinge} = 0$$

$$F(5) = F_2\left(3 + \Delta\right) - F_1\,(2.5)$$

$$5F = 235\,440\left(3\tfrac{2}{3}\right) - 220\,725\,(2.5)$$

$$\Rightarrow F = \underline{\underline{62293\ N}}$$

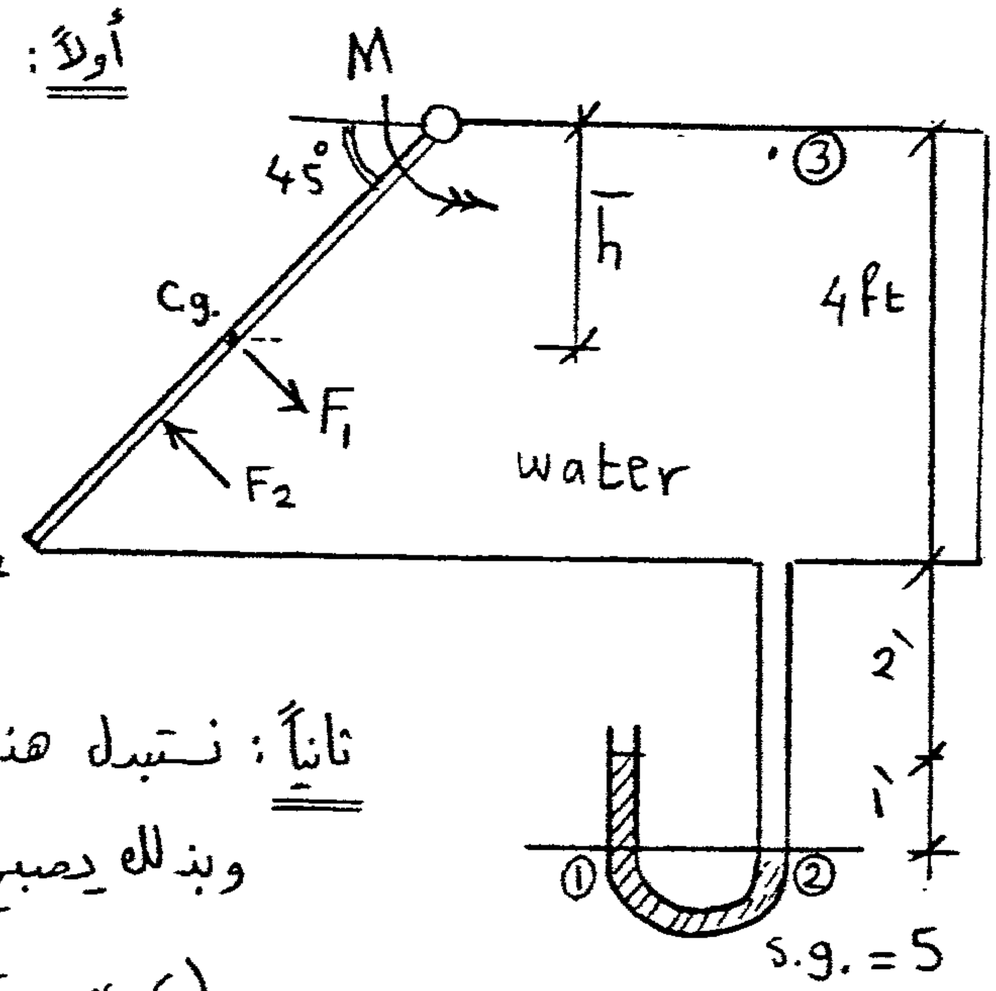
Find the Moment (M) to hold the gate closed (6ft wide)

(V.I.) أولاً: حساب الضغط فوق البوابه

$$P_1 = P_2$$

$$5\gamma_w (1) = P_3 + \gamma_w (7)$$

$$\therefore P_3 = -2\gamma_w = -2(62.4) = -124.8 \ \frac{lb}{ft^2}$$



ثانياً: نستبدل هذا الضغط بقوة فى ال C.g. F_1

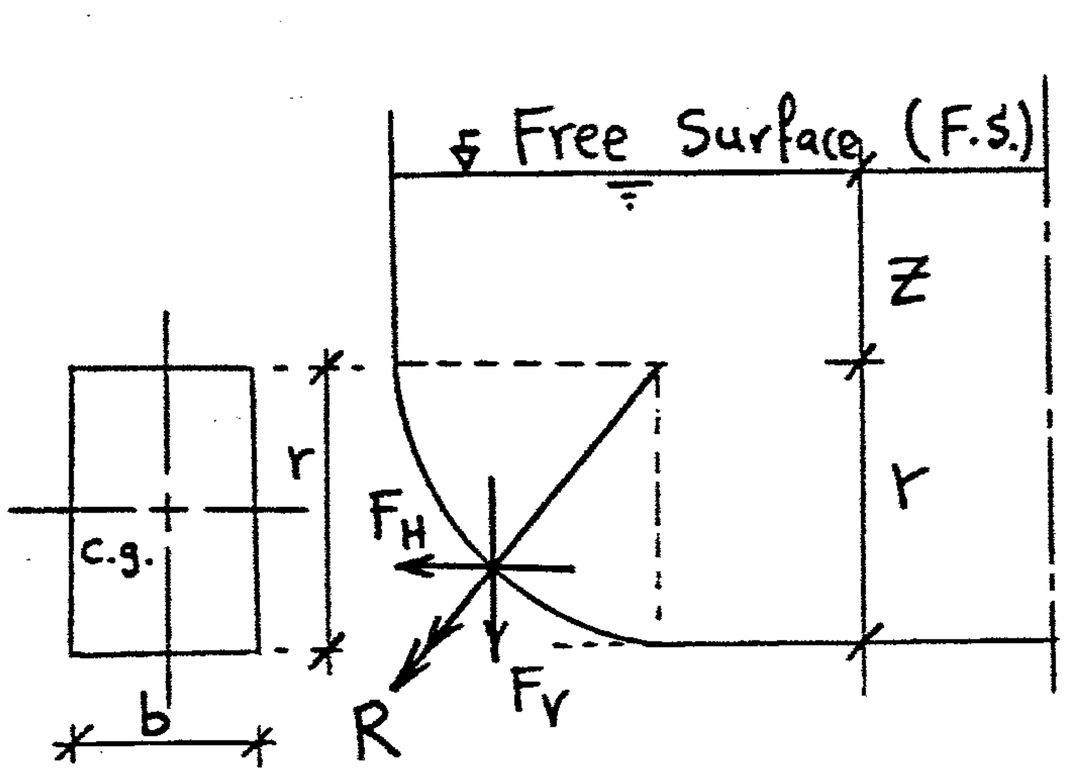
وبذلك يصبح السطح فوق البوابه New Free Surface

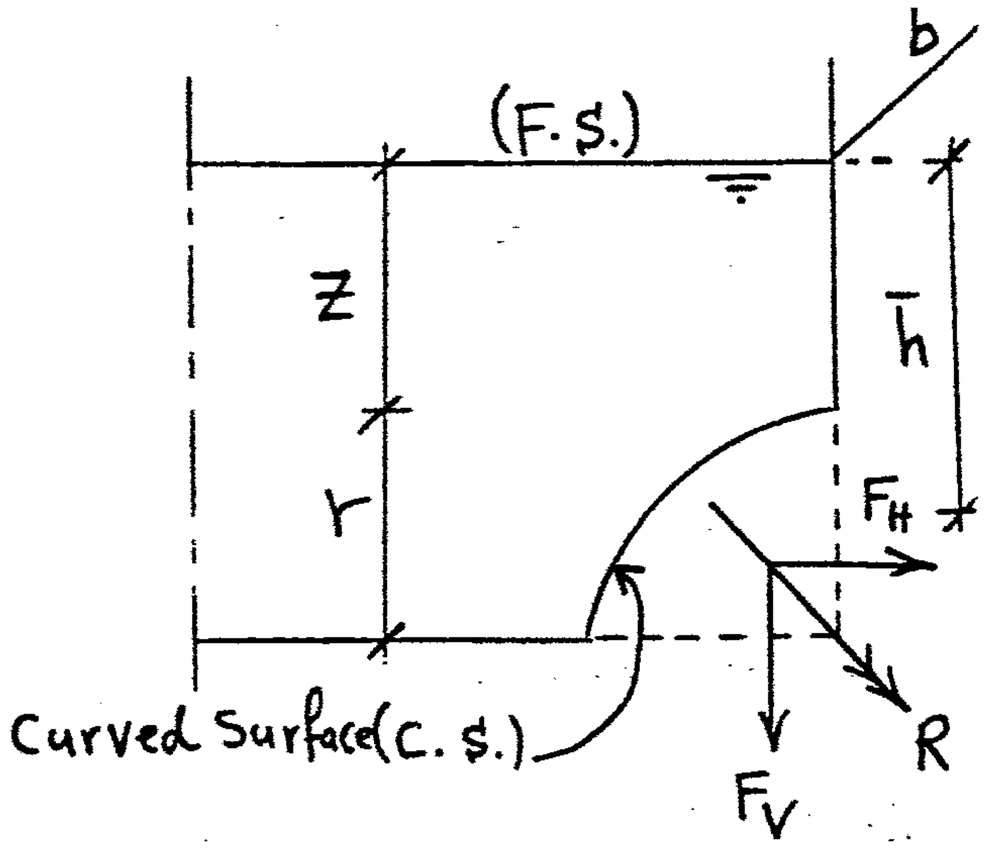
$$\therefore F_1 = P_3 A_{gate} = 124.8 \left(\frac{4}{\sin 45} * 6\right) = 4236 \ lb$$

$$F_2 = \gamma A \bar{h} = 62.4 \left(\frac{4}{\sin 45} * 6\right) * 2 = 4236 \ lb$$

$$M = 4236 * 5.656\left(\frac{2}{3}\right) - 4236 * 5.656\left(\frac{1}{2}\right) = \underline{3993 \ lb.ft}$$

5 Forces on Curved Surfaces





يؤثر الضغط عمودياً على أى سطح، فإذا كان السطح دائرياً مرت المحصلة R فى مركزه ولمعرفة اتجاه R نفترض وجود ثقب فى ال Curved Surface فيكون اتجاه خروج السائل هو اتجاه R

يتم تحليل المحصلة R إلى مركبتين، مركبة أفقية F_H ومركبة رأسية F_V

F_H (Horizontal Component)

المركبة الافقية F_H هى عبارة عن القوى المؤثرة على (مسقط ال Curved Surface على مستوى رأسى)

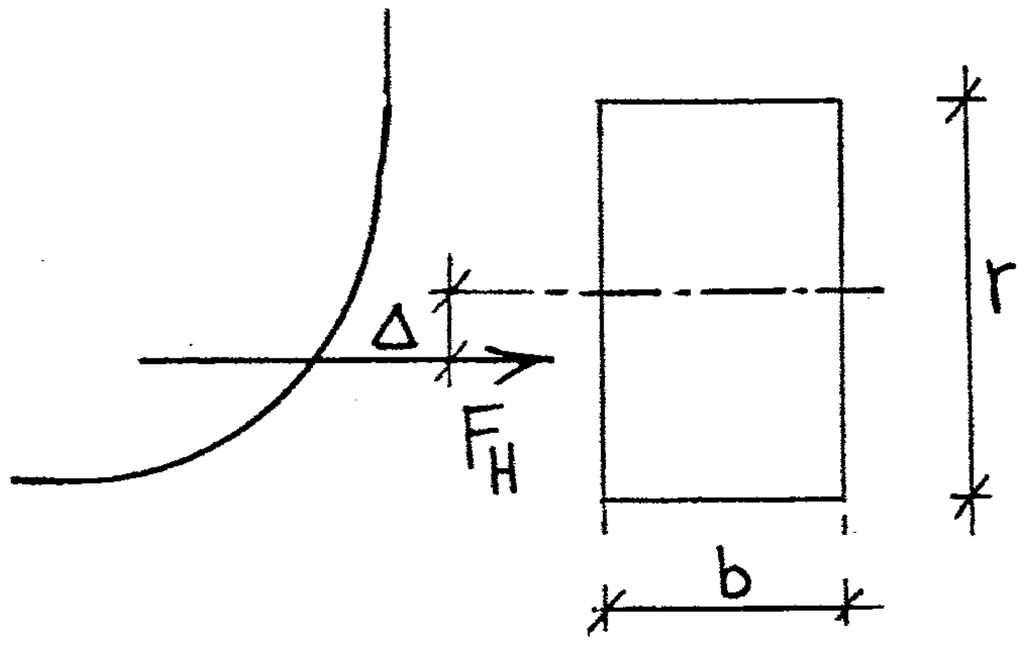
$$F = \gamma A \bar{h}$$

Where; $A = br \quad \bar{h} = Z + \frac{r}{2}$

$$\Delta = \frac{I_{c.g}}{A\bar{y}}$$

$$\bar{y} = \bar{h}$$

$$I_{c.g} = \frac{br^3}{12}$$



خطوات العمل

يتم اسقاط ال **Curved Surface** على مستوى رأسى ونتعامل مع المساحة المسقطة

اتجاه المركبة الأفقية عمودياً على المساحة المسقطة

وخط عملها يمر أسفل مركز المساحة المسقطة بمسافة Δ

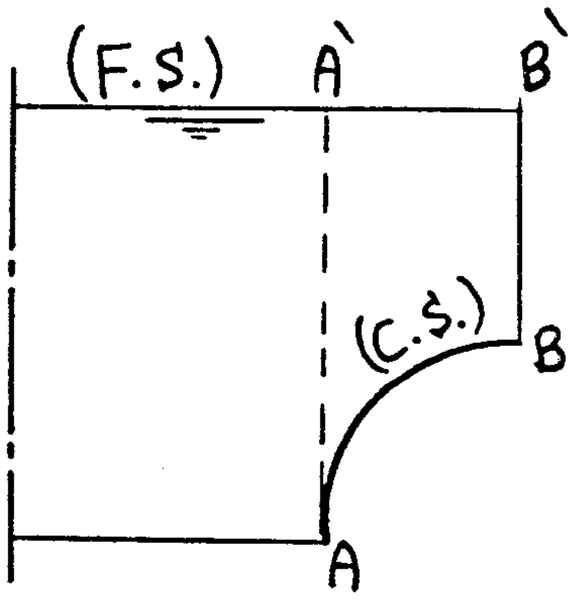
F_V (Vertical Component)

المركبة الرأسية F_V هى عبارة عن وزن السائل المحصور بين ال **Curved Surface** ومسقطه على ال **Free Surface**

$$F_V = \gamma \forall$$

خطوات العمل

يتم اسقاط أعلى وأسفل نقطة لل **Curved Surface** على ال **Free Surface** بشرط ألا تكون بينهما نقطة انقلاب

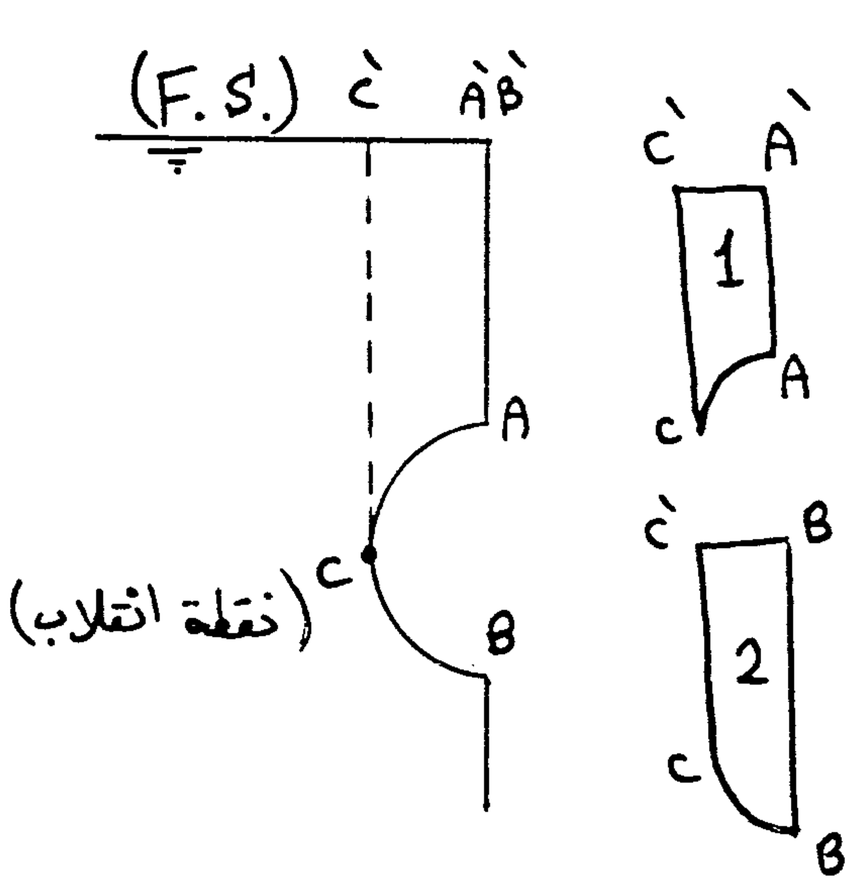


فى حالة عدم وجود نقطة انقلاب

يتم اسقاط أسفل نقطة **A** وأعلى نقطة **B** لل **Curved Surface** على ال **Free Surface** فى $\grave{A}$ ، B'

وتكون المركبة الرأسية F_V هى عبارة عن وزن السائل المحصور بين ال(C.S.) **AB**

ومسقطه على ال (F.S.) $\grave{A}\,B'$

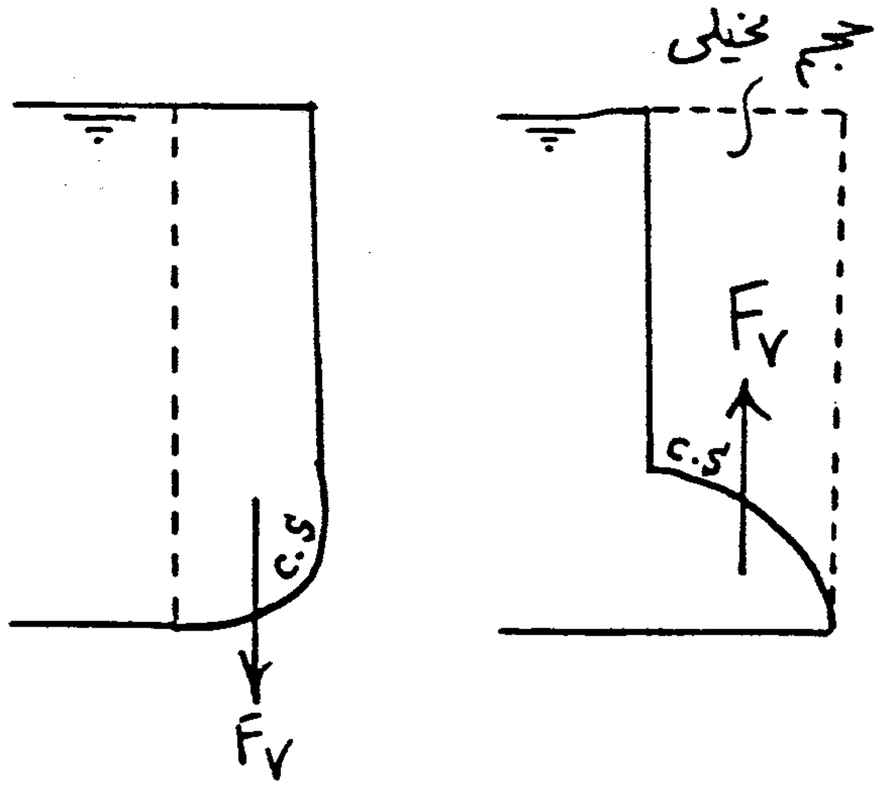


فى حالة وجود نقطة انقلاب Inflection point

يتم اسقاط أعلى نقطة **A** ونقطة الانقلاب **C** على ال (F.S.)

وكذلك اسقاط أسفل نقطة **B** ونقطة الانقلاب **C** على ال (F.S)

وتكون المركبة الرأسية هى عبارة عن الفرق فى الوزن بين 1,2



اتجاه F_V

لو كان السائل فوق ال C.S. تؤثر F_V رأسياً لأسفل ↓

لو كان السائل أسفل ال C.S. تؤثر F_V رأسياً لأعلى ↑

فى الحالة الثانية قد لا يتواجد أى سائل فوق

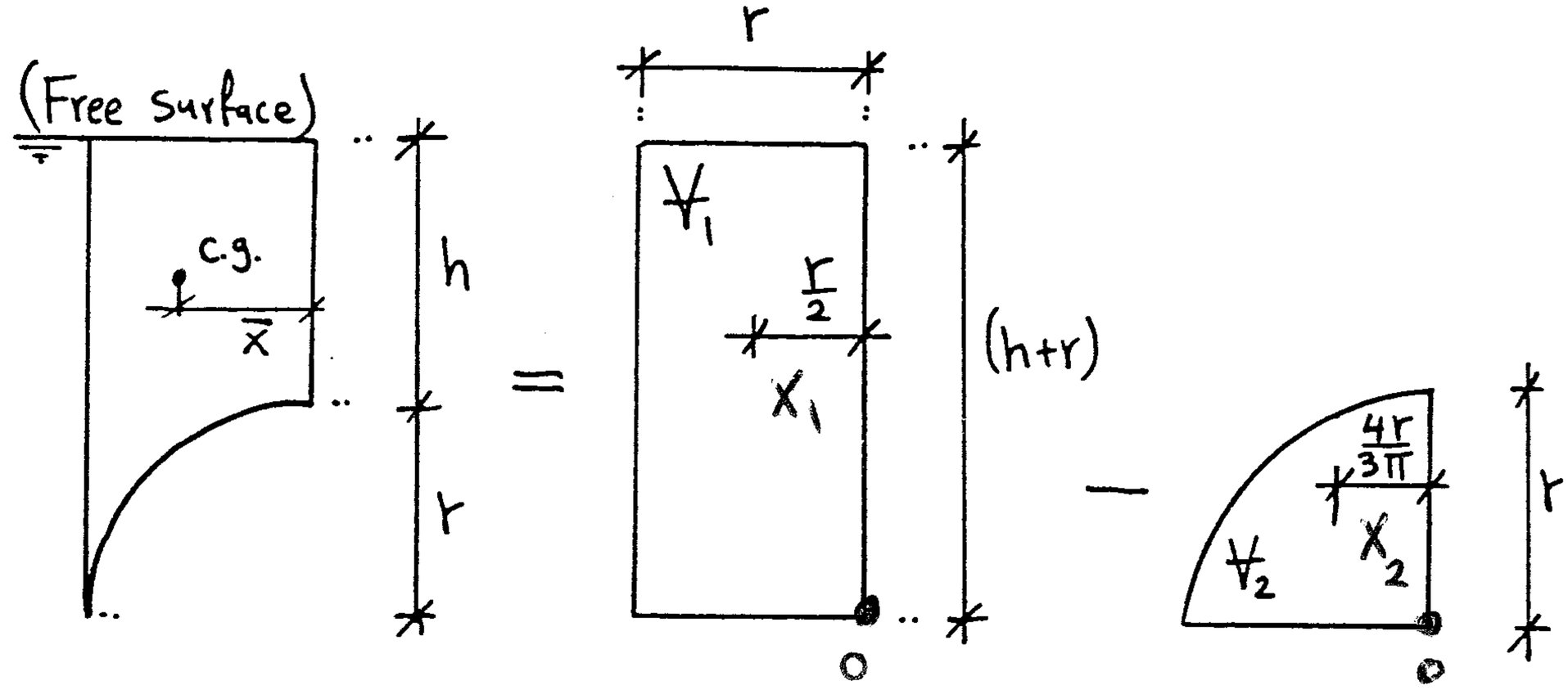
ال Curved Surface لكن القوى $F_V = \gamma V$

هى عبارة عن قوى مكافئة لوزن نفس السائل محتوياً على حجم تخيليّ فوق ال Curved Surface وحتى ال Free Surface واتجاهه لأعلى

خط عمل F_V

تؤثر القوى F_V فى مركز ثقل الحجم المحصور بين ال Curved Surface وال Free Surface

e.g.



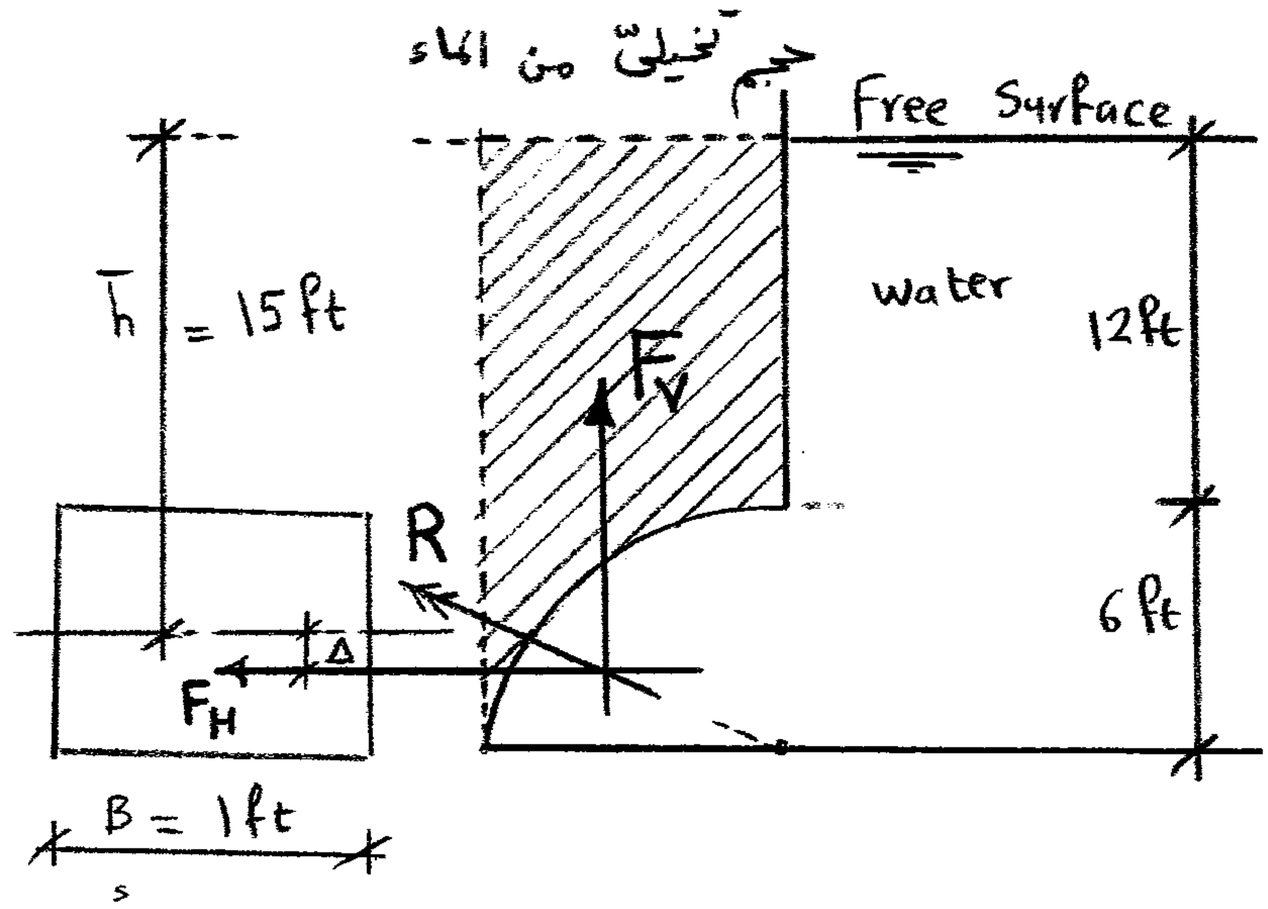
$$\forall_1 = (h+r)\, r\, b \qquad x_1 = \frac{r}{2}$$

$$\forall_2 = \frac{1}{4}\pi r^2 b \qquad x_2 = \frac{4r}{3\pi}$$

$$\forall = \forall_1 - \forall_2 \qquad F_V = \gamma \forall \downarrow$$

$$\bar{X} = \frac{\forall_1 X_1 - \forall_2 X_2}{\forall}$$

A tank wall has a shape as shown. Determine the Hzl and Vl components and locations of the force of water acting on the curved surface (4)



اعتماد الـ C.S على مستوى رأسي

Hzl force

$$F_H = \gamma A \bar{h}$$

$$= 62.4(6*1)*15$$

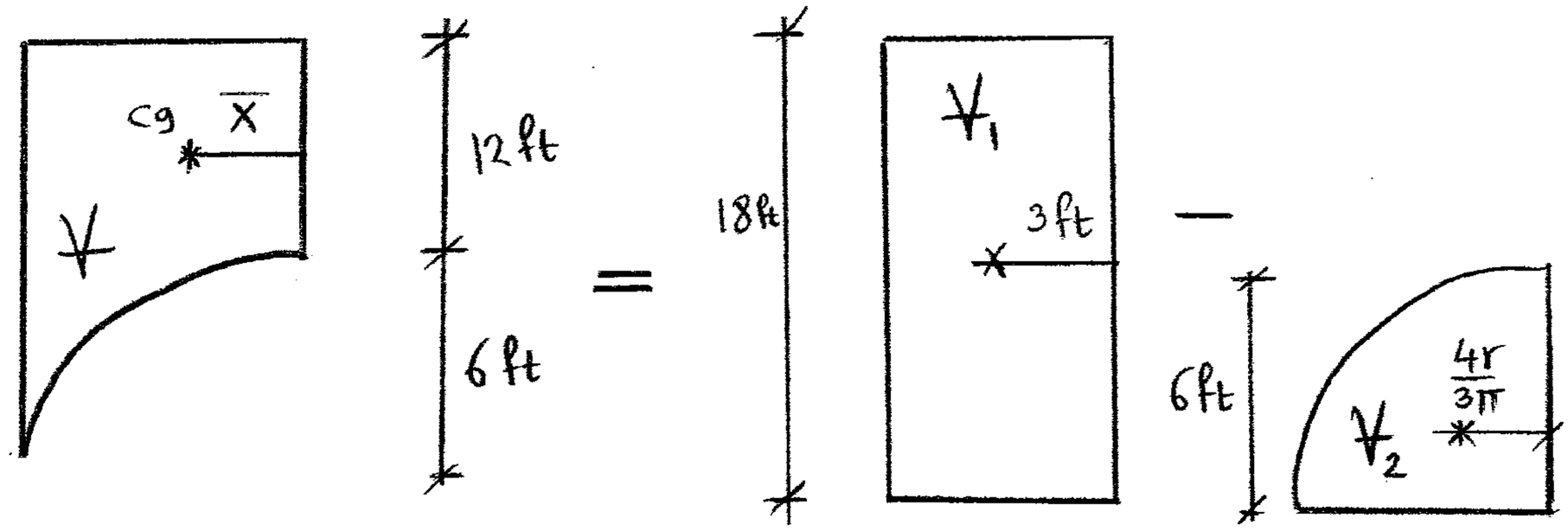
$$= \underline{\underline{5616 \text{ lb}}} \leftarrow$$

$$\Delta = \frac{I_{cg}}{A\bar{y}}$$

$$I_{cg} = \frac{1(6)^3}{12} = 18$$

$$\Delta = \frac{I_{cg}}{A\bar{h}} = \frac{18}{(1*6)15} = 0.2$$

Vl force



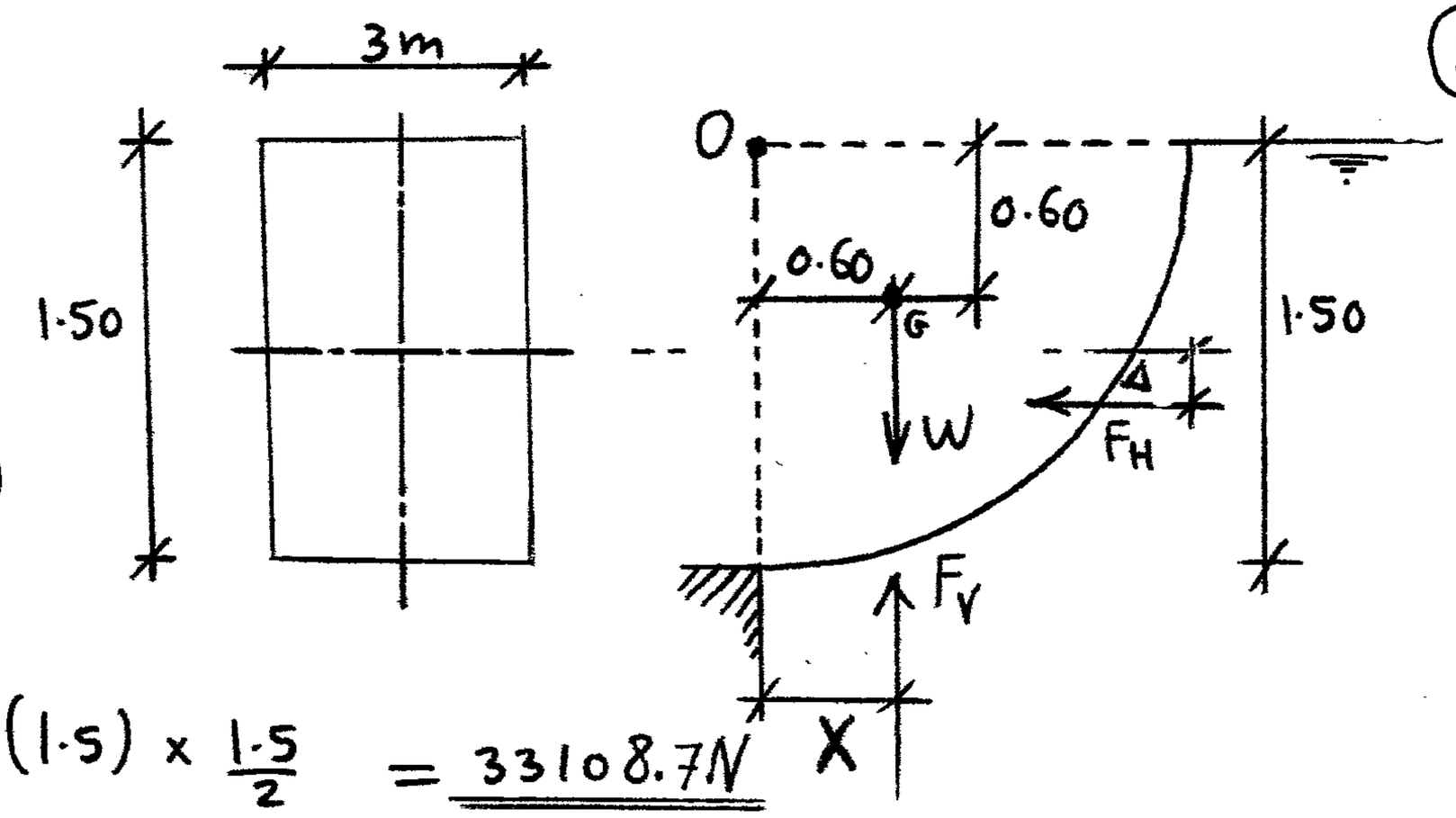
$$F_{V_1} = \gamma V_1 = 62.4\,[18*6*1] = 6739.2 \text{ lb} \downarrow$$

$$F_{V_2} = \gamma V_2 = 62.4\left[\frac{1}{4}\pi(6)^2*1\right] = 1764.32 \text{ lb} \uparrow$$

$$F_V = F_{V_1} - F_{V_2} = \underline{\underline{4974.88 \text{ lb}}} \downarrow$$

$$\bar{x} = \frac{F_{V_1}X_1 - F_{V_2}X_2}{F_V} = \frac{6739.2(3) - 1764.32\left(\frac{4(6)}{3\pi}\right)}{4974.88} = 3.16$$

[9]



$M = 6000\ kg$

$F_H = \gamma A \bar{h}$

$= 9810(3)(1.5) \times \frac{1.5}{2} = \underline{\underline{33108.7\,N}}$

$\Delta_y = \frac{I_{cg}}{A\bar{y}} = \frac{\frac{3(1.5)^3}{12}}{3(1.5)\left(\frac{1.5}{2}\right)} = 0.25$

$F_V =$ ↑

$= \gamma \forall = \gamma \frac{\pi r^2}{4} \times b$

$= 9810 \frac{\pi (1.5)^2}{4} \times 3 = \underline{\underline{52007\,N}}$

$X = \frac{4r}{3\pi} = \frac{4(1.5)}{3\pi} = 0.63662$

Check

Curved Surface الحصلة R دائماً عمودية على ال

$\sum M_{\text{center}} = 0$

∴ يجب أن تمر الحصلة دائماً في ال Center

∴ عزم الحصلة دائماً حول المركز تساوى صفر

$52007(0.63662) - 33108.7(1) \cong 0$

Resultant passes through O

Turning moment $= W \times 0.6 = 9.81 \times 6000 \times 0.6$

$= \underline{\underline{35316\ N.m}}$

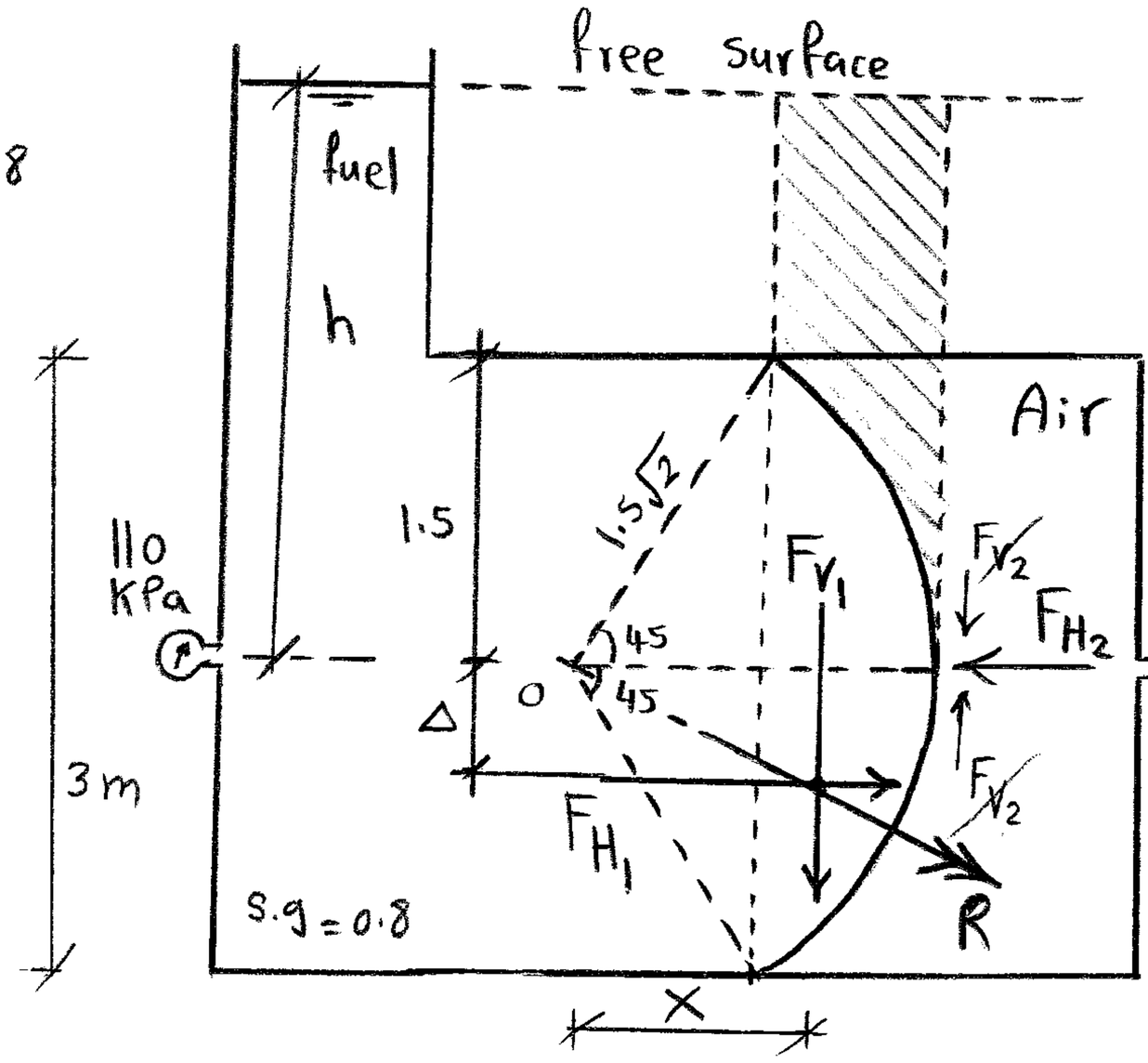
$R = \sqrt{F_H^2 + F_V^2} = \sqrt{(33108)^2 + (52007)^2} = \underline{\underline{61651\,N}}$

10 $B = 3m$, $S.g = 0.8$

$P = \gamma h$

$110 * 10^3 = 0.8\ (9810)\ h$

$\Rightarrow \quad h = 14.016\ m$



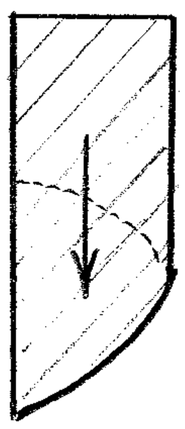
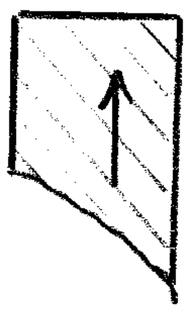
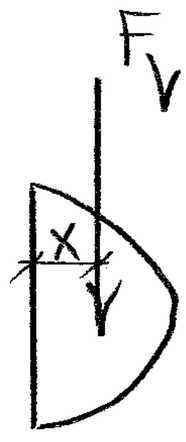
1st Fuel

$F = \gamma A \bar{h}$

$= 0.8\ (9810)\ [3 * 3]\ 14.016 = \underline{990 * 10^3\ N}$

$\Delta = \dfrac{I_{cg}}{A\bar{y}} = \dfrac{\frac{3(3)^3}{12}}{(3*3)\ 14.016} = \underline{0.05351\ m}$

VI force

$F_V = F_{V_1}$  $- F_{V_2}$  $=$ 

$A_{\text{segment}} = \frac{1}{2} r^2 (\theta^{rad} - \sin\theta)$

$= \frac{1}{2} (1.5\sqrt{2})^2 \left[90 * \frac{\pi}{180} - \sin 90^\circ \right] = 1.2843$

$F_V = \gamma \forall = 0.8\ (9810)\ [3 * 1.2843] = \underline{30237\ N}$

∵ R passes through the center of C.S. طريقة سهلة لإيجاد X

$\therefore \overset{\curvearrowright}{M_O} = F_{V_1} X - F_{H_1} \Delta = 0$

$X = \dfrac{F_{H_1} \Delta}{F_{V_1}} = \dfrac{990 * 10^3 * 0.05351}{30237} = \underline{1.752\ m}$

2nd Air

Vertical forces F_{V_2} Cancel each other

Hzl force acts in the C.g. of the projected area

$$F_{H_2} = P.A$$

$$= 110 * 10^3 * [3 * 3] = \underline{990 * 10^3 N}$$

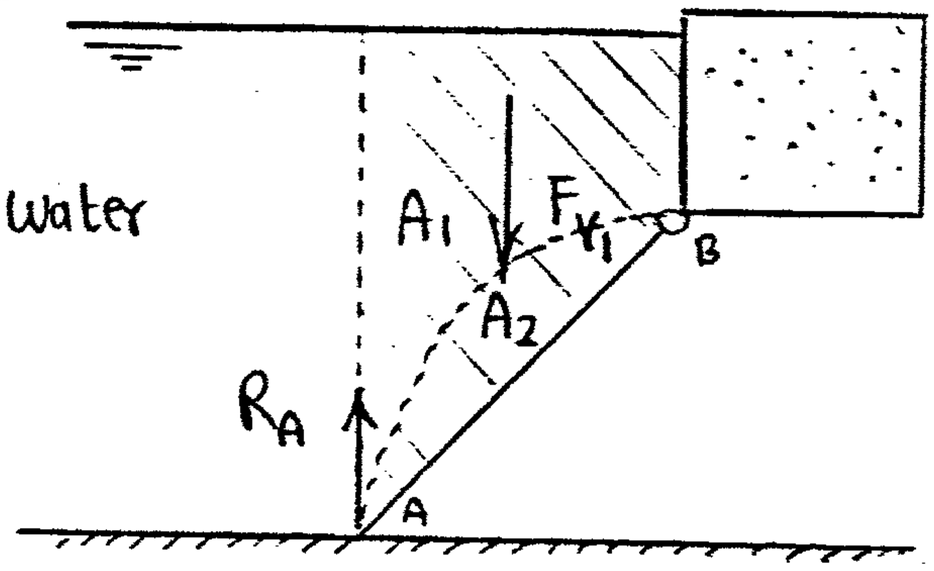
Resultant

$$F_H = F_{H_1} - F_{H_2} = 0 \quad , \quad F_V = 30237\ N.$$

$$R = \sqrt{F_H^2 + F_V^2} = \underline{30237\ N \downarrow}$$

8

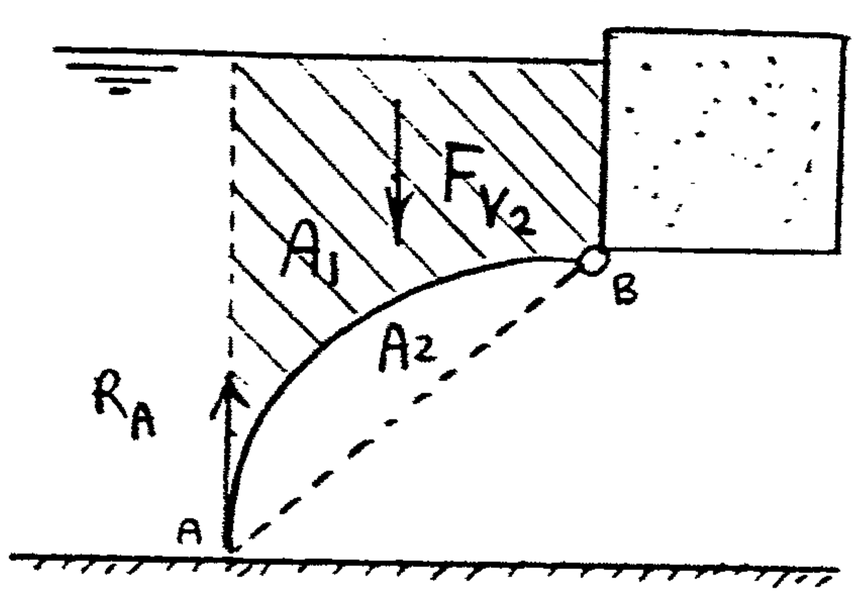
fig (a)



$$F_{V_1} = \gamma V_1$$

$$= \gamma (\underline{A_1 + A_2}) B$$

fig (b)



$$F_{V_2} = \gamma V_2$$

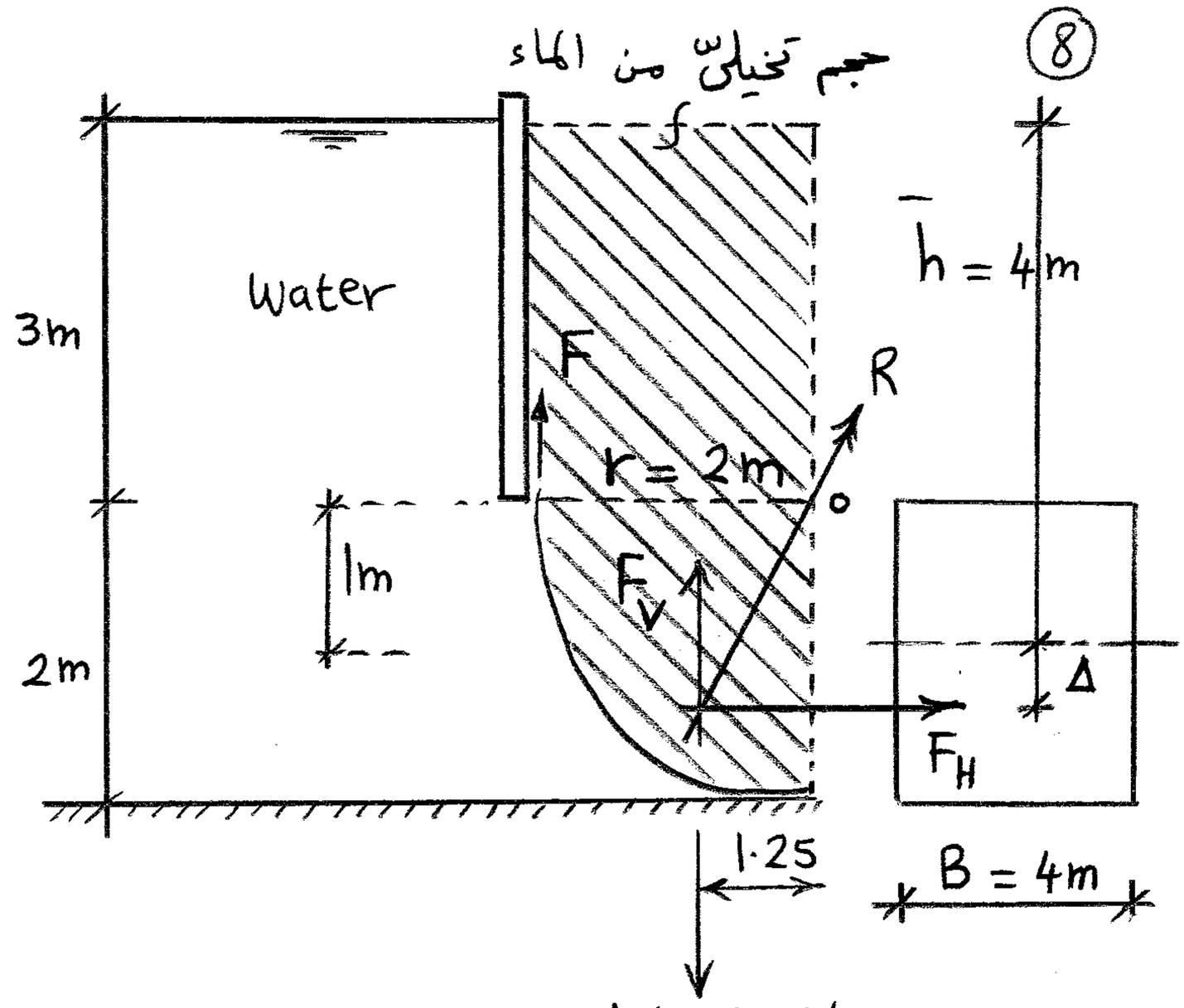
$$= \gamma \underline{A_1} B$$

$$\Rightarrow F_{V_1} > F_{V_2}$$

R_A in fig (B) will be less than R_A in fig (A)

III

$B = 4\,m$ wide



Hz l force

$$F_H = \gamma A \bar{h}$$

$$= 9810(4*2)\,4$$

$$= \underline{313\,920\ N} \rightarrow$$

VI force

$$F_V = \gamma_w \left[\square + \square \right]$$

$$= 9810 \left[3*2*4 + \frac{1}{4}\pi(2)^2 * 4 \right] = \underline{358\,716\ N} \uparrow$$

a) Neglecting the weight of the gate

No force is required to open the gate as the resultant of the water pressure passes through O (center of rotation)

b) The gate weight is 35 N

$$\Sigma M_O = 0$$

$$2F = 35 * 1.25$$

$$F = \underline{21.875\ N}$$

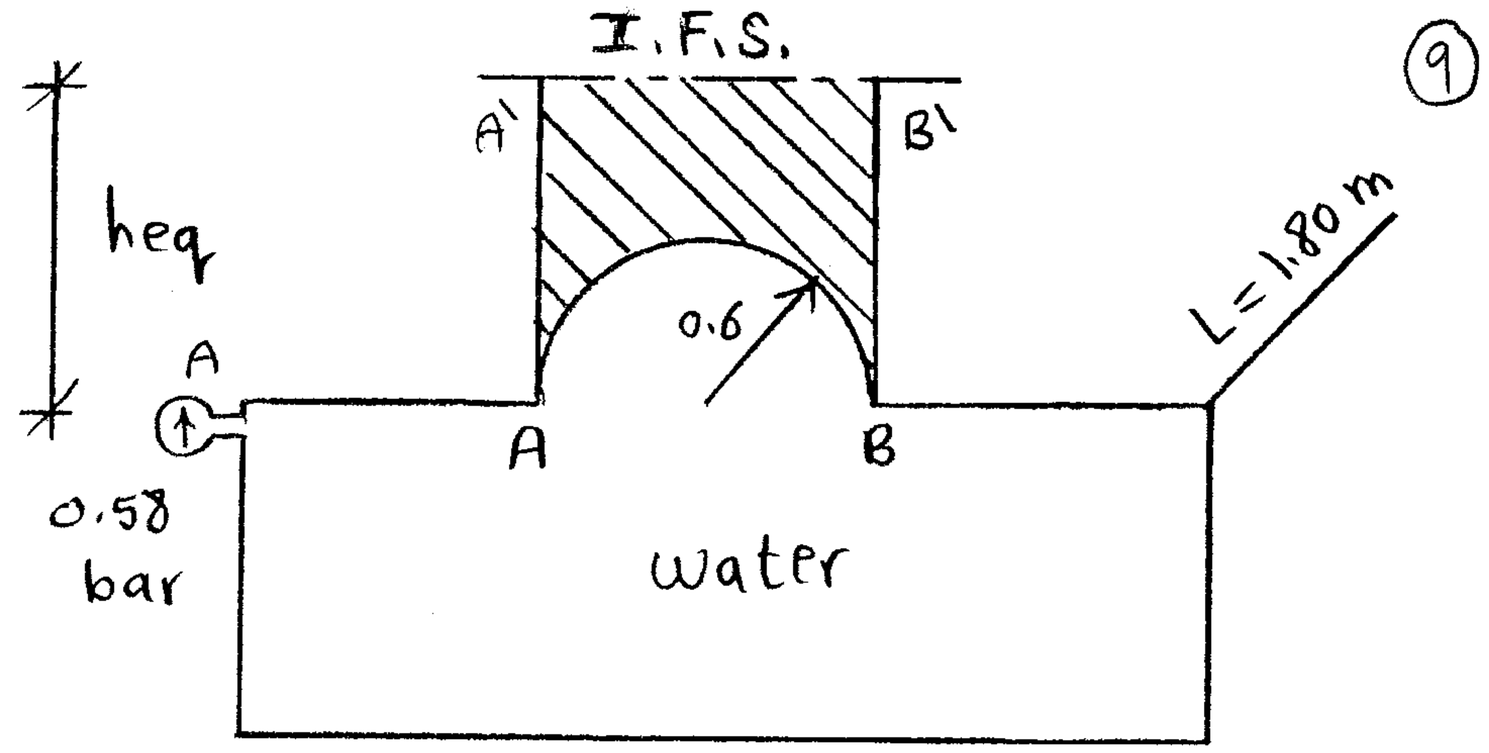
Cylinder

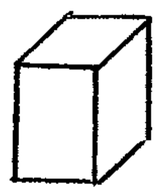
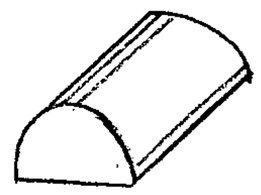
16 (a)

$P = \gamma_w h_{eq}$

$0.58 * 10^5 = 9810\ h_{eq}$

$\Rightarrow h_{eq} = 5.912\ m$

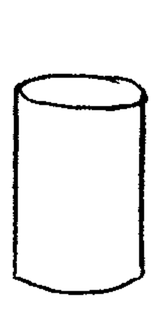


$F_{V_{water}} = \gamma_w [$  $-$  $]$

$= 9810 \left[1.2 * 5.912 * 1.80 - \frac{1}{2}\pi(0.6)^2 * 1.80 \right]$

$= \underline{\underline{115294\ N}}$

(b) Dome

$F_{V_{water}} = \gamma_w [$  $-$  $]$

$= 9810 \left[\pi(0.6)^2 * 5.912 - \frac{1}{2} * \frac{4}{3}\pi(0.6)^3 \right]$

$= \underline{\underline{61155\ N}}$

There is no hzl force in the above two problems as the hzl forces cancels each other.

$\therefore F_H = 0$

Examples

1- Reffering to fig. 9, determine the horizontal and vertical forces acting on the 2m diameter cylinder per meter length.

2- A cylindrical barrier 0.30m long and 0.60m diameter as shown in figure 8. Determine the magnitude of the horizontal and vertical components of the force due to water pressure exerted against the wall.

3- Determine the magnitude of the horizontal and vertical components of the force which will keep the cylinder in figure 10 just touching the tank floor. The cylinder is 1 m long and weighs 200 kg. The diameter of the cylinder is 2 m.

4- Determine the hydrostatic force acting on the radial gate as shown in figure 11, if the gate is 40 ft long. Show the line of action of the force acting on the gate.

5- In figure 12, calculate the absolute air pressure in the tank if the atmospheric pressure is 100 KN/m^2. Also, determine per m of length, the total horizontal force on ABC and the point of application of this force.

6- Calculate F to hold gate if R = 1.5, 2 ft

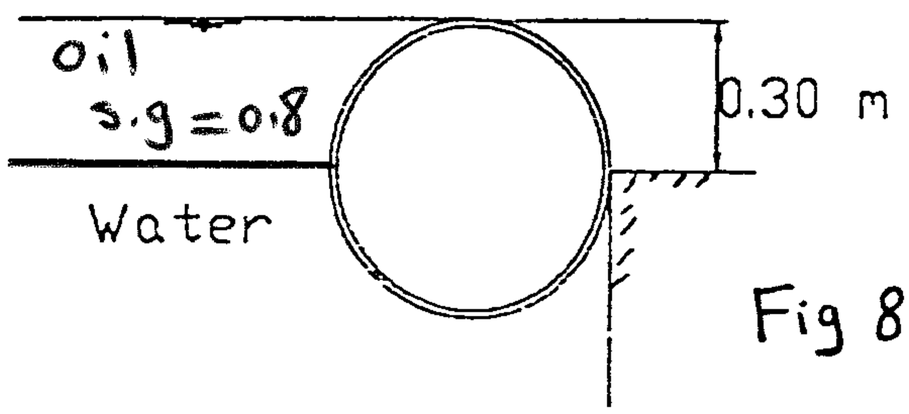


Fig 8

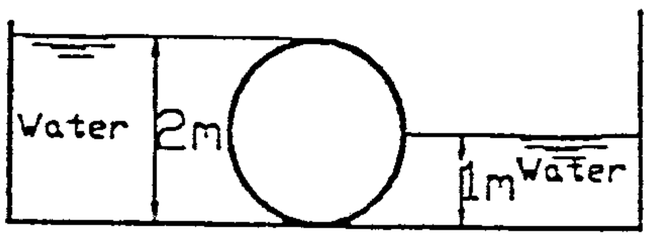


Fig 10

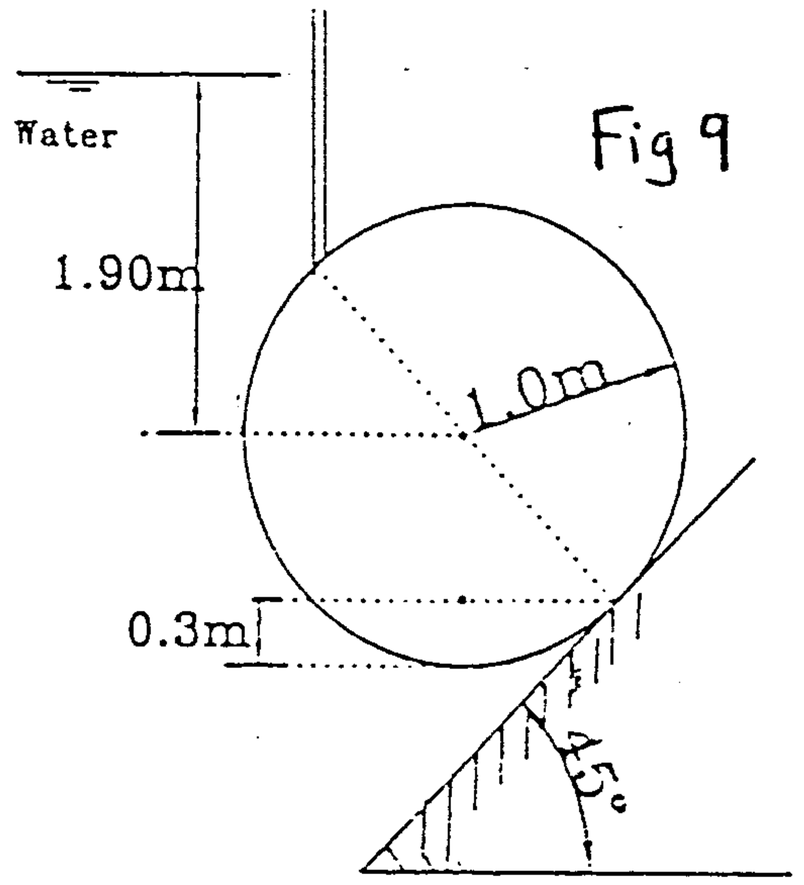


Fig 9

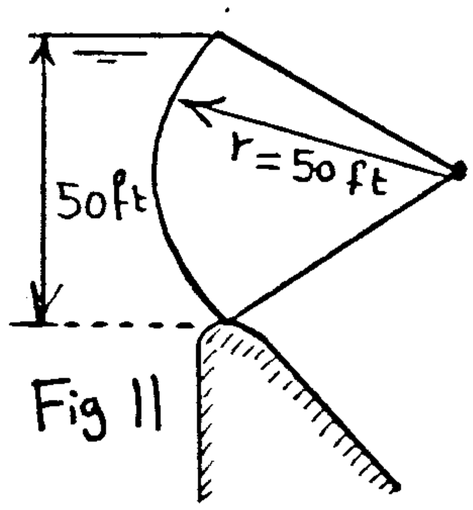


Fig 11

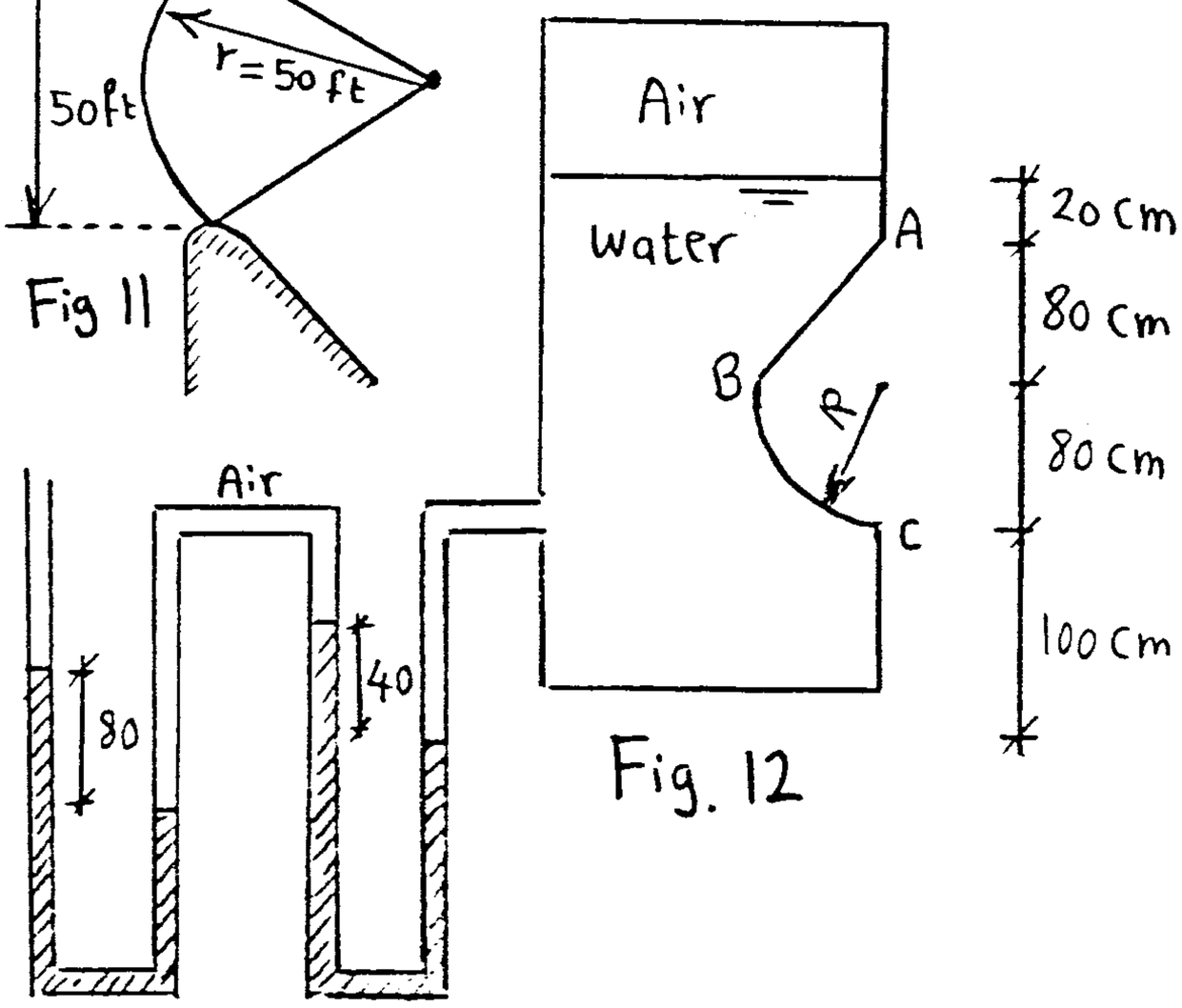
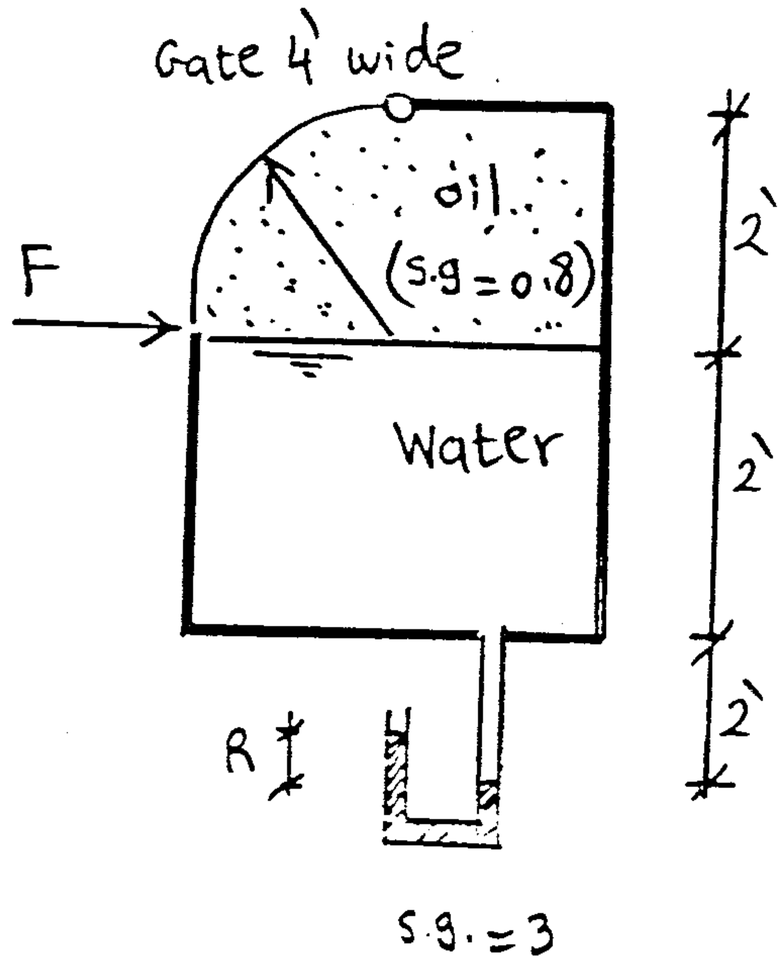
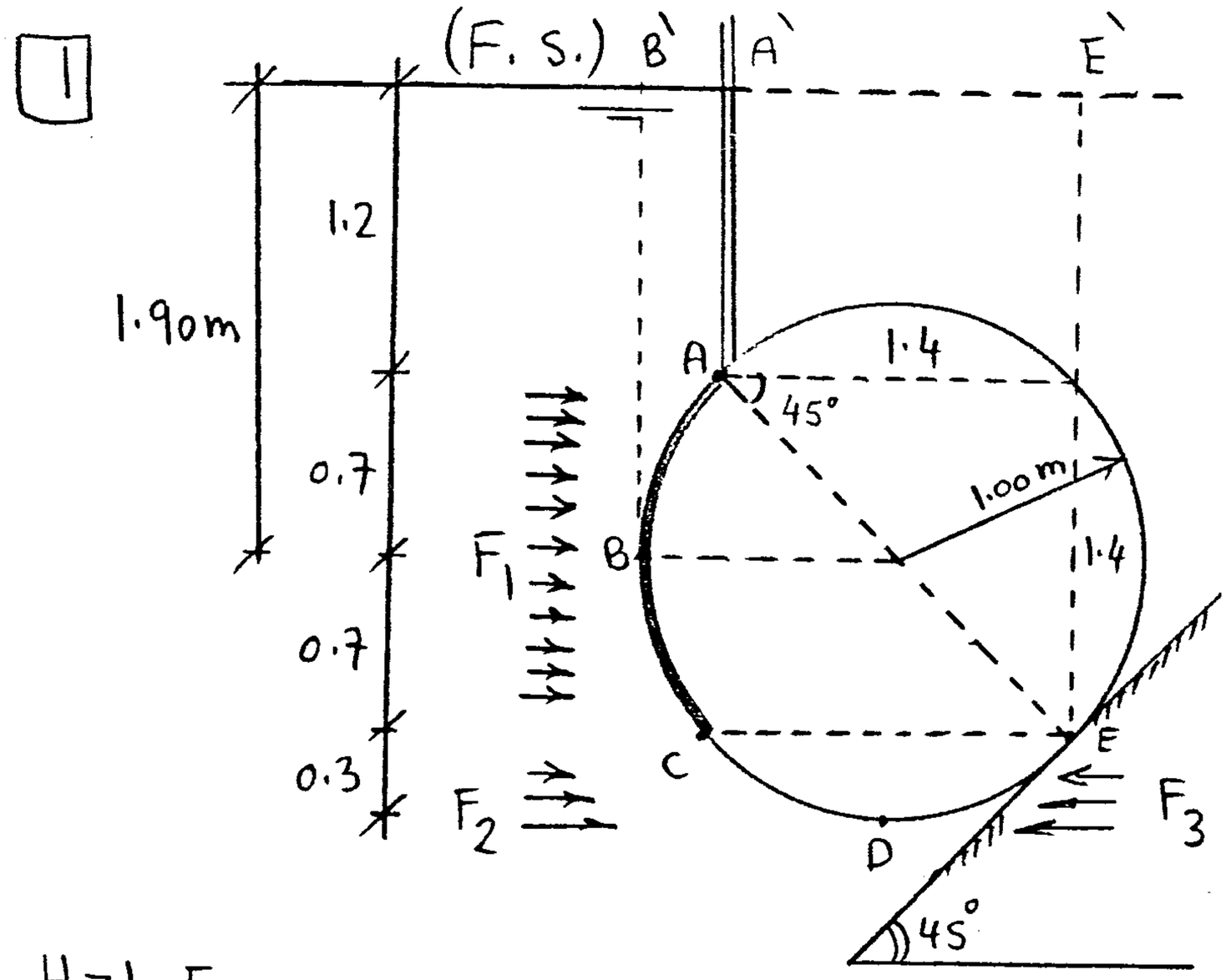


Fig. 12

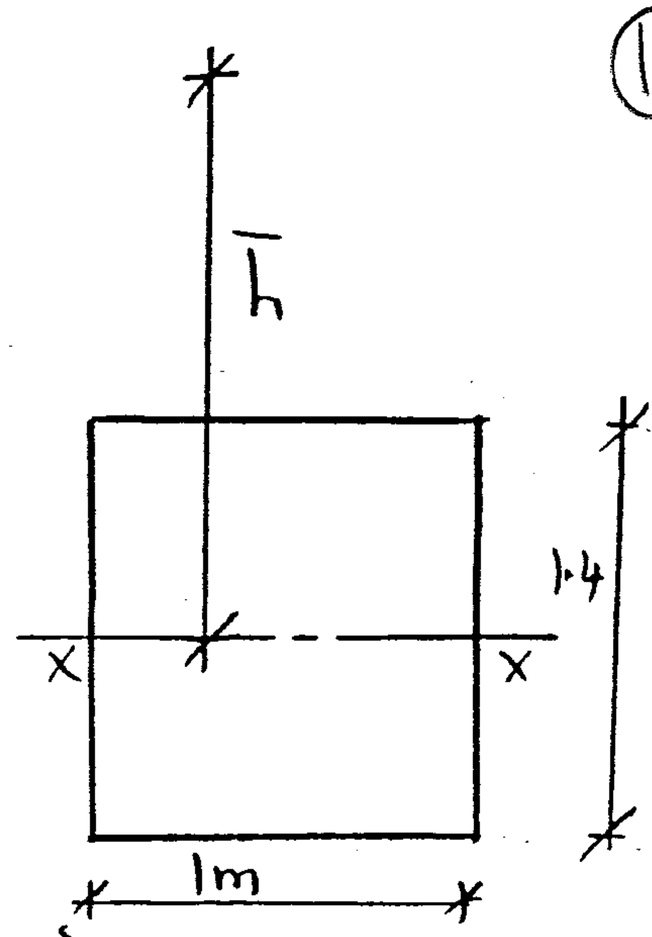


Problem (6)

(I)



(II)



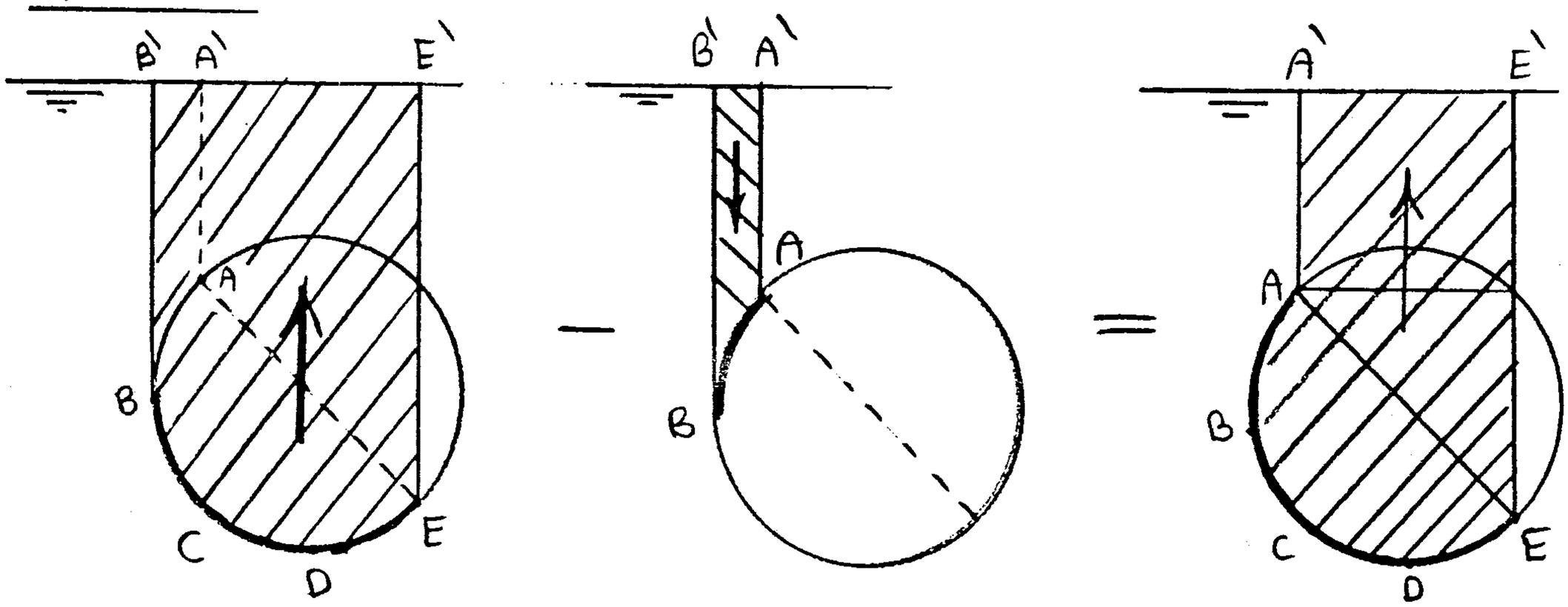
اسقاط ال C.S على مستوى رأسى

Hzl Force

$$F_H = F_1 + \cancel{F_2} - \cancel{F_3} = F_1$$

$$= \gamma A \bar{h} = 9810\,(1.4 * 1.0) * 1.9 = \underline{\underline{26095\ N}} \rightarrow$$

Vl Force



$$F_{V_1} \uparrow \quad - \quad F_{V_2} \downarrow \quad = \quad F_V \uparrow$$

$$F_V = \gamma\,(\square + \triangleleft + \frown)$$

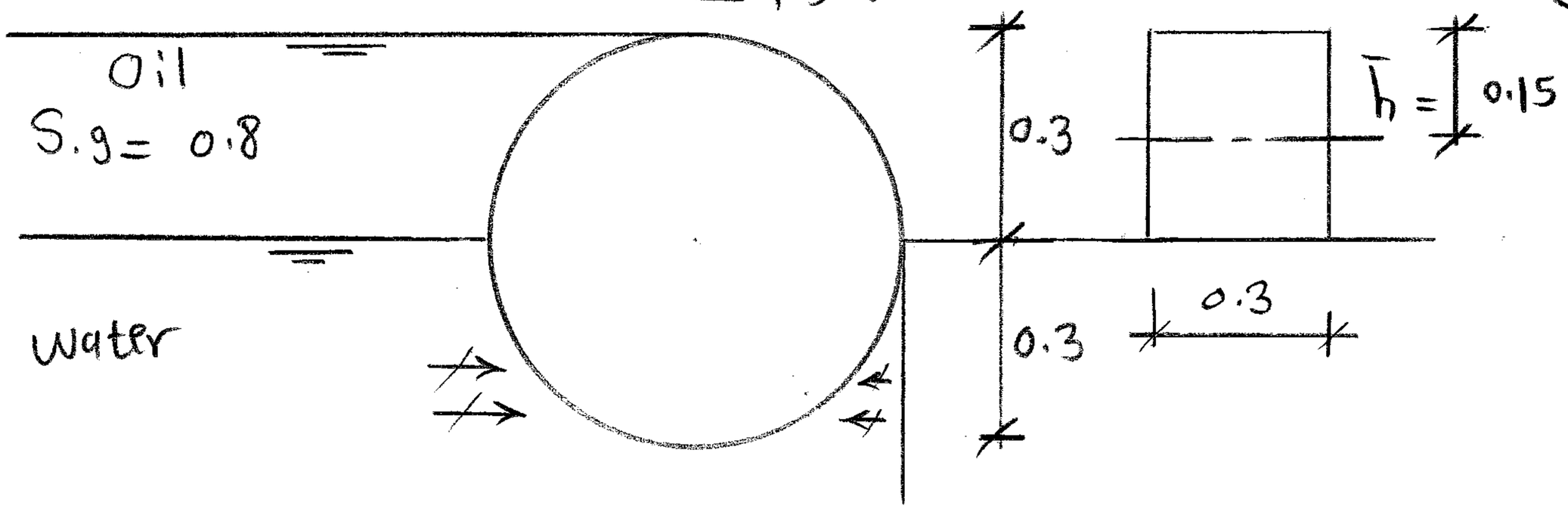
$$= 9810\left(1.2 * 1.4 + \tfrac{1}{2}(1.4)(1.4) + \tfrac{1}{2}\pi(1)^2\right) * 1$$

$$= \underline{\underline{41504\ N \uparrow}}$$

2

مثل رقم 14 في الـ Sheet

(12)



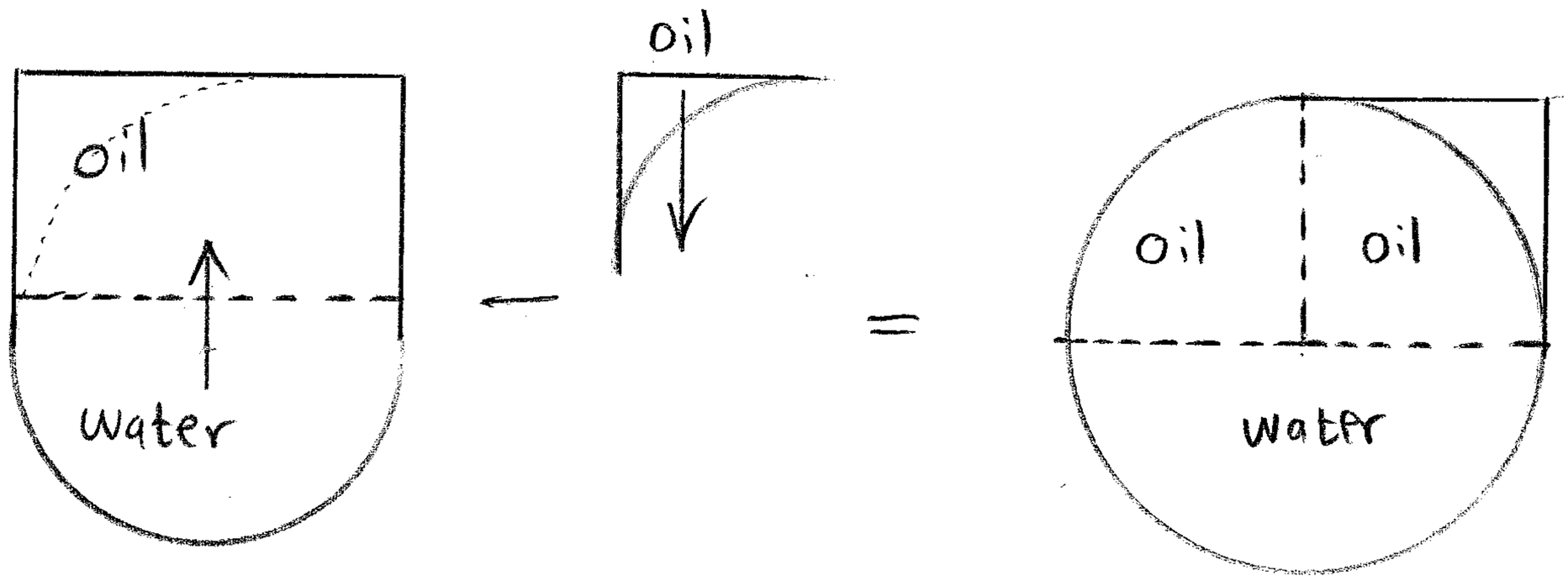
Horizontal Force

Horizontal forces of water Cancels each other

$$\therefore F_H = \gamma A \bar{h}$$

$$= 0.8(9810)(0.3 * 0.3)(0.15)$$

Vertical Force

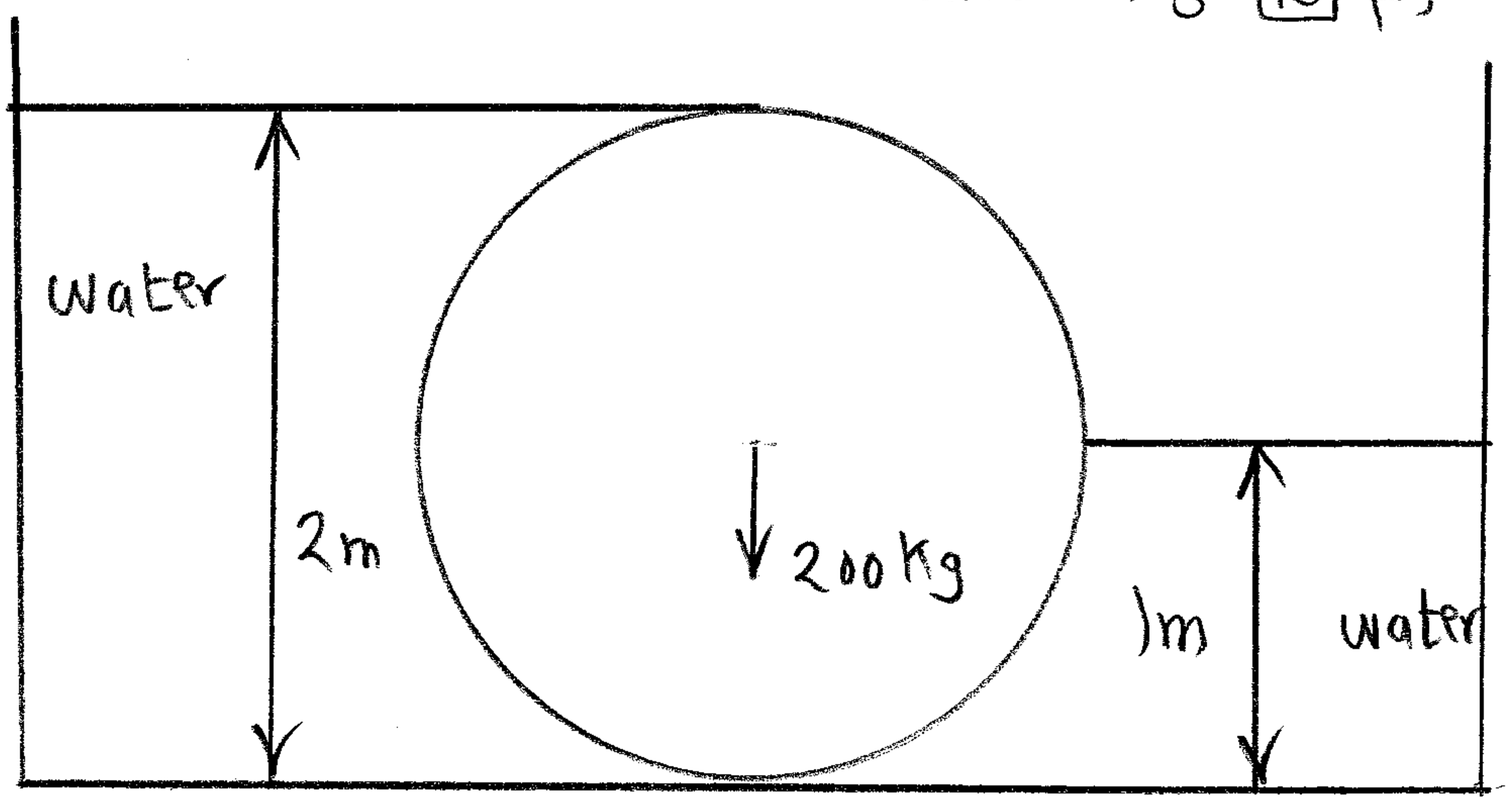


$$F_V = \gamma_{oil}\left(\frac{1}{4}\pi r^2 + r^2\right)b + \gamma_w\left(\frac{1}{2}\pi r^2\right)b$$

$$= 0.8(9810)\left(\frac{1}{4}\pi(0.3)^2 + (0.3)^2\right)0.3$$

$$+ 9810\left(\frac{1}{2}\pi(0.3)^2\right)0.3 = \underline{794\ N}$$

3



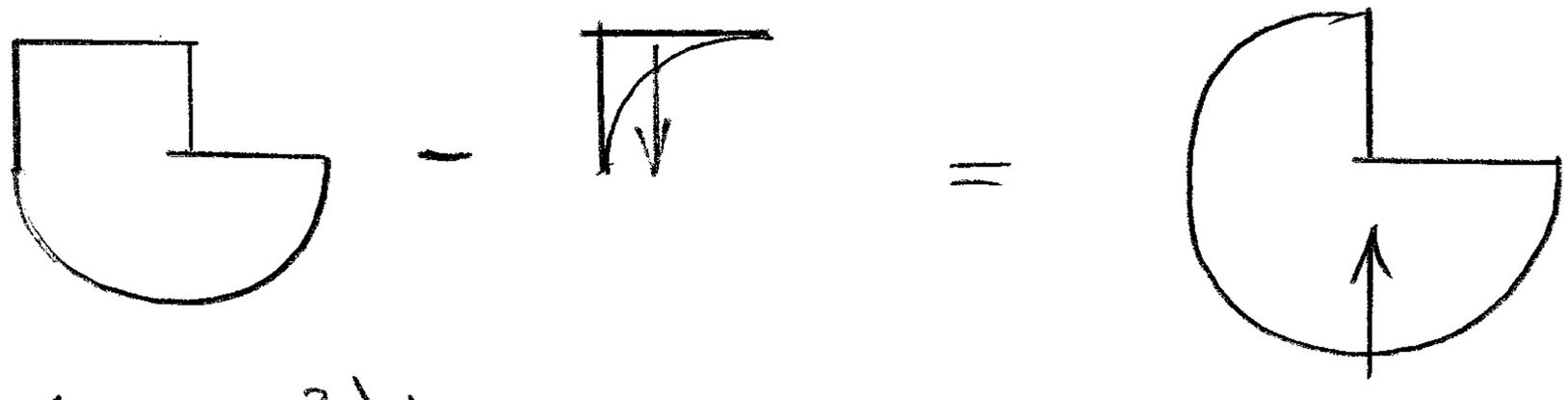
$B = 1\,m$

$$F_H = \gamma A_1 \bar{h}_1 - \gamma A_2 \bar{h}_2$$

$$= 9810(2*1)1 - 9810(1*1)0.5$$

$$= \underline{14715\ N}$$

$$F_V = F_{V_1} - F_{V_2}$$



$$= \gamma\left(\frac{3}{4}\pi r^2\right)b$$

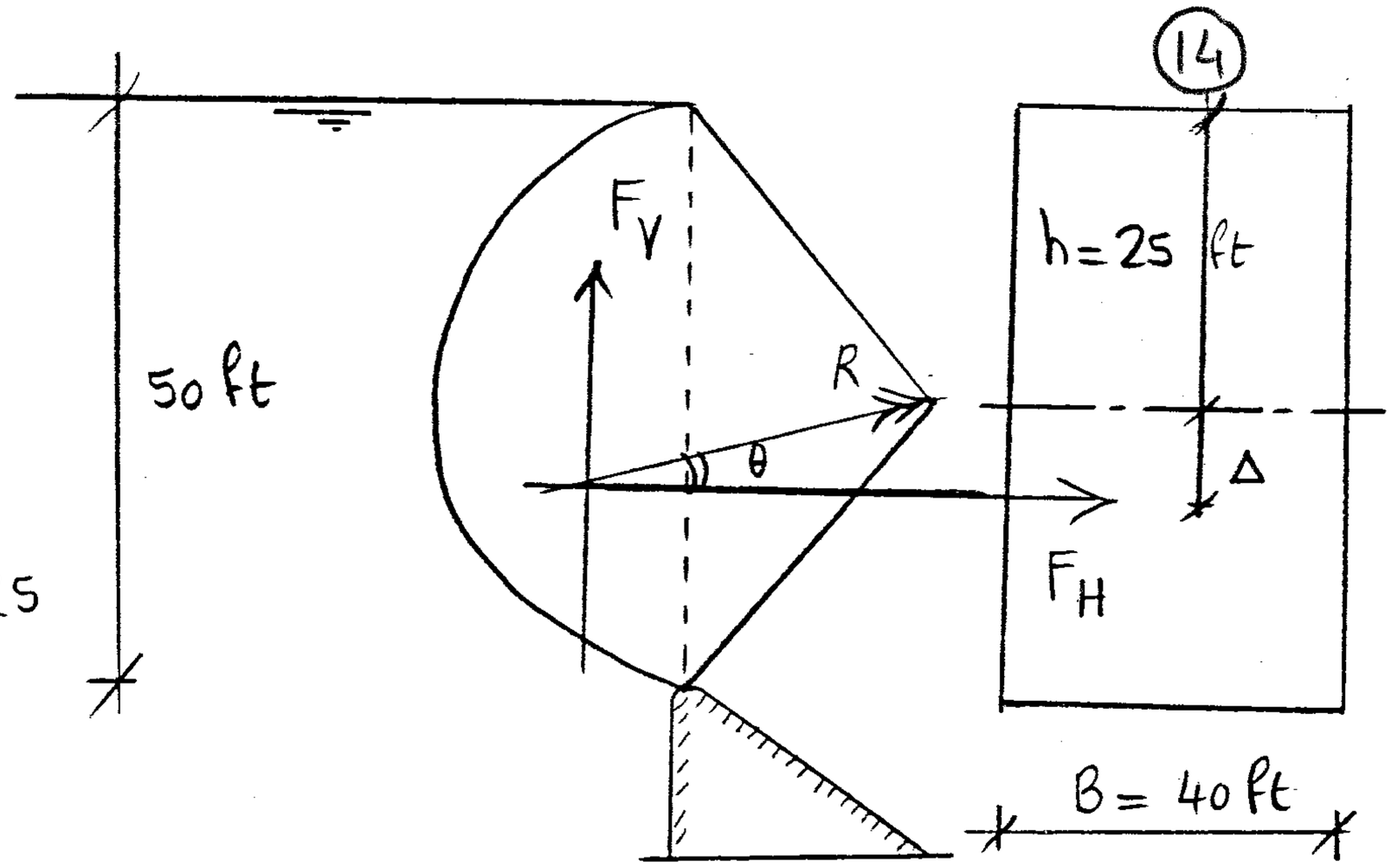
$$= 9810\left(\frac{3}{4}\pi(1)^2\right)1 = \underline{23114\ N}$$

$$W = 200 * 9.81 = \underline{1962\ N}$$

Horizontal Component = $\underline{14715\ N}$

Vertical Component = $23114 - 1962 = \underline{21152\ N}$

(4)



Hzl force

$$F = \gamma A \bar{h}$$

$$= 62.4(40*50)25$$

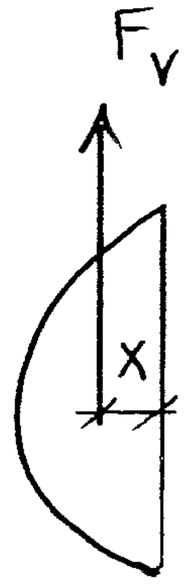
$$= \underline{3120\,000 \text{ lb}}$$

$$\Delta = \frac{I_{cg}}{A\bar{y}}$$

$$\Delta = \frac{\frac{40(50)^3}{12}}{(40*50)\ 25} = \underline{8.333 \text{ ft}}$$

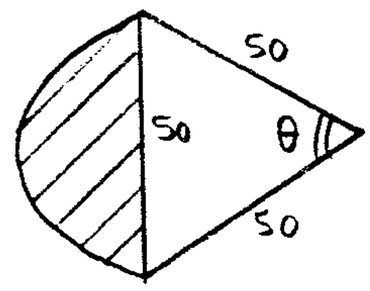
Vl force

$$F_V = F_{V_1} - F_{V_2} =$$



$$A_{\text{segment}} = \frac{1}{2} r^2 (\theta^{rad} - \sin\theta)$$

$$= \frac{1}{2}(50)^2 \left[60 * \frac{\pi}{180} - \sin 60^\circ\right] = 226.46$$



مثلث متساوى الاضلاع ⇐ $\theta = 60^\circ$

$$F_V = \gamma \forall = 62.4(226.46 * 40) = \underline{565264 \text{ lb}}$$

∵ R passes through the center of C.S. طريقه سهلة لإيجاد X

$$\therefore \overset{\curvearrowright}{M_o} = F_V X - F_H \Delta = 0$$

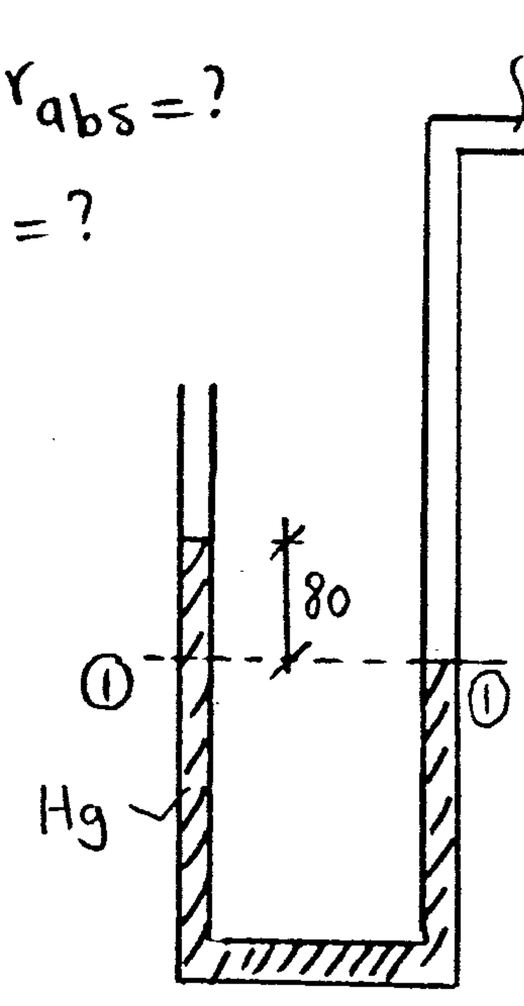
$$X = \frac{F_H . \Delta}{F_V} = \frac{3120\,000 * 8.333}{565264} = \underline{45.996 \text{ ft}}$$

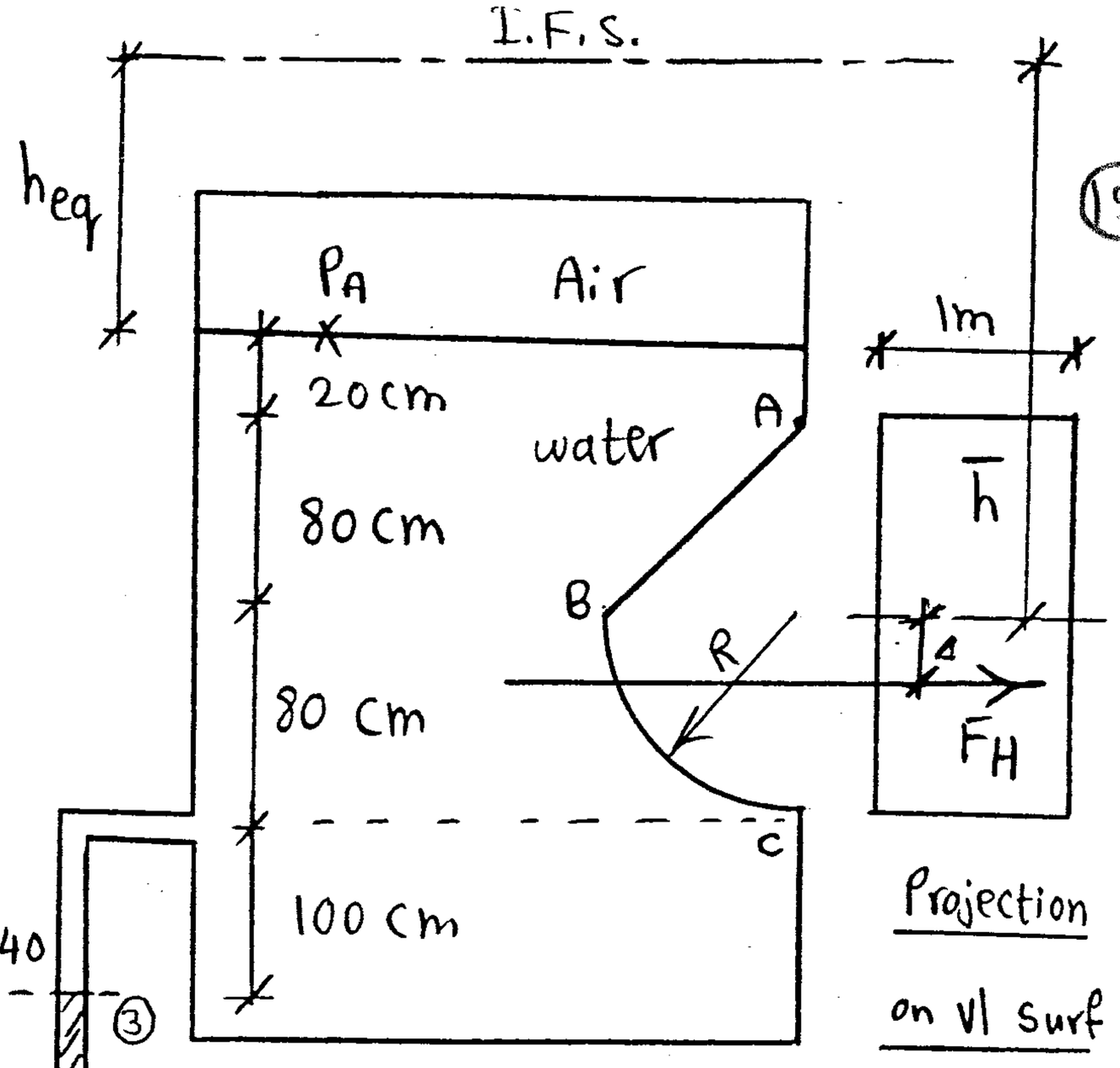
(5) Sheet الى [12] رقم

$P_{atm} = 100\ KN/m^2$

$P_{air_{abs}} = ?$

$F_H = ?$





$P_1 = 13.6\ (9810) * 0.8 = 106733\ N/m^2$

$P_2 = P_1 = 106733\ N/m^2$

$P_3 = P_2 + 13.6\ \gamma_w (0.4)$

$= 106733 + 13.6\ (9810)\ 0.4 = 160100\ N/m^2$

$P_A = 160100 - 9810\ (2.8) = 132632\ N/m^2$ (gage)

$P_{A_{abs}} = 132632 + 100 * 10^3 = \underline{\underline{232632}}\ N/m^2$ (abs)

$h_{eq} = \dfrac{P_{a\ gage}}{\gamma_w} = \dfrac{132632}{9810} = 13.52\ m$

$\bar{h} = 14.52$

$F_h = \gamma A \bar{h} = 9810\ (1.6 * 1) * (0.8 + 0.2 + 13.52) = \underline{\underline{227907}}$

$I_{c.g} = \dfrac{1\ (1.6)^3}{12} = 0.341\ m^4$

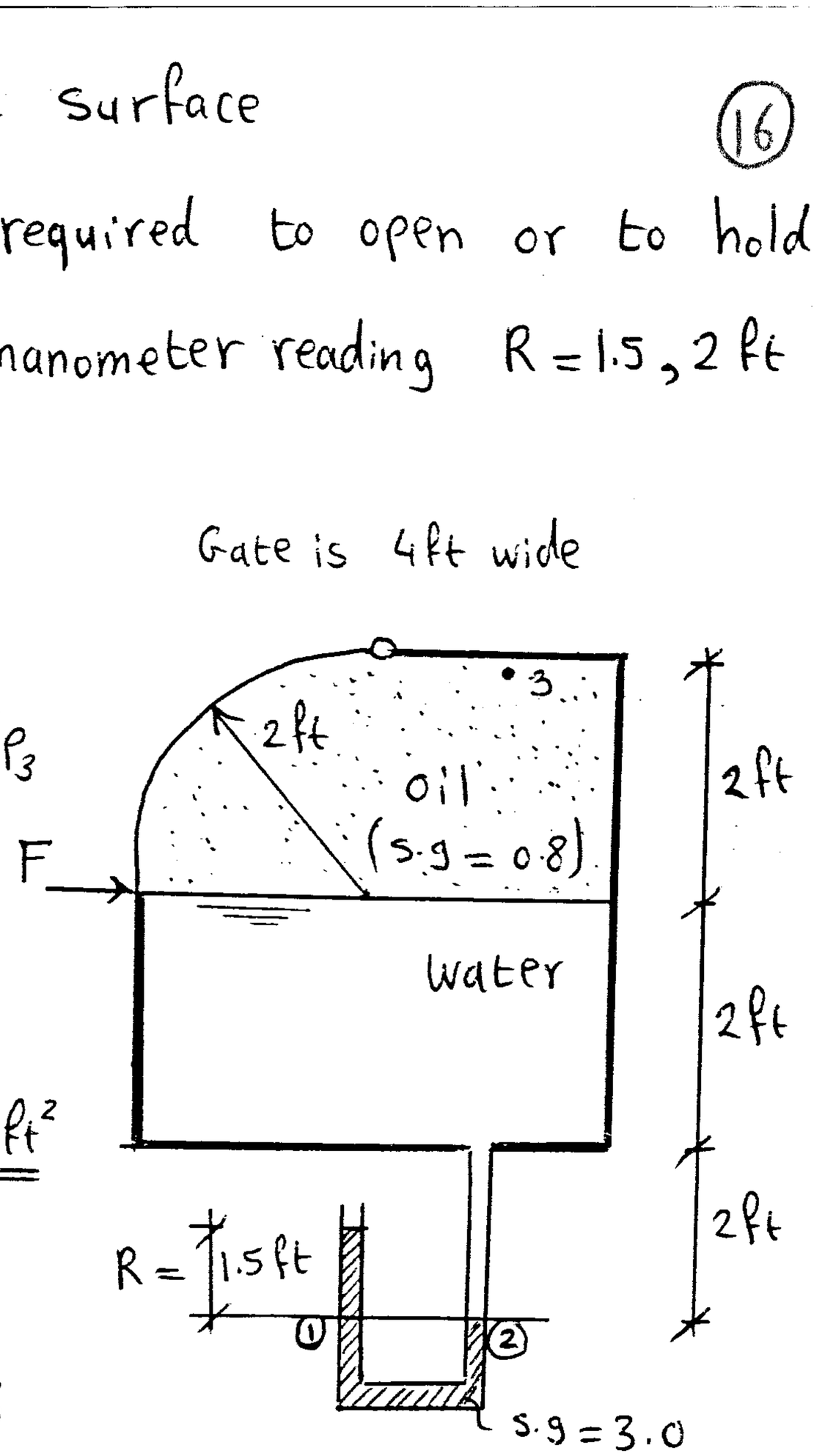
$\Delta = \dfrac{I_{c.g}}{A\bar{y}} = \dfrac{0.341}{1.6 * 14.52} = \underline{\underline{0.0147\ m}}$

Example — Curved surface

Calculate the force F required to open or to hold the gate shown if the manometer reading R = 1.5, 2 ft

Solution

If R = 1.5 ft



There is no free surface ⇒ calculate P_3

$\because P_1 = P_2$

$3\gamma_w(1.5) = P_3 + 0.8\gamma_w(2) + \gamma_w(4)$

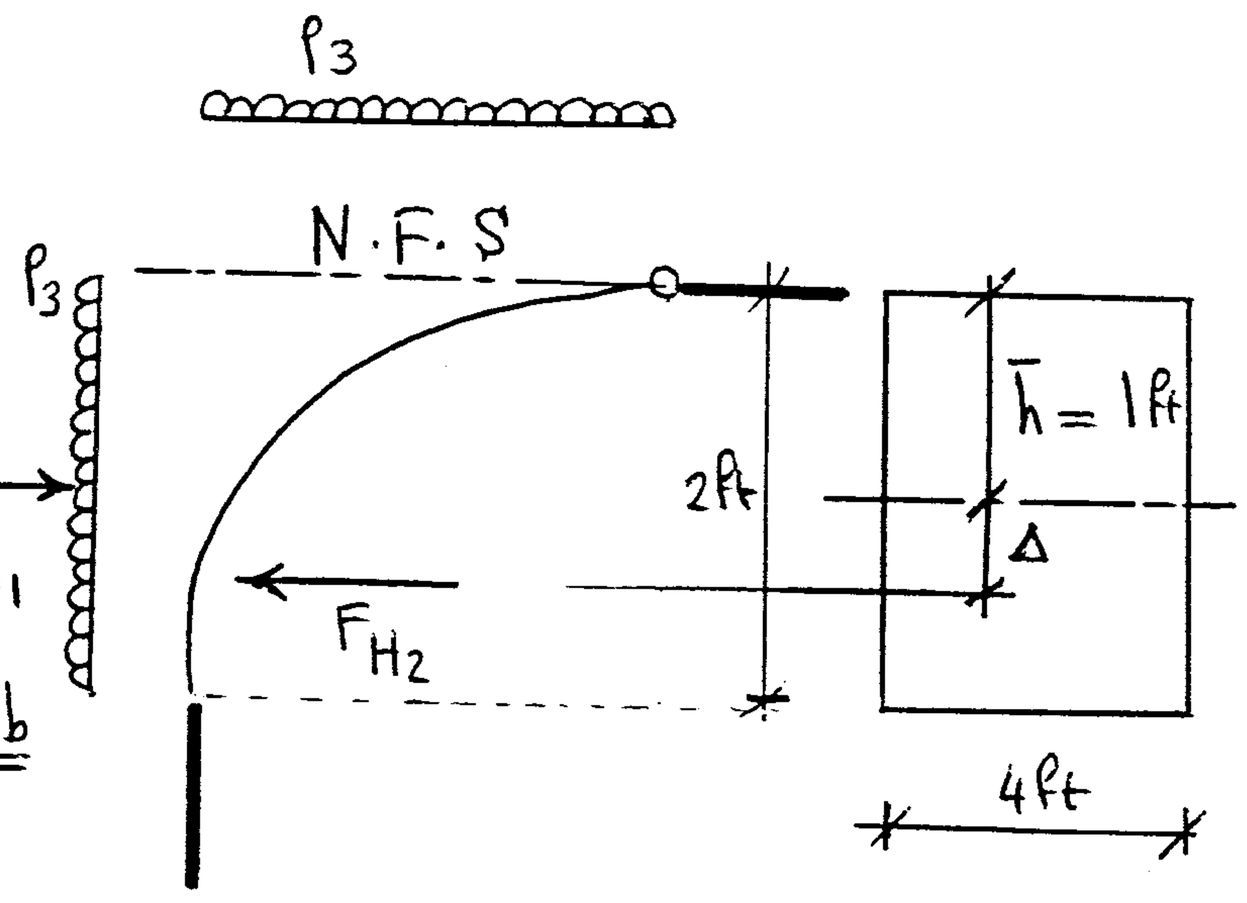
$\Rightarrow P_3 = -1.1\gamma_w = -\underline{68.64\ lb/ft^2}$

We cannot solve with I.F.s

نبدل الضغط السالب بـ distributed load

ونعتبر النقطه ③ هي New Free Surface (N.F.S)

To calculate F_H



$F_{H_1} = P_3 A$

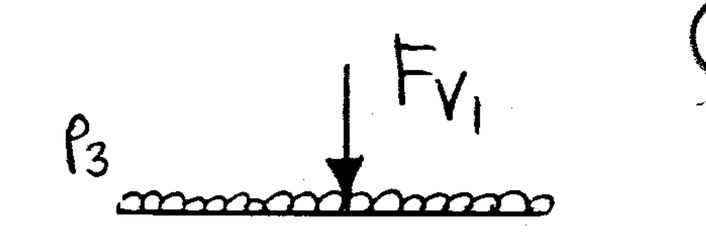
$= 68.64(2*4) = \underline{549.12\ lb}$

$F_{H_2} = \gamma A \bar{h}$

$= 0.8(62.4)[2*4]\ 1 = \underline{399.36\ lb}$

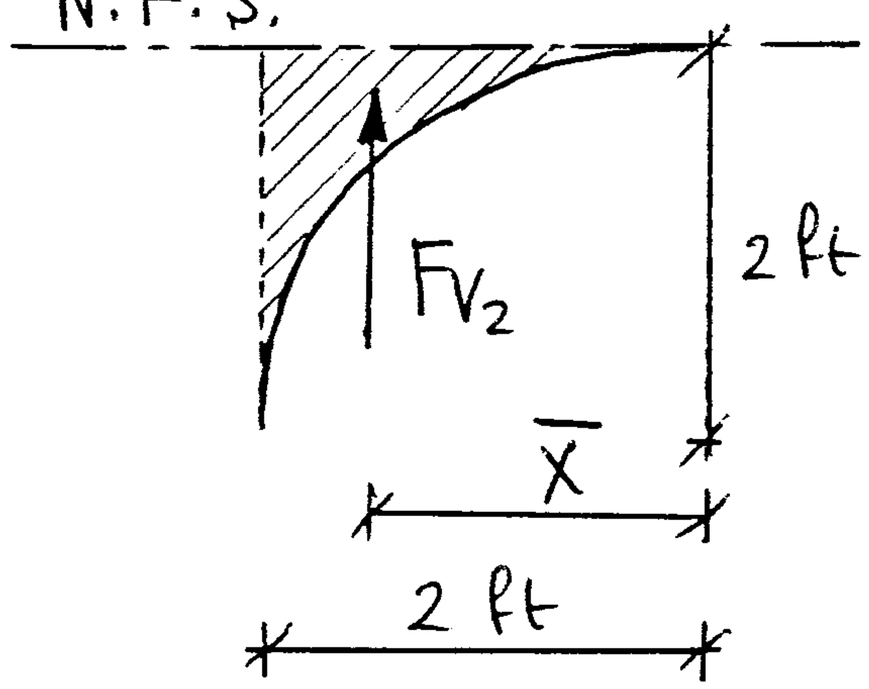
$\Delta = \frac{I_{cg}}{A\bar{y}} = \frac{\frac{4(2)^3}{12}}{(4*2)*1} = \underline{\frac{1}{3}\ ft}$

To calculate F_V



$$F_{V_1} = P_3 A$$

$$= 68.64\ (2*4) = \underline{549.12\ lb}$$

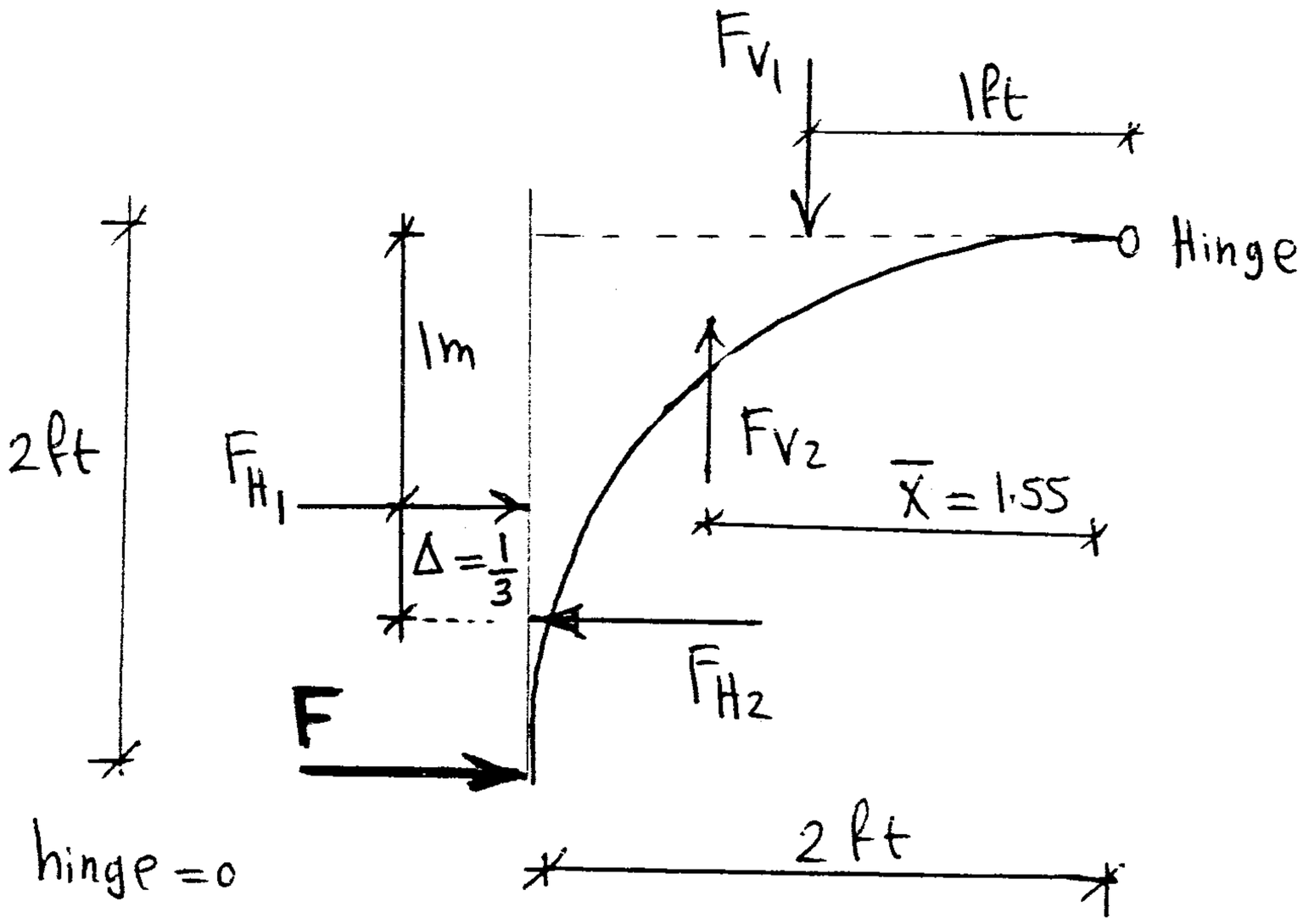


$$F_{V_2} = \gamma_{oil} \forall = \gamma_{oil}$$

$$= \gamma_{oil} \left[\square - \text{curved} \right]$$

$$= 0.8(62.4) \left[2*2 - \tfrac{1}{4}\pi(2)^2 \right] * 4 = \underline{171.4\ lb}$$

$$\bar{X} = \frac{A_1 X_1 - A_2 X_2}{A} = \frac{[2*2]\,1 - \left[\frac{1}{4}\pi 2^2\right]\frac{4(2)}{3\pi}}{[2*2] - \left[\frac{1}{4}\pi(2)^2\right]} = \underline{1.55}$$



Moment about hinge = 0

$$F(2) + F_{H_1}(1) + F_V(1) = F_{H_2}\left(\frac{4}{3}\right) + F_{V_2}(1.55)$$

$$\Rightarrow F = \checkmark$$

If manometer reading = 2ft

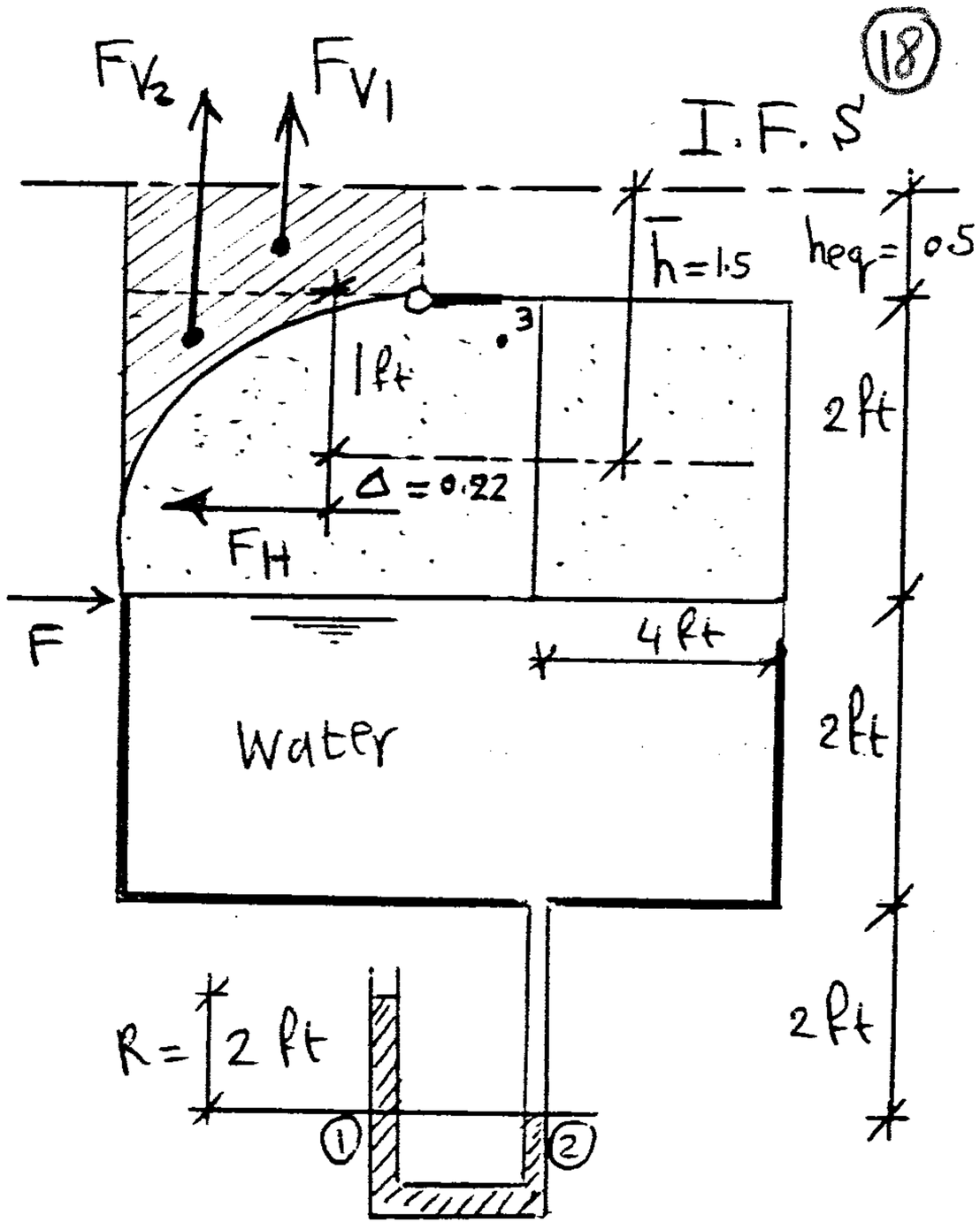
$\therefore P_1 = P_2$

$3\gamma_w(2) = P_3 + 0.8\gamma_w(2) + \gamma_w(4)$

$\Rightarrow P_3 = 0.4\gamma_w = 24.96$

Get the I.F.S.

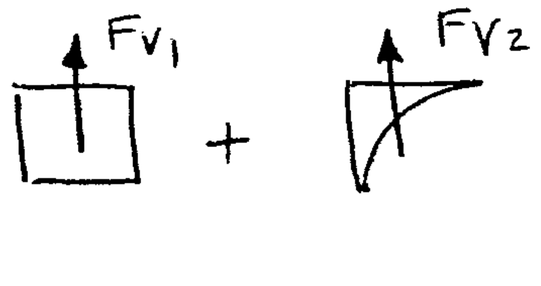
$$h_{eq} = \frac{P_3}{\gamma_{oil}} = \frac{24.96}{0.8(62.4)} = 0.5$$



$F_H = \gamma A \bar{h}$

$= 0.8(62.4)\,[2*4]\,1.5 = \underline{\underline{599.04\ lb}}$

$$\Delta = \frac{I_{cg}}{A\bar{y}} = \frac{\frac{4(2)^3}{12}}{4(2)\ 1.5} = \underline{\underline{0.222\ ft}}$$

$F_V = \gamma V =$  $=$ 

$F_{V_1} = 0.8(62.4)[2*0.5*4] = \underline{\underline{199.68\ lb}}$ or $P_3 A$

$24.96(2*4)$

$F_{V_2} = \underline{\underline{171.4\ lb}}$ as last problem

$\bar{X} = \underline{\underline{1.55\ ft}}$ as last problem

$\sum M_{\text{hinge}} = 0$

$F(2) = F_H(1.222) + F_{V_1}(1) + F_{V_2}(1.55)$ $\Rightarrow F = \checkmark$

13 $B = 1ft$ wide

$\Delta = 0.475mm * \frac{1}{0.3048} m = 1.56 ft$

There is no free surface $\Rightarrow$ Calculate P_3

$\because P_1 = P_2$

$13.6(62.4)\,1.56 = P_3 + 0.85\gamma_w(8)$

$\Rightarrow P_3 = 900\ lb/ft^2$

Get the I.F.S.

$$h_{eq} = \frac{P_3}{\gamma_{oil}} = \frac{900}{0.85(62.4)} = 16.97\ ft$$

$F_H = \gamma A \bar{h}$

$= 0.85(62.4)\,[4*1](2 + 16.97) = \underline{4024.67\ lb}$

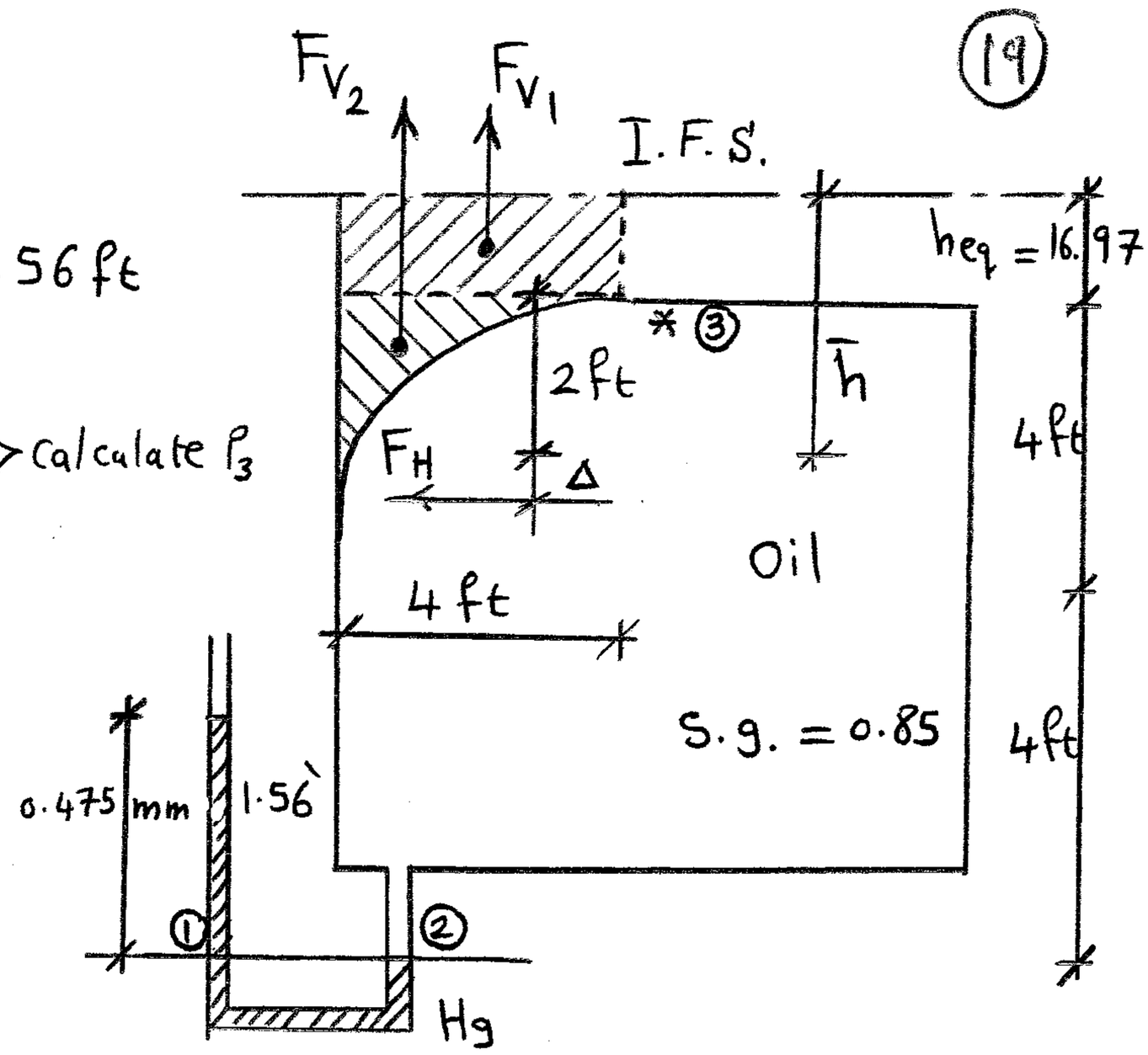
$F_{V_1} = 0.85(62.4)\,[4*16.97*1] = 3600\ lb$ or $P_3 A$

$F_{V_2} = \gamma_{oil} \forall = \gamma_{oil}$ 

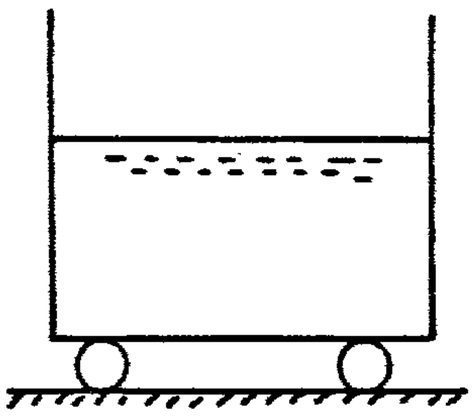
$= \gamma_{oil}\left[\square - \text{◠}\right]$

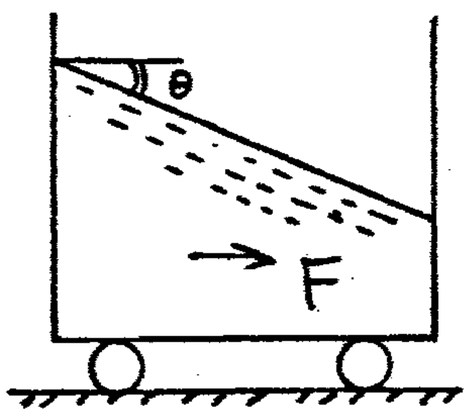
$= 0.85(62.4)\left[4*4 - \frac{1}{4}\pi(4)^2\right] * 1 = 182.1\ lb$

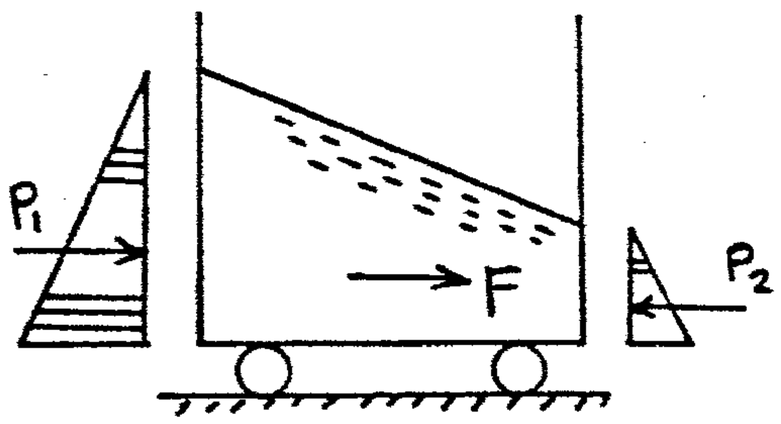
$F_V = F_{V_1} + F_{V_2} = \underline{3782.1\ lb}$



6 Fluid Masses Subjected to Linear Acceleration

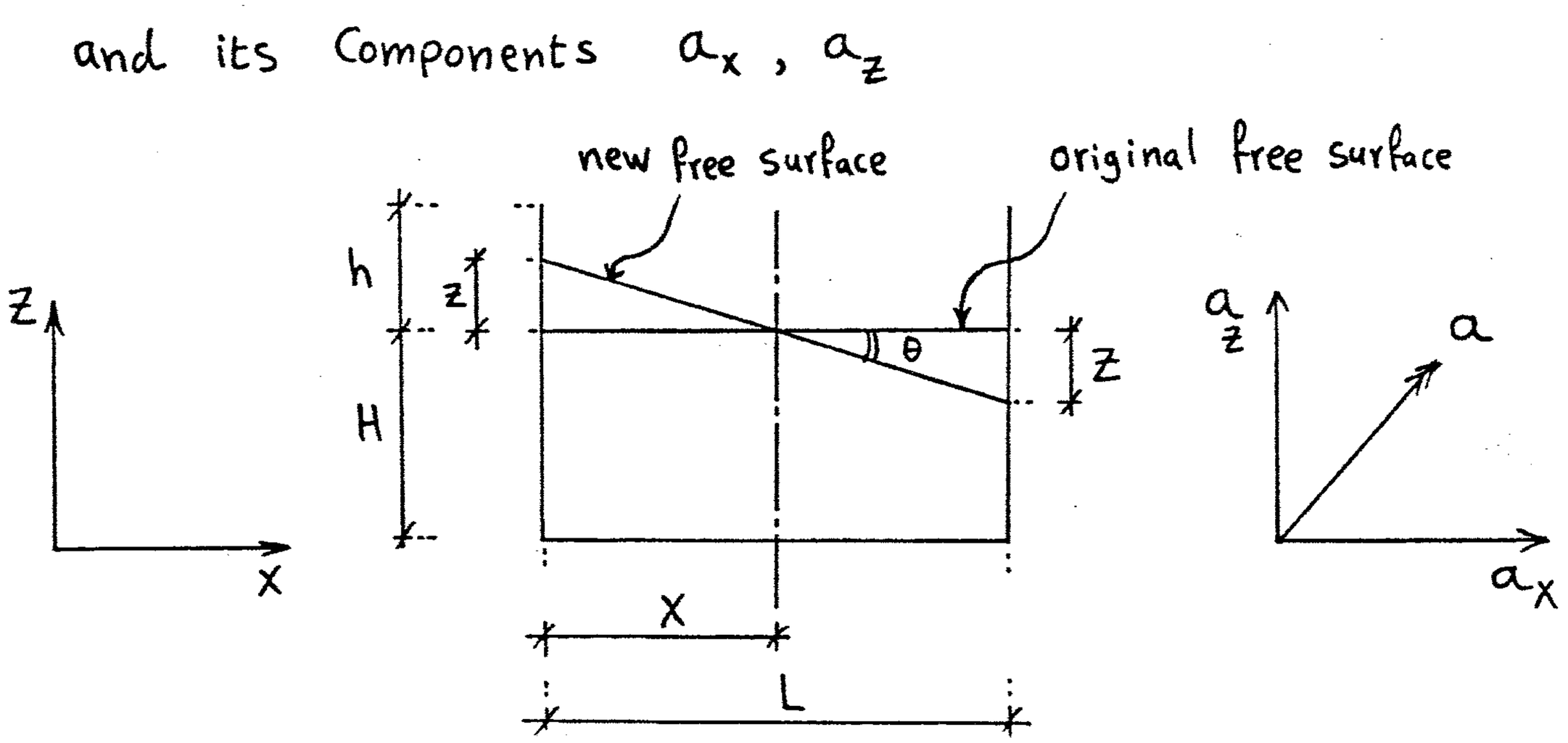






- إذا أعطى سائل فى إناء مفتوح عجلة منتظمة **uniform acceleration "a"**
- فإنه بعد زمن معين يتحرك الإناء كوحدة واحدة **moves as a solid body**
- وبالتالى لا توجد حركة نسبية بين جزيئات السائل وبعضها أو بينه وبين الاناء الحاوى له **(no shear stresses)**
- فى هذه الحالة يمكن تطبيق قوانين ال **Static fluid** لكن بإضافة تأثير العجلة.

Assume the acceleration (a) in a given direction and its Components a_x , a_z



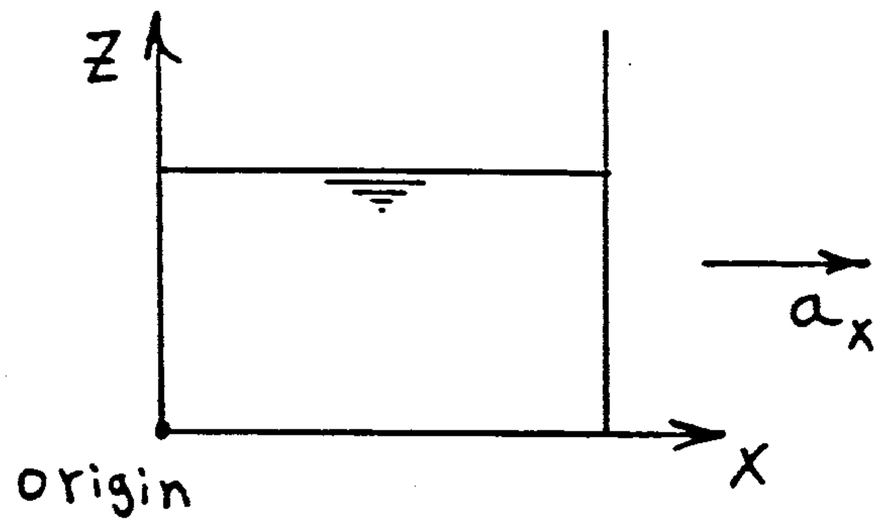
$$\tan\theta = \frac{z}{L/2}$$

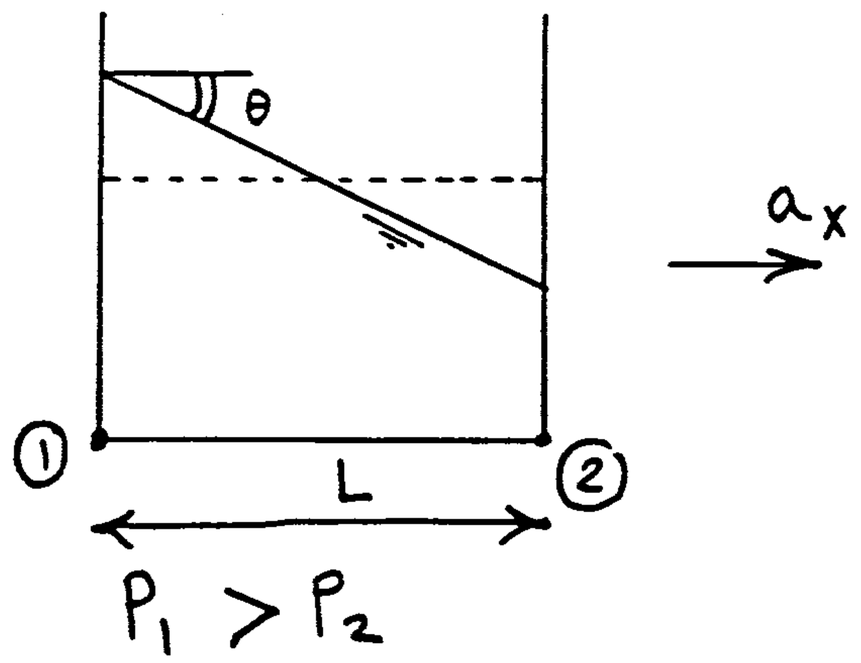
a_x : العجلة فى الاتجاه الأفقى

a_z : العجلة فى الاتجاه الرأسى

θ : الزاوية بين ال Free Surface قبل الحركة وال Free Surface بعد الحركة

[1] Horizontal acceleration





$P_1 > P_2$

$$\frac{\partial P}{\partial x} = -\frac{\gamma}{g} a_x$$

$$P_2 = P_1 - \frac{\gamma}{g} a_x L$$

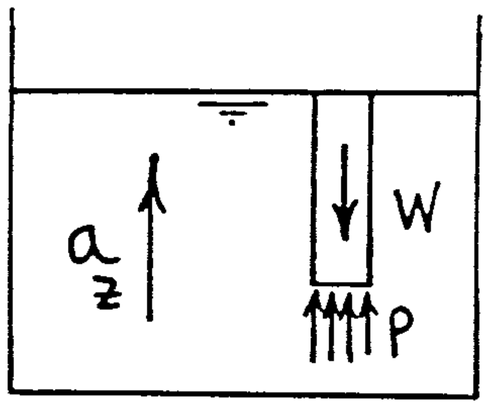
من هذه المعادلة نستنتج أن الضغط يتغير فى الاتجاه الأفقى وإشارة السالب تعنى أن الضغط يقل كلما اتجهنا فى اتجاه العجلة (إلى اليمين)

∴ أسطح تساوى الضغط ليست أفقية لكن تميل بزاوية θ

* If $a_x = 0 \Rightarrow \frac{\partial P}{\partial x} = 0$

فى حالة عدم وجود عجلة أفقية **ax** أو كان السائل يتحرك بسرعة منتظمة **(uniform velocity)** فإنه لا يوجد تغير فى الضغط فى الاتجاه الأفقى ويكون سطح السائل أفقى تماماً

[2] Vertical acceleration



$$\frac{\partial P}{\partial z} = -\frac{\gamma}{g}(g \pm a_z)$$

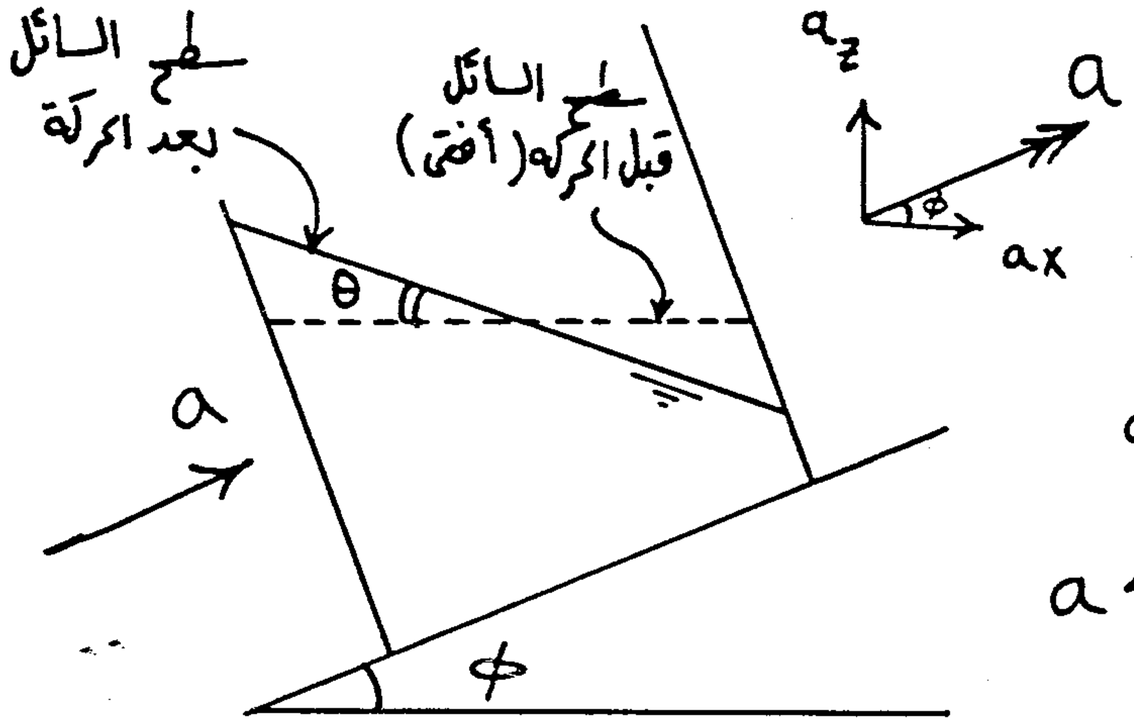
- إذا تحرك tank محتوياً على سائل رأسياً إلى أعلى بعجلة منتظمة a_z

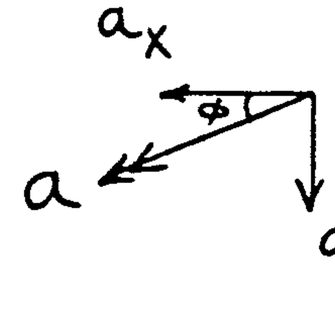
فإن الضغط يزداد طردياً مع زيادة العجلة الرأسي إلى أعلى ويظل سطح السائل أفقيا ما لم يتعرض إلى عجلة أفقية a_x

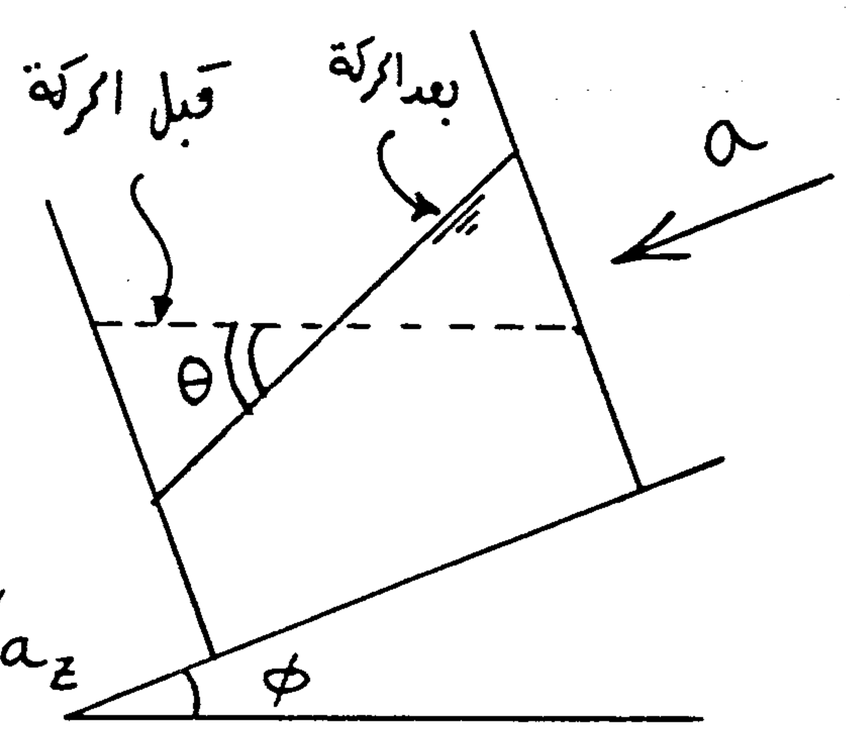
* If $a_z = 0 \Rightarrow \frac{\partial P}{\partial z} = -\gamma$

$$P = \frac{\gamma}{g}(g \pm a_z)h = \gamma' h \qquad \gamma' = \frac{\gamma}{g}(g \pm a_z) \quad (3)$$

3 Combined Horizontal and Vertical acceleration







$$\tan\theta = \frac{a_x}{g \pm a_z}$$

$a_x = a\cos\phi$

$a_z = a\sin\phi$

لو كان a_z لأعلى $\Leftarrow$ $a_z \rightarrow (+ve)$

لو كان a_z لأسفل $\Leftarrow$ $a_z \rightarrow (-ve)$

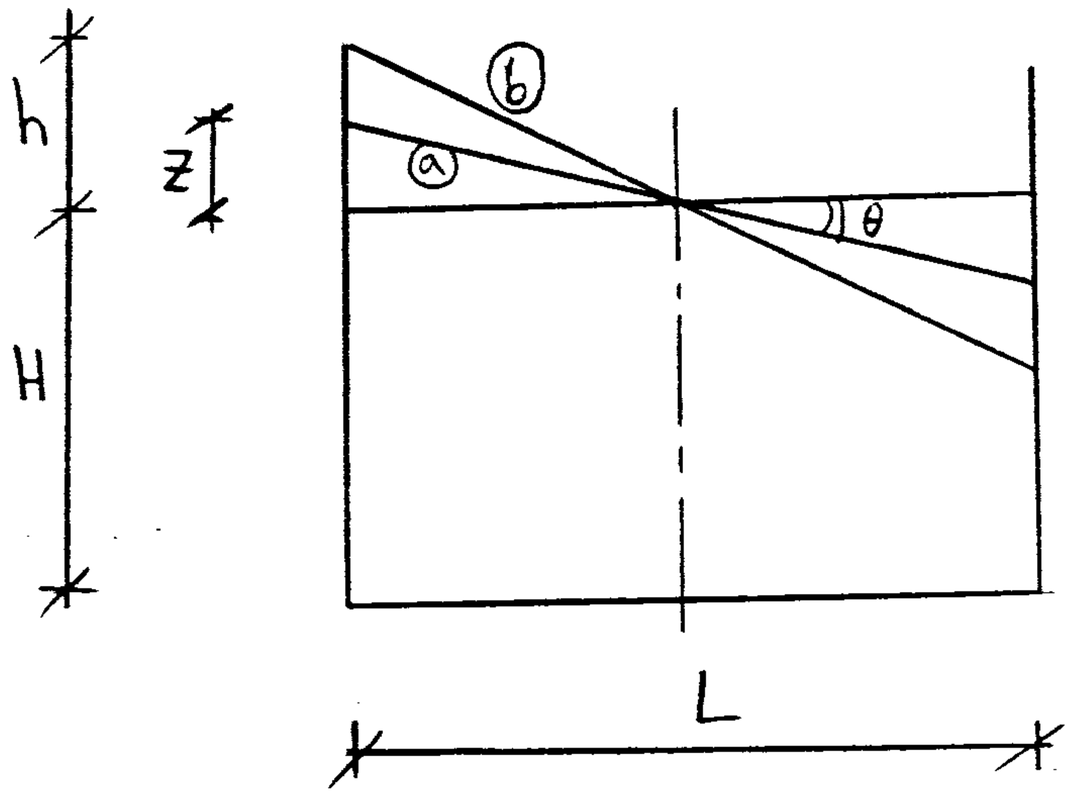
* If $a_x = 0$ $\Rightarrow \tan\theta = 0$

فى حالة عدم وجود عجلة أفقية فإن سطح السائل يظل أفقياً

* If $a_z = 0$ $\Rightarrow \tan\theta = \frac{a_x}{g}$

How to Know if liquid will be spilt?

$$\tan\theta = \frac{a_x}{g \pm a_z} = \frac{Z}{L/2}$$



$\Rightarrow$ get Z

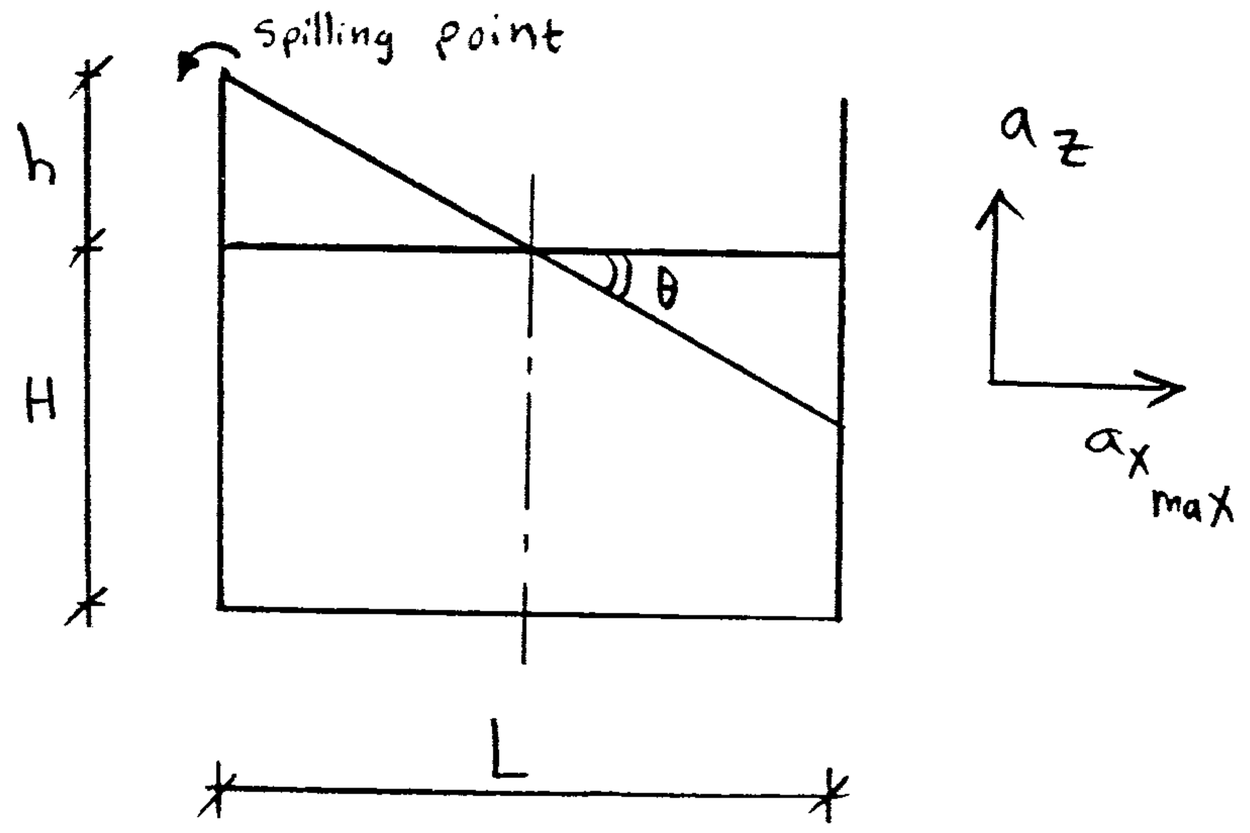
If $Z < h$ case ⓐ no liquid is spilled

If $Z = h$ case ⓑ liquid at the point of spilling

If $Z > h$ water is spilled $\Rightarrow$ 3 cases

How to get $(a_x)_{max}$ or max height of the Container to make the liquid at the spilling_point

$$\tan\theta = \frac{a_{x\,max}}{g \pm a_z} = \frac{h}{L/2}$$



$\Rightarrow$ get $a_{x\,max}$

$$\tan\theta = \frac{a_x}{g \pm a_z} = \frac{h_{max}}{L/2}$$

$\Rightarrow$ get h_{max}

When is liquid spilled ?

IF $z > h$ Liquid is spilled

or IF $a_x > a_{x_{max}}$ " " " } 3 Cases

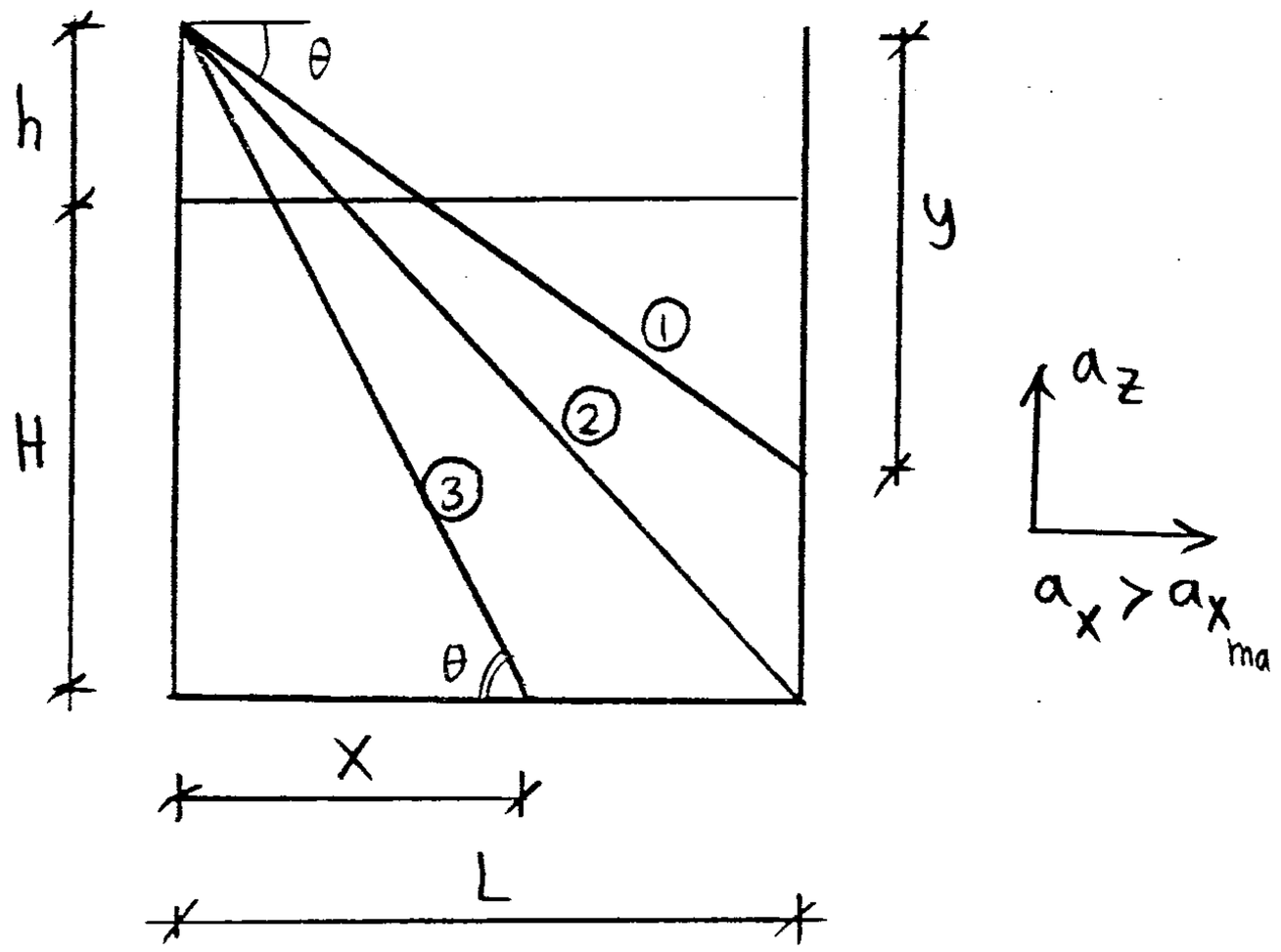
$$\tan\theta = \frac{a_x}{g \pm a_z} = \frac{y}{L}$$

$\Rightarrow$ get y

If $y < H+h$ $\Rightarrow$ Case 1

If $y = H+h$ $\Rightarrow$ Case 2

If $y > H+h$ $\Rightarrow$ Case 3



Case 3 $\tan\theta = \frac{a_x}{g \pm a_z} = \frac{H+h}{X}$

$\Rightarrow$ get X

How to get the Volume of spilled water?

for Case ①, ②

Volume spilled = Volume of air after motion − Volume of air before motic

$$\forall_{spilled} = \left[\frac{1}{2} L y - L h\right] b$$

Case ③

Volume spilled = Volume of water before motion − Volume of water after motior

$$\forall_{spilled} = \left[L H - \frac{1}{2} X (H+h)\right] b$$

Case of closed tank, find Pressure at (1) and (2)

$$\tan\theta = \frac{a_x}{g \pm a_z} = \checkmark$$

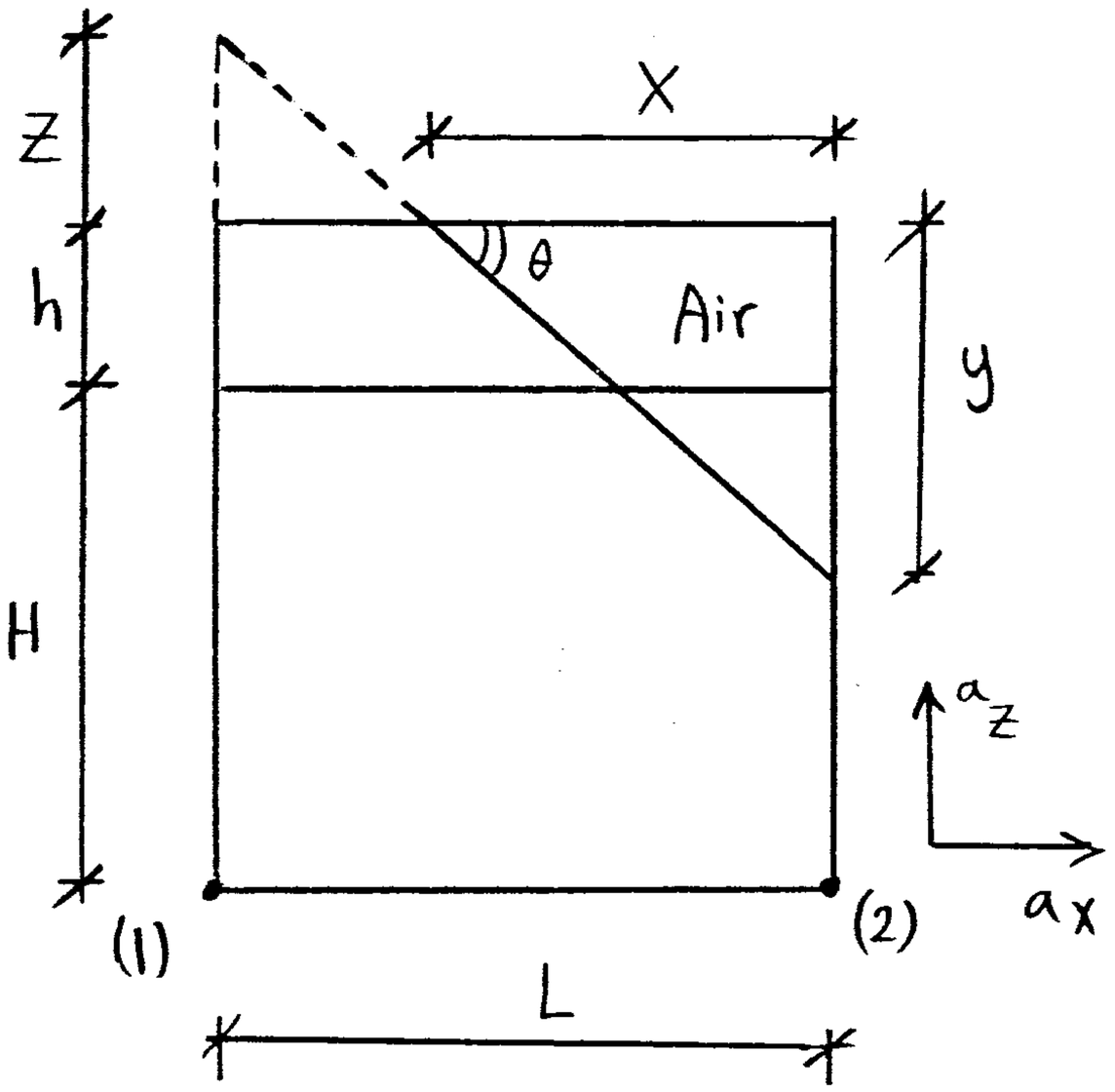
$$\tan\theta = \frac{y}{x} \rightarrow ①$$

∵ Area of air before motion = Area of air after motion

$$L\,h = \frac{1}{2}\,x\,y \rightarrow ②$$

from ①, ② ⇒ get X, y

⇒ get Z بحساب مثلثات



$$P_1 = P_{air} + \frac{\gamma}{g}(g \pm a_z)\left[H + h + Z\right]$$

$$P_2 = P_{air} + \frac{\gamma}{g}(g \pm a_z)\left[H + h - y\right]$$

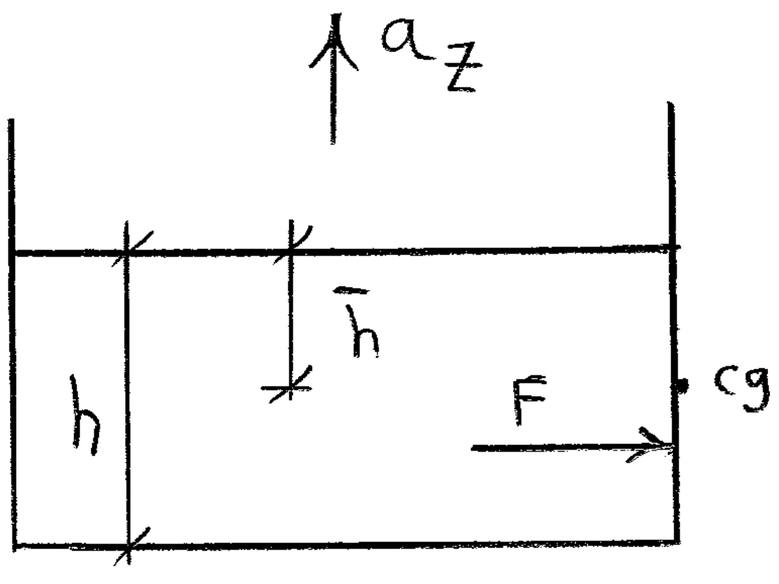
Forces on tank sides during acceleration

$$F = \gamma' A \bar{h}$$

$$\gamma' = \frac{\gamma}{g}(g \pm a_z)$$

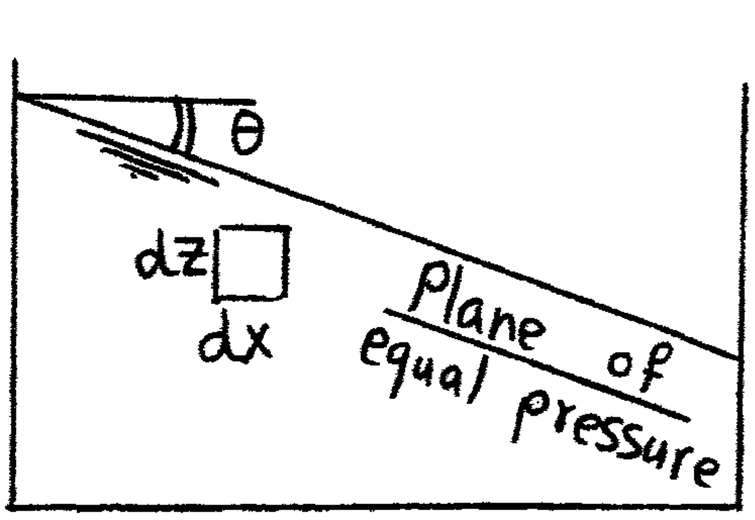
A = area of Side View

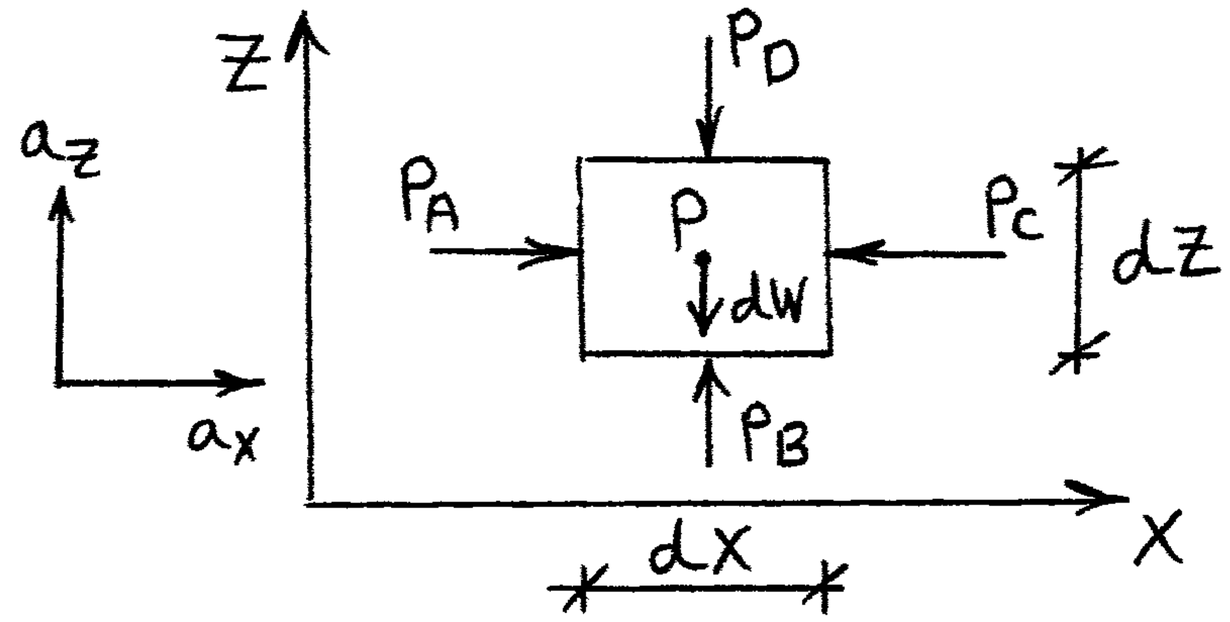
$$\bar{h} = \frac{h}{2}$$



Apply the basic hydrostatics equation to determine the pressure variation in the horizontal and vertical directions and the slope of the surface of constant pressure for any body fluid in rigid body motion.

Fluid Masses Subjected to linear Acceleration





Consider a Small fluid element with dimensions (dxdz)

$$\sum F_x = P_A\,dz - P_c\,dz \quad \ldots\ldots ①$$

$$\sum F_z = P_B\,dx - P_D\,dx - dW \quad \ldots\ldots ②$$

$$P_A = P - \frac{\partial P}{\partial x}\frac{dx}{2}, \quad P_C = P + \frac{\partial P}{\partial x}\cdot\frac{dx}{2}$$

$$P_B = P - \frac{\partial P}{\partial z}\frac{dz}{2}, \quad P_D = P + \frac{\partial P}{\partial z}\cdot\frac{dz}{2} \quad \Big\} \rightarrow ③$$

$$\sum F_x = dM\,a_x \rightarrow ④$$

$$P_A\,dz - P_c\,dz = dM\,a_x$$

$$\left(P - \frac{\partial P}{\partial x}\cdot\frac{dx}{2}\right)\cancel{dz} - \left(P + \frac{\partial P}{\partial x}\cdot\frac{dx}{2}\right)\cancel{dz} = \frac{\gamma}{g}\,dx\,\cancel{dz}\,a_x$$

$$-\frac{\partial P}{\partial x}\,\cancel{dx} = \frac{\gamma}{g}\,\cancel{dx}\,a_x$$

$$\boxed{\frac{\partial P}{\partial x} = -\frac{\gamma}{g} a_x} \quad \rightarrow (5)$$

$$\sum F_z = dM\, a_z \quad \rightarrow (6)$$

$$P_B\, dx - P_D\, dx - dW = dM\, a_z$$

$$\left(P - \frac{\partial P}{\partial z}\,\frac{dz}{2}\right) dx - \left(P + \frac{\partial P}{\partial z}\cdot\frac{dz}{2}\right) dx - \gamma\, dx\, dz = \frac{\gamma}{g}\, dx\, dz\, a_z$$

$$-\frac{\partial P}{\partial z}\cdot dz - \gamma\, dz = \frac{\gamma}{g}\, dz\, a_z$$

$$\boxed{\frac{\partial P}{\partial z} = -\frac{\gamma}{g}\left(g + a_z\right)} \quad \rightarrow (7)$$

$$dP = \frac{\partial P}{\partial x}\, dx + \frac{\partial P}{\partial z}\, dz = 0 \quad \rightarrow (8)$$

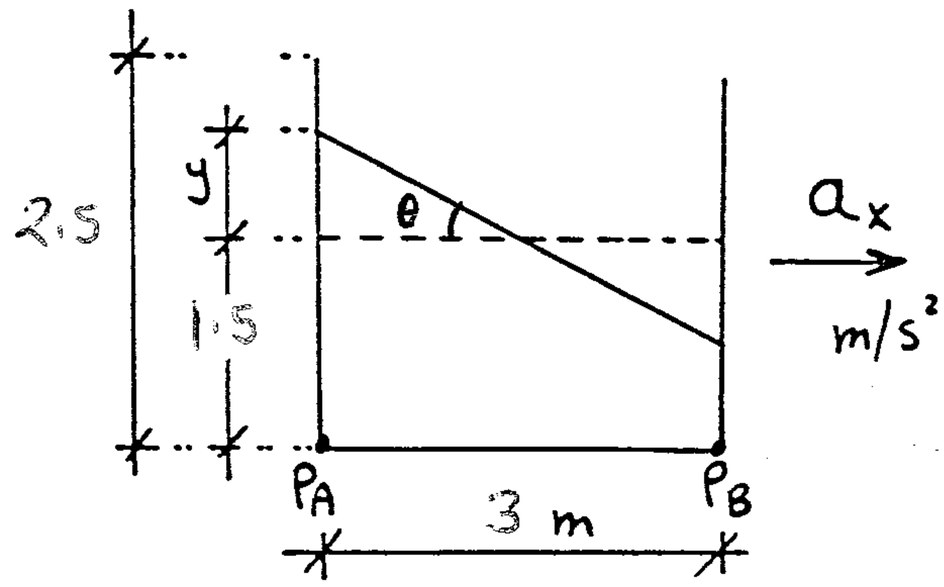
$$\frac{dz}{dx} = \frac{-\partial P/\partial x}{\partial P/\partial z}$$

$$\boxed{\tan\theta = \frac{dz}{dx} = \frac{-a_x}{g \pm a_z}} \quad \rightarrow (9)$$

19 A tank containing water moves horizontally with a constant linear acceleration of $3\ m/s^2$.
The tank is 3 m long, 2.5 m high and the depth of water when the tank is at rest is 1.5 m.
calculate a) the angle of the water surface to the horizontal,
b) the maximum pressure intensity on the bottom.

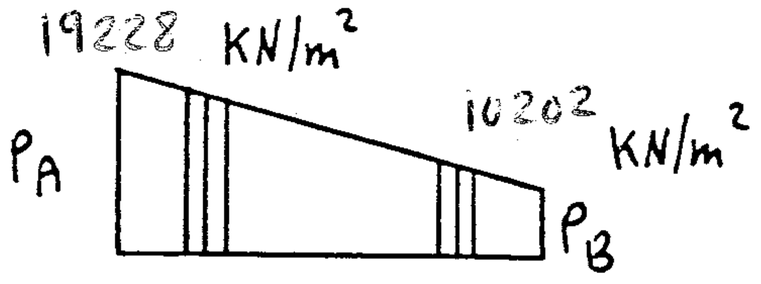
$$\tan\theta = \frac{a_x}{g}$$

$$\tan\theta = \frac{3}{9.81} \Rightarrow \theta = 17$$



$$\tan\theta = \frac{3}{9.81} = \frac{y}{x} = \frac{y}{1.5}$$

$$\therefore y = \frac{3 * 1.5}{9.81} = 0.46$$



$$P_A = \gamma h = 9810(1.5 + 0.46) = 19228\ N/m^2$$

$$P_B = \gamma h = 9810(1.5 - 0.46) = 10202\ N/m^2$$

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17 ⓑ $a_z = 9.81$ $P_a, P_b, P_c = ?$

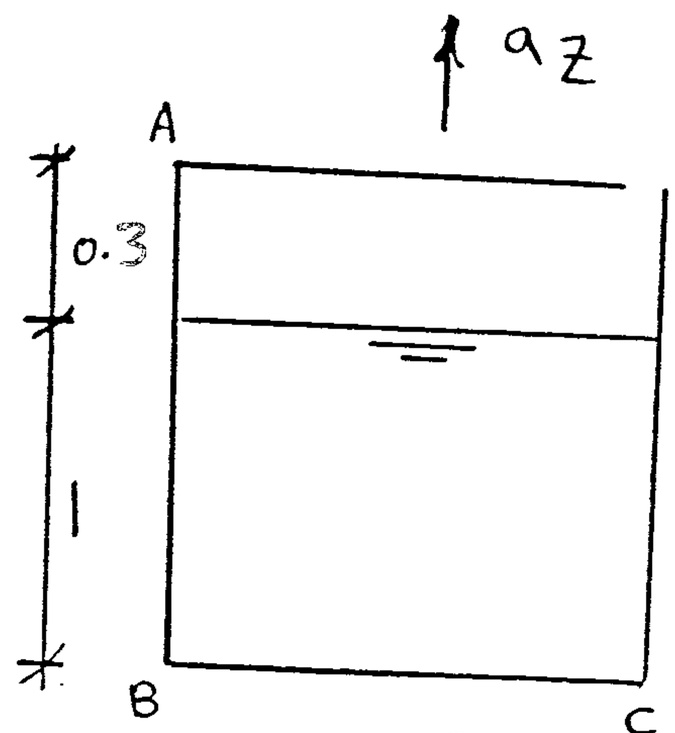
Sol

$$\tan\theta = 0$$

$$P_A = 0$$

$$P_B = P_C = \frac{\gamma}{g}(g \pm a_z)h$$

$$= \frac{9810}{9.81}(9.81 + 9.81)1 = \underline{19620\ N/m^2}$$



If acceleration ceases and the tank moves at Constant Velocity 6 m/s

إذا توقف التسارع وتحرك الخزان بسرعة ثابتة

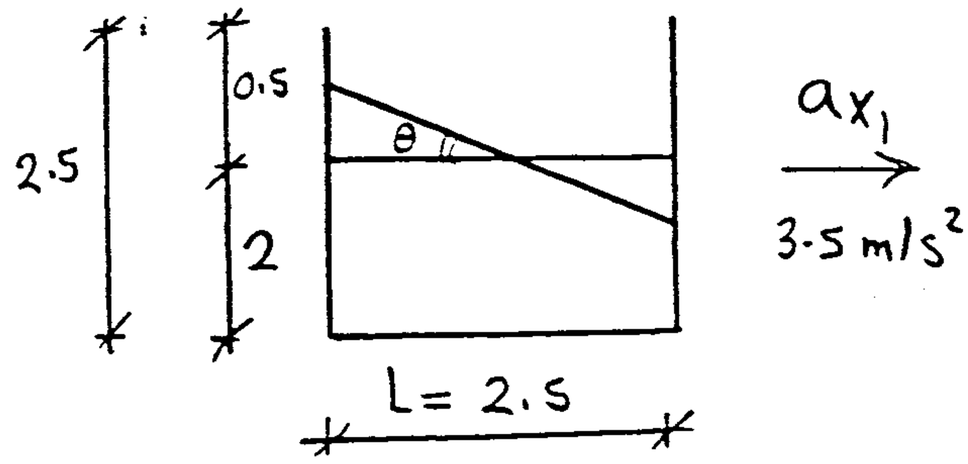
$$\Rightarrow a_z = 0 \Rightarrow P_B = P_C = \gamma h = 9810(1) = \underline{9810\ N/m^2}$$

A tank containing water moves horizontally with a constant linear acceleration of 3.5 m/s^2. (10)
The tank is 2.50 m long, 2.5 m high and the depth of water when the tank is a t rest is 2 m.
Calculate a) the angle of the water surface to the horizontal.

b) the volume of spilled water when the acceleration is increased by 25%.

c) the force acting on each side if a = 12 m/s^2.

(a) $a_x = 3.5\ m/s^2$ $\theta = ?$



$$\tan\theta = \frac{a_x}{g} = \frac{3.5}{9.81}$$

$$\therefore \theta = \underline{\underline{19.63^\circ}}$$

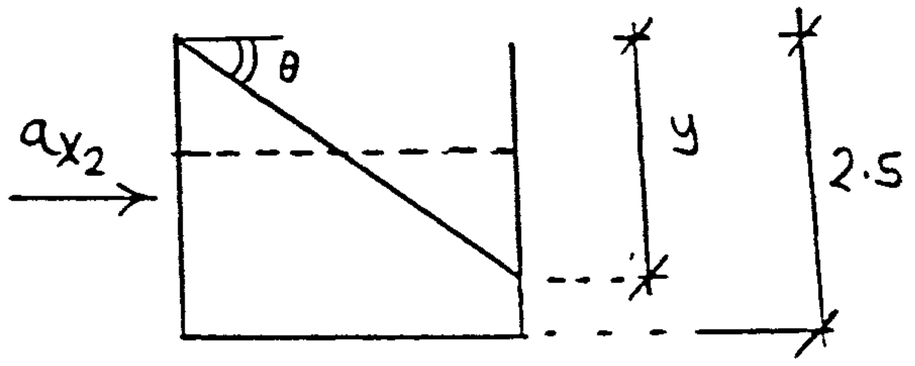
(b) If a_x is increased 25 %

$$a_{x_2} = 1.25\, a_{x_1} = 1.25 * 3.5 = 4.375\ m/s^2$$

$$\tan\theta = \frac{a_x}{g} = \frac{z}{L/2}$$

$$\frac{4.375}{9.81} = \frac{z}{2.5/2} \Rightarrow z = 0.557 > 0.5\ m$$

$\therefore$ Water will spill



$$\tan\theta = \frac{a_x}{g} = \frac{y}{L}$$

$$\frac{4.375}{9.81} = \frac{y}{2.5} \Rightarrow y = 1.11 < 2.5$$

$$\forall_{\text{water after acceleration}} = (2.5 * 2.5 * 1) - \left(\tfrac{1}{2} * 2.5 * 1.11 * 1\right) = 4.86\ m^3$$

$$\forall_{\substack{\text{water before}\\ \text{acceleration}}} = (2.5 * 2 * 1) = 5\ m^3$$

$$\forall_{\text{water spilt}} = \forall_{\text{water before acceleration}} - \forall_{\text{after acceleration}}$$

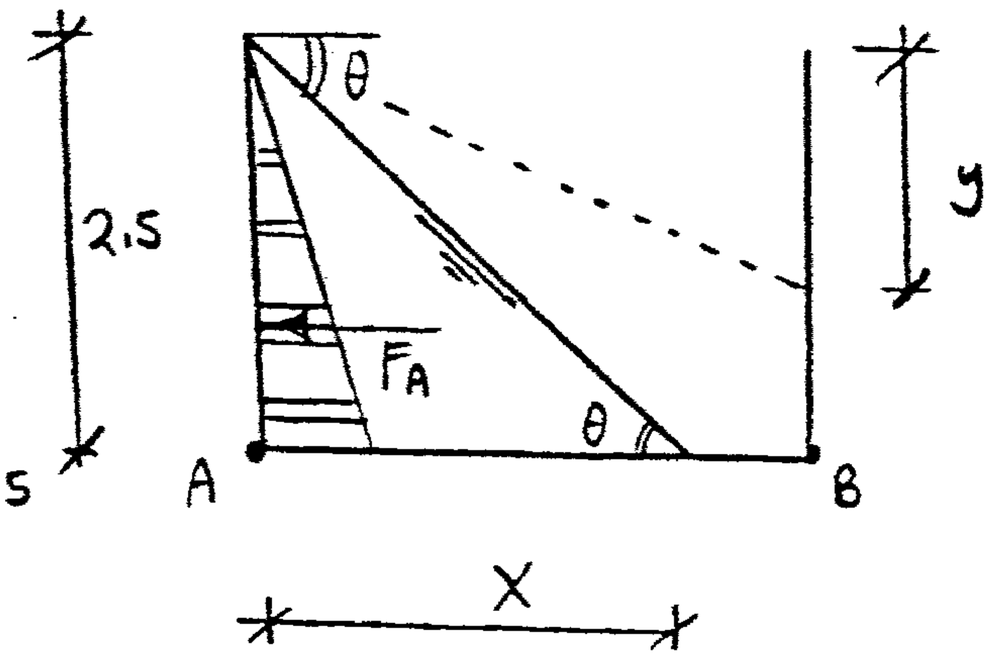
$$= 5 - 4.86 = \underline{\underline{0.14\ m^3}}$$

$$\%\ \text{Volume spilt} = \frac{0.14}{5} * 100 = 2.8\ \%$$

© $\underline{a_x = 12\ m/s^2}$

$$\tan\theta = \frac{a_x}{g} = \frac{y}{L}$$

$$\frac{12}{9.81} = \frac{y}{2.5} \Rightarrow y = 3.06 > 2.5$$



$$\therefore \tan\theta = \frac{a_x}{g} = \frac{2.5}{X}$$

$$\frac{12}{9.81} = \frac{2.5}{X} \Rightarrow X = 2.04$$

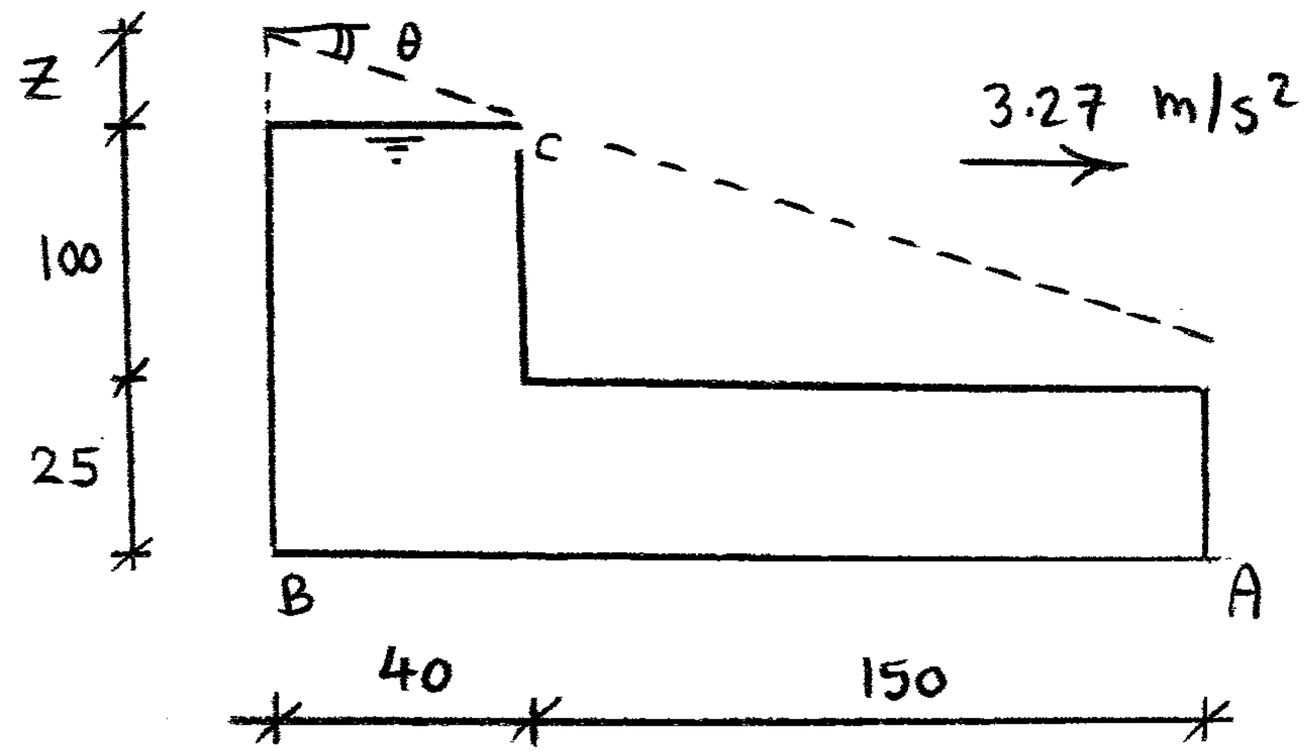
$$F_A = \gamma A \bar{h} = 9810\,(2.5 * 1)\,\frac{2.5}{2} = \underline{\underline{30656\ N}}$$

$$F_B = 0$$

20

$$\tan\theta = \frac{Z}{X} = \frac{a_x}{g}$$

$$\frac{Z}{40} = \frac{3.27}{9.81}$$



$$Z = \frac{40 * 3.27}{9.81} = 13.33\ m$$

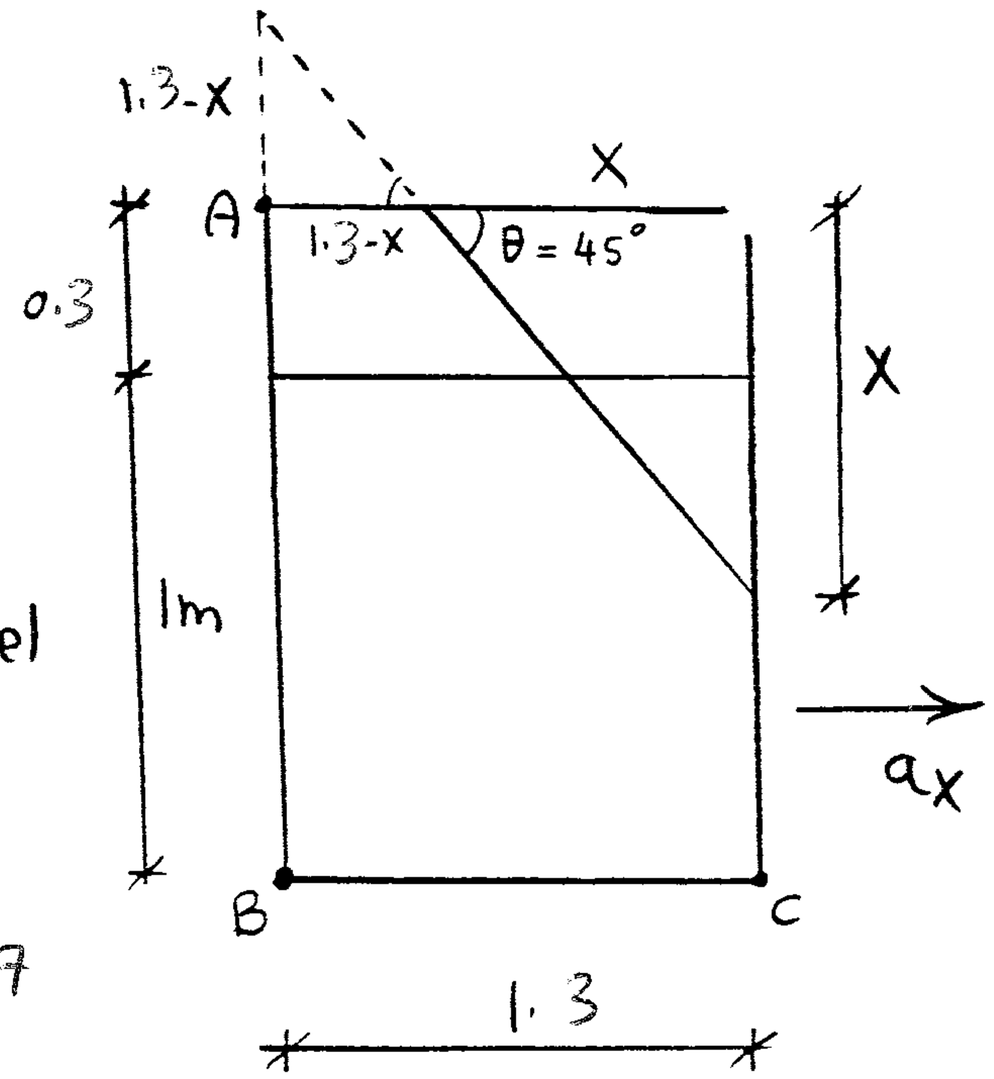
$$P_B = \gamma(h+z) = 9810\,(125 + 13.33) = \underline{\underline{1357017\ N/m^2}}$$

$$P_A = P_B - \frac{\gamma}{g} a_x L = 1357017 - \frac{9810}{9.81}\,(3.27)(190)$$

$$= \underline{\underline{735717\ N/m^2}}$$

17 (a)

$a_x = 9.81 \quad m/s^2 \qquad P_{a,b,c} = ?$



Sol

$$\tan\theta = \frac{a_x}{g} = \frac{9.81}{9.81} = 1$$

$$\therefore \theta = 45^\circ$$

$\because$ $\forall_{air}$ before accel = $\forall_{air}$ after accel

$$1.3 * 0.3 = \frac{1}{2} X^2$$

$$\therefore X = 0.883 \quad , 1.3 - X = 0.417$$

$$P_A = \gamma_w (0.417) = \underline{\underline{4091 \ N/m^2}}$$

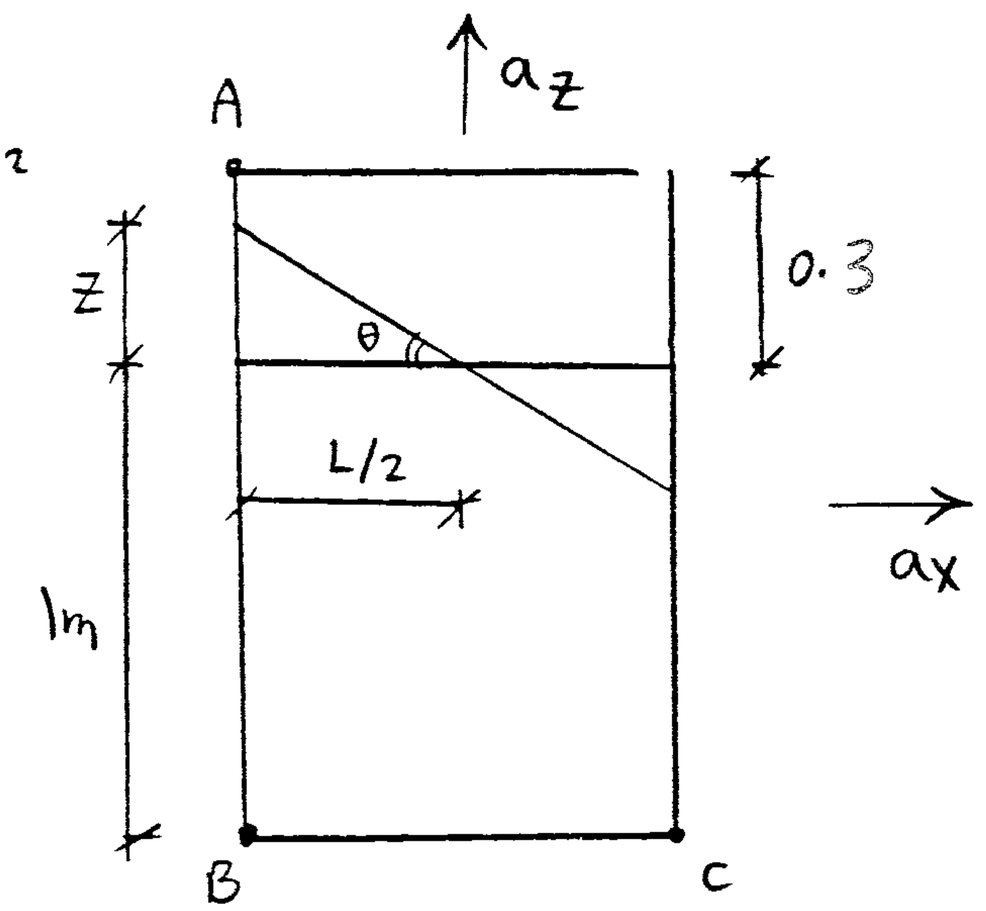
$$P_B = \gamma_w (0.417 + 1.3) = \underline{\underline{16\,844 \ N/m^2}}$$

$$P_C = P_B - \frac{\gamma}{g} a_x L = 16\,844 - \frac{9810}{9.81}(9.81)\,1.3 = \underline{\underline{4091 \ N/m^2}}$$

---

17 (c)

$a_x = 4.90 \quad m/s^2 \qquad a_z = 9.81 \quad m/s^2$



Sol

$$\tan\theta = \frac{a_x}{g \pm a_z} = \frac{Z}{L/2}$$

$$= \frac{4.90}{9.81 + 9.81} = \frac{Z}{1.3/2}$$

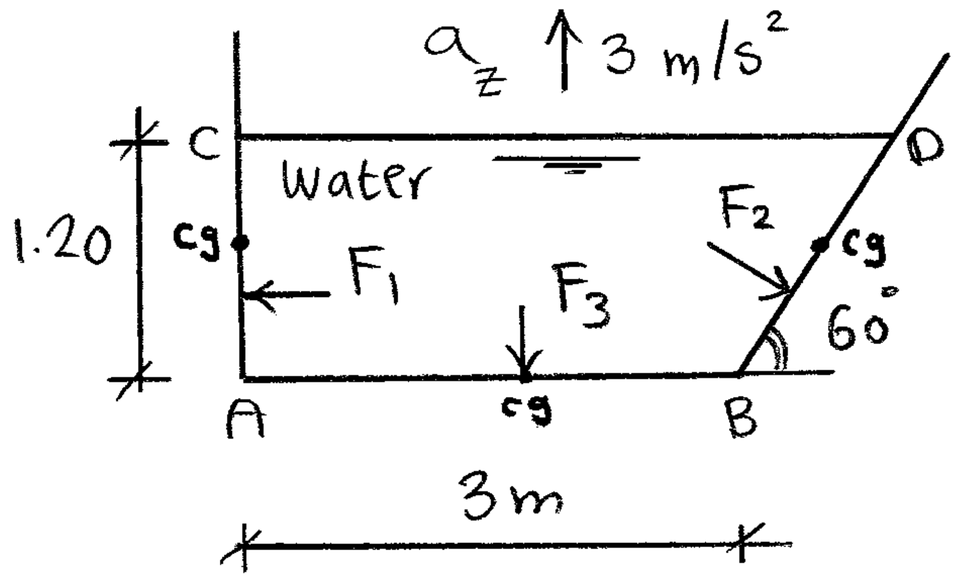
$$\Rightarrow Z = 0.162 < 0.3 \qquad \Rightarrow \underline{\underline{P_A = 0}}$$

$$P_B = \frac{\gamma}{g}(g \pm a_z) h = \frac{9810}{9.81}(9.81 + 9.81)(1 + 0.162) = \underline{\underline{22800 \ N/m^2}}$$

$$P_C = \frac{9810}{9.81}(9.81 + 9.81)(1 - 0.162) = \underline{\underline{16441 \ N/m^2}}$$

or// $P_C = P_B - \frac{\gamma}{g} a_x L = 16441 \ N/m^2$

18 Calculate the total forces on the ends and bottom of this container while at rest and when being accelerated vertically upward at 3 m/s^2. The container is 2 m wide. Repeat your calculations for a downward acceleration of 6 m/s^2.



Width = 2m

At rest

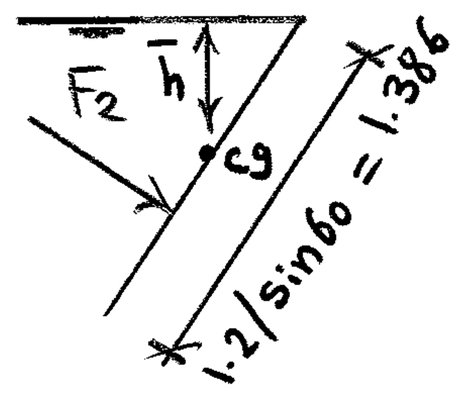
$P_A = P_B = \gamma h$

$= 9810(1.2) = 11772 \ N/m^2$

$F_1 = \gamma A\bar{h} = 9810(1.2*2)\frac{1.2}{2} = \underline{14126 \ N}$

$F_2 = \gamma A\bar{h} = 9810(1.386*2)\frac{1.2}{2} = \underline{16316 \ N}$

$F_3 = PA = 11772(3*2) = \underline{70632 \ N}$



Upward $a_z = 3 m/s^2$

$\gamma' = \frac{\gamma}{g}(g + a_z) = \frac{9810}{9.81}(9.81 + 3) = 12810 \ N/m^2$

$F_1 = \gamma' A\bar{h} = 12810(1.2*2)\frac{1.2}{2} = \underline{18446 \ N}$

$F_2 = \gamma' A\bar{h} = 12810(1.386*2)\frac{1.2}{2} = \underline{21305 \ N}$

$F_3 = PA = \gamma h.A = (12810*1.2)*(3*2) = \underline{92232 \ N}$

Downward $a_z = 6 m/s^2$

$\gamma' = \frac{\gamma}{g}(g - a_z) = \frac{9810}{9.81}(9.81 - 6) = 3810 \ N/m^2$

$F_1 = \gamma' A\bar{h} = 3810(1.2*2)\frac{1.2}{2} = \underline{5486 \ N}$

$F_2 = \gamma' A\bar{h} = 3810(1.386*2)\frac{1.2}{2} = \underline{6337 \ N}$

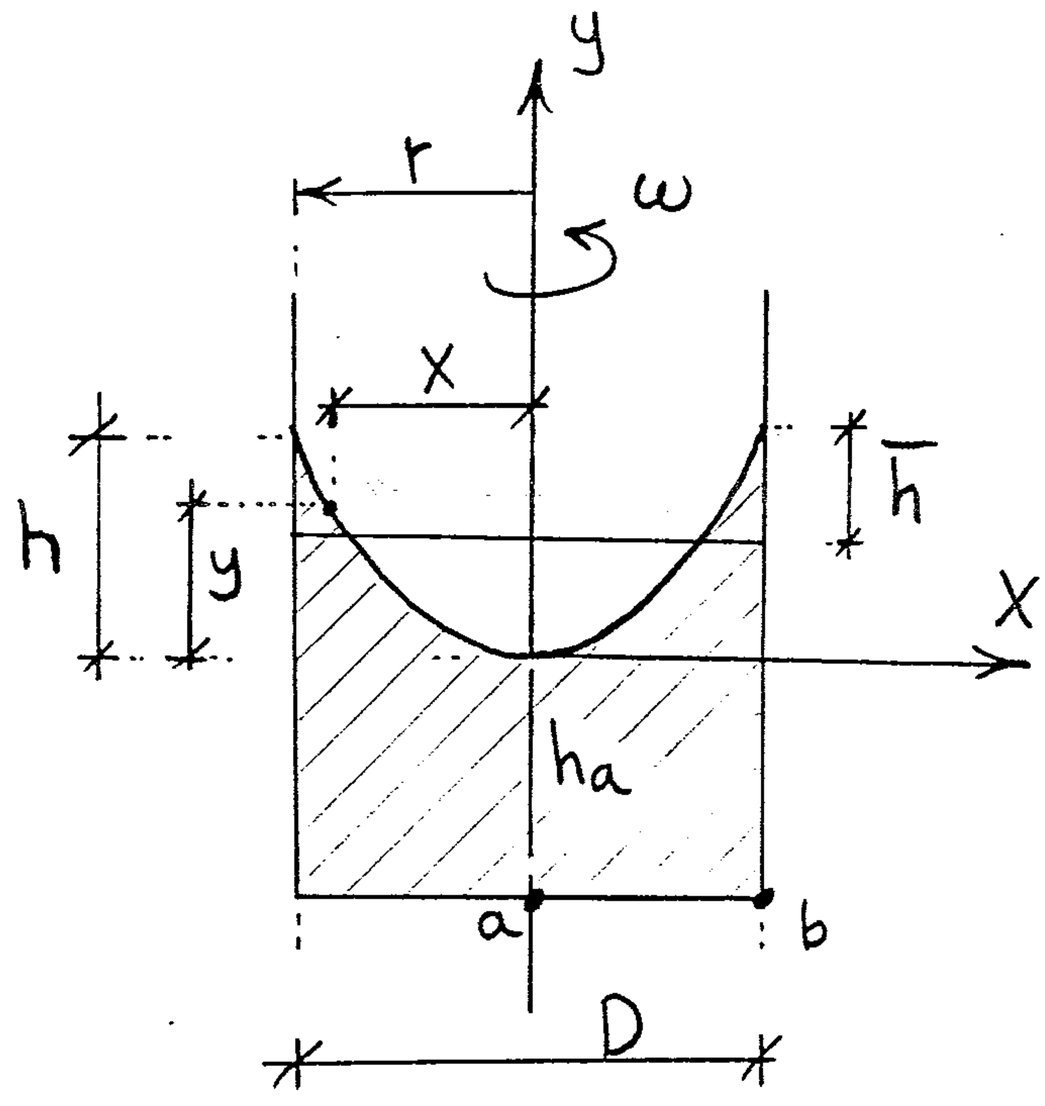
$F_3 = PA = \gamma' h.A = (3810*1.2)*(3*2) = \underline{27432 \ N}$

Uniform Rotation about Vertical Axis

$$\omega = \frac{2\pi N}{60}$$

N = number of revolutions Per minute (r.P.m.)

ω = Angular Velocity



$$y = \frac{\omega^2 x^2}{2g}$$

$$h = \frac{\omega^2 r^2}{2g}$$

في حالة عدم الانكاب

حجم السائل قبل الدوران = حجم السائل بعد الدوران

حجم الفراغ قبل الدوران = حجم الفراغ بعد الدوران

حجم الـ Paraboloid يساوى $\frac{1}{2}$ حجم الاسطوانه المشتركه معه فى القاعدة والارتفاع

$$\forall_{parab} = \frac{1}{2}\pi r^2 h$$

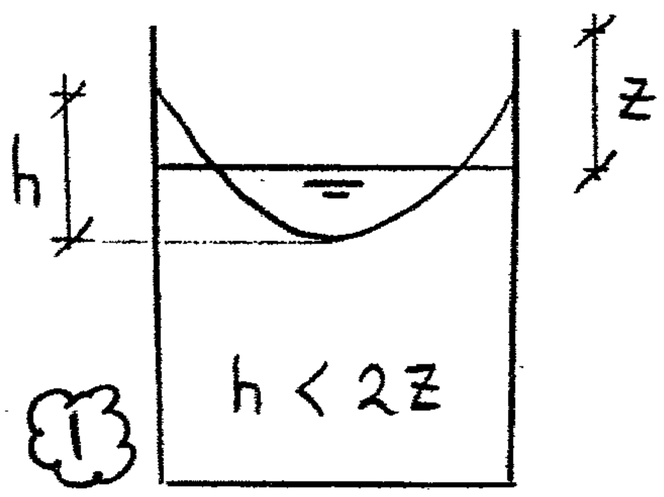
فى حالة عدم الانكاب يقسم الـ Paraboloid الى ارتفاعين متساويين

If No liquid is Spilled

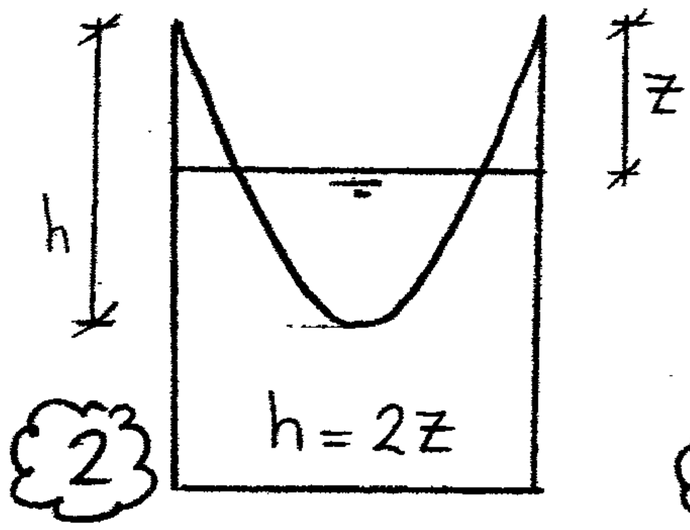
Volume of air before rotation = Volume of air after rotation

$$\pi r^2 \bar{h} = \frac{1}{2}\pi r^2 h \Rightarrow \boxed{\bar{h} = \frac{h}{2}}$$

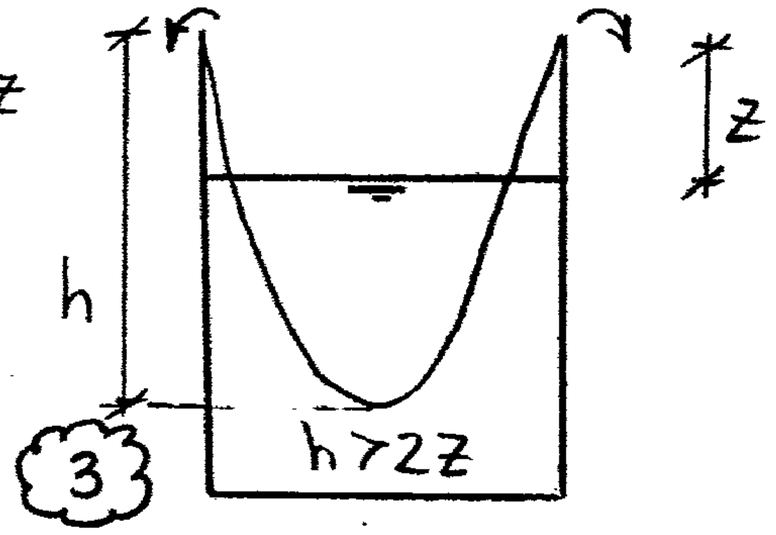
Open tank cases



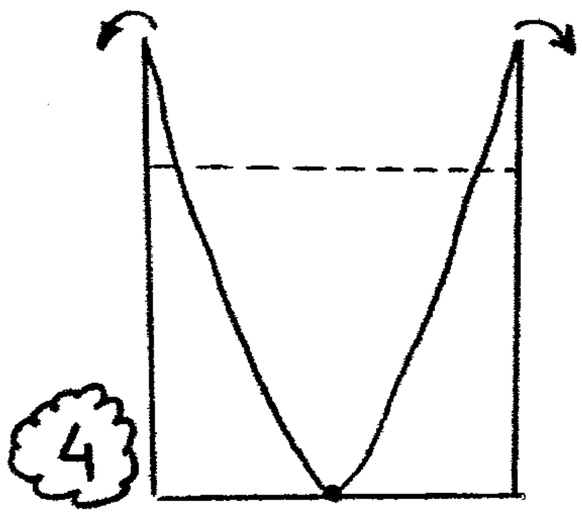
No spilling



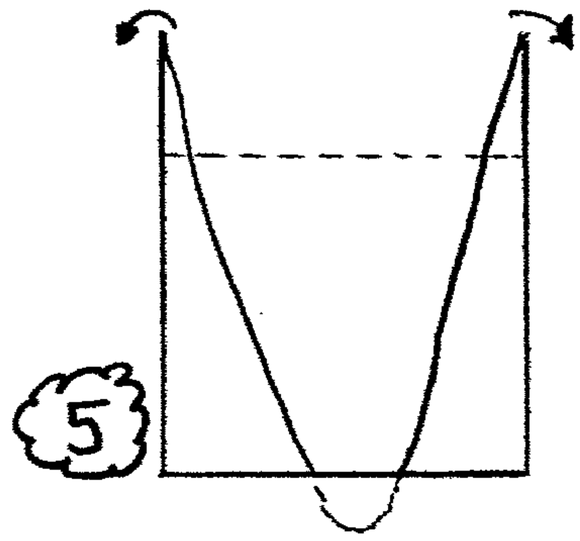
No spilling
(at the point of spilling)



Spilling

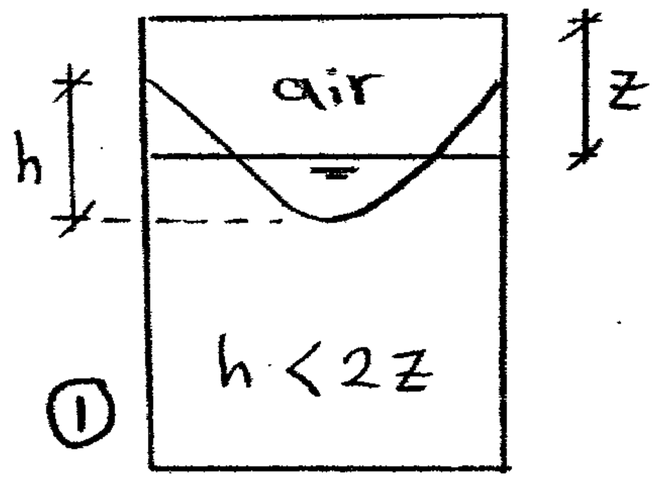


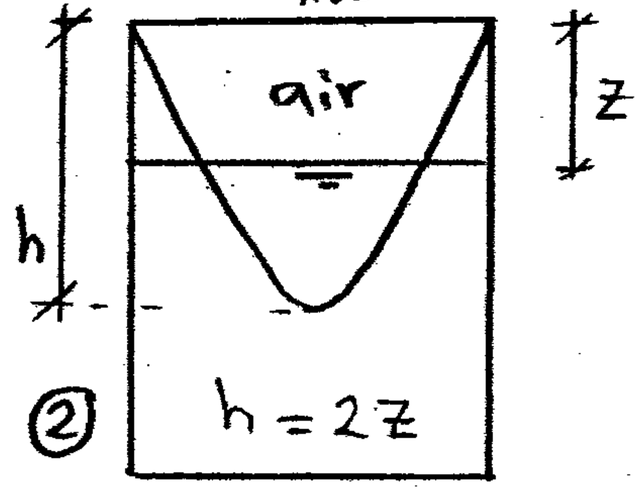
Point at the center
is just uncovered

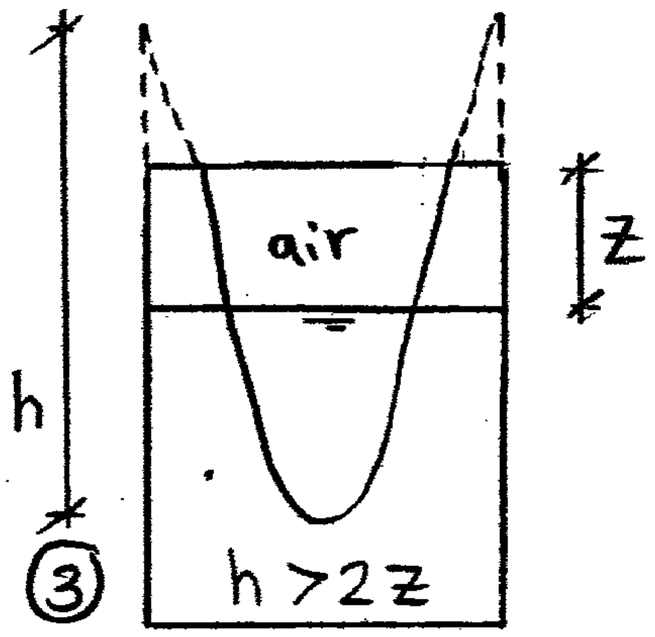


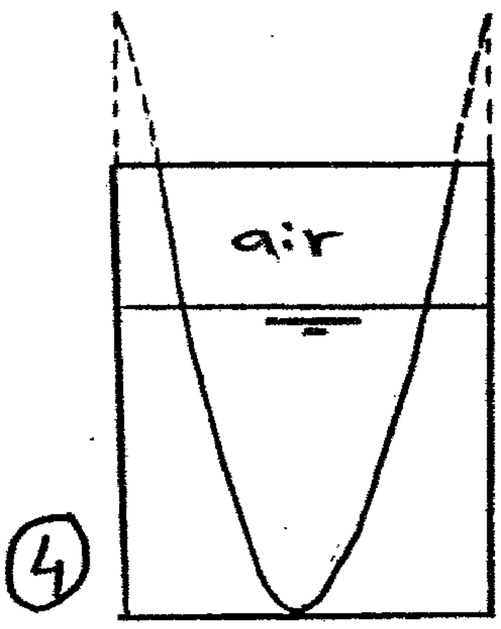
bottom of the tank
is uncovered

Closed tank cases



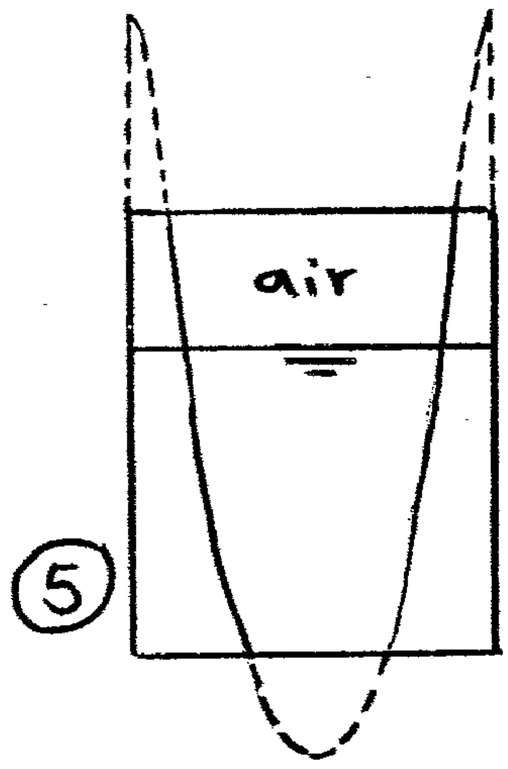






Point at the center
is just uncovered

Water starts to
touch the lid



bottom of the tank
is unconvered

21

(16)

An open cylindrical tank 2.0 meters high and 1.0 meter diameter contains 1.5 m of water. If the cylinder rotates about its geometric axis, find the constant angular velocity that can be applied when:

a)The water just starts spilling over

b)The point at the center of the base is just uncovered and the percentage of water left in the tank in this case.

خسبة المياه المتبقية بعد الحركة

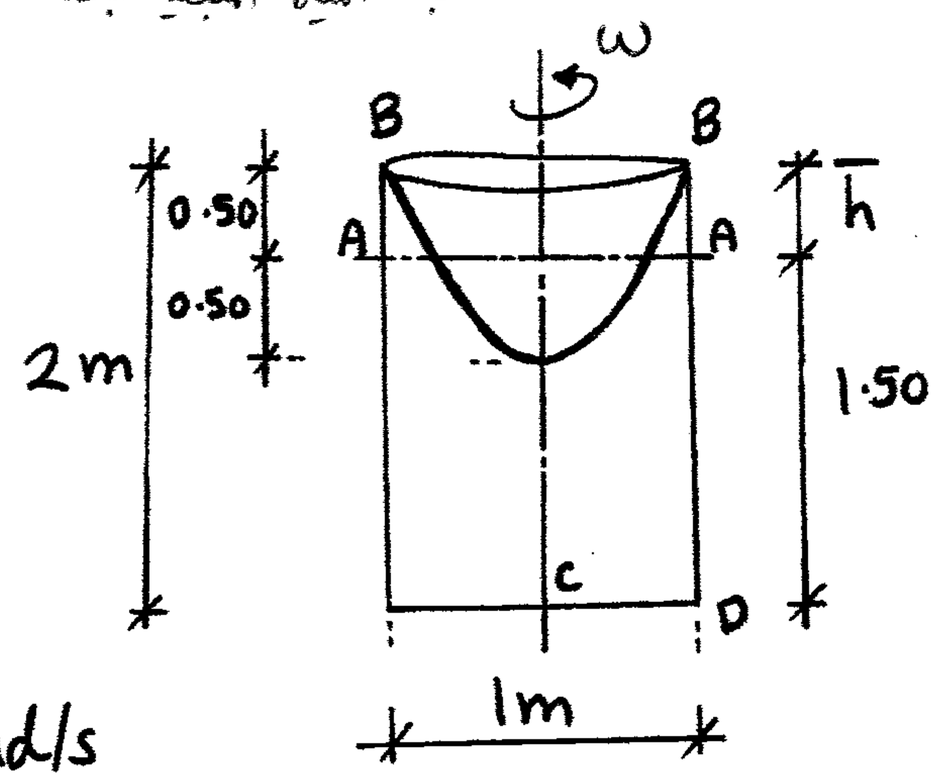
(a)

from fig $\Rightarrow \bar{h} = 2 - 1.50 = 0.5m$

$h = 2\bar{h} = 1m$

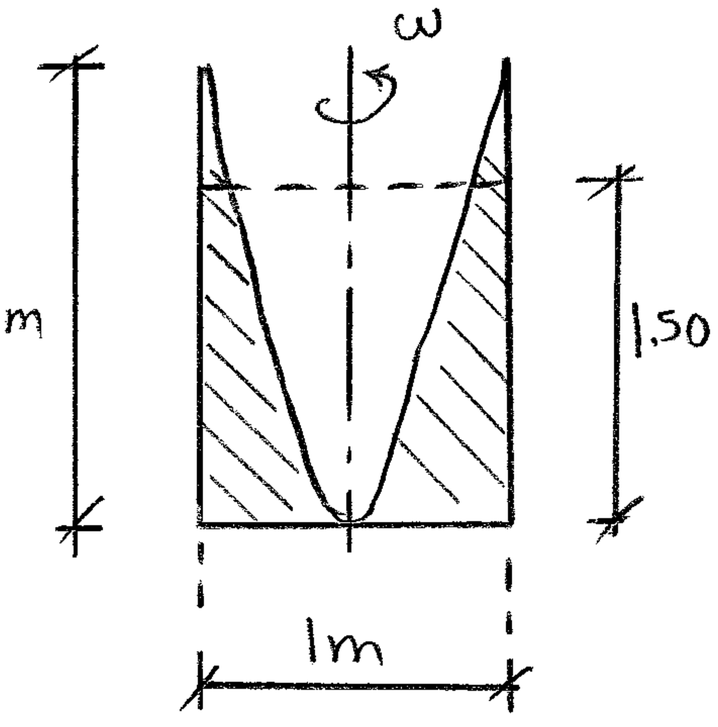
$$h = \frac{\omega^2 r^2}{2g}$$

$$1 = \frac{^{2}(0.50)^{2}}{2 * 9.81} \Rightarrow \omega = \underline{\underline{8.86 \text{ rad/s}}}$$



$$h = \frac{\omega^2 r^2}{2g}$$

$$2 = \frac{\omega^2 (0.5)^2}{2 * 9.81} \Rightarrow \omega = \underline{\underline{12.53 \text{ rad/s}}}$$



$$\forall_{left} = \forall_{Tank\ full} - \forall_{air\ after\ rotation}$$

$$\forall_{tank\ full} = \square = \pi r^2 h = \pi (0.5)^2 2 = 1.57\ m^3$$

$$\forall_{air\ after} = \cup = \frac{1}{2}\pi r^2 h = \frac{1}{2}\pi (0.5)^2 2 = 0.7854\ m^3$$

$$\forall_{left} = 1.57 - 0.7854 = 0.7854\ m^3$$

$$\forall_{before} = \square = \pi r^2 h = \pi (0.5)^2 1.5 = 1.178$$

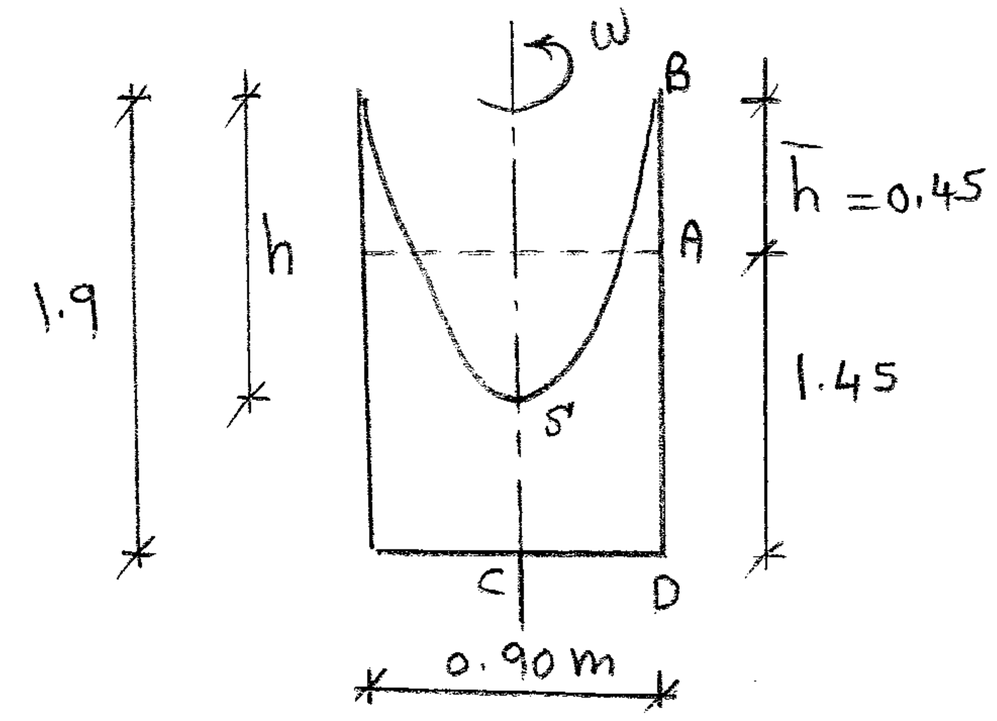
$$\%\ \text{water left} = \frac{\forall_{left}}{\forall_{before}} * 100 = \frac{0.7854}{1.178} * 100 = \underline{\underline{66.6\ \%}}$$

22

An open cylindrical tank, 1.9 m height and 0.9 m in diameter, contains 1.45 m of oil (S. G. = 0.9). If the cylinder rotates about its geometric axis, (17)

i-What constant angular velocity can be attained without spilling the oil?

ii- What is the pressure at the bottom of the tank at points C and D when w = 5.0 rad/sec



S.G. = 0.9

$$\forall_{\text{air before rotation}} = \forall_{\text{air after rotation}}$$

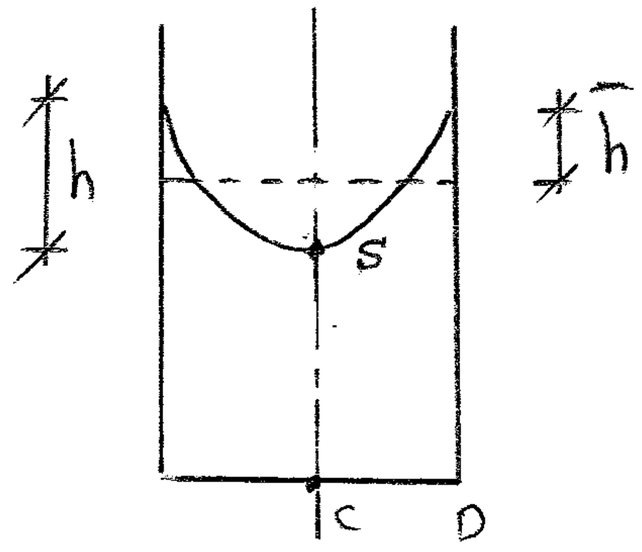
$$\bar{h} = \frac{h}{2}$$

$$h = 2\bar{h} = 2(0.45) = 0.9$$

$$h = \frac{\omega^2 r^2}{2g}$$

$$0.9 = \frac{\omega^2 (0.45)^2}{2(9.81)} \Rightarrow \omega = \underline{9.34 \text{ rad/sec}}$$

If $\omega = 5$ rad/sec



$$h = \frac{\omega^2 r^2}{2g} = \frac{(5)^2 (0.45)^2}{2(9.81)} = 0.258 \text{ m}$$

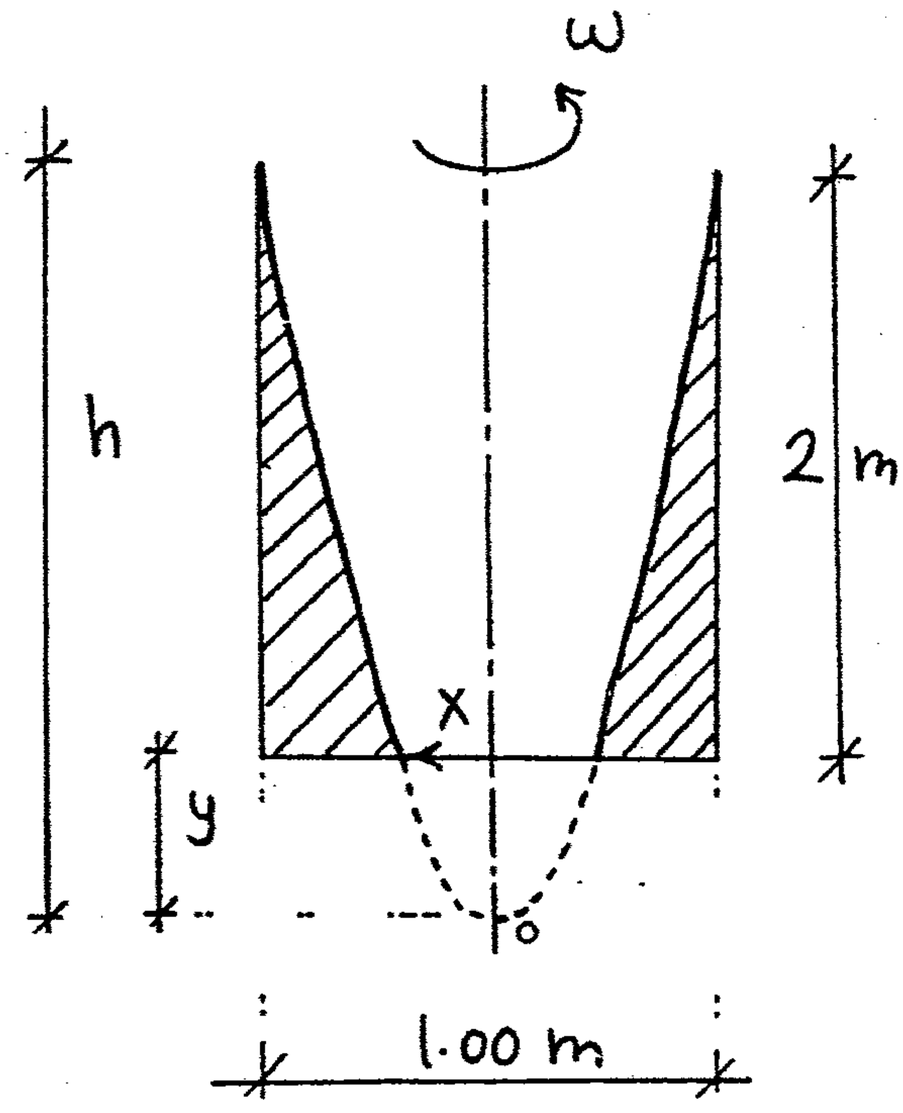
$$\bar{h} = \frac{h}{2} = \frac{0.258}{2} = 0.129$$

$$P_C = \gamma h_C = 0.9(9810)(1.45 - 0.129) = \underline{11\,663 \text{ N/m}^2}$$

$$P_D = \gamma h_D = 0.9(9810)(1.45 + 0.129) = \underline{13\,941 \text{ N/m}^2}$$

An open cylinderical tank 2 m high and 1 m in diameter is full of water. If the cylinder is rotated with an angular velocity of 2.5 rev/s. (18)

How much of the bottom of the tank is uncovered?



$$\omega = 2\pi n = 2\pi(2.5) = 15.7 \text{ rad/s}$$

$$h = \frac{\omega^2 r^2}{2g}$$

$$= \frac{(15.7)^2 (0.5)^2}{2 * 9.81} = 3.14$$

$$y = h - 2 = 3.14 - 2 = 1.14 \text{ m}$$

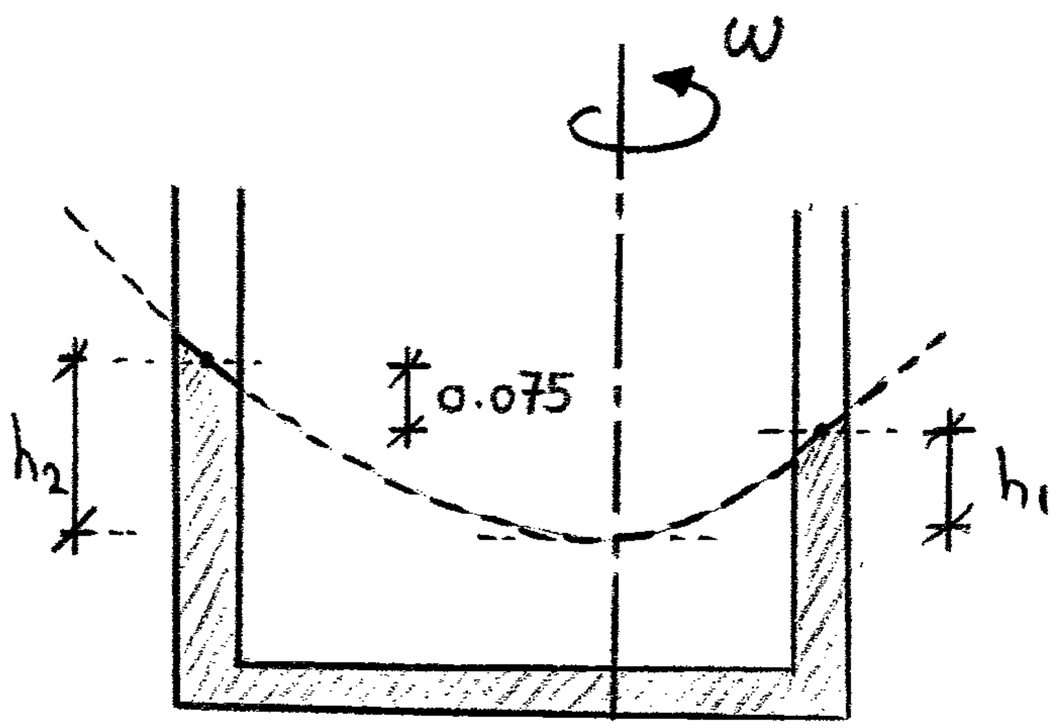
$$1.14 = \frac{(15.7)^2 x^2}{2 * 9.81} \Rightarrow x = 0.30 \text{ m}$$

$$\text{Uncovered Area} = \pi x^2 = \pi (0.3)^2 = \underline{0.285 \text{ m}^2}$$

24

$$h_2 - h_1 = 0.075 \text{ m}$$

$\omega = ?$



sol

$$h_1 = \frac{\omega^2 r^2}{2g} = \frac{\omega^2 (0.09)^2}{2(9.81)}$$

$$h_2 = \frac{\omega^2 (0.22)^2}{2(9.81)}$$

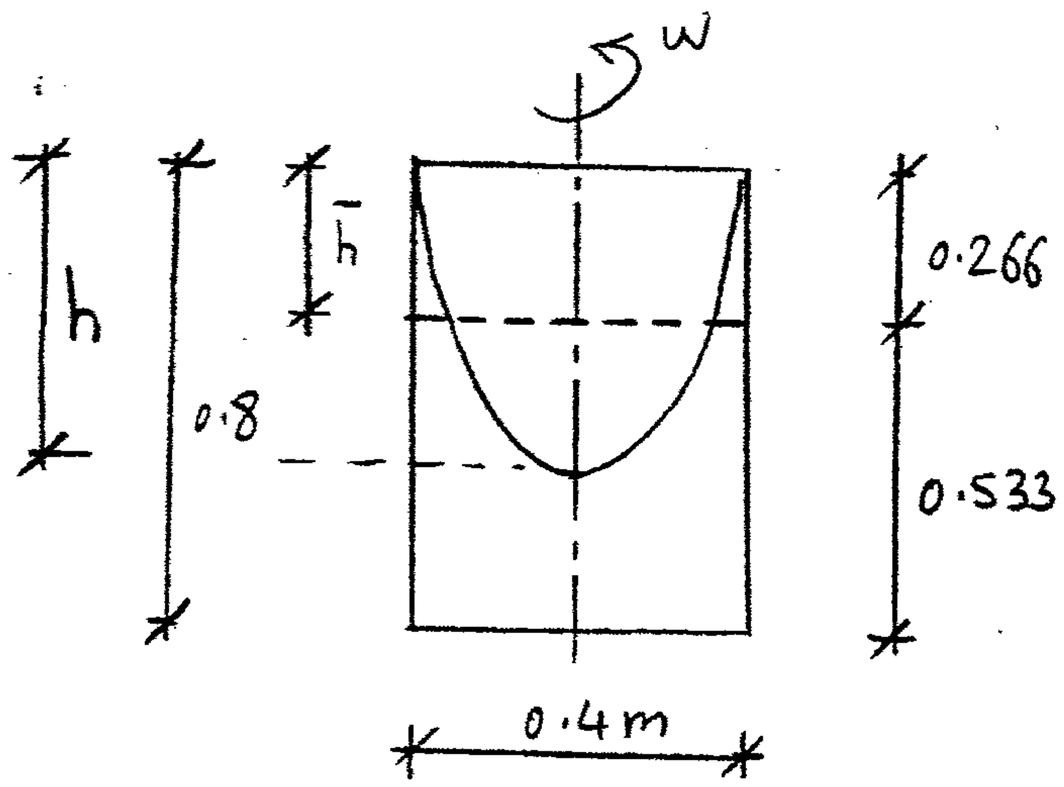
$$\frac{\omega^2 (0.22)^2}{2(9.81)} - \frac{\omega^2 (0.09)^2}{2(9.81)} = 0.075$$

$$\Rightarrow \omega = \underline{6.04 \text{ rad/s}}$$

A closed cylindrical container, 0.40 m diameter and 0.8 m high, two third of its height is filled with oil (S.G. = 0.85). The container is rotated about its vertical axis. Determine the speed of rotation when: a) the oil just start touching the lid.

b) the point at the center of the base is just clear of oil.

S.G. = 0.85



$\forall_{air\ before\ rot} = \forall_{air\ after\ rot}$

$\bar{h} = \frac{h}{2}$

$h = 2\bar{h} = 2(0.266) = 0.533$

$h = \frac{\omega^2 r^2}{2g}$

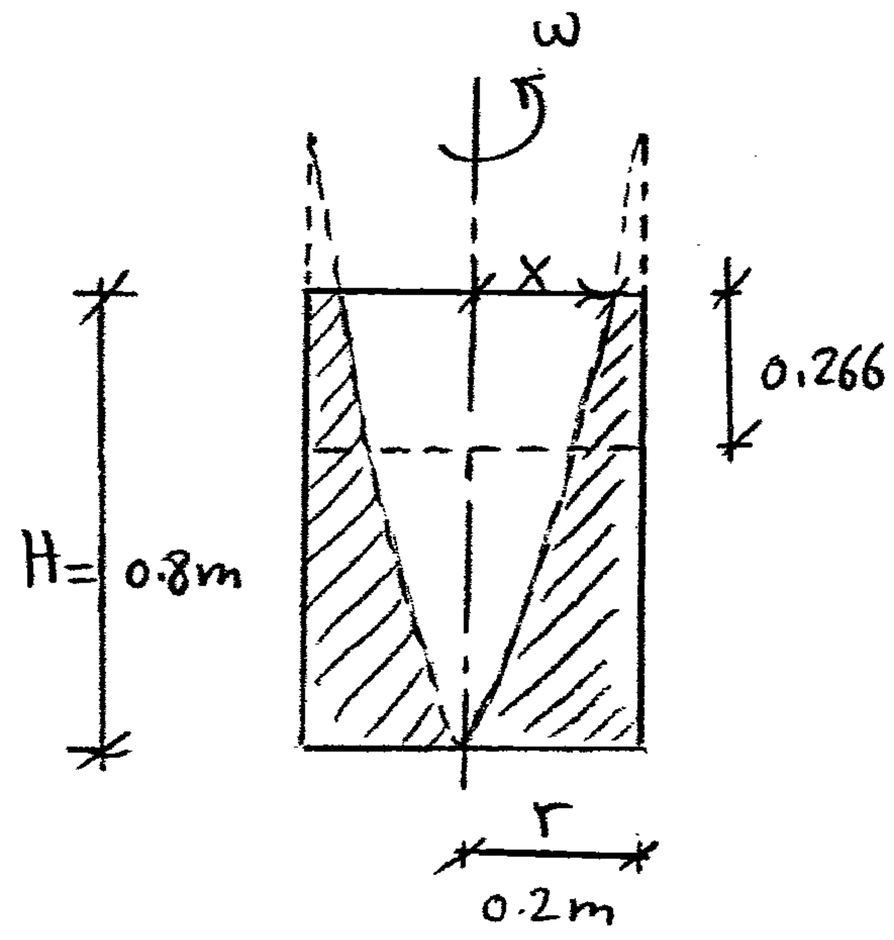
$0.533 = \frac{\omega^2 (0.2)^2}{2 * 9.81} \Rightarrow \omega = \underline{\underline{16.17\ rad/sec}}$

$\forall_{air\ before\ rotation} = \forall_{air\ after\ rotation}$

$\cancel{\pi} r^2 (0.266) = \frac{1}{2} \cancel{\pi} x^2 (0.8)$

$(0.2)^2 (0.266) = \frac{1}{2} x^2\ 0.8$

$\therefore \quad X = 0.163\ m$



$H = \frac{\omega^2 x^2}{2g}$

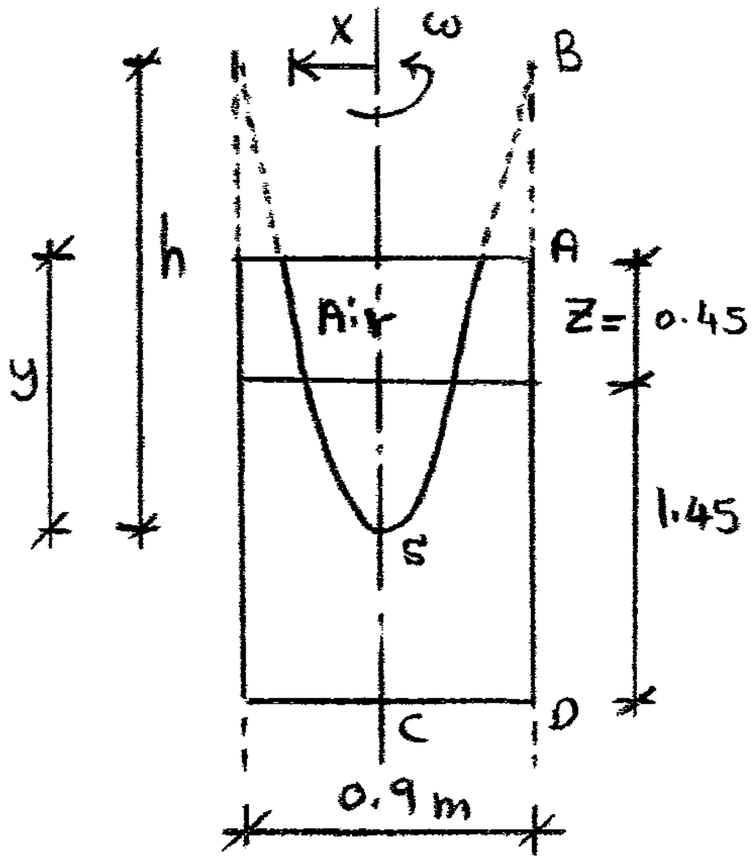
$0.8 = \frac{\omega^2 (0.163)^2}{2 * 9.81}$

$\Rightarrow \quad \omega = \underline{\underline{24.29\ \ rad/sec}}$

(23) A closed cylindrical tank with the air space subjected to a pressure of 14.8 psi. , 1.9 m height and 0.9 m in diameter, contains 1.45 m of oil (S. G. = 0.9). If the cylinder rotates about its geometric axis, (20)

a) When the angular velocity is 10 rad/sec, what are the pressures in bar at points C, D and S?

b) At what speed must the tank be rotated in order that the center of the bottom have zero depth?



a) $\omega = 10$ rad/sec

$$h = \frac{\omega^2 r^2}{2g} = \frac{(10)^2 (0.45)^2}{2(9.81)} = 1.03$$

$h > 2Z$ $\quad 1.03 > 2(0.45)$

∴ Water will touch the lid slbi

$$\forall_{air\ \text{before rotation}} = \forall_{air\ \text{after rotation}}$$

$$\frac{\pi (0.9)^2}{4} (0.45) = \frac{1}{2} \pi x^2 y$$

$$\therefore \quad x^2 y = 0.18225 \rightarrow (1)$$

$$\therefore \quad y = \frac{\omega^2 x^2}{2g} = \frac{(10)^2 x^2}{2(9.81)} = 5.09 x^2 \rightarrow (2)$$

from (1), (2)

$$5.09 x^2 = 0.18225 \Rightarrow x = 0.435$$

$$y = 0.9625$$

$$P_{air} = 14.8\ lb/in^2 = 14.8 * 144 = 2131.2\ lb/ft^2$$

$$= 2131.2 \frac{lb}{ft^2} \frac{4.44\ N}{1\ lb} * \frac{1^2\ ft^2}{(0.3048)^2\ m^2} = 101854 \frac{N}{m^2}$$

$$P_c = P_{air} + \gamma h_c \qquad h_c = 1.9 - y$$

$$= 101845 + 0.9(9810)(1.9 - 0.9625) = \underline{110122\ N/m^2}$$

$$P_d = P_{air} + \gamma h_d \qquad h_d = 1.9 - y + h$$

$$= 101845 + 0.9(9810)(1.9 - 0.9625 + 1.03) = \underline{119216\ N/m^2}$$

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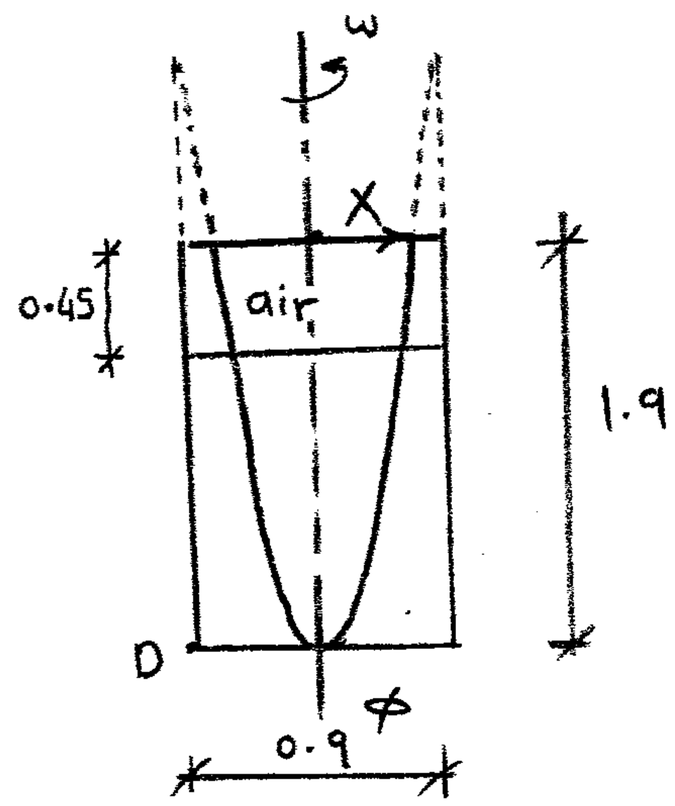
b) $\forall_{air\ before} = \forall_{air\ after}$

$$\frac{\pi (0.9)^2}{4} 0.45 = \frac{1}{2} \pi X^2 (1.9)$$

$$\therefore X = 0.31$$

$$y = \frac{\omega^2 X^2}{2g}$$

$$1.9 = \frac{\omega^2 (0.31)^2}{2(9.81)} \Rightarrow \omega = \underline{19.71\ rad/sec}$$

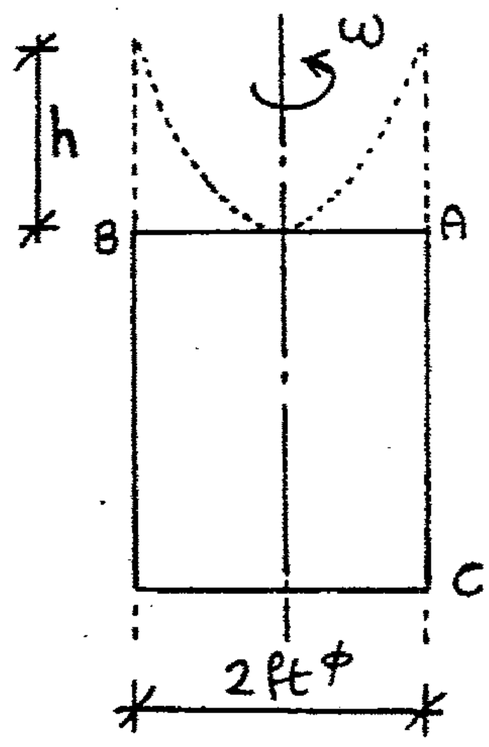


A closed vessel, 2 ft in diameter, is completely filled with water. If the vessel is rotated at 1200 rpm, what increase in pressure would occur at the top of the tank at the circumference?

$$\omega = \frac{2\pi N}{60} = \frac{2\pi (1200)}{60} = 125.6\ rad/s$$

$$h = \frac{\omega^2 r^2}{2g} = \frac{(125.6)^2 (1)^2}{2 * 32.2} = 245.2\ ft$$

$$P = \gamma h = 62.4\ (245.2) = 15300\ lb/ft^2$$



عند دوران سائل مغلق فإن الاسطح العليا عند A,B تصبح تحت ضغط ، ونظراً لوجود غطاء للإسطوانة فإن السائل لا يمكن أن يأخذ شكل ال Parabola لأن الغطاء يغلق عليه الخروج من الاسطوانة ، لذا يأوول سطح السائل إلى اتخاذ شكل ال Parabola وتصبح النقطة A تحت ضغط = $\frac{\gamma \omega^2 r^2}{2g}$

25 A tube AOB has the part OB, 300 mm long, bent upwards so that the angle OB is 45. AO is vertical and the end B is closed. The tube AOB is completely filled with water to a height of 230 mm above O. Find the number of revolutions per min of the tube about axis OA so that the pressure at B is the same as the pressure at O.

$$P_o = P_B$$

$$\gamma h_{AO} = \gamma h_{BC}$$

$$\therefore BC = AO = 230 \text{ mm}$$

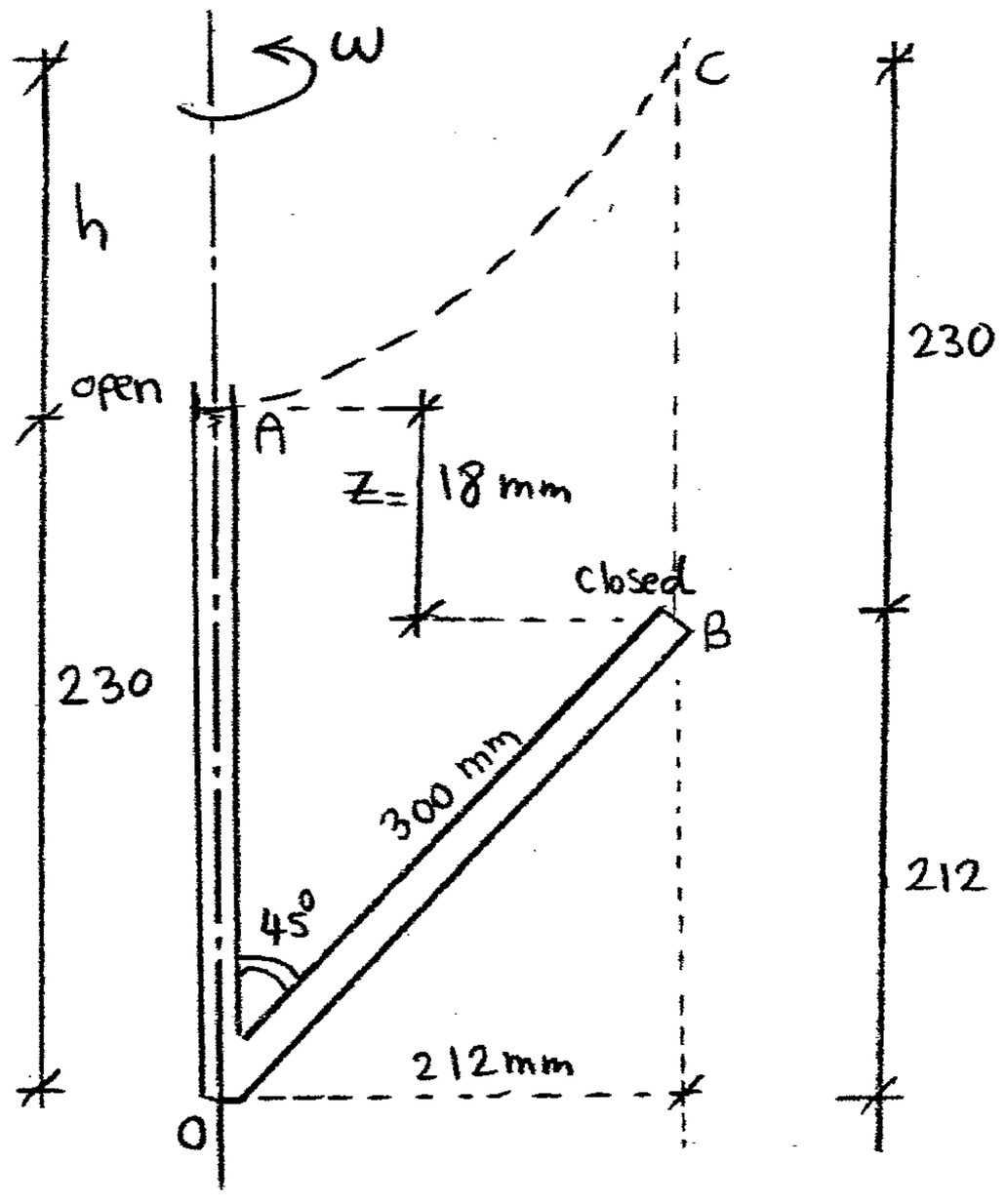
$$Z = 230 - 300 \cos 45 = 18 \text{mm}$$

$$h = \frac{\omega^2 r^2}{2g}$$

$$(0.23 - 0.018) = \frac{\omega^2 (0.212)^2}{2 * 9.81}$$

$$\Rightarrow \omega = 9.62 \text{ rad/s} = \frac{2\pi N}{60}$$

$$\therefore N = \underline{\underline{91.86 \text{ rev/min}}}$$



A closed cylindrical tank 2 m high and 1 m in diameter contains 1.5 m of water. When the angular velocity is constant at 20 rad/s, how much of the bottom of the tank is uncovered?

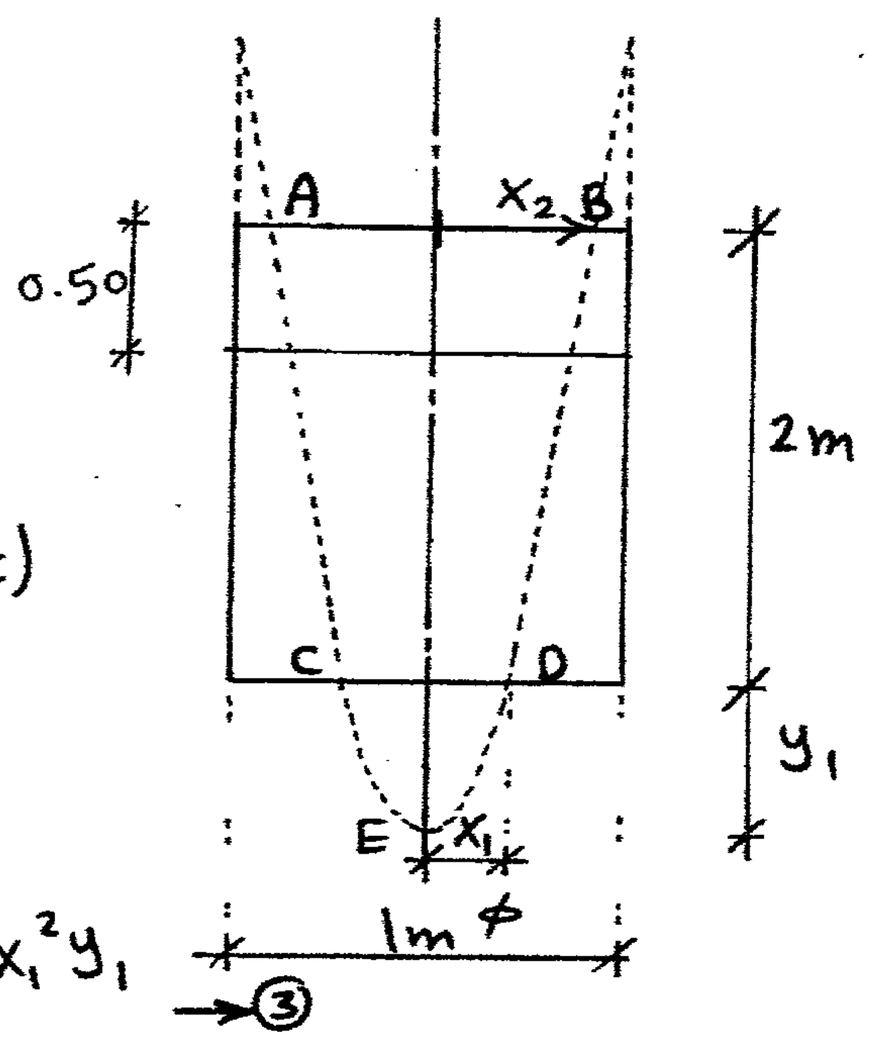
$$y_1 = \frac{(20)^2 x_1^2}{2 * 9.81} \rightarrow ①$$

$$2 + y_1 = \frac{(20)^2 x_2^2}{2 * 9.81} \rightarrow ②$$

$$V_{air\ (before\ rot)} = V_{air\ (after\ rot)}$$

$$\frac{\pi (1)^2}{4} * 0.5 = \underset{E}{\overset{A\ B}{\cup}} - \underset{E}{\overset{C\ D}{\cup}}$$

$$0.3927 = \frac{1}{2}\pi x_2^2 (2 + y_1) - \frac{1}{2}\pi x_1^2 y_1$$ → ③



from ①, ②, ③ $\Rightarrow x_1 = 0.116$

$$\text{Area} = \pi (0.116)^2 = \underline{\underline{0.0423 \text{ m}^2}}$$

7 Buoyancy & Floatation

Archimedes principle

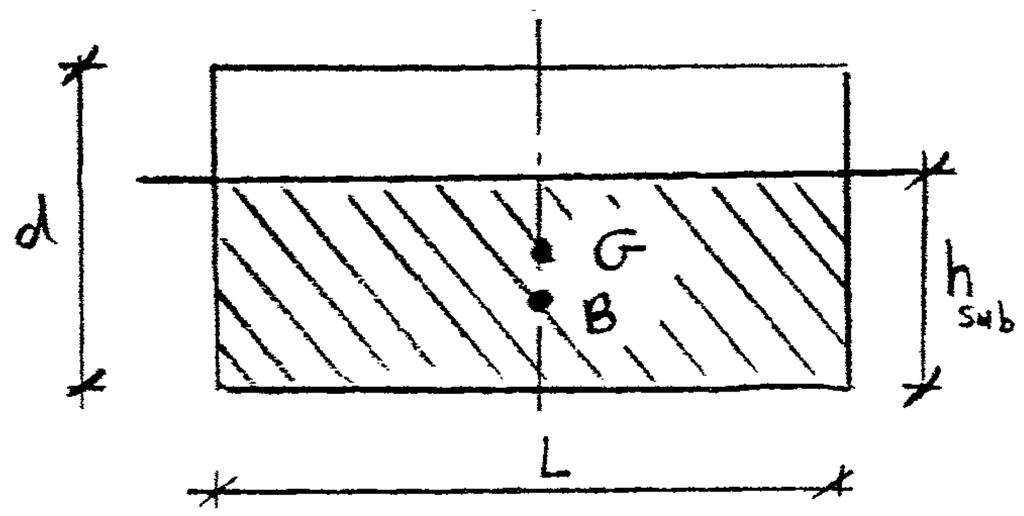
Any weight, floating or submerged in a liquid, is acted upon by a ***buoyant force*** equal to the weight of the liquid displaced, and acts through the center of gravity of the displaced liquid.

$\therefore$ **Weight of floating body = Weight of liquid displaced**

$$\gamma_b \forall_b = \gamma_w \forall_{sub}$$

$$S.g\ \gamma_w L b d = \gamma_w L b h_{sub}$$

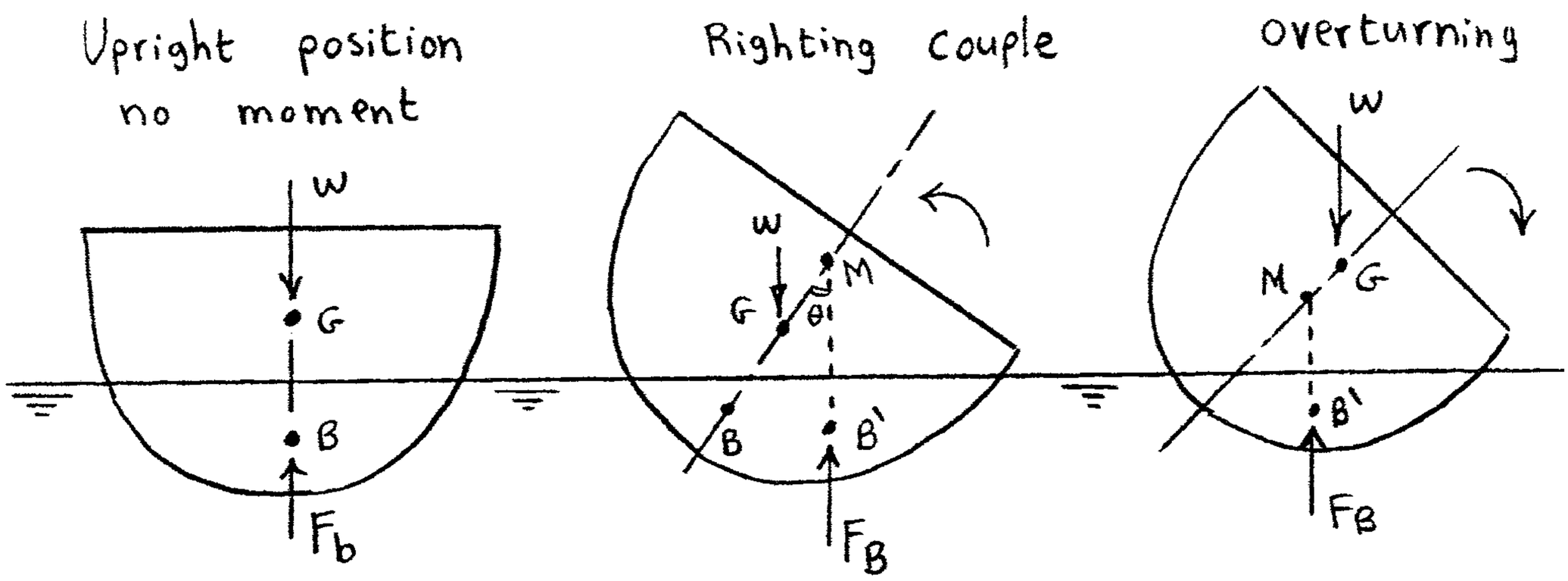
$$\therefore \boxed{h_{sub} = S.g\ d} \quad \text{(in this example)}$$



Center of Gravity (G) = Centroid of the whole body

Center of Buoyancy (B) = Centroid of the displaced liquid

Rotational stability of floating bodies



Original position — M above G — M under G

Stable — Stable — Unstable

* When the body is upright, point G and B lie on the same vertical
  $\Rightarrow$ no moment
* When the body is slightly rotated through a small angle θ the shape of the displaced volume gets different with an increase of volume towards one side.
  $\Rightarrow$ the centroid of the displaced volume B changes to B`
* Let a vertical through B` intersect the centerline at M
* The line of action of the buoyant force (acting through B`) forms a righting couple to return the body to its original position.
  $\Rightarrow$ the body is stable when point M is above G
* The point (M) is called the *metacenter.*

$$GM = BM - BG$$

GM = metacentric height

= +ve Stable

= −ve unstable

where

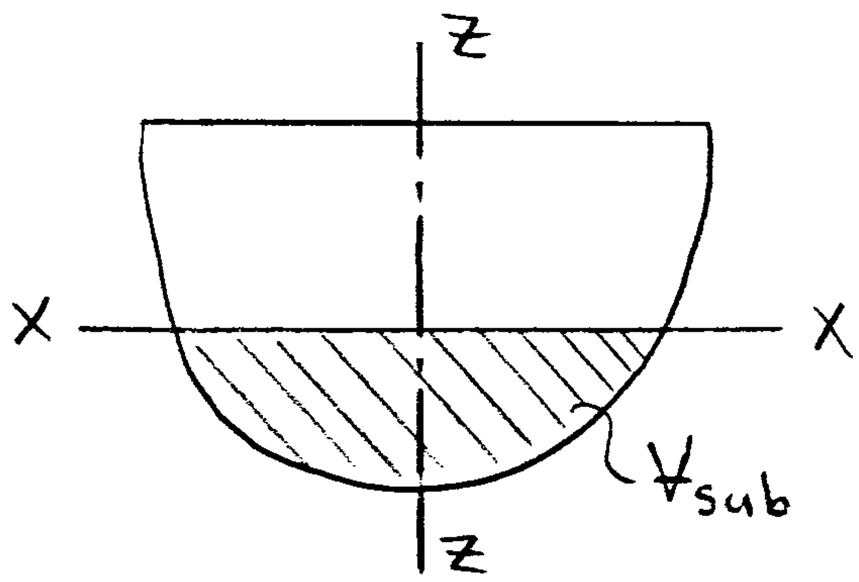
$$BM = \frac{I_y}{\forall_{sub}}$$

I_y = Moment of inertia around axis of rotation

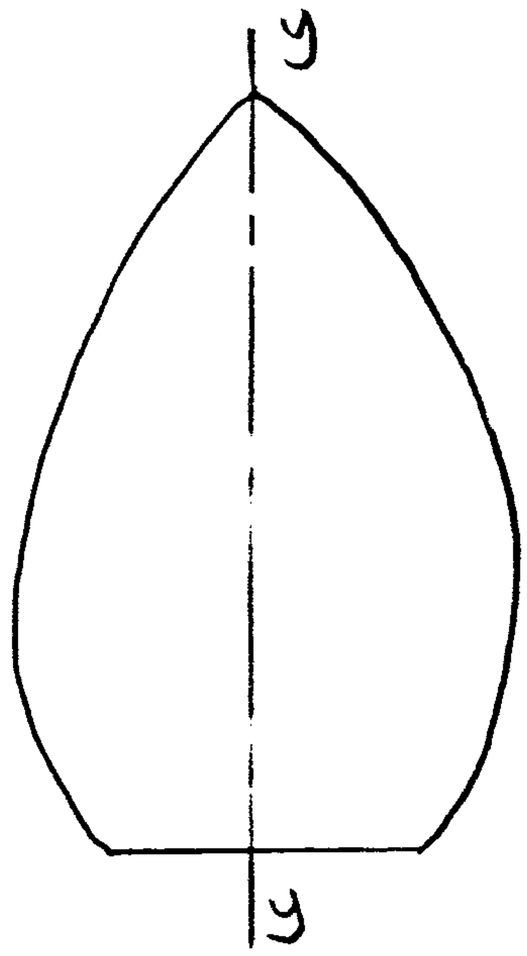
$\forall_{sub}$ = Submerged Volume

BG = distance between B and G

$$BG = \frac{d}{2} - \frac{h_{sub}}{2}$$ (in this example)



Elevation



plan

Center of Buoyancy

- It is the point of application of the force of buoyancy on the body.
- It is always the center of gravity of the volume of fluid displaced.

Types of equilibrium of Floating bodies

1.Stable equilibrium,
2.Unstable equilibrium and
3.Neutral equilibrium.

Stable Equilibrium

- It occurs when a body is tilted slightly by some external force, and then it returns back to its original position due to the weight and the upthrust.
- The position of metacentre M is higher than the center of gravity G.

Unstable Equilibrium

- It occurs when a body does not return to its original position from the slightly displaced angular position.
- The position of metacentre M is lower than G.

Neutral Equilibrium

- It occurs when a body, when given a small angular displacement, occupies a new position and remains at rest.
- The position of metacentre M coincides with G.

الرسم ضرورى مع كل تعريف

Metacentre

- The metacentre is the point of intersection of the axis of the body passing through the center of gravity (G) with the original centre of buoyancy (B) and a vertical line passing through the centre of buoyancy (B') of the tilted position of the body.
- The position of metacentre (M) remains practically constant for the small angle of tilt θ.

Metacentric Height:

- It is the distance between the centre of gravity of a floating body and the metacentre.
- GM = BM - BG

(4)

Will a beam of S.G. = 0.65 and length 1500 mm long with a cross section 136 mm wide and 96 mm height float in stable equilibrium in water with two sides horizontal?

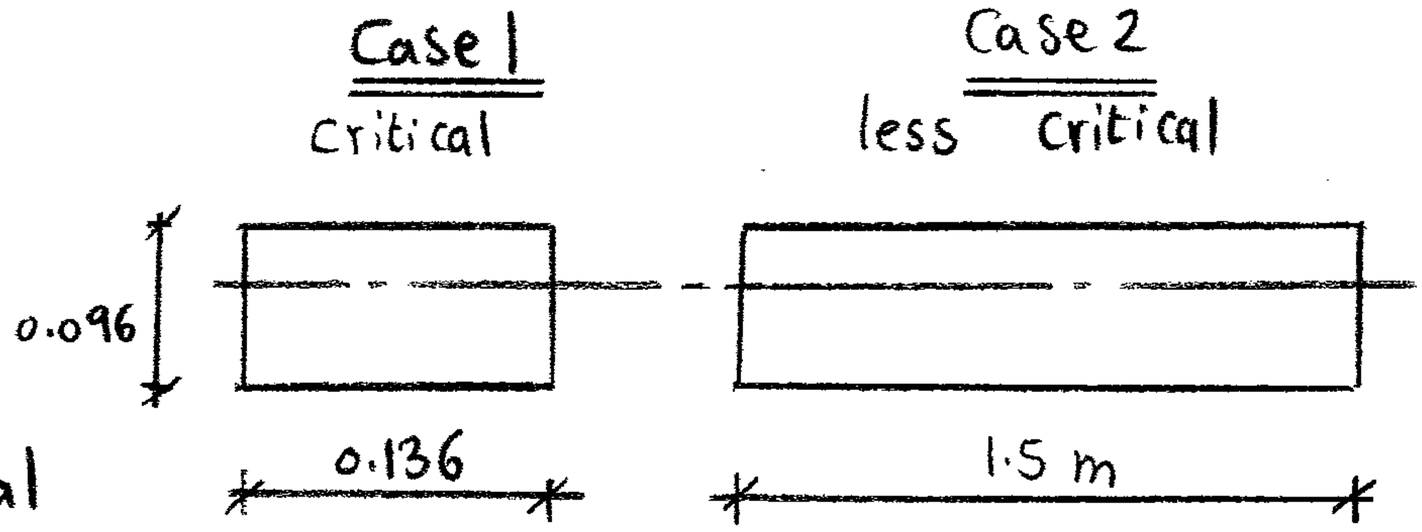
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Sol

2 Cases

0.136 < 1.5

Case 1 is more critical

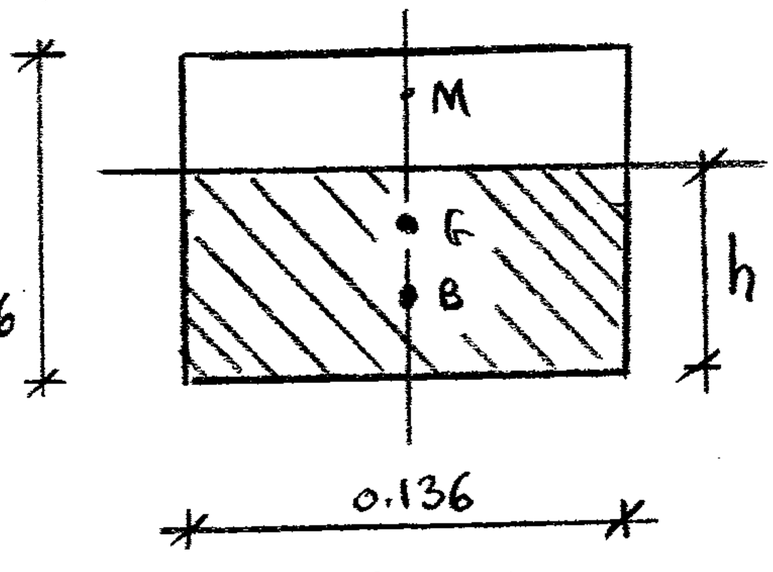


Weight of body = Weight of liquid displaced

$0.65 \gamma_w (1.5 * 0.136 * 0.096) = \gamma_w (1.5 * 0.136 * h)$

$\therefore h = S.g. * d$

$= 0.65 * 0.096 = 0.0624$



Elevation

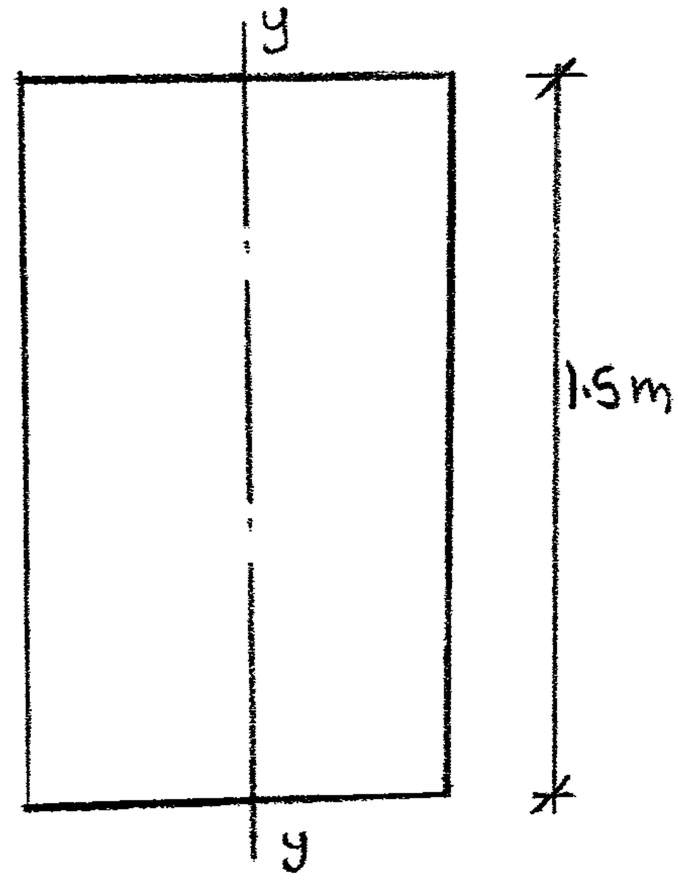
$BG = \frac{d}{2} - \frac{h}{2} = \frac{0.096 - 0.0624}{2} = 0.0168\ m$

$BM = \frac{I_y}{\forall_{sub}}$

$I_y = \frac{1.5 (0.136)^3}{12} = 3.144 * 10^{-4}\ m^4$

$\forall_{sub} = 0.136 * 1.5 * 0.0624 = 0.01273\ m^3$

$BM = \frac{I_y}{\forall_{sub}} = \frac{3.144 * 10^{-4}}{0.01273} = 0.0247\ m$



Plan

$GM = BM - BG$

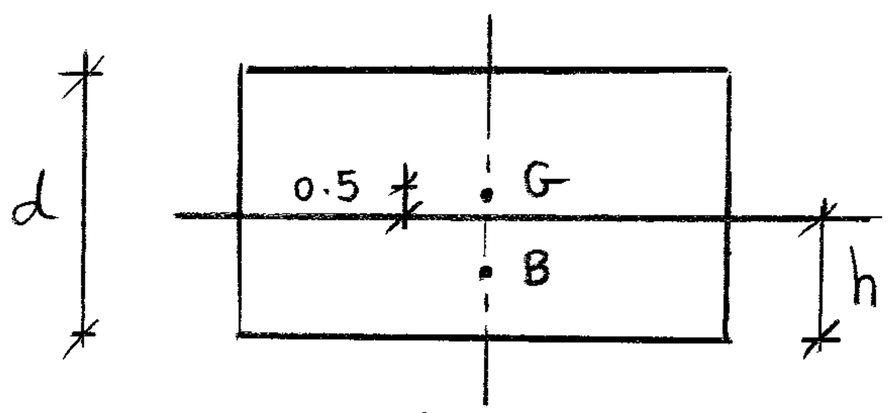
$= 0.0247 - 0.0168 = +7.9 * 10^{-3}\ m$

Metacentric height is +ve ⇒ Block is stable

28 A floating body 100 m wide and 150 m long has a gross weight of 60,000 ton. Its center of gravity is 0.5 m above the water surface.
Find the metacentric height and the restoring couple when the body is given a tilt as shown 0.5 m.

$W = 60{,}000\ ton_w * 10^3 * 9.81$

$= 588.6 * 10^6\ N$



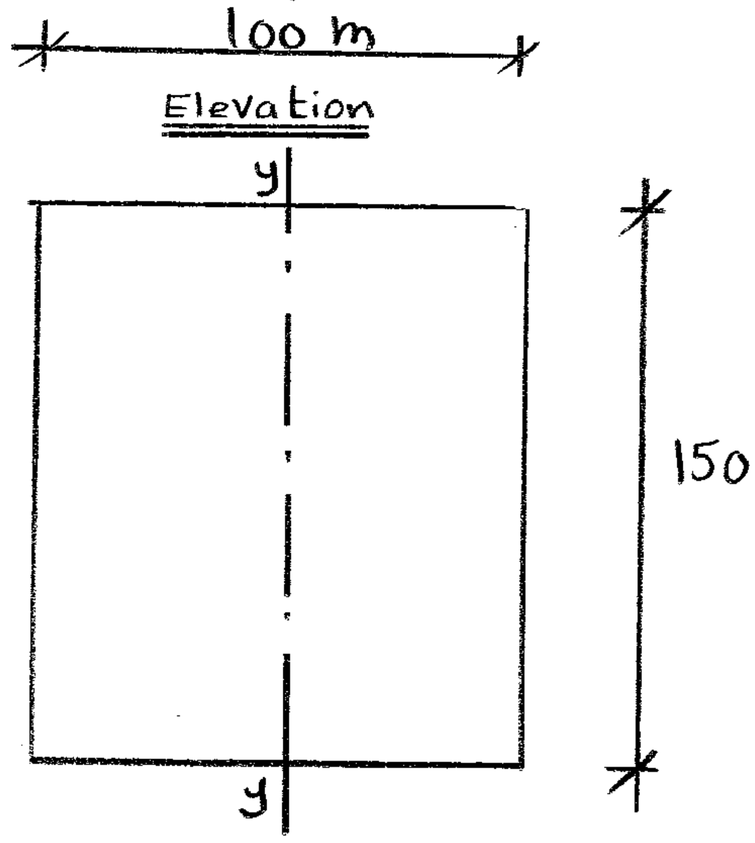
$W_{body} = W_{displaced}$

$W = \gamma_w \forall_{sub}$

$588.6 * 10^6 = 9810\ \forall_{sub}$

$\therefore \forall_{sub} = 60{,}000\ m^3$

$100 * 150 * h = 60{,}000 \Rightarrow h = \underline{4m}$



$BG = (h + 0.5) - \frac{h}{2} = 2.5\ m$

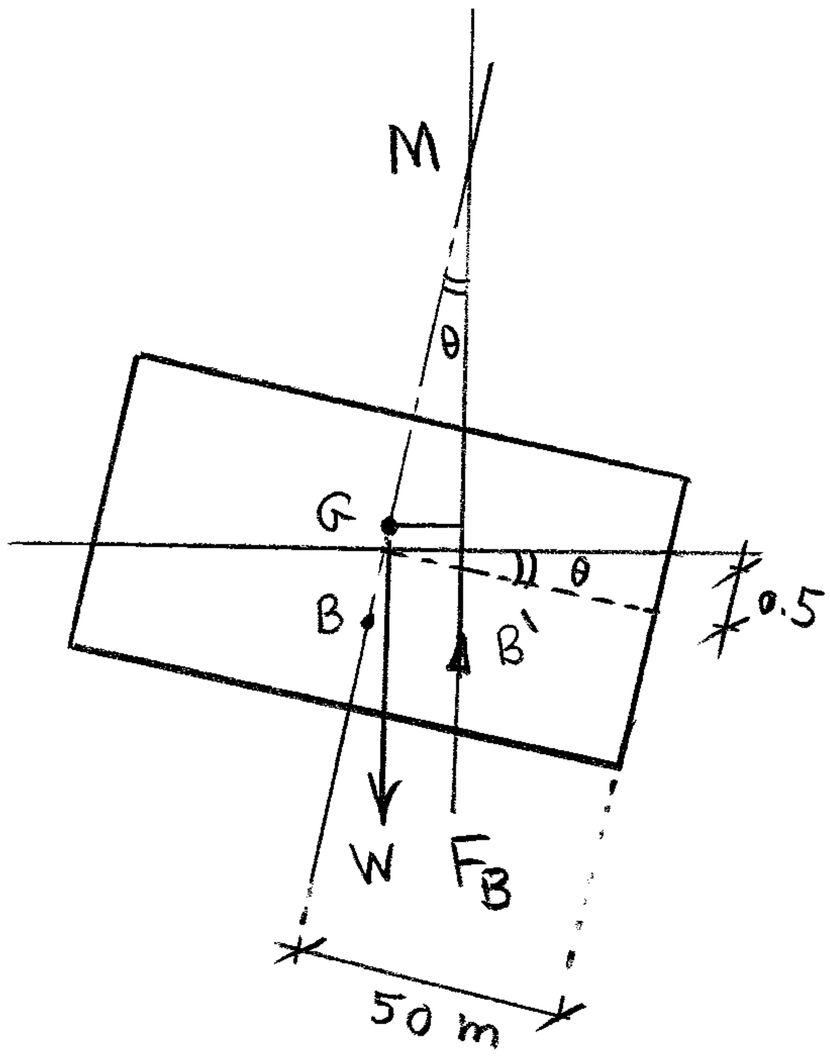
$BM = \frac{I_y}{\forall_{sub}}$

$I_y = \frac{150\ (100)^3}{12} = 12.5 * 10^6$

$BM = \frac{I_y}{\forall_{sub}} = \frac{12.5 * 10^6}{60 * 10^3} = 208.33$

$GM = BM - BG$

$= 208.33 - 2.5 = \underline{205.833}$



Restoring Couple

$\tan\theta = \frac{0.5}{50} \Rightarrow \theta = 0.573^\circ$

for very small angles θ

$M = W\ \overline{GM} \sin\theta = 60 * 10^3 * 205.833 * \sin 0.573 = \underline{123500\ mt}$

29 June 2004

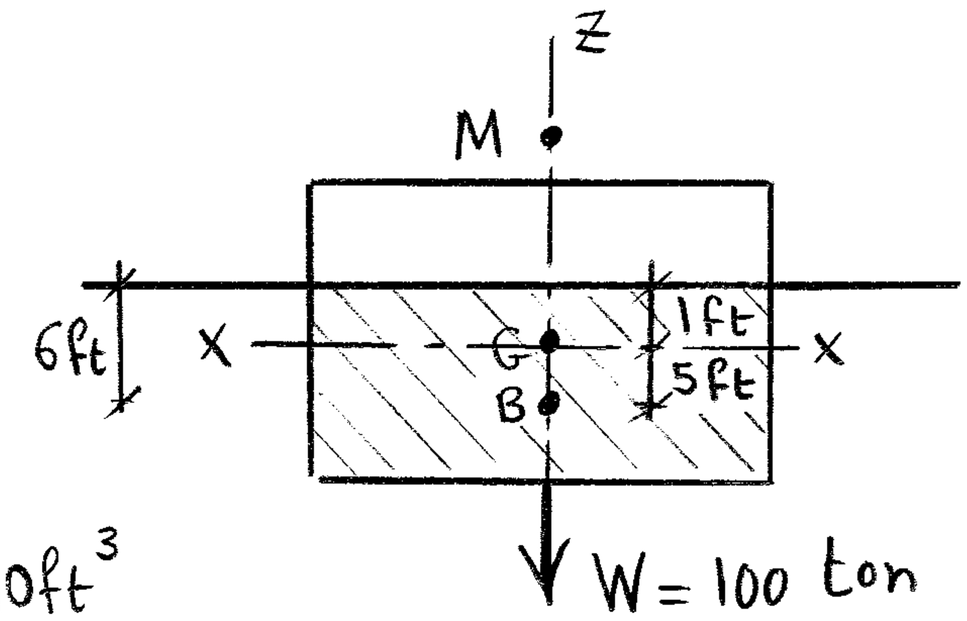
A ship displacing 1000ton has a horizontal cross-section at water-line as shown in the figure, its center of bouyancy is 6 ft below water surface and its center of gravity is 1 ft below the water surface. **Determine** its metacentric height for rolling (about y-axis) and for pitching (about x-axis).

$$W_{body} = W_{displaced}$$

$$1000ton = \gamma_w \forall_{sub}$$

$$1000ton = 1 \frac{ton_w}{m^3} * \forall_{sub}$$

$$\therefore \forall_{sub} = 1000m^3 = 1000 * \left(\frac{1}{0.3048}\right)^3 = 35310ft^3$$

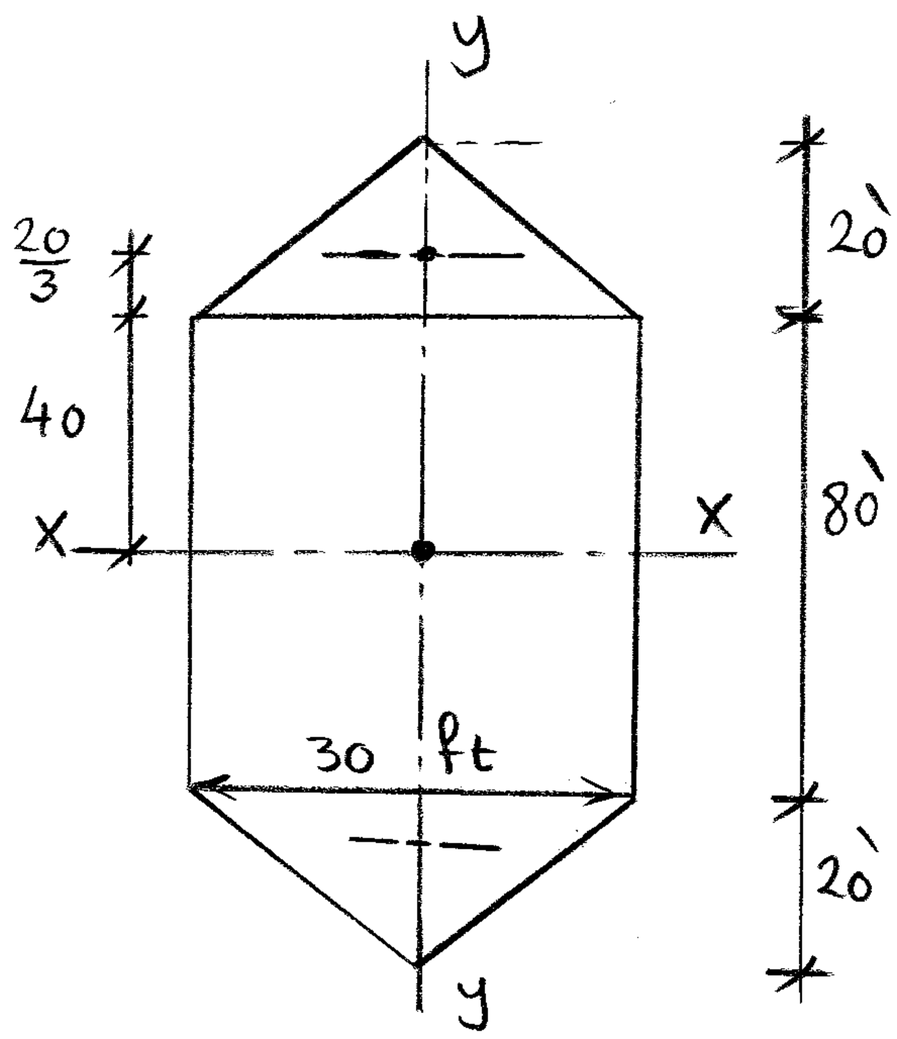


Elevation

$$BG = 5\ ft$$

$$BM = \frac{I_y}{\forall_{sub}} \quad \text{(rolling) about y-axis}$$

$$I_y = \frac{80(30)^3}{12} + 2\left[\frac{20*(30)^3}{48}\right] = 202500$$



Plan

$$BM = \frac{I_y}{\forall_{sub}} = \frac{202500}{35310} = 5.735$$

$$GM = BM - BG$$

$$= 5.735 - 5 = \underline{0.735\ ft}$$

$$BM = \frac{I_x}{\forall_{sub}} \quad \text{(pitching) about X-axis}$$

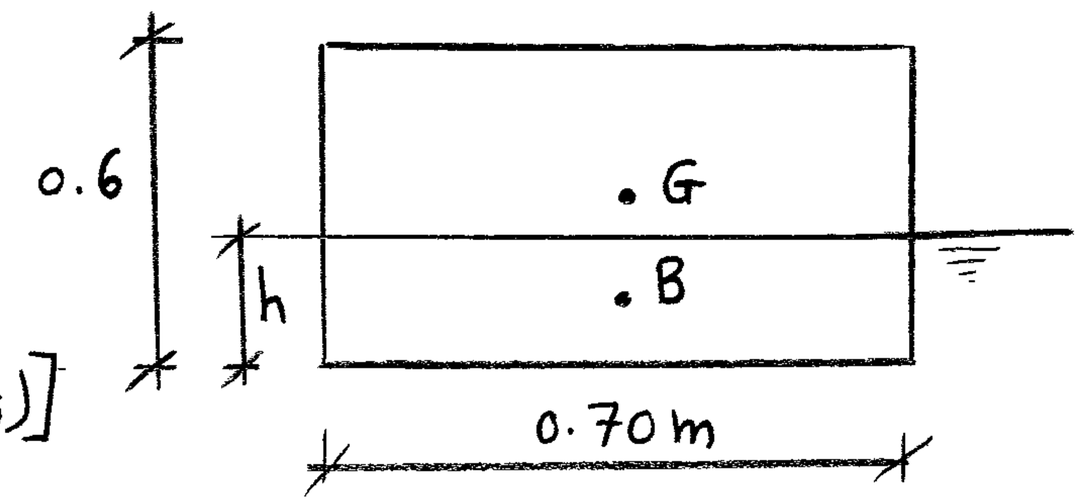
$$I_x = \frac{30(80)^3}{12} + 2\left[\frac{30(20)^3}{36} + \frac{1}{2}(30)(20)\left(40 + \frac{20}{3}\right)^2\right] = 2.6 * 10^6\ ft^4$$

$$BM = \frac{I_x}{\forall_{sub}} = \frac{2.6*10^6}{35310} = 73.63\ ft$$

$$GM = BM - BG = 73.63 - 5 = \underline{68.63\ ft}$$

(27) An empty tank rectangular in plan (with all sides closed) is 12.5 m long , and its cross section 0.70 m width x 0.60 m height. If the sheet metal weights 363 N/m^2 of the surface, and the tank is allowed to float in fresh water (Specific weight 9.81 KN/m^3) with the 0.60m wedge vertical. Show, whether the tank is stable or not?

$\omega = 363\ N/m^2$



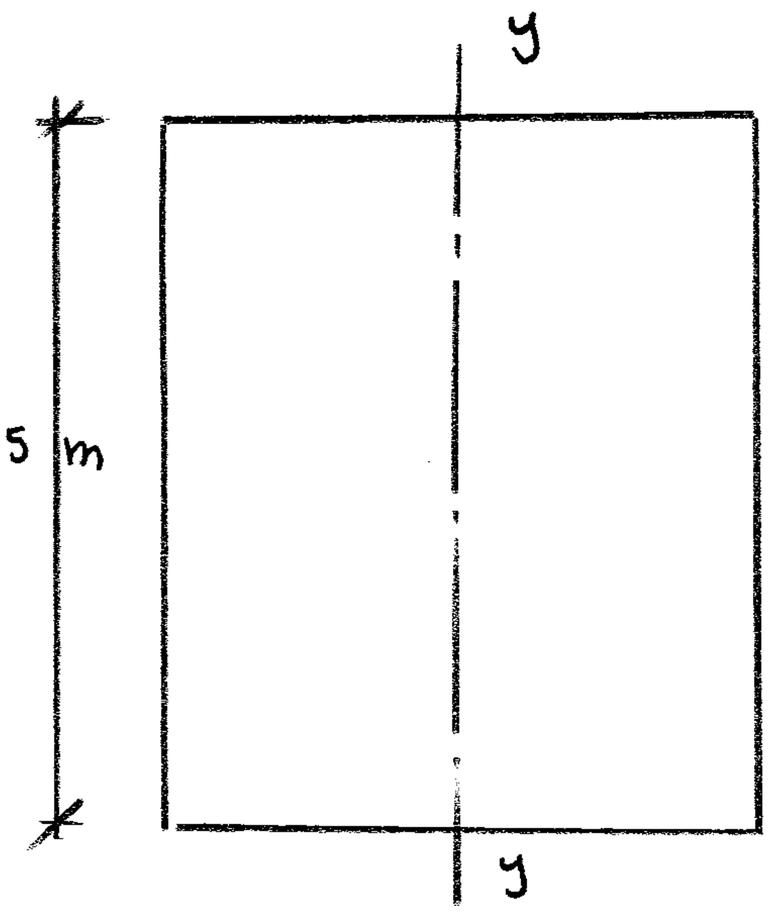
Elevation

$W_{body} = \omega A_{Total}$

$A_{Tot} = 2[0.6(0.7) + 0.6(12.5) + 0.7(12.5)]$

$= 33.34\ m^2$

$W_{body} = 363\ N/m^2 * 33.34 = \underline{12102\ N}$



$\therefore W_{body} = W_{displaced}$

$12102 = \gamma_w \forall_{sub}$

$V_{sub} = \frac{12102}{9810} = 1.233\ m^2$

$\therefore \forall_{sub} = 12.5 * 0.7 * h$

$\Rightarrow h = \frac{1.233}{12.5 * 0.7} = 0.14\ m$

$BG = 0.3 - \frac{h}{2} = 0.3 - \frac{0.14}{2} = 0.23\ m$

$BM = \frac{I_y}{\forall_{sub}} = \frac{\frac{12.5(0.7)^3}{12}}{1.233} = 0.29$

$GM = BM - BG$

$= 0.29 - 0.23 = 0.06$ (+ve) Stable

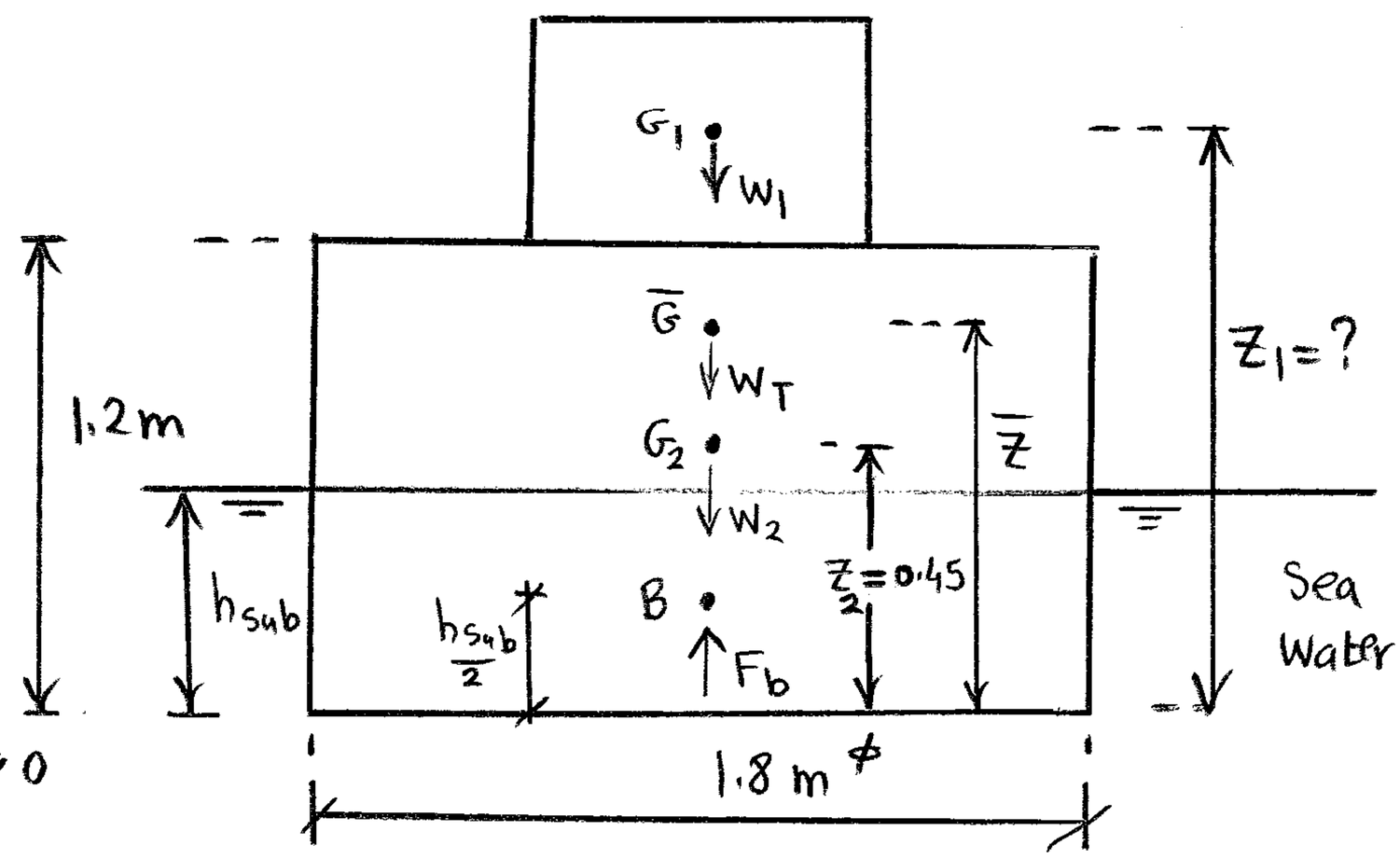
33 A cylindrical buoy 1.8 m diam., 1.2 m high and weighing 10 KN is in sea water of density 1025 kg/m3. Its center of gravity is 0.45 m from the bottom. If a load of 2 KN is placed on the top; find the maximum height of the C.G. of this load above the bottom if the buoy is to remain in stable equilibrium. (8)

$W_2 = 10\ KN$

$W_1 = 2\ KN$

$\gamma_{s.w} = 1025\ kg/m^3$

$Z_1 = ?$



for Stability

$GM = BM - BG \geq 0$

$BG = \bar{Z} - \frac{h_{sub}}{2}$

$\bar{Z} = \frac{W_1 Z_1 + W_2 Z_2}{W_T}$

$W_1 = 2\ KN$ $\quad W_2 = 10\ KN$ $\quad W_T = 12\ KN$

$Z_1 = ?$ $\quad Z_2 = 0.45$ $\quad \bar{Z} = ?$

$\bar{Z} = \frac{2 Z_1 + 10(0.45)}{12} = \frac{Z_1}{6} + 0.375$

$\therefore \quad F_b = W$

$\gamma_{sw} \forall_{sub} = 12\ KN$

$\gamma_{sw} = 1025 \frac{kg}{m^3} * 9.81 = 10055 \frac{N}{m^3}$

$10055 * \left[\frac{\pi (1.8)^2}{4} h_{sub}\right] = 12 * 10^3 \Rightarrow h_{sub} = \underline{\underline{0.469m}}$

$$BG = \overline{Z} - \frac{h_{sub}}{2}$$

$$* \; BG = \left(\frac{Z_1}{6} + 0.375\right) - \frac{0.469}{2} = \frac{Z_1}{6} + 0.14$$

$$BM = \frac{I_y}{\forall_{sub}}$$

$$I_y = \frac{\pi d^4}{64} = \frac{\pi (1.8)^4}{64} = 0.515 \; m^4$$

$$\forall_{sub} = \frac{\pi (1.8)^2}{4} (0.469) = 1.193 \; m^3$$

$$* \; BM = \frac{I_y}{\forall_{sub}} = \frac{0.515}{1.193} = 0.432$$

$$* \; GM = BM - BG \geq 0$$

$$0.432 - \left(\frac{Z_1}{6} + 0.14\right) \geq 0$$

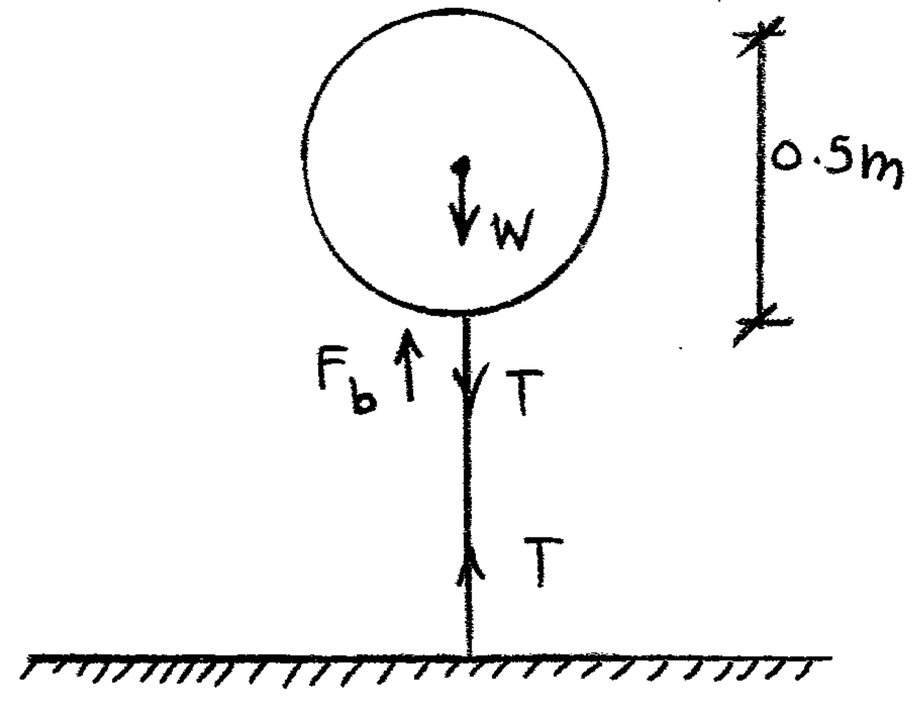
$$\therefore \; Z_1 < \underline{\underline{1.75 \; m}}$$

26

A spherical Buoy شمندورة (floating ball) has a 0.50 m in diameter, weights 500 N, and is anchored to the seafloor with a cable. Although the buoy normally floats on the surface, at certain times the water depth increases so that the buoy is completely immersed. What is the tension on the cable. ?

$W = 500\ N \qquad T = ?$

Solution



$$F_b = \gamma V$$

$$= 9810 \left[\frac{4}{3} \pi (0.25)^3 \right]$$

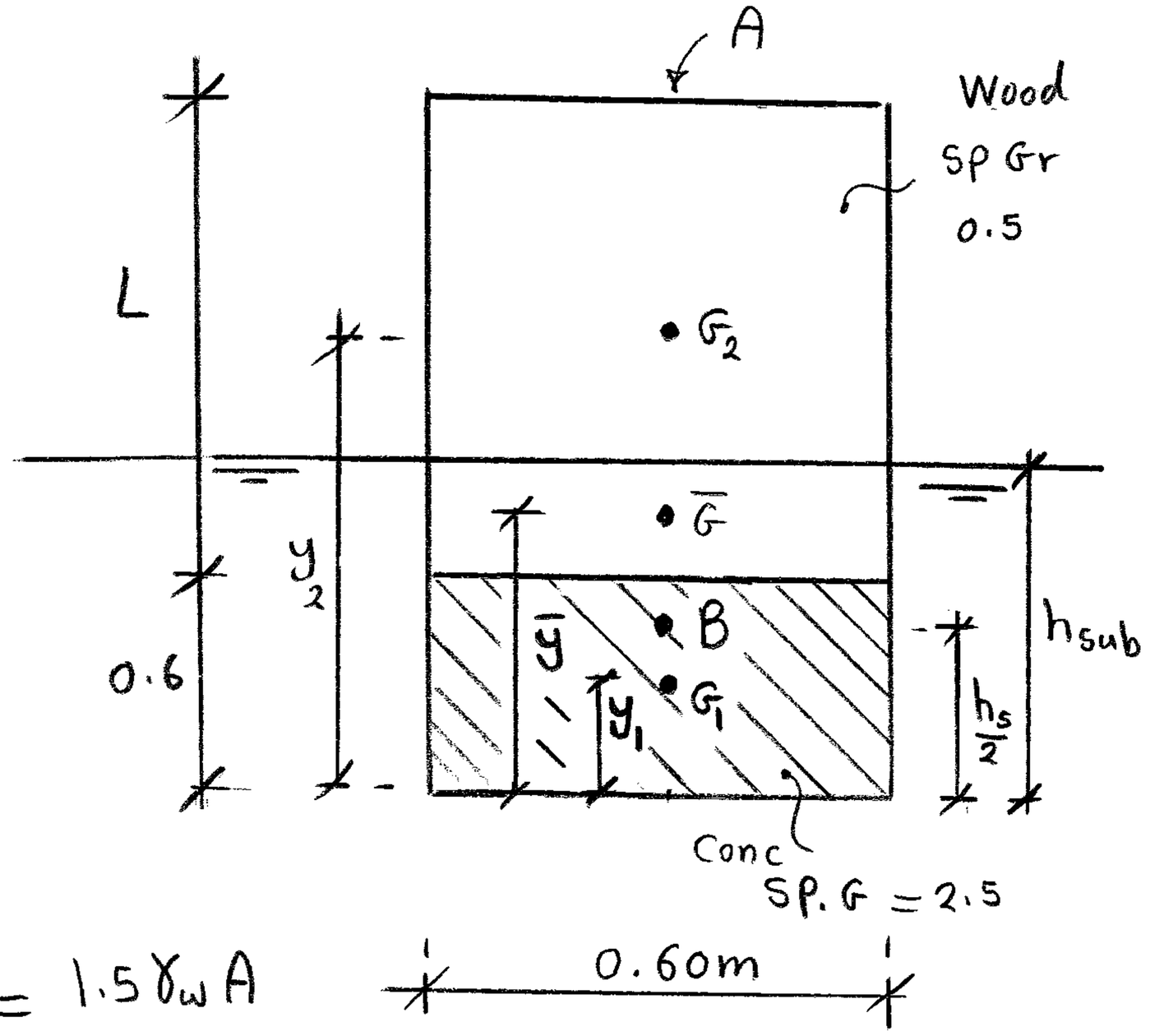
$$= 642\ N$$

$$W + T = F_b$$

$$500 + T = 642$$

$$\Rightarrow \quad T = \underline{142\ N}$$

30 b A wooden cylinder 60 cm in diameter, S.G. = 0.50 has a concrete cylinder 60 cm long of the same diameter, S.G. = 2.50, attached to one end. Determine the length of wooden cylinder for the system to float in stable equilibrium with its axis vertical.



for stability

$$GM = BM - BG$$

$$BG = \bar{y} - \frac{h_s}{2}$$

$$\bar{y} = \frac{W_1 y_1 + W_2 y_2}{W_T}$$

$$W_1 = \gamma_{conc} \forall_{conc}$$

$$= 2.5\gamma_w * (A * 0.6) = 1.5\gamma_w A$$

$$y_1 = \frac{0.6}{2} = 0.3$$

$$W_2 = \gamma_{wood} \forall_{wood} = 0.5\gamma_w * (A * L)$$

$$y_2 = 0.6 + \frac{L}{2}$$

$$W_T = W_1 + W_2 = 1.5\gamma_w A + 0.5\gamma_w A L$$

$$\bar{y} = \frac{1.5\gamma_w A (0.3) + 0.5\gamma_w A L (0.6 + \frac{L}{2})}{\gamma_w A (1.5 + L)}$$

$$\therefore \bar{y} = \frac{0.45 + 0.3L + 0.25L^2}{1.5 + 0.5L}$$

$$\therefore F_b = W$$

$$\gamma_w \forall_{sub} = W_1 + W_2$$

$$\gamma_w A h_{sub} = \gamma_w A (1.5 + 0.5L)$$

$$\therefore h_{sub} = 1.5 + 0.5L$$

$$* BG = \bar{y} - \frac{h_s}{2}$$

$$= \frac{0.45 + 0.3L + 0.25L^2}{1.5 + 0.5L} - \frac{1.5 + 0.5L}{2}$$

$$BM = \frac{I_y}{\forall_{sub}}$$

$$I_y = \frac{\pi d^4}{64} \quad , \quad \forall_{sub} = \frac{\pi d^2}{4} \cdot h_{sub}$$

$$* BM = \frac{I_y}{\forall_{sub}} = \frac{\pi d^4/64}{\frac{\pi d^2}{4} h_{sub}} = \frac{d^2}{16 h_{sub}} = \frac{d^2}{16(1.5+0.5L)}$$

$$GM = BM - BG \qquad \text{or} \qquad BM = BG$$

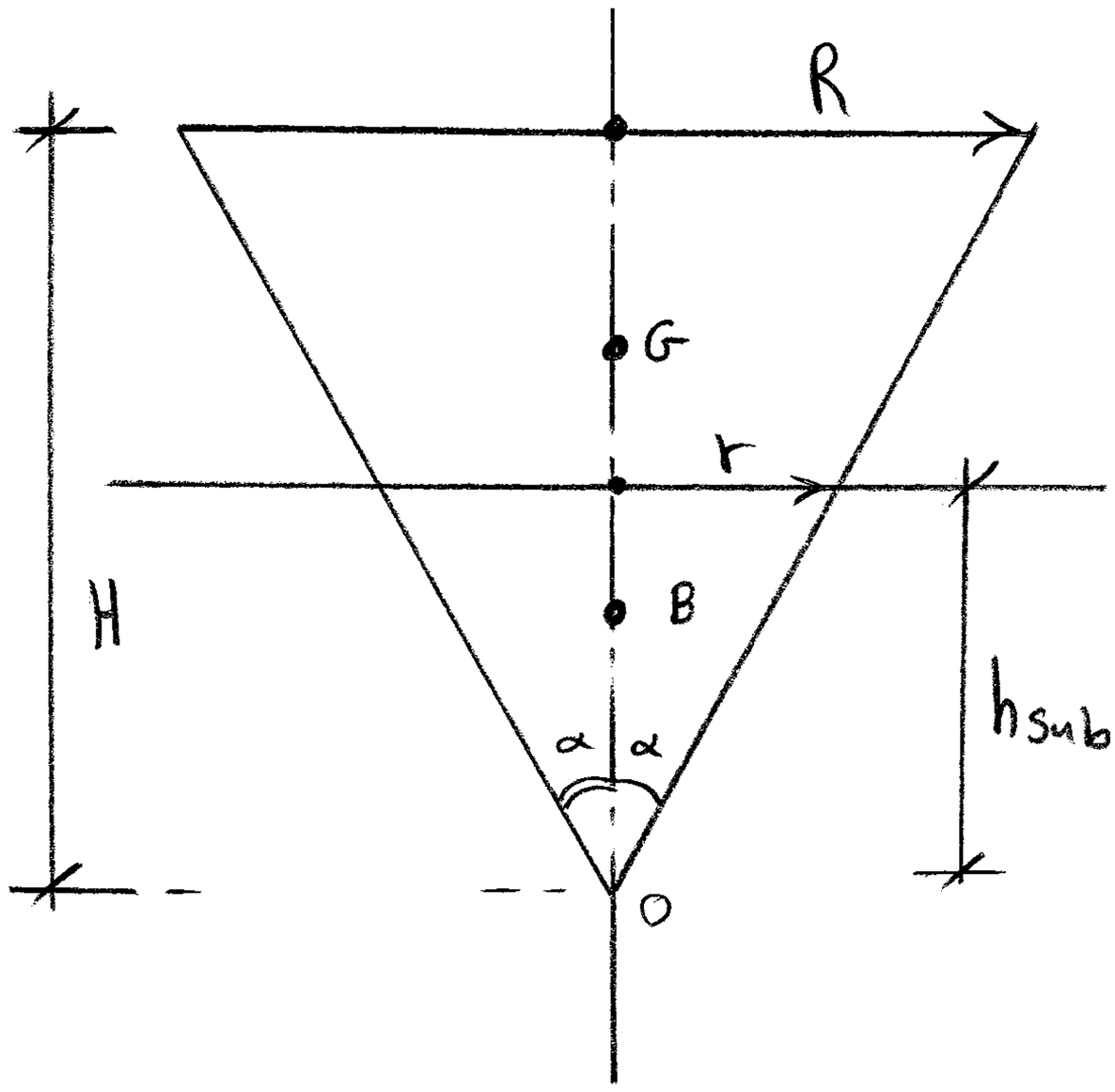
$$\frac{d^2}{16(1.5+0.5L)} = \frac{0.45 + 0.3L + 0.25L^2}{1.5 + 0.5L} - \frac{(1.5+0.5L)^2}{2(1.5+0.5L)}$$

$d = 0.6$

$$\Rightarrow \frac{(0.6)^2}{16} = 0.45 + 0.3L + 0.25L^2 - \left(1.125 + 0.75L + 0.125L^2\right)$$

$$\therefore 0.125L^2 - 0.45L - 0.6975 = 0 \qquad \Rightarrow \underline{L = 4.77\ m}$$

31 A right solid cone with apex angle equal to 60° is of density **k** relative to that of the liquid in which it floats with apex downwards. Determine what range of **k** is compatible with stable equilibrium.



For Stability

$GM = BM - BG \geq 0$

$BG = \frac{3}{4}H - \frac{3}{4}h_{sub}$

$\because \quad F_b = W$

$\gamma_{body} \forall_{body} = \gamma_{fluid} \forall_{sub}$

$\therefore \quad K = \frac{\gamma_{body}}{\gamma_{fluid}}$

$\therefore \quad K \gamma_f \forall_b = \gamma_f \forall_s$

$K \forall_b = \forall_s$

$K \frac{1}{3}\pi R^2 H = \frac{1}{3}\pi r^2 h_{sub}$

$K R^2 H = r^2 h_{sub}$

$K (H^2 \tan^2\alpha) H = (h_{sub}^2 \tan^2\alpha) h_{sub}$

$K H^3 = h_{sub}^3$

$\Rightarrow \quad h_{sub} = K^{1/3} H$

$\tan\alpha = \frac{r}{h_{sub}} = \frac{R}{H}$

* $r = h_{sub} \tan\alpha$

* $R = H \tan\alpha$

$BM = \frac{I_y}{\forall_{sub}}$

$I_y = \frac{\pi r^4}{4}$

$$\forall_{sub} = \frac{1}{3}\pi r^2 h_{sub}$$

$$* \quad BM = \frac{I_y}{\forall_{sub}} = \frac{\pi r^4/4}{\frac{1}{3}\pi r^2 h_{sub}} = \frac{3}{4}\frac{r^2}{h_{sub}}$$

$$\therefore \quad r = h_{sub} \tan\alpha$$

$$\therefore \quad BM = \frac{3}{4}\frac{(h_{sub}\tan\alpha)^2}{h_{sub}} = \frac{3}{4} h_{sub} \tan^2\alpha$$

$$* \quad BM \geq BG$$

$$\frac{3}{4} h_{sub} \tan^2\alpha \geq \frac{3}{4}(H - h_{sub})$$

$$\therefore \quad h_{sub}(1 + \tan^2\alpha) \geq H$$

$$\therefore \quad h_{sub} = K^{1/3} H$$

$$\therefore \quad K^{1/3} H (1 + \tan^2\alpha) \geq H$$

$$K^{1/3} \geq \left(\frac{1}{1 + \tan^2\alpha}\right)$$

$$\therefore \quad \alpha = \frac{\text{apex angle}}{2} = \frac{60}{2} = 30^\circ$$

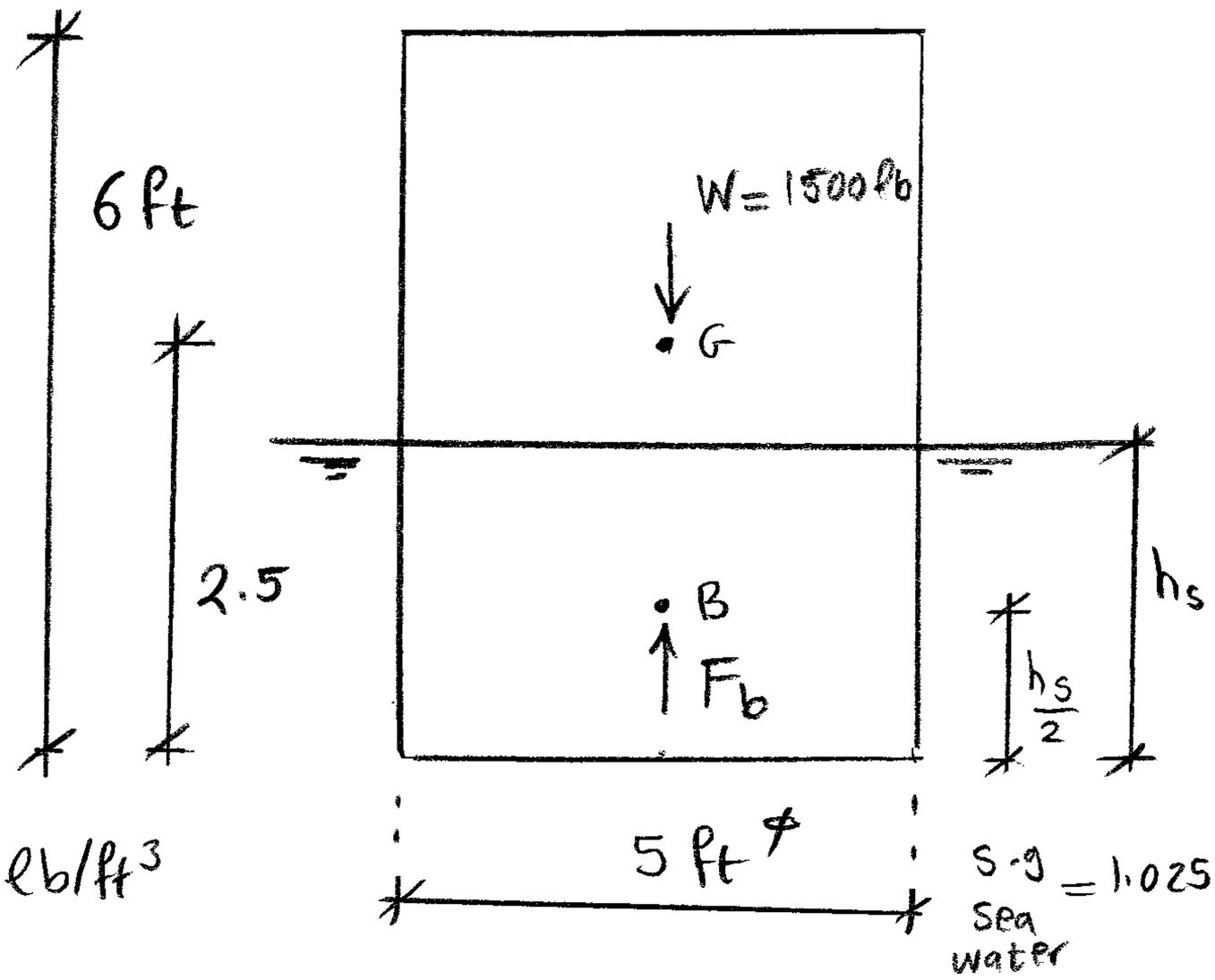
$$\therefore \quad K \geq \left(\frac{1}{1 + \tan^2 30}\right)^3 = 0.42$$

but if K is > 1 the cone will sink

$\therefore$ K ranges from $0.42 \rightarrow 1$

(15)

32 A cylindrical buoy is 5 feet diameter and 6 feet high. It weighs 1500 Lb and its C.G. is 2.5 feet above the base and is on the axis. Show that the buoy will not float with its axis vertical in sea water. If one end of a vertical chain is fastened to the centre of the base, find the tension in the chain in order that the buoy may just float with its axis vertical



for stability

$GM = BM - BG \geq 0$

$BG = 2.5 - \frac{h_s}{2}$

$F_b = W$

$F_b = \gamma_{sw} V_{sub}$

$\gamma_{sw} = 1.025\,(62.4) = 63.96$ lb/ft^3

$V_{sub} = \frac{\pi d^2}{4} h_{sub}$

$\therefore \quad F_b = W$

$63.96\left(\frac{\pi (5)^2}{4} h_{sub}\right) = 1500$ lb $\Rightarrow$ $h_{sub} = \underline{1.194}$ ft

$* BG = 2.5 - \frac{h_s}{2} = \underline{1.903 \text{ ft}}$

$BM = \frac{I_y}{V_{sub}}$

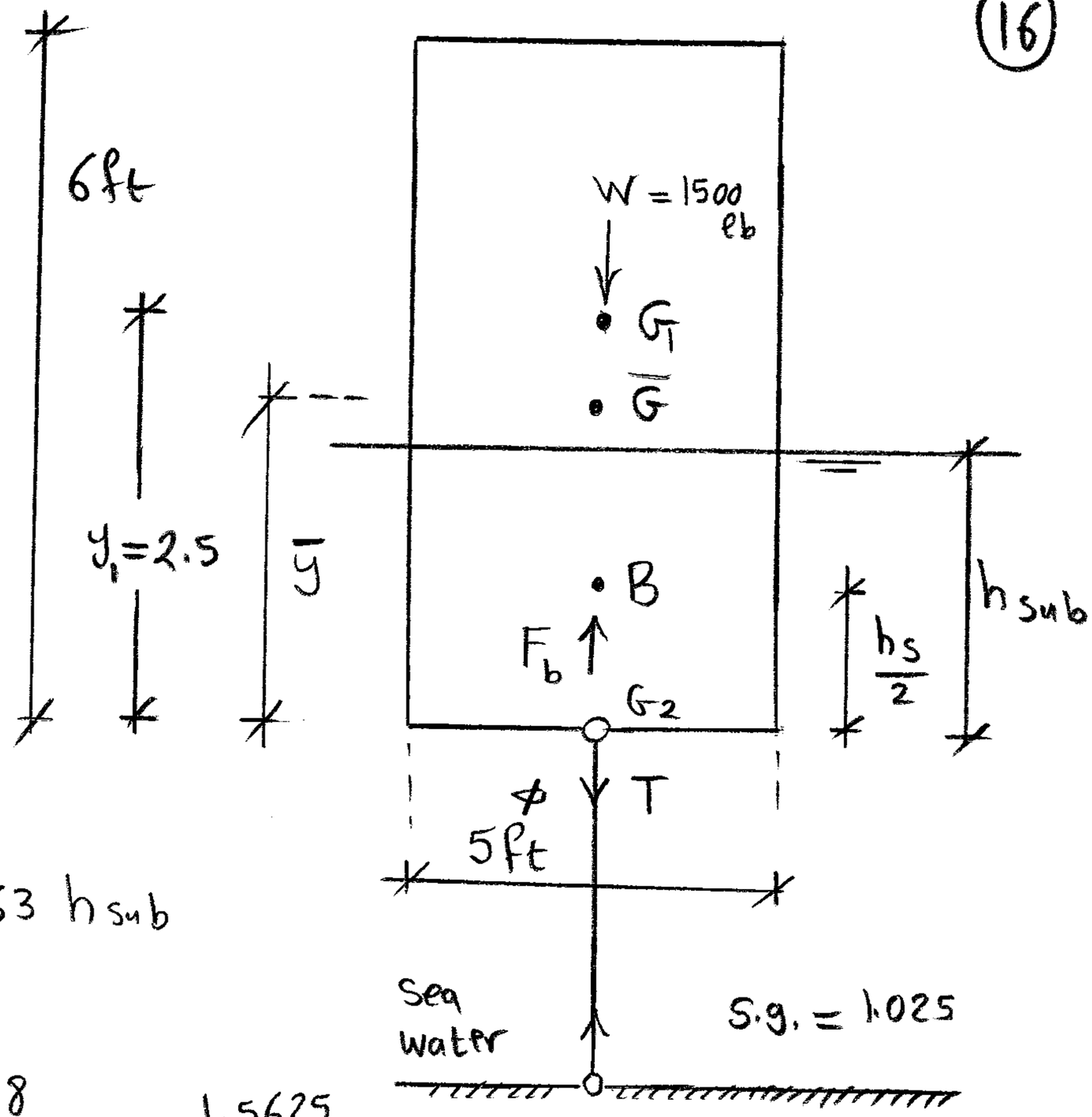
$I_y = \frac{\pi d^4}{64} = \frac{\pi (5)^4}{64} = 30.68$

$V_{sub} = \frac{\pi d^2}{4} * h_{sub} = \frac{\pi (5)^2}{4} * 1.194 = 23.44$

$* BM = \frac{I_y}{V_{sub}} = \frac{30.68}{23.44} = \underline{1.31 \text{ ft}} < BG$

unstable

Tension in the cable = ?



for stability

$$BM \geq BG$$

$$BM = \frac{I_y}{\forall_{sub}}$$

$$I_y = \frac{\pi d^4}{64} = 30.68$$

$$\forall_{sub} = \frac{\pi (5)^2}{4} h_{sub} = 19.63\, h_{sub}$$

$$* \; BM = \frac{I_y}{\forall_{sub}} = \frac{30.68}{19.63\, h_{sub}} = \frac{1.5625}{h_{sub}}$$

$$BG = \bar{y} - \frac{h_s}{2}$$

The chain will change the location of the C.G. of the body

$$\bar{y} = \frac{W y_1 + T y_2}{W + T}$$

$W = 1500$ lb $T = ?$ $W + T = 1500 + T$

$y_1 = 2.5$ ft $y_2 = 0$ $\bar{y} = ?$

$$\bar{y} = \frac{1500(2.5) + T(0)}{1500 + T}$$

$$= \frac{3750}{1500 + T}$$

$$F_b = T + W$$

$$\gamma_{s.w} \forall_{sub} = T + 1500$$

$$1.025(62.4).\ \frac{\pi(5)^2}{4} h_{sub} = T + 1500$$

$$\therefore\ h_{sub} = \frac{T + 1500}{1255.85}$$

$$* \ BG = \bar{y} - \frac{h_s}{2}$$

$$= \frac{3750}{1500 + T} - \frac{T + 1500}{2(1255.85)}$$

$$\therefore\ BM = \frac{1.5625}{h_{sub}} = \frac{1.5625\ (1255.85)}{(T + 1500)}$$

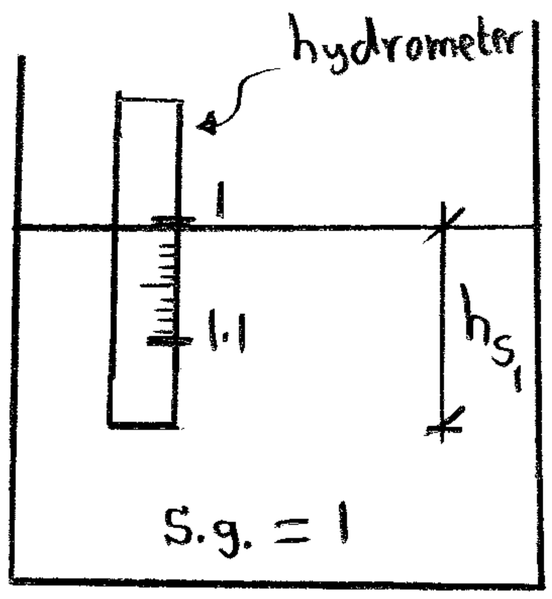
$$* \quad BM = BG$$

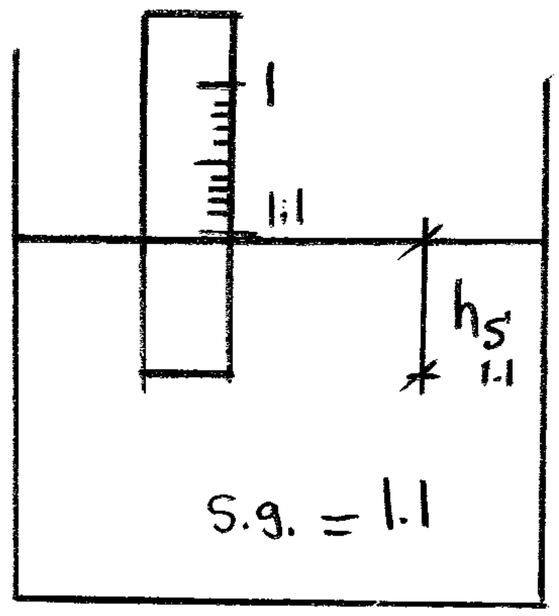
$$\frac{1.5625\ (1255.85)}{T + 1500} = \frac{3750}{T + 1500} - \frac{T + 1500}{2\ (1255.85)}$$

$$\frac{(T + 1500)^2}{2\ (1255.85)} = 1787.7$$

$$\therefore\ T = \underline{\underline{619\ lb}}$$

34 A hydrometer weighs 0.035 N and has a stem 5 mm in diameter. Compute the distance between specific gravity markings 1.0 and 1.1.





A hydrometer is a device used for measuring the s.g. of a fluid, by inserting a graduated cylinder in the fluid concerned, then reading the scale on the water surface

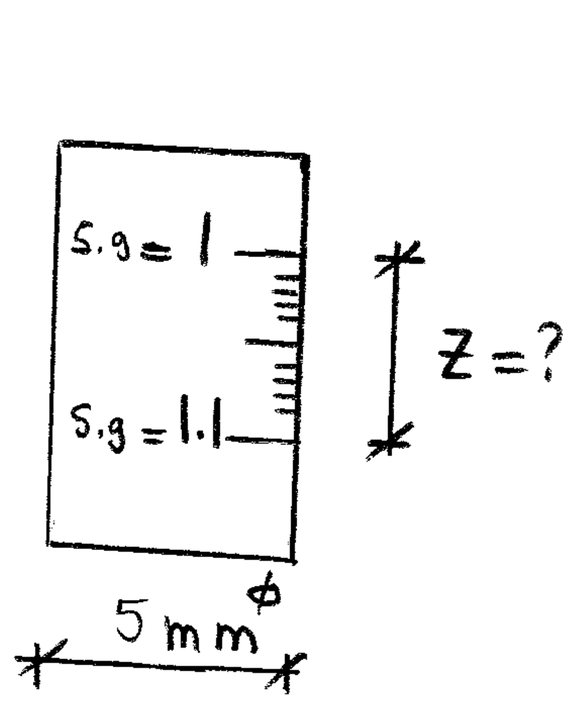
$W = 0.035\ N$ $\quad Z = ?$

$\therefore F_b = W$

$\gamma_{fluid}\ \forall_{sub} = W$

$S.g\ \gamma_w \frac{\pi d^2}{4} h_{sub} = W$

$S.g\ (9810)\ \pi \frac{(5*10^{-3})^2}{4} h_{sub} = 0.035\ N$



$h_{sub} = \frac{0.1817}{S.g.}$

$Z = h_{s_1} - h_{s_{1.1}}$

$= \frac{0.1817}{1} - \frac{0.1817}{1.1} = 0.0165\ m = \underline{16.5\ mm}$

8 Fundamentals of Fluid Flow

Types of fluid flow

1- Steady and unsteady flow

a- Steady flow

It occurs when velocity, acceleration,.. etc doesn't change with time

$$\frac{\partial V}{\partial t} = 0$$

b- Unsteady flow

It occurs when velocity or acceleration,.. etc changes with time

e.g. flow in a pipe whose valve is opening or closing

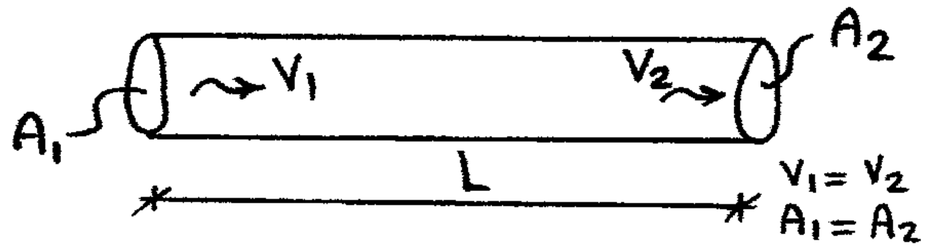
$$\frac{dV}{dt} \neq 0$$

2-Uniform and Non-uniform flow

a- Uniform flow

It occurs when velocity and cross-section remains constant over a given length

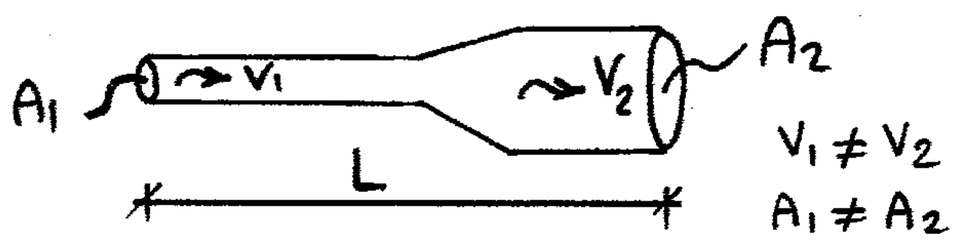
$$\frac{\partial V}{\partial L} = 0 \quad , \quad \frac{\partial A}{\partial L} = 0$$



b- Non-uniform flow

It occurs when velocity or cross-section changes over a given length

$$\frac{dV}{dL} \neq 0 \quad , \quad \frac{dA}{dL} \neq 0$$

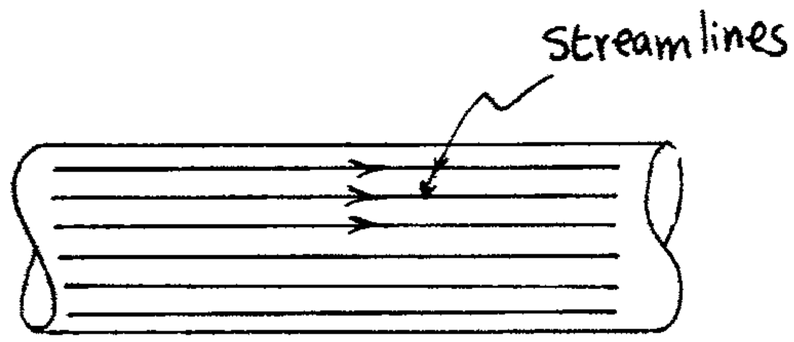


3- Laminar and turbulent flow

a- Laminar flow

It occurs when fluid particles in parallel paths and do not intersect

e.g. flow through capillary tubes, ground water, and blood in veins.

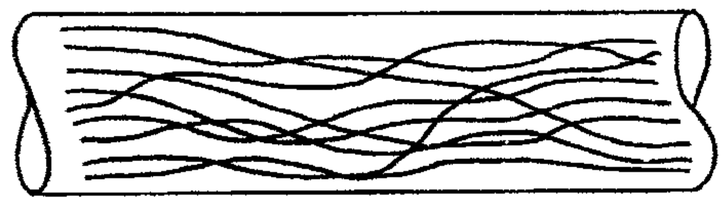


$R_n < 2000$

b- Turblent flow

It occurs when fluid particles move in random motion

e.g. Nearly in all flow in pipes

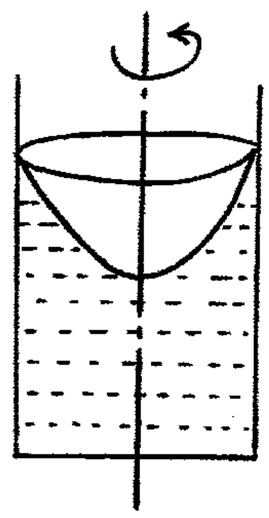


$R_n > 4000$

4- Rotational and Irrotational flow

a- Rotational flow

It occurs when fluid particles have a rotation about an axis



b- Irrotational flow

It occurs when fluid particles don't have a rotation about an axis

5- Compressible and incompressible flow

a- Compressible flow

It occurs when the density of the fluid changes from point to point

e.g. Flow of gases through orifices and nozzles

b- Incompressible flow

It occurs when the density is constant for fluid flow

e.g. Liquid are generally considered flowing incompressibly

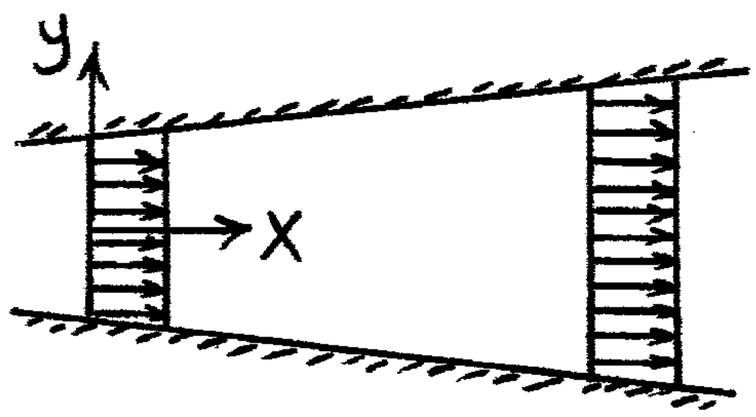
6- One, two three dimensional flow

a- One dimensional flow

It occurs when the velocity is a function of time and one co-ordinate.

$v = f(x, t)$

e.g. Flow through a straight uniform diameter pipe



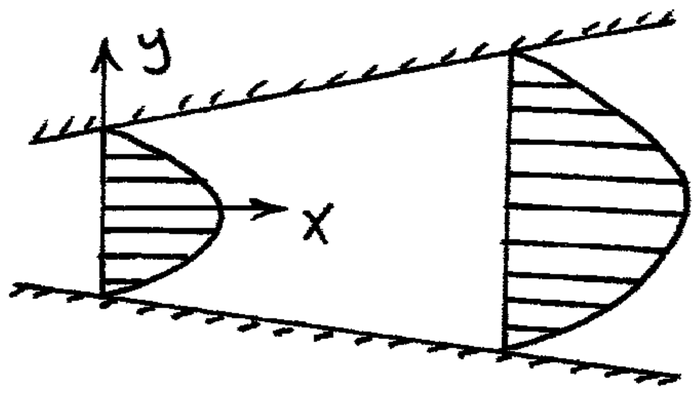
The flow is never truly 1 dimensional, because viscosity causes the fluid velocity to be zero at the boundaries.

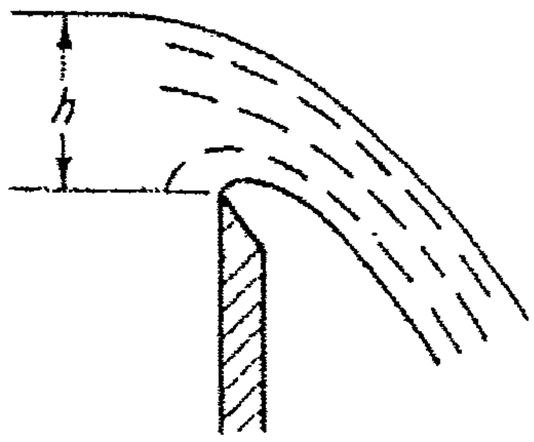
b- Two dimensional flow

It occurs when the velocity is a function of time and two co-ordinates

$v = f(x, y, t)$

e.g. Flow in the main stream of a wide river



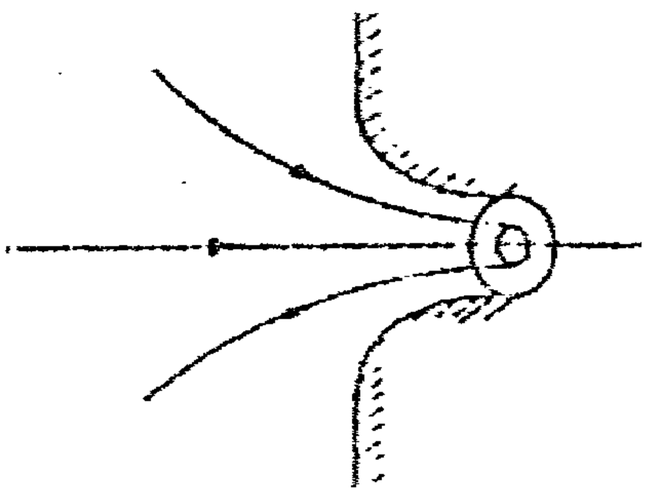


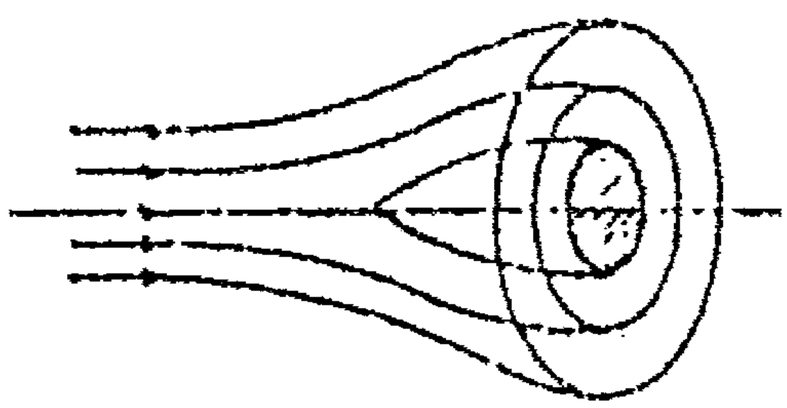
c- Three dimensional flow

It occurs when the velocity is a function of time and three co-ordinates

$v = f(x, y, z, t)$

e.g. Flow in a converging or diverging pipe

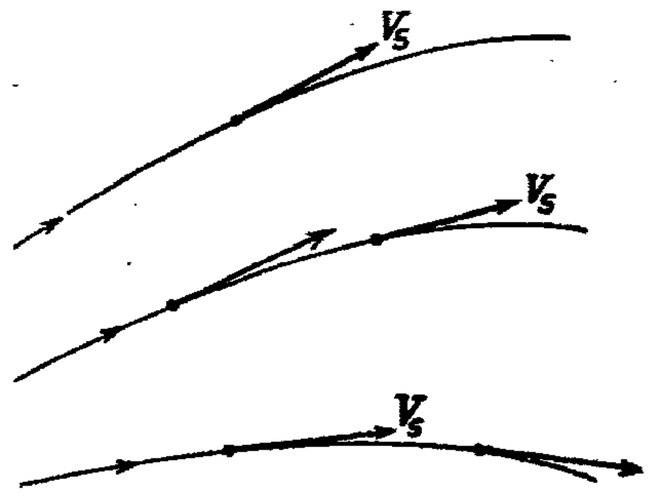




7- Stream lines and streamtubes

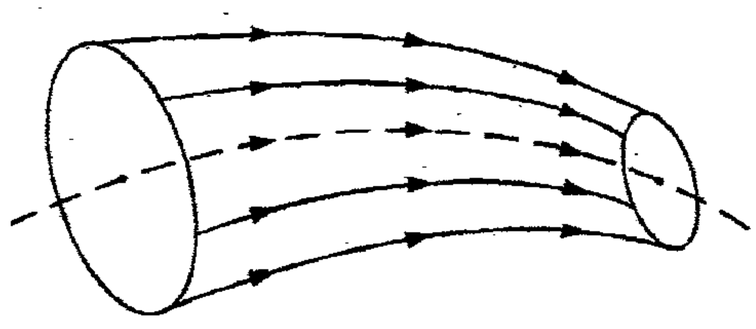
a- Streamlines

- Streamlines are imaginary curves drawn to show the direction of fluid flow
- The tangent at any point gives the velocity direction



b- Streamtubes

- A stream tube is a fluid mass bounded by a group of streamlines



8- Ideal and Real Fluids

a- Ideal Fluids

- It is a fluid that has no viscosity, and incompressible
- Shear resistance is considered zero
- Ideal fluid does not exist in nature

e.g. Water and air are assumed ideal

b- Real Fluids

- It is a fluid that has viscosity, and compressible
- It offers resistance to its flow

e.g. All fluids in nature

9- Viscous and inviscid flow

a- Viscous flow

- It occurs for fluids that have viscosity which offers shear resistance to the flow
- A part of the total energy is lost in flow

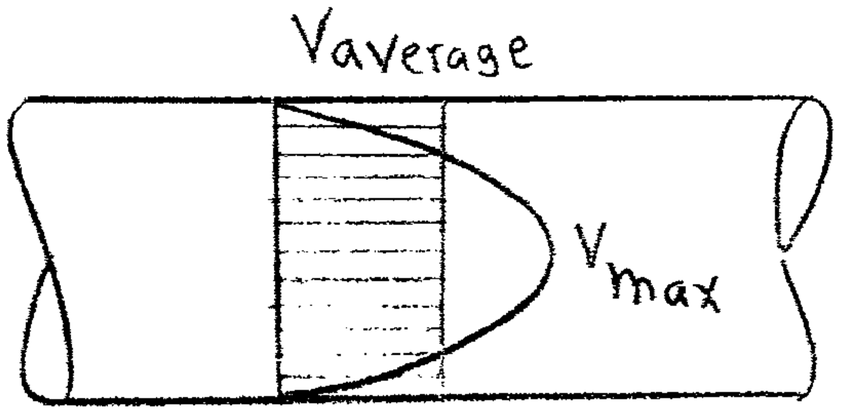
b- Inviscid flow

- It occurs for fluids that have no viscosity
- No shear resistance to the flow
- The total energy remains constant.

10- Mean velocity and Discharge

a- Mean velocity

It is the average velocity passing a given section



$$V_{mean} = \frac{Q}{A}$$

b- Discharge

It is the rate of Volume of liquid passing a given cross-section

$$Q = \frac{\forall}{t} = A.V$$

The Continuity equation

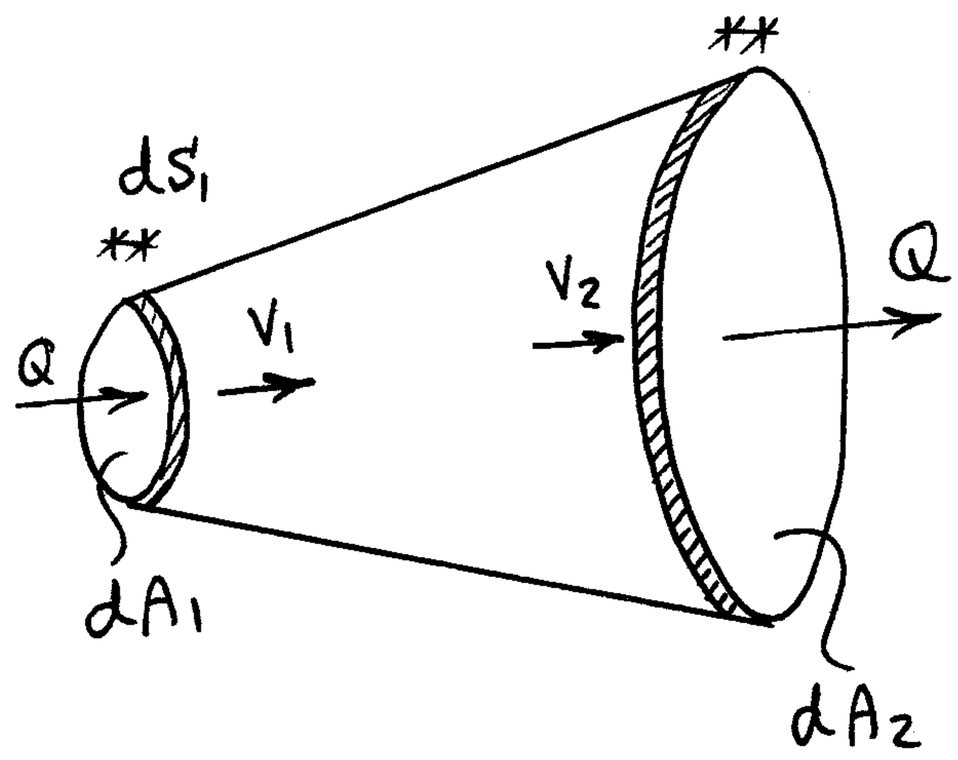
If no fluid is added or removed from the Pipe in any length then the Mass passing across diffrent Sections Shall be the Same

$$dM_1 = dM_2$$

$$\rho_1 \, dA_1 \, \frac{dS_1}{dt} = \rho_2 \, dA_2 \, \frac{dS_2}{dt}$$

$$\rho_1 \, dA_1 \, V_1 = \rho_2 \, dA_2 \, V_2$$

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$



For incompressible fluids $\rho_1 = \rho_2$

$$\boxed{A_1 V_1 = A_2 V_2}$$

$\begin{bmatrix} m^3/Sec \\ cm^3/sec \\ ft^3/sec \end{bmatrix}$ L^3/T

Q = Discharge = Area * Velocity $= A \cdot V$

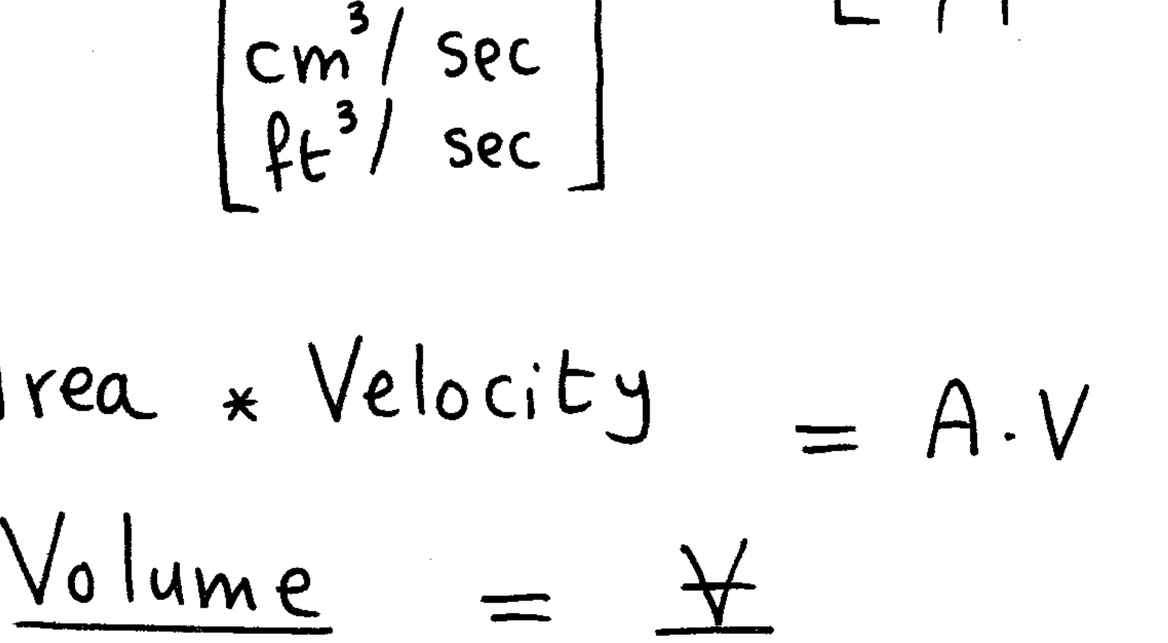
= Flow rate = $\frac{\text{Volume}}{\text{Time}} = \frac{\forall}{t}$

Q = Constant

Input = Out put

e.g

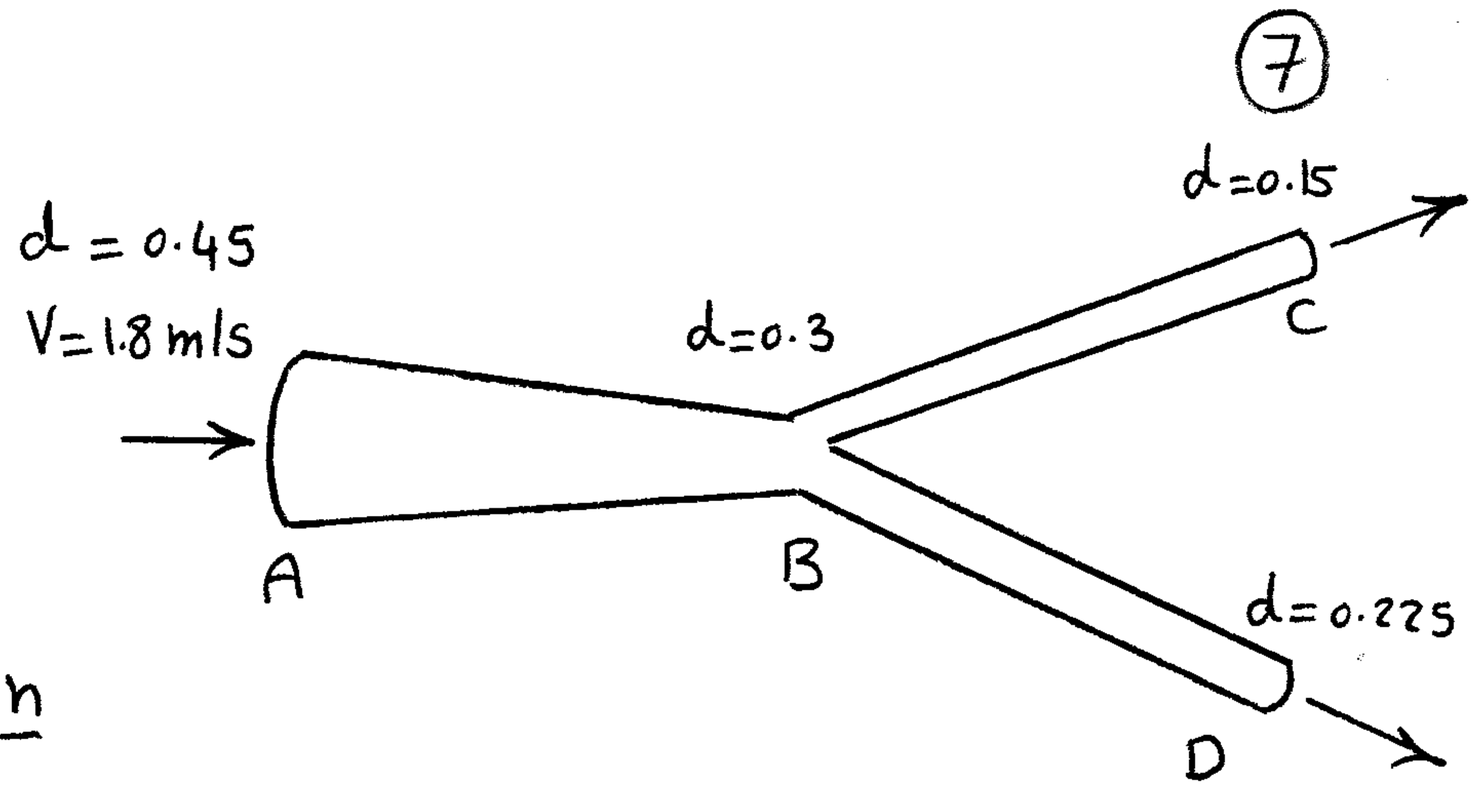
$$Q_1 = Q_2 + Q_3$$



(3)

$Q_C = ?$ $Q_D = ?$

$V_B = ?$ $V_C = ?$



Applying Continuity eqn

$$Q_A = A.V = \frac{\pi d^2}{4} * V$$

$$= \frac{\pi (0.45)^2}{4} * 1.8 = 0.286 \ m^3/s$$

$$Q_D = AV = \frac{\pi (0.225)^2}{4} * 3.6 = \underline{\underline{0.143 \ m^3/s}}$$

$$Q_A = Q_c + Q_D$$

$$0.286 = Q_c + 0.143 \Rightarrow Q_c = \underline{\underline{0.143 \ m^3/s}}$$

$$V_c = \frac{Q_c}{A_c} = \frac{0.143}{\frac{\pi (0.15)^2}{4}} = \underline{\underline{8.09 \ m/s}}$$

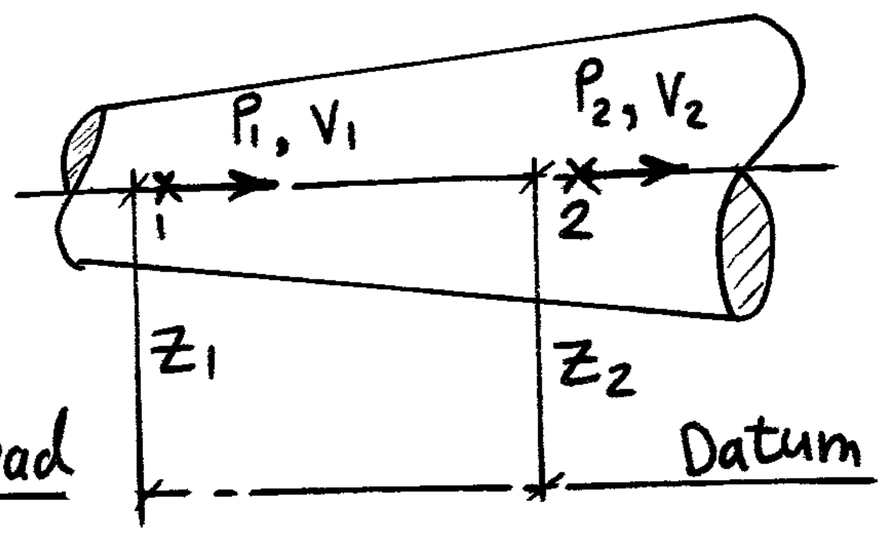
$$Q_B = Q_A = 0.286 \ m^3/s$$

$$V_B = \frac{Q_B}{A_B} = \frac{0.286}{\frac{\pi (0.3)^2}{4}} = \underline{\underline{4.04 \ m/s}}$$

Fluid Dynamics

For any Fluid element it has

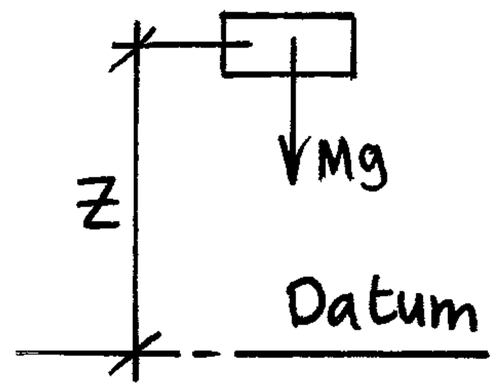
Three energies or heads



1- Potential energy or Potential head

$$P.E = Mg\,Z$$

$$\text{Potential head} = \frac{P.E}{\text{unit Weight}} = \frac{MgZ}{Mg} = Z$$



2- Kinetic energy or Velocity head

$$K.E. = \frac{MV^2}{2}$$

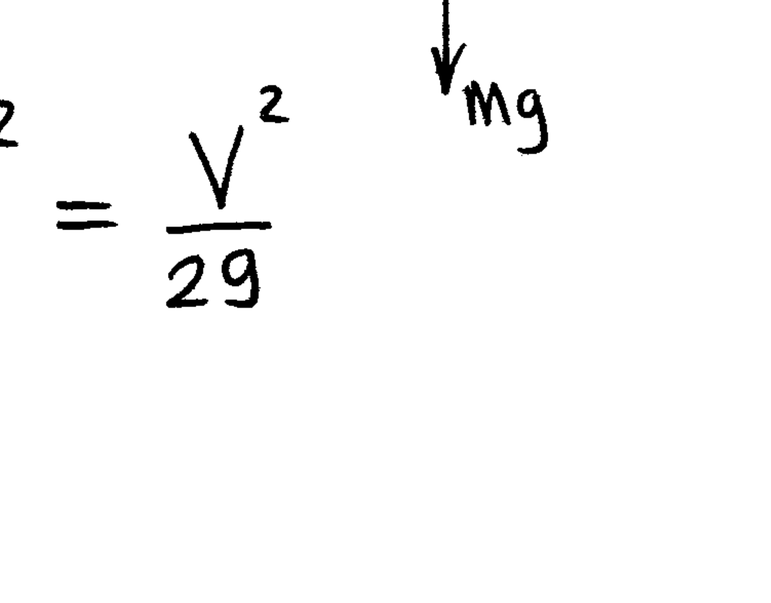


$$\text{Velocity head} = \frac{K.E.}{\text{unit Weight}} = \frac{MV^2}{2Mg} = \frac{V^2}{2g}$$

$$\frac{V^2}{2g} = \frac{L^2T^{-2}}{LT^{-1}} = (L)$$

3- Pressure energy or Pressure head

$$\text{Pressure energy} = (\gamma h).A.X$$



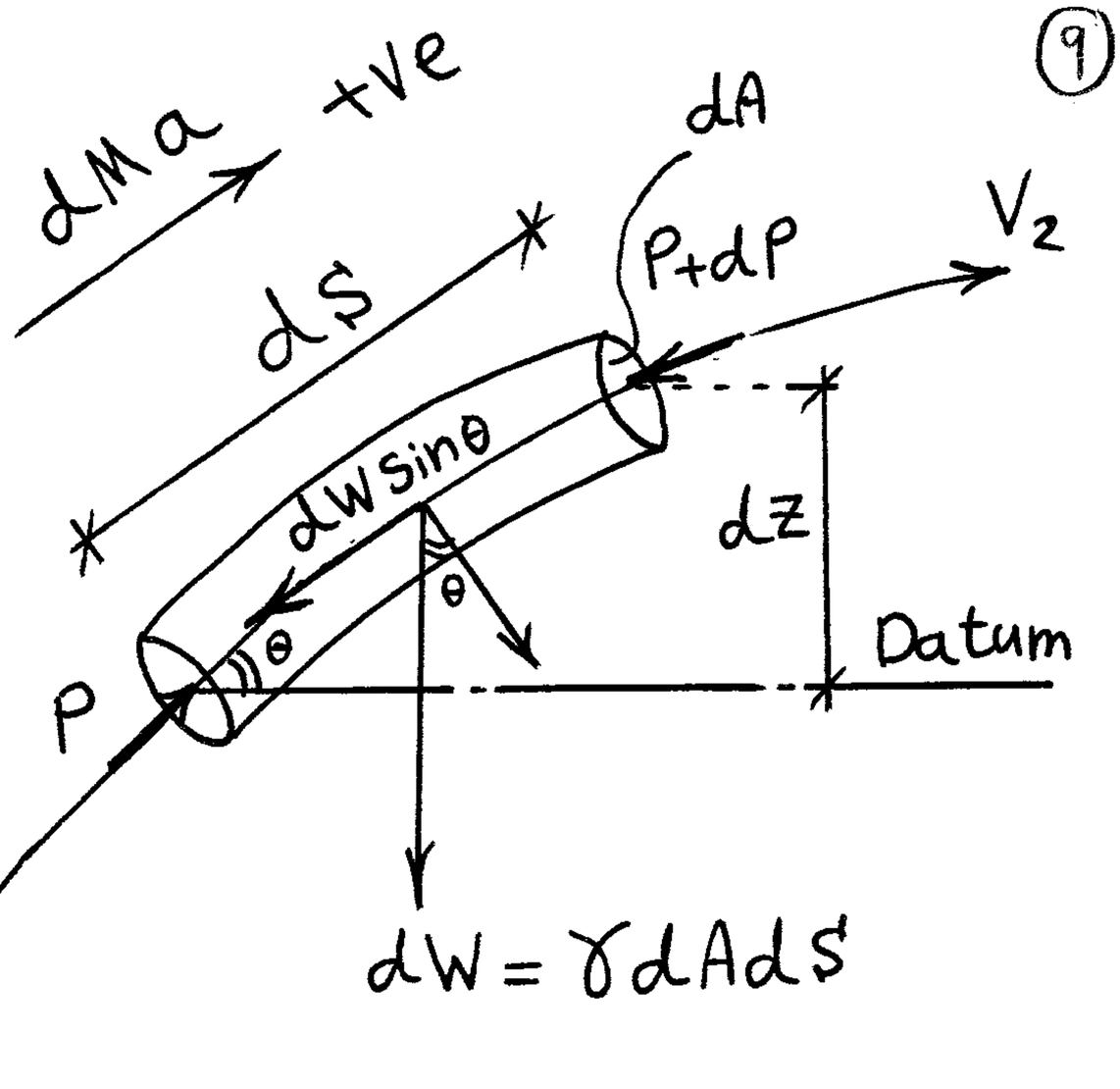
$$\text{Pressure head} = \frac{\text{Pressure energy}}{\text{unit weight}}$$

$$= \frac{\gamma h A X}{\gamma A X} = h = (L)$$

Ideal Fluid

Euler & Bernoulli's eqn

Consider a fluid element of cross-Section dA and length dS moving along a Streamline.



$dW = \gamma dA dS$

$\sin\theta = \frac{dz}{dS}$

Applying Newton 2nd law

$$P dA - (P + dP) dA - dW \sin\theta = dM a$$

$$\cancel{P dA} - \cancel{P dA} - dP dA - \gamma \cancel{dA} \cancel{dS} \left(\frac{dz}{\cancel{dS}}\right) = \rho \cancel{dA} dS \frac{dV}{dt}$$

$$-dP - \gamma dz = \rho dS \frac{dV}{dt} \qquad \div \gamma$$

$$-\frac{dP}{\gamma} - dz - \frac{\rho}{\gamma}\left(\frac{dS}{dt}\right) dV$$

Euler's eqn

$$\boxed{\frac{dP}{\gamma} + dz + \frac{V dV}{g} = 0}$$

Equation of Steady motion along a Streamline

By integration of Euler's equation

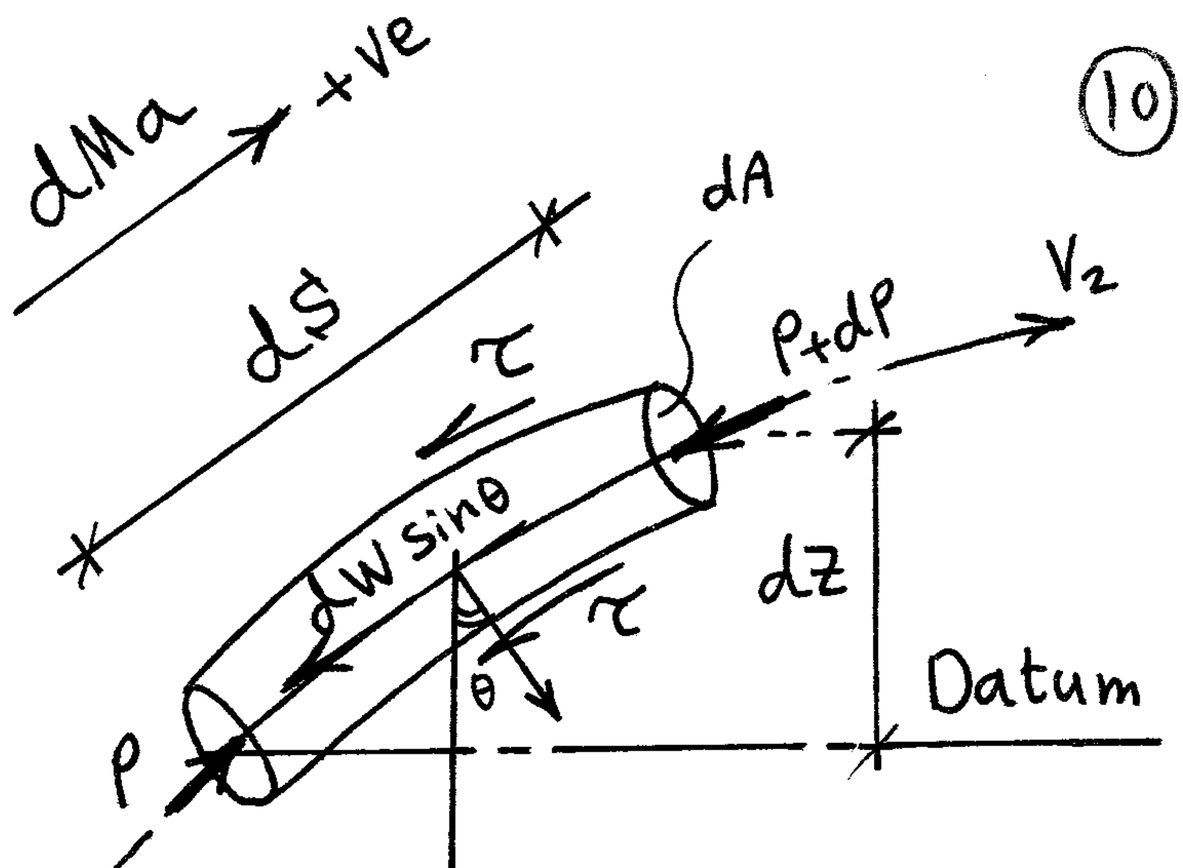
Bernoulli's eqn

$$\boxed{\frac{P}{\gamma} + Z + \frac{V^2}{2g} = \text{Constant}}$$

Pressure head + Position head + Velocity head = Total head

Real Fluid

Real fluid has an additional force acting caused by Friction



$$F = \tau\, dA = \tau(2\pi r)\, dS$$

Applying Newton 2nd law

$$P\,dA - (P + dP)\,dA - dW \sin\theta - \tau(2\pi r)\,dS = dMa$$

$$-dP\,\cancel{dA} - \gamma\,\cancel{dA}\,\cancel{dS}\left(\frac{dz}{\cancel{dS}}\right) - \tau\left(2\frac{\cancel{dA}}{r}\right)dS = \rho\,\cancel{dA}\,dS\,\frac{dV}{dt}$$

$\because dA = \pi r^2 \qquad \div \gamma$

$$-\frac{dP}{\gamma} - dz - \frac{2\tau\, dS}{\gamma r} = \frac{V\,dV}{g}$$

$$\frac{dP}{\gamma} + dz + d\left(\frac{V^2}{2g}\right) = -\frac{2\tau\, dS}{\gamma r}$$

$$\int_1^2 \frac{dP}{\gamma} + \int_1^2 dz + \int_1^2 d\left(\frac{V^2}{2g}\right) = \int_1^2 -\frac{2\tau\, dS}{\gamma r}$$

$$\left(\frac{P_1}{\gamma} + Z_1 + \frac{V_1^2}{2g}\right)_{①} - \frac{2\tau L}{\gamma r} = \left(\frac{P_2}{\gamma} + Z_2 + \frac{V_2^2}{2g}\right)_{②}$$

Dims of $\frac{2\tau L}{\gamma r} = \frac{N/m^2 * m}{N/m^3 * m} = m$

H_1 (Total energy at Sec 1) $- h_L$ (head lost ① → ②) $= H_2$ (T.E. at Sec 2)

Total Energy Line T.E.L. T.E.L.

هو خط يمثل مجموع الـ٣ طاقات في معادلة Bernoulli $\left(\frac{P}{\gamma} + \frac{V^2}{2g} + z\right)$

وحيث أن مجموعهم ثابت فإنه يظل أفقياً وموازى لل datum فى حالة عدم وجود losses

نعتبر الـ flow دائماً Ideal إلا إذا ذكر وجود losses

Hydraulic Gradient Line H.G.L. H.G.L.

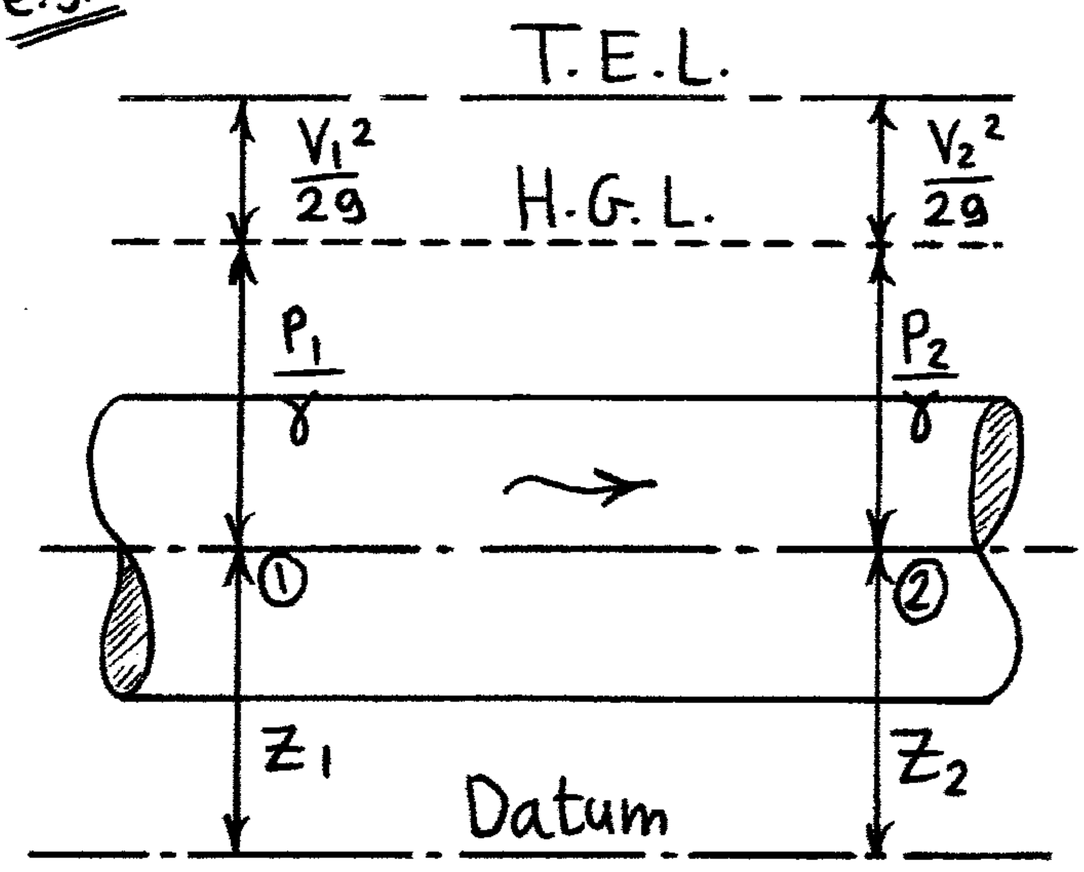
هو عبارة عن خط يمثل مجموع Static head $\left(\frac{P}{\gamma} + z\right)$ وبالتالى فهو

يبعد بمقدار Dynamic head $\left(\frac{V^2}{2g}\right)$ عن الـ T.E.L.

هو الخط الذى يمر بالأماكن التى عندها الضغط يساوى صفر (Imaginary free Surface)

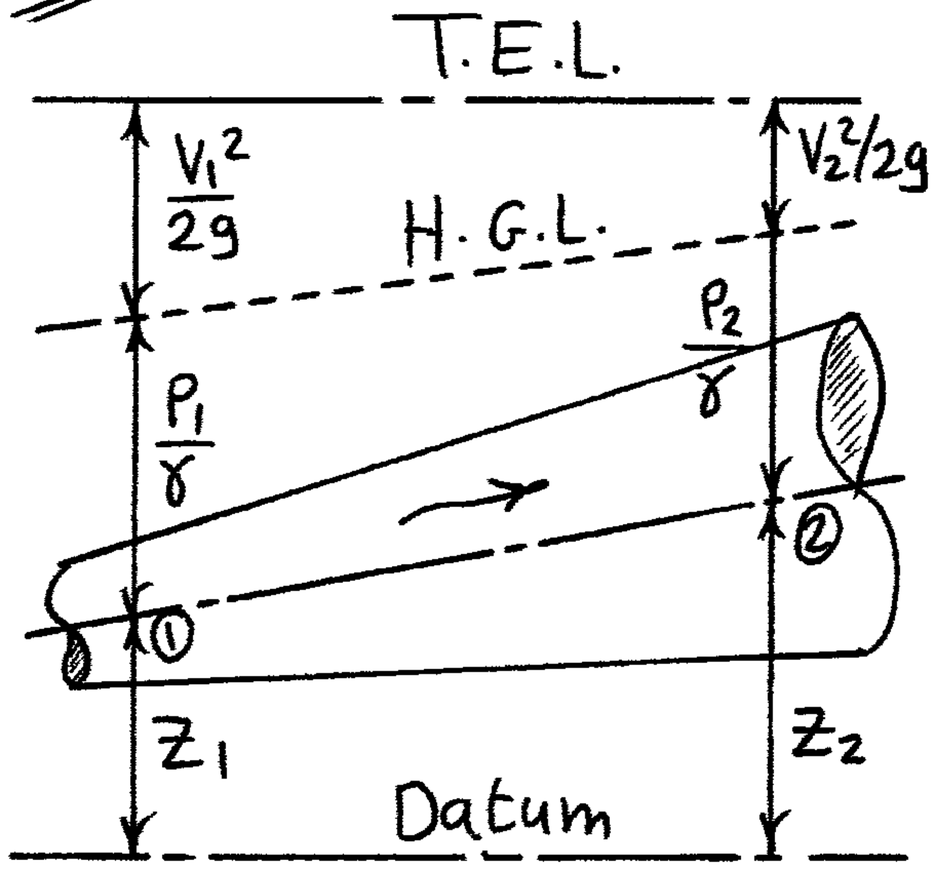
Examples on Ideal Fluid

e.g.1

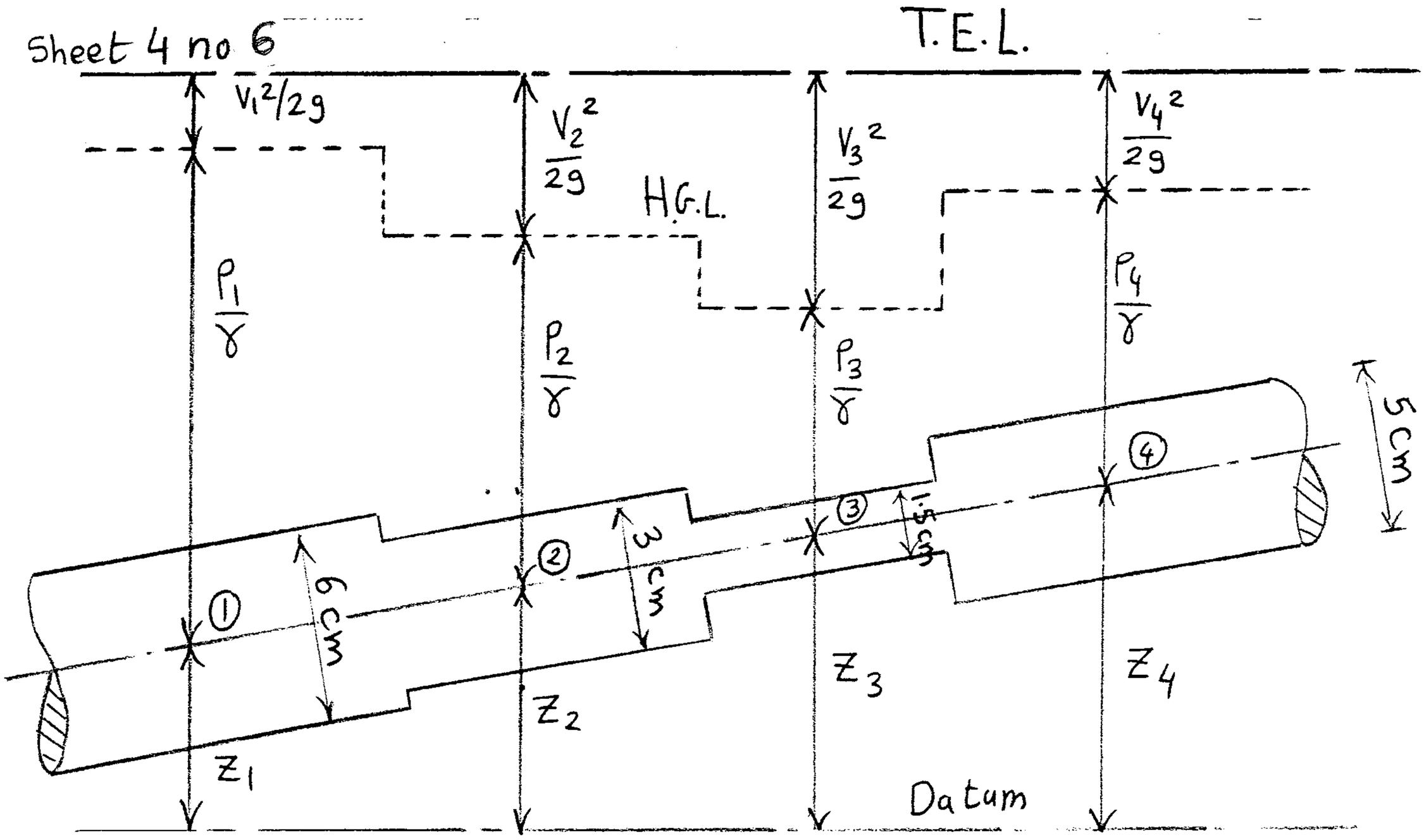


فى حالة أنه مقطع الأنبوبة يزيد فإنه السرعة تقل وبما أنه الـ T.E.L أفقى فإنه الـ H.G.L يزيد تدريجياً

e.g.2

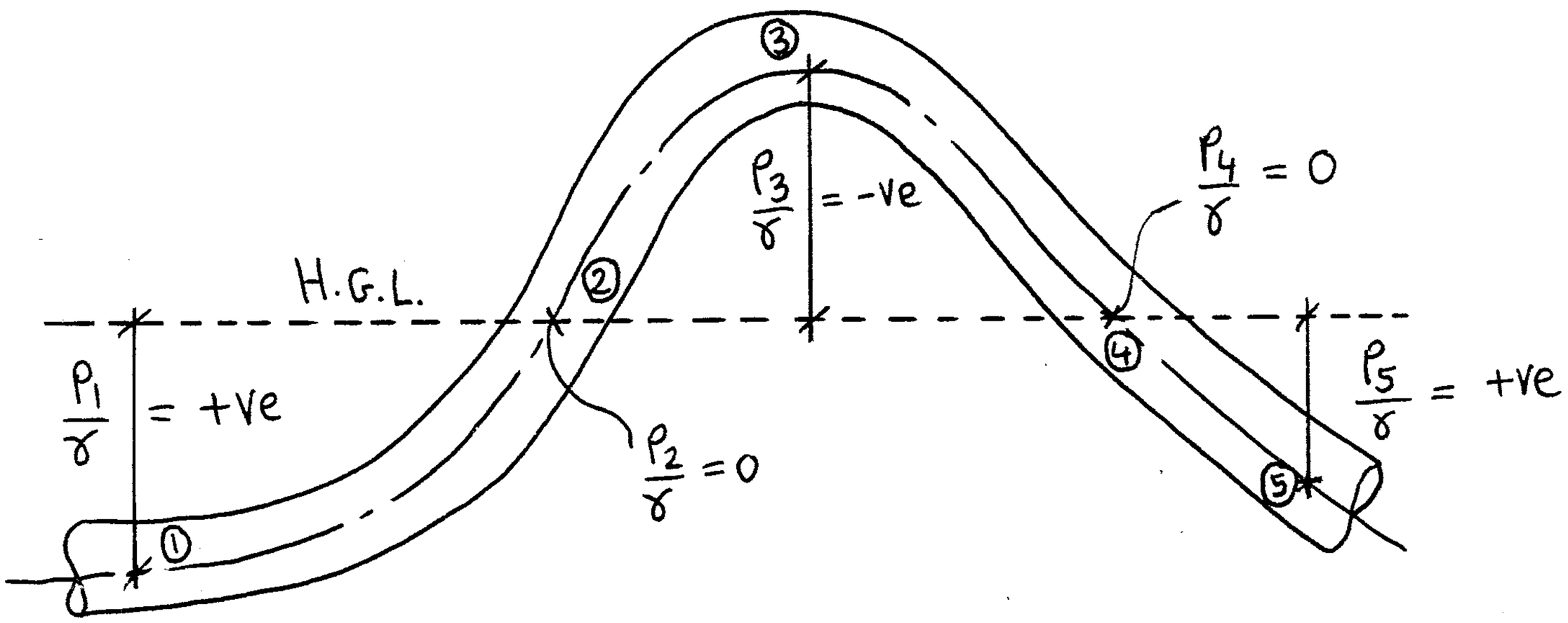


فى حالة أنه مقطع الانبوبة ثابت فإنه السرعة تظل ثابته وبما أنه T.E.L أفقى فإنه الـ H.G.L يكونه أفقى وموازى للـ T.E.L



فى حالة أنه مقطع الأنبوبة يضيق فجأة فإنه السرعه تزيد فجأة فيقل الـ H.G.L فجأة والعكس صحيح

Example on H.G.L

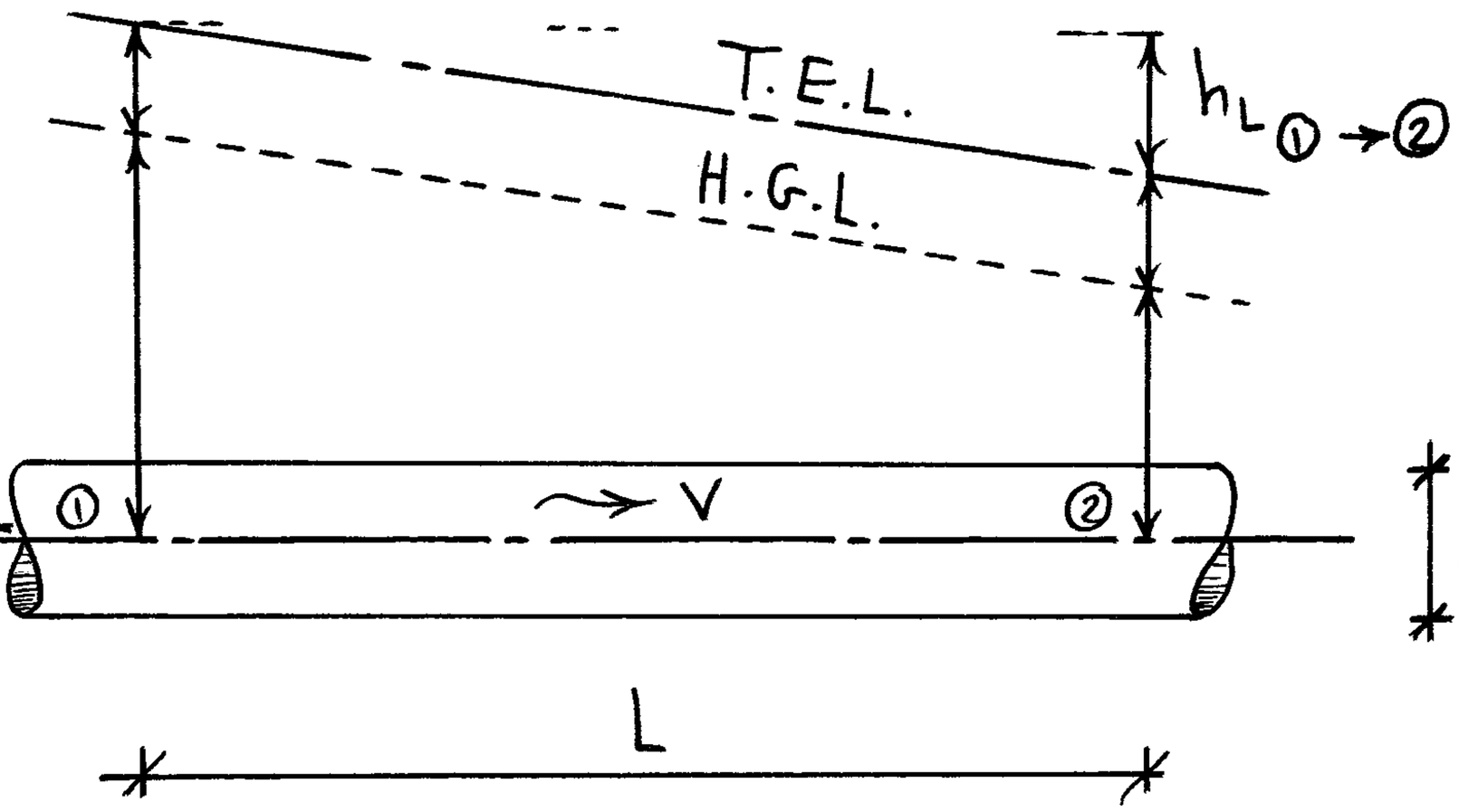


1) H.G.L above pipe's Centerline have +ve Pressure
2) H.G.L below pipe's Centerline have -ve Pressure
3) Centerline intersecting with H.G.L. have Zero Pressure

Example on Real Fluid

Losses

1- Main Losses (Friction Losses)

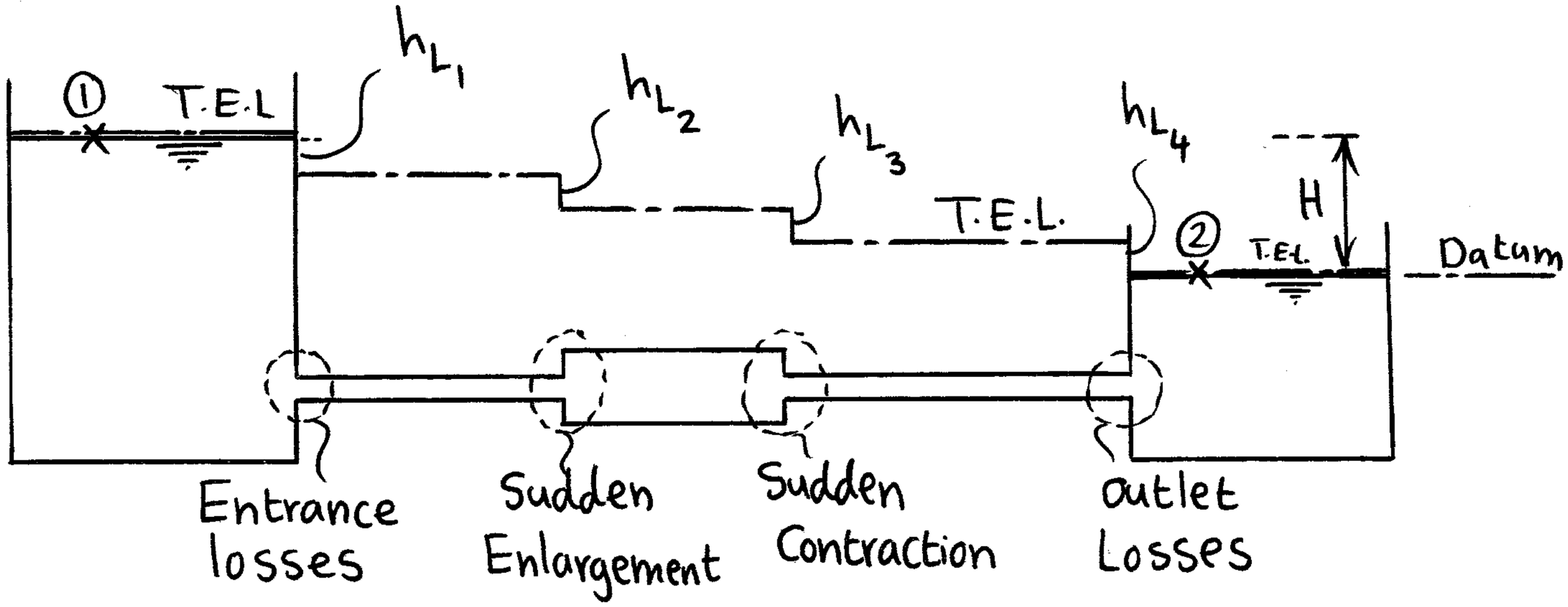


في حالة وجود Friction فإن الـ T.E.L يقل تدريجياً

معامل الاحتكاك

$$h_{L_{①\to②}} = \frac{f L}{d}\left(\frac{V^2}{2g}\right) = K\frac{V^2}{2g}$$

2- Secondary Losses (Minor - Local) losses



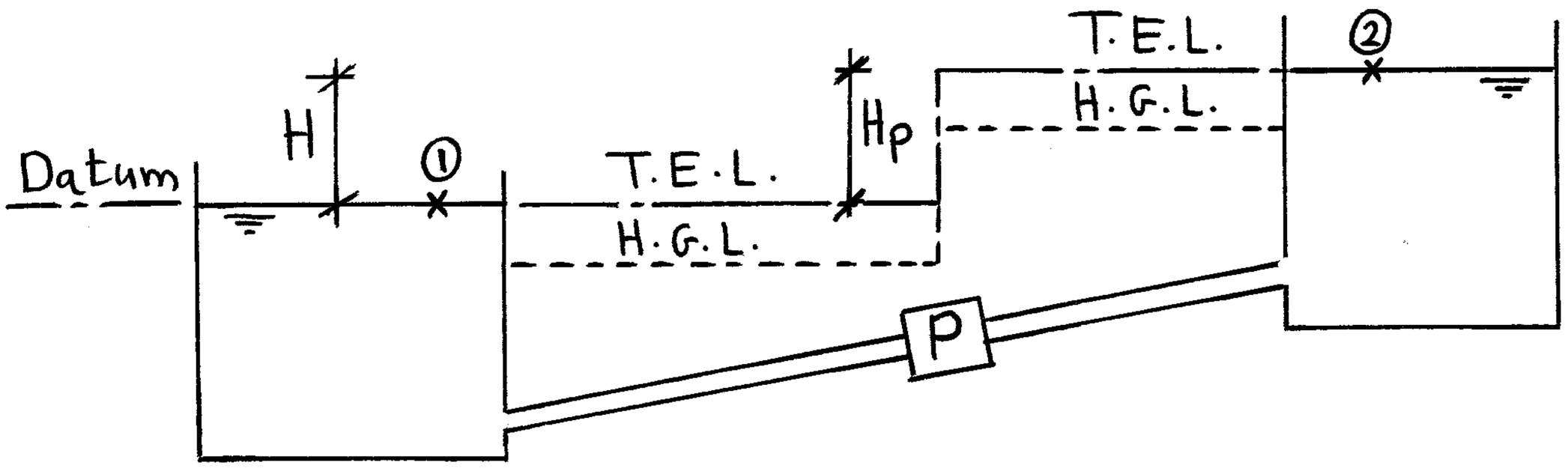
$$\sum h_L = h_{L_1} + h_{L_2} + h_{L_3} + h_{L_4}$$

Apply Bernoulli between ①,②

$$\frac{P_1}{\gamma}_{=0} + \frac{V_1^2}{2g}_{\approx 0} + z_1 = \frac{P_2}{\gamma}_{=0} + \frac{V_2^2}{2g}_{\approx 0} + z_2{}_{=0} + \sum h_L$$

$$H = \sum h_L$$

Pumps (Adds energy to the System)



Applying Bernoulli eqn between ①, ②

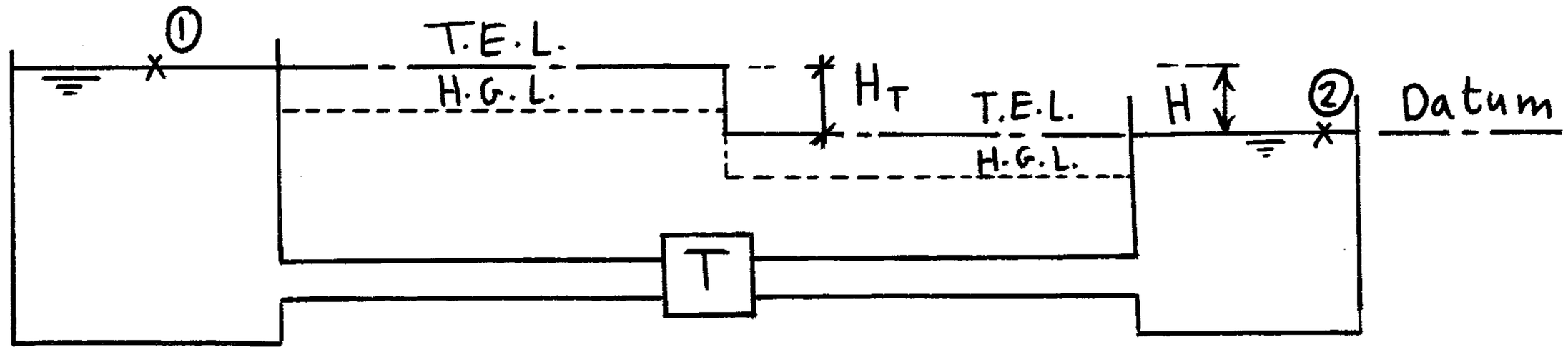
$$\frac{P_1}{\gamma}^{0} + \frac{V_1^2}{2g}_{\approx 0} + Z_1^{=0} + H_p = \frac{P_2}{\gamma}_{=0} + \frac{V_2^2}{2g}_{\approx 0} + Z_2$$

$$H_p = H$$

تعمل الـ Pump على إضافة طاقة Energy إلى الـ System

وبالتالى يزيد الـ T.E.L كما يزيد معه الـ H.G.L.

Turbines (Extracts energy from the System)



Applying Bernoulli eqn between ①, ②

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 - H_T = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

$$H = H_T$$

تعمل الـ Turbine على استخراج طاقة Energy من الـ System

وبالتالى يقل الـ T.E.L. كما يقل معه الـ H.G.L.

Bernoulli's General equation

Energy at A	+ Energy added	− Energy lost	− Energy Extracted	= Energy at B

$$\left(\frac{P_A}{\gamma}+\frac{V_A^2}{2g}+Z_A\right)+H_p-\Sigma H_L-H_T=\left(\frac{P_B}{\gamma}+\frac{V_B^2}{2g}+Z_B\right)$$

طريقة حل مسائل Bernoulli

١- يتم رسم المسألة واختيار Datum لكل equation ويفضل أن يكون اختيار الـ Datum عند أقل منسوب لتجنب الاشارات السالبة

٢- تطبيق Bernolli بين النقطتين فى نفس السائل

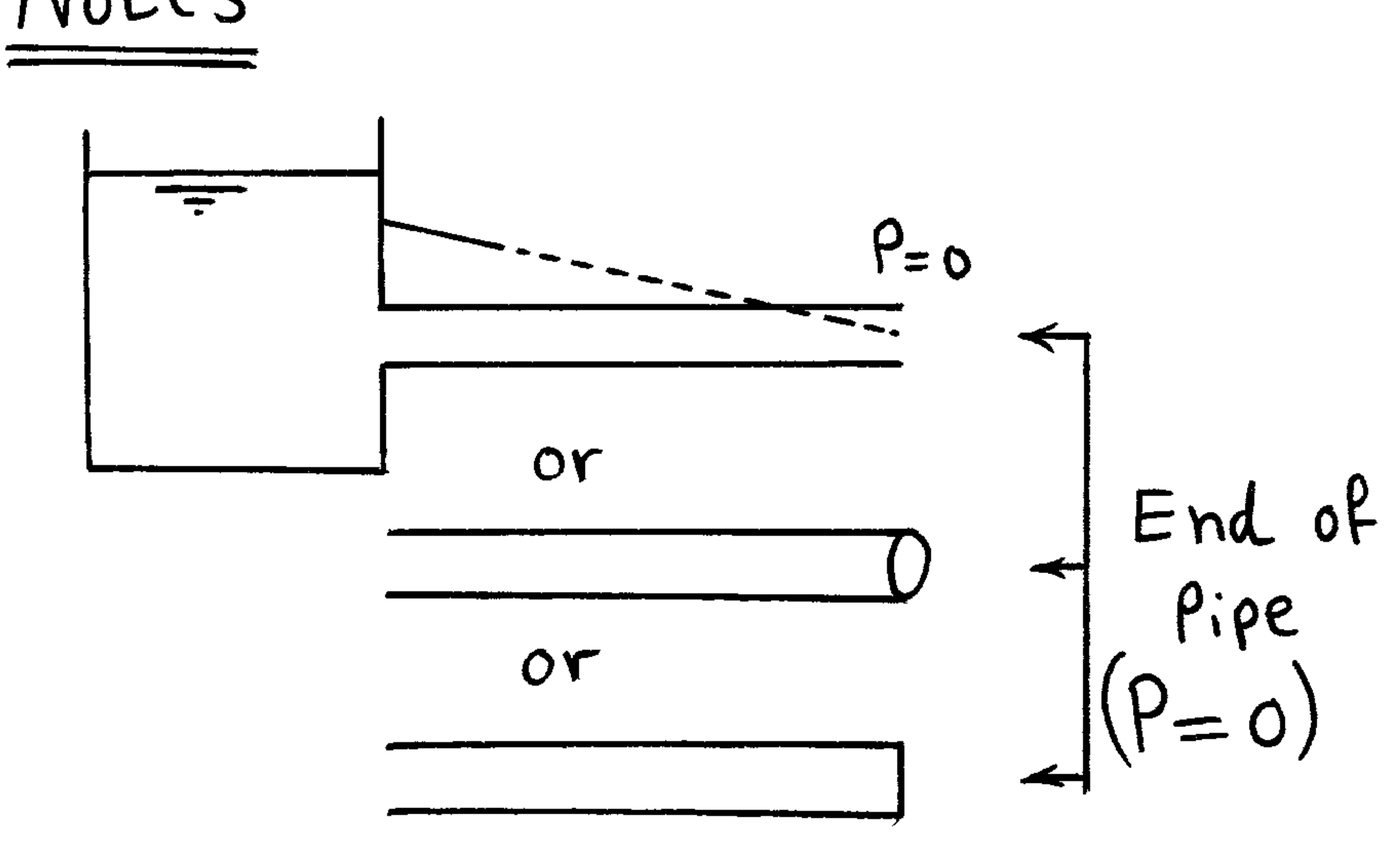
٣- لو لم تكن السرعة معلومة يتم عمل علاقة تربط بينهما V_2 و V_1 عن طريق قانون الـ Continuity $Q=A_1V_1=A_2V_2$

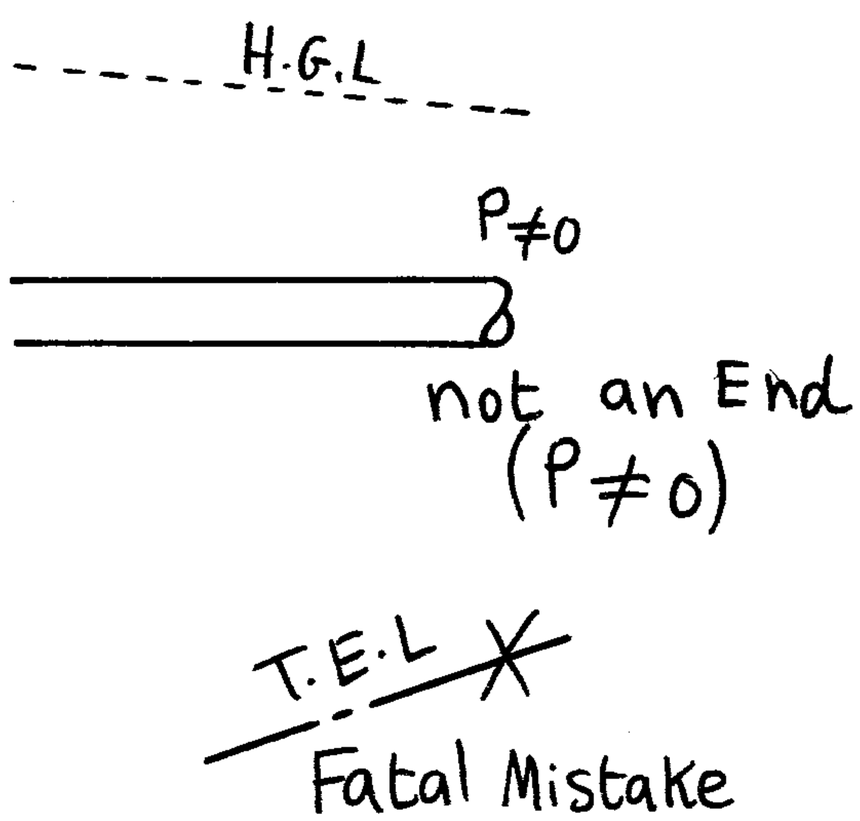
٤- يتم طرح أى losses

٥- يتم طرح أى head ناتج عن وجود Turbine

٦- يتم زيادة أى head ناتج عن وجود Pump

Notes

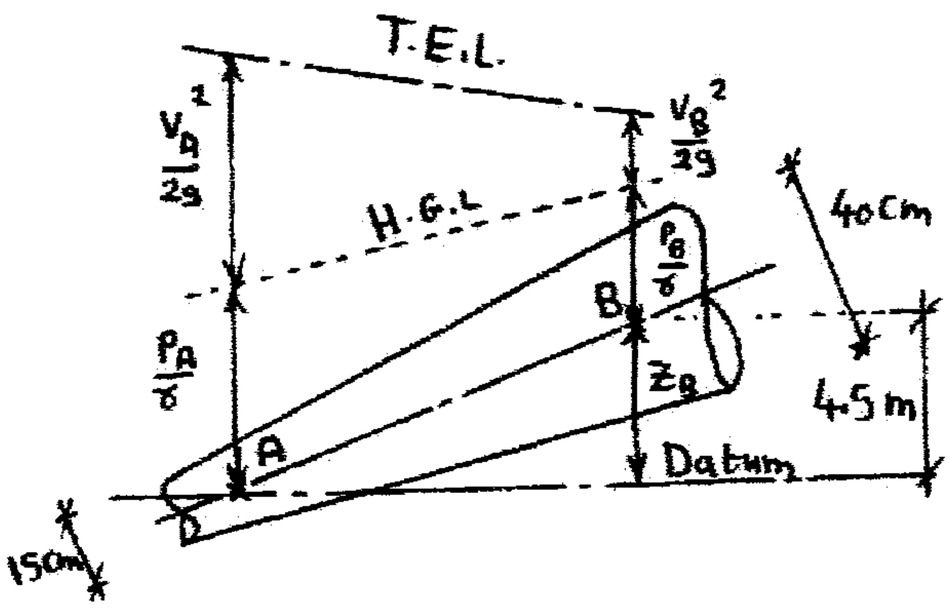




4

16

An inclined pipe carrying water gradually changes from 15 cm at A to 40 cm at B which is 4.5 m vertically above A. If the pressure at A and B are respectively 0.70 kg/cm^2 and 0.5 kg/cm^2 and the discharge is 150 liters/sec.
Determine a) the direction of flow b) the head loss between the sections.



$P_A = 0.7\ Kg_w/cm^2$

$= 0.7 * 9.81 \times 10^4 = \underline{\underline{68670\ N/m^2}}$

$P_B = 0.5\ Kg_w/cm^2$

$= 0.5 \times 9.81 \times 10^4 = \underline{\underline{49050\ N/m^2}}$

$Q = 150\ lit/sec = 150 \times 10^{-3}\ m^3/s$

$\therefore E_A = \frac{P}{\gamma} + \frac{V^2}{2g} + Z$

$V_A = \frac{Q}{A_A} = \frac{150 \times 10^{-3}}{\frac{\pi (0.15)^2}{4}} = 8.48\ m/s$

Taking a datum at A

Total Energy at A $E_A = \frac{68670}{9810} + \frac{(8.48)^2}{2 * 9.81} + 0 = \underline{\underline{10.67\ m}}$

$V_B = \frac{Q}{A_B} = \frac{150 \times 10^{-3}}{\frac{\pi (0.4)^2}{4}} = 1.19\ m/s$

Total Energy at B $E_B = \frac{49050}{9810} + \frac{(1.19)^2}{2 * 9.81} + 4.5 = \underline{\underline{9.57\ m}}$

$\because E_A > E_B$

$\therefore$ Water flow from A $\rightarrow$ B

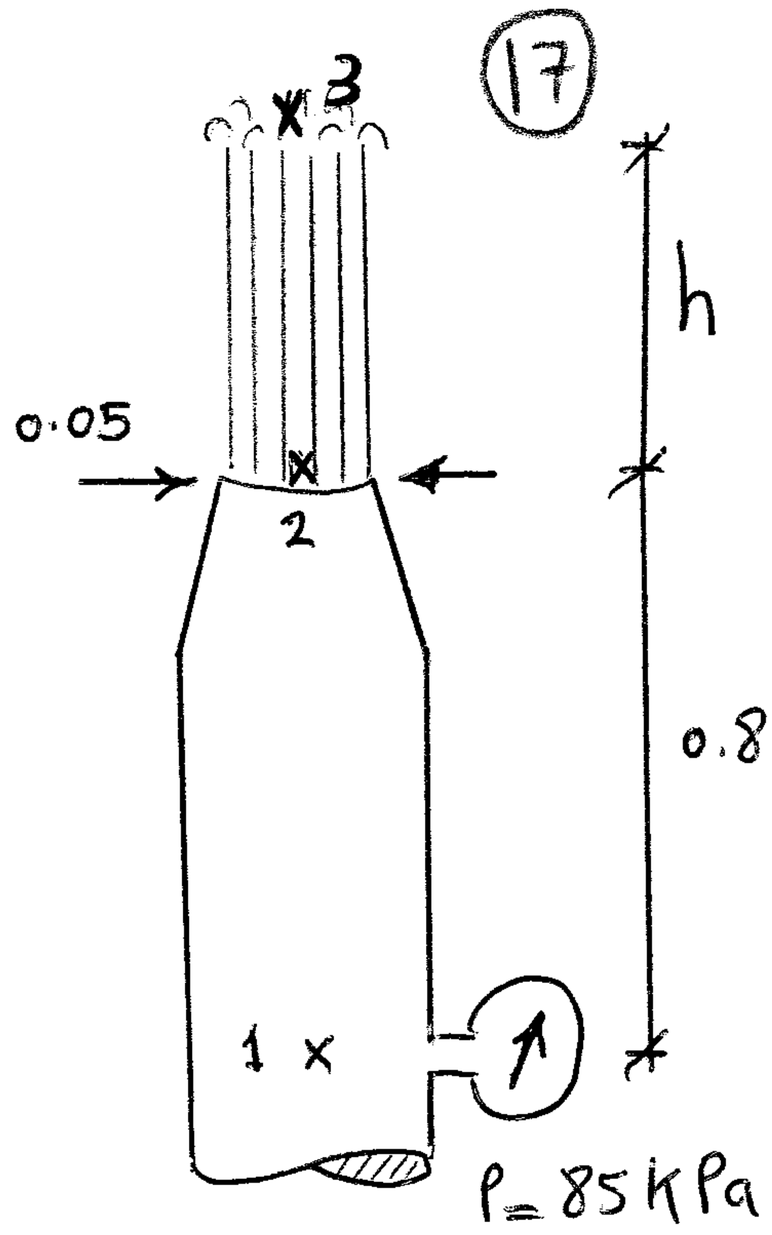
head Loss = Total Energy at A − Total Energy at B

$h_L = E_A - E_B = 10.67 - 9.57 = \underline{\underline{1.1\ m}}$

Jan 2003

[5] مثل سابقة

$Q = ?$, $h = ?$



Sol

Continuity eqn bet 1, 2

$$d_1^2 V_1 = d_2^2 V_2$$

$$(0.1)^2 V_1 = (0.05)^2 V_2$$

$$V_2 = 4 V_1$$

Bernoulli eqn bet ①, ② datum at ①

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

$$\frac{85 * 10^3}{9810} + \frac{V_1^2}{2(9.81)} + 0 = 0 + \frac{(4V_1)^2}{2(9.81)} + 0.8$$

$$\Rightarrow V_1 = 3.21 \text{ m/s}^2 \qquad \Rightarrow V_2 = 4V_1 = 12.82 \text{ m/s}$$

$$\Rightarrow Q = V_1 A_1 = 3.21 \frac{\pi (0.1)^2}{4} = \underline{0.025 \text{ m}^3/\text{s}}$$

Bernoulli eqn bet ②, ③ datum at ②

$$\frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + z_3$$

$$h = \frac{V_2^2}{2g} = \frac{(12.82)^2}{2(9.81)} = \underline{8.37 \text{ m}}$$

Another solution using Newton's eqn

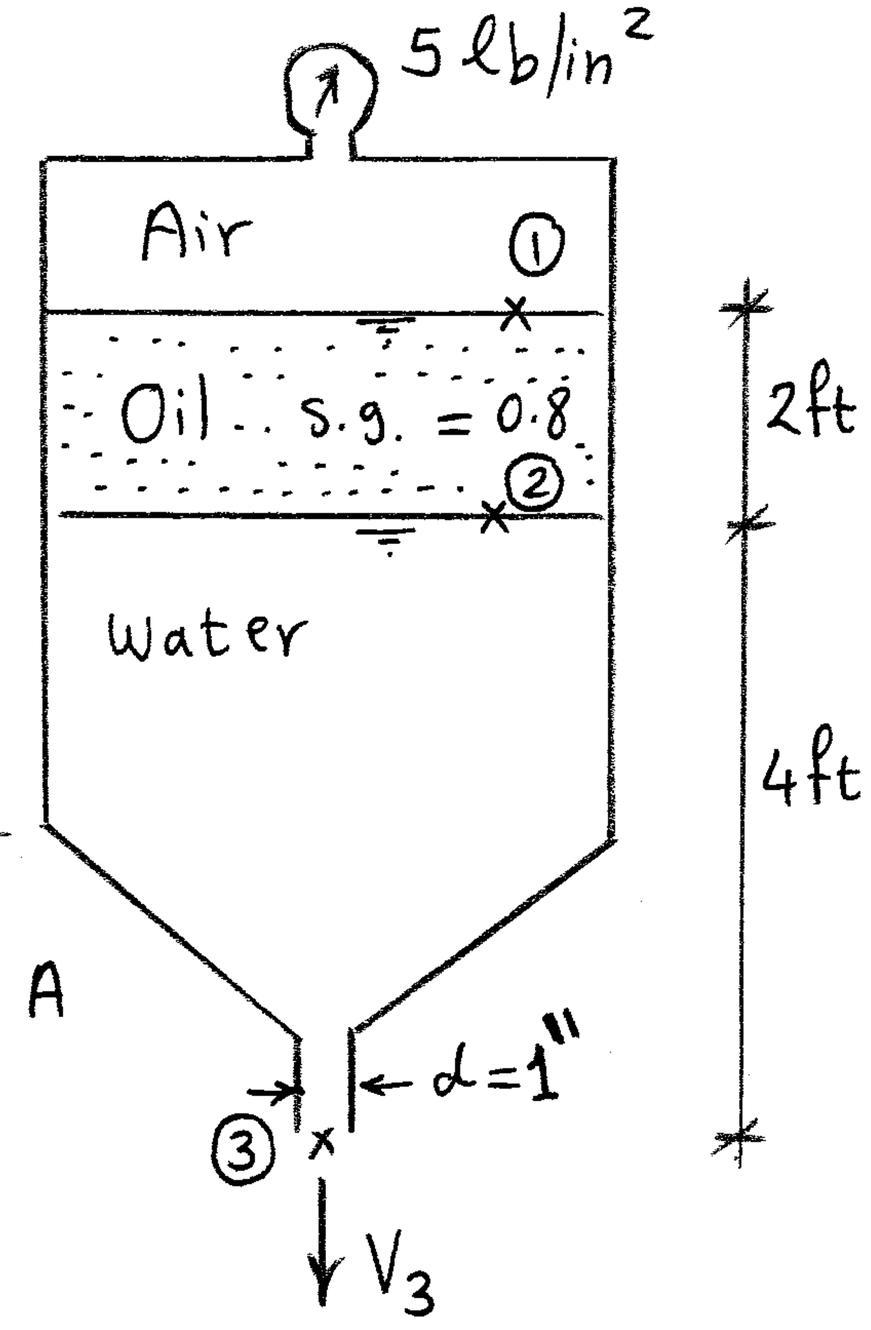
$$V_f^2 = V_{in}^2 - 2gh$$

$$0 = (12.82)^2 - 2(9.81)h \qquad \Rightarrow h = 8.37 \text{ m}$$

7 A cylindrical tank contains air, oil, and water as shown. A pressure of 5 lb/in^2 is maintained on the oil surface. **What** is the velocity of the water leaving the 1.0-inch diameter pipe (neglect the kinetic energy of the fluids in the tank above elevation A).

$$P_{air} = 5\ \frac{lb}{in^2} = 5*144 = 720\ lb/ft^2$$

$V_3 = ?$



Sol

$$P_2 = P_{air} + \gamma_{oil}\, h_{oil}$$

$$= 720 + 0.8(62.4)\,2 = 819.8\ \frac{lb}{ft^2}$$

Bernoulli eqn between ②, ③ Datum at ③

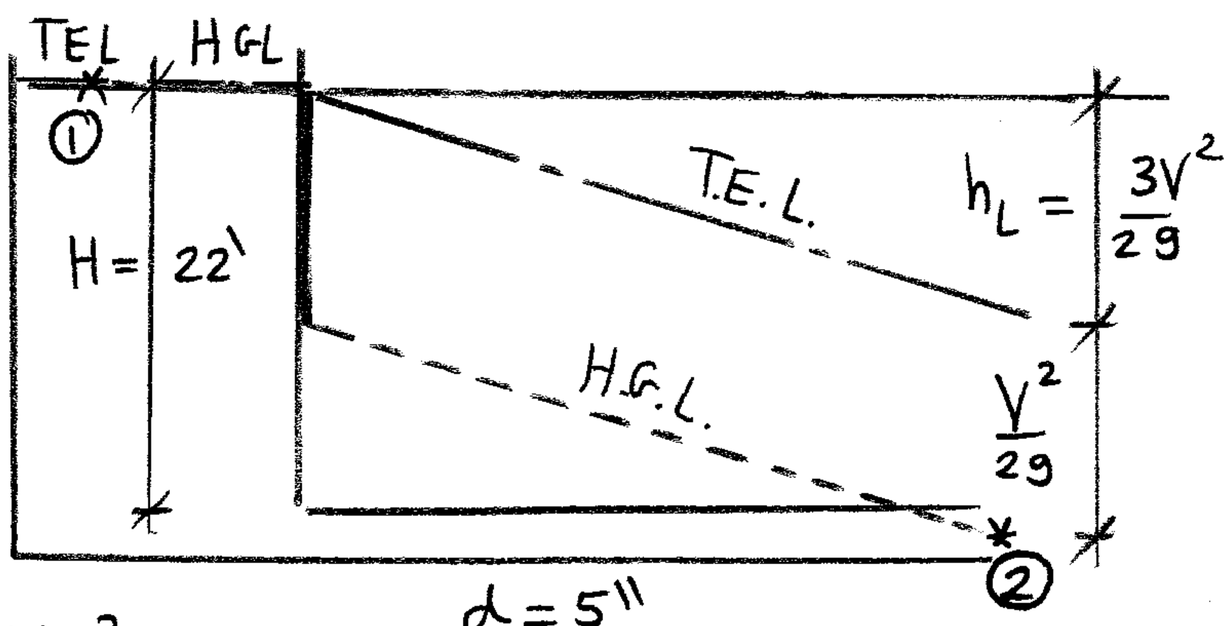
$$\frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 = \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + Z_3$$

$$\frac{819.8}{62.4} + 4 = \frac{V_3^2}{2(32.2)} \Rightarrow V_3 = 33.2\ ft/s$$

8

The losses in the shown figure equals $3(V^2/2g)$ft, when H is 22ft.
What is the discharge passing in the pipe? Draw the TEL and the HGL.

$Q = ?$



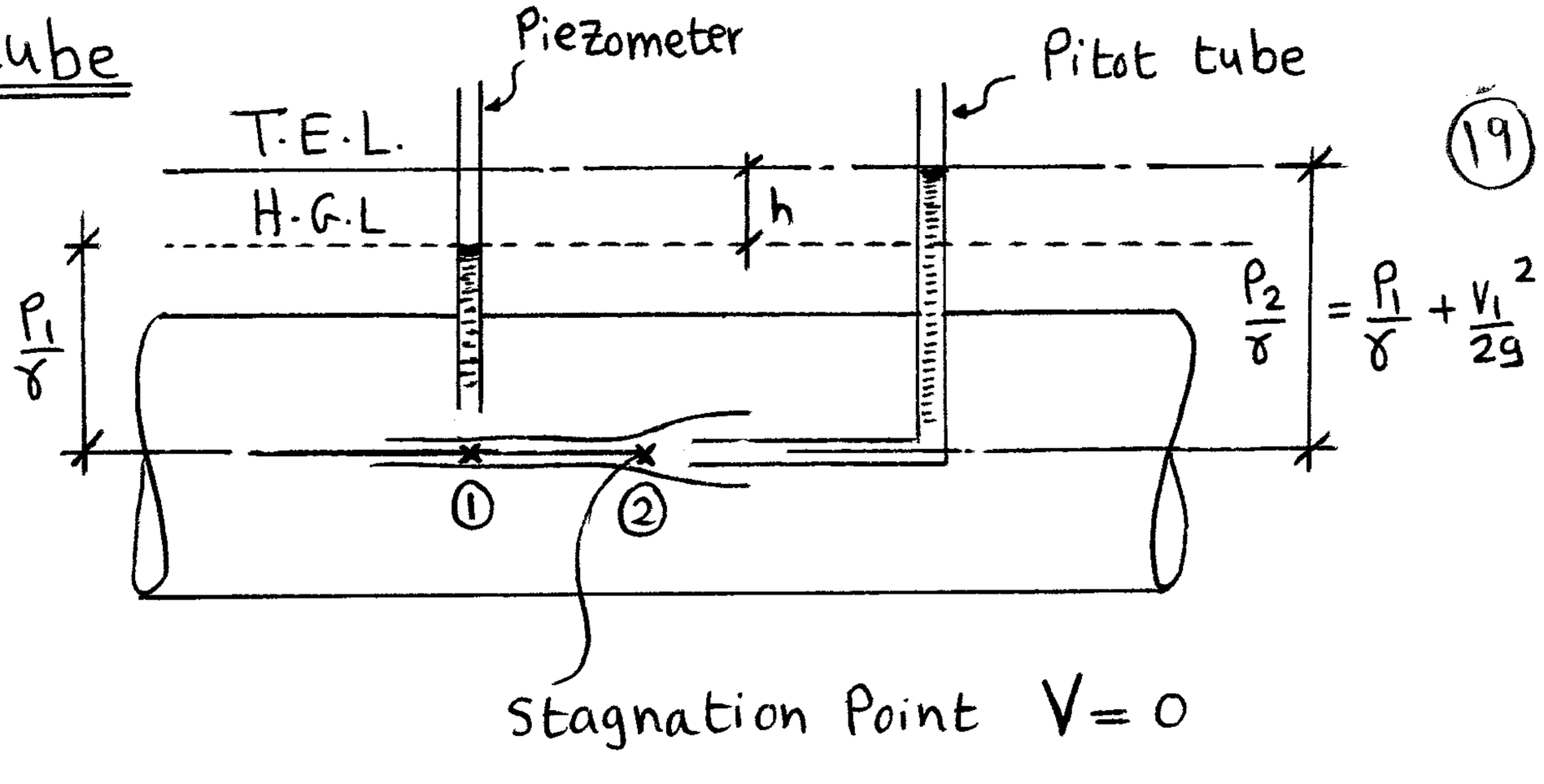
Bernoulli eqn between ①, ②

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + \Sigma h_L$$

$$22 = \frac{V_2^2}{2g} + \frac{3V_2^2}{2g} = \frac{4V_2^2}{2(32.2)} \Rightarrow V_2 = 18.82\ ft/s$$

$$\therefore Q = AV = \frac{\pi\,(5/12)^2}{4} * 18.82 = 2.56\ ft^3/s$$

Pitot tube



* The Pitot tube is used for measuring Velocity head of a flowing liquid
* It is Placed with its opening in the direction of flow
* The liquid flows up the tube until all its kinetic Energy is Converted to Potentral Energy

Applying Bernoullis eqn between ①, ②

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + \cancel{Z_1} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + \cancel{Z_2}$$

$\frac{V_2^2}{2g} = 0$

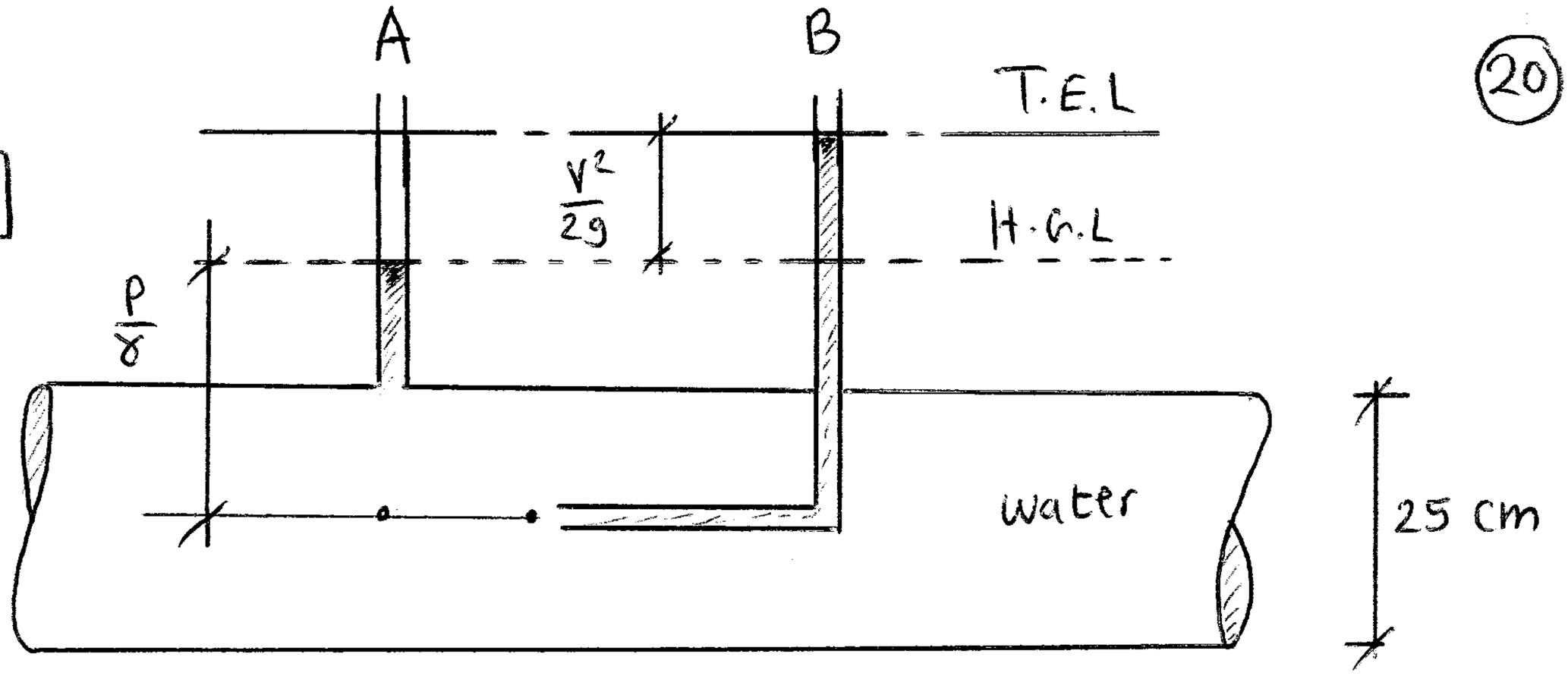
$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} = \frac{P_2}{\gamma}$$

$$\frac{P_2 - P_1}{\gamma} = \frac{V_1^2}{2g}$$

$$\boxed{h = \frac{V_1^2}{2g}}$$

(20)

9



$P = 30 \text{ kPa} = 30 * 10^3 \text{ N/m}^2$

$Q = 50 \text{ lit/s} = 0.05 \text{ m}^3/\text{s}$

To what height will water rise in tubes A and B ?

Sol

Tube A (Piezometer)

$$h_A = \frac{P}{\gamma} = \frac{30 * 10^3}{9810} = \underline{3.06 \text{ m}}$$

Tube B (Pitot tube)

$$h_B = \frac{P}{\gamma} + \frac{V^2}{2g}$$

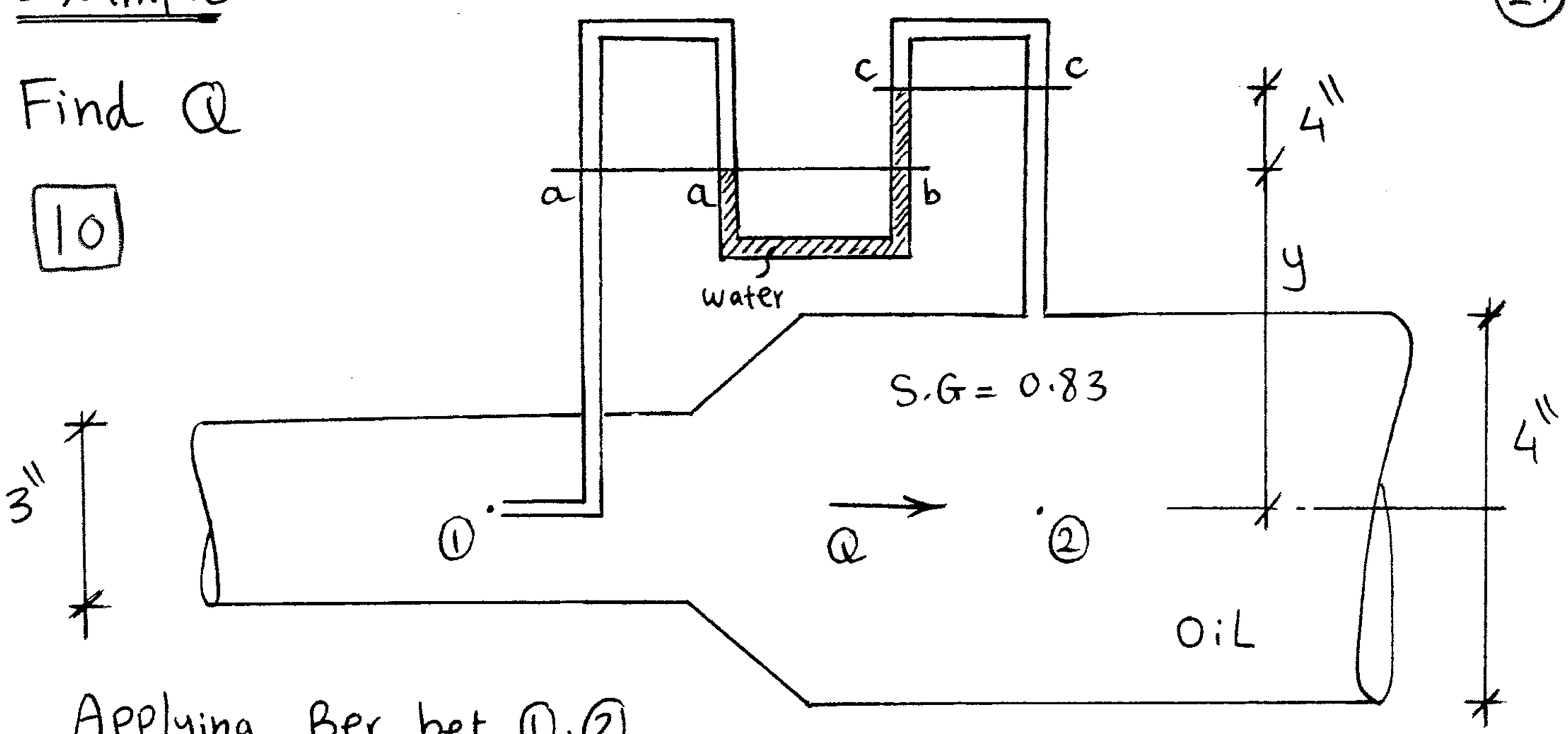
$$V = \frac{Q}{A} = \frac{0.05}{\frac{\pi (0.25)^2}{4}} = 1.02$$

$$h_B = \frac{30*10^3}{9810} + \frac{(1.02)^2}{2(9.81)} = \underline{3.11 \text{ m}}$$

Example

Find Q

10



Applying Ber bet ①, ②

$$\frac{P_1}{\gamma_{oil}} + \frac{V_1^2}{2g}_{=0} = \frac{P_2}{\gamma_{oil}} + \frac{V_2^2}{2g}$$

$$\frac{P_1 - P_2}{\gamma_{oil}} = \frac{V_2^2}{2g} \rightarrow \text{(I)}$$

$$P_1 - \gamma_{oil} y - \gamma_w \left(\frac{4''}{12}\right) + \gamma_{oil}\left(\frac{4''}{12}\right) + \gamma_{oil} y = P_2$$

$$P_1 - P_2 = \gamma_w \left(\frac{4}{12}\right) - \gamma_{oil}\left(\frac{4}{12}\right) \qquad \div \gamma_{oil}$$

$$\frac{P_1 - P_2}{\gamma_{oil}} = \frac{4}{12}\left[\frac{\gamma_w}{\gamma_{oil}} - 1\right] = \frac{4}{12}\left(\frac{\gamma_w}{0.83\gamma_w} - 1\right) = 0.068 \text{ ft} \rightarrow \text{(II)}$$

from (I), (II)

$$\frac{V_2^2}{2(32.2)} = 0.068 \Rightarrow V_2 = 2.1 \text{ ft/s}$$

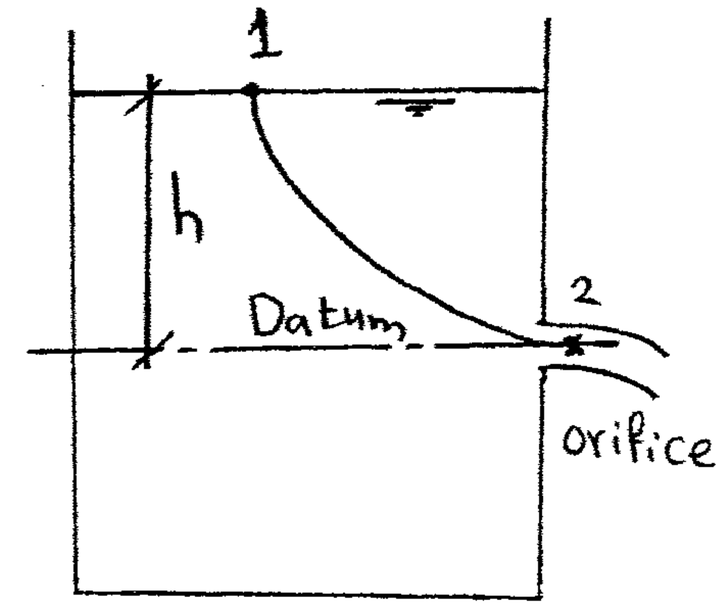
$$Q = A_2 V_2 = \frac{\pi \left(\frac{4}{12}\right)^2}{4} * 2.1 = \underline{0.183 \text{ ft}^3/\text{s}}$$

9 Flow through Orifices

Coefficient of Velocity C_v

Applying Bernoulli eqn bet ①,②

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$



$$0 + 0 + h = 0 + \frac{V_2^2}{2g} + 0$$

$$\boxed{V_2 = \sqrt{2gh}} \Rightarrow \boxed{V_{th} = \sqrt{2gh}}$$

$V_{act} < V_{th}$ ((due to loss of energy by friction))

Actual Velocity = C_v * theoretical Velocity

$$\boxed{V_{act} = C_v V_{th} = C_v \sqrt{2gh}}$$

Coefficient of Velocity = $C_v = \frac{V_{act}}{V_{th}} < 1$

Coefficient of discharge C_d

$Q = aV$

$$\boxed{Q = a\sqrt{2gh}}$$ theoretical discharge.

Actual Discharge = C_d * Theoretical discharge

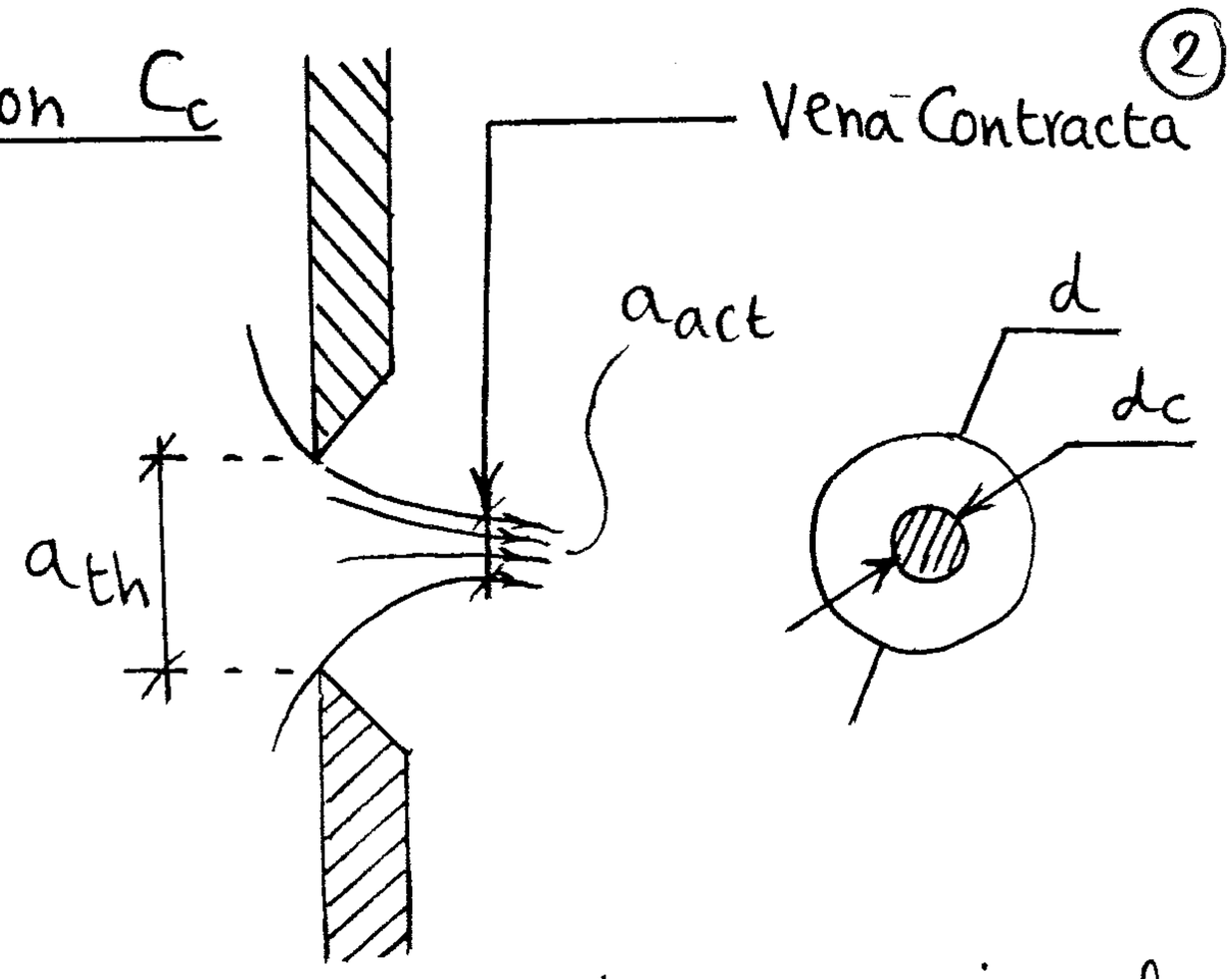
$$\boxed{Q_{act} = C_d Q_{th} = C_d a_{th} \sqrt{2gh}}$$

Coefficient of discharge $C_d = \frac{Q_{act}}{Q_{th}} < 1$

Coefficient of Contraction C_c

$$a_{th} = \frac{\pi d^2}{4}$$

a_{act} = area at the Vena Contracta



Vena Contracta

It is the Section at the minimum diameter jet of Water discharging from the orifice at which the Stream-lines Starts to become parallel

$a_{act} < a_{th}$ ((due to the Converging effect of Streamlines))

Actual area a_{act} = C_c * theoretical area
(Area of Contraction a_c)

$$\boxed{a_{act} = C_c\, a_{th}}$$

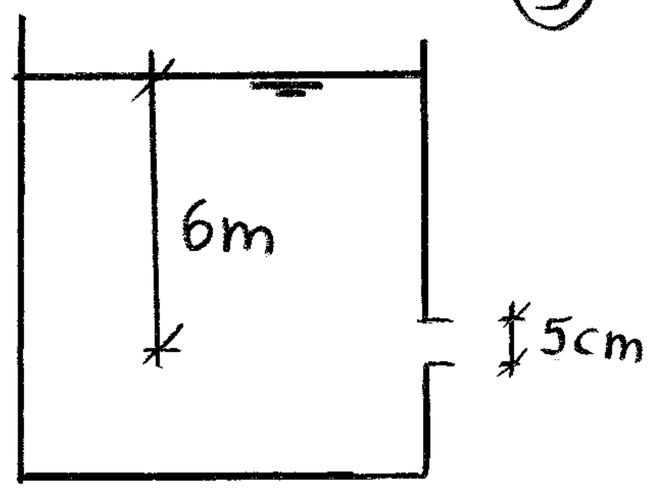
Coefficient of Contraction $C_c = \frac{a_{act}}{a_{th}} < 1$

$$Q_{act} = a_{act}\, V_{act}$$

$$C_d\, Q_{th} = C_c\, a_{th}\, C_v\, V_{th}$$

$$\boxed{C_d = C_c\, C_v}$$

(3)



II $\forall = 2.7\ m^3$, $t = 3 min$

$V_{vena\ contracta} = 9.5\ m/s$ $\quad C_v, C_c, C_d = ?$

Sol

$$Q_{act} = \frac{\forall}{t} = \frac{2.7}{3*60} = 0.015\ m^3/s$$

$$V_{th} = \sqrt{2gh} = \sqrt{2*9.81*6} = 10.85\ m/s$$

$$C_V = \frac{V_{act}}{V_{th}} = \frac{9.5}{10.85} = \underline{\underline{0.875}}$$

$$a_{act} = \frac{Q_{act}}{V_{act}} = \frac{0.015}{9.5} = 1.58*10^{-3}\ m^2$$

$$a_{th} = \frac{\pi d_{th}^2}{4} = \frac{\pi (0.05)^2}{4} = 1.96*10^{-3}\ m^2$$

$$C_c = \frac{a_{act}}{a_{th}} = \frac{1.58*10^{-3}}{1.96*10^{-3}} = \underline{\underline{0.8}}$$

$$C_d = C_c\ C_V = 0.8 * 0.875 = \underline{\underline{0.7}}$$

Another Solution

$$Q_{th} = V_{th}\ a_{th} = 10.85 * 1.96 * 10^{-3} = 0.0213$$

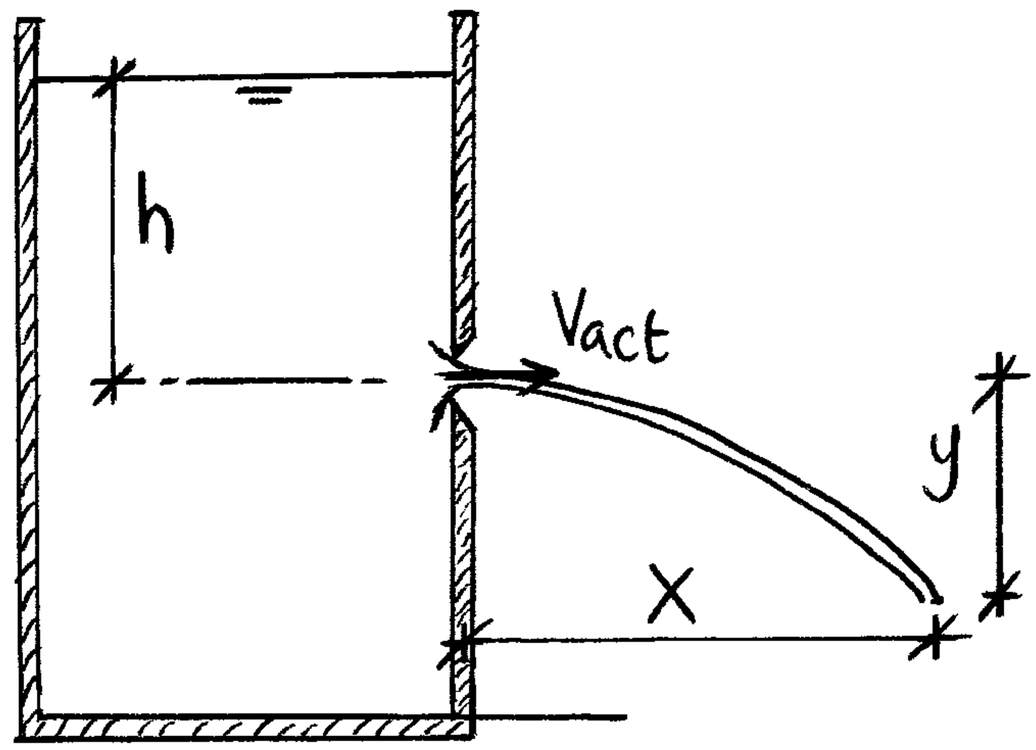
$$C_d = \frac{Q_{act}}{Q_{th}} = \frac{0.015}{0.0213} = 0.7$$

Determination of the Coefficient of Velocity ④ C_v

Applying Newton's equation

$$S = V_o t + \frac{1}{2} a t^2$$

∵ no acceleration in hzl direction



∴ $a_x = 0$, $S = x$ لا توجد عجلة أفقية

$$X = V_{o_x} t = V_{act} t$$

$$\boxed{t = \frac{X}{V_{act}}} \rightarrow ①$$

∵ no initial velocity in Vl direction

$V_{oy} = 0$, $S = y$, $a_y = g$

$$\boxed{y = \frac{1}{2} g t^2} \rightarrow ②$$

from ①, ②

$$y = \frac{1}{2} g \frac{X^2}{V_{act}^2} \rightarrow ③$$

$$\because V_{act} = C_v V_{th} = C_v \sqrt{2gh}$$

$$V_{act}^2 = C_v^2 (2gh) \rightarrow ④$$

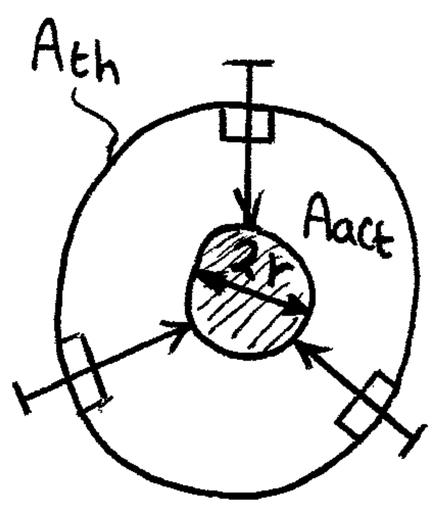
from ③, ④

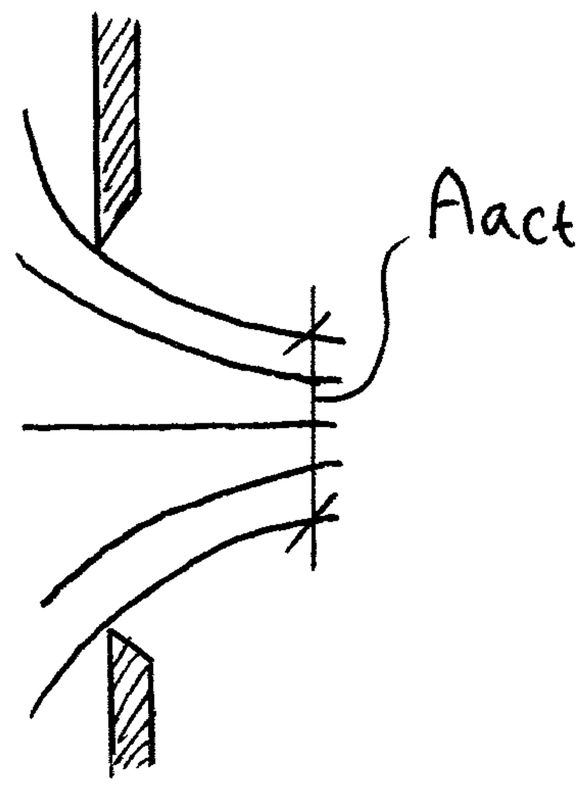
$$y = \frac{1}{2} \frac{g X^2}{C_v^2 (2gh}$$

$$C_v^2 = \frac{X^2}{4yh} \quad \text{or} \quad \boxed{C_v = \frac{X}{2\sqrt{yh}}}$$

Determination of the Coefficient of Contraction C_c

$$C_c = \frac{A_{act}}{A_{th}}$$





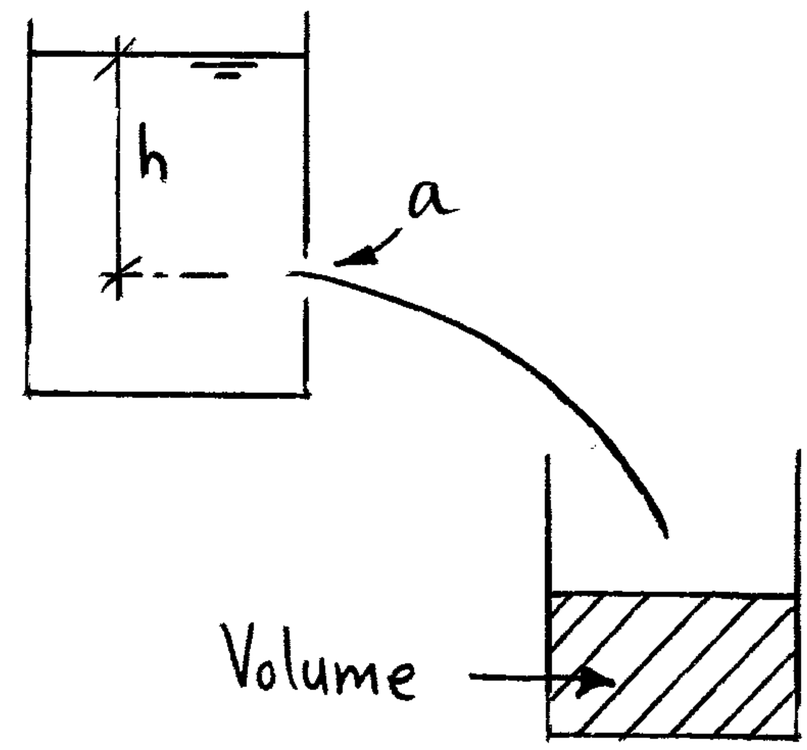
Determination of the Coefficient of Discharge C_d

$$Q_{th} = a_{th} V_{th}$$

$$Q_{th} = a_{th} \sqrt{2gh}$$

$$Q_{act} = \frac{\text{Volume}}{\text{time}}$$

$$C_d = \frac{Q_{act}}{Q_{th}}$$



13

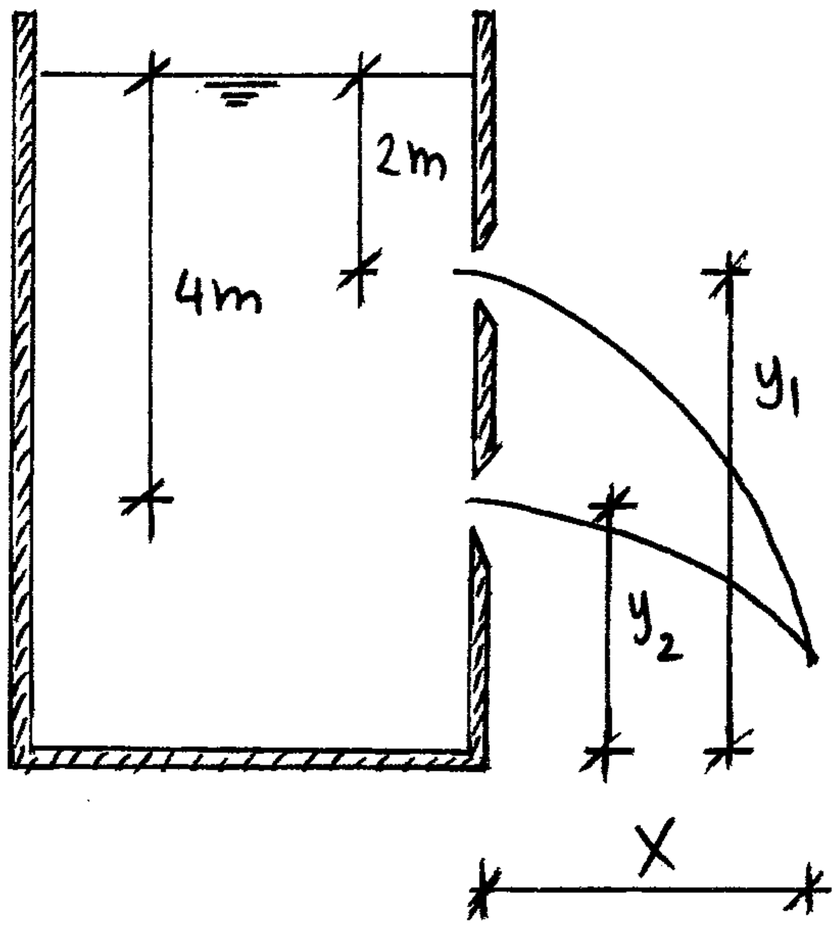
$C_v = 0.9$ for the two orifices

Find y_1, y_2

$$C_v^2 = \frac{X^2}{4yh}$$

$$(0.9)^2 = \frac{X^2}{4y_1(2)} \rightarrow ①$$

$$(0.9)^2 = \frac{X^2}{4y_2(4)} \rightarrow ②$$



From ①, ② $\quad y_1 = 2y_2$

$\therefore y_1 - y_2 = 2\,m$

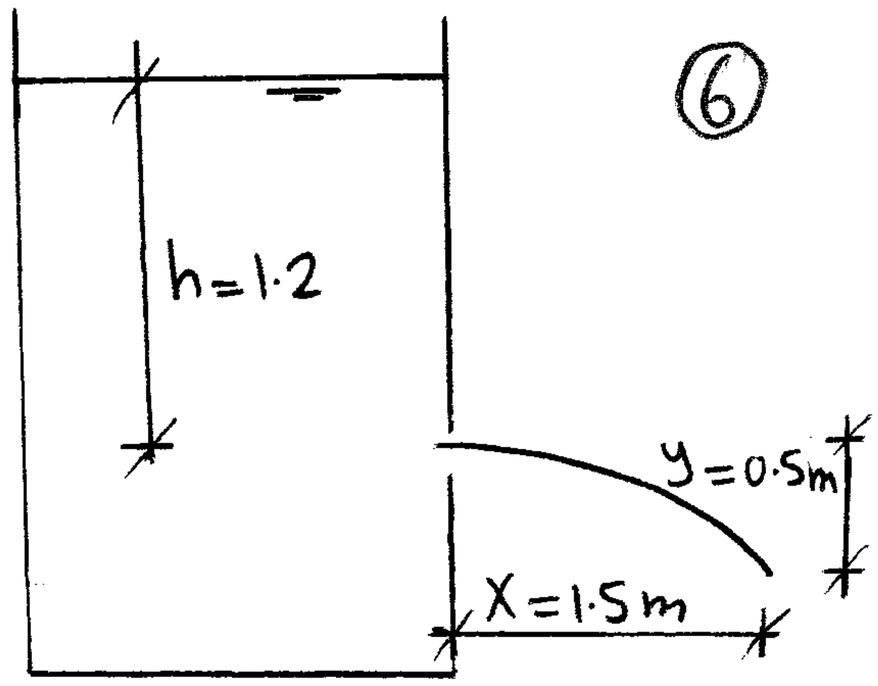
$\therefore y_1 = 4\,m$

$y_2 = 2\,m$

$d_{th} = 3.5$ cm

$Q_{act} = 2.8 \times 10^{-3}$ m^3/s

$C_v = ?$ $C_c = ?$ $C_d = ?$



⑥

Sol

$$V_{th} = \sqrt{2gh} = \sqrt{2*9.81*1.2} = 4.85 \text{ m/s}$$

$$a_{th} = \frac{\pi d_{th}^2}{4} = \frac{\pi (0.035)^2}{4} = 0.00096 \text{ m}^2$$

$$Q_{th} = a_{th} V_{th} = 0.00096 (4.85) = 0.0046 \text{ m}^3/\text{s}$$

$$C_d = \frac{Q_{act}}{Q_{th}} = \frac{2.8 \times 10^{-3}}{4.6 \times 10^{-3}} = 0.6$$

$$C_v = \frac{X}{2\sqrt{yh}} = \frac{1.5}{2\sqrt{(0.5)(1.2)}} = 0.97$$

$$C_c = \frac{C_d}{C_v} = \frac{0.6}{0.97} = 0.62$$

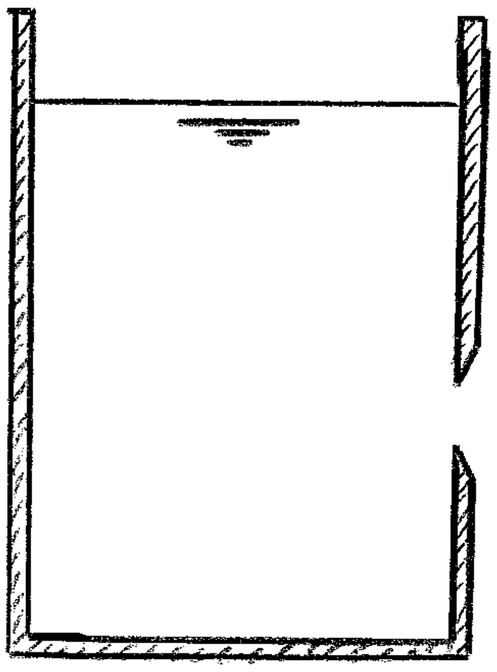
or

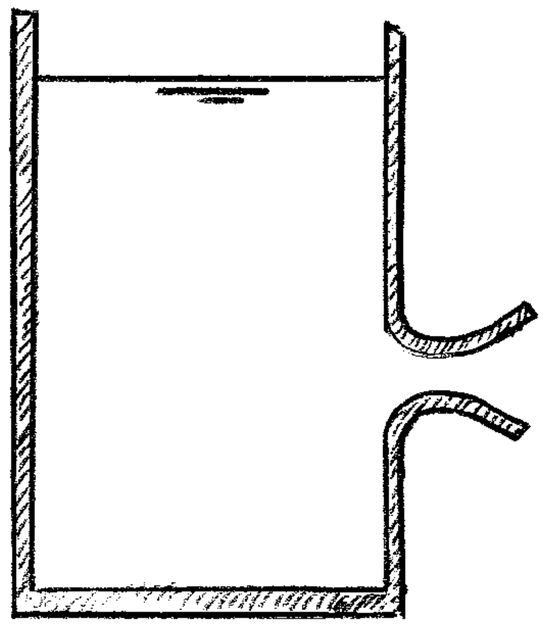
$$V_{act} = C_v V_{th} = 0.97 (4.85) = 4.7 \text{ m/s}$$

$$a_{act} = \frac{Q_{act}}{V_{act}} = \frac{2.8 \times 10^{-3}}{4.7} = 0.0006$$

$$C_c = \frac{a_{act}}{a_{th}} = \frac{0.0006}{0.00096} = 0.62$$

Types of orifices





Sharp edged orifice

$C_c \simeq 0.6$

Bell mouthed orifice

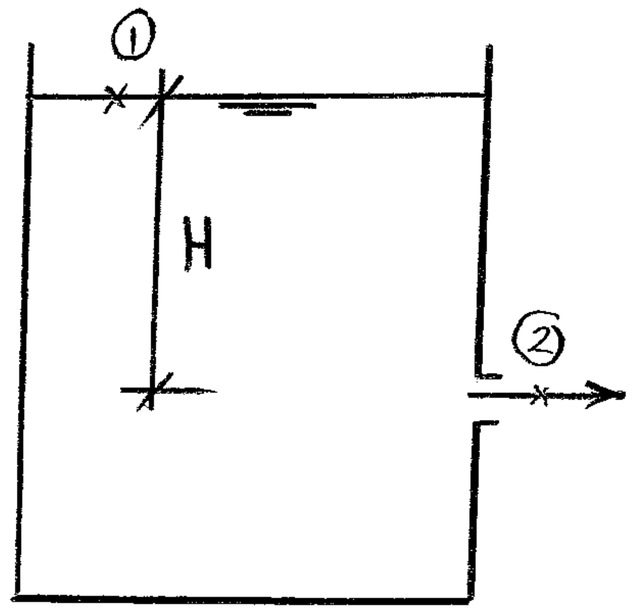
$C_c \simeq 1$

Head loss in an orifice

$$V_{th} = \sqrt{2gH}$$

$$V_{act} = C_v\sqrt{2gH}$$

$$V_{act}^2 = C_v^2\ 2gH \quad \rightarrow (1)$$



Apply Bernoulli eqn between ①, ② Datum at ②

$$\frac{P_1}{\gamma} + \frac{V_{1\,act}^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_{2\,act}^2}{2g} + Z_2 + h_L$$

$$0 + 0 + H = 0 + \frac{V_{2\,act}^2}{2g} + 0 + h_L$$

$$V_{act}^2 = 2g(H - h_L) \rightarrow (2)$$

from ①, ②

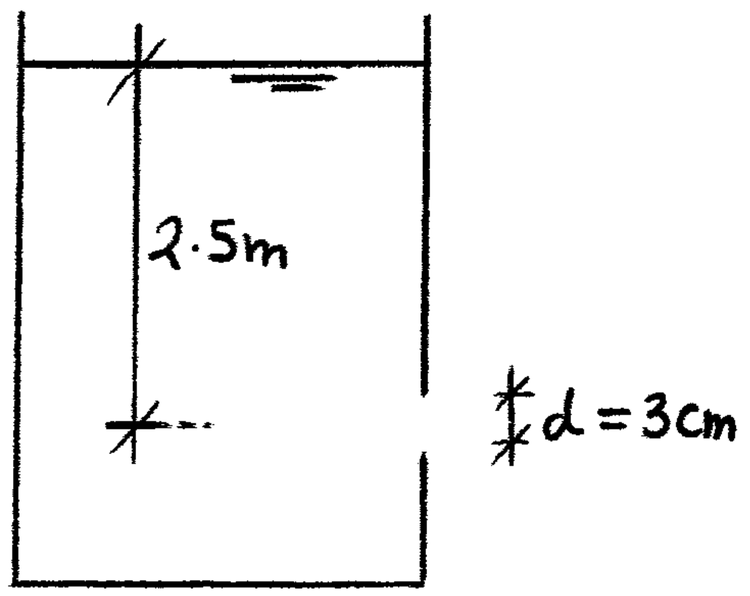
$$C_v^2\ 2gH = 2g(H - h_L)$$

$$\boxed{h_{L_{1\rightarrow 2}} = H(1 - C_v^2)}$$

12 **Find** the discharge and head loss when using sharp- edged orifice ($C_c = 0.64$). If C_V of the orifice is 0.94 and the water head over the center line of the orifice is 2.5 m. (8)

$C_c = 0.64$ $\quad C_V = 0.94$

$Q = ?$ $\quad h_L = ?$



Sol//

$$a_{th} = \frac{\pi (0.03)^2}{4} = 7.06 * 10^{-4}$$

$$V_{th} = \sqrt{2gh} = \sqrt{2(9.81)(2.5)} = 7 \text{ m/s}$$

$$C_d = C_c C_V = 0.64(0.94) = 0.6$$

$$Q = C_d a_{th} V_{th} = 2.96 * 10^{-3} \text{ m}^3/\text{s}$$

$$h_L = H(1 - C_V^2) = 2.5\left(1 - (0.94)^2\right) = 0.291 \text{ m}$$

---

14 If water flows steadily through the large tank shown in the figure.
Determine the water depth h_A.

$h_A = ?$

Sol//

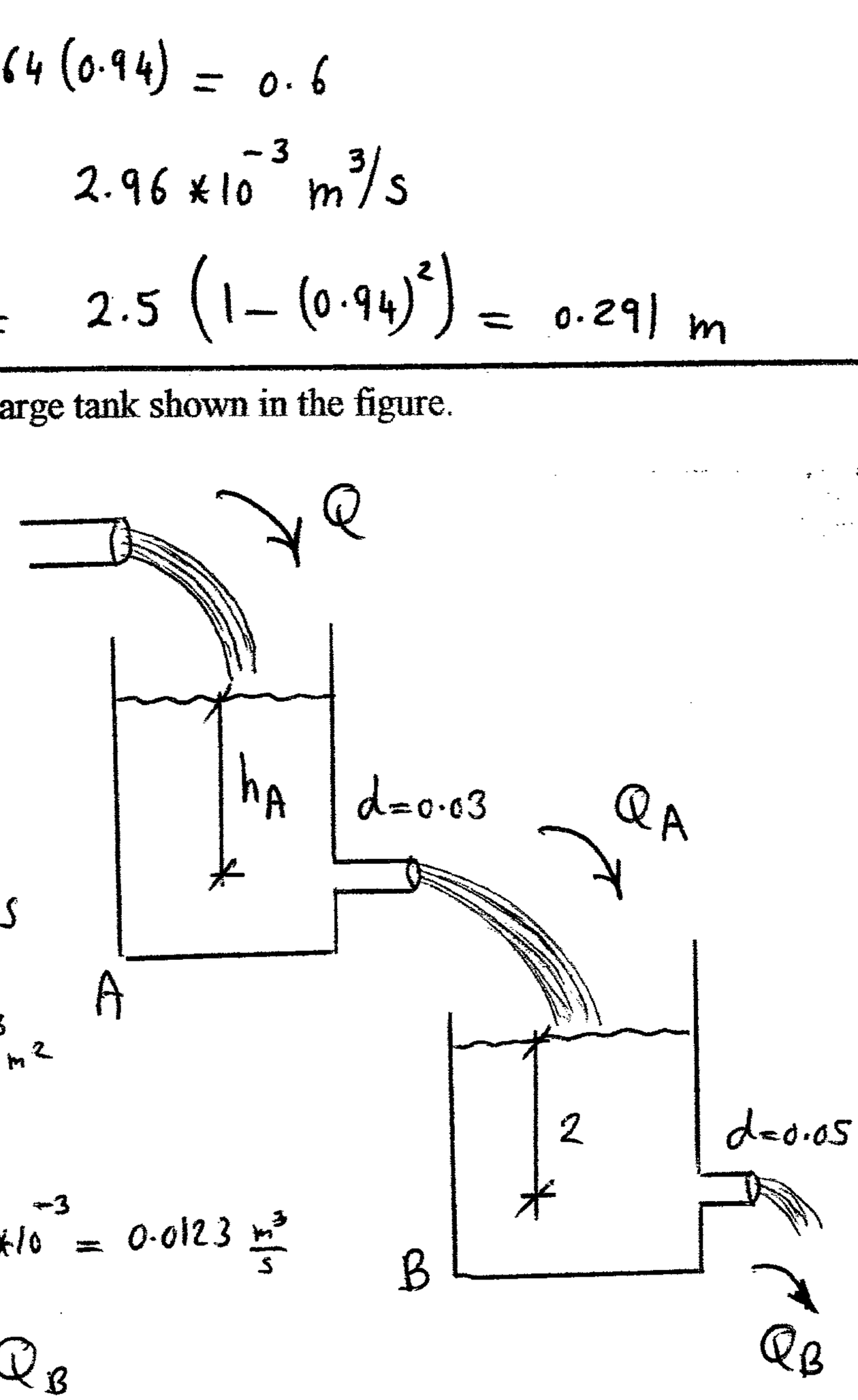
$$V_B = \sqrt{2gh} = \sqrt{2(9.81)\,2} = 6.26 \text{ m/s}$$

$$A_B = \frac{\pi (0.05)^2}{4} = 1.96 * 10^{-3} \text{ m}^2$$

$$Q_B = V_B A_B = 6.26 * 1.96 * 10^{-3} = 0.0123 \frac{\text{m}^3}{\text{s}}$$

$$\therefore \quad Q_A = Q_B$$

$$\frac{\pi (0.03)^2}{4} \sqrt{2(9.81)h} = 0.0123 \quad \Rightarrow \quad h = \underline{15.43 \text{ m}}$$



Flow through Large Orifices

from Continuity equation

$$dQ = V dA$$

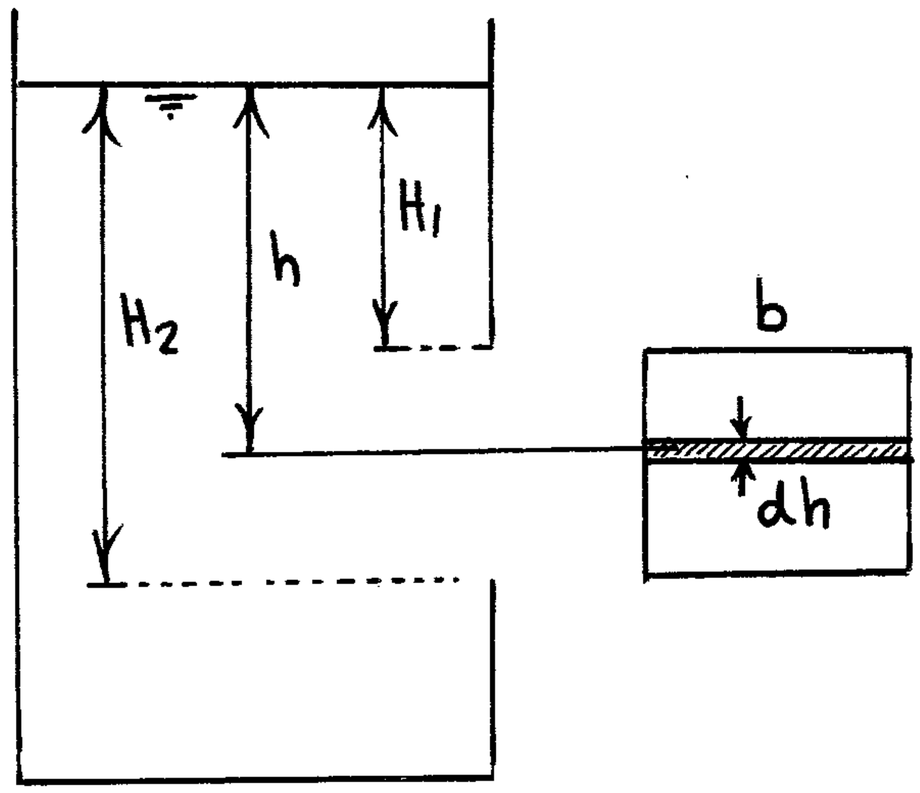
where ; $V = \sqrt{2gh}$

$$dA = b\,dh$$

$$dQ = \sqrt{2gh}\; b\,dh$$

$$Q = b\sqrt{2g} \int_{H_1}^{H_2} h^{1/2}\, dh$$

$$Q = \frac{2}{3} b\sqrt{2g} \left(H_2^{3/2} - H_1^{3/2} \right)$$



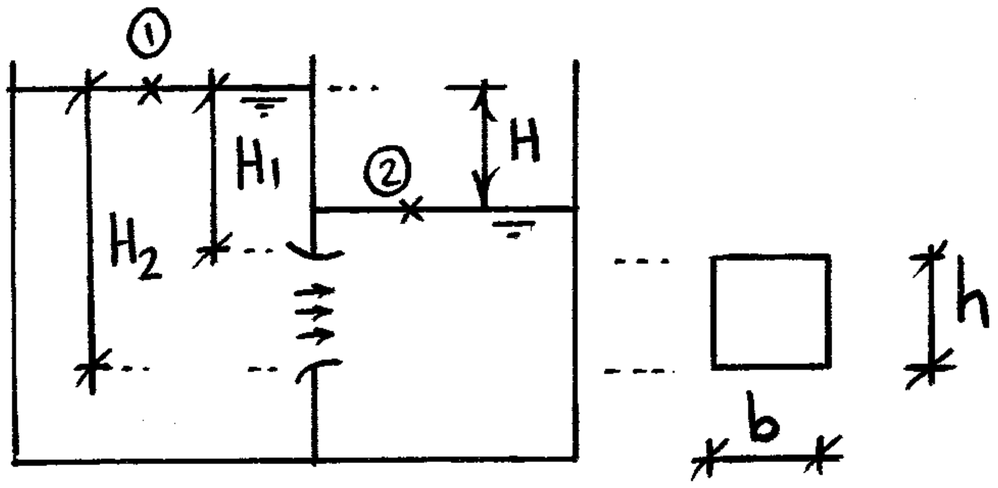
$$\boxed{Q = \frac{2}{3} C_d\, b\sqrt{2g} \left(H_2^{3/2} - H_1^{3/2} \right)}$$

Discharge through Fully Submerged Large orifices (drowned)

$$Q = AV$$

$$= bh\left(\sqrt{2gH}\right)$$

$$= b(H_2 - H_1) * \sqrt{2gH}$$



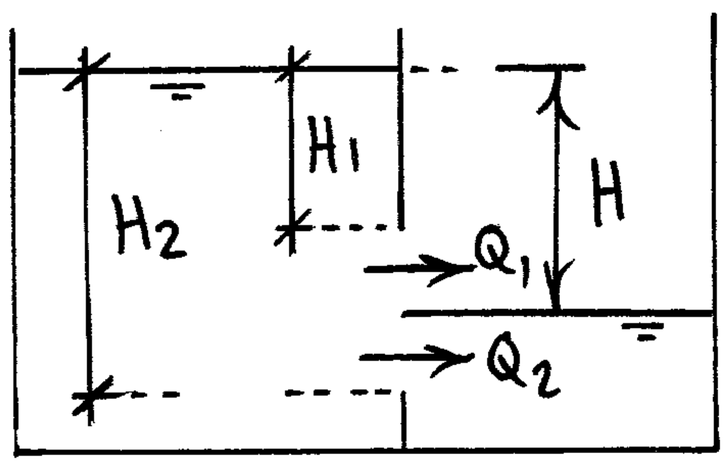
$$\boxed{Q_{act} = C_d\, b\,(H_2 - H_1) * \sqrt{2gH}}$$

Discharge through Partially Submerged Orifice

$$Q_1 = \frac{2}{3} C_d\, b\sqrt{2g} \left(H^{3/2} - H_1^{3/2} \right)$$

$$Q_2 = C_d\, b\,(H_2 - H) * \sqrt{2gH}$$

$$Q_{act} = Q_1 + Q_2$$



(10)

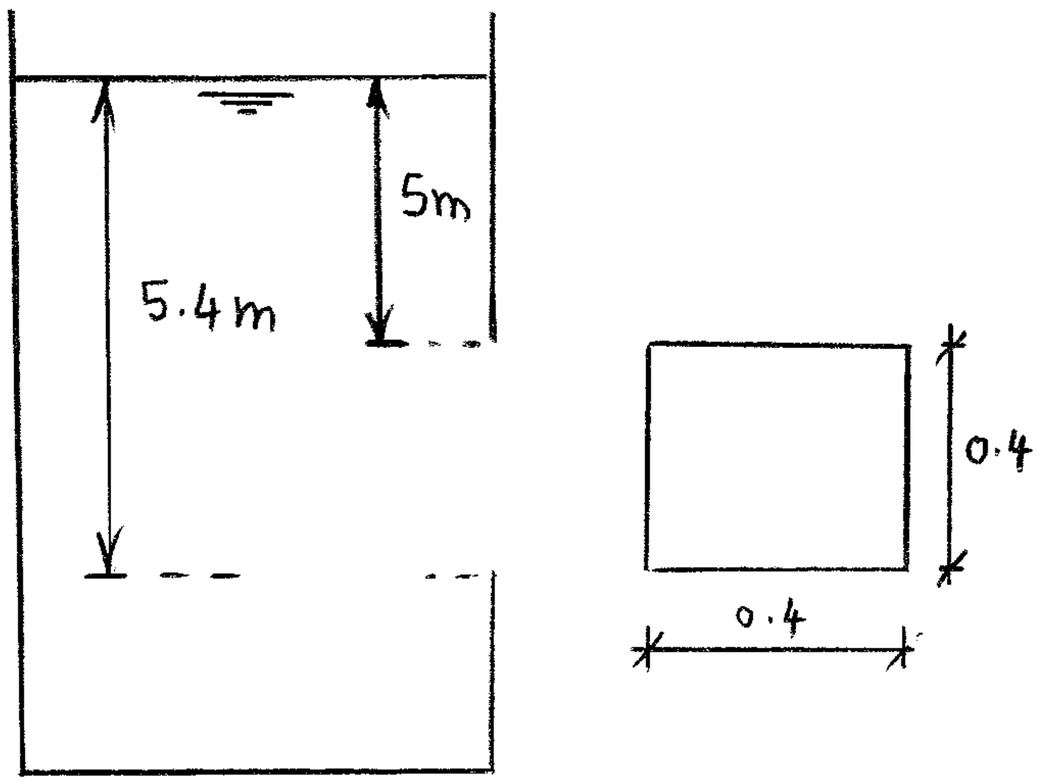
A big square orifice in the side of the tank has a side length of 40 cm. If the height of water over the top end of the orifice is 5 m, **Determine** the discharge.
What will be the difference in discharge if treated as a small orifice? ($C_d = 0.63$)
Resolve if the orifice mentioned was in the bottom of the tank. **Comment**.

$C_d = 0.63 \qquad Q = ?$

$$Q_1 = \frac{2}{3} C_d b \sqrt{2g} \left[H_2^{\frac{3}{2}} - H_1^{\frac{3}{2}}\right]$$

$$= \frac{2}{3}(0.63)\,0.4\sqrt{2(9.81)}\left[5.4^{\frac{3}{2}} - 5^{\frac{3}{2}}\right]$$

$$= \underline{1.01808 \ m^3/s}$$



If treated as a small orifice

$$Q_2 = C_d\, a \sqrt{2gh}$$

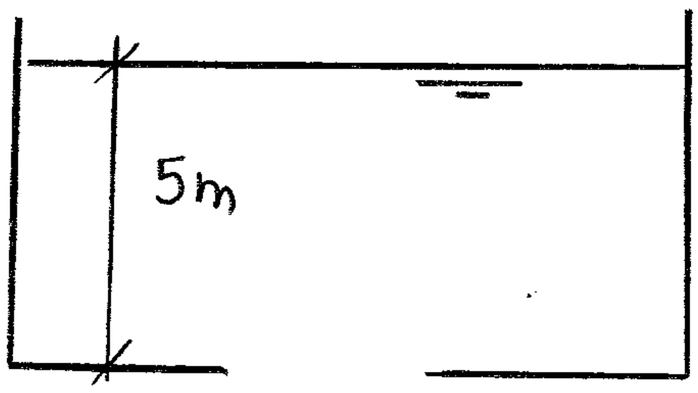
$$= 0.63\ (0.4)(0.4)\sqrt{2(9.81)\left(\frac{5+5.4}{2}\right)} = 1.01815$$

$$\Delta Q = Q_1 - Q_2 = \underline{7 * 10^{-5}\ m^3/s} = 70\ cm^3/s$$

If the orifice was placed at the bottom of the tank

V = Constant

∴ Can be treated as a Small orifice



$$Q = C_d\, a \sqrt{2gh}$$

$$= 0.63\ (0.4 * 0.4)\sqrt{2(9.81)\ 5} = \underline{0.998\ m^3/s}$$

Example:

A rectangular orifice 1.5 m wide and 1.2 m deep is fitted in one side of a large tank. The water level on one side of the orifice is 2 m above the top edge of the orifice, while on the other side of the orifice, water level is 0.4 m below its top edge. Calculate the discharge through the orifice. Take $C_d = 0.62$.

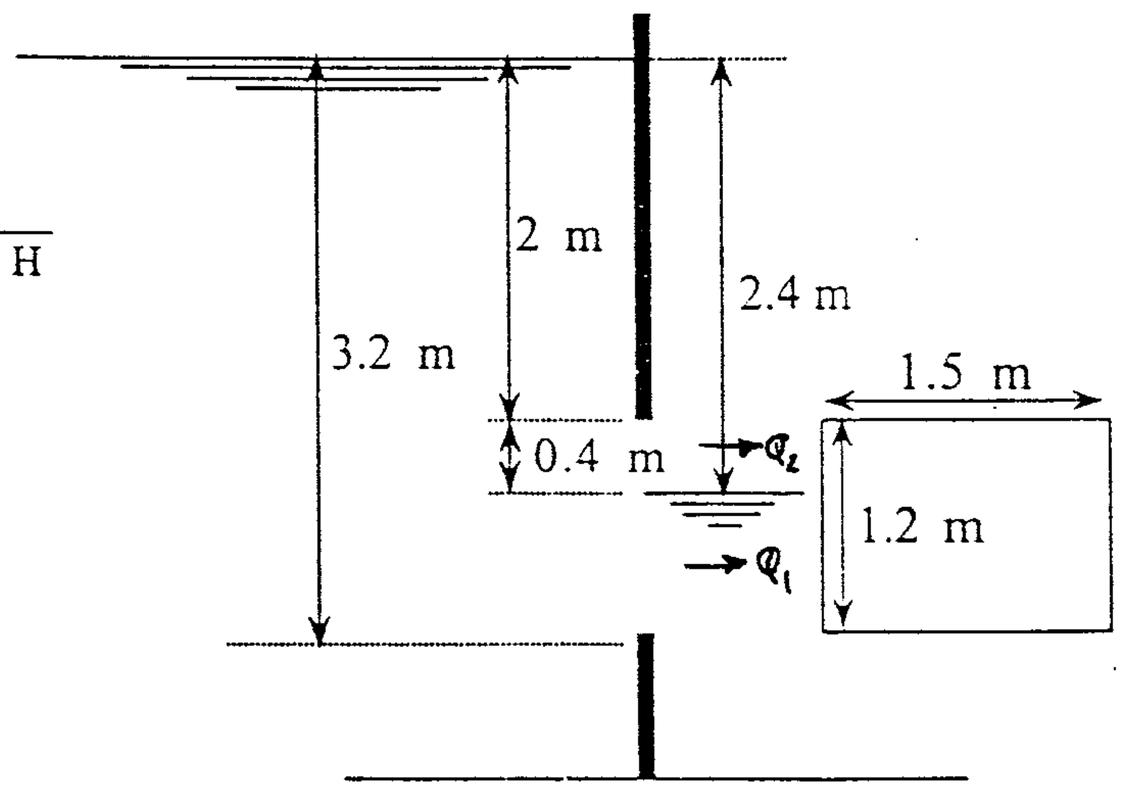
Solution:

Partially submerged large orifice

$b = 1.5$ m , $d = 1.2$ m

$H_1 = 2$ m , $H_2 = 2$ m + 1.2 m = 3.2 m

$H = 2.4$ m



$$Q_1 = C_d\, b\, (H_2 - H)\, \sqrt{2gH}$$

$$Q_2 = \frac{2}{3} C_d\, b \sqrt{2g} \left(H^{3/2} - H_1^{3/2}\right)$$

$$Q_1 = 0.62 \times 1.5\,(3.2 - 2.4) \times \sqrt{2 \times 9.8 \times 2.4}$$

$$= 5.1\ m^3/s$$

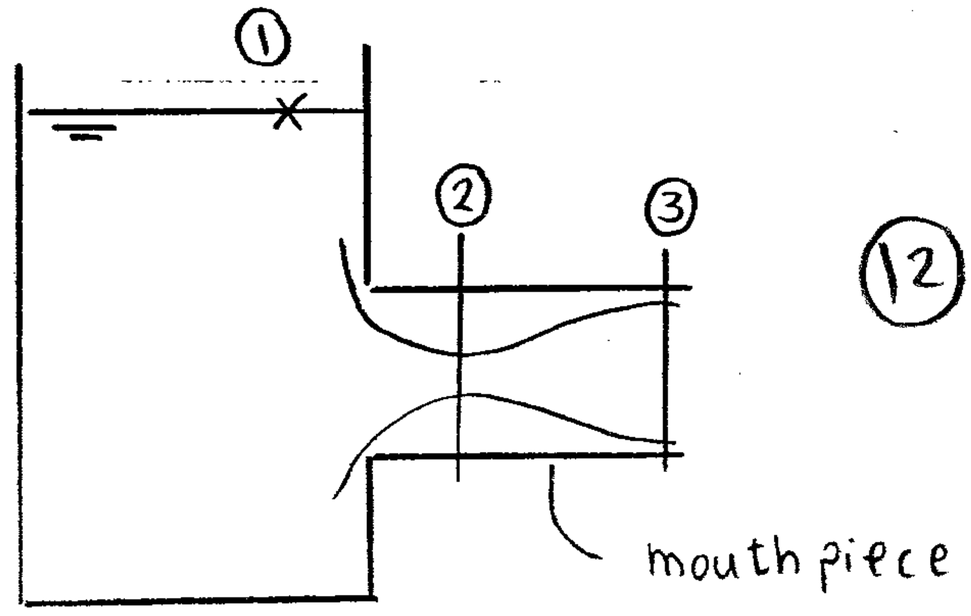
$$Q_2 = \frac{2}{3} \times 0.62 \times 1.5 \sqrt{2 \times 9.8} \left(2.4^{3/2} - 2^{3/2}\right)$$

$$= 2.44\ m^3/s$$

$$Q_t = Q_1 + Q_2$$

$$= 5.1 + 2.44 = \underline{\underline{7.54\ m^3/s}}$$

Flow through mouthpieces



It is a Small tube attached to an orifice

neglect head losses from ① → ②

$$h_{L_{2\to3}} = \frac{(V_2 - V_3)^2}{2g}$$ (sudden enlargment)

$$C_c = \frac{a_{act}}{a_{th}} = \frac{a_2}{a_3}$$

$$\because \; a_2 V_2 = a_3 V_3$$

$$C_c a_3 V_2 = a_3 V_3$$

$$V_2 = \frac{V_3}{C_c}$$

$$\therefore \; h_{L_{2\to3}} = \frac{\left(\frac{V_3}{C_c} - V_3\right)^2}{2g}$$

$$h_{L_{2\to3}} = \left(\frac{1}{C_c} - 1\right)^2 \frac{V_3^2}{2g}$$ head loss through mouthpiece

If we neglect h_L from ② → ③

and applying Bernoulli eqn between ②, ③

$$\frac{P_2}{\gamma} + \frac{V_2^2}{2g} + 0 = 0 + \frac{V_3^2}{2g} + 0$$

$$\frac{P_2}{\gamma} = \frac{V_3^2 - V_2^2}{2g} = -ve$$ due to -ve pressure, discharge increases

Check Cavitation

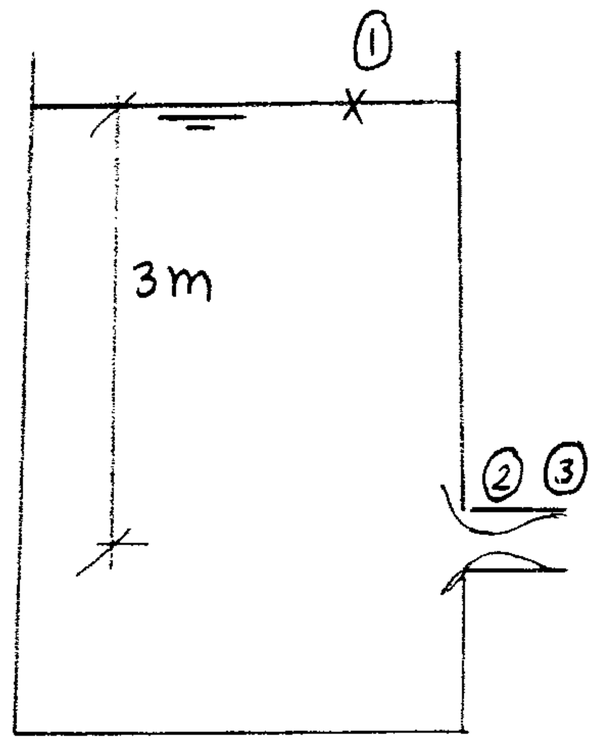
Example:

17 Water is discharged through an external mouthpiece of 25 cm^2 area under a head of 3 m. Find the discharge through the mouthpiece and the pressure at the venacontracta. $C_C = 0.62$. (13)

Solution:

$a = 25\ cm^2$ $\quad C_c = 0.62$

$Q = ?$ $\quad P_2 = ?$



Bernoulli eqn between ①, ③

$$0 + 0 + 3 = 0 + \frac{V_3^2}{2g} + 0 + h_{L_{2\to3}}$$

$$3 = \frac{V_3^2}{2g} + \left(\frac{1}{C_c} - 1\right)^2 \frac{V_3^2}{2g}$$

$$3 = \frac{V_3^2}{2(9.81)} \left[1 + \left(\frac{1}{0.62} - 1\right)^2\right]$$

$\Rightarrow V_3 = 6.54\ m/s$

$$\therefore Q = V_3 a_3 = 6.54 * (25 * 10^{-4}) = \underline{0.0163\ m^3/s}$$

Bernoulli eqn between ①, ②

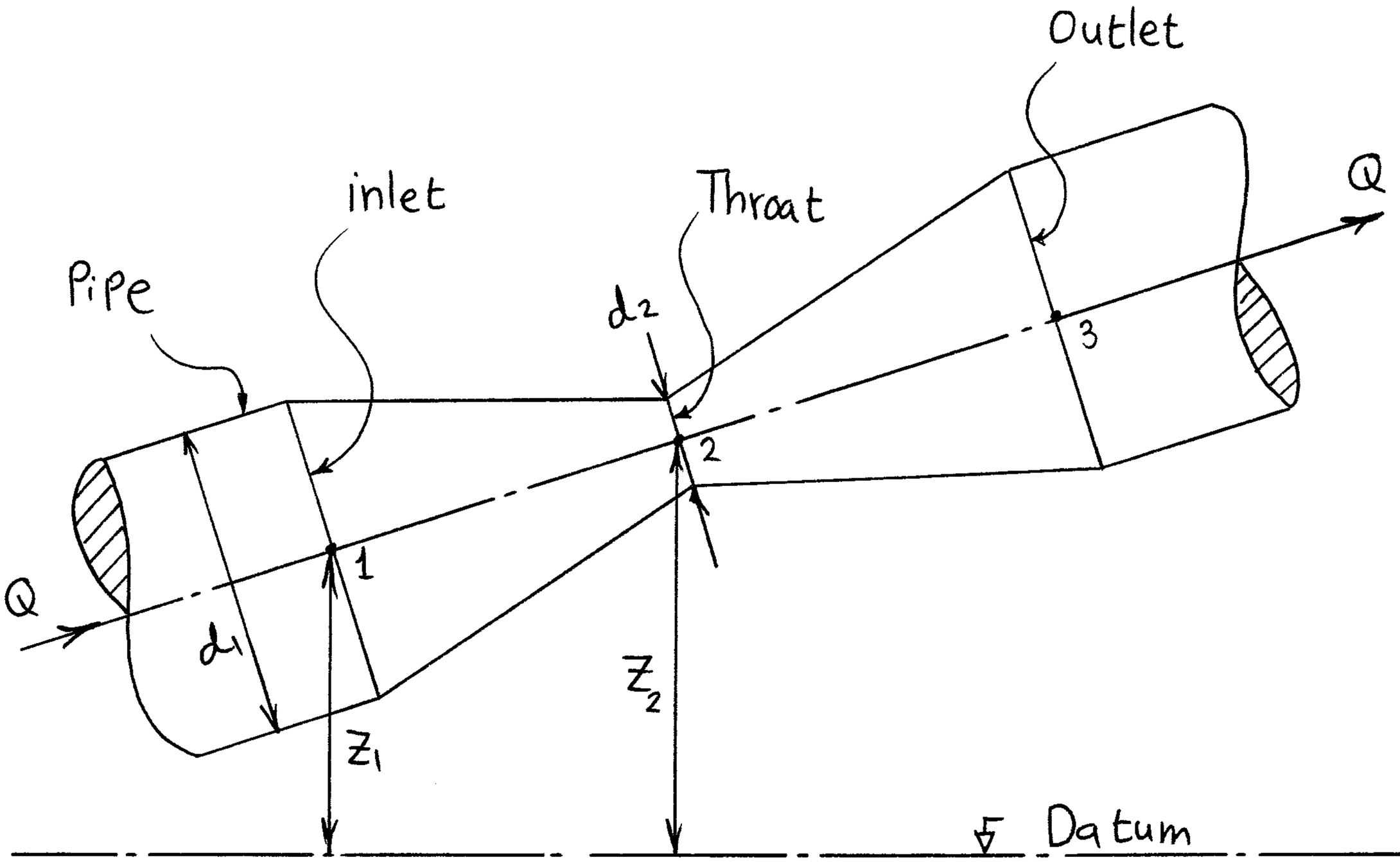
$$0 + 0 + 3 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + 0$$

$$\because V_2 = \frac{V_3}{C_c} = \frac{6.54}{0.62} = 10.55$$

$$\frac{P_2}{\gamma} = 3 - \frac{(10.55)^2}{2(9.81)} \Rightarrow P_2 = \underline{-26221\ Pa}$$

Flow through a Venturimeter

It is a device used for measuring discharge in Pipes



Applying Bernoulli equation between ①, ②

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

Let $H = \left(\frac{P_1}{\gamma} + Z_1\right) - \left(\frac{P_2}{\gamma} + Z_2\right)$

$$\frac{V_2^2}{2g} - \frac{V_1^2}{2g} = \left(\frac{P_1}{\gamma} + Z_1\right) - \left(\frac{P_2}{\gamma} + Z_2\right) = H$$

$$V_2^2 - V_1^2 = 2gH \quad \rightarrow ①$$

$\because A_1V_1 = A_2V_2$

$$\therefore V_1 = V_2 \frac{A_2}{A_1} \quad \rightarrow ②$$

$$V_2^2 - V_2^2 \frac{A_2^2}{A_1^2} = 2gH$$

$$V_2^2 \left[1 - \frac{A_2^2}{A_1^2} \right] = 2gH$$

$$V_2^2 \left(\frac{A_1^2 - A_2^2}{A_1^2} \right) = 2gH$$

$$V_2 = \frac{A_1 \sqrt{2gH}}{\sqrt{A_1^2 - A_2^2}}$$

$$Q_{th} = V_2 A_2 = \frac{A_1 A_2 \sqrt{2gH}}{\sqrt{A_1^2 - A_2^2}}$$

$$\boxed{Q_{act} = \frac{C_d A_1 A_2 \sqrt{2gH}}{\sqrt{A_1^2 - A_2^2}}}$$

<u>To get Q</u>

① get $A_1 = \frac{\pi d_1^2}{4}$ at inlet

② get $A_2 = \frac{\pi d_2^2}{4}$ at outlet

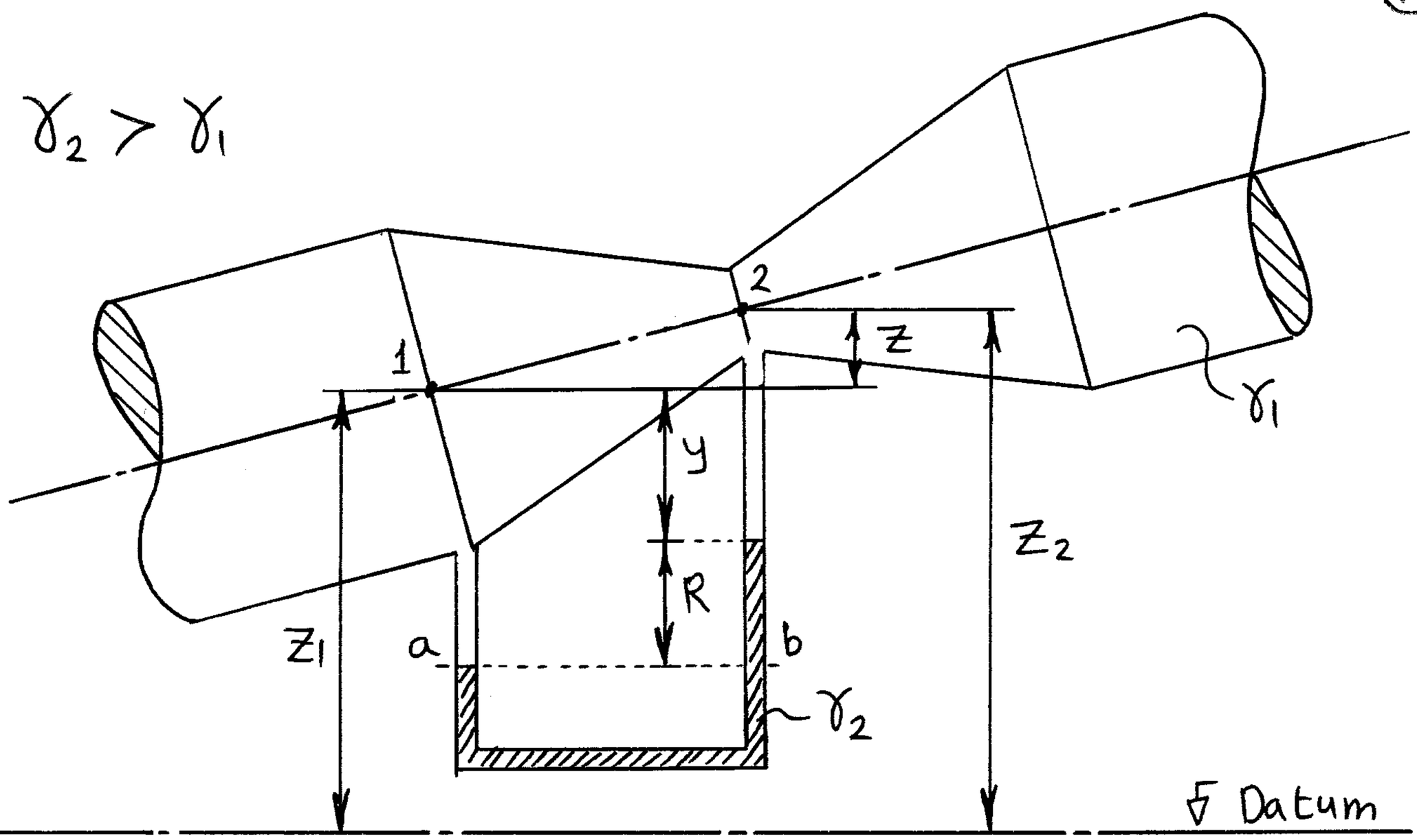
③ get $H = \left(\frac{P_1}{\gamma} + Z_1 \right) - \left(\frac{P_2}{\gamma} + Z_2 \right)$

لحساب H إما ① نقيس P_1 ، P_2 ، Z_1 ، Z_2

أو ② نستخدم Manometer

Using Manometer with Venturimeter

$\gamma_2 > \gamma_1$



$$P_a = P_1 + \gamma_1 y + \gamma_1 R$$

$$P_b = P_2 + \gamma_1 z + \gamma_1 y + \gamma_2 R$$

$$P_a = P_b$$

$$P_1 + \cancel{\gamma_1 y} + \gamma_1 R = P_2 + \gamma_1 z + \cancel{\gamma_1 y} + \gamma_2 R$$

$$P_1 - P_2 - \gamma_1 z = R(\gamma_2 - \gamma_1) \qquad \div \gamma_1$$

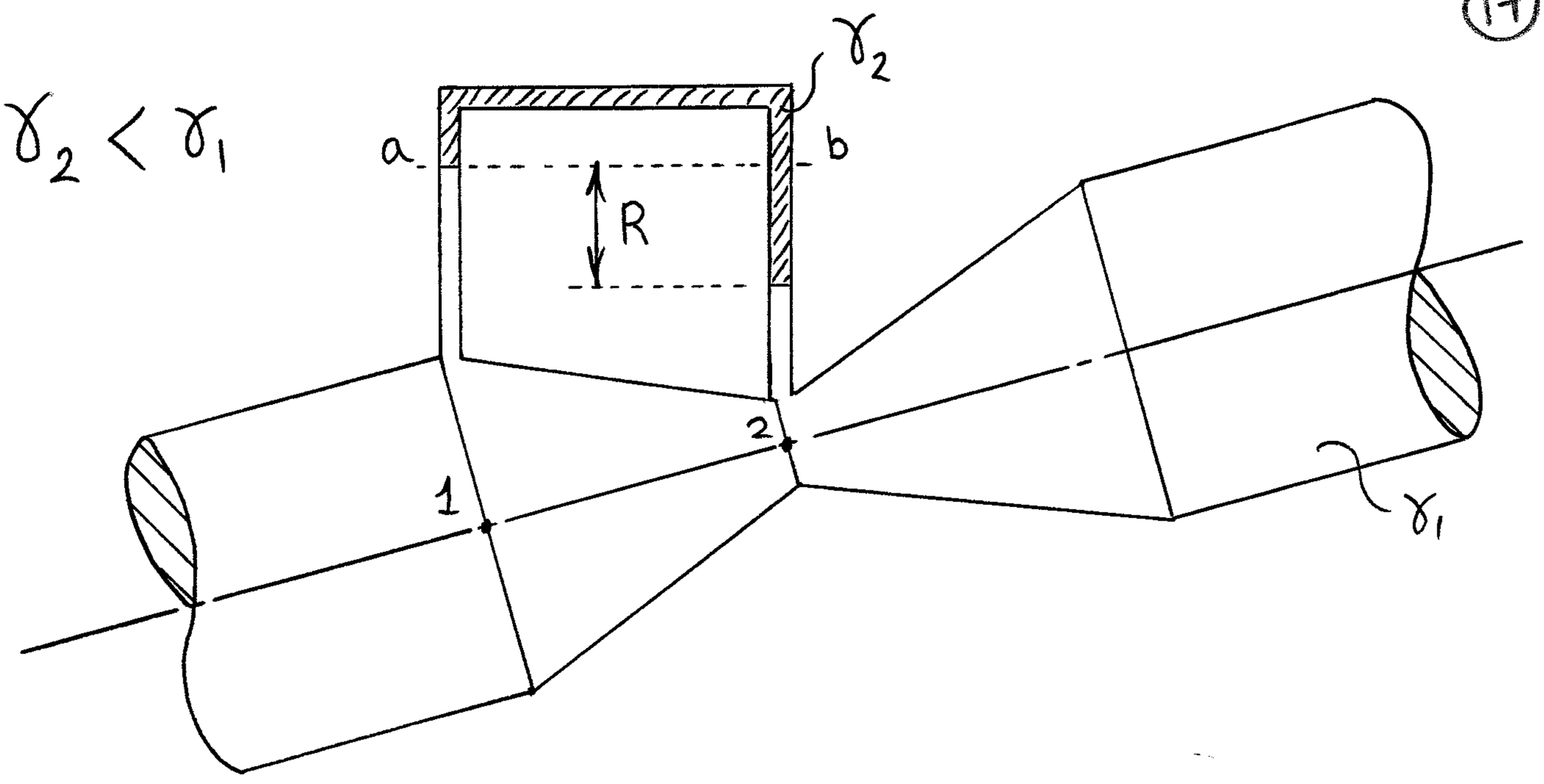
$$\frac{P_1}{\gamma_1} - \frac{P_2}{\gamma_1} - z = R\left(\frac{\gamma_2}{\gamma_1} - 1\right)$$

$$\because z = z_2 - z_1 \qquad \Rightarrow \qquad -z = z_1 - z_2$$

$$\frac{P_1}{\gamma} - \frac{P_2}{\gamma} + z_1 - z_2 = R\left(\frac{\gamma_2}{\gamma_1} - 1\right)$$

$$\boxed{H = \left(\frac{P_1}{\gamma} + z_1\right) - \left(\frac{P_2}{\gamma} + z_2\right) = R\left(\frac{\gamma_2}{\gamma_1} - 1\right)}$$

Using an Inverted U-tube Manometer



$$H = \left(\frac{P_1}{\gamma} + Z_1\right) - \left(\frac{P_2}{\gamma} + Z_2\right) = R\left(1 - \frac{\gamma_2}{\gamma_1}\right)$$

Head loss in Venturimeter

$$\frac{P_1}{\gamma} + \frac{V_{1act}^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_{2act}^2}{2g} + Z_2 + h_L$$

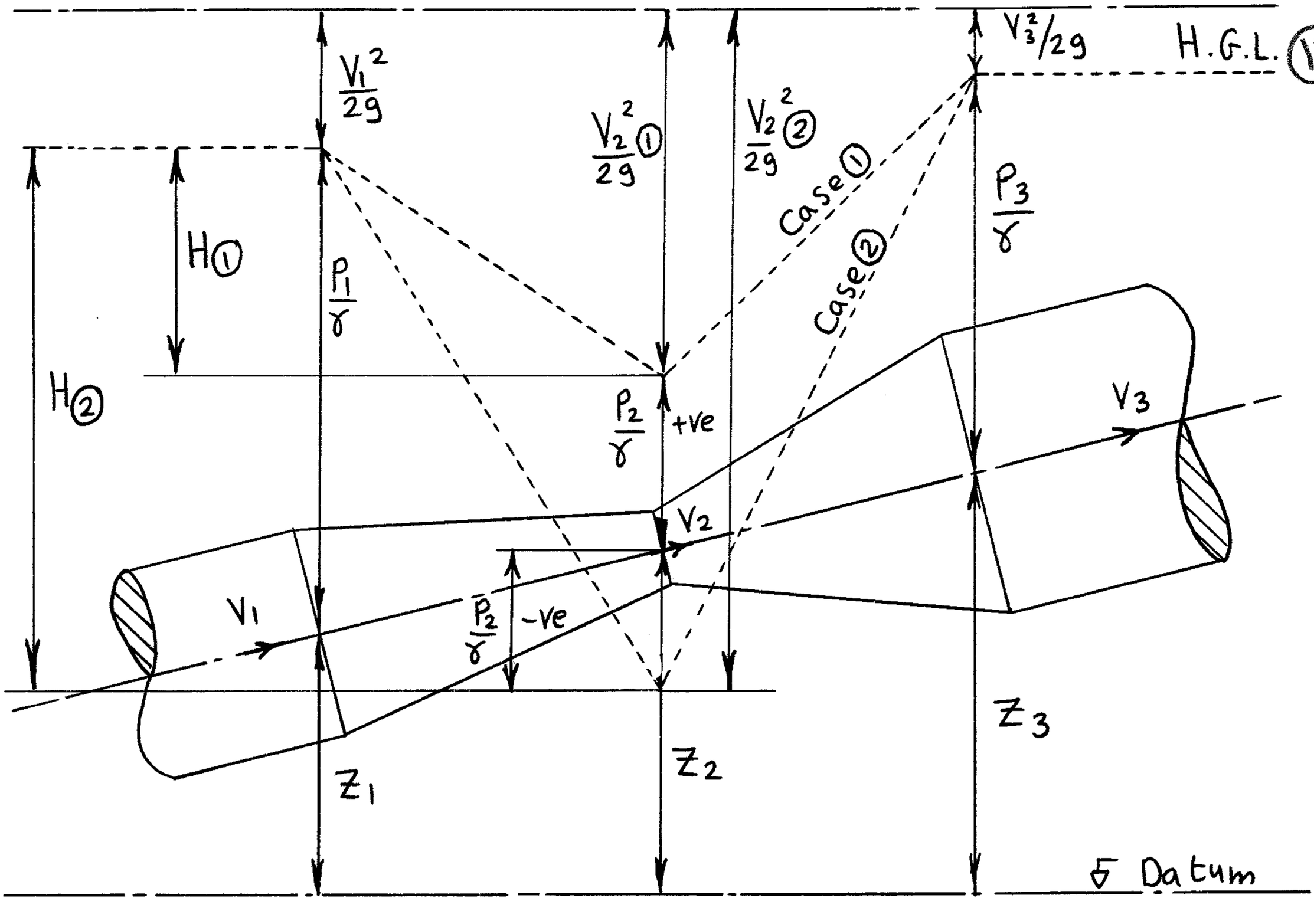
$$\left(\frac{P_1}{\gamma} + Z_1\right) - \left(\frac{P_2}{\gamma} + Z_2\right) = \frac{V_{2act}^2 - V_{1act}^2}{2g} + h_L$$

$$H = \frac{\left(\frac{Q_{act}^2}{A_2^2} - \frac{Q_{act}^2}{A_1^2}\right)}{2g} + h_L$$

$$h_L = H - \frac{Q_{act}^2}{2g}\left(\frac{A_1^2 - A_2^2}{A_1^2 A_2^2}\right)$$

$$\because Q_{act} = \frac{C_d A_1 A_2 \sqrt{2gH}}{\sqrt{A_1^2 - A_2^2}}$$

$$h_L = H\left(1 - C_d^2\right)$$

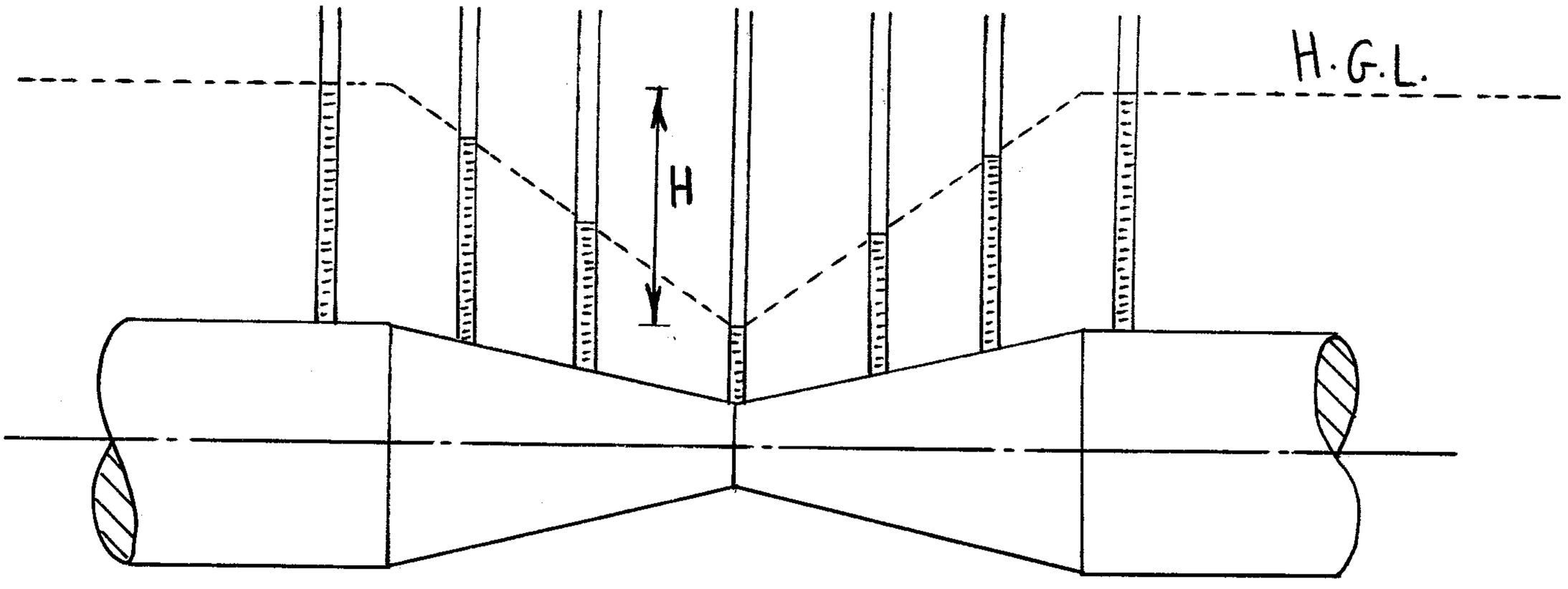


Case ①

$\frac{P_2}{\gamma}$ +ve , H small , V_2 Small

Case ②

$\frac{P_2}{\gamma}$ -ve , H big , V_2 big



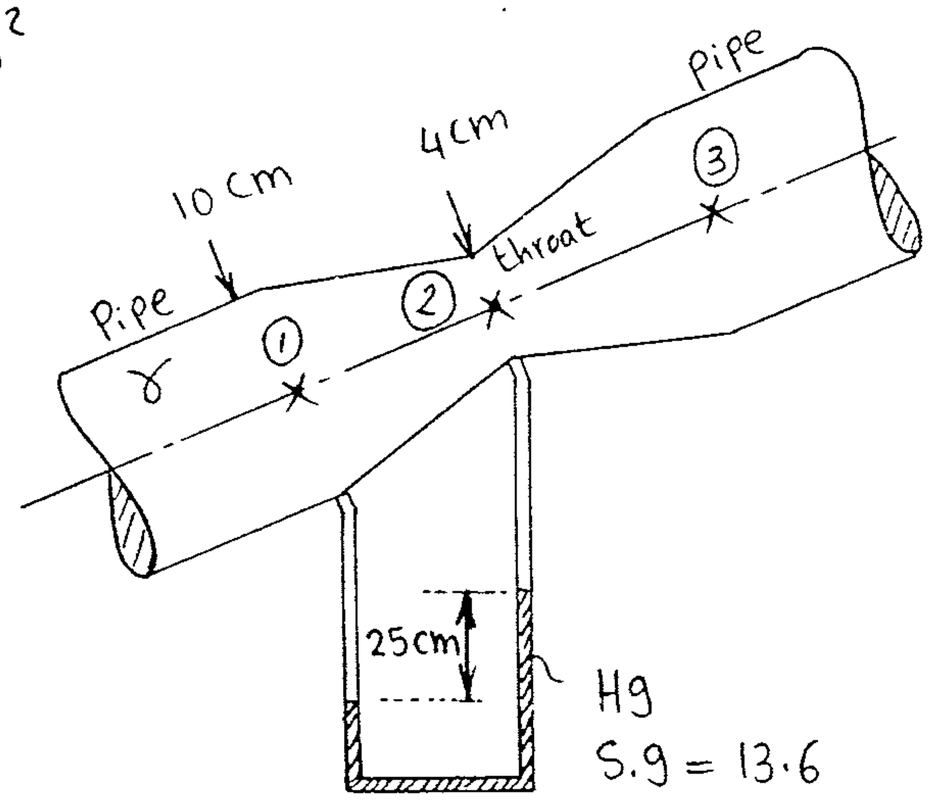
19

A venturimeter of throat diameter 4 cm is fitted into a pipeline of 10 cm diameter. The coefficient of discharge is 0.96. Calculate the flow through the meter when the reading on the mercury-water manometer connected across the upstream and throat is 25 cm. If the energy loss in the downstream divergent cone of the meter is $10(Vp^2/2g)$ Calculate the head loss across the meter (Vp is the velocity in the pipeline).

(19)

$C_d = 0.96$, $h_{L_{2\to3}} = \frac{10\,Vp^2}{2g}$

$h_{L_{1\to3}} = ?$



$A_1 = \frac{\pi(0.1)^2}{4} = 0.00785\ m^2$

$A_2 = \frac{\pi(0.04)^2}{4} = 0.00125\ m^2$

$H = R\left(\frac{\gamma_2}{\gamma_1} - 1\right) = 0.25\left(\frac{13.6}{1} - 1\right) = 3.15\ m$

$Q = \frac{A_1 A_2 \sqrt{2gH}}{\sqrt{A_1^2 - A_2^2}} = \frac{(0.00785)(0.00125)\sqrt{2 * 9.81 * 3.15}}{\sqrt{(0.00785)^2 - (0.00125)^2}}$

$= 0.01\ m^3/s$

Head loss through the meter = $h_{L_{1\to3}}$

$h_{L_{1\to3}} = h_{L_{1\to2}} + h_{L_{2\to3}}$

$h_{L_{1\to2}} = H(1 - C_d^2) = 3.15(1 - (0.96)^2) = 0.247$

$V_1 = \frac{Q}{A_1} = \frac{0.01}{0.00785} = 1.27\ m/s$

$h_{L_{2\to3}} = \frac{10V_1^2}{2g} = \frac{10(1.27)^2}{2 * 9.81} = 0.83$

$h_{L_{1\to3}} = 0.247 + 0.83 = \underline{\underline{1.07\ m}}$

(18) Calculate the flow rate of gasoline (S.G. = 0.85) through the pipeline using first the gauge readings and then the manometer reading.

1st Using Gauge

$$A_1 = \frac{\pi (0.3)^2}{4} = 0.07$$

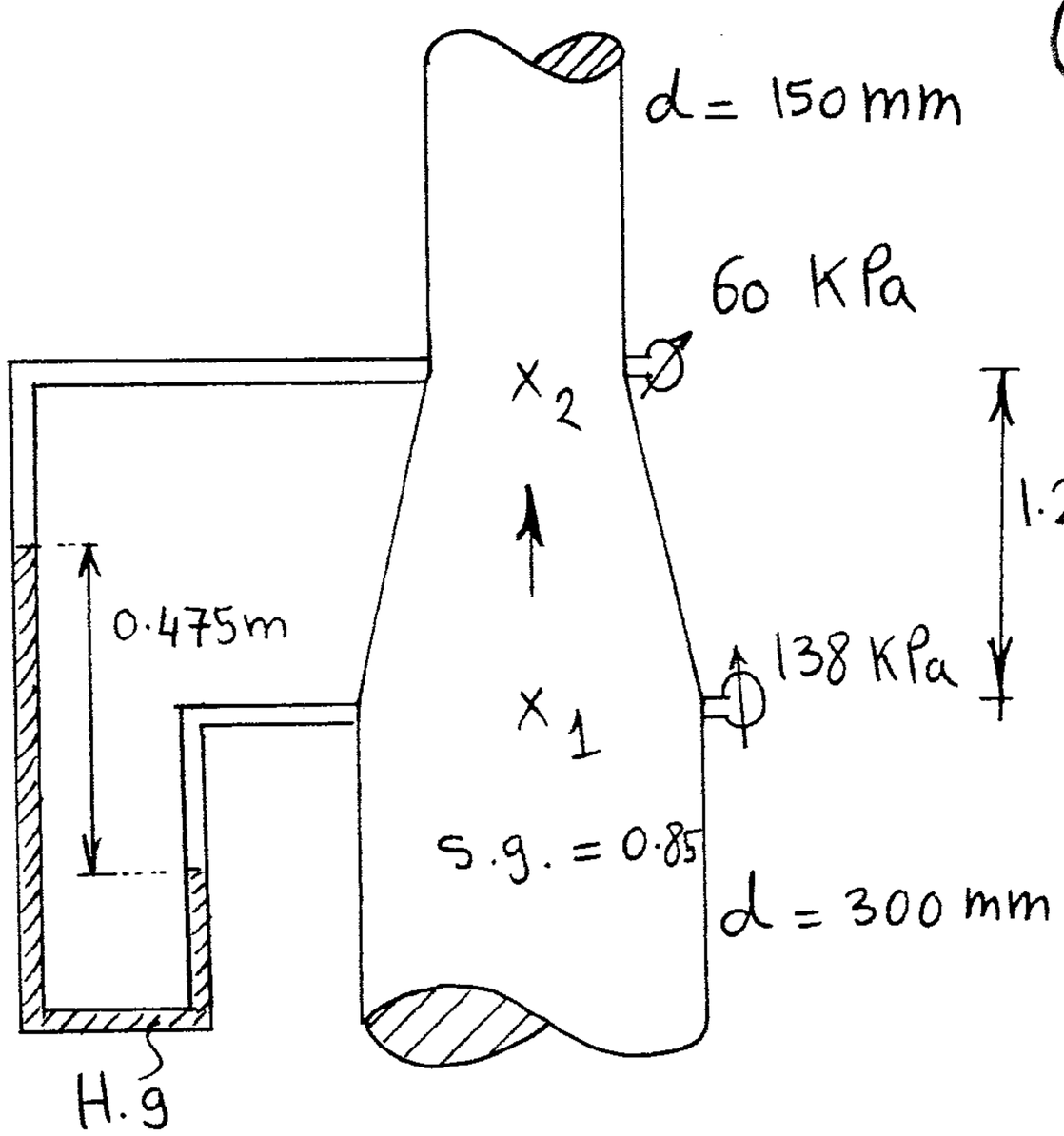
$$A_2 = \frac{\pi (0.15)^2}{4} = 0.017$$

$$H = \left(\frac{P_1}{\gamma} + Z_1\right) - \left(\frac{P_2}{\gamma} + Z_2\right)$$

$$= \frac{138 \times 10^3}{(0.85)9810} + 0 - \frac{60 \times 10^3}{(0.85)9810} - 1.2$$

$$= 8.15 \text{ m}$$

(20)



$$Q = \frac{A_1 A_2 \sqrt{2gH}}{\sqrt{A_1^2 - A_2^2}}$$

$$= \frac{(0.07)(0.017)\sqrt{2 * 9.81 * 8.15}}{\sqrt{(0.07)^2 - (0.017)^2}} = 0.23 \text{ m}^3/\text{s}$$

2nd Manometer Reading

$$H = R\left(\frac{\gamma_2}{\gamma_1} - 1\right)$$

$$= 0.475\left(\frac{13.6\,\gamma_w}{0.85\,\gamma_w} - 1\right) = 7.125 \text{ m}$$

$$Q = \frac{A_1 A_2 \sqrt{2gH}}{\sqrt{A_1^2 - A_2^2}} = \frac{0.07(0.017)\sqrt{2 * 9.81 * 7.125}}{\sqrt{(0.07)^2 - (0.017)^2}}$$

$$= 0.2 \text{ m}^3/\text{s}$$

Cavitation التكهف

When a liquid flows into a region where its pressure is reduced to vapour pressure, the liquid boils and bubbles are formed that collapse near a solid boundary causing explosion and the flow stops discharging. (21)

عندما يتدفق سائل فى منطقة ذات ضغط منخفض وكان هذا الضغط يساوى الVapour pressure فإن السائل يغلي ويبدأ فى التبخر ، ونتيجة لذلك تتكون فقاعات كثيرة من بخار السائل فيحملها السائل إلى منطقة ذات ضغط عالى حيث تنهار هذه الفقاعات.

إذا حدث انهيار للفقاعات بجانب الجدار فإن السائل تحت الضغط العالى يُسرع فى ملئ هذا الفراغ مسبباً نقر (Pitting) فى الجدار فتتكون كهوف.

هذا التتابع قد يسبب انهيار للجدار وبالتالى يقف السائل عن التدفق.

Vapour Pressure

It is the pressure at which the liquid begins to boil then vaporizes

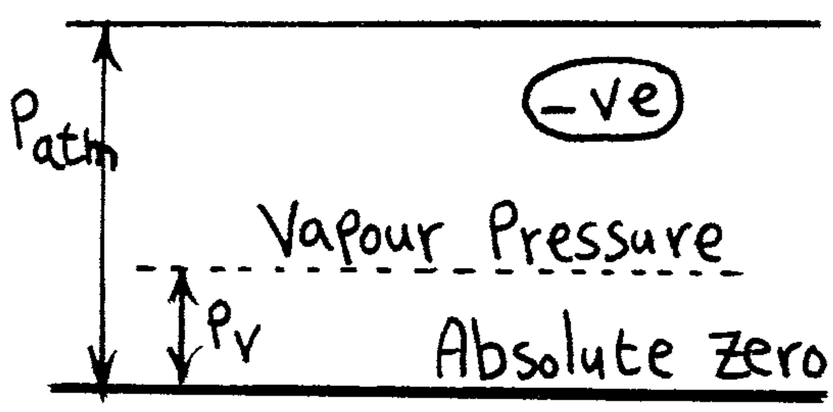
$P_{Vapour\ (abs)} = P_V$

$P_{Vapour\ (gauge)} = -(P_{atm} - P_V)$

e.g

$P_{Vapour\ (water)} = 2.3$ KPa (abs)

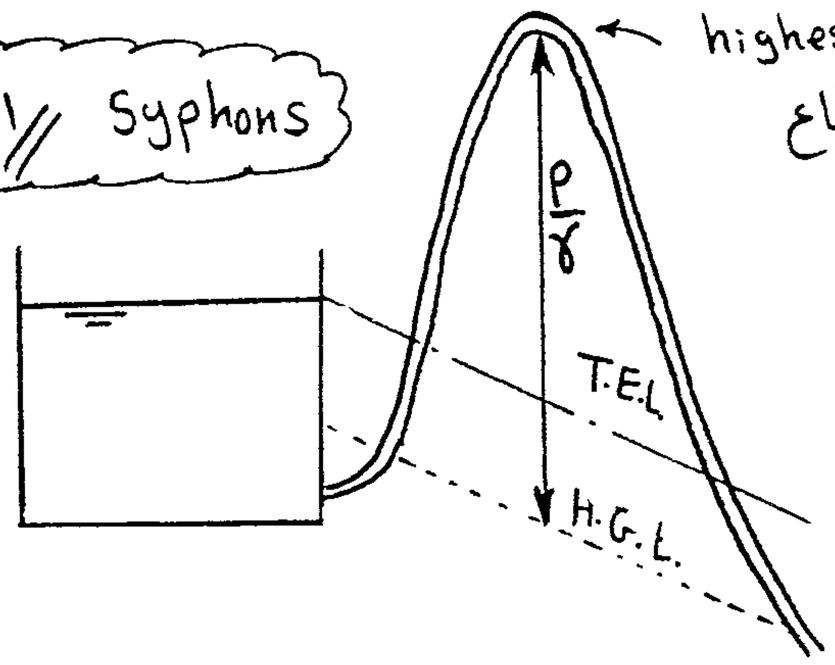
$= -(101.3 - 2.3) = -99$ KPa (gauge)

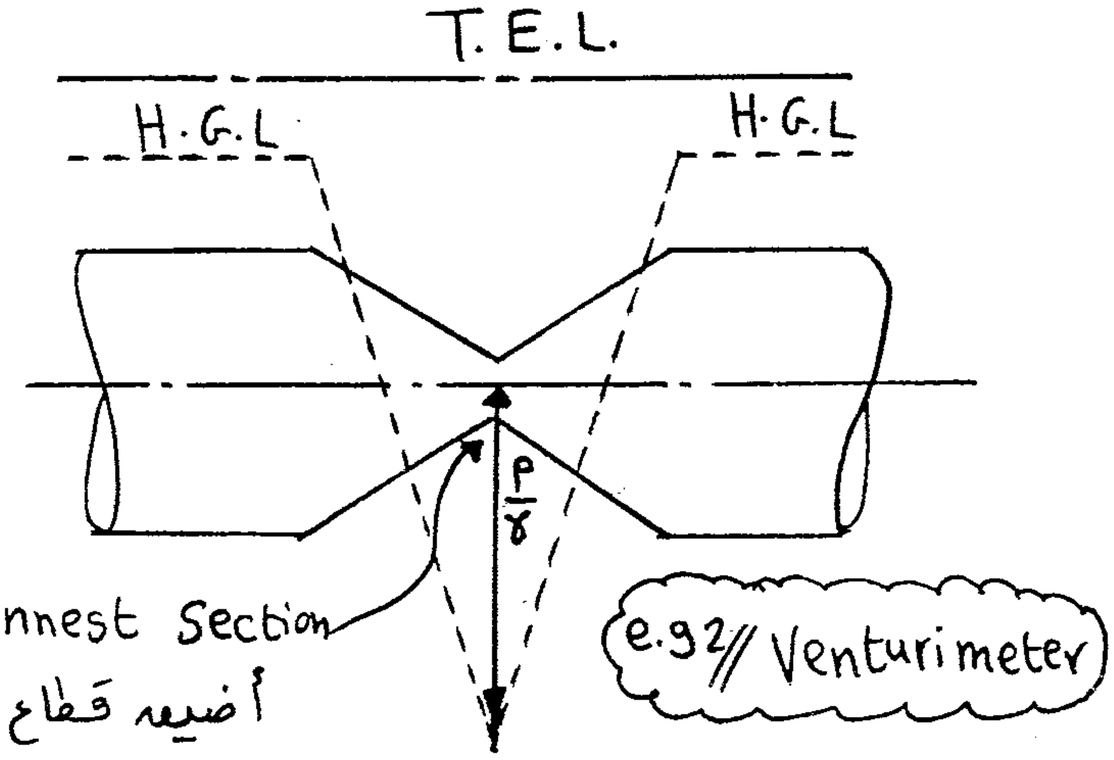


* If P_V not given $\Rightarrow$ $P_V = -8\gamma_w$ N/m^2 (Pa)

e.g 1// Syphons

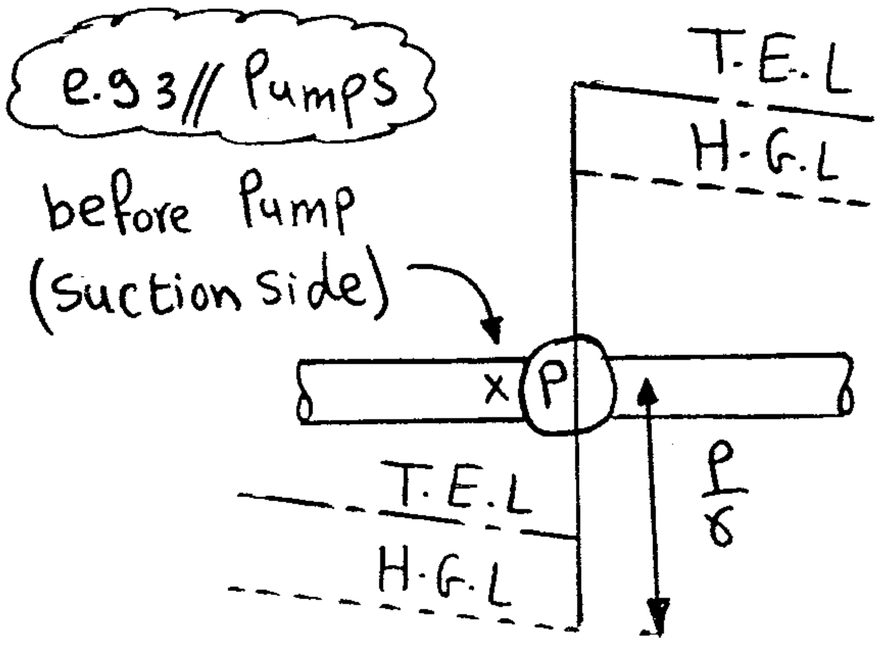
highest section
أعلى قطاع

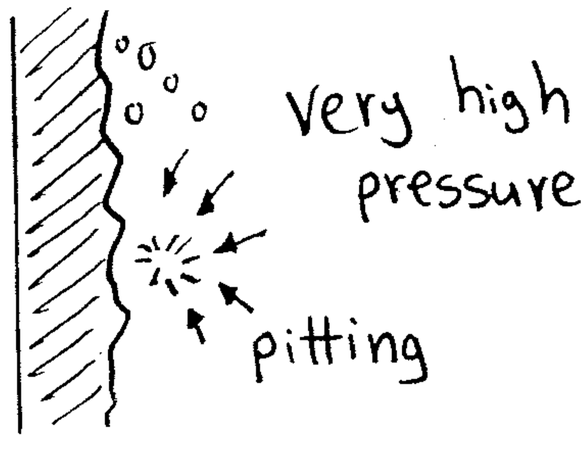




e.g 2// Venturimeter

e.g 3// Pumps



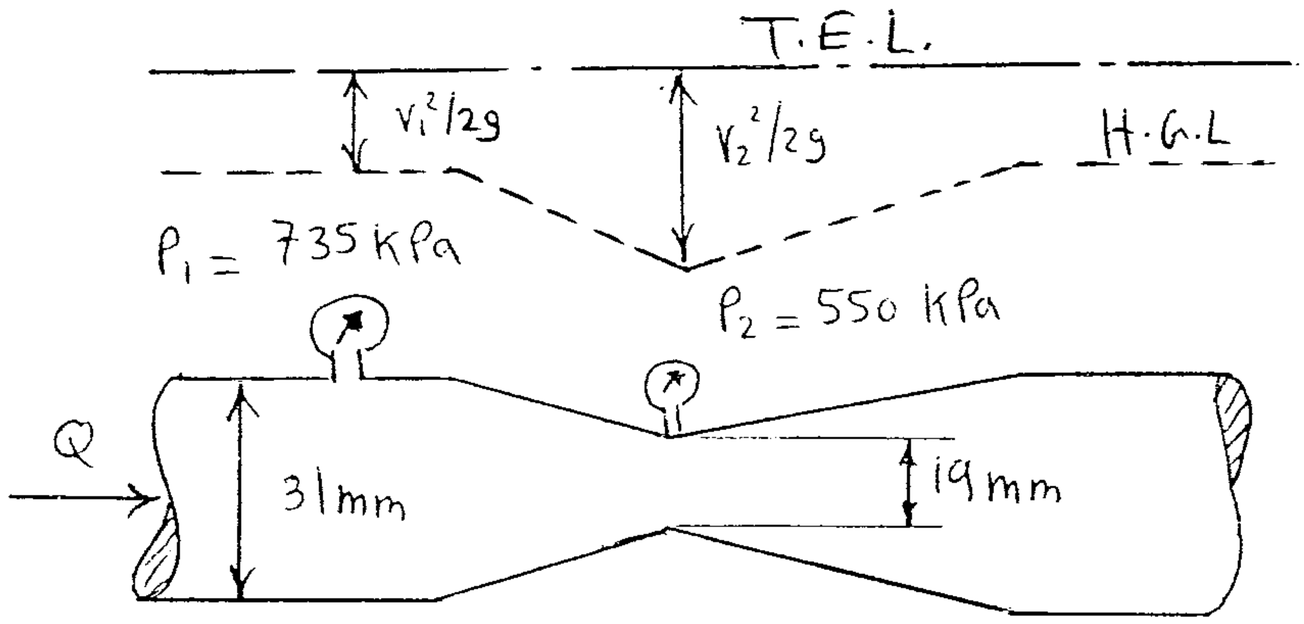


20

Determine the flowrate through the shown Venturi-meter, if ideal conditions exist. For what flowrate through the Venturi-meter, will cavitation begin if p_1 = 275 KPa gage, atmospheric pressure is 101 KPa (abs), and the vapour pressure is 5.2 KPa abs (22)

$\gamma_f = 9.1\ kN/m^3$

$Q = ?$



Sol

$$A_1 = \frac{\pi (0.031)^2}{4} = 7.55 * 10^{-4}\ m^2$$

$$A_2 = \frac{\pi (0.019)^2}{4} = 2.83 * 10^{-4}$$

$$H = \left(\frac{P_1}{\gamma} + z_1\right) - \left(\frac{P_2}{\gamma} + z_2\right)$$

$$= \frac{735 * 10^3}{9110} - \frac{550 * 10^3}{9110} = 20.31$$

$$Q = \frac{A_1 A_2 \sqrt{2gH}}{\sqrt{A_1^2 - A_2^2}} = \underline{\underline{6.11 * 10^{-3}\ m^3/s}}$$

If $P_1 = 275$ KPa, $P_{atm} = 101$ KPa abs, $P_v = 5.2$ KPa abs

Find Q at the begining of Cavitation

Sol

Cavitation will occur if P_2 reachs the Vapour pressure

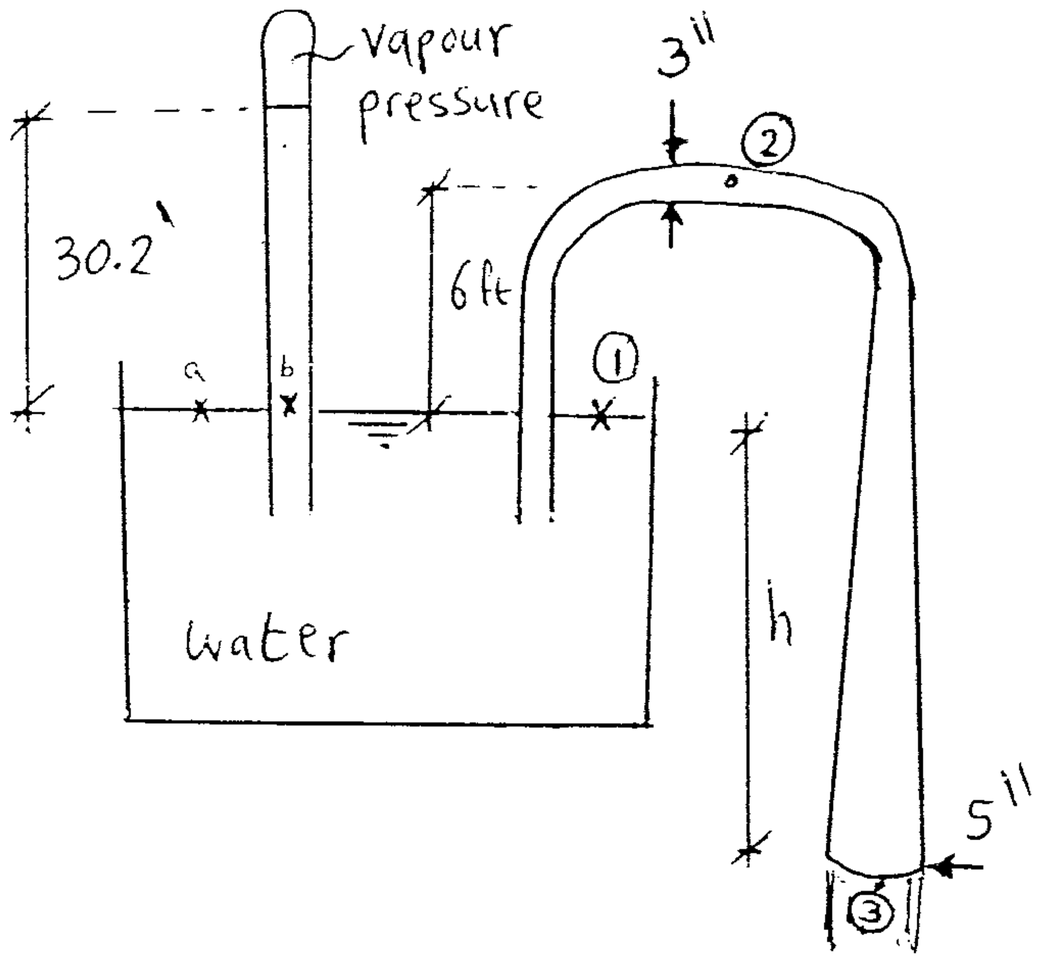
$$P_{v_{gage}} = P_{v_{abs}} - P_{atm} = (5.2 - 101) * 10^3 = -95.8 * 10^3\ Pa$$

$$H = \left(\frac{P_1}{\gamma} + z_1\right) - \left(\frac{P_2}{\gamma} + z_2\right) = \frac{275 * 10^3}{9110} - \frac{-95.8 * 10^3}{9110} = 40.7$$

$$Q = \frac{A_1 A_2 \sqrt{2gH}}{\sqrt{A_1^2 - A_2^2}} = \underline{\underline{8.64 * 10^{-3}\ m^3/s}}$$

21

Water is siphoned from the shown tank in figure. The water barometer indicates a reading of 30.20 ft. (local atmospheric pressure) (23)
Determine the maximum value of "h" allowed without cavitation occurring.
Hint: the pressure of the vapor in the closed end of the barometer equals the vapor pressure



$h_{max} = ?$ without Cavitation

Sol

$$P_{Vapour} = P_b - \gamma h$$

$$= 0 - 62.4(30.2)$$

$$= \underline{-1884.5 \ lb/ft^2}$$

Cavitation will occur if point ② reachs Vapour pressure

Apply Bernoulli eqn between ①, ② datum at 1

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

$$0 = \frac{-1884.5}{62.4} + \frac{V_2^2}{2(32.2)} + 6 \quad \ldots$$

$$\Rightarrow V_2 = \underline{39.47 \ ft/s}$$

$$\because d_3^2 V_3 = d_2^2 V_2$$

$$(5)^2 V_3 = (3)^2 (39.47) \quad \Rightarrow V_3 = \underline{14.21 \ ft/s}$$

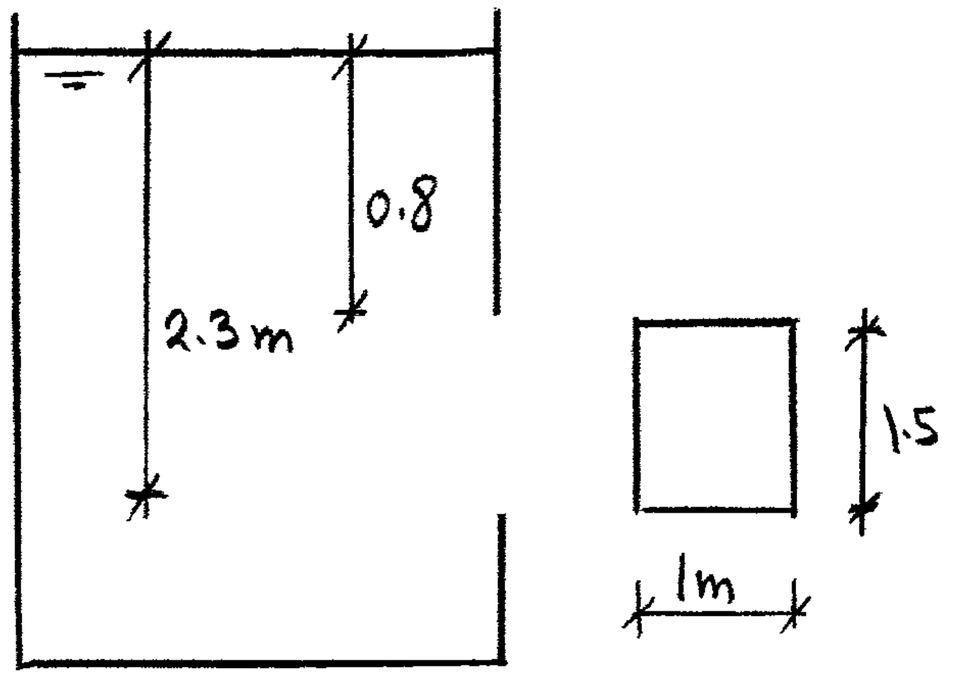
Apply Bernoulli eqn between ①, ③ datum at 3

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + z_3$$

$$h = \frac{(14.21)^2}{2(32.2)} = \underline{3.13}$$

16 Large orifice

$C_d = 0.6 \quad Q = ?$



$$Q = \frac{2}{3} C_d b \sqrt{2g} \left[H_2^{\frac{3}{2}} - H_1^{\frac{3}{2}} \right]$$

$$= \frac{2}{3}(0.6)(1)\sqrt{2*9.81}\left[2.3^{3/2} - 0.8^{3/2}\right]$$

$$= \underline{\underline{4.91 \ m^3/s}}$$

If treated as a Small orifice

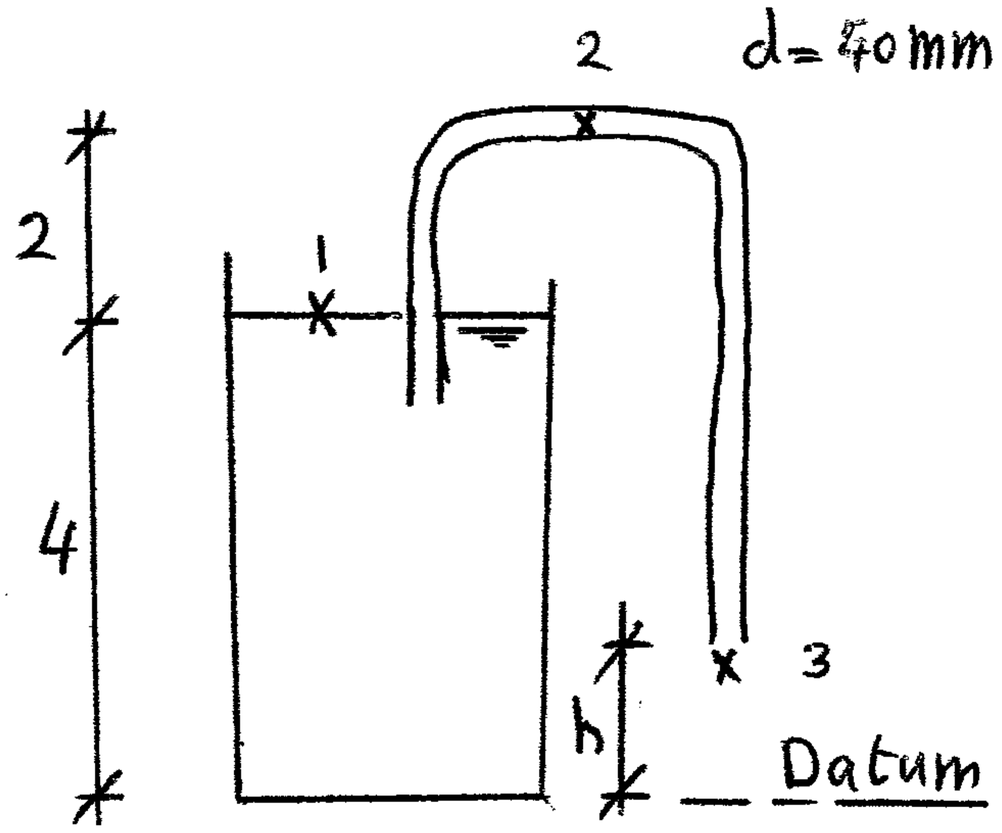
$$Q = a\sqrt{2gh} = 1 * 1.5 \sqrt{2 * 9.81 * \left(\frac{0.8+2.3}{2}\right)} = 8.27 \ m^3/s$$

$$\% \text{ error} = \frac{8.27 - 4.91}{4.91} * 100 = \underline{\underline{68.5\ \%}}$$

---

22 The tube will collapse

if $P_{outside} - P_{inside} > 40$ KPa



$\because$ Outside pressure $= P_{atm} = 0$

$\because$ Minimum pressure inside $= P_2$

$\therefore P_{2_{min}} = -40 * 10^3$ Pa

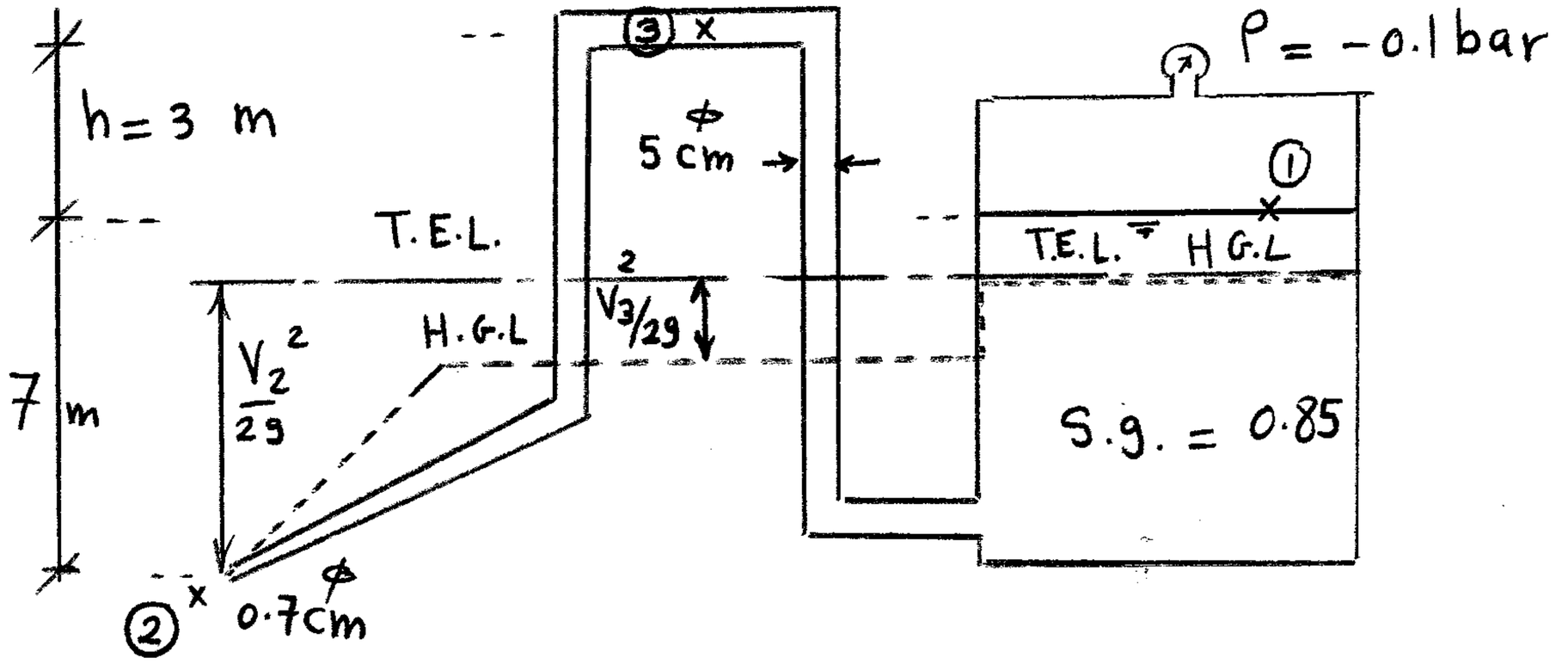
Bernoulli eqn bet ②, ③

$$\frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + z_3$$

$$\frac{-40*10^3}{9810} + \frac{V^2}{2g} + 6 = 0 + \frac{V^2}{2g} + h$$

$\therefore h = \underline{\underline{4.92 \ m}}$

23



b) discharge in ℓ/s

Bernoulli eqn between ①, ② Datum at ②

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{-0.1 * 10^5}{0.85 * 9810} + 0 + 7 = 0 + \frac{V_2^2}{2(9.81)} + 0 \quad \Rightarrow V_2 = 10.66 \text{ m/s}$$

$$Q = A_2 V_2 = \frac{\pi (0.7 * 10^{-2})^2}{4} * 10.66 = 4.1 * 10^{-4} \frac{m^3}{s} = \underline{0.41 \text{ lit/s}}$$

c) Pressure P_3 at the top of the Pipe in Pascal

$$V_3 d_3^2 = V_2 d_2^2$$

$$V_3 (5)^2 = 10.66 (0.7)^2 \quad \Rightarrow V_3 = 0.2 \text{ m/s}$$

Bernoulli eqn between ①, ③ Datum ①

$$\frac{-0.1 * 10^5}{0.85 * 9810} + 0 + 0 = \frac{P_3}{0.85 (9810)} + \frac{(0.2)^2}{2 (9.81)} + 3$$

$\therefore P_3 = -35\,032$ Pa

a) Starting pressure in cm Hg

$$h_{Hg} = \frac{P_3}{\gamma_{Hg}} = \frac{35\,032}{13.6 * 9810} = 0.262 \text{ m} = \underline{26.2 \text{ cm Hg}}$$

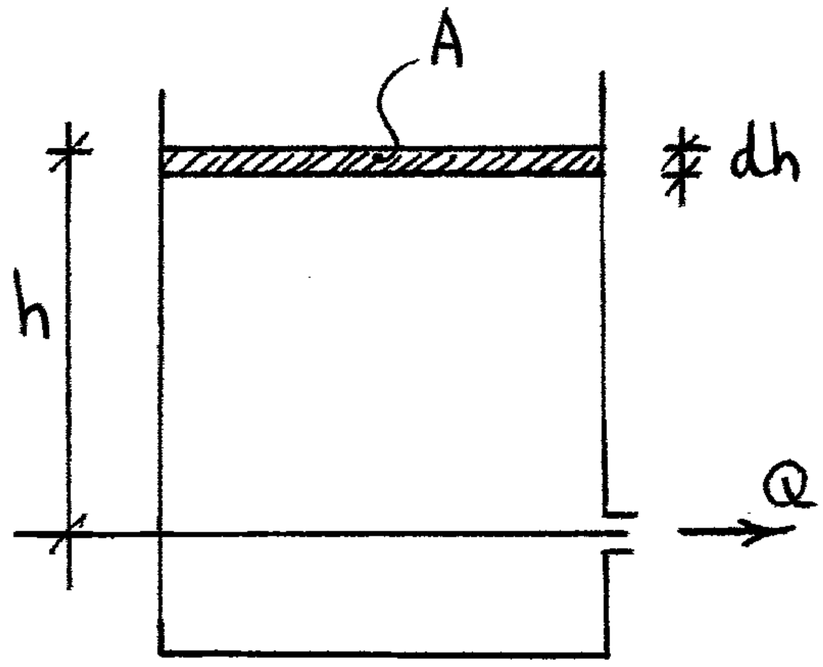
10 Discharge under variable head

after time $= dt$

Volume out $= Q\,dt$

Volume $= A\,dh$

$$\boxed{Q\,dt = -A\,dh}$$



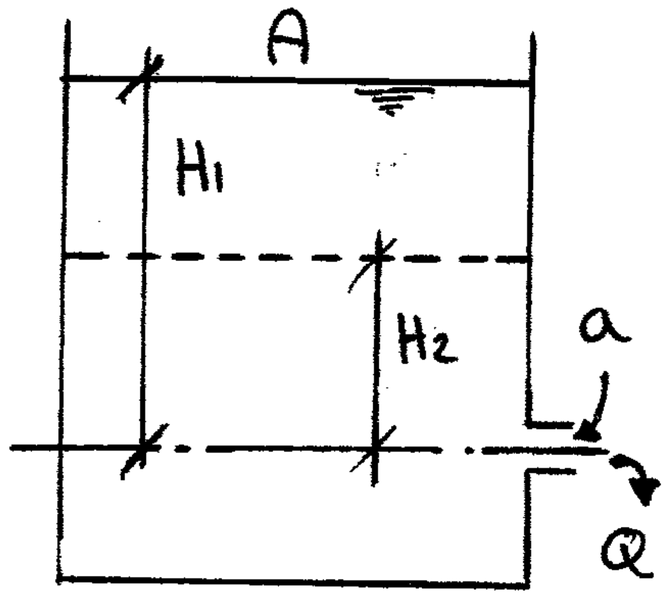
Time required to empty the tank

$$Q\,dt = -A\,dh$$

$$C_d a\sqrt{2gh}\,dt = -A\,dh$$

$$\int_0^T dt = \frac{-A}{C_d a\sqrt{2g}} \int_{H_1}^{H_2} h^{-\frac{1}{2}}\,dh$$

$$T = \frac{-2A}{C_d a\sqrt{2g}} \left(H_2^{\frac{1}{2}} - H_1^{\frac{1}{2}} \right)$$



$$\boxed{T = \frac{2A}{C_d a\sqrt{2g}} \left(\sqrt{H_1} - \sqrt{H_2} \right)}$$

H_1 = initial head in tank

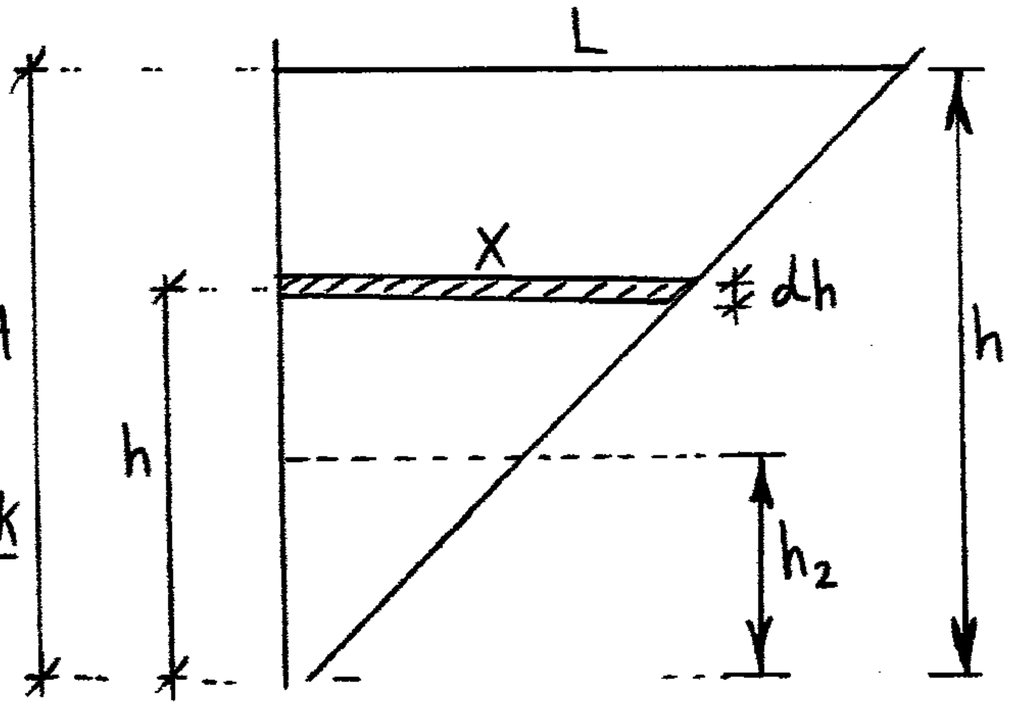
H_2 = final head in tank

If it is required to empty the tank $H_2 = 0$

Triangular Tank

$$\frac{X}{h} = \frac{L}{H}$$

$$X = \frac{L}{H} h$$



Time required to empty the tank

$$Q\,dt = -A\,dh$$

$$C_d a\sqrt{2gh}\,dt = -BX\,dh$$

$$\int_0^T dt = \int \frac{-B\left(\frac{L}{H}\right) h\,dh}{C_d a\sqrt{2g}\,h^{1/2}}$$

$$T = \frac{-BL\int_{h_1}^{h_2} h^{1/2}\,dh}{H C_d a\sqrt{2g}}$$

$$\frac{-A\,\frac{2}{3}\left|h^{\frac{3}{2}}\right|_{h_1}^{h_2}}{H C_d a\sqrt{2g}}$$

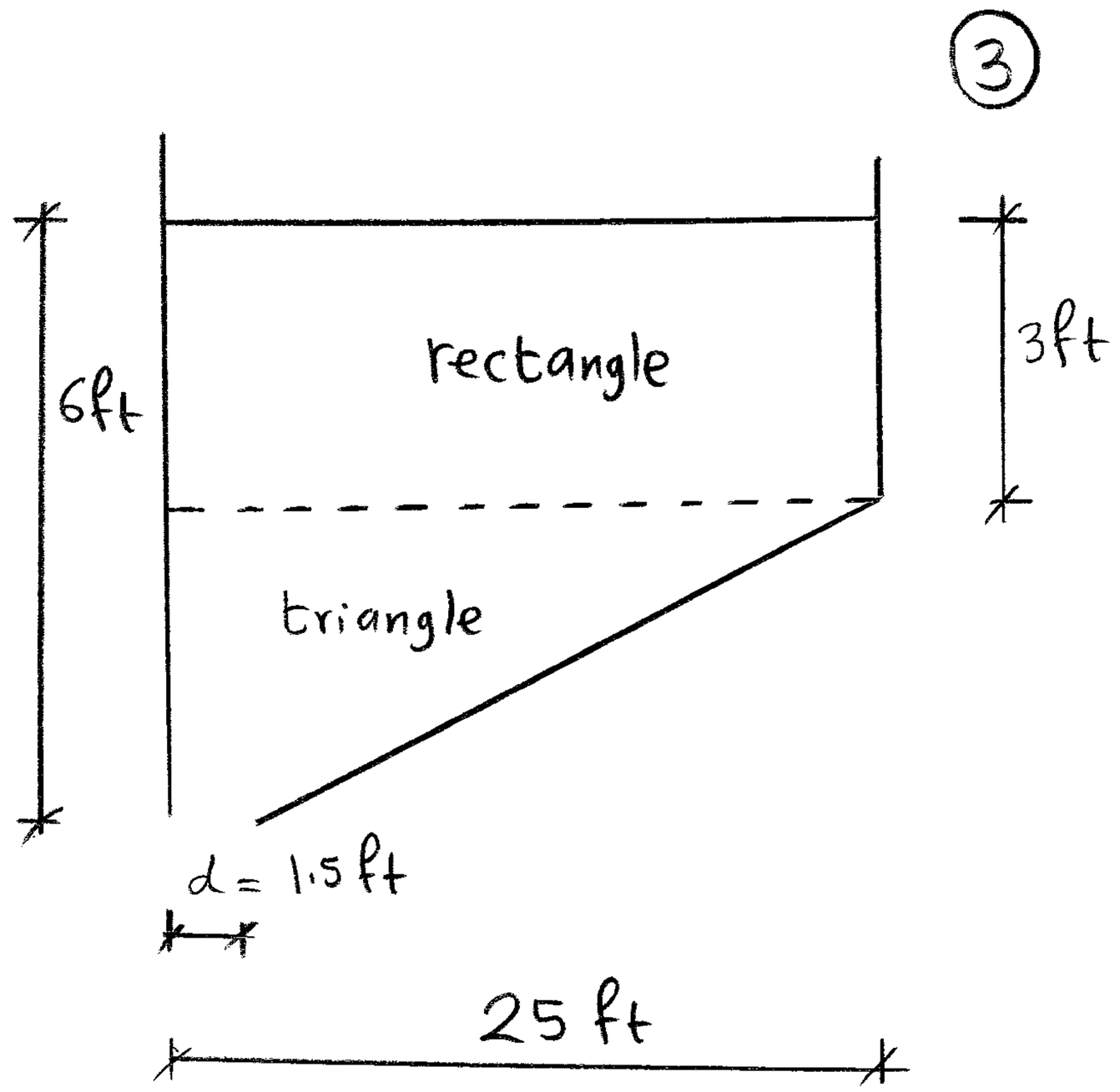
$$\boxed{T = \frac{2A\left(h_1^{\frac{3}{2}} - h_2^{\frac{3}{2}}\right)}{3H C_d a\sqrt{2g}}}$$

1

Swimming pool

$B = 10ft$ $d = 1.5 ft$

$C_d = 0.65$ $T_{empty} = ?$



Sol

Rectangular Part

$$T_1 = \frac{2A}{C_d a \sqrt{2g}} \left(\sqrt{H_1} - \sqrt{H_2} \right)$$

$$= \frac{2 (25 * 10) \left[\sqrt{6} - \sqrt{3} \right]}{0.65 \frac{\pi (1.5)^2}{4} * \sqrt{2 * 32.2}} = \underline{38.92 \text{ sec}}$$

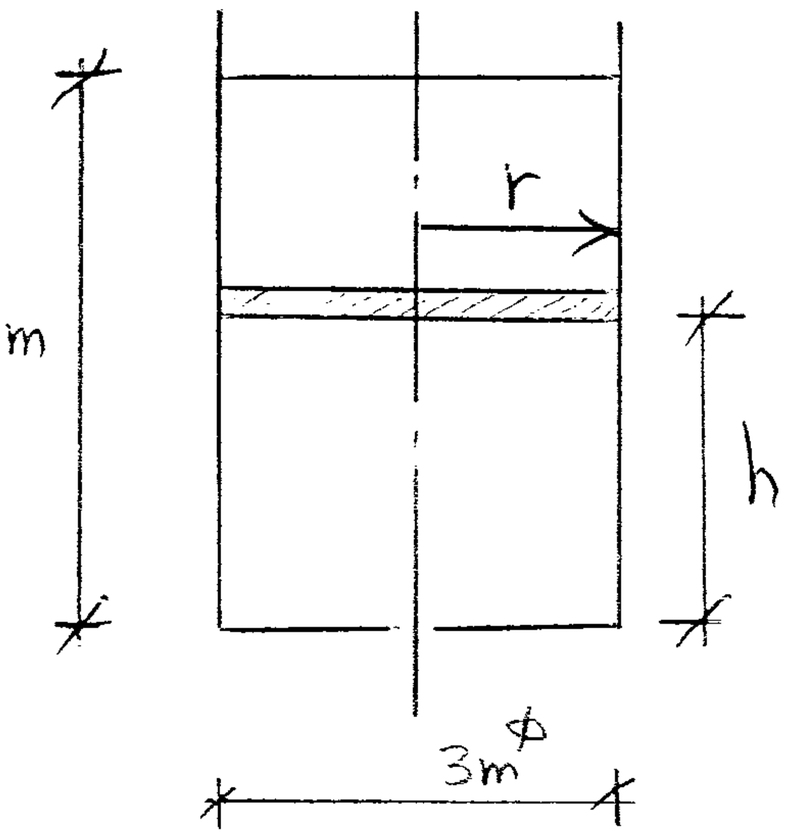
Triangular Part

$$T_2 = \frac{2 A \left(h_1^{\frac{3}{2}} - h_2^{\frac{3}{2}} \right)}{3 H C_d a \sqrt{2g}}$$

$$= \frac{2 (25 * 10) \left(3^{3/2} - 0 \right)}{3 (3) 0.65 \pi \frac{(1.5)^2}{4} \sqrt{2 * 32.2}} = 31.32$$

$$T = T_1 + T_2 = \underline{70.24 \text{ secs}}$$

Calculate the time required to empty a cylinder tank 3 m diameter and 4 m height through a circular orifice 5 cm diameter and $C_d = 0.62$ placed at the bottom of the tank. The tank was originally full. What will be effect of connecting a vertical pipe of length 0.5 m to the orifice at the bottom on the time of emptying the tank? (4)



$d = 5\,cm \qquad C_d = 0.62$

$T_{empty} = ?$

Sol

$$Q\,dt = -A\,dh$$

$$C_d\, a \sqrt{2gh}\, dt = -\pi r^2 dh$$

$$\int_0^T dt = \frac{-\pi r^2}{C_d\, a \sqrt{2g}} \int_{h=4}^{h=0} h^{-\frac{1}{2}}\, dh$$

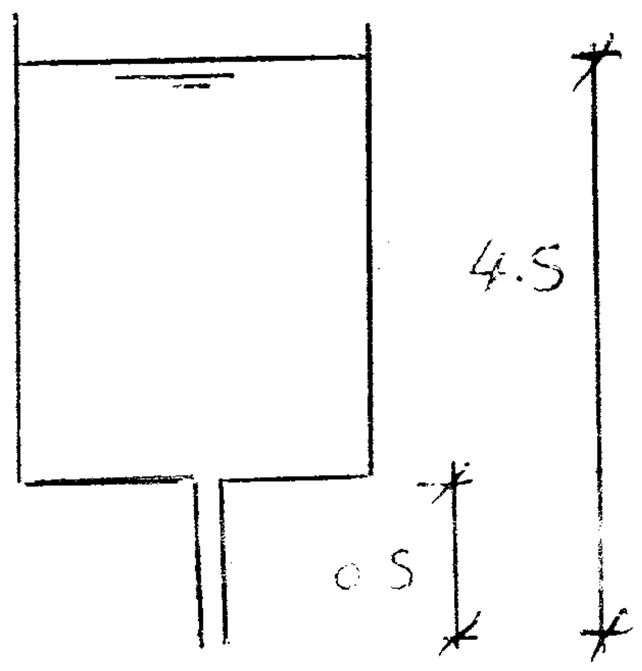
$$T = \frac{2\pi r^2}{C_d\, a \sqrt{2g}} \left| h^{+\frac{1}{2}} \right|_0^4$$

$$= \frac{2\pi (1.5)^2 \left(4^{\frac{1}{2}} - 0^{\frac{1}{2}}\right)}{0.62 \;\frac{\pi (0.05)^2}{4}\; \sqrt{2 * 9.81}} = \underline{\underline{243 \text{ sec}}}$$

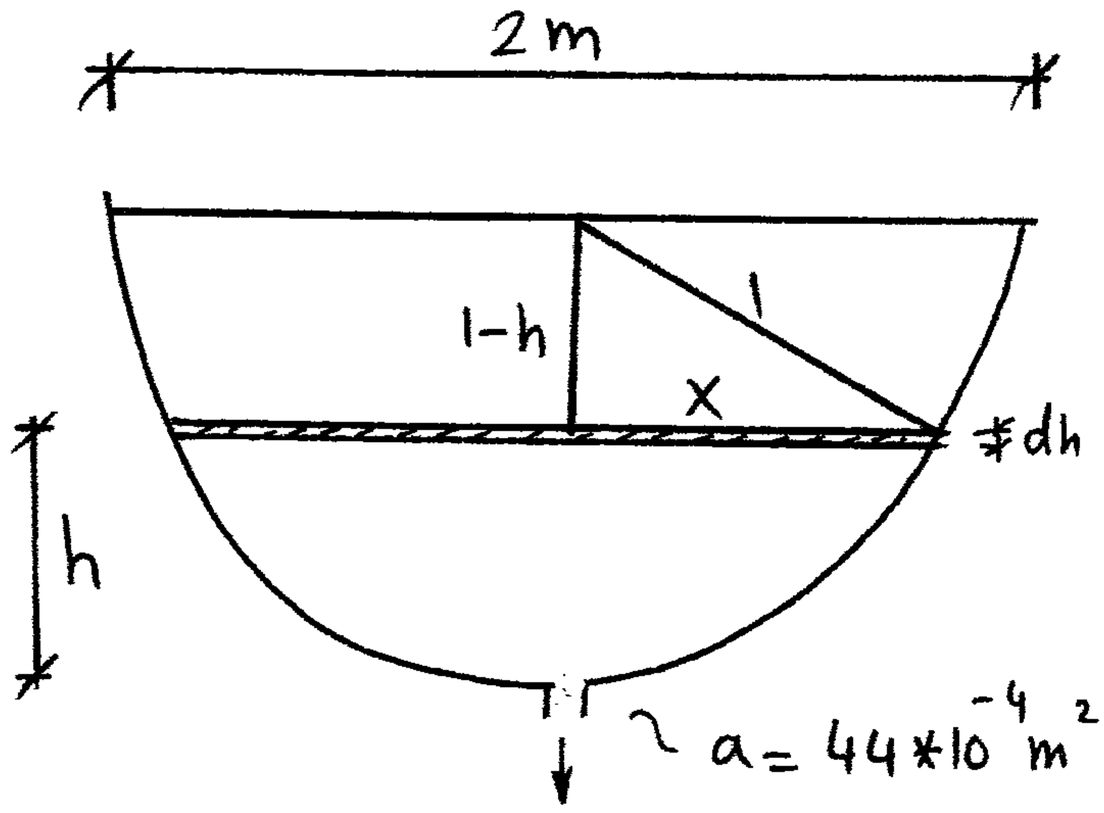
IF a pipe of length 0.5m is installed

$$T = \frac{2\pi (1.5)^2 \left[(4.5)^{1/2} - (0.5)^{1/2}\right]}{0.62 \;\frac{\pi (0.05)^2}{4}\; \sqrt{2 * 9.81}}$$

$$= \underline{\underline{3707 \text{ sec}}}$$



Calculate the time required emptying a hemi-spherical vessel full of water with an orifice at the bottom. The diameter line of the vessel is 2 m. Take the area of the orifice is 44 cm^2 and C_d=0.60.



$C_d = 0.6$

$Q\,dt = -A\,dh$

$A = \pi x^2$

$\therefore x^2 + (1-h)^2 = 1$

$x^2 + 1 - 2h + h^2 = 1$

$x^2 = 2h - h^2 \quad \rightarrow ①$

$Q\,dt = -A\,dh$

$C_d\, a\sqrt{2gh}\, dt = -\pi x^2\, dh$

$$\int_0^T dt = \frac{-\pi}{C_d a\sqrt{2g}} \int_{h=1}^{h=0} \left(\frac{2h - h^2}{h^{1/2}}\right) dh$$

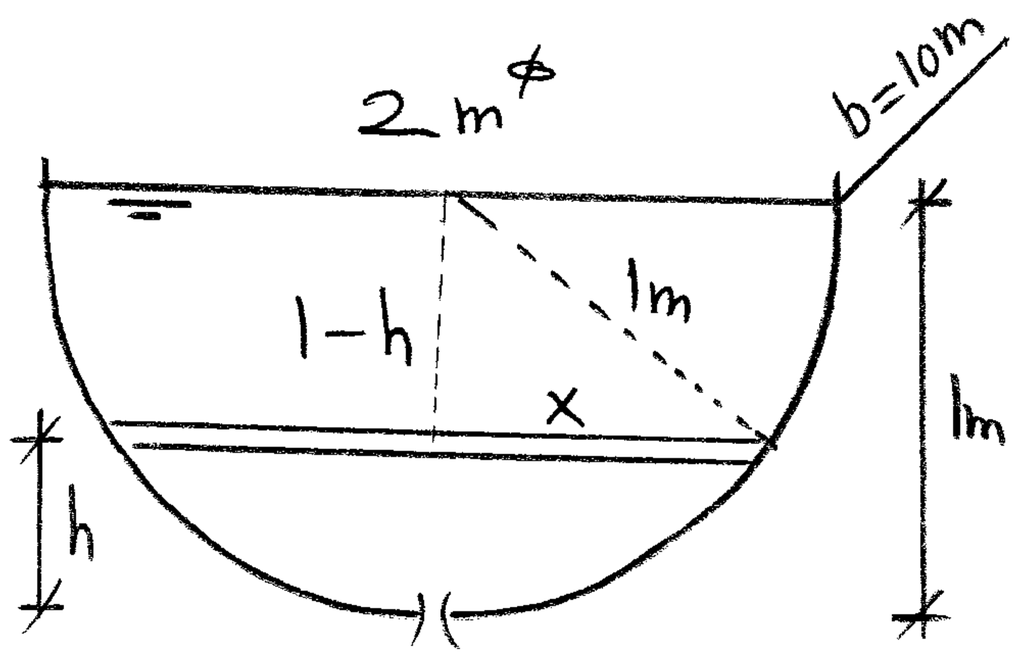
$$\int_1^0 \left(2h^{1/2} - h^{3/2}\right) dh$$

$$T = \frac{-\pi}{C_d\, a\sqrt{2g}} \left| \frac{2}{3} * 2h^{3/2} - \frac{2}{5} h^{5/2} \right|_1^0$$

$$= \frac{-\pi}{0.6\,(44*10^{-4})\sqrt{2*981}} \left| 0 - \left(\frac{4}{3} - \frac{2}{5}\right) \right| = \underline{\underline{250.7 \text{ sec}}}$$

(6)

An orifice is fitted at the bottom of a boiler tank drum (cylinder) for the purpose of emptying it. The drum is horizontal and half full of water. It is 10 m long and 2 m in diameter. Find the time required to empty the boiler, if the diameter of the orifice is 50 mm. (Take $C_d = 0.8$).



$b = 10\ m \quad T = ?$

$d_{orifice} = 50mm \qquad C_d = 0.8$

Sol//

$$A = 2bx$$

$$\because x^2 + (1-h)^2 = 1$$

$$x^2 + 1 - 2h + h^2 = 1$$

$$x^2 = 2h - h^2$$

$$\therefore A = 2b\sqrt{2h - h^2}$$

$$Q\,dt = -A\,dh$$

$$C_d\, a\sqrt{2gh}\,dt = -2b\sqrt{2h - h^2}\,dh$$

$$\int_0^T dt = \frac{-2b}{C_d\, a\sqrt{2g}} \int_1^0 \sqrt{2 - h}\,dh$$

$$T = \frac{-2(10)}{0.8\ \frac{\pi(0.05)^2}{4}\sqrt{2 * 9.81}} \left(-\frac{2}{3}\right) \Big| (2-h)^{\frac{3}{2}} \Big|_1^0$$

$$= 1916.32\ \left[(2-0)^{3/2} - (2-1)^{\frac{3}{2}}\right]$$

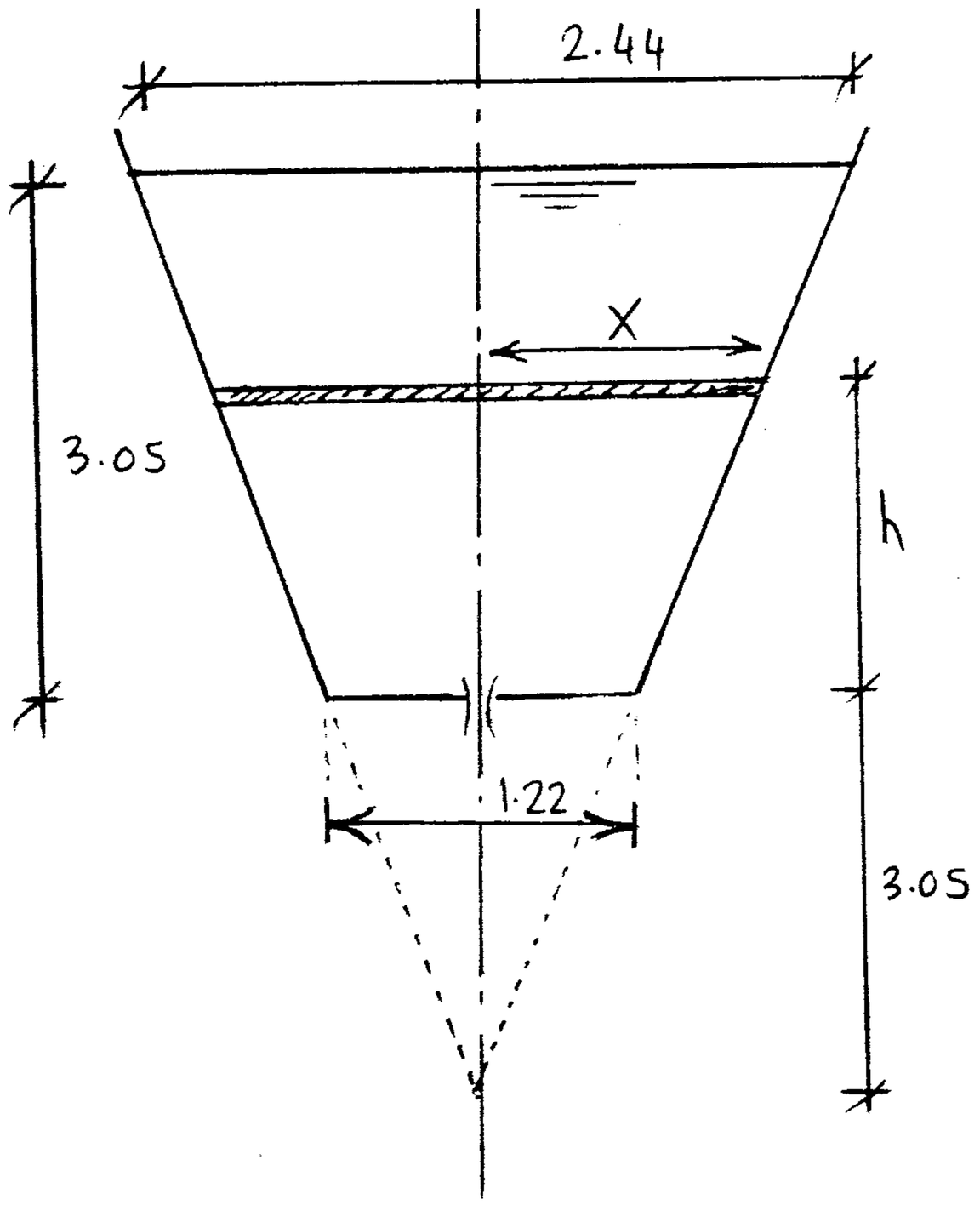
$$= \underline{3504\ \text{Sec}}$$

(7)

The shown tank has the form of a frustum of a cone, with 2.44 m diameter at the top and 1.22 m in diameter at the bottom. The full depth is 3.05 meters. The bottom has an orifice in the center with a coefficient of discharge 0.60

WHAT is the diameter of the orifice that would empty the full tank in 6 minutes.

Mid term Nov 2000



$C_d = 0.6$

$t = 6\ min = 6*60 = 360\ sec$

$$Q\,dt = -A\,dh$$

$$C_d\, a\sqrt{2gh}\,dt = -\pi x^2\,dh$$

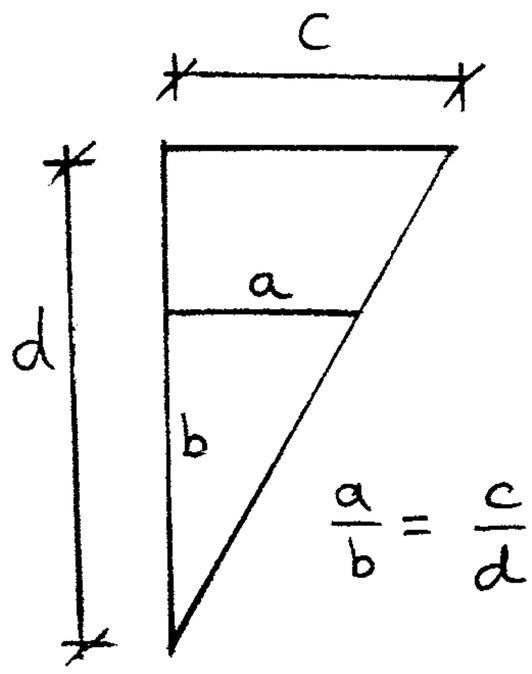
$$\frac{X}{(3.05+h)} = \frac{1.22}{6.1}$$

$$X = \frac{3.05+h}{5}$$

$$C_d\, a\sqrt{2gh}\,dt = -\frac{\pi}{25}(3.05+h)^2\,dh$$

$$0.6\,a\sqrt{2g}\int_0^{360} dt = \frac{-\pi}{25}\int_{3.05}^{0}\left(\frac{h^2+6.1h+9.3}{h^{1/2}}\right)dh$$

$$0.6\,a\sqrt{2*9.81}\,(360) = \frac{-\pi}{25}\int_{3.05}^{0}\left(h^{3/2}+6.1h^{1/2}+9.3h^{-1/2}\right)dh$$



$$\frac{a}{b} = \frac{c}{d}$$

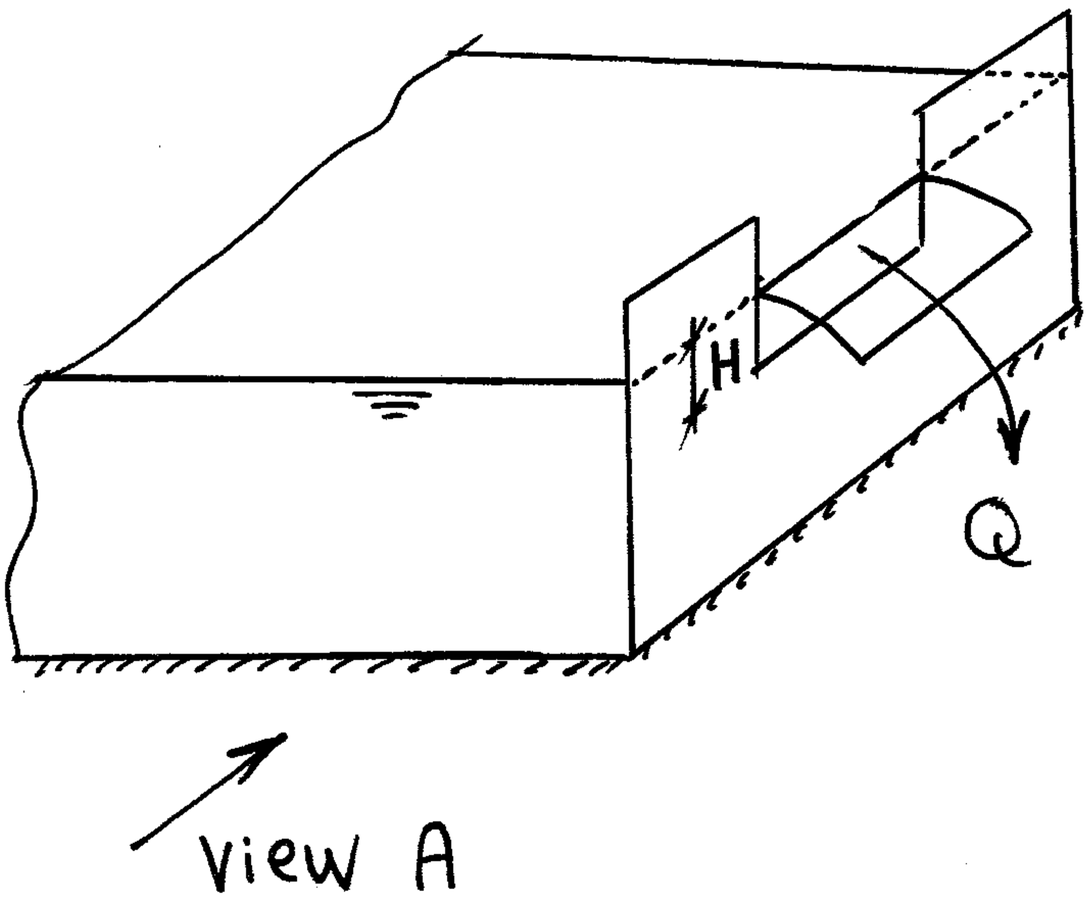
$$a = \frac{1}{7613}\left|\frac{2}{5}h^{5/2}+\frac{2}{3}6.1h^{3/2}+\frac{2}{1}9.3h^{1/2}\right|_0^{3.05}$$

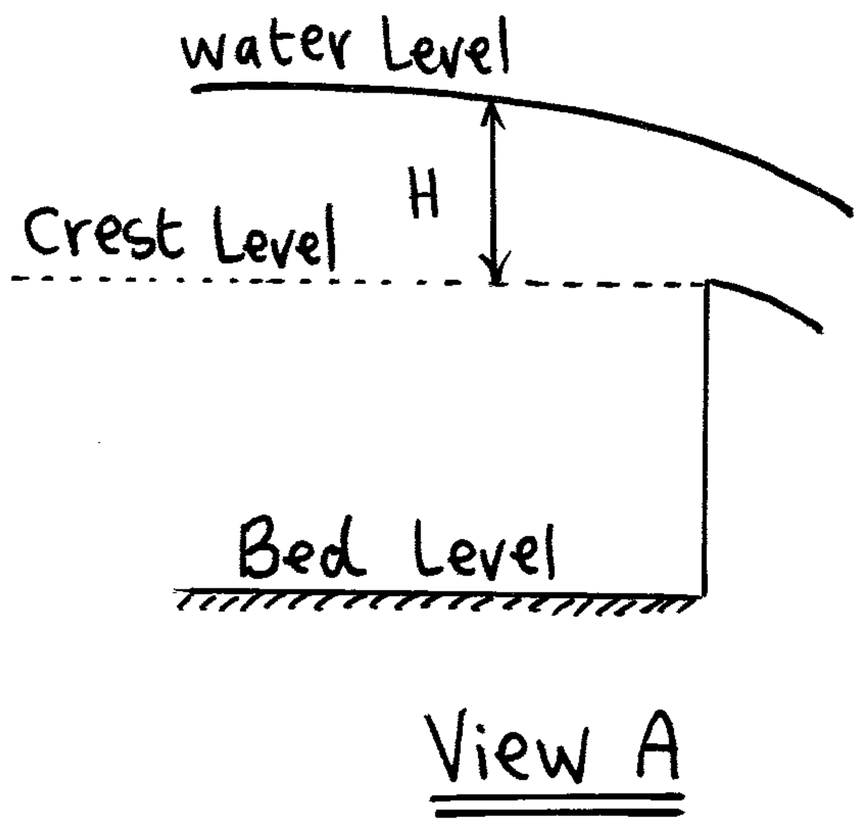
$$a = 0.008\ m^2$$

$$\frac{\pi d^2}{4} = 0.008 \Rightarrow \underline{\underline{d = 0.1\ m}}$$

Flow Over Notches

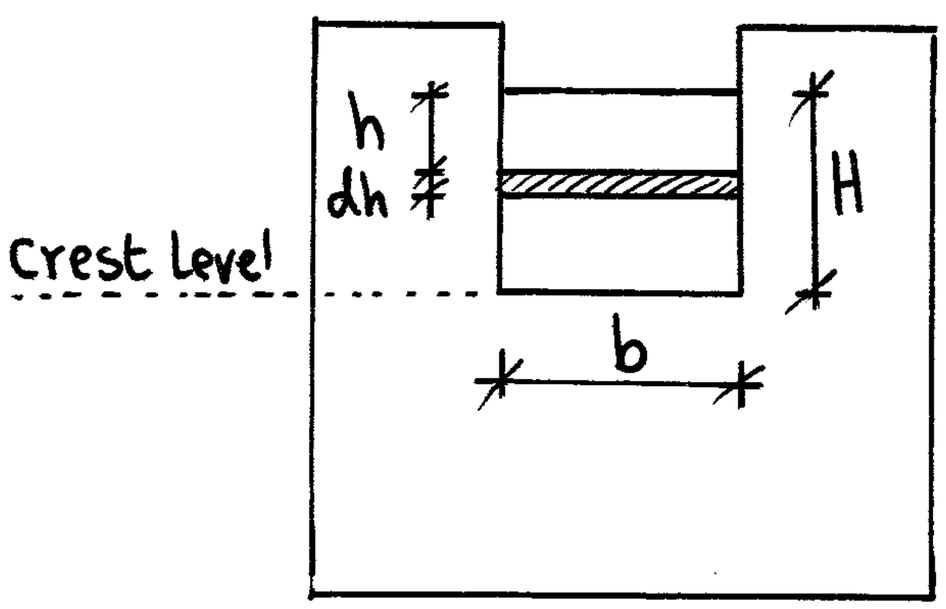
Rectangular notch





View A

View B



View B

From Continuity equation

$$dQ = V\,dA$$

where; $V = \sqrt{2gh}$

$$dA = b\,dh$$

$$dQ = \sqrt{2gh}\; b\,dh$$

$$Q = b\sqrt{2g}\int_0^{H_p} h^{\frac{1}{2}}\,dh$$

$$Q = \frac{2}{3}\, b\sqrt{2g}\; H^{3/2}$$

$$\boxed{Q_{act} = \frac{2}{3}\, C_d\, b\sqrt{2g}\; H^{3/2}}$$

Triangular Notch (V-Notch)

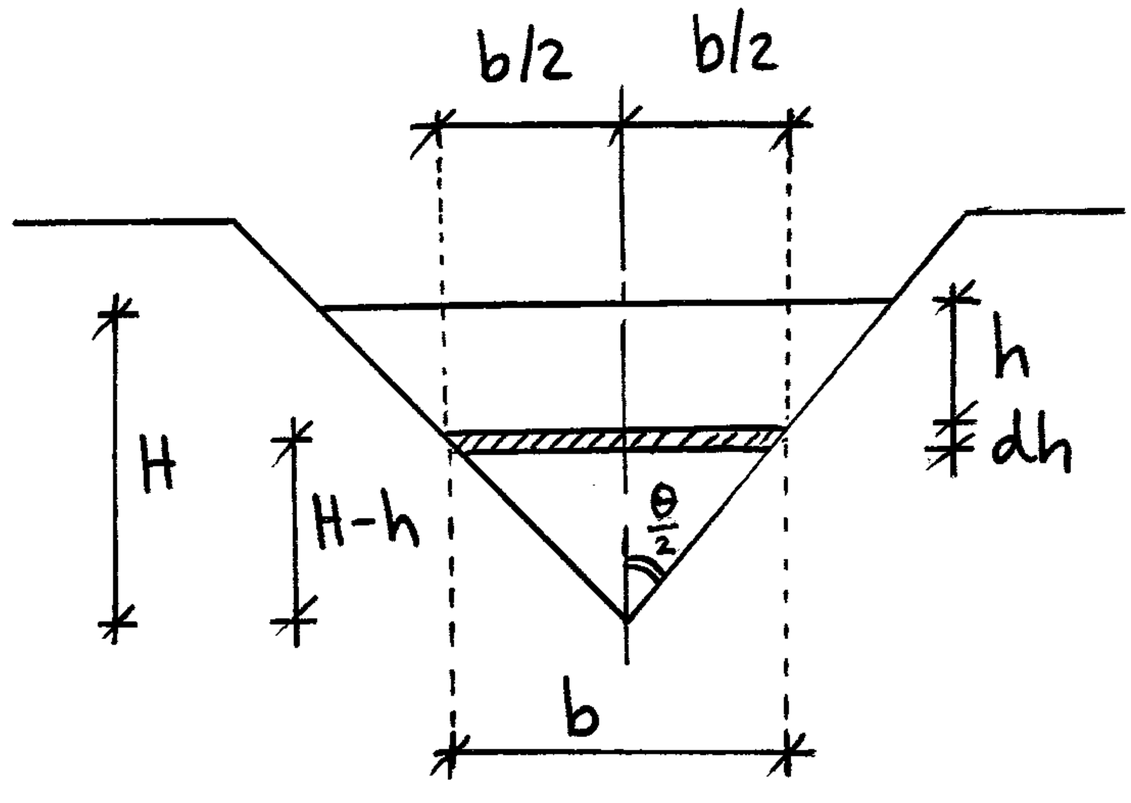
from Continuity equation

$$dQ = V\,dA$$

where; $V = \sqrt{2gh}$

$$dA = b\,dh$$

$$b = 2(H-h)\tan\frac{\theta}{2}$$



$$\therefore dQ = \sqrt{2gh}\left[2(H-h)\tan\frac{\theta}{2}\right]dh$$

$$Q = 2\sqrt{2g}\tan\frac{\theta}{2}\int_0^H (H-h)\,h^{1/2}\,dh$$

$$\int_0^H \left(Hh^{1/2} - h^{3/2}\right)dh$$

$$\left|\frac{2}{3}Hh^{3/2} - \frac{2}{5}h^{5/2}\right|_0^H$$

$$Q = 2\sqrt{2g}\tan\frac{\theta}{2}\left(\frac{2}{3}H^{5/2} - \frac{2}{5}H^{5/2}\right)$$

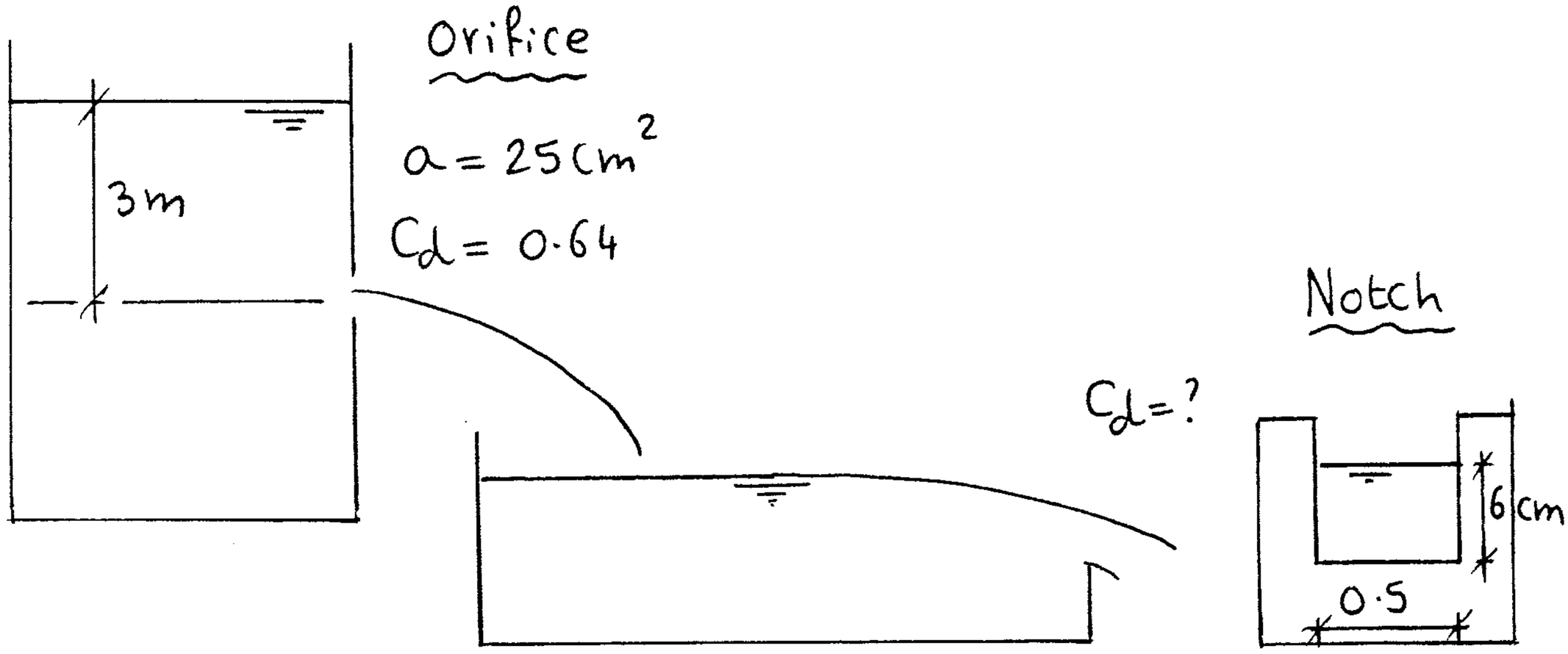
$$Q = \frac{8}{15}\sqrt{2g}\tan\frac{\theta}{2}\,H^{5/2}$$

$$\boxed{Q_{act} = \frac{8}{15}C_d\sqrt{2g}\tan\frac{\theta}{2}\,H^{5/2}}$$

at $\theta = 90^\circ \Rightarrow \tan\frac{90}{2} = 1$

$$Q_{act} = \frac{8}{15}C_d\sqrt{2g}\,H^{5/2}$$

2 Water in tank discharges through an orifice of 25 cm^2 cross-sectional area and coefficient of discharge is 0.64. The axis of the orifice is 3m below the free surface of water in tank. The water, which has passed through the orifice, flows over a rectangular notch of breadth 0.5 m. The head over the notch is maintained at 6 cm. Find the coefficient of discharge for the notch. (10)



Orifice

$$Q_{act} = C_d a \sqrt{2gh}$$

$$= 0.64 \left(25 \times 10^{-4}\right) \sqrt{2 * 9.81 * 3} = 0.0122 \ m^3/s$$

Notch

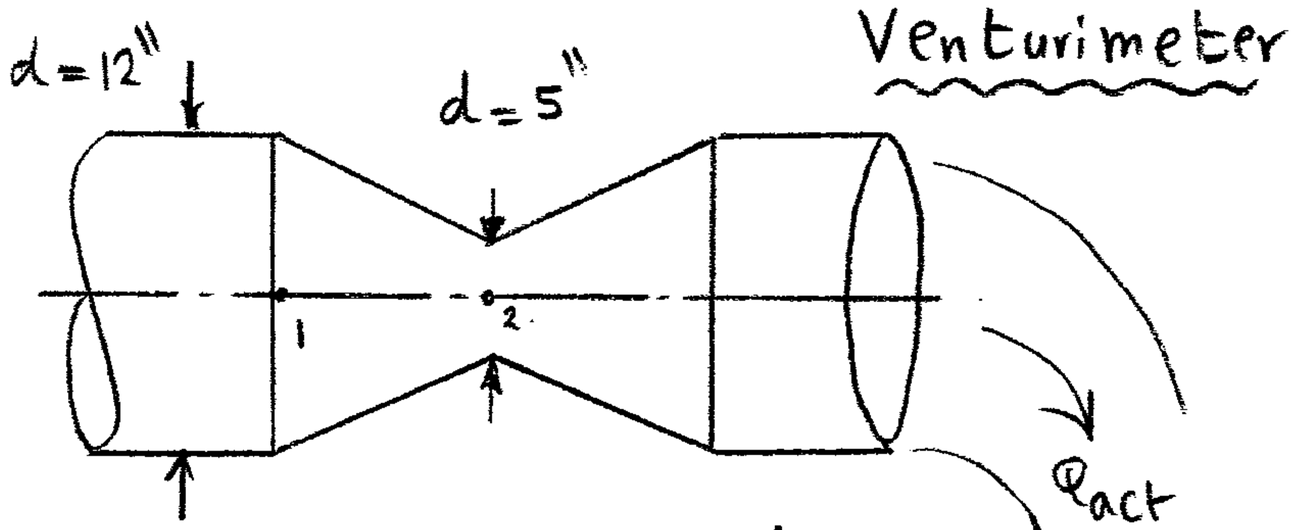
$$Q_{act} = \frac{2}{3} C_d B \sqrt{2g} \ H^{\frac{3}{2}}$$

$$0.0122 = \frac{2}{3} C_d (0.5) \sqrt{2 * 9.81} \ (0.06)^{\frac{3}{2}}$$

$$\therefore C_d = \underline{0.565}$$

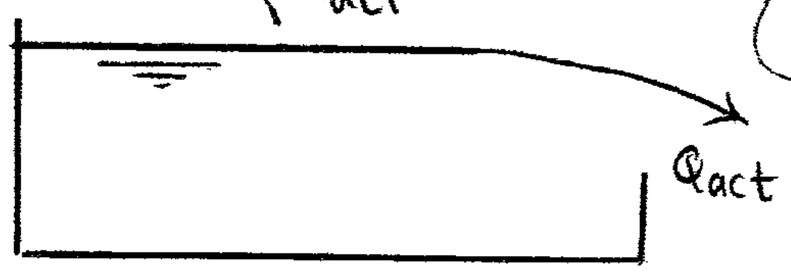
3 During a test in the laboratory, the water passes through a horizontal venturimeter and then flows over a right-angled V-notch. The head at notch being registered. The inlet and the outlet diameters for the venturimeter are 12 inches and the throat diameter is 5 inches. When a steady head of 0.604 ft is recorded over the V-notch the differential pressure head between inlet and throat was found to be 1.075 ft of water. If the V-notch coefficient of discharge is 0.6, determine the coefficient of discharge of the venturimeter.

(11)

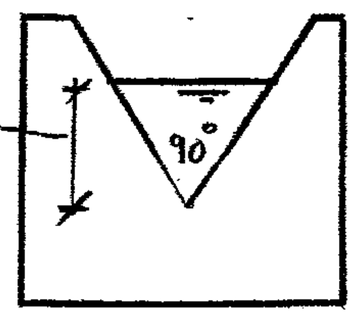


Venturimeter

notch



$h = 0.604$ ft



$\left(\frac{P_1}{\gamma_w} - \frac{P_2}{\gamma_w}\right) = 1.075$ ft

$C_{d_{notch}} = 0.6 \qquad C_{d_{vent}} = ?$

notch

$$Q_{act} = \frac{8}{15} C_d \sqrt{2g} \tan\left(\frac{\theta}{2}\right) h^{5/2}$$

$$= \frac{8}{15}(0.6)\sqrt{2(32.2)} \tan\left(\frac{90}{2}\right)(0.604)^{5/2} = 0.728 \text{ ft}^3/\text{s}$$

Venturimeter

$$A_1 = \frac{\pi\left(\frac{12}{12}\right)^2}{4} = 0.785 \text{ ft}^2$$

$$A_2 = \frac{\pi\left(\frac{5}{12}\right)^2}{4} = 0.136 \text{ ft}^2$$

$$H = \left(\frac{P_1}{\gamma} + z_1\right) - \left(\frac{P_2}{\gamma} + z_2\right) = \left(\frac{P_1}{\gamma} - \frac{P_2}{\gamma}\right) = 1.075 \text{ ft}$$

$$Q_{th} = \frac{A_1 A_2 \sqrt{2gH}}{\sqrt{A_1^2 - A_2^2}} = \frac{(0.785)(0.136)\sqrt{2(32.2)(1.075)}}{\sqrt{(0.785)^2 - (0.136)^2}} = 1.15$$

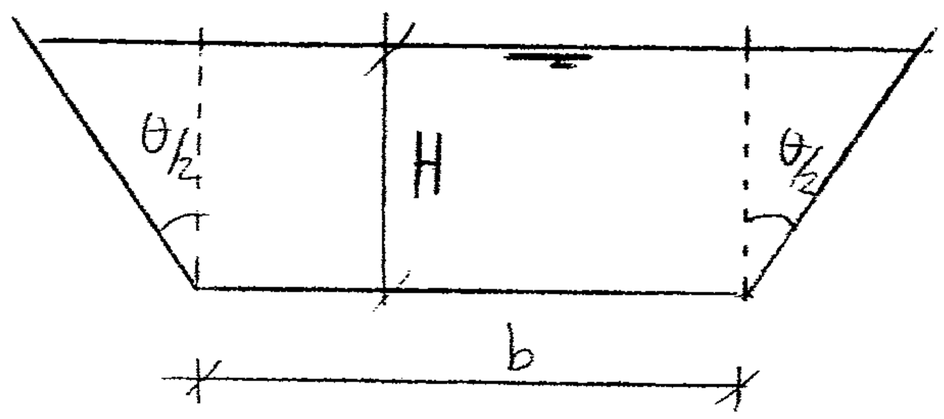
$$C_{d_{venturimeter}} = \frac{Q_{act}}{Q_{th}} = \frac{0.728}{1.15} = 0.63$$

Develop an expression for the discharge through a trapezoidal notch having a base width b and sides making an angle θ to the vertical.

(12)

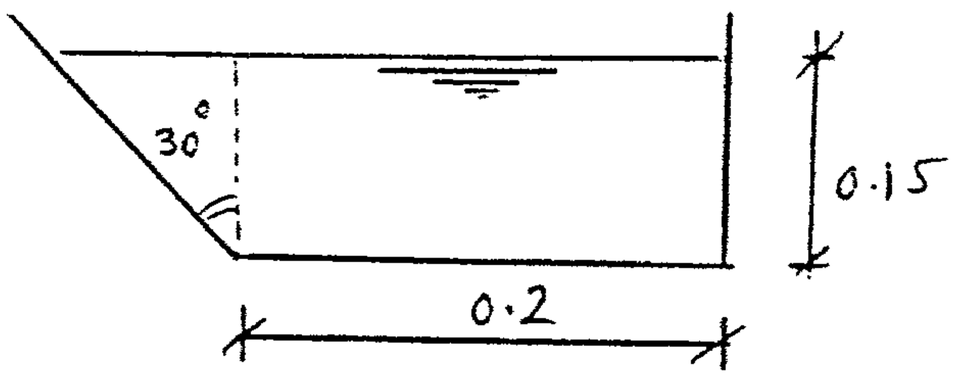
$$Q_{Trapezoidal} = Q_{rec} + Q_{tri}$$

$$Q_{Trap} = \frac{2}{3} b\sqrt{2g}\, H^{\frac{3}{2}} + \frac{8}{15}\sqrt{2g}\tan\frac{\theta}{2}\, H^{\frac{5}{2}}$$



---

[5] What is the theoretical rate of flow in m^3/sec from the notch shown in the figure, if the total head above the sill of the notch is 0.15 m?



Sol

$$Q = Q_{rect} + \frac{1}{2} Q_{tri}$$

$$= \frac{2}{3} b\sqrt{2g}\, H^{3/2} + \frac{1}{2}\,\frac{8}{15}\sqrt{2g}\,\tan\frac{\theta}{2}\, H^{5/2}$$

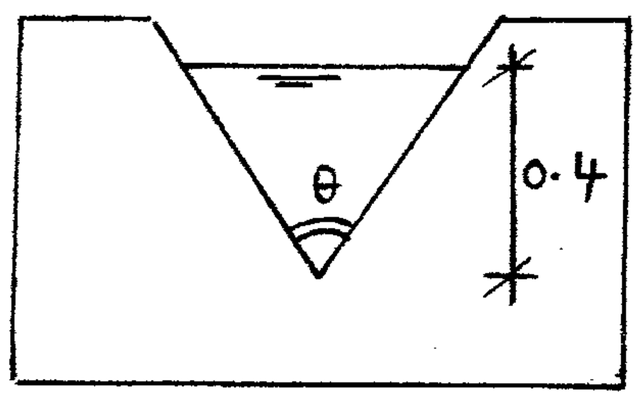
$$= \frac{2}{3}(0.2)\sqrt{2(9.81)}\,(0.15)^{3/2} + \frac{8}{30}\sqrt{2(9.81)}\tan 30\,(0.15)^{5/2}$$

$$= \underline{0.0402\ m^3/s}$$

---

[4] $\theta = 60°$ $\quad C_d = 0.6$ $\quad$ % error = ?

$h = 0.4$ $\quad$ error = 0.4 cm = 0.004 m



Sol

$$Q = \frac{8}{15} C_d \sqrt{2g}\tan\frac{\theta}{2}\, H^{\frac{5}{2}}$$

$$= \frac{8}{15}(0.6)\sqrt{2*9.81}\tan\frac{60}{2}(0.4)^{\frac{5}{2}} = 0.0828$$

$$dQ = \frac{8}{15} C_d \sqrt{2g}\tan\frac{\theta}{2} * \frac{5}{2} H^{\frac{3}{2}}\, dH$$

$$= \frac{8}{15}(0.6)\sqrt{2*9.81}\tan\frac{60}{2} * \frac{5}{2}(0.4)^{\frac{3}{2}}\, 0.004 = 2.07 * 10^{-3}$$

$$\%\ \text{error} = \frac{dQ}{Q} = \frac{0.00207}{0.0828} \times 100 = \underline{2.5\ \%}$$

Rest of Sheet 4

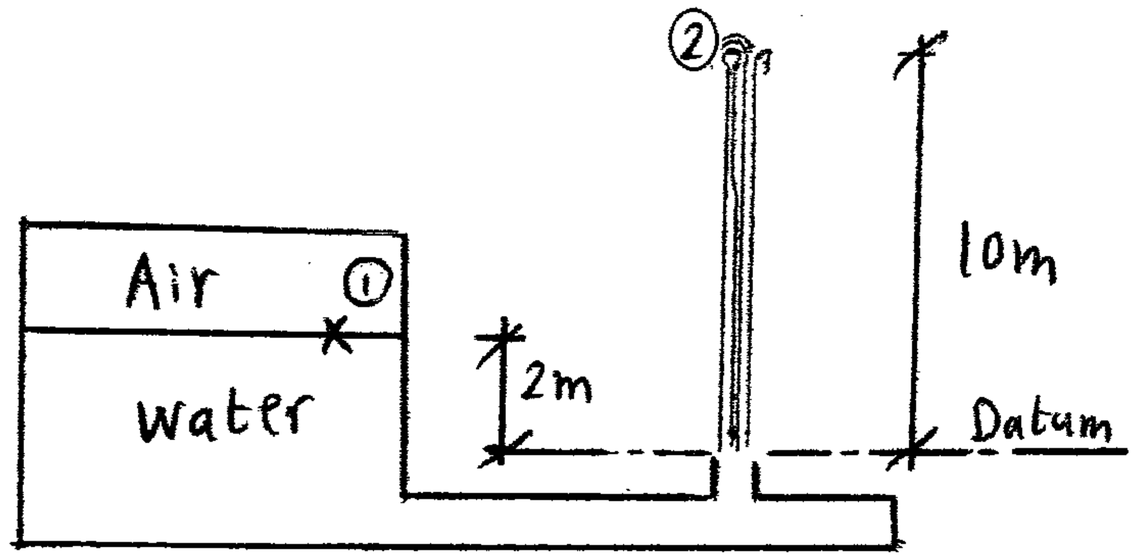
5- Using the shown system, calculate the required air pressure above the water surface to cause the jet to rise 10 meters from the nozzle

$P_{air} = ?$

Applying Ber ①,②

$$\frac{P_{air}}{\gamma_w} + 0+2 = 0+0+10$$

$$P_{air} = 8\gamma_w = \underline{\underline{78480\ Pa}}$$

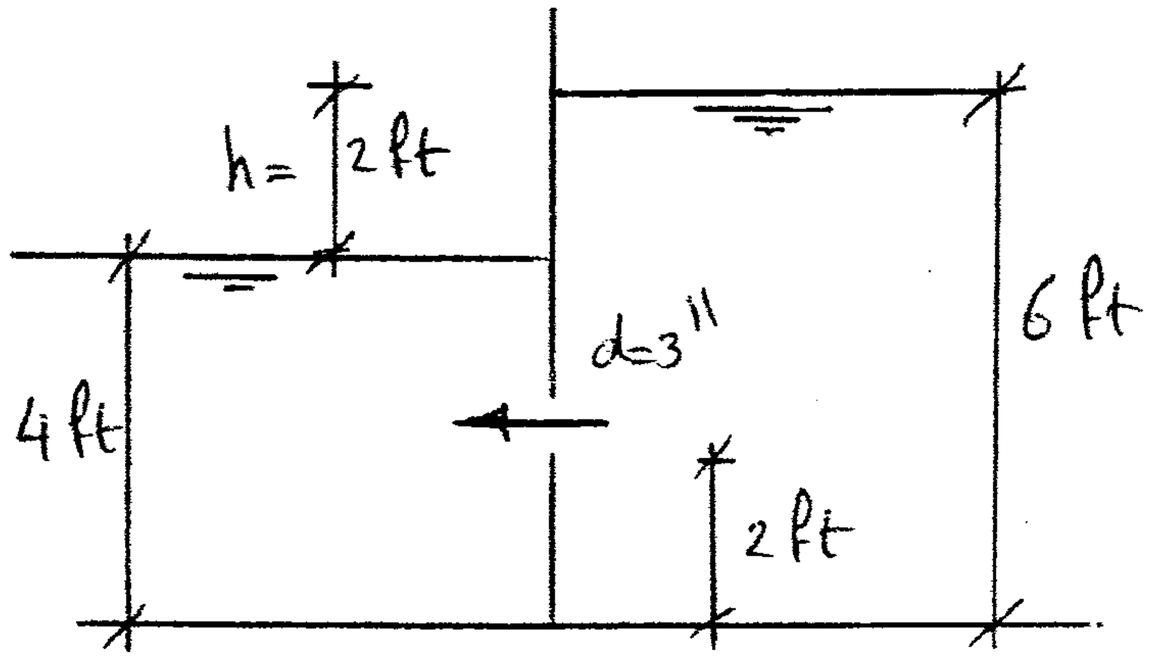


15- Determine the flowrate through the submerged orifice shown in the figure if the contraction coefficient C_d is 0.68

Submerged orifice

$C_d = 0.68$ $\quad Q = ?$

Sol//



$$Q = C_d\ A\ V$$

$$= C_d \frac{\pi d^2}{4} \sqrt{2gh}$$

$$Q = 0.68 \frac{\pi \left(\frac{3}{12}\right)^2}{4} \sqrt{2(32.2)2} = \underline{\underline{0.379\ ft^3/s}}$$

11 Pumps, Turbines & Jets

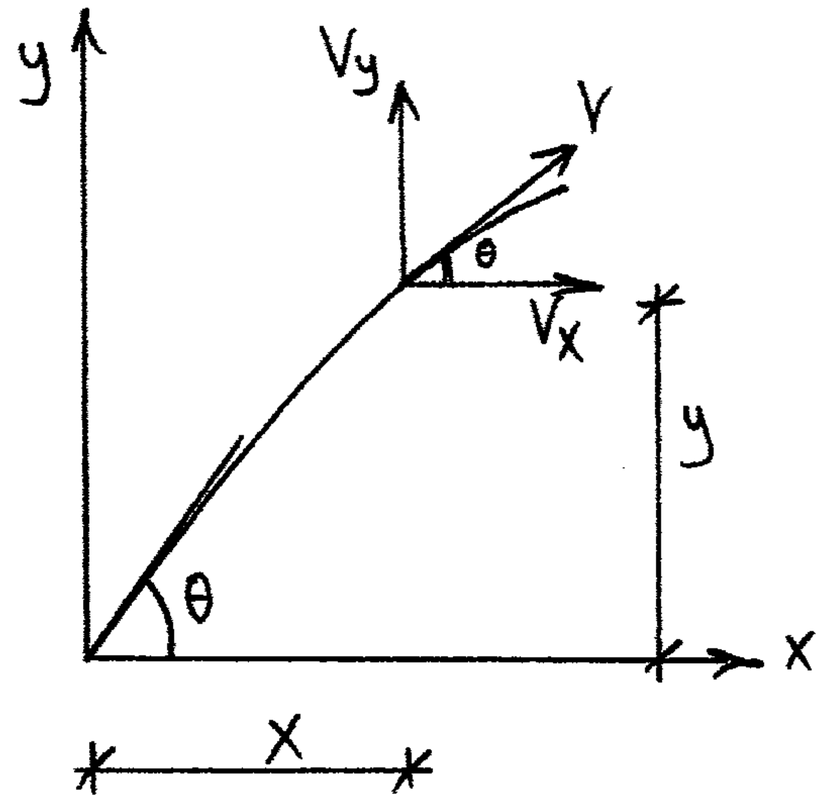
Free jet

Newton's equations

$$V = V_0 + at \quad \rightarrow ①$$

$$S = V_0 t + \frac{1}{2} a t^2 \quad \rightarrow ②$$

$$V^2 = V_0^2 + 2as \quad \rightarrow ③$$



يتم تحليل سرعة الـ Free jet الى مركبتين V_x , V_y

$$V_x = V \cos\theta$$

$$V_y = V \sin\theta$$

Horizontal direction

لا يوجد عجلة فى الاتجاه الأفقى

There is no acceleration in the hzl direction

$$X = V_{o_x} t$$

$$V_x = V_{o_x}$$

السرعة ثابته فى الاتجاه الأفقى

Vertical direction

عجلة الجاذبيه هى العجلة المؤثره فى الاتجاه الرأسى

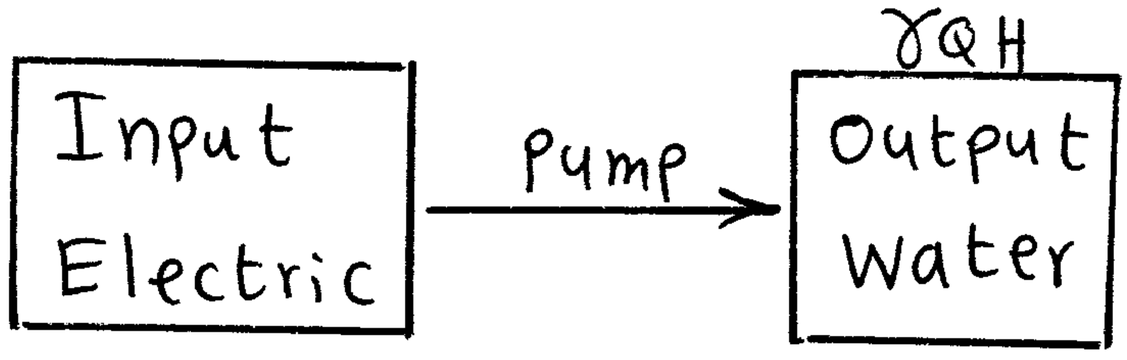
The acceleration in the Vertical direction is the gravity

$$Y = V_{oy} t - \frac{1}{2} g t^2$$

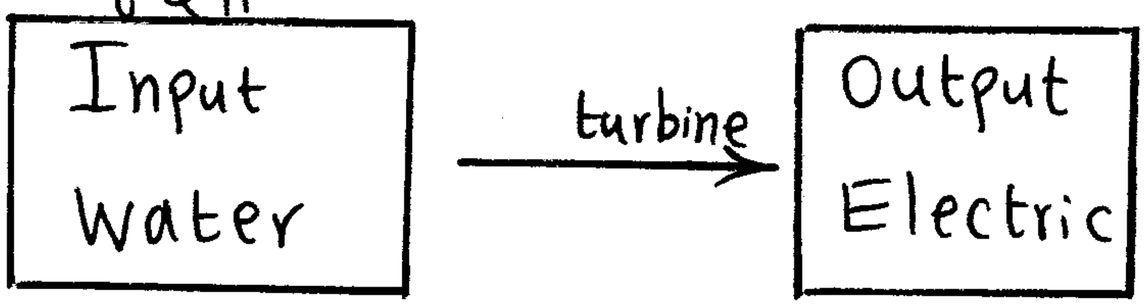
$$V_y = V_{oy} - gt$$

$$V_y^2 = V_{oy}^2 - 2gy$$

Efficiency



$$\eta = \frac{\text{output}}{\text{input}} = \frac{\gamma Q H}{\text{input}}$$



$$\eta = \frac{\text{Output}}{\text{input}} = \frac{\text{output}}{\gamma Q H}$$

Pump's power

$$P = \frac{\gamma Q H}{\eta} = \ldots\ldots \text{watt}$$

$$HP = \frac{\gamma Q H}{\eta k} = \ldots\ldots \text{horsepower}$$

Turbine's power

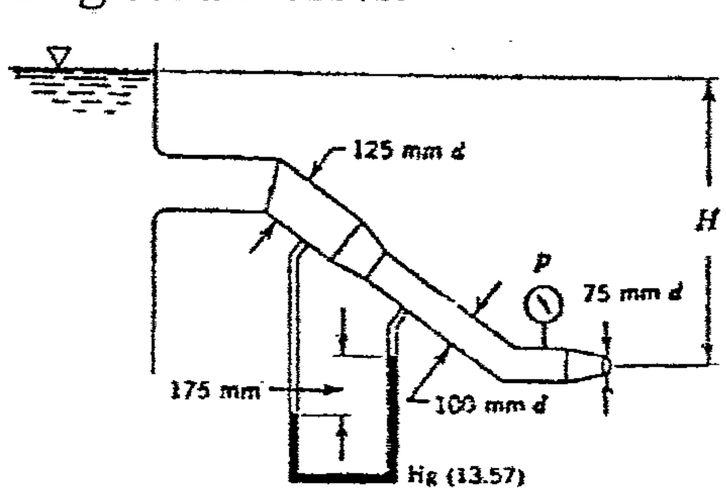
$$P = \gamma Q H \eta$$

γ	1000	9810	62.4
k	75	735	550

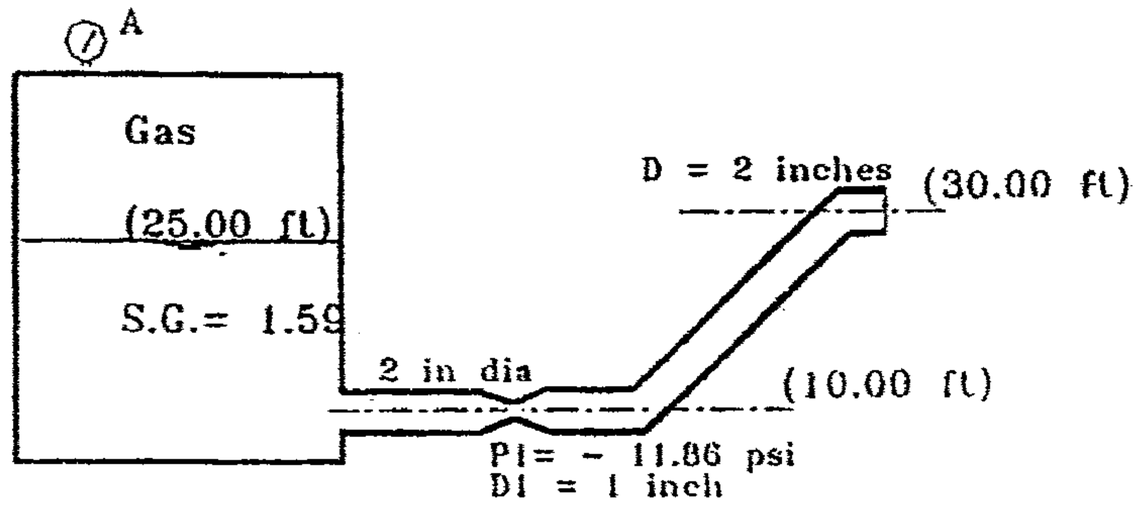
$$HP = \frac{\gamma Q H \eta}{k}$$

Extra problems chosen from past examinations

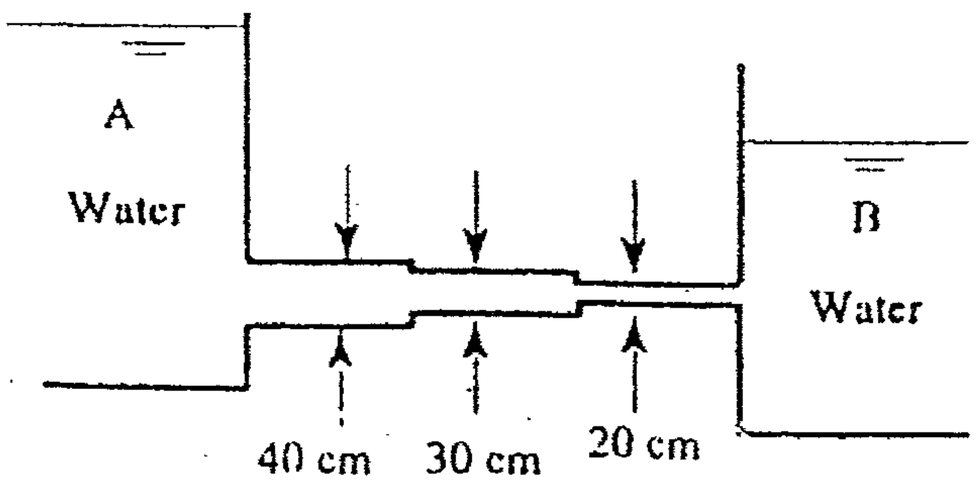
12 - Water is flowing in the shown figure. **Calculate** H (m) and P (KPa).



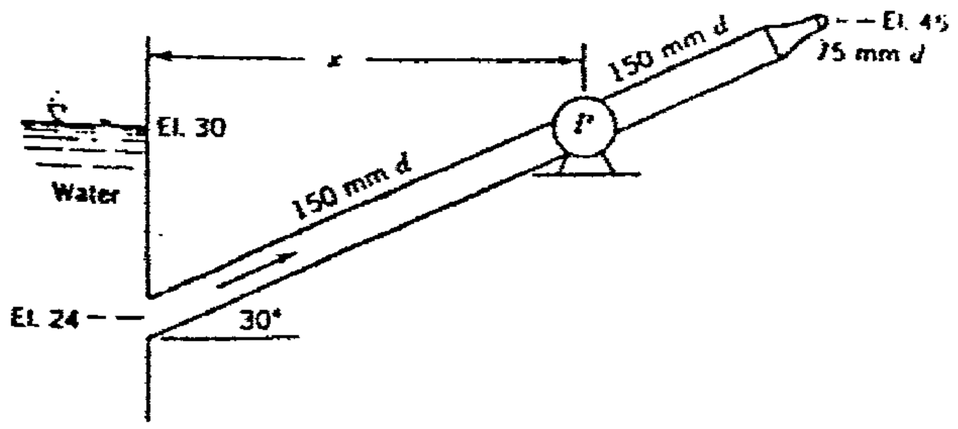
13 - **Find** the flow rate that passes through the system shown in figure, and **what** is the corresponding head, which is read by the gage 'A'. Take $P_1 = -11.86$ psi. Also **draw** T.E.L and H.G.L. Neglect all losses.



14 - Water flows at a rate of 0.06 m^3/s from reservoir A to reservoir B through three pipes connected in series, as shown in figure. If the head loss along the pipe is $0.3V^2/2g$, where V is the velocity of flow in the pipe. Neglecting all minor losses, **determine** the difference between water surface elevations in the reservoirs. **Sketch** the T.E.L and H.G.L.



15 - **Compute** the pump power required maintaining a flow rate of 0.1 m^3/s in the shown figure if the barometric pressure is 98.6 KPa and the vapor pressure is 6.9 KPa. **Calculate** the maximum possible X for reliable operation.



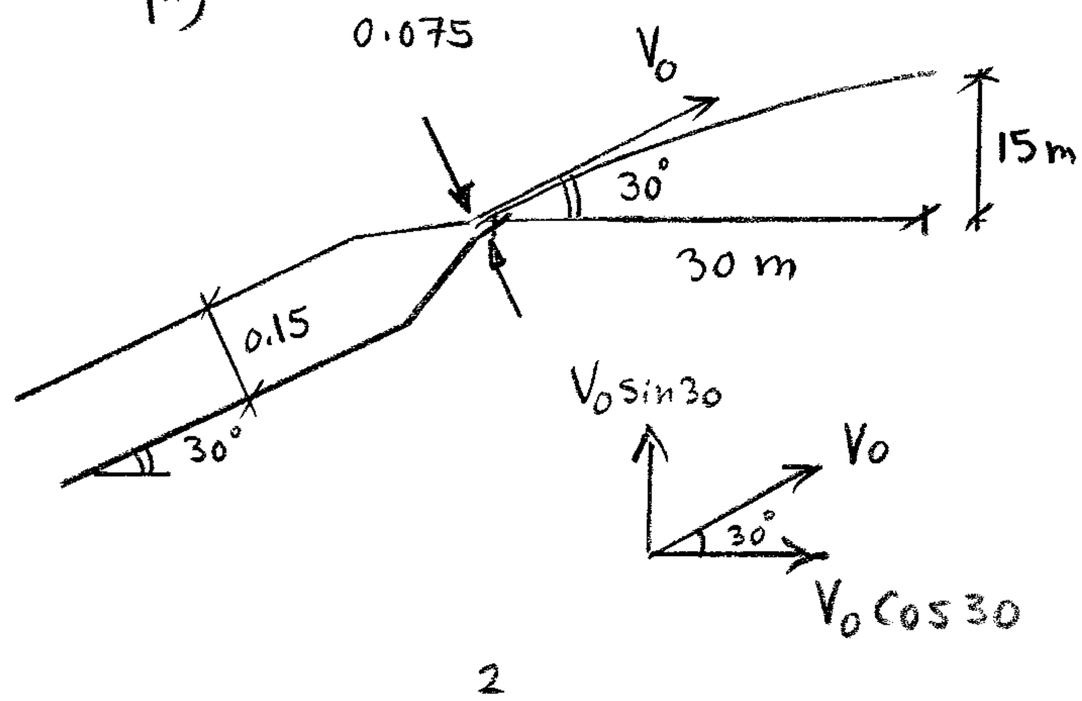
6

$$S = V_0 t + \frac{1}{2} a t^2 \quad \text{(Newton's eqn)}$$

$$\therefore \; X = V_0 \cos\theta \, t \rightarrow (1)$$

$$y = V_0 \sin\theta \, t - \frac{1}{2} g t^2 \rightarrow (2)$$

$$t = \frac{X}{V_0 \cos\theta}$$



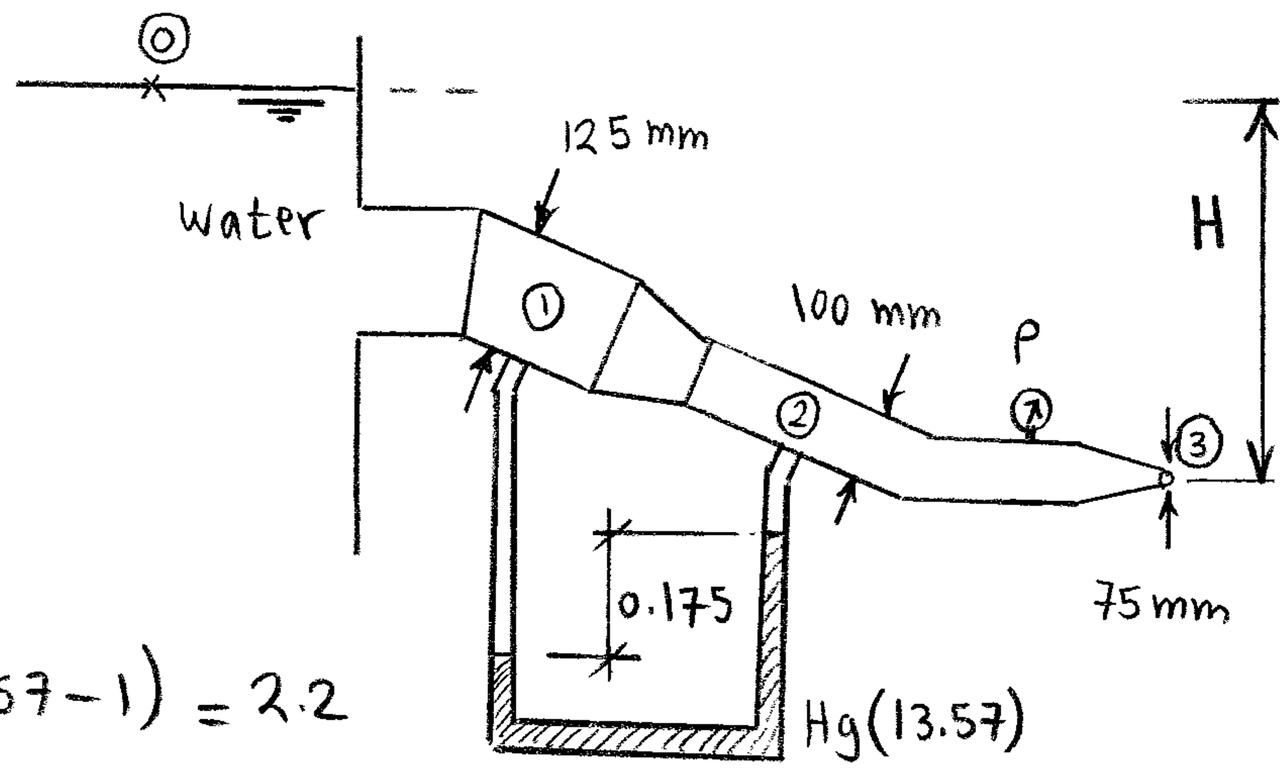
$$y = V_0 \sin\theta \left(\frac{X}{V_0 \cos\theta}\right) - \frac{1}{2} g \left(\frac{X}{V_0 \cos\theta}\right)^2$$

$$15 = \tan 30 * 30 - \frac{1}{2}(9.81)\left(\frac{30}{V_0 \cos 30}\right)^2 \Rightarrow V_0 = 50.36 \text{ m/s}$$

$$Q = V_0 A_0 = 50.36 \; \frac{\pi (0.075)^2}{4} = \underline{\underline{0.22 \text{ m}^3/\text{s}}}$$

12

$$Q = \frac{A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}}$$



$$A_1 = \pi (0.125)^2/4 = 0.0123$$

$$A_2 = \pi (0.1)^2/4 = 7.85 * 10^{-3}$$

$$h = R\left(\frac{\gamma_2}{\gamma_1} - 1\right) = 0.175(13.57 - 1) = 2.2$$

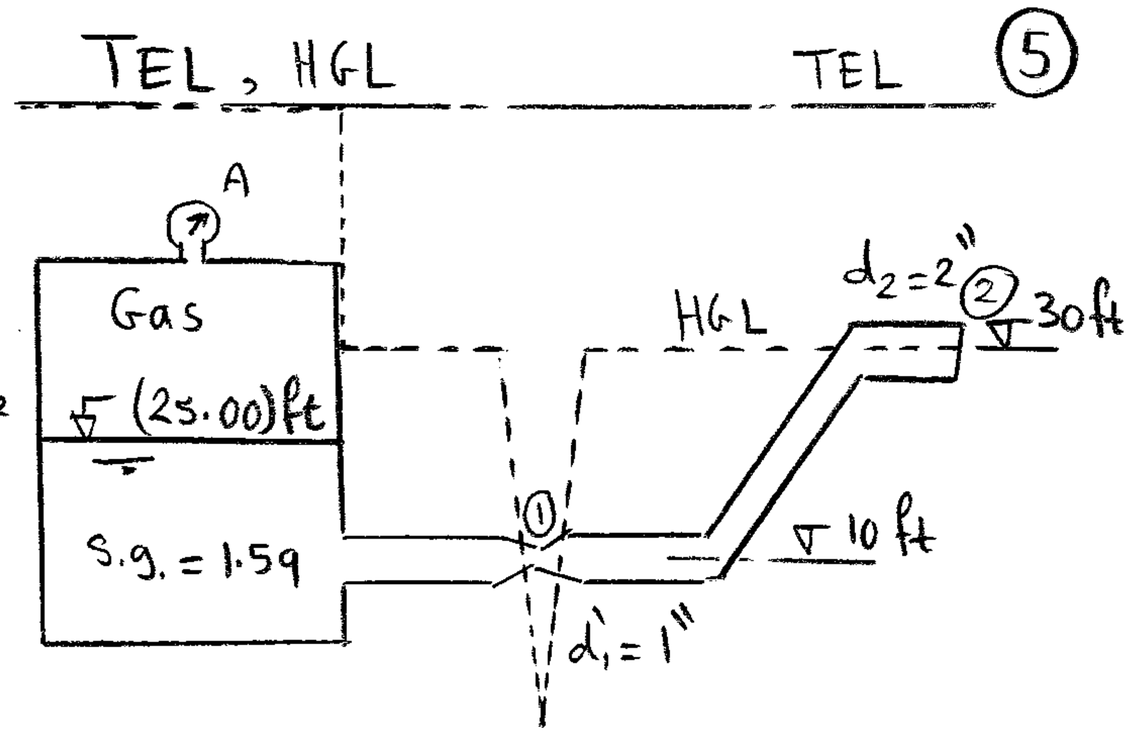
$$Q = \frac{0.0123 * (7.85 * 10^{-3}) \sqrt{2(9.81)2.2}}{\sqrt{(0.0123)^2 - (7.85 * 10^{-3})^2}} = \underline{\underline{0.0671 \text{ m}^3/\text{s}}}$$

$$V_3 = \sqrt{2gH} \quad , \quad V_3 = \frac{Q}{A_3}$$

$$\frac{0.0671}{\frac{\pi (0.075)^2}{4}} = \sqrt{2 * 9.81 * H} \Rightarrow H = \underline{\underline{11.78 \text{ m}}}$$

13

$Q = ?$ $\quad P_A = ?$



$P_1 = -11.86 \text{ psi} = -11.86 * 144 = -1707.8 \frac{lb}{ft^2}$

Continuity eqn

$$V_1 A_1 = V_2 A_2$$

$$V_1 (1)^2 = V_2 (2)^2$$

$$V_1 = 4 V_2$$

Bernoulli eqn between ①, ②

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{-1707.8}{1.59 * 62.4} + \frac{16 V_2^2}{2 * 32.2} + 10 = 0 + \frac{V_2^2}{2 * 32.2} + 30$$

$\therefore \quad V_2 = 12.64 \text{ ft/s}$

$$Q = A_2 V_2$$

$$= \frac{\pi \left(\frac{2}{12}\right)^2}{4} * 12.64 = \underline{\underline{0.276 \text{ ft}^3/\text{s}}}$$

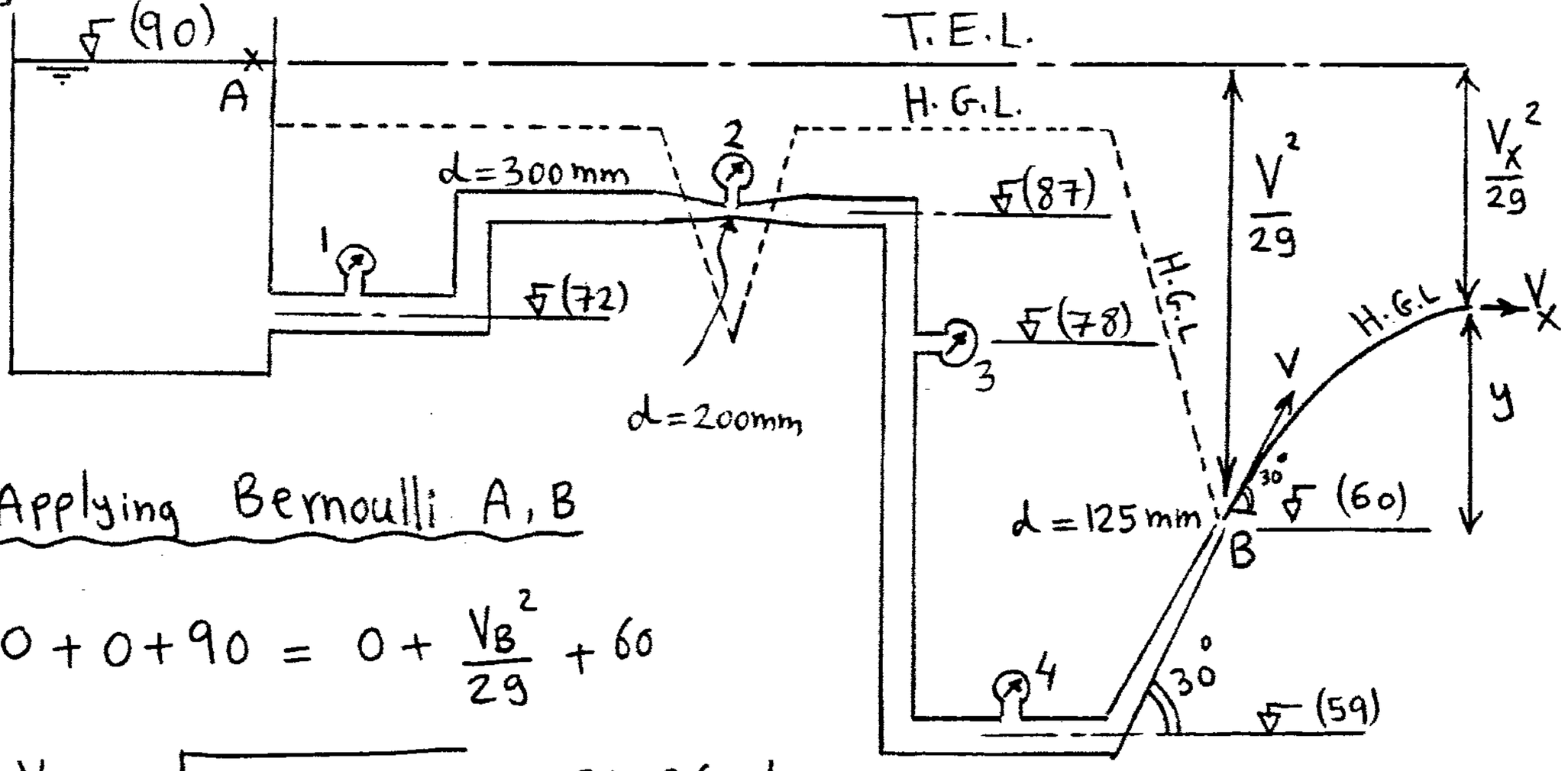
Bernoulli eqn between Ⓐ, ②

$$\frac{P_A}{1.59 * 62.4} + 0 + 25 = 0 + \frac{(12.64)^2}{2(32.2)} + 30$$

$\therefore P_A = \underline{\underline{724.2 \text{ lb/ft}^2}}$

7

6



Applying Bernoulli A, B

$$0+0+90 = 0+\frac{V_B^2}{29}+60$$

$$V_B = \sqrt{2*9.81*30} = 24.26\ m/s$$

$$Q = V_B A_B = 24.26\ \frac{\pi (0.125)^2}{4} = 0.3\ m^3/s$$

$$V_1 = \frac{Q}{A_1} = \frac{0.3}{\frac{\pi (0.3)^2}{4}} = 4.24\ m/s$$

$$V_2 = \frac{Q}{A_2} = \frac{0.3}{\frac{\pi (0.2)^2}{4}} = 9.55\ m/s$$

Applying Bernoulli between A, 1

$$0+0+90 = \frac{P_1}{9810}+\frac{(4.24)^2}{2*9.81}+72 \Rightarrow P_1 = \underline{168000\ Pa}$$

Applying Bernoulli between A, 4

$$0+0+90 = \frac{P_4}{9810}+\frac{(4.24)^2}{2*9.81}+59 \Rightarrow P_4 = \underline{295100\ Pa}$$

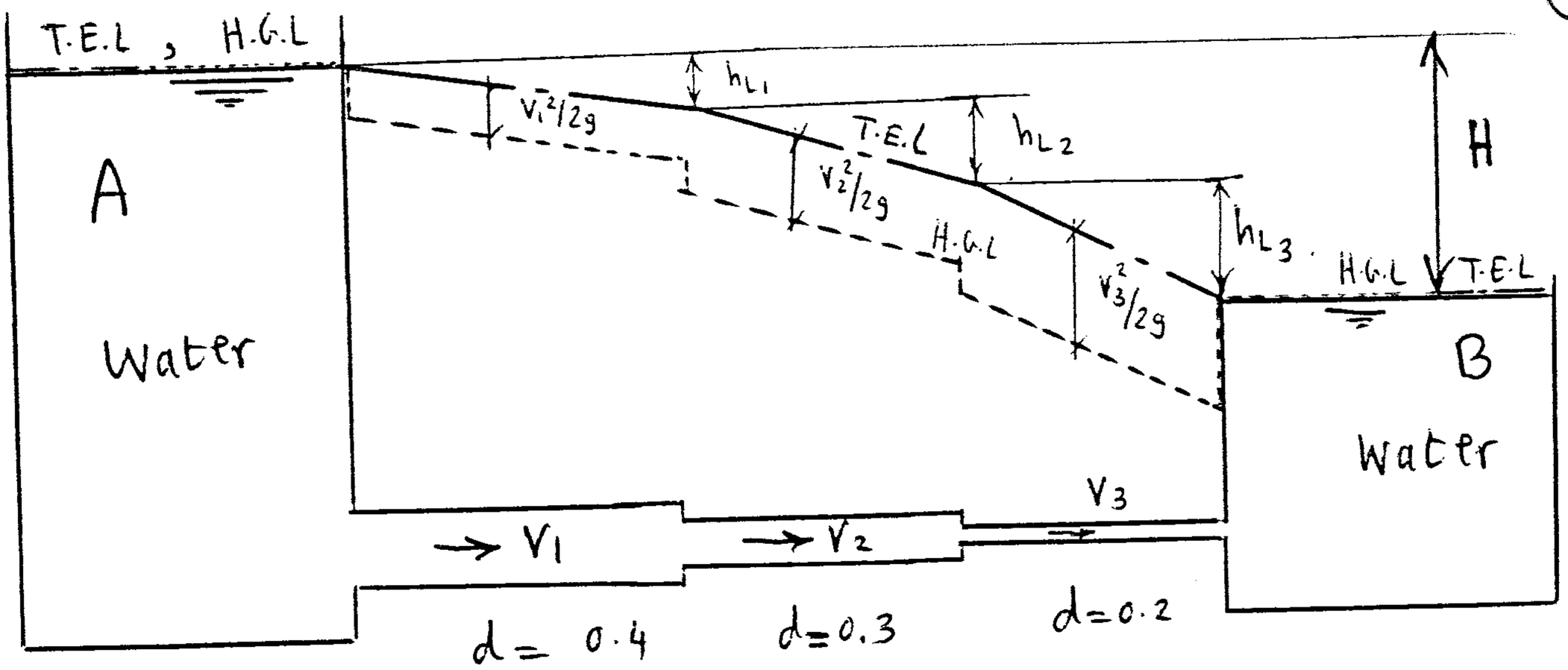
$$V_y^2 = V_{oy}^2 - 2gy$$

$$V_y = 0 \quad , \quad V_{oy} = V_B \sin\theta = 24.26 \sin 30$$

$$0 = (24.26 \sin 30)^2 - 2*9.81 \times y \Rightarrow y = 7.5\ m$$

14

7



$h_L = 0.3 \frac{V^2}{2g}$, $Q = 0.06\ m^3/s$ $H = ?$

Sol

$$V_1 = \frac{Q}{A} = \frac{0.06}{\frac{\pi (0.4)^2}{4}} = 0.477\ m/s$$

$$V_2 = \frac{Q}{A} = \frac{0.06}{\frac{\pi (0.3)^2}{4}} = 0.849\ m/s$$

$$V_3 = \frac{Q}{A} = \frac{0.06}{\frac{\pi (0.2)^2}{4}} = 1.91\ m/s$$

Applying Bernoulli eqn between A, B Datum at B

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + Z_B + \underbrace{0.3\frac{V_1^2}{2g} + 0.3\frac{V_2^2}{2g} + 0.3\frac{V_3^2}{2g}}_{\Sigma h_L}$$

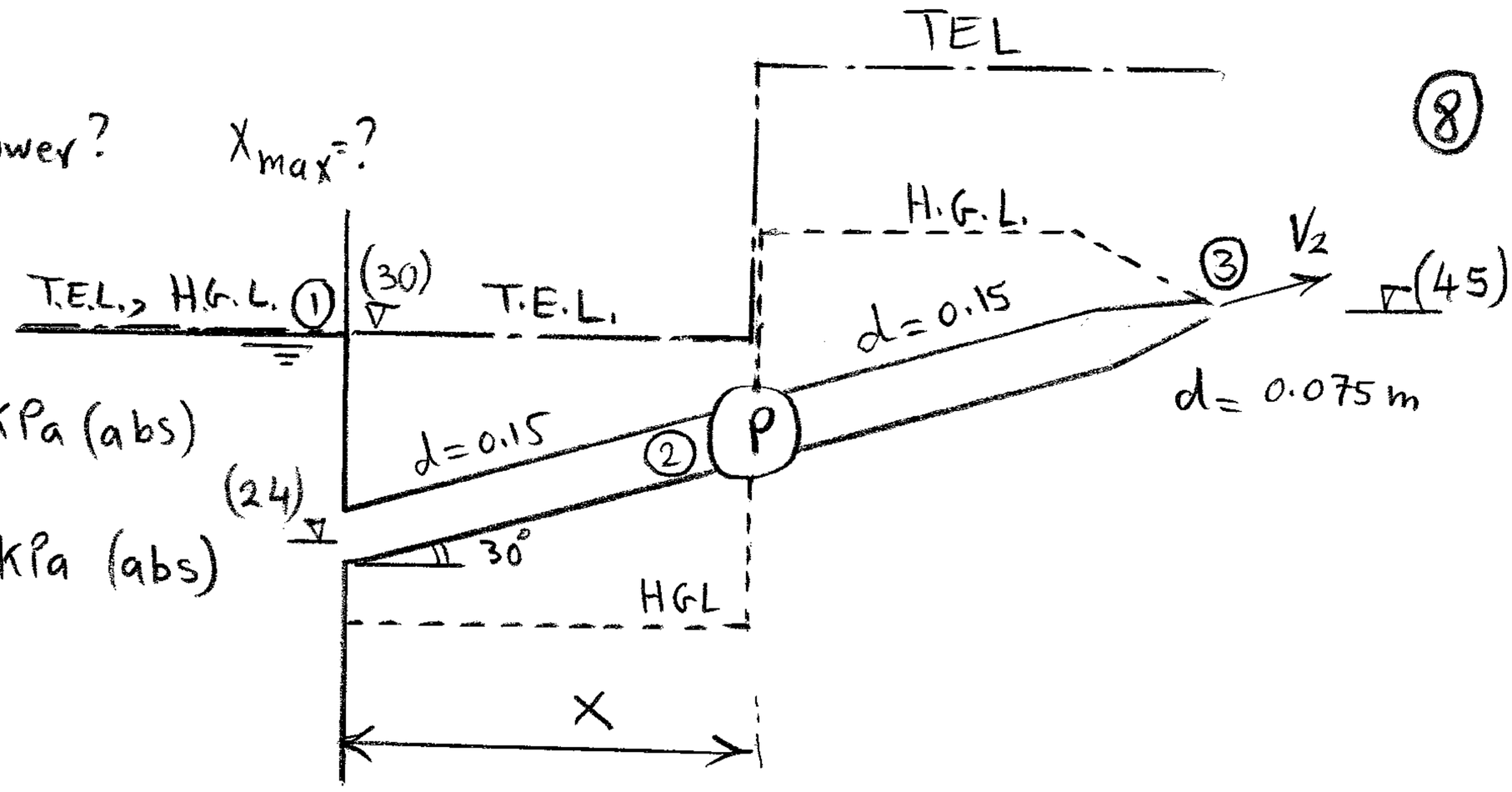
$$H = \frac{0.3}{2(9.81)} \left[(0.477)^2 + (0.849)^2 + (1.91)^2 \right] = \underline{\underline{0.07\ m}}$$

15 Pump Power? X_{max}=?

$Q = 0.1\ m^3/s$

$P_{atm} = 98.6$ KPa (abs)

$P_v = 6.9$ KPa (abs)



Solution

$$V_2 = \frac{Q}{A_2} = \frac{0.1}{\frac{\pi (0.15)^2}{4}} = 5.66\ m/s$$

$$V_3 = \frac{Q}{A_3} = \frac{0.1}{\frac{\pi (0.075)^2}{4}} = 22.63\ m/s$$

Bernoulli eqn between ①, ③

$$0 + 0 + 30 + h_p = 0 + \frac{(22.63)^2}{2(9.81)} + 45 \quad \Rightarrow h_p = 41.1\ m$$

$$Power = \gamma Q h_p = 9810\ (0.1)\ 41.1 = \underline{40319\ watt}$$

for no Cavitation $\Rightarrow$ max -ve pressure at ② is Vapor pressure

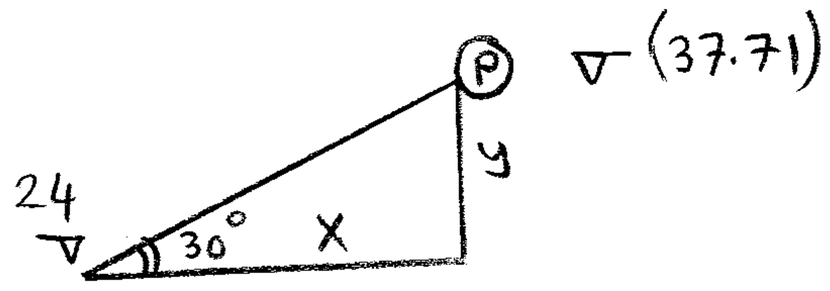
$$P_{\substack{vapor \\ (abs)}} = P_{atm} + P_{\substack{vapor \\ (gage)}}$$

$$6.9 * 10^3 = 98.6 * 10^3 + P_{gage} \quad \Rightarrow P_{gage} = -91.7 * 10^3$$

Bernoulli eqn between ①, ②

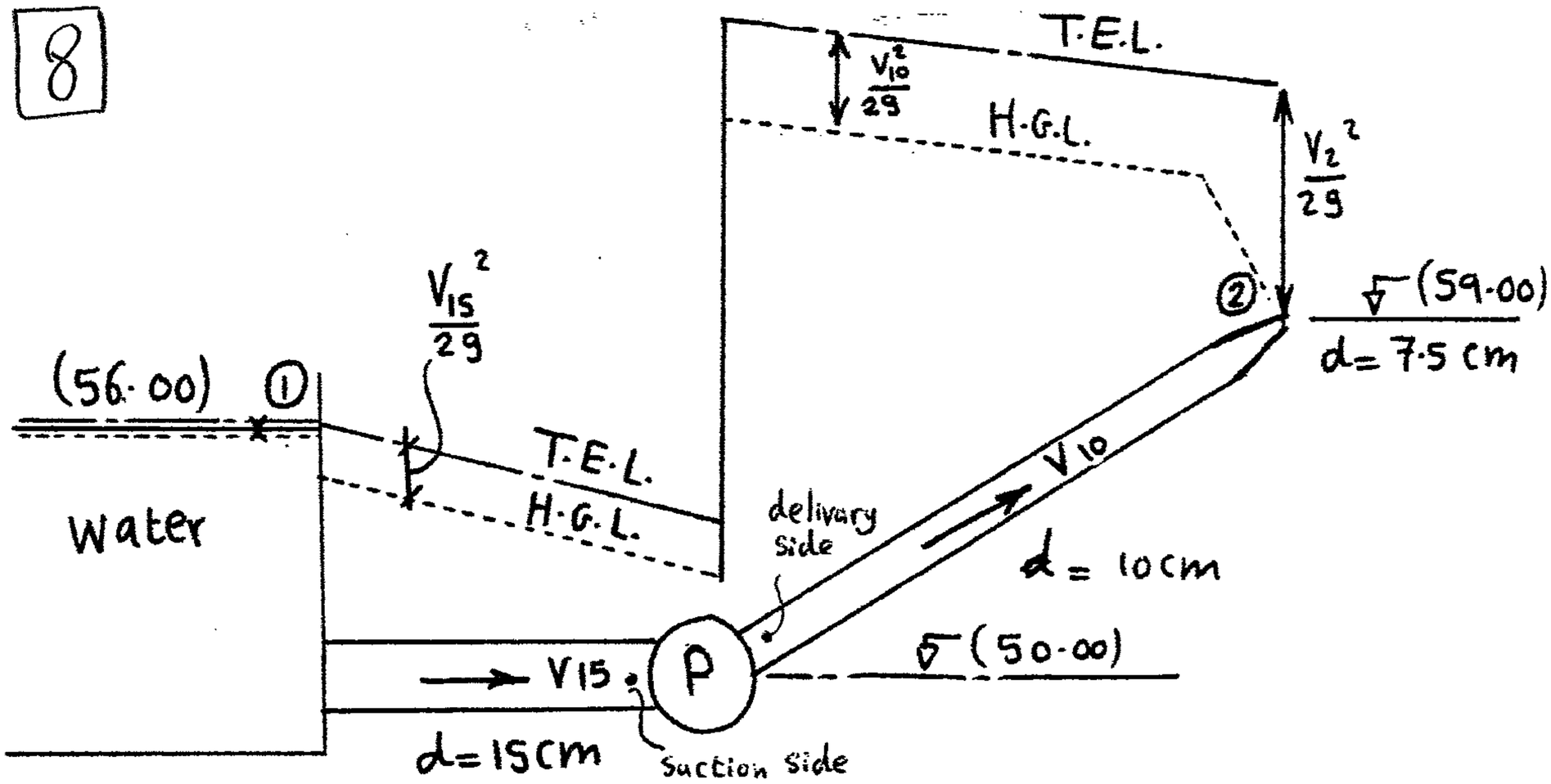
$$0 + 0 + 30 = \frac{-91.7 * 10^3}{9810} + \frac{(5.66)^2}{2(9.81)} + Z_2 \quad \Rightarrow Z_2 = 37.71$$

$$X = \frac{y}{\tan 30} = \frac{(37.71 - 24)}{\tan 30} = \underline{23.75\ m}$$



8

9



$Q = ?$ $\quad H_P = 24\ m$ $\quad h_{L_{15}} = \frac{8 V_{15}^2}{2g}$ $\quad h_{L_{10}} = \frac{15 V_{10}^2}{2g}$ $\quad \frac{P_S}{\gamma} = ?$

$$A_{10} = \frac{\pi (0.1)^2}{4} = 7.85 * 10^{-3}\ m^2$$

$$A_{15} = \frac{\pi (0.15)^2}{4} = 0.0177\ m^2$$

$$A_{7.5} = \frac{\pi (0.075)^2}{4} = 4.4 * 10^{-3}\ m^2$$

Applying Bernoulli eqn between ①, ②

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 + H_P = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_{L_{15}} + h_{L_{10}}$$

$$56 + 24 = \frac{Q^2}{2g A_{7.5}^2} + 59 + \frac{8 Q^2}{2g A_{15}^2} + \frac{15 Q^2}{2g A_{10}^2}$$

$\therefore Q = 0.0359\ m^3/s$

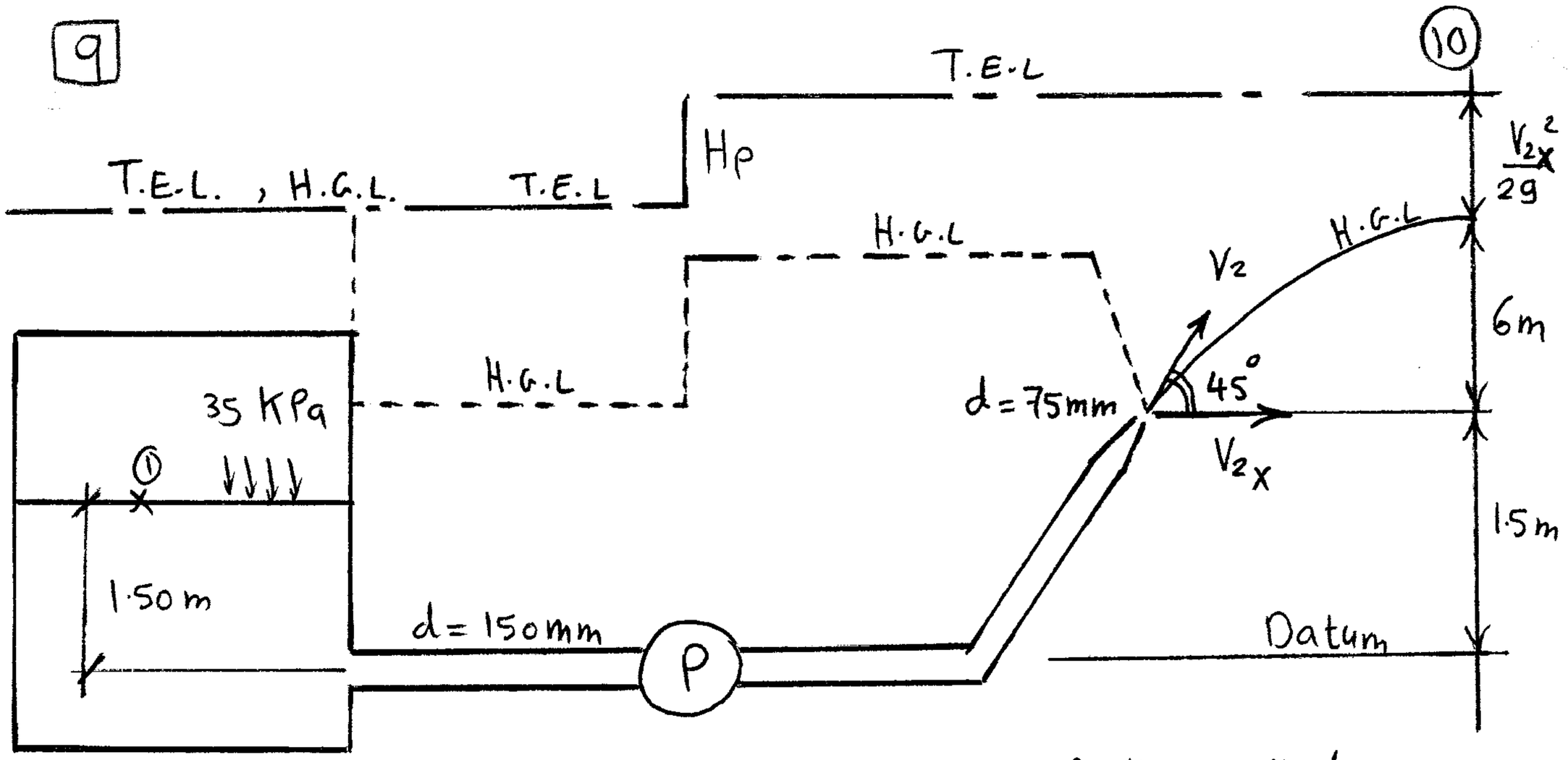
$$V_{15} = \frac{Q}{A_{15}} = 2.03\ m/s$$

Applying Bernoulli eqn between ①, ⑤

$$0 + 0 + 56 = \frac{P_S}{\gamma} + \frac{V_{15}^2}{2g} + 50 + \frac{8 V_{15}^2}{2g}$$

$\therefore \frac{P_S}{\gamma} = \underline{\underline{4.1\ m}}$

9



$$V_y^2 = V_{oy}^2 - 2gy$$

$$0 = (V_2 \sin 45)^2 - 2(9.81)6$$

$$\Rightarrow V_2 = \underline{\underline{15.34}}$$

Another method

$$0 + \frac{V_2^2}{2g} + 0 = 0 + \frac{V_{2x}^2}{2g} + 6$$

$$\frac{V_2^2 - V_2^2 \cos^2 45}{2(9.81)} = 6$$

$$\Rightarrow V_2 = 15.34 \text{ m/s}$$

Applying Bernoulli eqn between ①, ②

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 + H_p = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

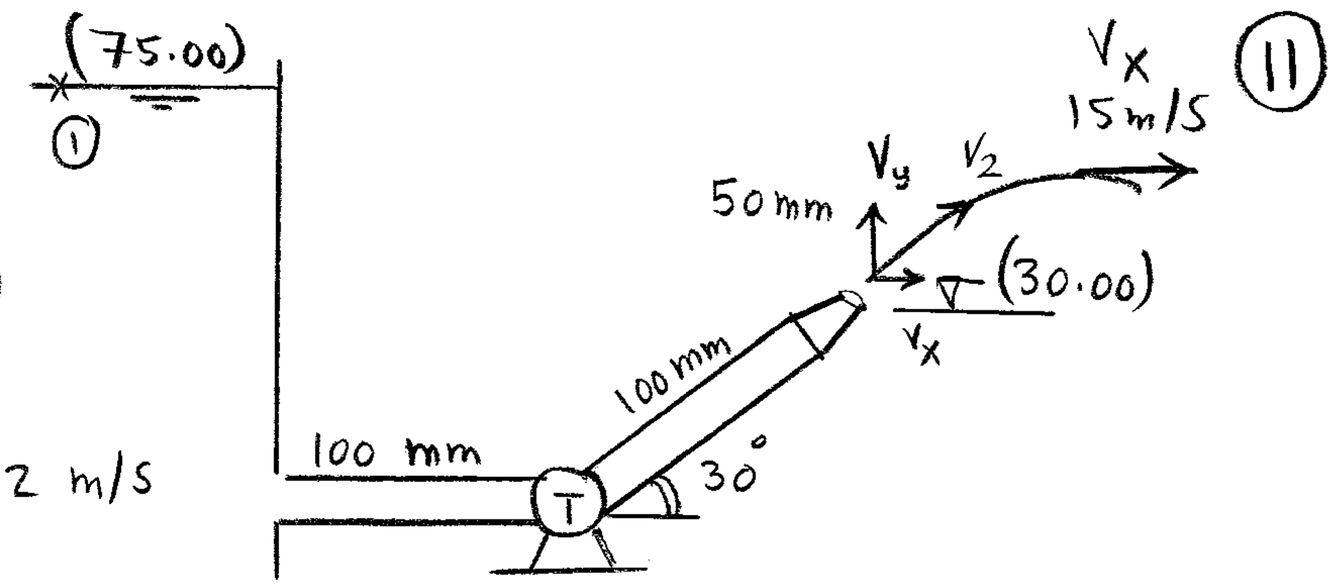
$$\frac{35*10^3}{9810} + 0 + 1.5 + H_p = 0 + \frac{(15.34)^2}{2*9.81} + 1.5$$

$$\Rightarrow H_p = 8.42 \text{ m}$$

$$Q = AV = \frac{\pi (0.075)^2}{4} (15.34) = 0.067 \text{ m}^3/\text{s}$$

$$HP = \frac{\gamma Q H_p}{\eta K} = \frac{9810\,(0.067)(8.42)}{0.7 \times 735} = \underline{\underline{10.75 \text{ horsepower}}}$$

10 Power = ?



$$V_x = 15 \text{ m/s} \quad \text{(constant)}$$

$$V_2 = \frac{V_x}{\cos\theta} = \frac{15}{\cos 30} = 17.32 \text{ m/s}$$

Bernoulli between ①, ②

$$0 + 0 + 75 = 0 + \frac{(17.32)^2}{2(9.81)} + 30 + h_T$$

$$\therefore h_T = 29.71$$

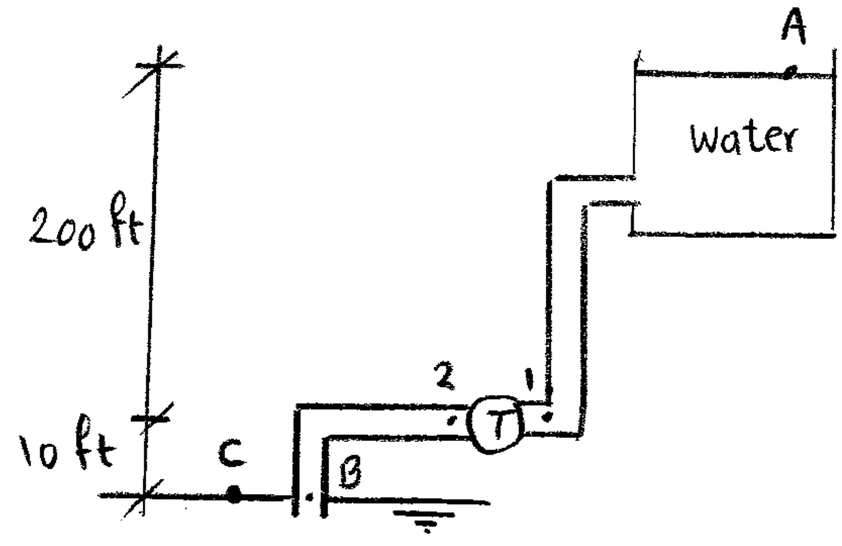
$$\text{Power} = \gamma Q h_T = 9810\left(17.32 * \frac{\pi (0.05)^2}{4}\right) * 29.71 = \underline{9912 \text{ watt}}$$

---

11 $h_{L_{A\to 1}} = \frac{6V^2}{2g}$, $h_{L_{2\to B}} = 0.5\frac{V^2}{2g}$

$Q = 40 \text{ ft}^3/\text{s}$ $\quad d = 24'' = 2 \text{ ft}$

$HP = ?$



$$V = \frac{Q}{A} = \frac{40}{\pi (2)^2 / 4} = 12.72 \text{ ft/s}$$

Bernoulli eqn between A, C datum at C

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + Z_A = \frac{P_C}{\gamma} + \frac{V_C^2}{2g} + Z_C + h_{L_{AC}} + h_T$$

$$200 = 6\frac{V^2}{2g} + 0.5\frac{V^2}{2g} + h_T$$

$$h_T = 200 - \frac{6.5(12.72)^2}{2(32.2)} = 183.63$$

$$HP = \frac{\gamma Q h_T}{k} = \frac{62.4(40)(183.63)}{550} = \underline{833.4 \text{ horsepower}}$$

$1\,ft = 12\,inch$, $inch = 2.54\,cm$ $\qquad 1\,ft = 0.3048\,m$

$1\,slug = 14.59\,kg$ $\qquad 1\,m^3 = 1000\,litre$, $1\,litre = 1000\,cm^3$

$g = 9.81\,m/s^2 = 981\,cm/s^2 = 32.2\,ft/s^2$

$\rho = 1000\,kg/m^3 = 1\,gm/cm^3 = 1.94\,slug/ft^3$

$\gamma = 9810\,N/m^3 = 981\,dyne/cm^3 = 62.4\,lb/ft^3$

$\gamma = \rho g$, $\gamma = \frac{W}{V}$, $\rho = \frac{M}{V}$

$1\,kg = 2.205\,lb$ $\qquad lb = 4.44\,N$ $\qquad N = 10^5\,dyne$

Specific Volume $= \frac{1}{\rho}$ or $\frac{1}{\gamma}$

S.G. $=$ r.d. $= \frac{\rho_{liquid}}{\rho_{water}} = \frac{\gamma_{fluid}}{\gamma_{water}}$ $\qquad$ S.g mercury (Hg) $= 13.6$

$Psi = 144\,Psf$ $\qquad Psi = \frac{lb}{in^2}$ $\qquad Psf = \frac{lb}{ft^2}$

$\omega = \frac{2\pi N}{60}$, $\omega = 2\pi n$, $V = \omega r$

$Power = T.\omega$, $T = F.r$, $F = \tau.A$, $\tau = \mu \frac{V}{y}$

$HP = \frac{watt}{735}$, $K = \frac{\Delta P}{-\frac{\Delta V}{V}} = \frac{\Delta P}{+\frac{\Delta \rho}{\rho}} = \frac{\Delta P}{+\frac{\Delta \gamma}{\gamma}}$

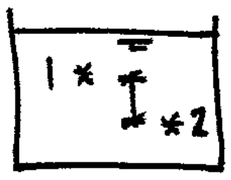
$Poise = \frac{dyne}{cm^2}.s$, $Pa.s = \frac{N}{m^2}.s$ $\qquad Poise = 0.1\,Pa.s$

$Stoke = 10^{-4}\,m^2/s$, $Stoke = cm^2/s$ $\qquad \tau = \mu \frac{dV}{dy}$

$h = \frac{4\sigma \cos\theta}{\gamma d}$ Capillary tube

$\Delta P = \frac{4\sigma}{d}$ water drop , $\Delta P = \frac{8\sigma}{d}$ Soapy bubble

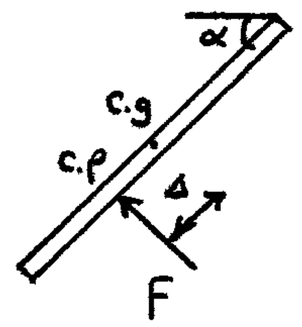
$P = \gamma h$ $\quad P_2 = P_1 + S.g\,\gamma_w\, h$

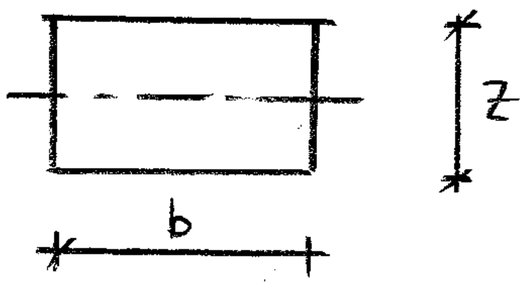


$P_{atm} = 0$ (gauge) $\quad P_{atm} = 101.3 * 10^3$ Pa (absolute)

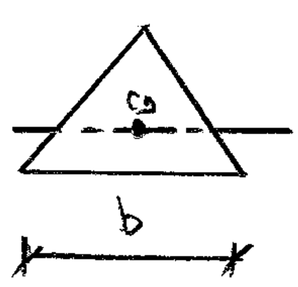
$P_{abs} = P_{atm} + P_{gauge}$ $\quad bar = 10^5$ Pa

$F = \gamma A \bar{h}$ $\quad \Delta = \frac{I_{cg}}{A\bar{y}}$ $\quad \bar{y} = \frac{\bar{h}}{\sin\alpha}$

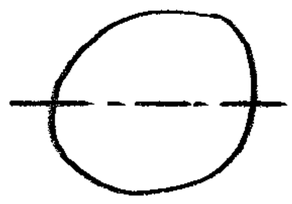




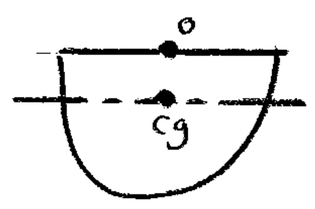
$I_{cg} = \frac{bz^3}{12}$



$I_{cg} = \frac{bz^3}{36}$



$I_{cg} = \frac{\pi d^4}{64}$



$I_{cg} = 0.11\, r^4$

Curved Surface

$F_H = \gamma A \bar{h}$ $\quad \Delta = \frac{I_{cg}}{A\bar{h}}$ $\quad F_V = \gamma V$ $\quad \bar{x} = \frac{A_1 x_1 \pm A_2 x_2}{A_1 \pm A_2}$

$R = \sqrt{F_H^2 + F_V^2}$ $\quad \tan\theta = \frac{F_V}{F_H}$

$\tan\theta = \frac{a_x}{g \pm a_z} = \frac{z}{L/2}$ or $\frac{y}{L}$ or $\frac{H+h}{x}$, $F = \gamma' A \bar{h}$

$P_2 = P_1 - \frac{\gamma}{g} a_x L$, $P = \gamma' h$, $\gamma' = \frac{\gamma}{g}(g \pm a_z)$

$\omega = \frac{2\pi N}{60}$ $\quad y = \frac{\omega^2 x^2}{2g}$ $\quad h = \frac{\omega^2 r^2}{2g}$ $\quad V_{parabloid} = \frac{1}{2}\pi r^2 h$

If no water is spilled $V_{water\ bef} = V_{water\ after}$, $V_{air\ bef} = V_{air\ after}$, $\bar{h} = \frac{h}{2}$

Volume of water spilled $= V_{water\ bef} - V_{water\ after} = V_{air\ after} - V_{air\ before}$

$W = F_b$, $W = \gamma_{body} V_{body}$, $F_b = \gamma_{liquid} V_{submerged}$

$GM = BM - BG \geq 0$ $\quad BM = \frac{I_y}{V_{sub}}$ $\quad moment = W\, GM \sin\theta$

$\bar{z} = \frac{W_1 z_1 + W_2 z_2}{W_1 + W_2}$ $\quad V_{cone} = \frac{1}{3}\pi r^2 h$, $V_{sphere} = \frac{4}{3}\pi r^3$

cumecs = m^3/s ③

$$Q = A_1 V_1 = A_2 V_2 \quad \text{(Continuity equation)} \qquad cfs = ft^3/s$$

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 + h_{pump} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_{loss} + h_{turbine} \quad \text{(Bernoulli)}$$

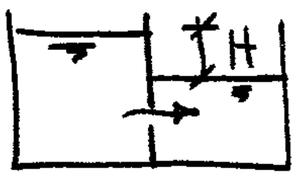
$$T.E = \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 \quad , \quad H.G = \frac{P}{\gamma} + Z \quad , \quad HG = T.E - \frac{V^2}{2g}$$

<u>orifice</u> $V_{th} = \sqrt{2gh}$ $Q_{act} = C_d Q_{th}$ $V_{act} = C_v V_{th}$

$a_{act} = C_c a_{th}$ $C_d = C_c C_v$ $C_v = \frac{X}{2\sqrt{yh}}$

<u>Large orifice</u> $Q = \frac{2}{3} C_d B \sqrt{2g} \left(H_2^{3/2} - H_1^{3/2}\right)$ $h_{L\,orifice} = H\left(1 - C_v^2\right)$

<u>Submerged orifice</u> $Q = a\sqrt{2gH}$



<u>mouth piece</u> $h_L = \frac{(V_2 - V_3)^2}{2g} = \frac{V_3^2}{2g}\left(\frac{1}{C_c} - 1\right)^2$

<u>Venturi-meter</u> $Q = \frac{C_d A_1 A_2 \sqrt{2gH}}{\sqrt{A_1^2 - A_2^2}}$

$h_L = H\left(1 - C_d^2\right)$

$H = \left(\frac{P_1}{\gamma} + Z_1\right) - \left(\frac{P_2}{\gamma} + Z_2\right)$

$= R\left(\frac{\gamma_2}{\gamma_1} - 1\right)$ U-tube

$= R\left(1 - \frac{\gamma_2}{\gamma_1}\right)$ inverted U-tube

<u>Variable head</u>

$Q\,dt = -A\,dh$ $T_{\substack{\text{rectangular}\\\text{tank}}} = \frac{2A}{C_d a\sqrt{2g}}\left(\sqrt{H_1} - \sqrt{H_2}\right)$

<u>Rectangular notch</u> $Q = \frac{2}{3} C_d b \sqrt{2g}\, h^{3/2}$

<u>triangular notch</u> $Q = \frac{8}{15} C_d \tan\frac{\theta}{2} \sqrt{2g}\, h^{5/2}$

$S = V_0 t + \frac{1}{2} a t^2$, $V_f^2 = V_0^2 + 2aS$ (Newton's equations)

Power $= \frac{\gamma Q H_p}{\eta}$ watt , $HP = \frac{\gamma Q H_p}{\eta K}$ horse

γ	9810	62.4
K	735	550

12 The Momentum Equation

From Newton's 2nd law

Linear Impulse = Change in linear Momentum

$$(\Sigma F)\, t = M\,(\Delta V)$$

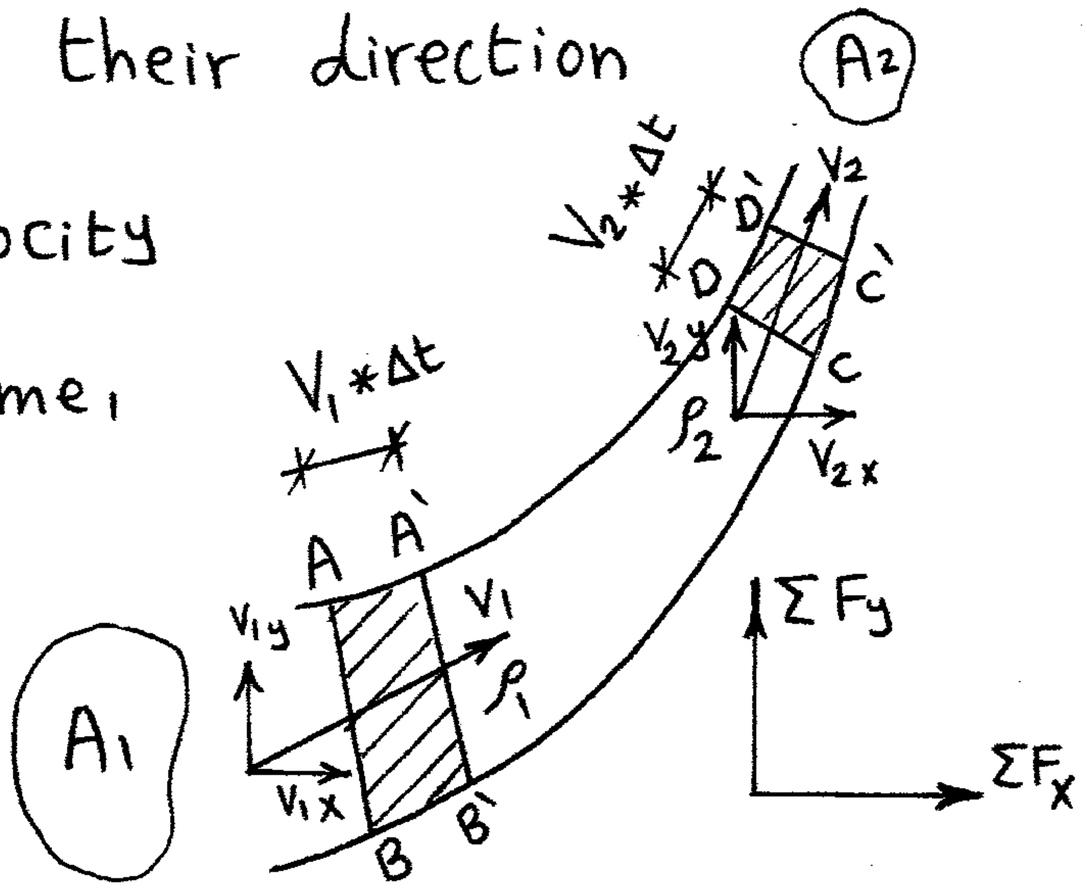
These are Vector quantities which are added or Subtracted according to their direction

Momentum = Mass * Velocity

$$\text{Mass}_{AA'BB'} = \rho_1 * \text{Volume}_1$$

$$M_{AA'BB'} = \rho_1 A_1 (V_1 \Delta t)$$

$$M_{CC'DD'} = \rho_2 A_2 (V_2 \Delta t)$$



X - direction

Change in Momentum in X direction $= M_{CC'DD'} * V_{2x} - M_{AA'BB'} * V_{1x}$

$$\Sigma F_x\, t = M_{CC'DD'}\, V_{2x} - M_{AA'BB'}\, V_{1x}$$

$$\Sigma F_x = \frac{(\rho_2 A_2 V_{2x} \Delta t)\, V_{2x} - (\rho_1 A_1 V_{1x} \Delta t)\, V_{1x}}{\Delta t}$$

$$\Sigma F_x = \rho_2 A_2 V_{2x} V_{2x} - \rho_1 A_1 V_{1x} V_{1x}$$

For incompressible Fluid $\rho_1 = \rho_2 = \rho$

For Steady State Condition $A_1 V_1 = A_2 V_2 = Q$

$$\boxed{\Sigma F_x = \rho Q (V_2 - V_1)_x}$$

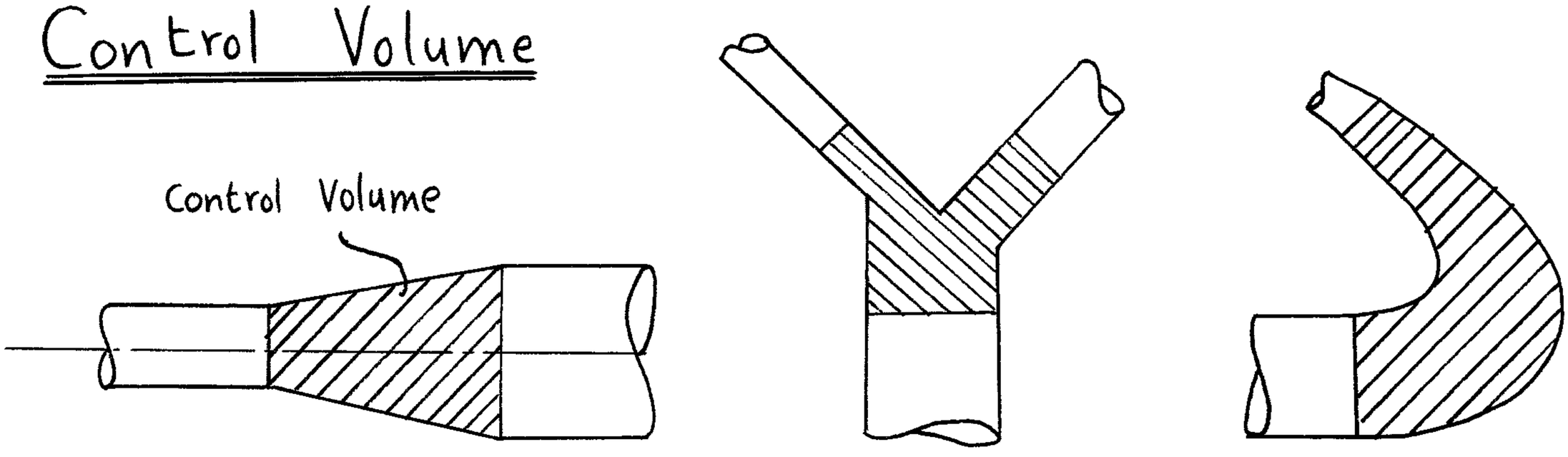
Similarly in y direction

$$\sum F_y = \rho Q (V_2 - V_1)_y$$

In any direction

$$\boxed{\sum F = \rho Q [V_{final} - V_{initial}]}$$

Control Volume



هو حجم من السائل حصل خلاله تغير فى كمية الحركة وبالتالى تولدت قوة تؤثر على هذا الـ Control Volume

$$\sum \text{Forces} = \text{Rate of Change in Momentum}$$

المؤثر على Control Volume فى أى اتجاه = فى نفس الاتجاه

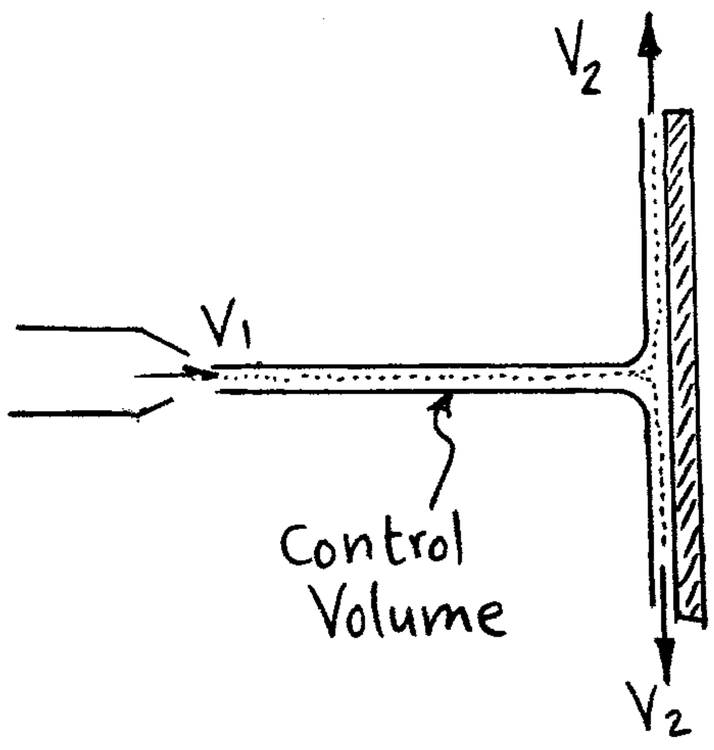
$$\sum \text{Forces} = (\rho Q V)_{final} - (\rho Q V)_{initial}$$

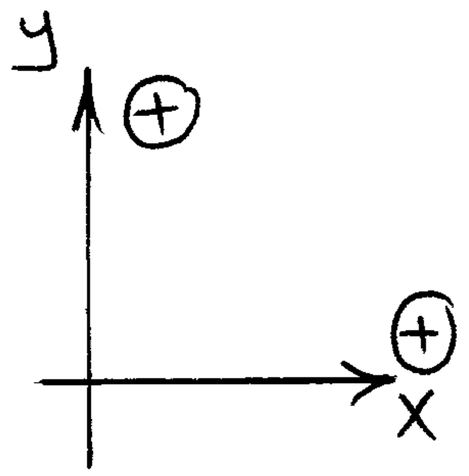
تطبق هذه المعادلة فى أى اتجاه

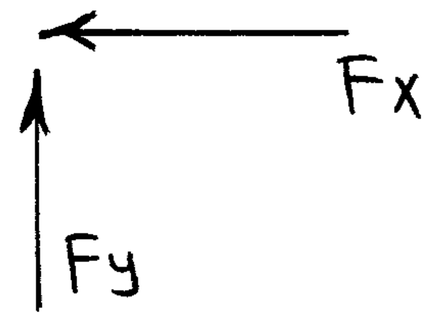
فى جميع مسائل الـMomentum يجب التعويض عن الضغط بالـGage Pressure

Example

Let F_x, F_y are Force Components on Water







في بداية كل مسألة يتم فرض المحاور الموجبه x,y واتجاه رد الفعل Reaction المتوقع

Momentum equation in X-direction

$$-F_x = (\ 0\ ) - (\rho Q V_1)$$

$$\therefore F_x = \rho Q V_1$$

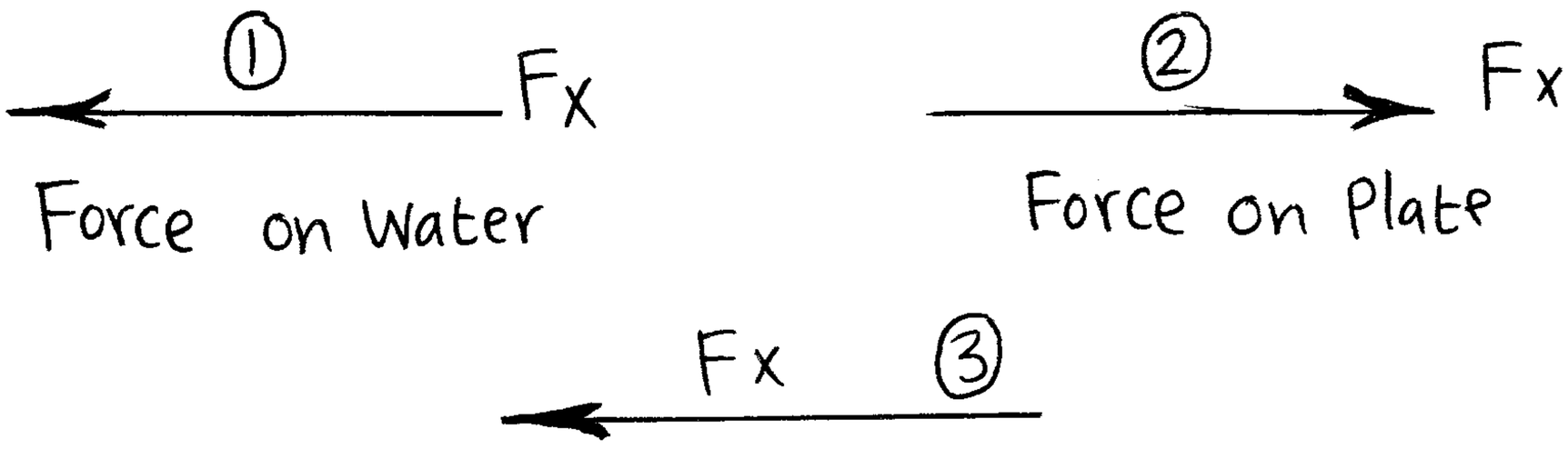
* لو طلعت اشارة F_x أو F_y موجبة

← معناها أنه الفرض كان سليم

Momentum equation in Y-direction

$$+F_y = (\rho Q V_2 - \rho Q V_2) - (\ 0\ )$$

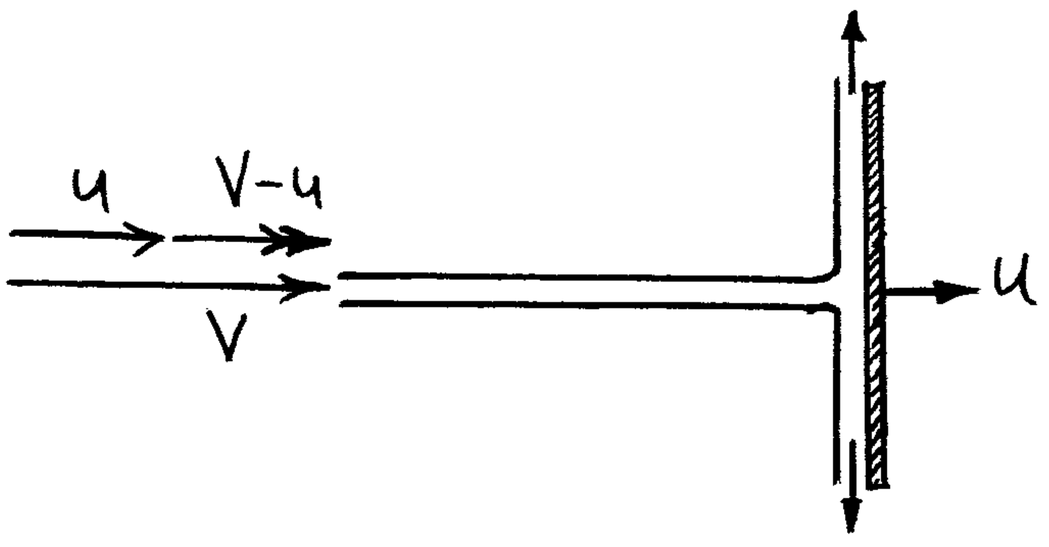
$$F_y = 0$$



Moving Plates

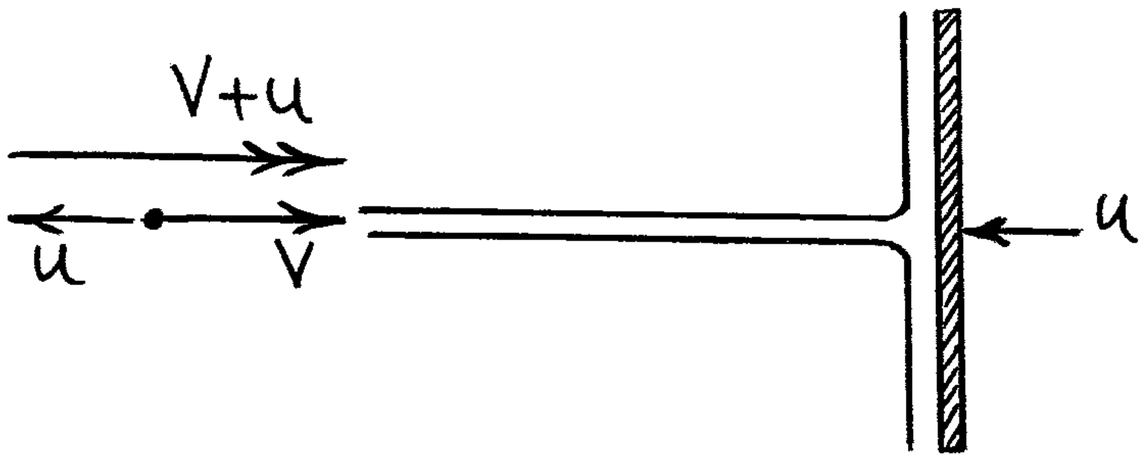
$$F_x = \rho Q (V-u)$$

where $Q = A_{jet}(V-u)$



$$F_x = \rho Q (V+u)$$

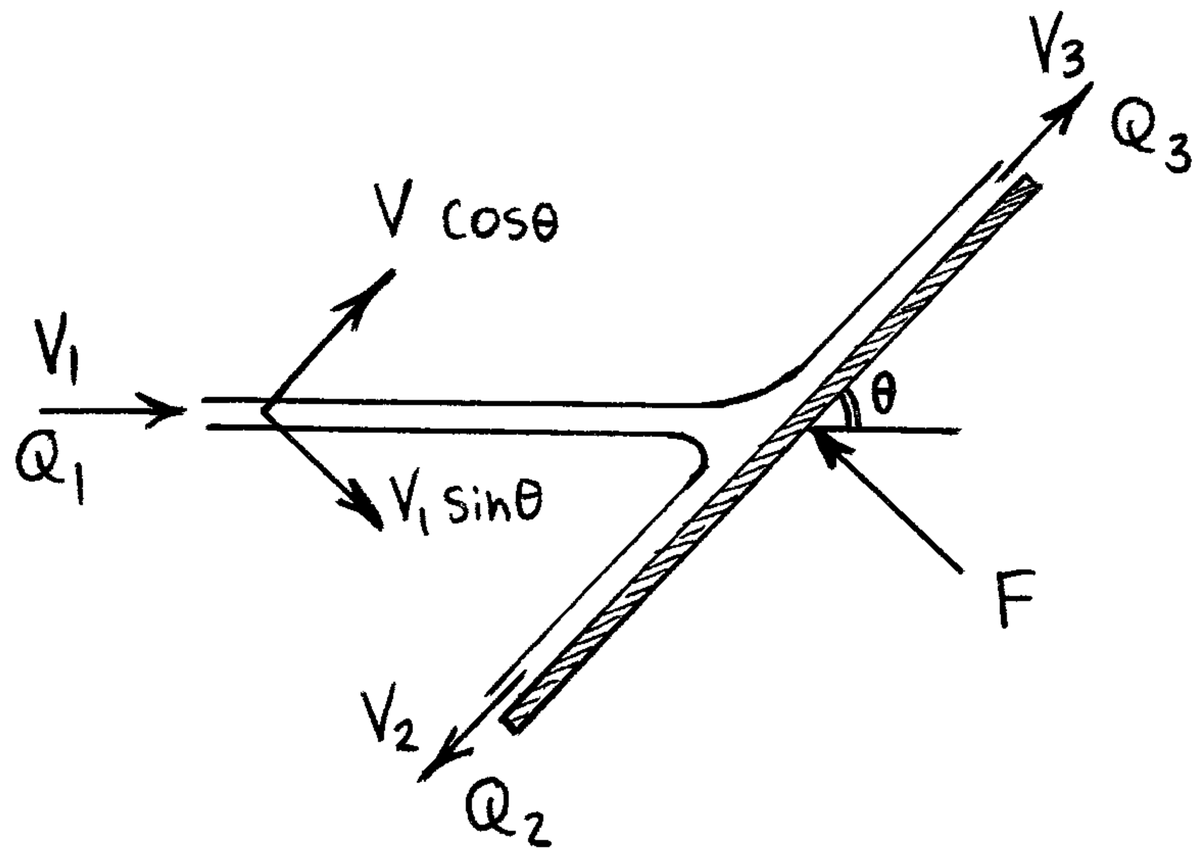
where $Q = A_{jet}(V+u)$



Solving ⊥ to the Plate

$$\Sigma F = \rho Q (\cancel{V_f}^{\,0} - V_I)$$

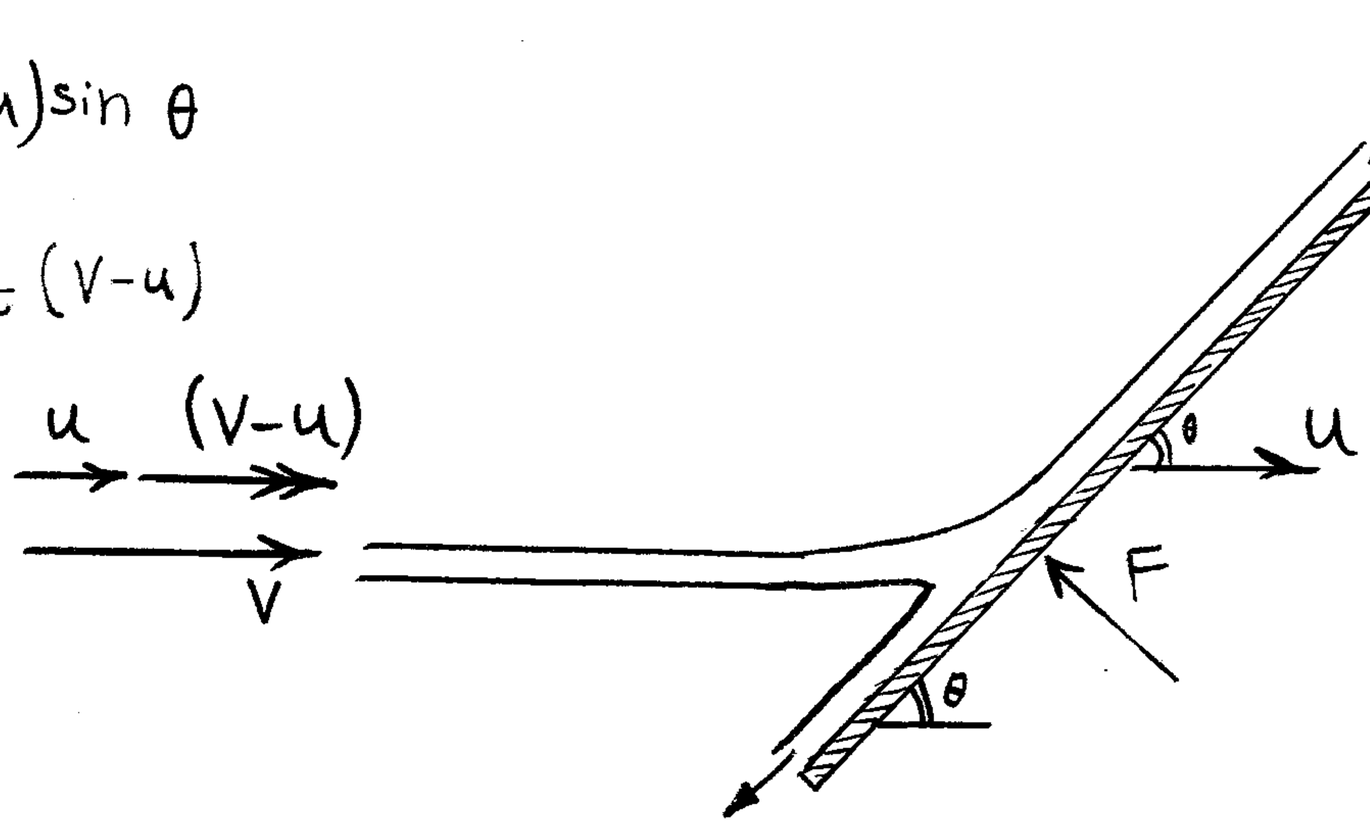
$$F = \rho Q V_1 \sin\theta$$



$$\Sigma F = \rho Q (V_f - V_I)$$

$$F = \rho Q (V-u)\sin\theta$$

where $Q = A_{jet}(V-u)$



Applications

1- Direct Connection

عندما يذكر أنه الوصلة رأسية يدخل الوزن W رأسياً

$$\Sigma F_x = \rho Q \left(V_{f_x} - V_{I_x} \right)$$

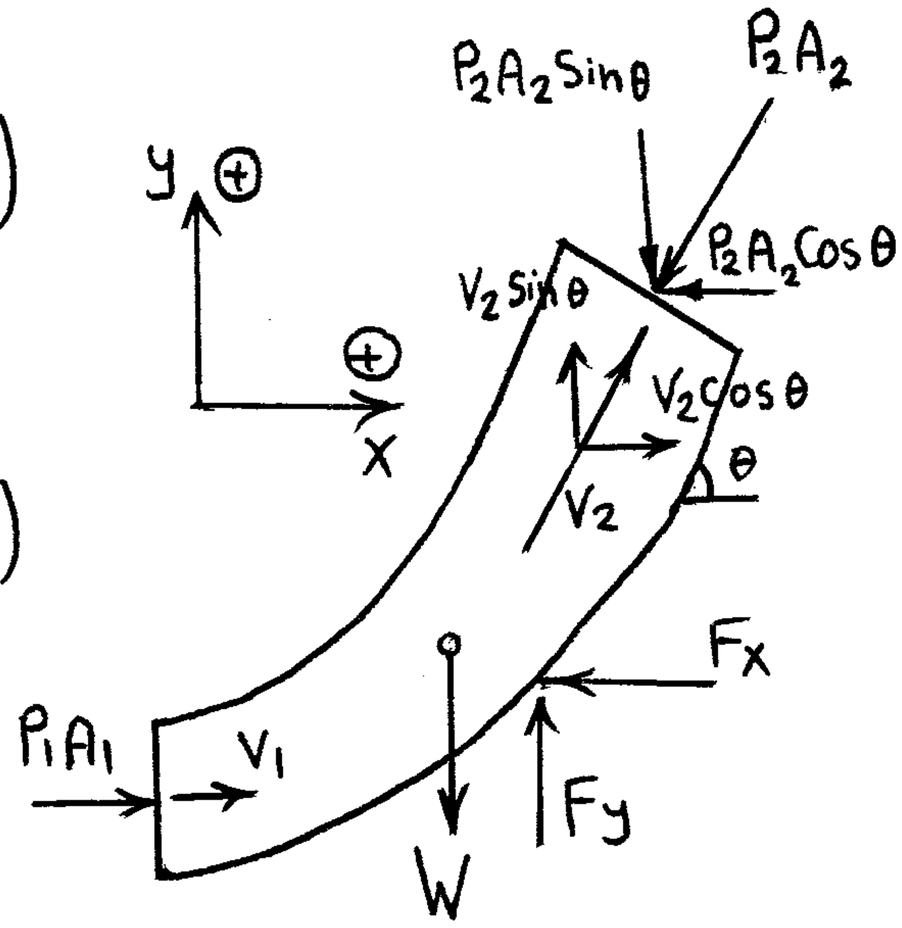
$$-F_x + P_1 A_1 - P_2 A_2 \cos\theta = \rho Q \left(V_2 \cos\theta - V_1 \right)$$

$$\Sigma F_y = \rho Q \left(V_{f_y} - V_{I_y} \right)$$

$$F_y - W - P_2 A_2 \sin\theta = \rho Q \left(V_2 \sin\theta - 0 \right)$$

$$\text{Direction of } F_R = \tan^{-1} \frac{F_x}{F_y}$$

$$F_R = \sqrt{F_x^2 + F_y^2}$$



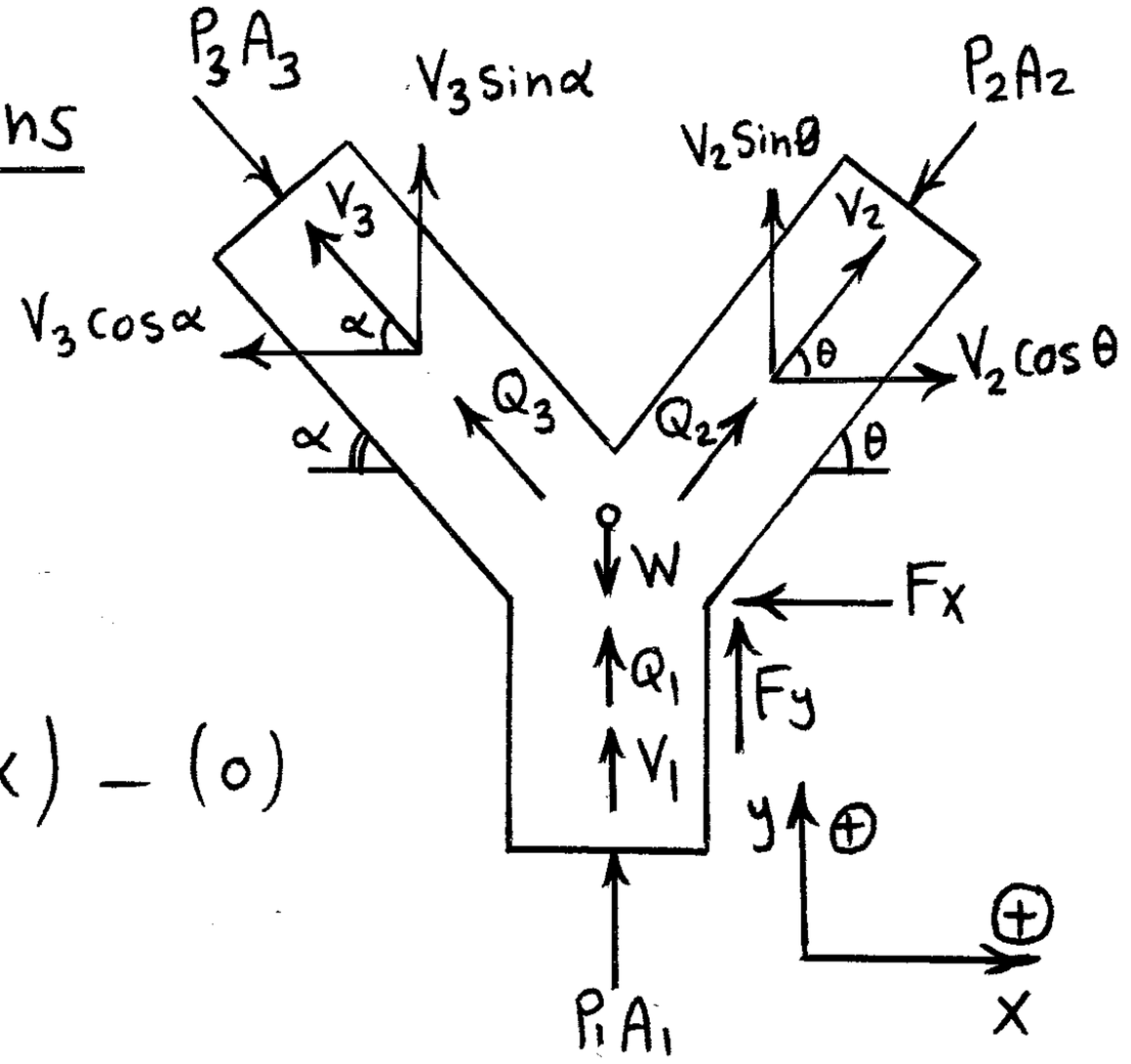
2- Connection with 2 directions

$$Q_1 = Q_2 + Q_3$$

$$\Sigma F_x = \rho Q \left(V_{f_x} - V_{I_x} \right)$$

$$-F_x - P_2 A_2 \cos\theta + P_3 A_3 \cos\alpha$$

$$= \left(\rho Q_2 V_2 \cos\theta - \rho Q_3 V_3 \cos\alpha \right) - (0)$$



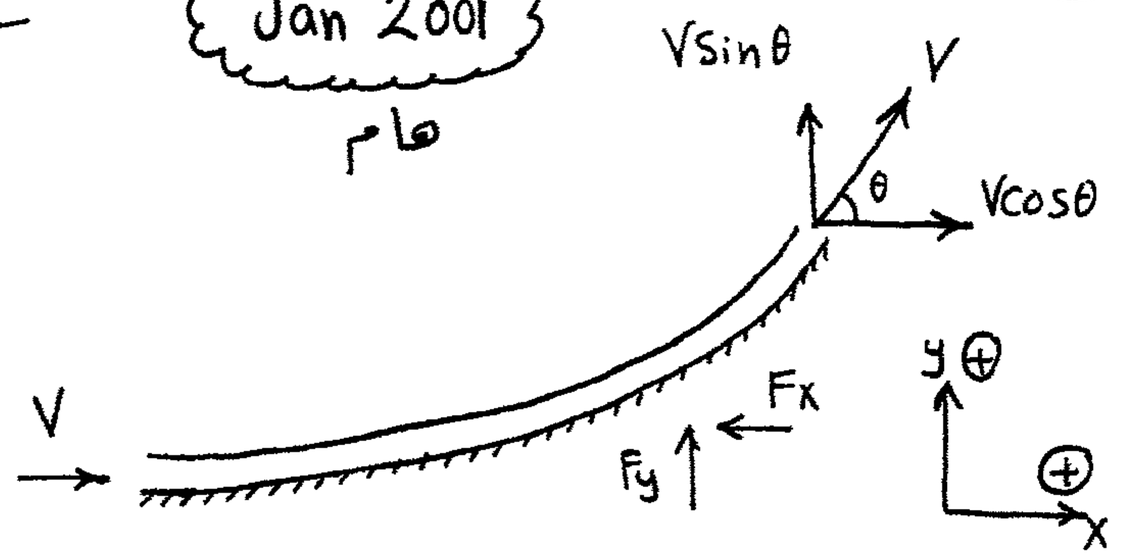
$$\Sigma F_y = \rho Q \left(V_{f_y} - V_{I_y} \right)$$

$$F_y - W - P_2 A_2 \sin\theta - P_3 A_3 \sin\alpha = \left(\rho Q_2 V_2 \sin\theta + \rho Q_3 V_3 \sin\alpha \right)$$

$$+ P_1 A_1 \qquad - \left(\rho Q_1 V_1 \right)$$

Force on a fixed curved Plate

Jan 2001 عام



1

$$\Sigma F_x = \rho Q (V_f - V_i)$$

$$-F_x = \rho Q (V\cos\theta - V)$$

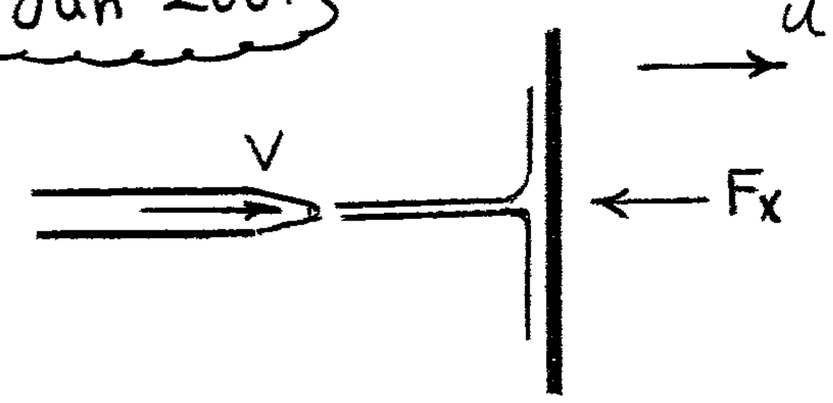
$$F_x = \rho Q V (1 - \cos\theta)$$

$$+F_y = \rho Q (V\sin\theta - 0)$$

$$F_y = \rho Q V \sin\theta$$

2

Jan 2001 عام



$$Q = 0.05\ m^3/s \qquad d = 50mm$$

$$A = \frac{\pi (0.05)^2}{4} = 1.96 * 10^{-3}\ m^2$$

$$V = \frac{Q}{A} = 25.4\ m/s$$

i- Plate is stationary $u = 0$

$$F_x = \rho Q V = 1000\ (0.05)\ 25.4 = \underline{1270\ N}$$

ii- Plate is moving at 12 m/s in direction of flow $u = 12\ m/s$

$$Q = A(V-u) = 1.96 * 10^{-3} (25.4 - 12) = 0.026\ m^3/s$$

$$F_x = \rho Q (V-u) = 1000\ (0.026)(25.4-12) = \underline{348\ N}$$

iii- Plate is moving at 12m/s against flow direction $u = 12\ m/s$

$$Q = A(V+u) = 1.96 * 10^{-3} (25.4 + 12) = 0.073\ m^3/s$$

$$F_x = \rho Q (V+u) = 1000\ (0.073)(25.4 + 12) = \underline{2730\ N}$$

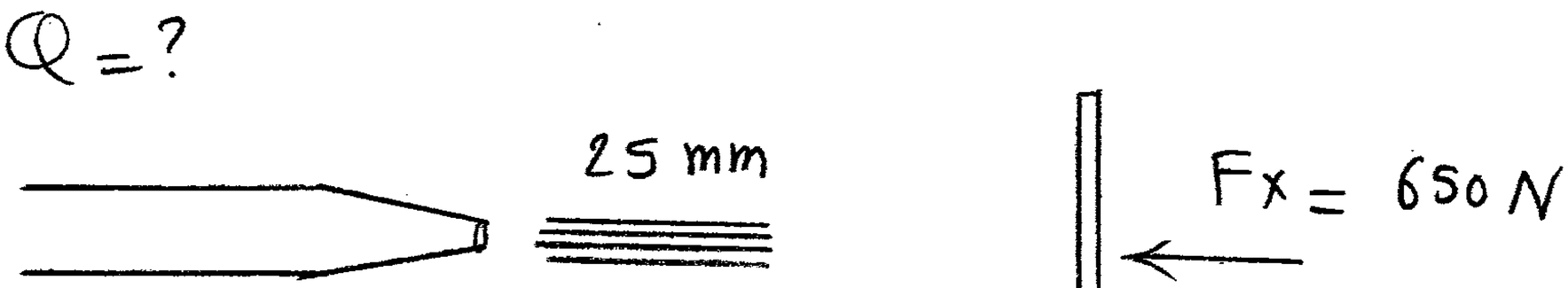
iV- Plate has a velocity of 30 m/s in flow direction $u = 30$

$\because u > V$ Velocity of Plate > flow Velocity

$\Rightarrow$ water will not strike the plate $\Rightarrow F_x = 0$

(7)

3 The force exerted by a 25mm diameter jet against a flat plate normal to the axis of the jet is 650N. What is the flow in m^3/s?



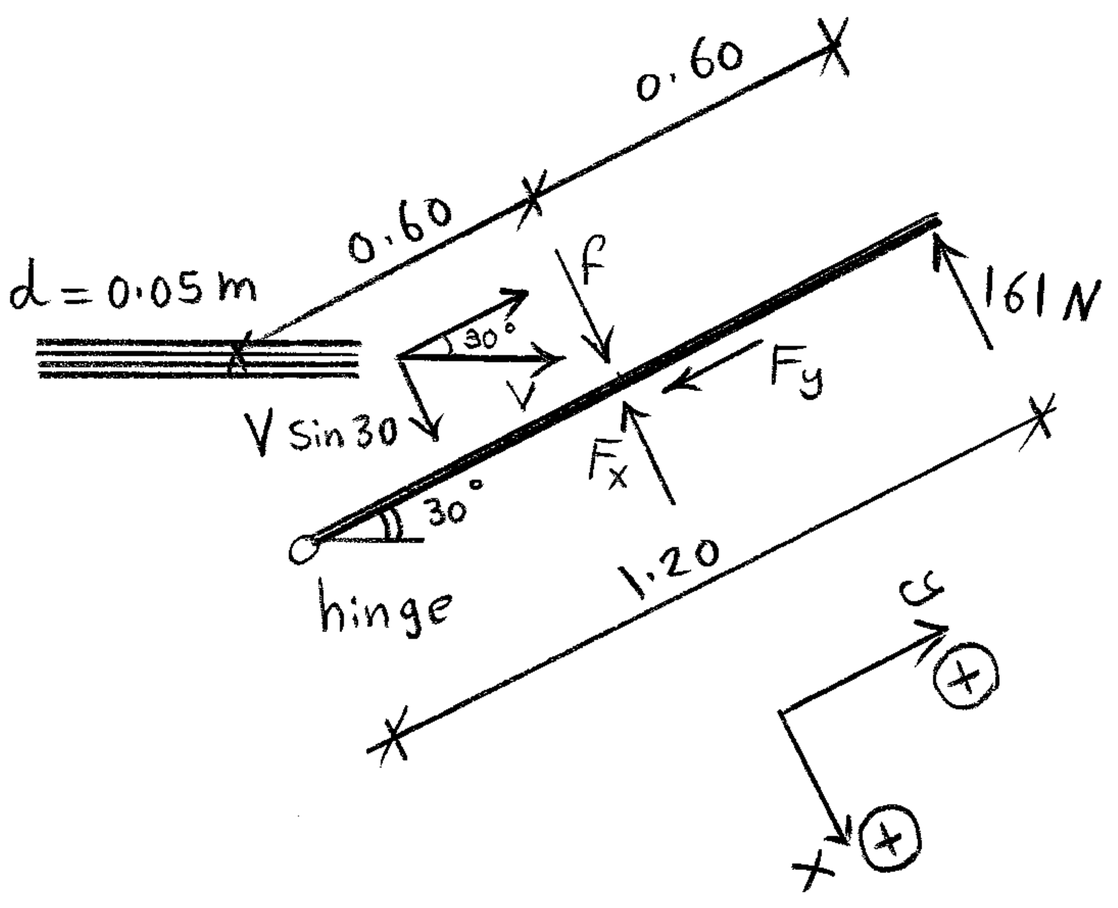
$$\Sigma F_x = \rho Q V_{f_x} - \rho Q V_{i_x}$$

$$-F_x = 0 - \rho Q V$$

$$650 = 1000\, Q\, \frac{Q}{A}$$

$$650 = 1000\, \frac{Q^2}{\frac{\pi (0.25)^2}{4}} \Rightarrow Q = 0.178\ m^3/s$$

4 A water jet 50 mm diameter strikes a 1.20 m plate which is at an angle of 30° with the stream's direction. If the force applied at the edge of the plate, to maintain equilibrium, is 161 N, calculate the rate of flow. Neglect the weight of the plate.



Q = ? to maintain equilibrium

$$\Sigma M_{hinge} = 0$$

$$161\,(1.2) = f\,(0.6)$$

$$\Rightarrow f = 322\ N \searrow \text{(action)}$$

$$\therefore F_x = 322\ N \nwarrow \text{(reaction)}$$

$$\Sigma F_x = \cancel{\rho Q V_{f_x}} - \rho Q V_{i_x}$$

$$-F_x = 0 - \rho Q V \sin 30 = -\rho \frac{Q^2}{a} \sin 30$$

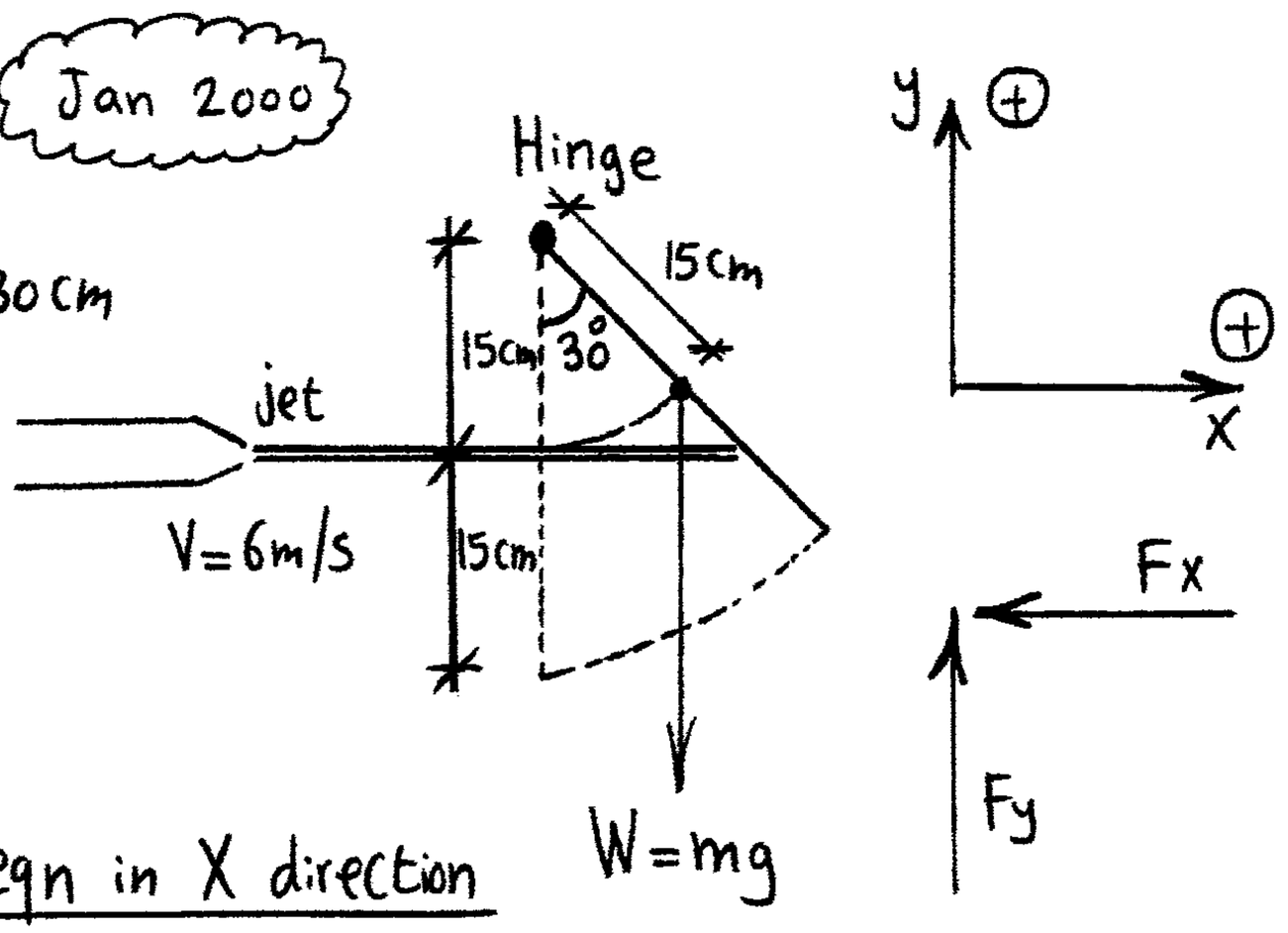
$$322 = 1000\, \frac{Q^2}{\frac{\pi (0.05)^2}{4}} \sin 30 \Rightarrow Q = \underline{0.0355\ m^3/s}$$

5 A square plate of uniform thickness and length of side 30 cm hangs vertically from hinges at its top edge. When a horizontal jet strikes the plate at its center, the plate is deflected and comes to rest at an angle of 30° to the vertical. The jet is 25 mm diameter and has a velocity of 6 m/s. Calculate the mass of the plate and give the distance along the plate, from the hinge, of the point at which the jet strikes the plate in its deflected position.

Jan 2000

$d_{jet} = 25\,mm$

Square Plate length of side $= 30\,cm$



Let F_x, F_y are the Force Components on the liquid

Applying the Momentum eqn in X direction

$$-F_x = (0)_f - \rho Q V_i$$

$$F_x = \rho Q V_i = 1000 * \frac{\pi}{4}(0.025)^2 * 6 * 6 = \underline{17.67\ N}$$

Applying the Momentum eqn in y direction

$$F_y = (0) - (0) = 0$$

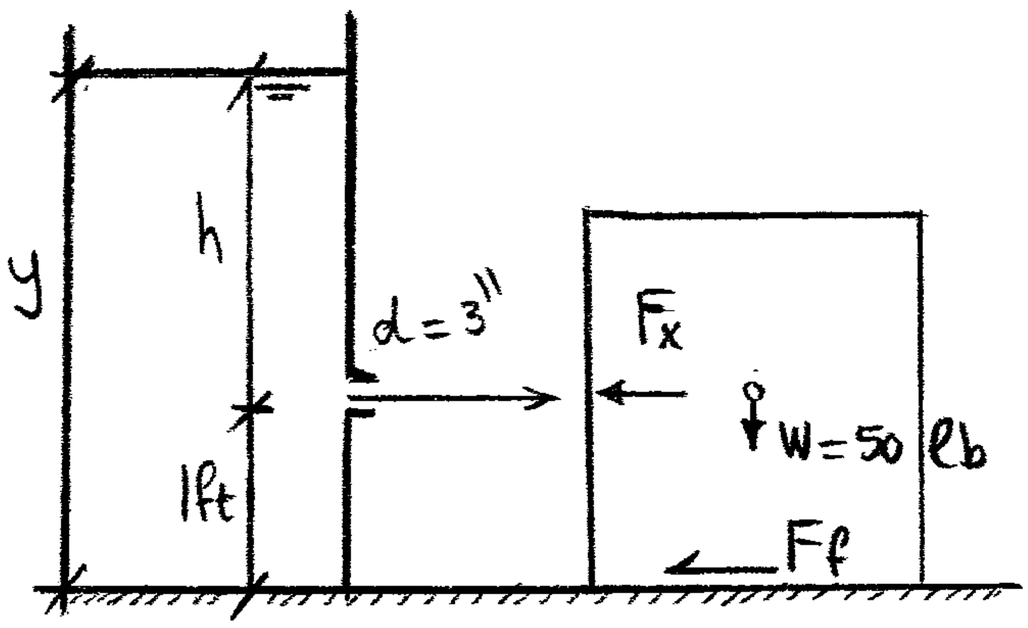
ΣM @ Hinge $= 0$

$$mg\,(15 \sin 30) = F_x\,(15)$$

$$m = \frac{17.67\,(15)}{9.81\,(15)\sin 30} = \underline{3.60\ Kg}$$

Distance from the hinge at which the jet strikes the Plate $= \frac{15}{\cos 30} = \underline{17.3\,cm}$

6 Water flows from a large tank through an orifice of 3-in-diameter and against a block, as shown in fig. the water jet strikes the block at the vena contracta. The block weighs 50 lb, and the coefficient of friction between the block and the floor is 0.57. The orifice's coefficient of discharge C_d is 0.60, and it's coefficient of contraction C_c is 0.62.Find the minimum height to which water must rise in the tank (y in the figure) in order to start the block moving (to the right).



$\mu = 0.57$ (Coefficient of friction)

$C_d = 0.6$, $C_c = 0.62$

$y = ?$ to start block moving

Sol For the block to start moving

$\therefore F_x = F_f$

$F_f = \mu W = 0.57\,(50) = \underline{\underline{28.5\ lb}}$

$\Sigma F_x = \rho Q V_f - \rho Q V_i$

$-28.5 = 0 - 1.94\,QV$

$\therefore QV = 14.69$

$Q_{act} = C_d\, a \sqrt{2gh}$, $V_{act} = C_v \sqrt{2gh}$

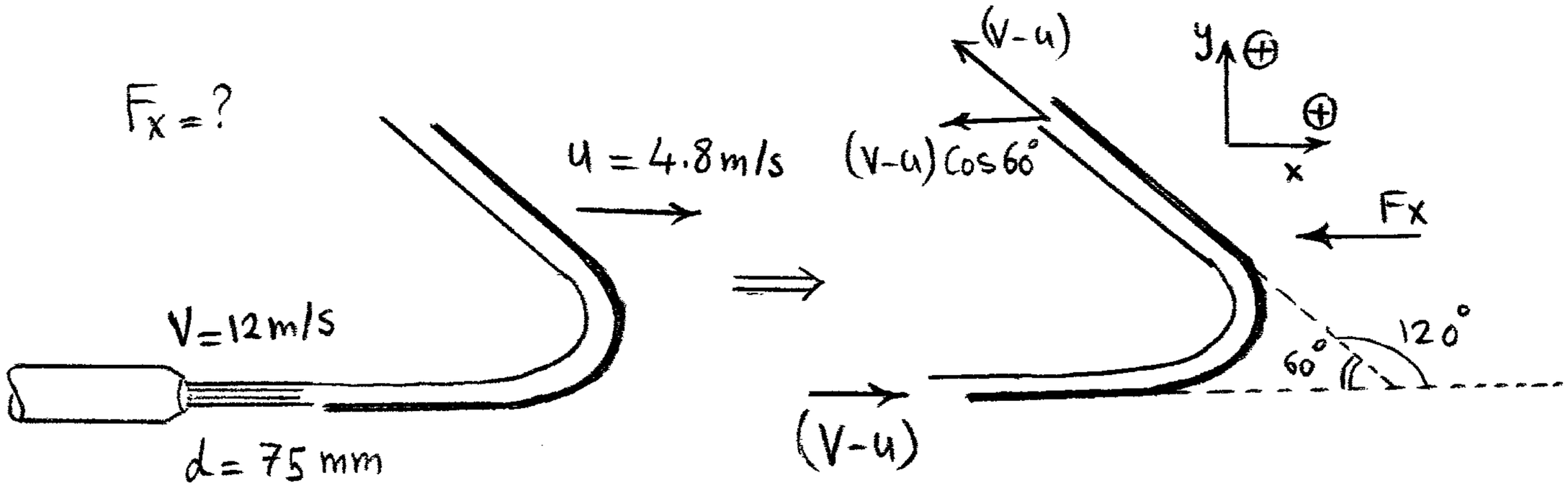
$C_v = \dfrac{C_d}{C_c} = \dfrac{0.6}{0.62} = 0.968$

$Q_{act} V_{act} = C_d\, C_v\, a\, (2gh)$

$14.69 = 0.6\,(0.968)\,\dfrac{\pi\,(3/12)^2}{4} * 2 * 32.2\,(y-1)$

$\Rightarrow y = \underline{\underline{9\ ft}}$

no. (7, 12). A jet of water 75 mm in diameter with a velocity of 12 m/sec meets a vane having a velocity of 4.8 m/sec in the direction of the jet. If water meets the vane tangentially and the deflection angle is 120° , find the force of water exerted on the vane.



Sol

$$Q = A(V-u)$$

$$= \frac{\pi (0.075)^2}{4} * (12 - 4.8) = 0.0318 \ m^3/s$$

$$\Sigma F_x = \rho Q V_{f_x} - \rho Q V_{i_x}$$

$$-F_x = -\rho Q (V-u)\cos 60 - \rho Q (V-u)$$

$$F_x = \rho Q (V-u)[1 + \cos 60]$$

$$= 1000\,(0.0318) * (12-4.8)\,[1 + \cos 60] = 343.44 \leftarrow \text{reaction}$$

∴ Force in direction of jet (action) → $F = \underline{343.44\ N}$

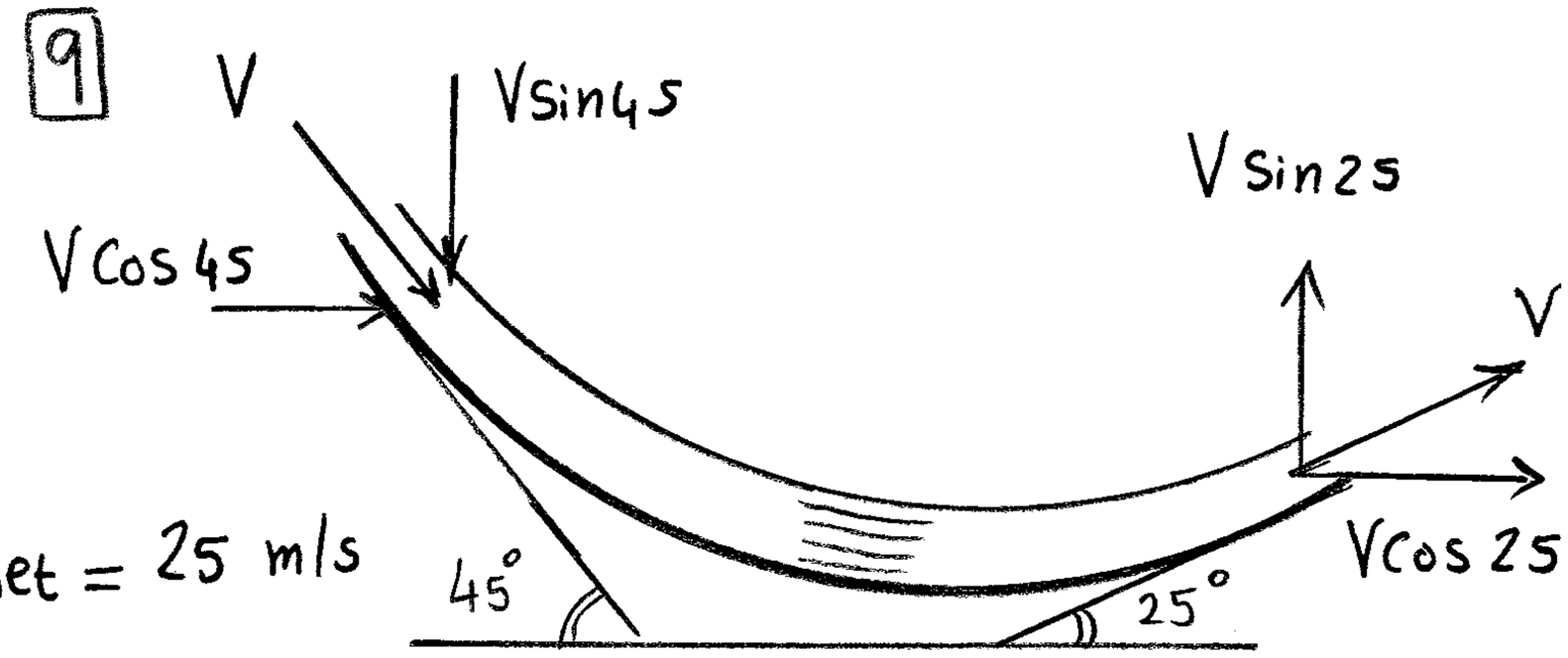
Find the work done / sec

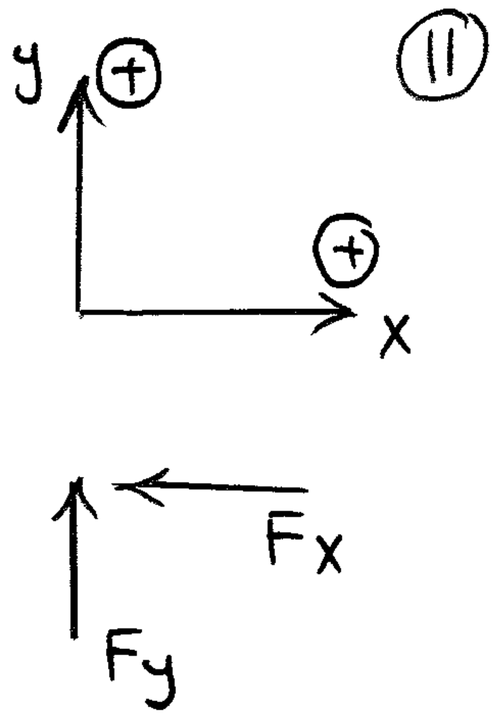
$$\frac{\text{Work done}}{\text{time}} = \text{Power} = \text{Force} * \text{Velocity}$$

$$= F_x * u$$

$$= 343.44 * 4.8 = \underline{1648.5 \text{ watt}}$$

9





11

$V_{jet} = 25\ m/s$

$A_{jet} = 75mm * 25\ mm$

$F_x = ?$ $F_y = ?$

$Q = A.V = (0.075 * 0.025) * 25 = 0.047\ m^3/s$

$\sum F_x = \rho Q V_{f_x} - \rho Q V_{i_x}$

$-F_x = \rho Q (V\cos 25 - V\cos 45)$

$-F_x = 1000(0.047)(25\cos 25 - 25\cos 45) = 233.4$

$\therefore F_x = \underset{\text{reaction}}{\xrightarrow{233.4}}$, $\underset{\text{action}}{\xleftarrow{233.4}}$ (Force_x exerted on Vane)

$\sum F_y = \rho Q V_{f_y} - \rho Q V_{i_y}$

$+F_y = \rho Q (V\sin 25 - V\sin 45)$

$F_y = 1000(0.047)(25\sin 25 - 25\sin 45) = -333$

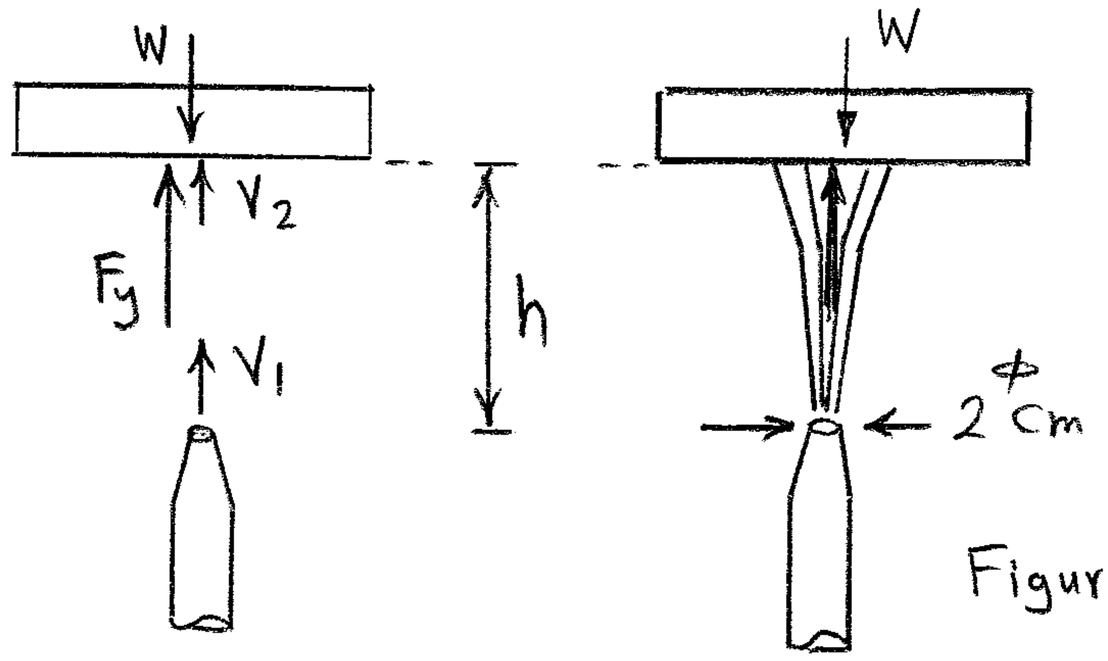
$\therefore F_y = \underset{\text{reaction}}{\downarrow 333}$, $\underset{\text{action}}{\uparrow 333}$ (Force_y exerted on Vane)

A liquid jet is issuing upward against a flat board of weight 1 lb and supporting it as indicated in the figure. Determine the equilibrium height of the board above the nozzle exit, if the discharge is 1.5 l/s and the nozzle diameter is 2 cm.

$W = 1\ lb$

$Q = 1.5\ l/s$

Find h at equilibrium



Figure

Solution

at equilibrium F_y due to jet of water = W

$$\therefore F_y = \rho Q V_2$$

$$\therefore W = \rho Q V_2$$

$$1\ lb * 4.44 \frac{N}{lb} = 1000 \left(1.5 * 10^{-3}\right) * V_2$$

$$\therefore V_2 = 2.96\ m/s$$

$$\therefore V_1 = \frac{Q}{A} = \frac{1.5 * 10^{-3}}{\frac{\pi (0.02)^2}{4}} = 4.77\ m/s$$

Applying Bernoulli eqn between ①, ② or Newton's equation

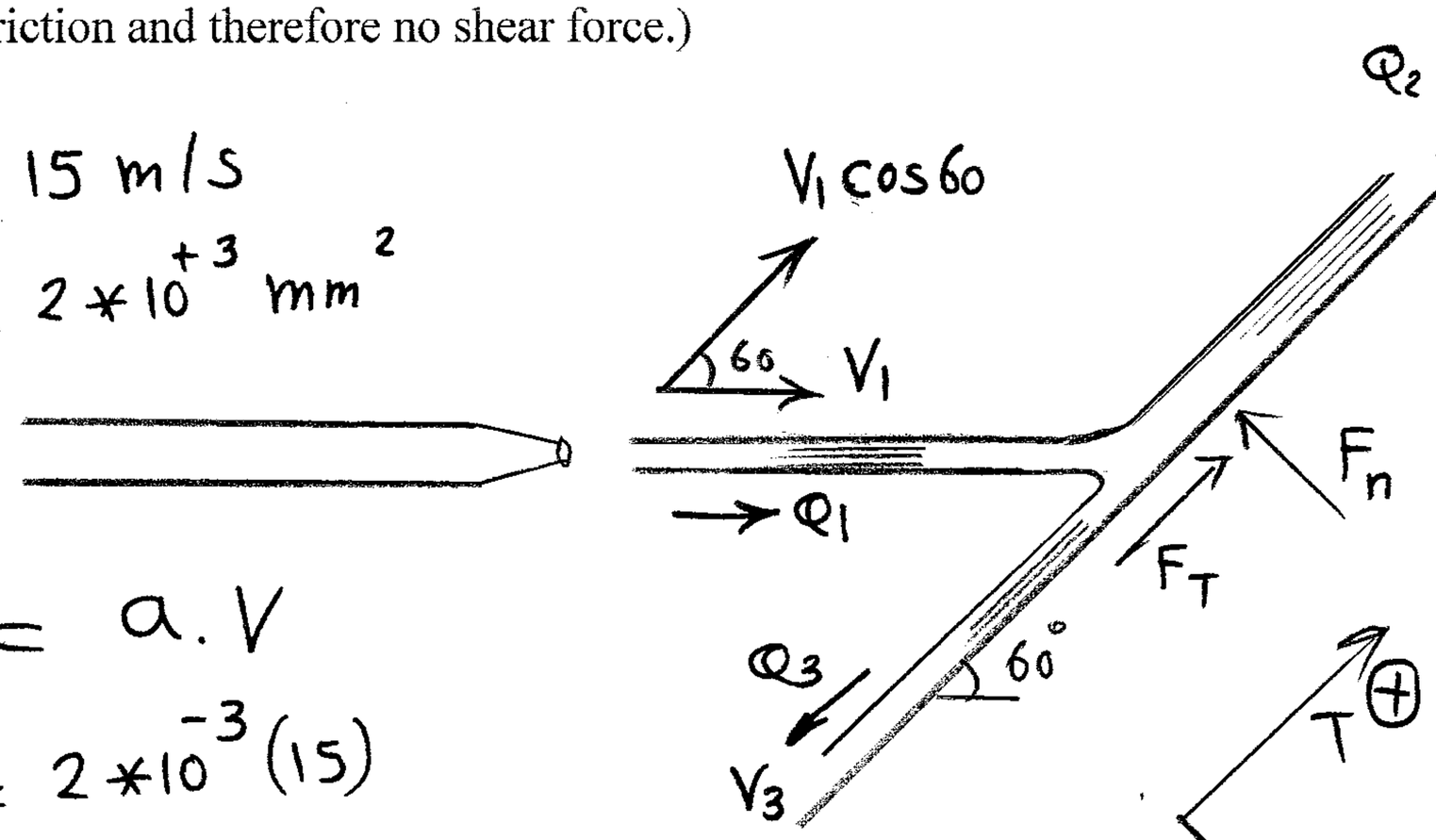
$$V_2^2 = V_1^2 - 2gh$$

$$(2.96)^2 = (4.77)^2 - 2(9.81)h \quad \Rightarrow h = \underline{0.715\ m}$$

A horizontal jet of water $2*10^3$ mm^2 cross-section and flowing at a velocity of 15 m/s hits a flat plate at 60° to the axis (of the jet) and to the horizontal. The jet is such that there is no side spread. If the plate is stationary, calculate a) the force exerted on the plate in the direction of the jet and b) the ratio between the quantity of fluid that is deflected upwards and that downwards. (Assume that there is no friction and therefore no shear force.)

$V_1 = 15$ m/s

$a_1 = 2*10^{+3}$ mm^2



$Q_1 = a.V$

$= 2*10^{-3}(15)$

$= 0.03$ m^3/s

∵ There is no shear force $\Rightarrow F_T = 0$

$\Sigma F_T = \rho Q V_{f_T} - \rho Q V_{i_T}$

$0 = (\rho Q_2 V_2 - \rho Q_3 V_3) - (\rho Q_1 V_1 \cos 60)$

∵ There is no friction $\Rightarrow V_1 = V_2 = V_3 = 15$ m/s

$\therefore \quad Q_2 - Q_3 = Q_1 \cos 60$

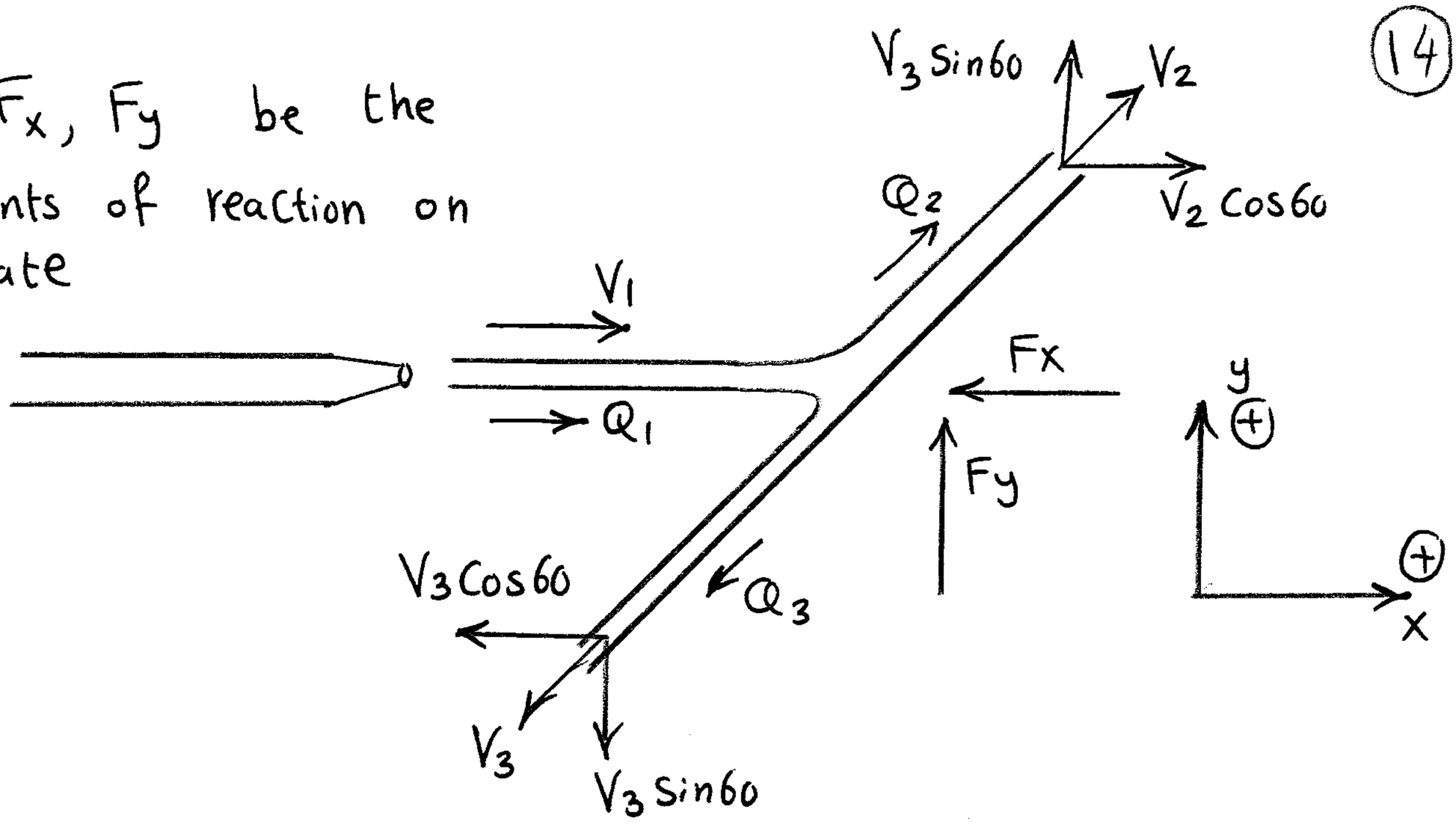
$\because \quad Q_1 = Q_2 + Q_3 \quad$ or $\quad Q_3 = Q_1 - Q_2$

$\therefore \quad Q_2 - (Q_1 - Q_2) = Q_1 \cos 60$

$2Q_2 = Q_1 (1 + \cos 60)$

$Q_2 = \frac{Q_1}{2}(1 + \cos 60)$	$= \frac{0.03}{2}(1 + \cos 60) = 0.0225$
$Q_3 = \frac{Q_1}{2}(1 - \cos 60)$	$= \frac{0.03}{2}(1 - \cos 60) = 7.5*10^{-3}$

Let F_x, F_y be the Components of reaction on the plate



$$\sum F_x = \rho Q V_{f_x} - \rho Q V_{i_x}$$

$$-F_x = (\rho Q_2 V_2 \cos 60 - \rho Q_3 V_3 \cos 60) - (\rho Q_1 V_1)$$

$$-F_x = \{1000\,(0.0225)(1.5 \cos 60) - 1000(7.5 * 10^{-3})\,1.5 \cos 60\}$$

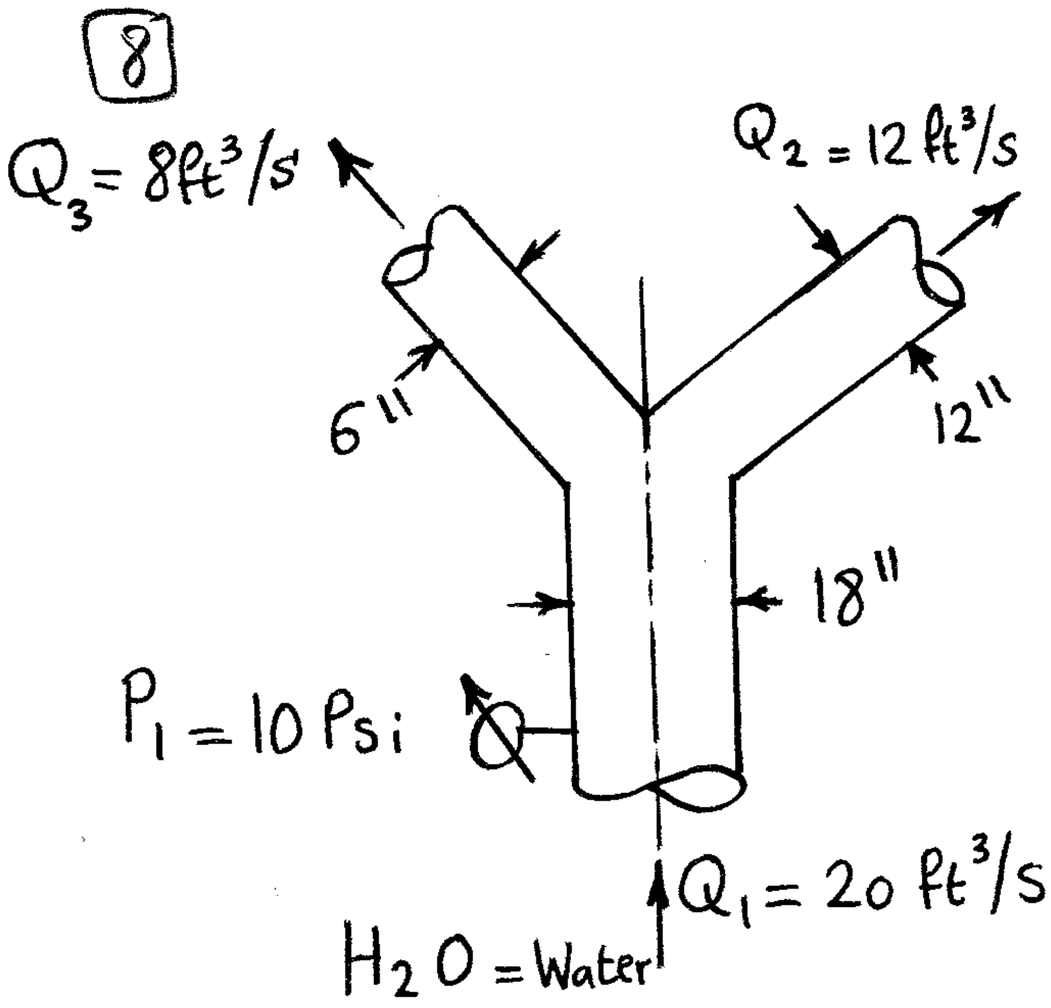
$$- 1000\,(0.03)(1.5)$$

$$\therefore F_x = \underline{33.75\ N} \quad \text{(reaction)}$$

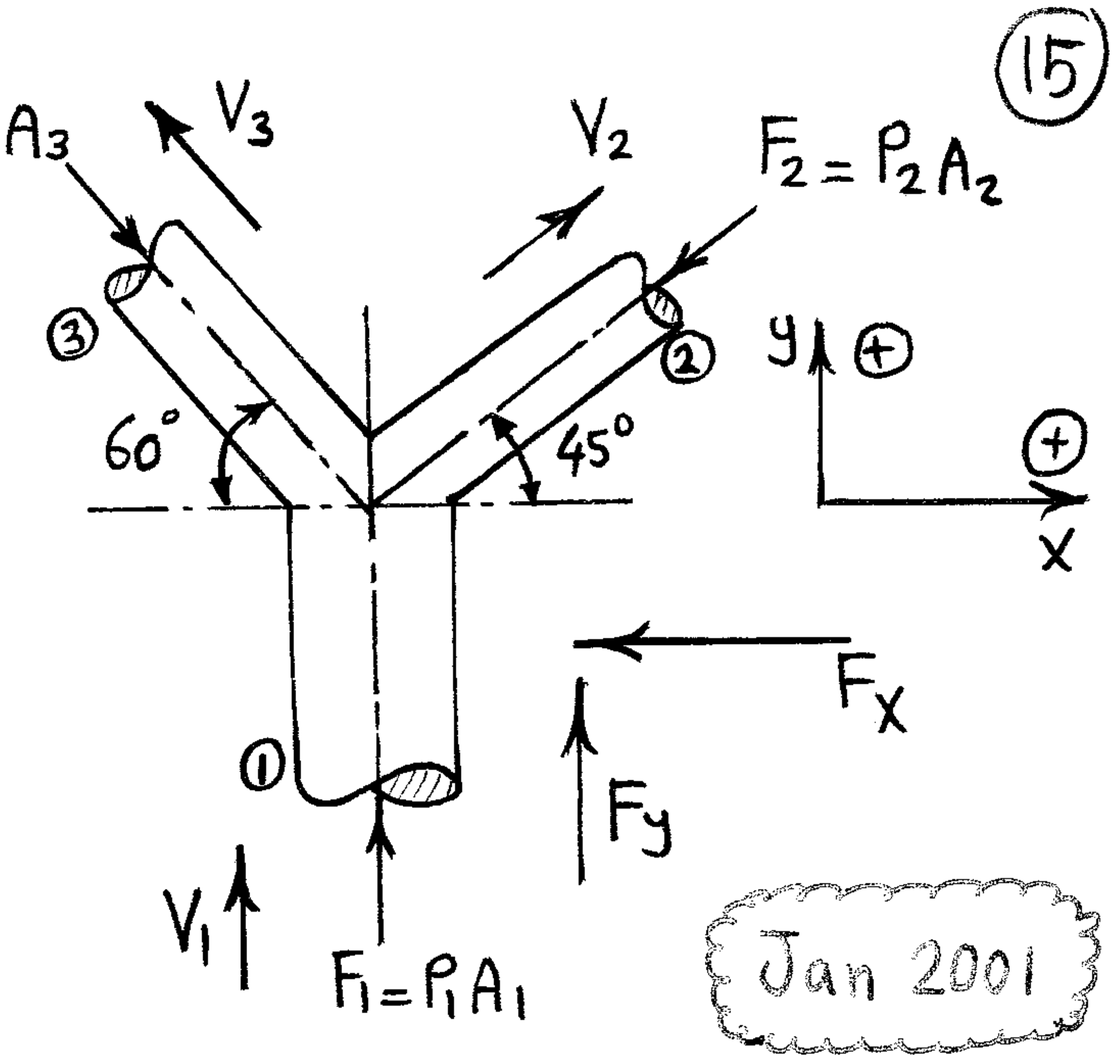
$\therefore$ The force exerted on the Plate in the direction of the jet is $\underline{33.75\ N}$

hzl Plane

8



15



Jan 2001

Let F_x, F_y are force Components on the liquid

$P_1 = 10 Psi = 10 \times 144 = 1440 \ lb/ft^2$

$V_1 = Q_1/A_1 = 11.32 \ ft/s$ $\qquad$ $V_3 = Q_3/A_3 = 40.74 \ ft/s$

$V_2 = Q_2/A_2 = 15.3 \ ft/s$

Applying Bernoulli eqn between ①, ②

$$\frac{1440}{62.4} + \frac{(11.32)^2}{2*32.2} = \frac{P_2}{62.4} + \frac{(15.3)^2}{2*32.2} \Rightarrow P_2 = 1337.3 \ lb/ft^2$$

Applying Bernoulli eqn between ①, ③

$$\frac{1440}{62.4} + \frac{(11.32)^2}{2*32.2} = \frac{P_3}{62.4} + \frac{(40.74)^2}{2*32.2} \Rightarrow P_3 = -44.04 \ lb/ft^2$$

Applying Momentum eqn in X direction

* $\rho_w = 1.94 \ slug/ft^3$

$$-F_x - P_2A_2\cos 45 + P_3A_3\cos 60 = (\rho Q_2 V_2 \cos 45 - \rho Q_3 V_3 \cos 60) - (0)$$

$$\therefore F_x = \underline{\underline{-687.75 \ lb}}$$

Applying Momentum eqn in Y direction

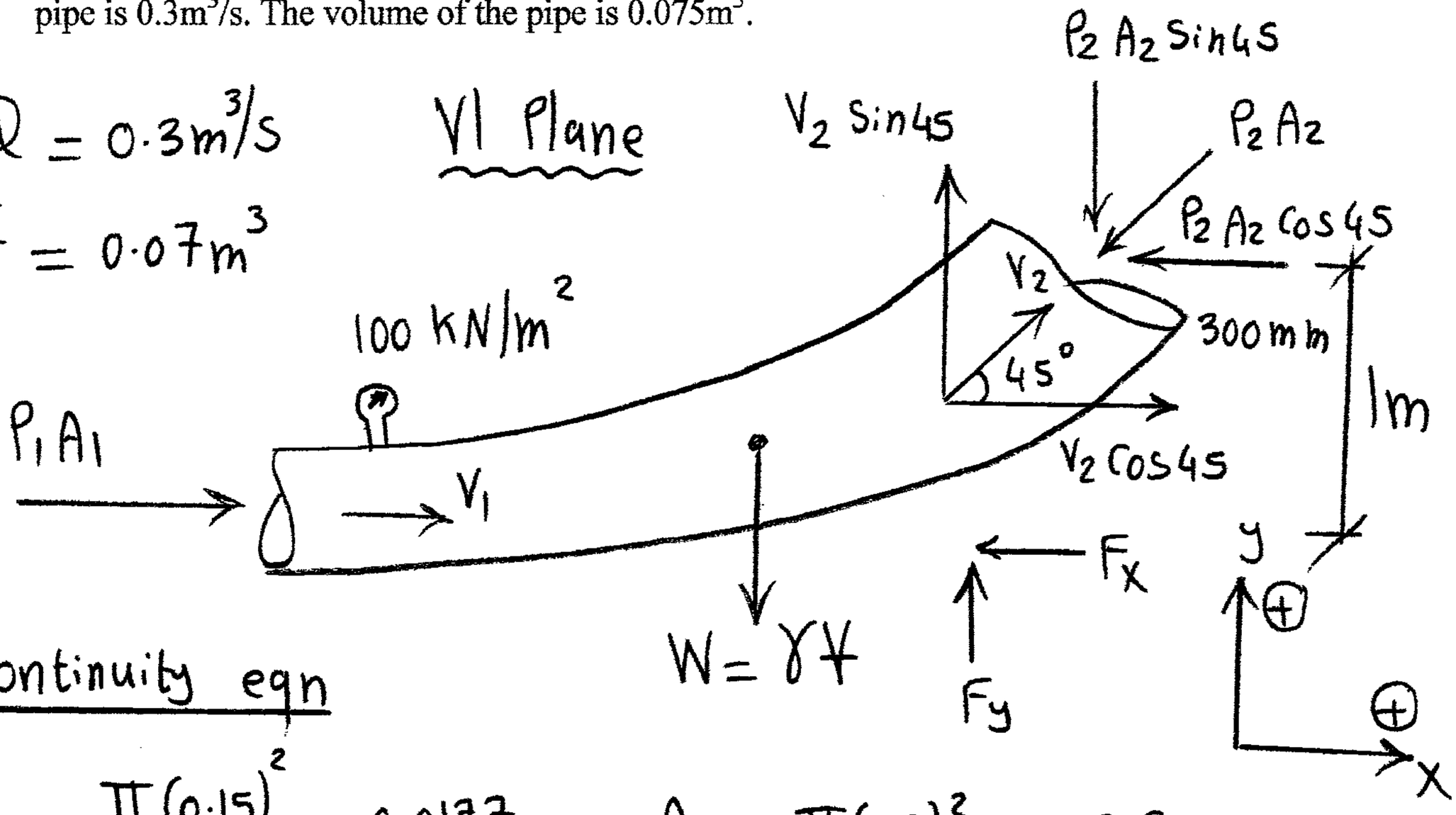
$$F_y + P_1A_1 - P_2A_2 \sin 45 - P_3A_3 \sin 60 = (\rho Q_2 V_2 \sin 45 + \rho Q_3 V_3 \sin 60) - (\rho Q_1 V_1)$$

$$\therefore F_y = \underline{\underline{-1449.25 \ lb}}$$

10 The outlet pipe from a pump is a bend of 45 rising in the vertical plane (i.e. and internal angle of 135). The bend is 150mm diameter at its inlet and 300mm diameter at its outlet. The pipe axis at the inlet is horizontal and at the outlet it is 1m higher. By neglecting friction, calculate the force and its direction if the inlet pressure is 100kN/m^2 and the flow of water through the pipe is 0.3m^3/s. The volume of the pipe is 0.075m^3. (16)

$Q = 0.3 m^3/s$

$\forall = 0.07 m^3$



V1 Plane

Continuity eqn

$A_1 = \frac{\pi (0.15)^2}{4} = 0.0177$

$A_2 = \frac{\pi (0.3)^2}{4} = 0.707$

$V_1 = Q/A_1 = 16.95 \text{ m/s}$ $V_2 = Q/A_2 = 0.424 \text{ m/s}$

Bernoulli eqn

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{100 * 10^3}{9810} + \frac{(16.95)^2}{2(9.81)} + 0 = \frac{P_2}{9810} + \frac{(0.424)^2}{2(9.81)} + 1$$

$\Rightarrow P_2 = 233.7 * 10^3 \text{ N/m}^2$

Momentum eqn

$$\Sigma F_x = \rho Q V_{f_x} - \rho Q V_{i_x}$$

$$+P_1A_1 - P_2A_2\cos 45 - F_x = \rho Q V_2 \cos 45 - \rho Q V_1$$

$$(100*10^3)(0.0177) - (233.7*10^3)(0.707)\cos 45 - F_x = 1000(0.3)0.424\cos 45$$

$$- 1000(0.3)16.95 \qquad \Rightarrow F_x = \underline{\underline{-110\ kN}}$$

$$\Sigma F_y = \rho Q V_{f_y} - \rho Q V_{i_y}$$

$$F_y - W - P_2A_2\sin 45 = \rho Q V_2 \sin 45 - 0$$

$$F_y - 9810(0.07) - 233.7*10^3(0.707)\sin 45 = 1000(0.3)0.424\sin 45$$

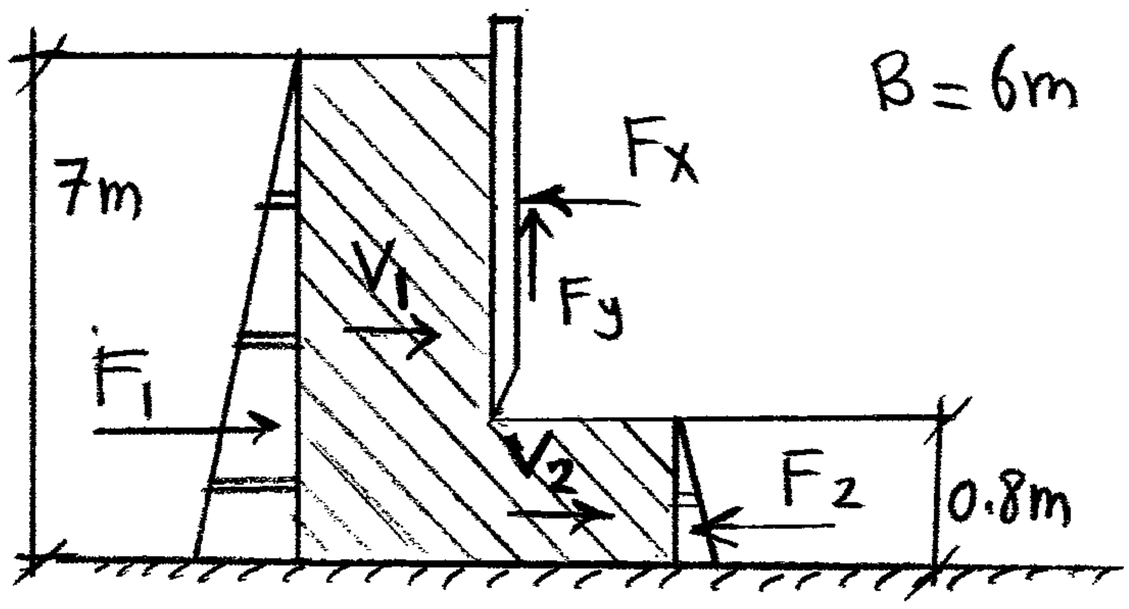
$$\Rightarrow F_y = \underline{\underline{117.6\ kN}}$$

A sluice gate is used to control the flow of water in a horizontal rectangular channel with 6 m wide. The gate is lowered so that the stream flowing under it has a depth of 800 mm and a velocity of 12 m/s. The depth upstream of the sluice gate is 7.0 m. Determine the force exerted by the water on the gate assuming uniform velocity distribution in the channel and neglecting all losses.

$$V_2 = 12\ m/s$$

$$Q = A_2V_2 = 6(0.8)12 = 57.6$$

$$V_1 = \frac{Q}{A_1} = \frac{57.6}{7*6} = 1.37$$



$$\Sigma F_x = \rho Q V_{f_x} - \rho Q V_{i_x}$$

$$F_1 - F_x - F_2 = \rho Q V_2 - \rho Q V_1$$

$$\gamma A_1\bar{h}_1 - F_x - \gamma A_2\bar{h}_2 = \rho Q V_2 - \rho Q V_1$$

$$9810(7*6)\frac{7}{2} - F_x - 9810(0.8*6)\frac{0.8}{2} = 1000(57.6)(1.37-12)$$

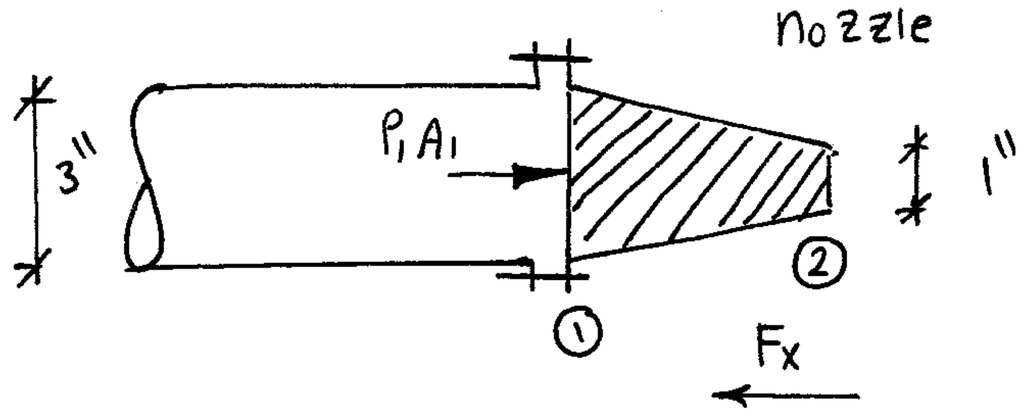
$$\Rightarrow F_x = \underline{\underline{2035\ kN}}$$

11

S.g. = 0.85

P_1 = 100 Psi

= 100 * 144 = 14400 lb/ft²



$A_1 = \frac{\pi \left(\frac{3}{12}\right)^2}{4} = 0.05 \text{ ft}^2$

$A_2 = \frac{\pi \left(\frac{1}{12}\right)^2}{4} = 5.45 * 10^{-3} \text{ ft}^2$

Applying Bernoulli eqn between ①, ②

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + \cancel{Z_1} = \cancel{\frac{P_2}{\gamma}} + \frac{V_2^2}{2g} + \cancel{Z_2}$$

$$\frac{14400}{62.4} + \frac{Q^2}{2g A_1^2} = \frac{Q^2}{2g A_2^2} \Rightarrow Q = 0.67 \text{ ft}^3/\text{s}$$

$V_1 = \frac{Q}{A_1} = 13.4$ ft/s

$V_2 = \frac{Q}{A_2} = 122.94$ ft/s

Applying Momentum eqn between ①, ②

$$\Sigma F_x = \rho Q (V_f - V_i)$$

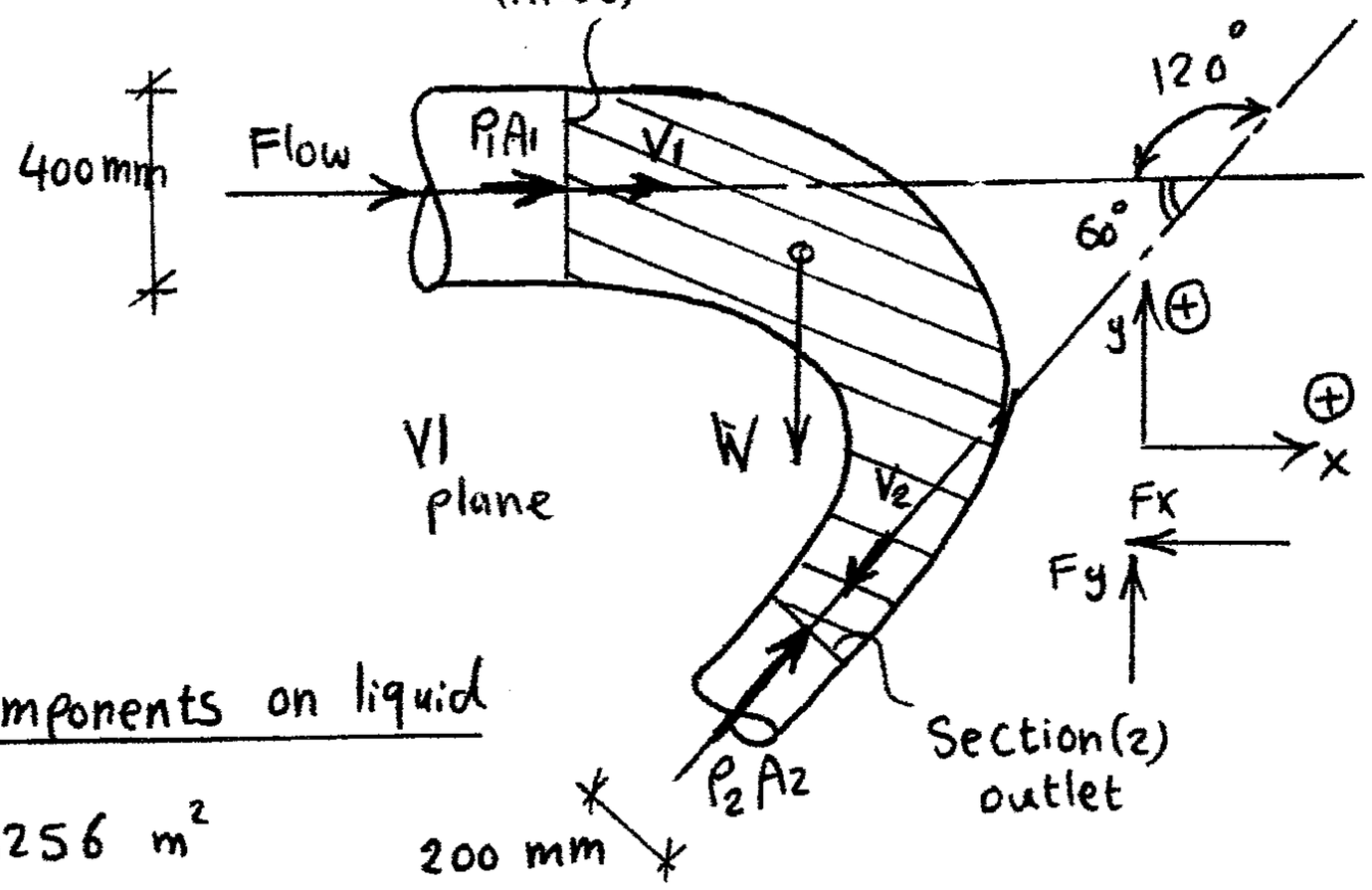
$$P_1 A_1 - F_x = \rho Q V_2 - \rho Q V_1$$

$$14400\ (0.05) - F_x = 1.94\ (0.67)\ 122.94 - 1.94\ (0.67)\ 13.4$$

$\Rightarrow F_x =$ 577.62 lb

13. A conveying elbow turns water through an angle of 120° in a vertical plane. The flow cross-sectional diameter is 400 mm at the elbow inlet, section 1, and 200 mm at the elbow outlet, section 2. The elbow flow passage volume is 0.20 m^3 between sections 1&2. The water flow rate is 0.40 m^3/s and the elbow inlet and outlet pressures are 150 KPa and 90KPa, respectively. Determine the magnitude and direction of the horizontal and vertical components of reaction force exerted by the water on the elbow.

Jan 1999



Volume $= 0.2 m^3$

$Q = 0.4 m^3/s$

$P_1 = 150 * 10^3$ Pa

$P_2 = 90 * 10^3$ Pa

Let F_x, F_y be Force Components on liquid

$$A_1 = \frac{\pi (0.4)^2}{4} = 0.1256 \ m^2$$

$$A_2 = \frac{\pi (0.2)^2}{4} = 0.0314 \ m^2$$

$$V_1 = \frac{Q}{A_1} = \frac{0.4}{0.1256} = 3.18 \ m/s \qquad V_2 = \frac{Q}{A_2} = \frac{0.4}{0.0314} = 12.73 \ m/s$$

$$W = \gamma \forall = 9810(0.2) = 1962 \ N$$

Applying the Momentum eqn in X-direction

$$P_1 A_1 + P_2 A_2 \cos 60^\circ - F_x = (-\rho Q V_2 \cos 60) - (\rho Q V_1)$$

$$150 * 10^3 (0.1256) + 90 * 10^3 (0.0314) \cos 60^\circ - F_x = [-1000(0.4)12.73 \cos 60] - [1000(0.4)\ 3.18]$$

$\therefore F_x =$ _____ N

Applying the Momentum eqn in Y-direction

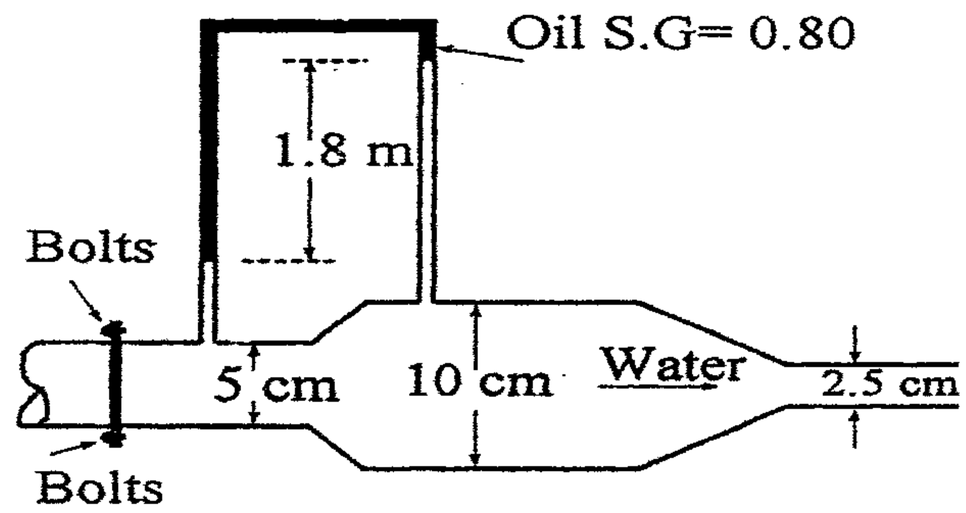
$$P_2 A_2 \sin 60 + F_y - W = (-\rho Q V_2 \sin 60) - (0)$$

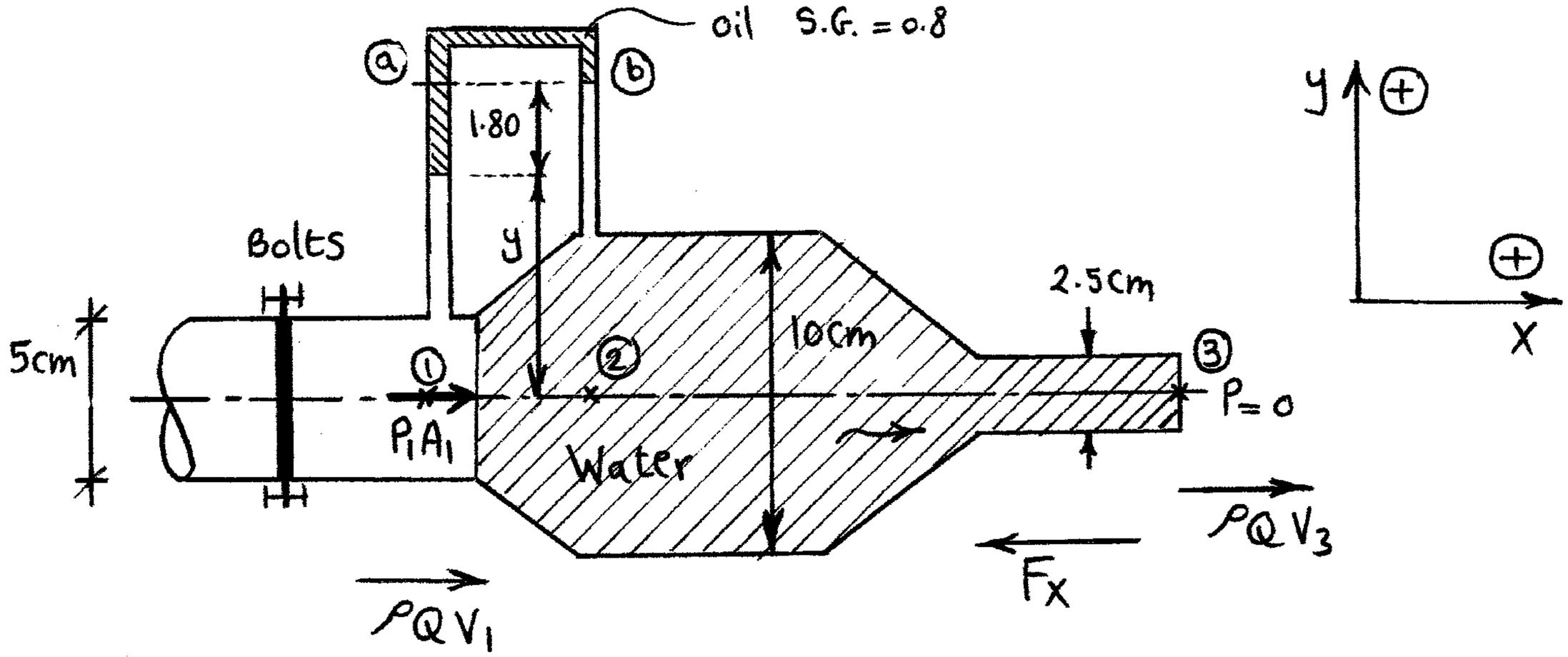
$$90 * 10^3 (0.0314) \sin 60 + F_y - 1962 = [-1000 (0.4) 12.74 \sin 60] - [0]$$

$\therefore F_y =$ _____ N

(20)

Find the tension in the bolts in the shown figure. Assume ideal flow





$$Q = \frac{A_1 A_2 \sqrt{2gH}}{\sqrt{A_2^2 - A_1^2}}$$

$$A_1 = \frac{\pi (0.05)^2}{4} = 0.00196 \ m^2$$

$$A_2 = \frac{\pi (0.1)^2}{4} = 0.00785 \ m^2$$

$$H = R\left(1 - \frac{\gamma_2}{\gamma_1}\right) = 1.8\left(1 - \frac{0.8}{1}\right) = 0.36$$

$$Q = \frac{(0.00196)(0.00785)\sqrt{2(9.81)\ 0.36}}{\sqrt{(0.00785)^2 - (0.00196)^2}} = 0.00538 \ m^3/s$$

$$V_1 = \frac{Q}{A_1} = \frac{0.00538}{0.00196} = \underline{\underline{2.74 \text{ m/s}}}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.00538}{0.00785} = \underline{\underline{0.686 \text{ m/s}}}$$

$$V_3 = \frac{Q}{A_3} = \frac{0.00538}{\frac{\pi}{4}\left(\frac{2.5}{100}\right)^2} = 10.96 \text{ m/s}$$

Applying Bernoulli equation between ①, ③

$$\frac{P_1}{\gamma_w} + \frac{V_1^2}{2g} = 0 + \frac{V_3^2}{2g}$$

$$\frac{P_1}{9810} = \frac{(10.96)^2 - (2.74)^2}{2 * 9.81} \Rightarrow P_1 = 56307 \text{ N/m}^2$$

Let F_x be the Force Component on the liquid

Applying the Momentum eqn in X direction

$$-F_x + P_1 A_1 = \rho Q V_3 - \rho Q V_1$$

$$-F_x + 56307 \frac{\pi}{4}\left(\frac{5}{100}\right)^2 = 1000(0.00538)\left[10.96 - 2.74\right]$$

$$F_x = \underline{\underline{66.33 \text{ N}}}$$

$F_x = 66.33$ N ← Force on liquid

F_x → Force on Flange Bolts

13 Pipe Flow

Types of losses

- Main losses (Friction)
- Secondary losses (minor losses – local)

a) Friction losses

$$h_f = \frac{F L V^2}{2 g d}$$

Darcy – Weisbach

1st General eqn. (Laminar + Turblent)

L: length of pipe

V: Velocity

d: diameter

h_f: head loss due to friction

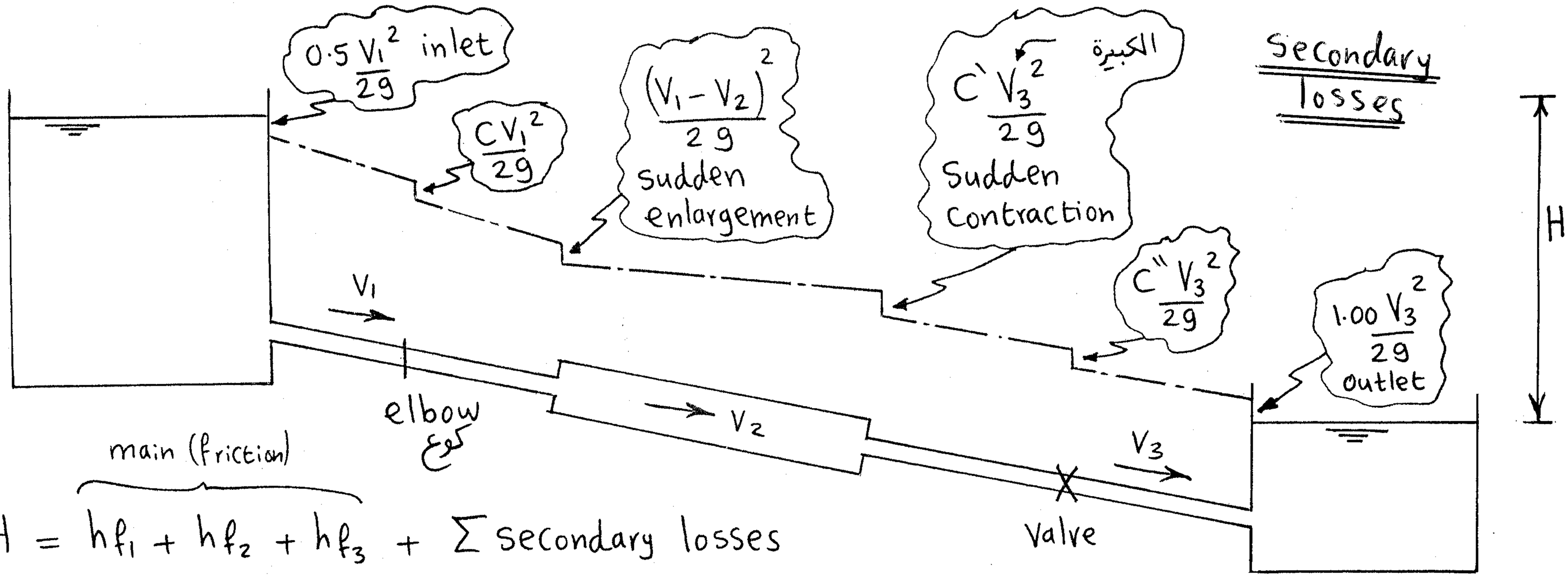
F: friction factor معامل الاحتكاك

IF F is not given ⟹ Moody Chart

$$\because V = \frac{Q}{A} = \frac{4Q}{\pi d^2}$$

$$h_f = \frac{8 F L Q^2}{\pi^2 g d^5}$$

2nd General eqn. (Laminar + Turblent)



$$H = h_{f_1} + h_{f_2} + h_{f_3} + \sum \text{secondary losses}$$

$$= \frac{F_1 L_1 V_1^2}{2g d_1} + \frac{F_2 L_2 V_2^2}{2g d_2} + \frac{F_3 L_3 V_3^2}{2g d_3} + \frac{0.5 V_1^2}{2g} + \frac{C V_1^2}{2g} + \frac{(V_1-V_2)^2}{2g} + \frac{C' V_3^2}{2g} + \frac{C'' V_3^2}{2g} + \frac{V_3^2}{2g}$$

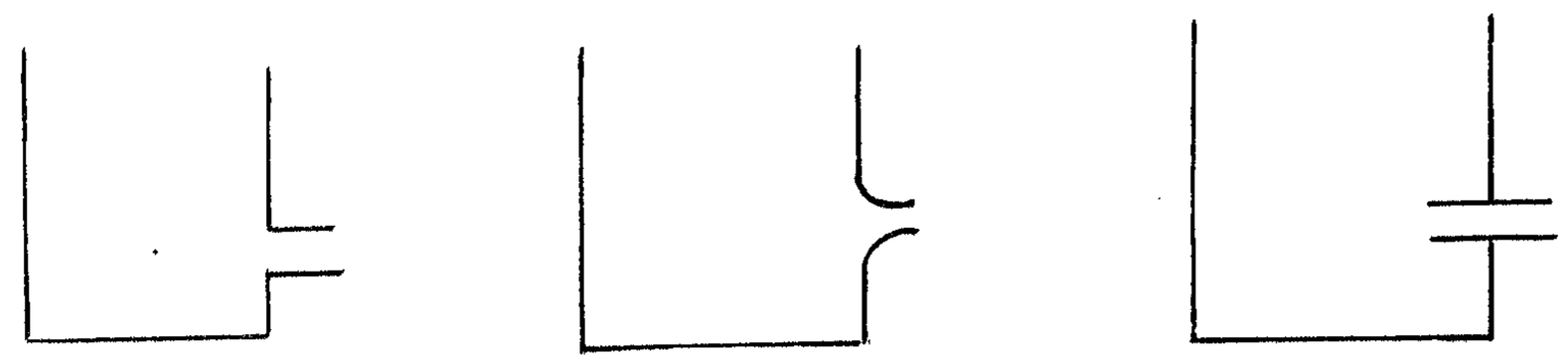
$$V_2 = V_1 \left(\frac{d_1}{d_2}\right)^2$$

$$V_3 = V_1 \left(\frac{d_1}{d_3}\right)^2$$

$\therefore V_1 = \checkmark$

$$\therefore Q = V_1 \frac{\pi d_1^2}{4}$$

Inlet Losses

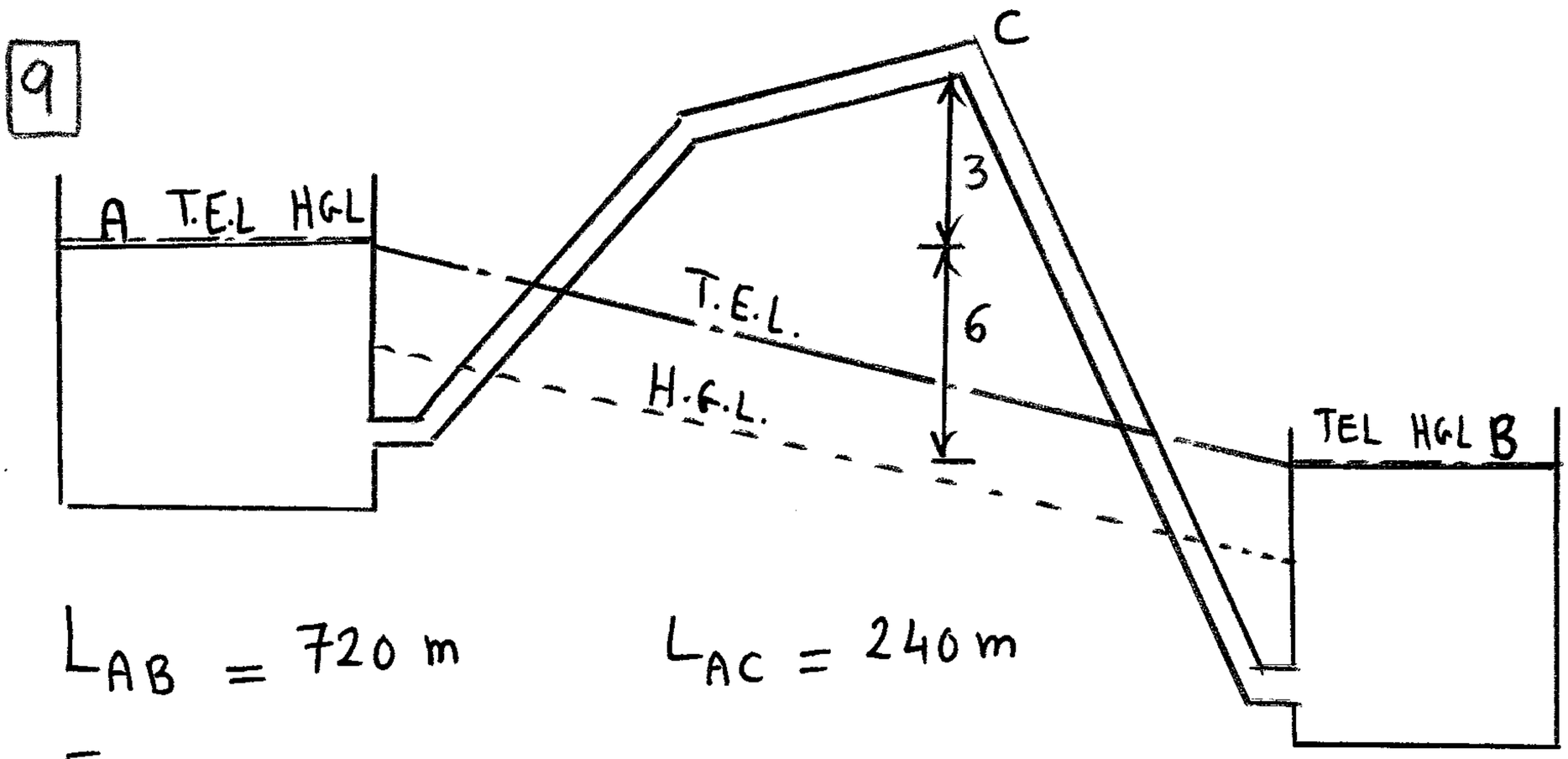


$C = 0.5$ Sharp

$C = 0.04$ rounded

$C = 0.8$ projecting

9



$L_{AB} = 720$ m $\qquad L_{AC} = 240$ m $\qquad Q = ?$

$F = 0.04 \qquad d = 1.2$ m $\qquad P_c = ?$

Applying Bernoulli eqn between A,B Datum at B

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + Z_B + h_{L_{AB}}$$

$$0 + 0 + 6 = 0 + 0 + 0 + \frac{8FLQ^2}{\pi^2 g d^5}$$

$$6 = \frac{8(0.04)\,720\,Q^2}{\pi^2 (9.81)\,(1.2)^5} \Rightarrow Q = \underline{2.5\ m^3/s}$$

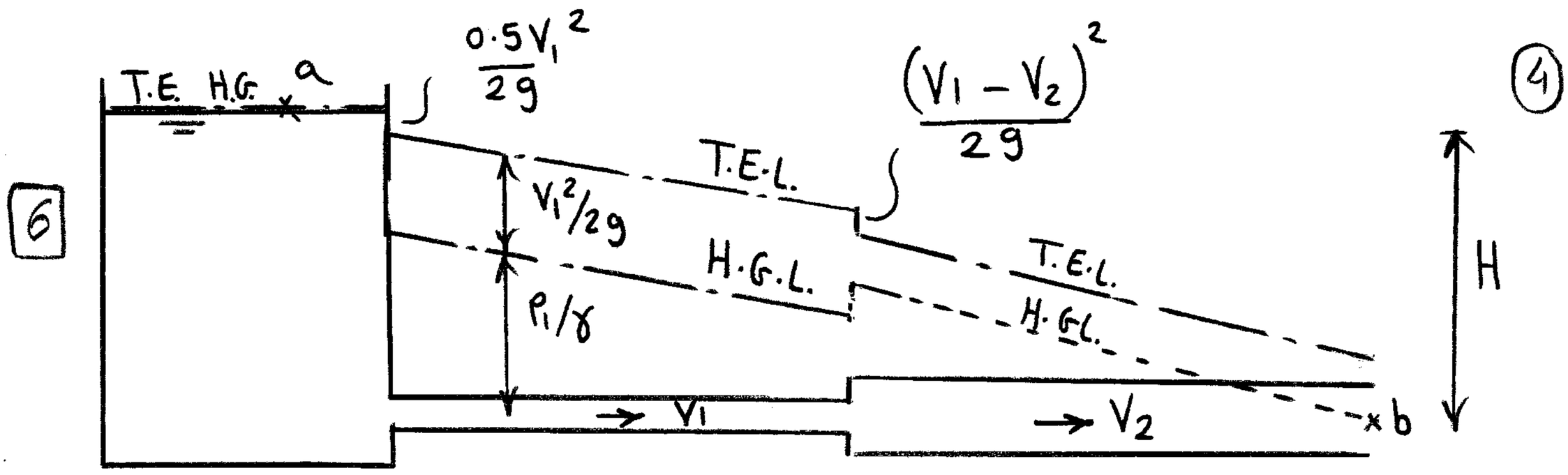
$$\therefore V = \frac{Q}{A} = \frac{2.5}{\pi \frac{(1.2)^2}{4}} = 2.21\ m/s$$

Applying Bernoulli eqn between A,C Datum at A

$$0 + 0 + 0 = \frac{P_c}{\gamma} + \frac{V_c^2}{2g} + Z_c + \frac{FLV^2}{2gd}$$

$$\frac{P_c}{\gamma} = -\left\{ \frac{(2.21)^2}{2(9.81)} + 3 + \frac{0.04(240)(2.21)^2}{2(9.81)\,1.2} \right\}$$

$$P_c = \underline{-51408\ N/m^2}$$



$H = ?$

$Q = 2.8$ lit/s

$L = 15$ m

$d = 50$ mm

$F = 0.0192$

$L = 24$ m

$d = 75$ mm

$F = 0.0232$

$$V_1 = \frac{Q}{A_1} = \frac{0.0028}{\frac{\pi(0.05)^2}{4}} = 1.43 \text{ m/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.0028}{\frac{\pi(0.075)^2}{4}} = 0.633 \text{ m/s}$$

Applying Bernoulli eqn between a, b Datum at b

$$\frac{P_a}{\gamma} + \frac{V_a^2}{2g} + Z_a = \frac{P_b}{\gamma} + \frac{V_b^2}{2g} + Z_b + \sum h_L \text{ sec + friction}$$

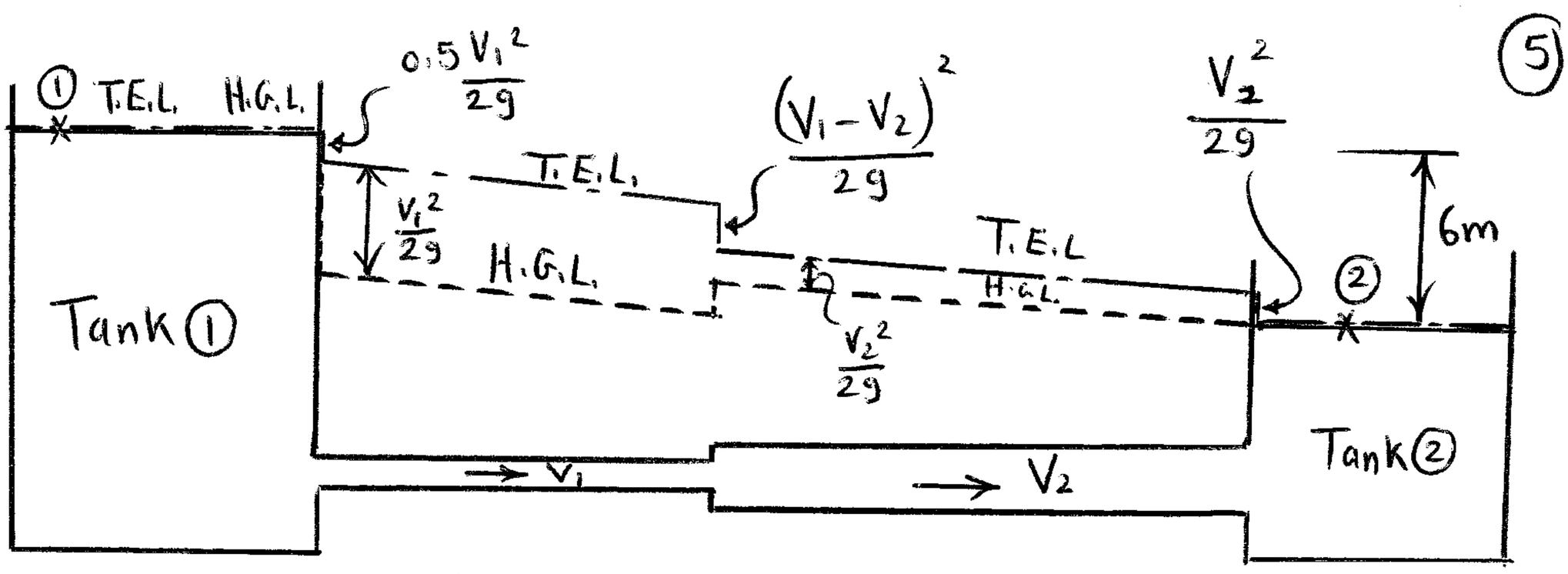
$$0 + 0 + H = 0 + \frac{V_b^2}{2g} + 0 + \sum h_L$$

$$H = \frac{V_2^2}{2g} + \underbrace{\frac{0.5V_1^2}{2g} + \frac{(V_1-V_2)^2}{2g}}_{\text{Secondary}} + \underbrace{\frac{F_1 L_1 V_1^2}{2g d_1} + \frac{F_2 L_2 V_2^2}{2g d_2}}_{\text{friction}}$$

$$H = \frac{(0.633)^2 + 0.5(1.43)^2 + (1.43-0.633)^2}{2(9.81)} + \frac{0.0192(15)(1.43)^2}{2(9.81)(0.05)} + \frac{0.0232(24)(0.633)^2}{2(9.81)(0.075)}$$

$$H = \underline{\underline{0.857 \text{ m}}}$$

7



5

$Q = ?$ $\quad L_1 = 6m$, $d_1 = 150\,mm$ $\quad L_2 = 15\,m$, $d_2 = 225\,mm$ $\quad F = 0.04$

$$V_1 = \frac{Q}{A_1} = \frac{Q}{\pi\frac{(0.15)^2}{4}} = 56.6\,Q$$

$$V_2 = \frac{Q}{A_2} = \frac{Q}{\pi\frac{(0.225)^2}{4}} = 25.15\,Q$$

Apply Bernoulli equation between ①, ② Datum at ②

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \sum h_L$$

$$0 + 0 + 6 = 0 + 0 + 0 + h_{L_{sec}} + h_{L_{friction}}$$

$$6 = \left\{0.5\frac{V_1^2}{2g} + \frac{(V_1-V_2)^2}{2g} + \frac{V_2^2}{2g}\right\} + \frac{8F_1L_1Q^2}{\pi^2 g d^5} + \frac{8FLQ^2}{\pi^2 g d^5}$$

$$6 = \frac{1}{2(9.81)}\left[0.5(56.6Q)^2 + (56.6-25.15)^2Q^2 + (25.15Q)^2\right] + \frac{8(0.04)6\,Q^2}{\pi^2(9.81)(0.15)^5}$$

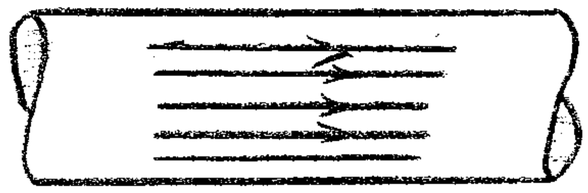
$$+ \frac{8(0.04)15\,Q^2}{\pi^2(9.81)(0.225)^2} \Rightarrow \underline{\underline{Q = \quad m^3/s}}$$

Types of flow

Laminar flow هادی

Fluid particles move in streamlines which are parallel to each other.

$R_n \leqslant 2000$



Turblent flow مضطرب

Fluid particles move with high velocity in random motion

Streamlines are not parallel to each other

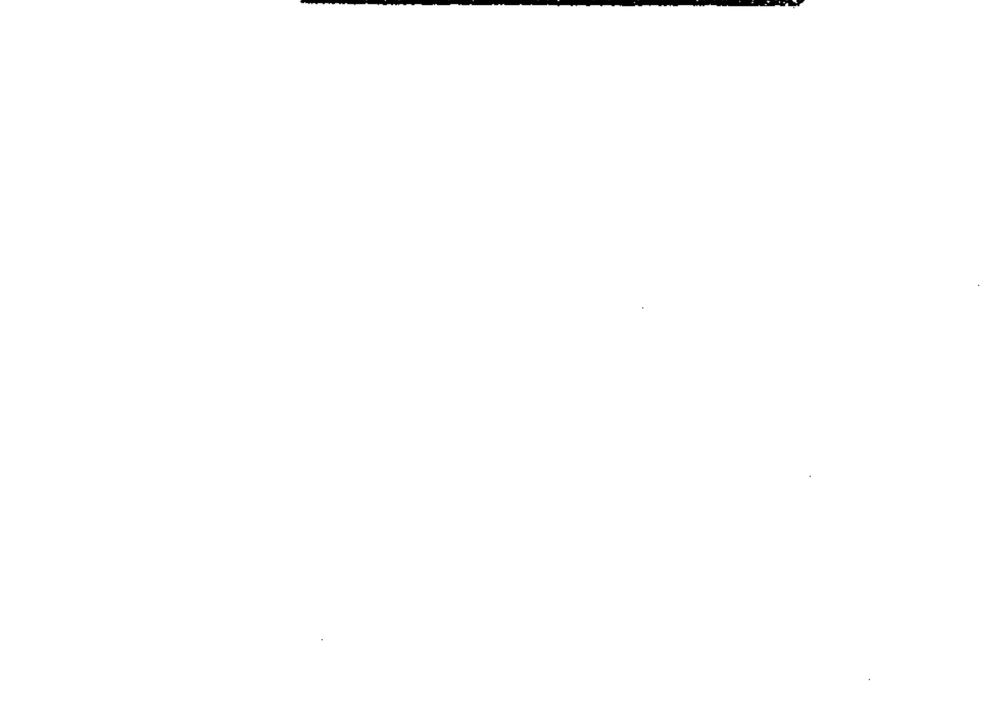
$R_n \geqslant 4000$



Transition flow انتقالی

$2000 < R_n < 4000$

Reynold's experiment



Laminar flow (Steady) low velocity

Transition

Turblent flow (unsteady) high velocity

$$R_n = \frac{\rho V d}{\mu} = \frac{V d}{\nu}$$

Reynold's number

where; ρ : density

V : Velocity

d : diameter of the pipe

μ : dynamic Viscosity ميو

ν : Kinematic Viscosity نيو

$$\nu = \frac{\mu}{\rho}$$

For laminar flow only

$R_n < 2000$

$$h_f = \frac{32 \mu L V}{\gamma d^2}$$

Hagen – Pousille

where; μ = dynamic Viscosity

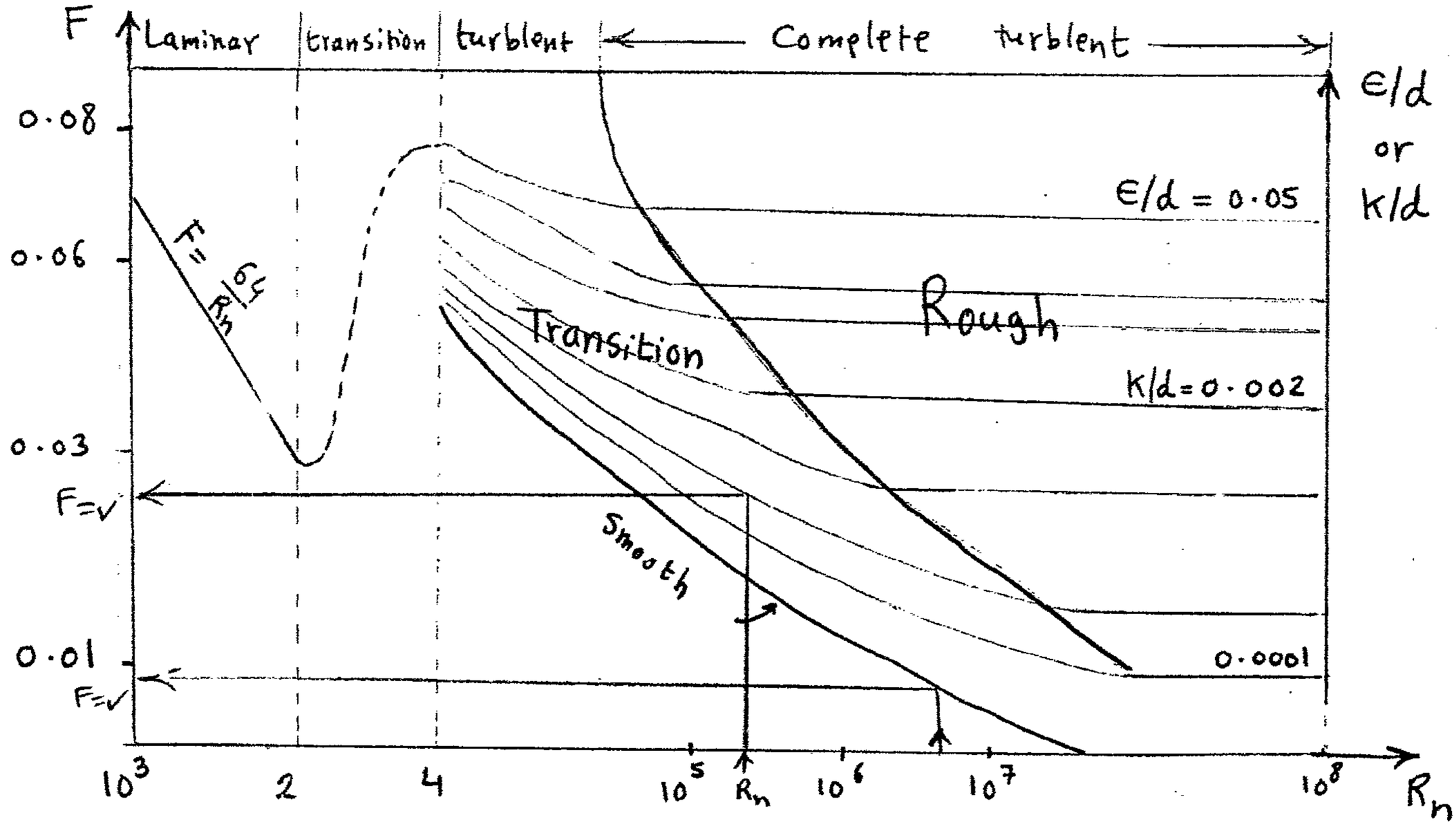
$$F = \frac{64}{R_n}$$

For Laminar + Turblent flow

$$h_f = \frac{F L V^2}{2 g d} = \frac{8 F L Q^2}{\pi^2 g d^5}$$

Darcy

Moody chart



Reynolds no $R_n = \frac{\rho V d}{\mu} = \frac{Vd}{\nu}$

K or ϵ = roughness

ϵ/d = relative roughness

or K/d

Given

① ϵ/d & R_n $\Rightarrow$ get F

② Smooth pipe and R_n $\Rightarrow$ get F

Smooth turblent Zone . (F is function in R_n) Smooth curve

$$\frac{1}{\sqrt{F}} = 2 \text{ Log} \left(\frac{R_n \sqrt{F}}{2.51} \right)$$

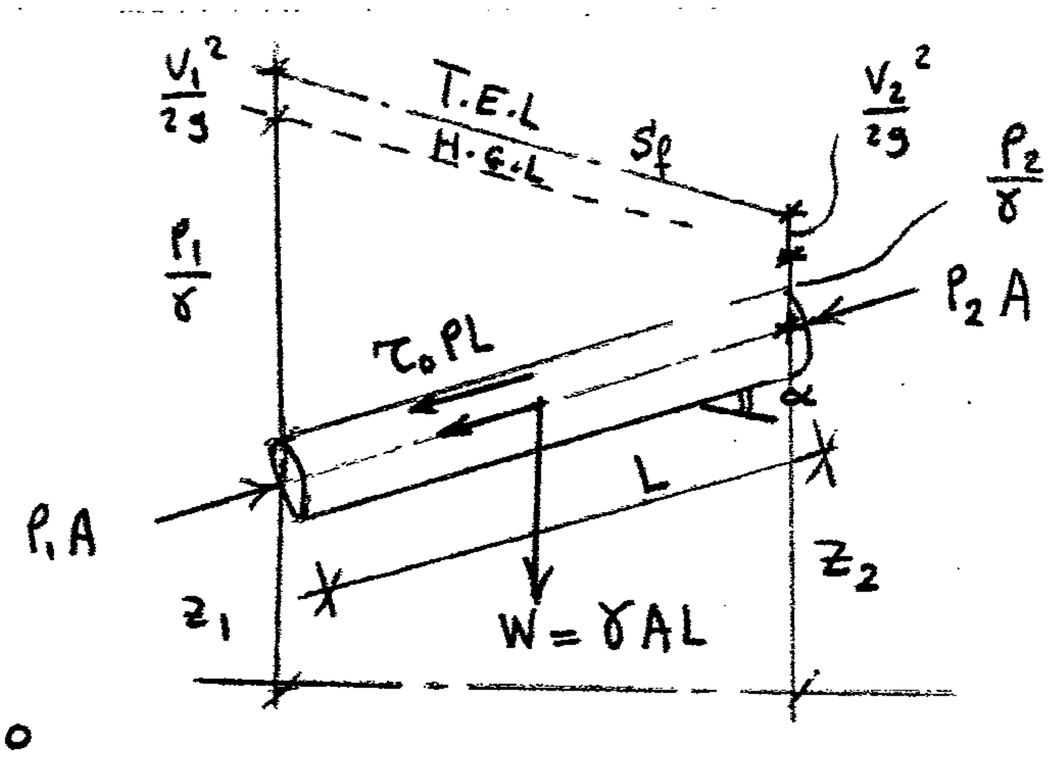
Rough turblent Zone . (F is function of ϵ/d) Rough lines

$$\frac{1}{\sqrt{F}} = 2 \text{ Log} \left(3.7 \frac{d}{\epsilon} \right)$$

Rough and Smooth turb. Zone Transition curves

$$\frac{1}{\sqrt{F}} = -2 \text{ Log} \left[\frac{\epsilon}{3.7d} + \frac{2.51}{R_n \sqrt{F}} \right]$$

Shear Stress in pipes



$\sin\alpha = \frac{Z_2 - Z_1}{L}$

τ_o : Shear Stress at boundary

$P_1 A - P_2 A - \tau_o P L - \gamma AL \sin\alpha = 0$

↖ perimeter

$\div \gamma A$

$$\frac{P_1}{\gamma} - \frac{P_2}{\gamma} - \frac{L(Z_2 - Z_1)}{L} = \frac{\tau_o P L}{\gamma A} \quad \longrightarrow (1)$$

$$h_L = \left(\frac{P_1}{\gamma} + Z_1\right) - \left(\frac{P_2}{\gamma} + Z_2\right) \quad \longrightarrow (2)$$

$$\therefore h_L = \frac{\tau_o P L}{\gamma A}$$

حنفظ

$$\therefore \boxed{\tau_o = \gamma \frac{A}{P} \frac{h_L}{L} = \gamma R S_f}$$

shear stress at walls of the pipe (or boundaries)

R = Hydraulic radius

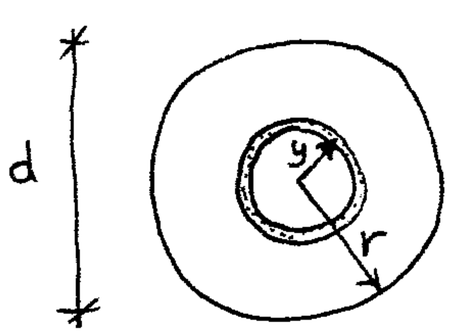
$$R = \frac{\text{Area}}{\text{Wetted perimeter}} = \frac{A}{P}$$ حنفظ

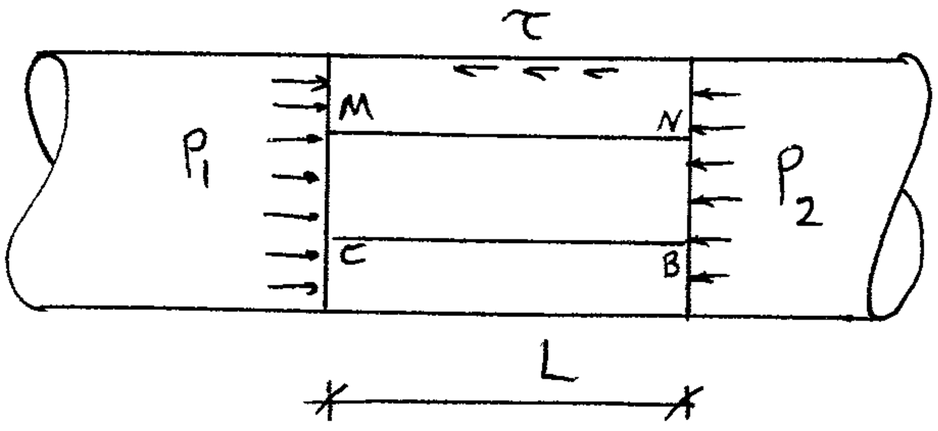
in pipes $R = \frac{\pi d^2}{4} \cdot \frac{}{\pi d} = \frac{d}{4}$ ←

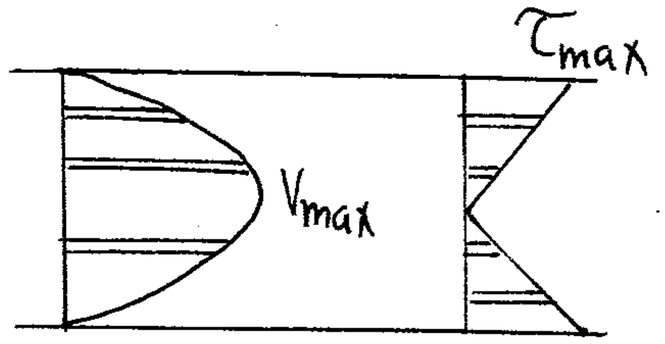
S_f = slope of the T.E.L

$S_f = \frac{h_L}{L}$ ←

Velocity and Shear stress distribution in pipes (Laminar)







$$\tau = \mu \frac{dV}{dy}$$

$$F_{shear} = -\mu \frac{dV}{dy} * 2\pi y L \quad \rightarrow (1)$$

$$F_{pipe} = (P_1 - P_2)\pi y^2$$

$$= \gamma h_L \pi y^2 \quad \rightarrow (2)$$

from (1), (2)

$$\mu \frac{dV}{dy} * 2\pi y L = \gamma h_L \pi y^2$$

$$dV = -\frac{\gamma h_L}{2\mu L} y\, dy$$

$$\int dV = -\frac{\gamma h_L}{2\mu L} \int y\, dy$$

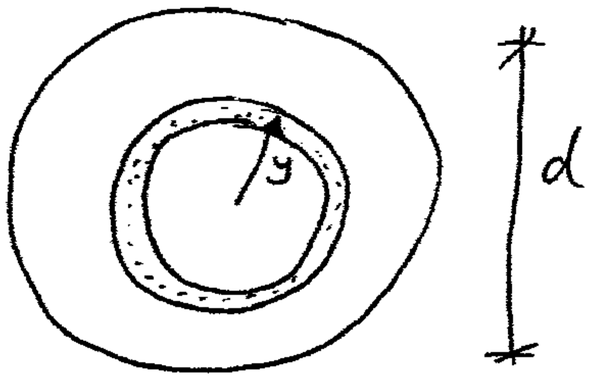
$$V = -\frac{\gamma h_L}{2\mu L} \frac{y^2}{2} + C$$

at $y = 0 \Rightarrow V = V_{max} \Rightarrow C = V_{max}$

$$\boxed{V = V_{max} - \frac{\gamma h_L}{4\mu L} y^2}$$ Velocity at any point

at $y = 0$ (at the center) $\boxed{V_{max} = \frac{\gamma h_L d^2}{16 \mu L}}$ max Velocity

Head loss in pipes (Laminar)



$$Q = \int V \, dA$$

$$dA = 2\pi y \, dy$$

$$V = V_{max} - \frac{\gamma h_L}{4\mu L} y^2$$

$$= \frac{\gamma h_L d^2}{16\mu L} - \frac{\gamma h_L y^2}{4\mu L}$$

$$= \frac{\gamma h_L}{4\mu L}\left(\frac{d^2}{4} - y^2\right)$$

$$Q = \int_0^{D/2} \frac{\gamma h_L}{4\mu L}\left(\frac{d^2}{4} - y^2\right) . \; 2\pi \; y \, dy$$

$$Q = \frac{\pi d^4}{128\mu} \cdot \frac{\gamma h_L}{L}$$

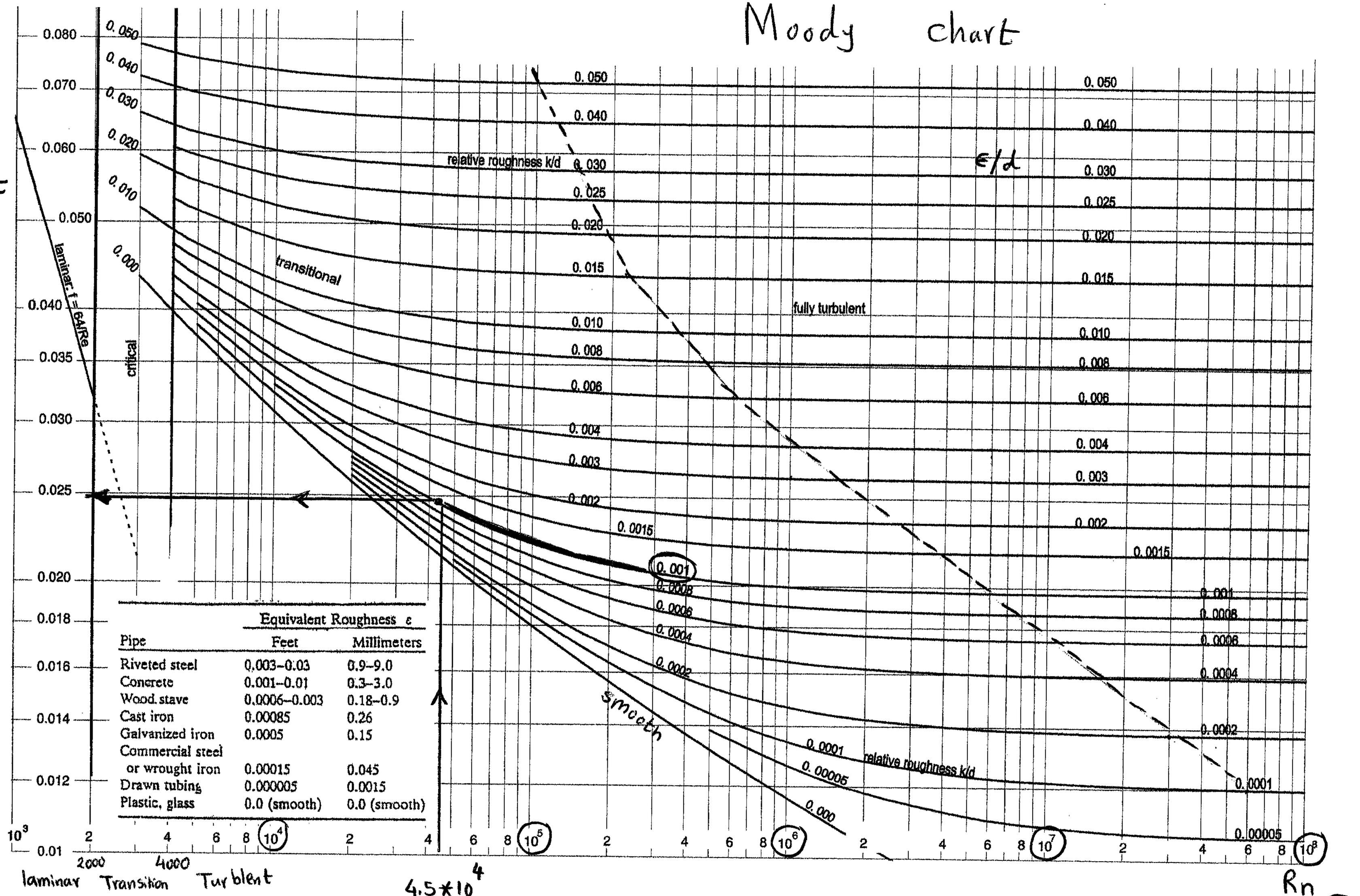
$\because$ V_{mean} (average Velocity) $= \frac{Q}{A}$

$$V_{mean} = \frac{\pi d^4}{128\mu} \cdot \frac{\gamma h_L}{L} * \frac{4}{\pi d^2} =$$

$$V_{mean} = \frac{d^2 \gamma h_L}{32 \mu L}$$

$$\therefore \; h_L = \frac{32 \mu L V_{mean}}{\gamma d^2}$$

Moody chart



Pipe	Equivalent Roughness ε Feet	Millimeters
Riveted steel	0.003–0.03	0.9–9.0
Concrete	0.001–0.01	0.3–3.0
Wood stave	0.0006–0.003	0.18–0.9
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Commercial steel or wrought iron	0.00015	0.045
Drawn tubing	0.000005	0.0015
Plastic, glass	0.0 (smooth)	0.0 (smooth)

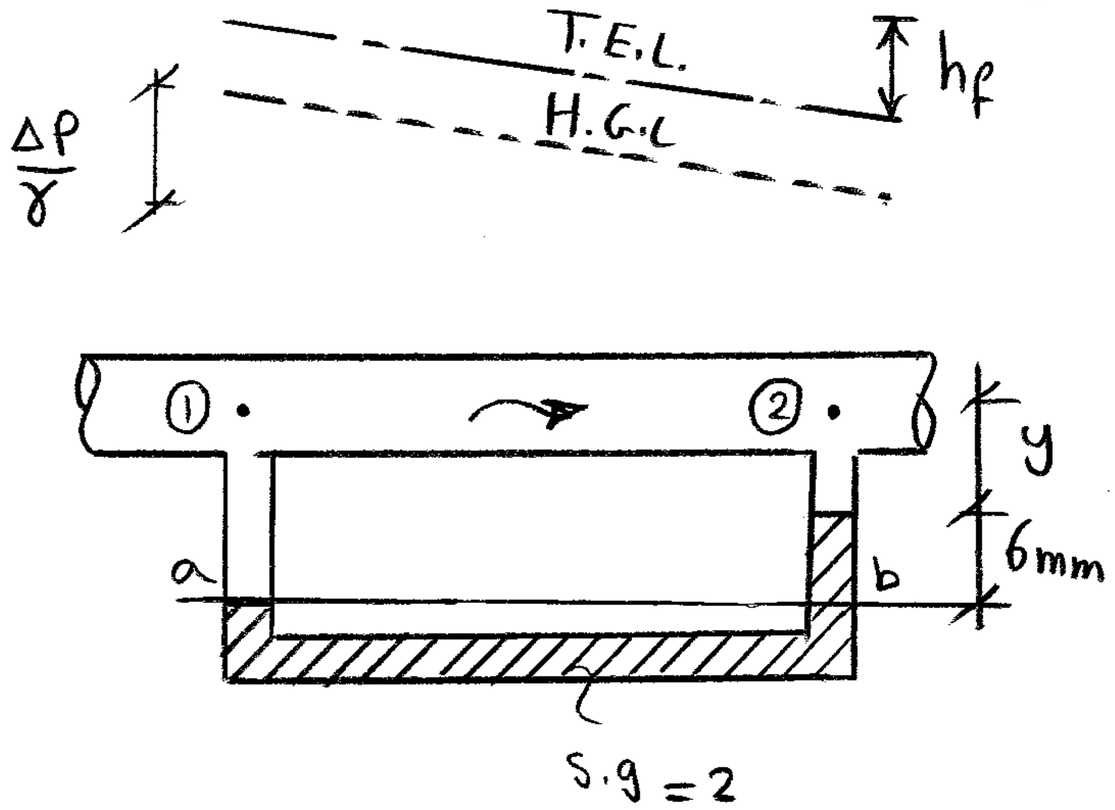
1

$\mu = 0.002$ Pa.s

$\rho = 1000$ Kg/m^3

$d = 4$cm , $L = 2$m

$V_{mean} = ?$ $V_{max} = ?$



$$h_f = \frac{\Delta P}{\gamma} = \frac{P_1 - P_2}{\gamma}$$

$$P_1 + \cancel{\gamma_w y} + \gamma_w\left(\frac{6}{1000}\right) - 2\gamma_w\left(\frac{6}{1000}\right) - \cancel{\gamma_w y} = P_2$$

$$\therefore \quad \frac{P_1 - P_2}{\gamma_w} = 6 * 10^{-3}$$

$$\therefore \quad h_f = \frac{32 \mu L V}{\gamma d^2} \qquad \text{(Laminar flow)}$$

$$6 * 10^{-3} = \frac{32(0.002)\,2\,V_{mean}}{9810\,(0.04)^2} \Rightarrow V_{mean} = \underline{0.736 \text{ m/s}}$$

$$V_{max} = 2\,V_{mean} = \underline{1.472 \text{ m/s}}$$

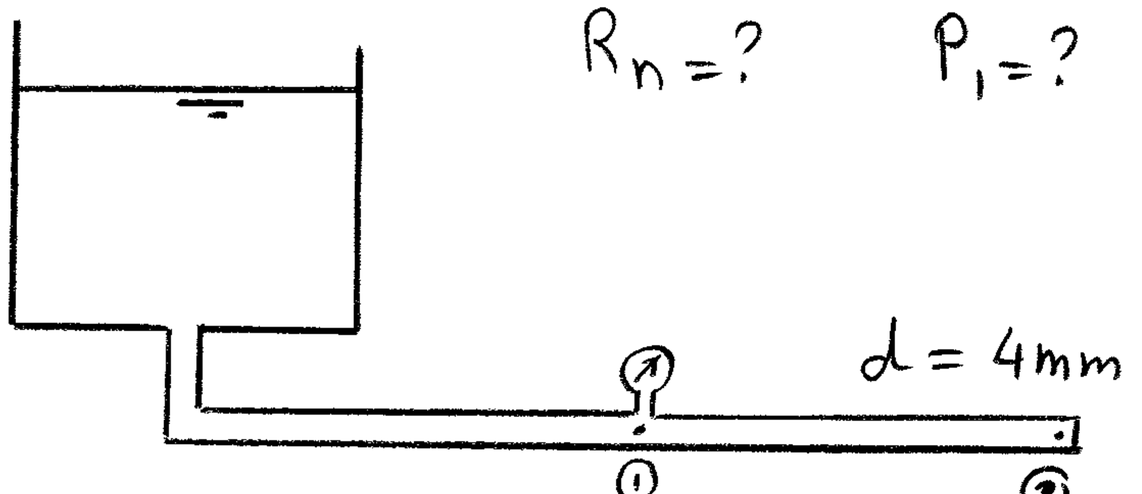
3

$\mu = 0.015$ Pa.s

$\rho = 1200$ Kg/m^3

$V = 1$m/s

$R_n = ?$ $P_1 = ?$



$$R_n = \frac{\rho V d}{\mu} = \frac{1200(1)\,4*10^{-3}}{0.015} = \underline{320} < 2000$$

laminar flow

Bernoulli eqn between ①, ②

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \frac{32 \mu L V}{\gamma d^2}$$

$$\frac{P_1}{9810} + \frac{1^2}{2(9.81)} + 0 = 0 + \frac{1^2}{2(9.81)} + 0 + \frac{32(0.015)3(1)}{9810\,(4*10^{-3})^2} \Rightarrow P_1 = \underline{90{,}000 \text{ Pa}}$$

2 $\mu = 0.009\ N.s/m^2$ $\quad \rho = 1260\ kg/m^3$

$L = 6.5\ m$ $\quad d = 0.01\ m$ $\quad Q = 1.8\ lit/min$

a) $R_n = ?$, type of flow

$$V = \frac{Q}{A} = \frac{1.8 * 10^{-3}}{60} * \frac{1}{\frac{\pi (0.01)^2}{4}} = 0.382\ m/s$$

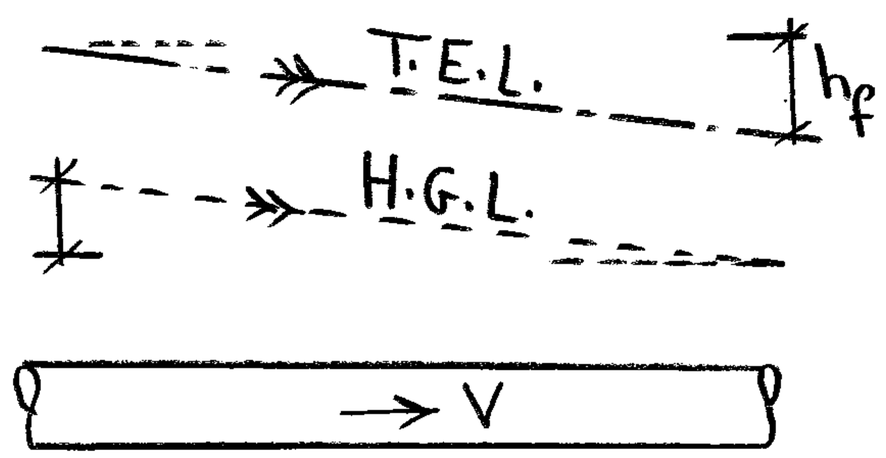
$$R_n = \frac{\rho V d}{\mu} = \frac{1260 * 0.382 * 0.01}{0.009} = \underline{534.7}$$

$R_n < 2000 \Rightarrow$ laminar flow

---

b) Pressure loss = ?

$$\frac{\Delta P}{\gamma} = h_f = \frac{32 \mu L V}{\gamma d^2}$$



$$= \frac{32 (0.009)\ 6.5\ (0.382)}{1260 * 9.81 * (0.01)^2} = \underline{0.58\ m}$$

---

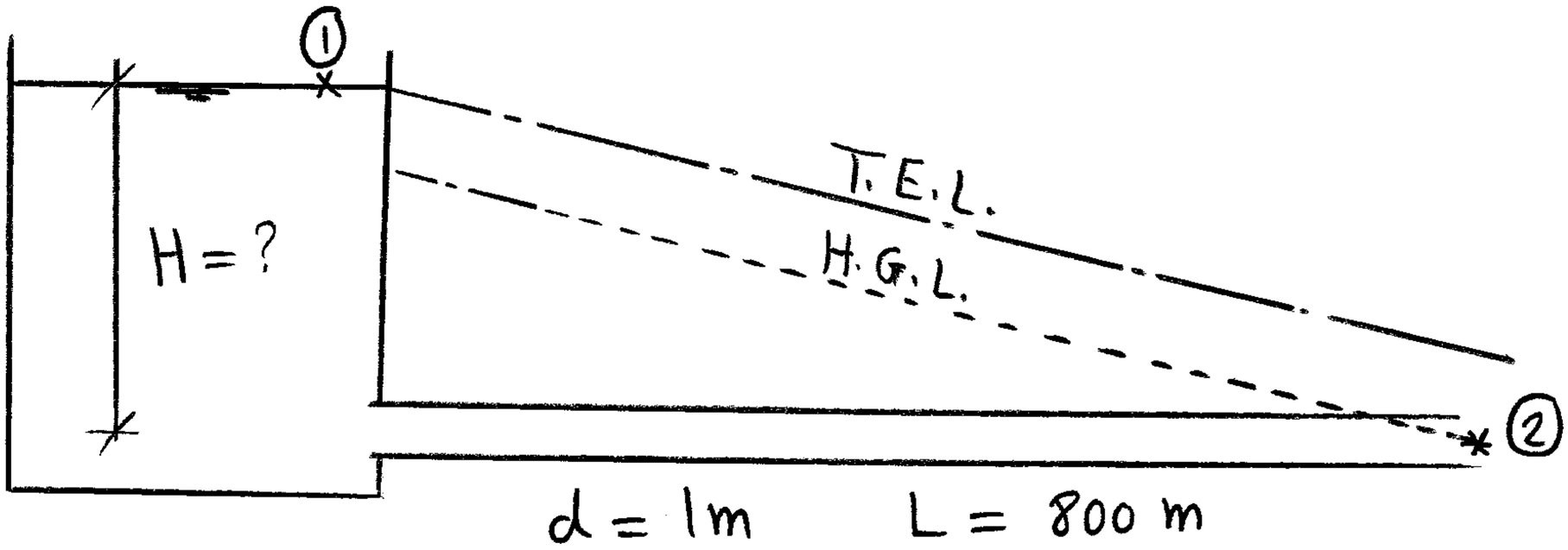
c) max Q for laminar flow

$$R_n = \frac{\rho V d}{\mu}$$

$$2000 = \frac{1260 * V * 0.01}{0.009} \Rightarrow V = 1.42\ m/s$$

$$Q_{max\ laminar} = A \cdot V = \frac{\pi (0.01)^2}{4} * 1.42 = 1.122\ \frac{m^3}{s} = \underline{6.7\ lit/min}$$

4



Smooth pipe $\nu = 10^{-6}\ m^2/s$ $V = 1\ m/s$ T.E.L H.G.L

$$R_n = \frac{\rho V d}{\mu} = \frac{V d}{\nu} = \frac{1 * 1}{10^{-6}} = 10^6$$

$R_n > 4000 \Rightarrow$ Turblent flow $\Longrightarrow$ Moody chart

$R_n = 10^6 \xrightarrow{\text{Smooth pipe}} F = 0.012$

$$h_L = \frac{F L V^2}{2 g d} = \frac{0.012\ (800)\ (1)^2}{2\ (9.81)\ (1)} = 0.49\ m$$

Bernoulli ①, ②

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$$0 + 0 + H = 0 + \frac{V^2}{2g} + 0 + h_L$$

$$H = \frac{(1)^2}{2\ (9.81)} + 0.49 = \underline{0.54\ m}$$

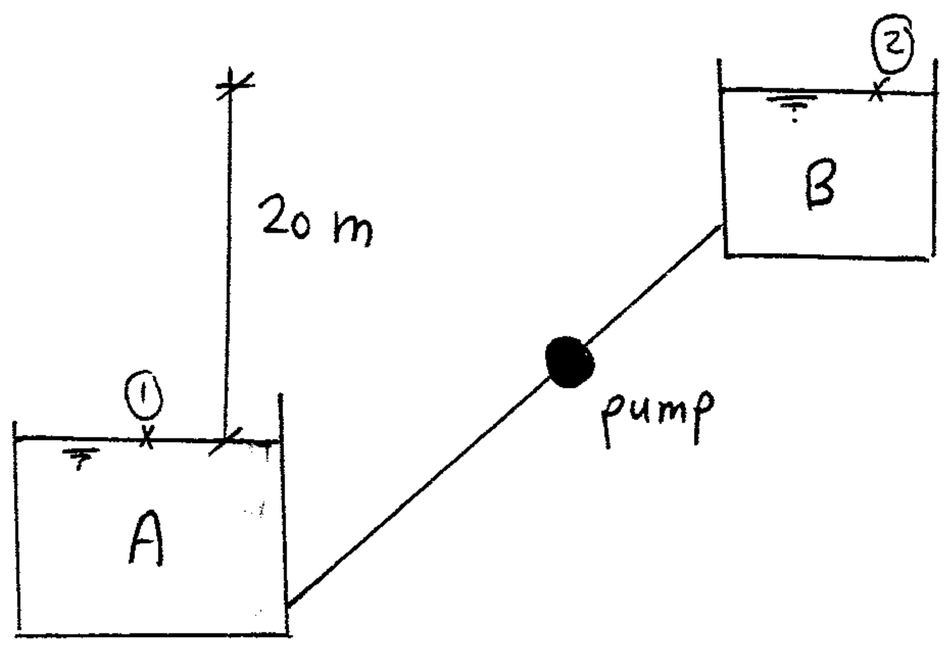
$S.g = 1.2$ $\quad \mu = 2.4 * 10^{-3}$ Pa.s

$V = 1.8$ m/s $\quad \eta = 50\%$

$\epsilon = 0.005$ cm $= 0.005 * 10^{-2}$ m

$d = 0.05$ m $\quad L = 48$ m

$HP_{pump} = ?$



solution

$$R_n = \frac{\rho V d}{\mu} = \frac{(1.2 * 1000) * 1.8 * 0.05}{2.4 * 10^{-3}} = 45000 > 4000$$

∴ Flow is turblent (go to Moody chart)

Calculate $\frac{\epsilon}{d} = \frac{0.005 * 10^{-2}\ m}{0.05\ m} = 0.001$

$\frac{\epsilon}{d} = 0.001$, $R_n = 4.5 \times 10^4$ $\xrightarrow{\text{Moody}}$ $F = 0.025$

Applying Bernoulli eqn between ①, ②

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 + h_p = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + \Sigma h_L$$

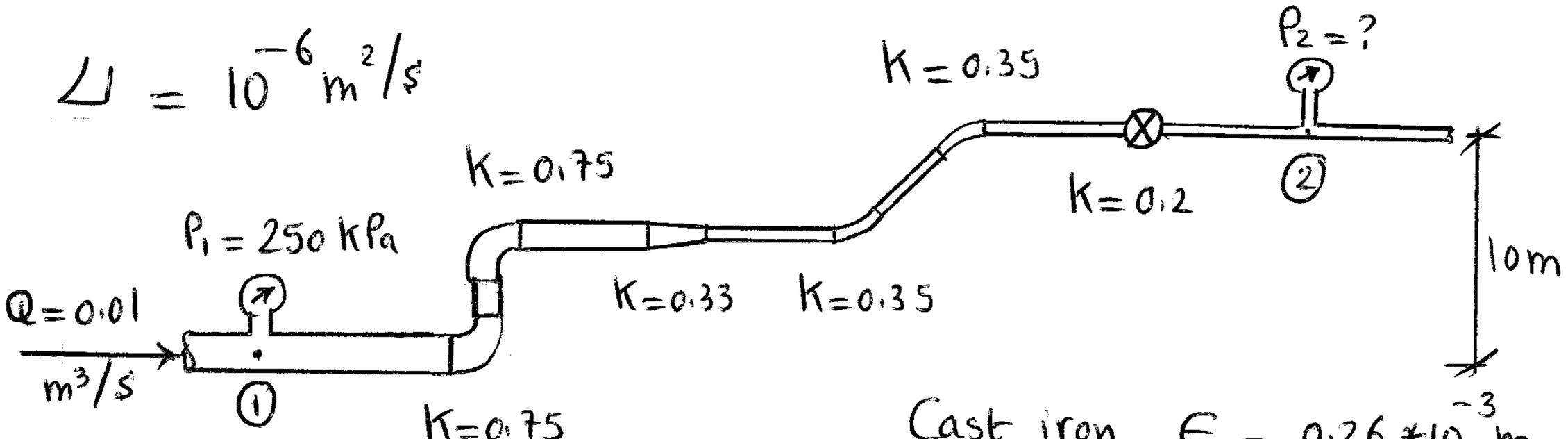
$$h_p = 20 + \frac{FLV^2}{2gd}$$

$$= 20 + \frac{0.025\ (48)(1.8)^2}{2 * 9.81 * 0.05} = 23.96\ m$$

$$HP = \frac{\gamma Q h_p}{\eta_K} = \frac{1.2 * 9810 \left[1.8\ \frac{\pi (0.05)^2}{4}\right] 23.96}{0.5 * 735} = 2.71 \text{ horsepower}$$

5 (17)

For the piping system shown in the figure, determine the pressure p_2. There are 30 m of 75 cm-ID cast iron pipe and 100 m of 15 cm-ID cast iron pipe. Water flow rate is 0.01 m^3/s, the gauge pressure at 1 is 250 KPa (υ- water = 1x 10^{-6} m^2/s, ε-cast iron = 0.26x10^{-3} m)



Cast iron $\epsilon = 0.26 * 10^{-3}$ m

$L_1 = 30$ m $\quad d_1 = 0.75$m $\Rightarrow A_1 = \pi(0.75)^2/4 = 0.442$

$L_2 = 100$ m $\quad d_2 = 0.15$ m $\Rightarrow A_2 = \pi(0.15)^2/4 = 0.0177$

$V_1 = \frac{Q}{A_1} = \frac{0.01}{0.442} = 0.0226$, $V_2 = \frac{Q}{A_2} = \frac{0.01}{0.0177} = 0.566$ m/s

$R_{n_1} = \frac{V_1 d_1}{\upsilon} = \frac{0.0226(0.75)}{10^{-6}} = 1.7 * 10^4 \xrightarrow[\frac{\epsilon}{d_1} = \frac{0.26*10^{-3}}{0.75} = 0.0003]{\text{Moody}} F_1 = 0.028$

$R_{n_2} = \frac{V_2 d_2}{\upsilon} = \frac{0.566(0.15)}{10^{-6}} = 8.5 * 10^4 \xrightarrow[\frac{\epsilon}{d_2} = \frac{0.26*10^{-3}}{0.15} = 0.0017]{\text{Moody}} F_2 = 0.025$

Bernoulli eqn between ①, ② Datum at ①

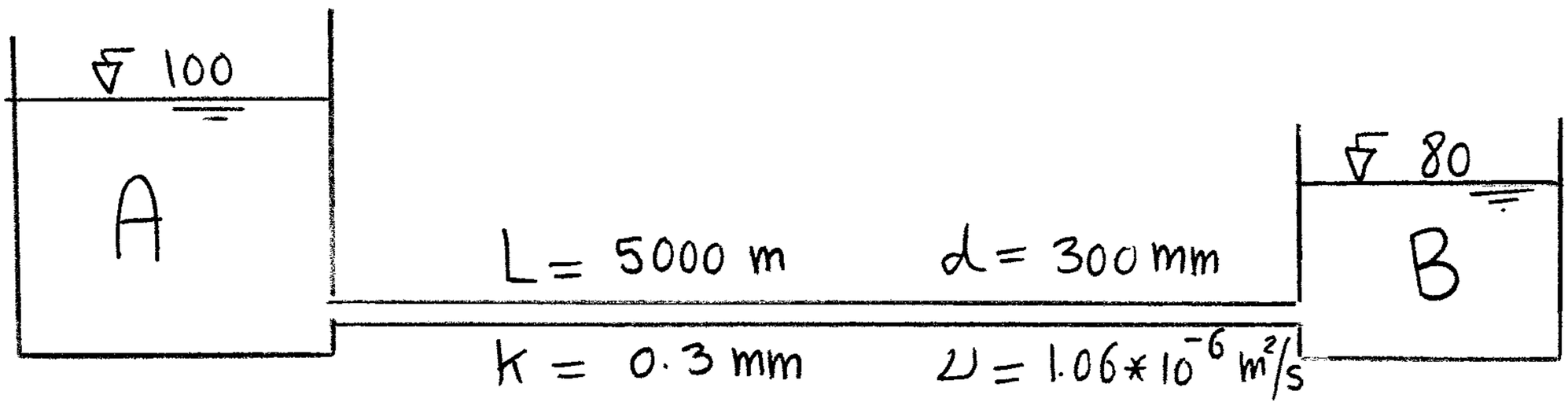
$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \sum h_L \quad \begin{matrix}\nearrow \text{secondary} \\ \searrow \text{main}\end{matrix}$$

$$\frac{250*10^3}{9810} + \frac{V_1^2}{2(9.81)} + 0 = \frac{P_2}{9810} + \frac{V_2^2}{2(9.81)} + 10 + \underbrace{\frac{F_1 L_1 V_1^2}{2g d_1} + \frac{F_2 L_2 V_2^2}{2g d_2}}_{\text{main}}$$

$$+ \left\{ 0.75\frac{V_1^2}{2g} + 0.75\frac{V_1^2}{2g} + 0.33\frac{V_2^2}{2g} + 0.35\frac{V_2^2}{2g} + 0.35\frac{V_2^2}{2g} + 0.2\frac{V_2^2}{2g} \right\} \text{secondary}$$

$\Rightarrow P_2 =$

Using Moody chart, Find the discharge Q (neglect minor losses)



$h_f = 100 - 80 = 20\ m$

$\therefore \boxed{h_f = \frac{F L V^2}{2 g d}}$ Darcy F, V unknowns

∵ Water is flowing from tank to tank ⇒ turblent flow

Assume Complete turblence (F depends on ϵ only)

$\frac{\epsilon}{d} = \frac{0.3\ mm}{300\ mm} = 0.001 \xrightarrow[\text{chart}]{\text{Moody}} F = 0.02$

$\xrightarrow{\text{Darcy}} \quad 20 = \frac{0.02\ (5000)\ V^2}{2\ (9.81)\ 0.3} \quad \Rightarrow V = 1.085\ m/s$

Check

$R_n = \frac{\rho V d}{\mu} = \frac{V d}{\nu} = \frac{1.085 * 0.3}{1.06 * 10^{-6}} = 3 * 10^5$

$\xrightarrow{\text{Moody}} F = 0.021$

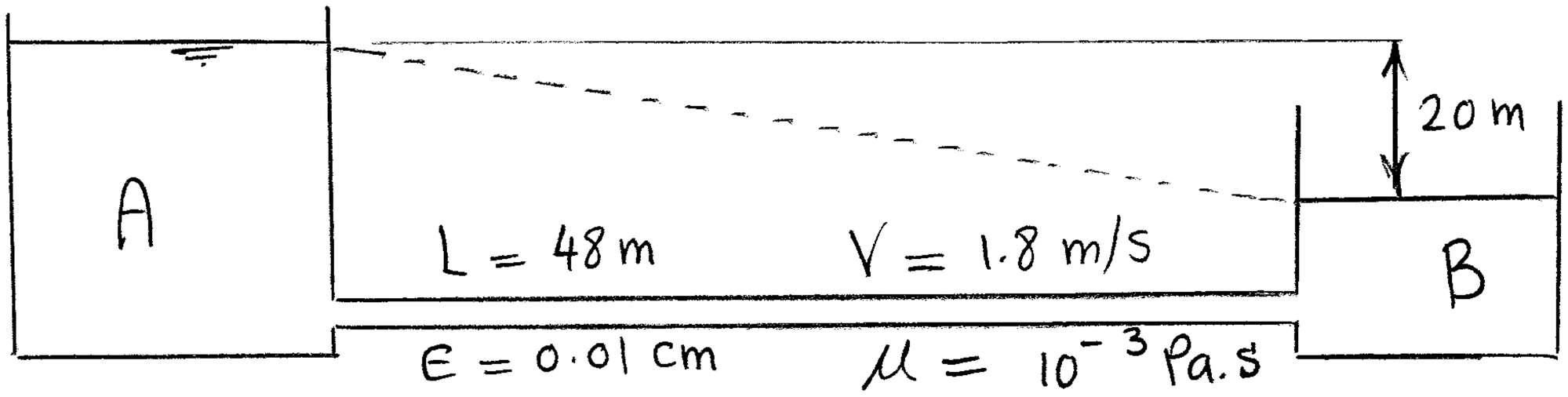
$20 = \frac{0.021\ (5000)\ V^2}{2\ (9.81)\ 0.3} \quad \Rightarrow V = 1.06\ m/s$

$Q = AV = \frac{\pi (0.3)^2}{4} * 1.06 = \underline{\underline{0.075\ m^3/s}}$

يتم عمل Check واحد على الأقل (المفروض عمل Checks حتى تثبت قيمة F ، V)

∵ الفرق صغير ⇐ لا داعي لعمل Check ②

Find the diameter of the pipeline using Moody chart



$R_n = \frac{\rho V d}{\mu}$, $\frac{\epsilon}{d}$ $\quad h_L = \frac{F L V^2}{2 g d}$

في هذه الحالة R_n , $\frac{\epsilon}{d}$ غير معلوم لأن d مجهولة

هناك حلين ① فرض d (حل صعب)

② فرض F (حل سهل)

assume $F = 0.02$

$\xrightarrow{\text{Darcy}}$ $20 = \frac{0.02\,(48)\,(1.8)^2}{2\,(9.81)\,d} \Rightarrow d = 7.92 * 10^{-3}\ m$

Check ①

$R_n = \frac{\rho V d}{\mu} = \frac{1000\,(1.8)\;7.92 * 10^{-3}}{10^{-3}} = 14256 = 1.4 * 10^4$

$\frac{\epsilon}{d} = \frac{0.01 * 10^{-2}}{7.92 * 10^{-3}} = 0.012 \xrightarrow{\text{Moody}} F = 0.042$

$\therefore 20 = \frac{0.042\,(48)\,(1.8)^2}{2\,(9.81)\,d} \Rightarrow d = \underline{0.0166\ m}$

Check ②

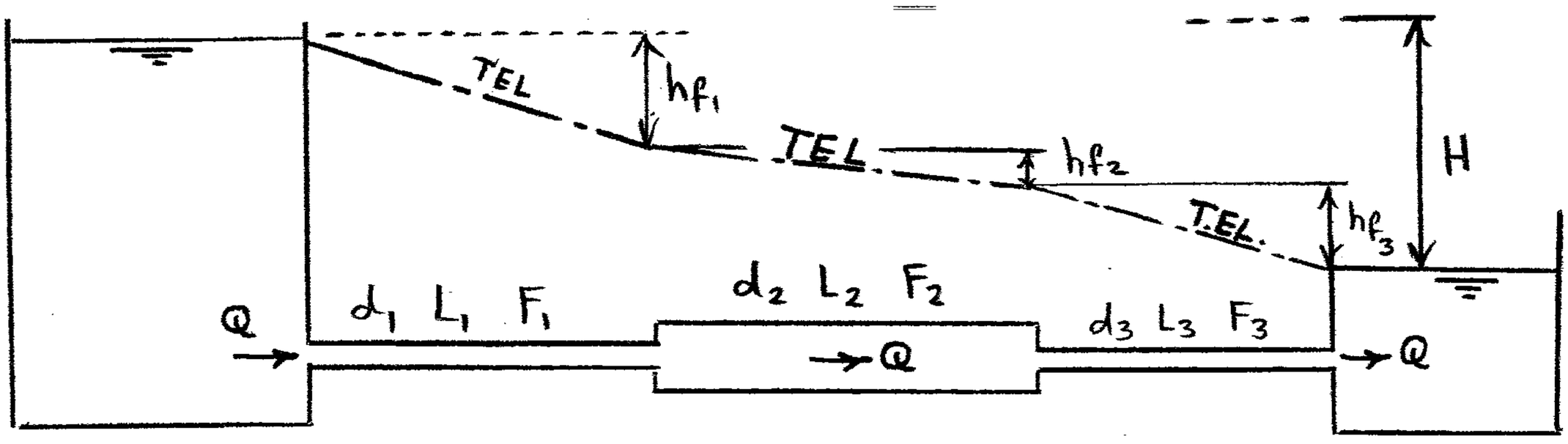
$R_n = \frac{\rho V d}{\mu} = \frac{1000\,(1.8)\,(0.0166)}{10^{-3}} = 29880 = 3 * 10^4$

$\frac{\epsilon}{d} = \frac{0.01 * 10^{-2}}{0.0166} = 0.006 \xrightarrow{\text{Moody}} F = 0.036$

$\therefore 20 = \frac{0.036\,(48)\,(1.8)^2}{2\,(9.81)\,d} \Rightarrow d = \underline{0.0143\ m}$

14 Pipes in Series and in Parallel

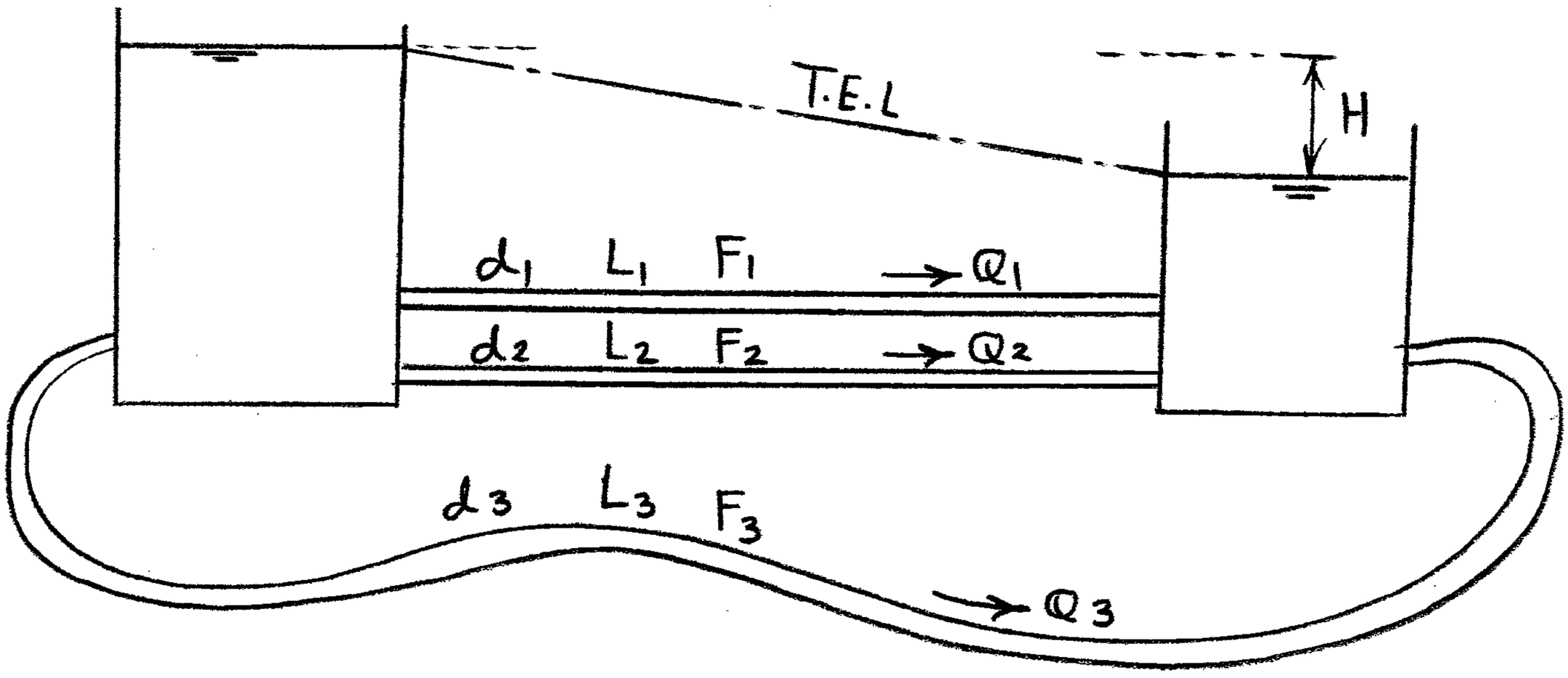
Pipes in Series



$$Q_1 = Q_2 = Q_3$$

$$H = h_{f_1} + h_{f_2} + h_{f_3}$$

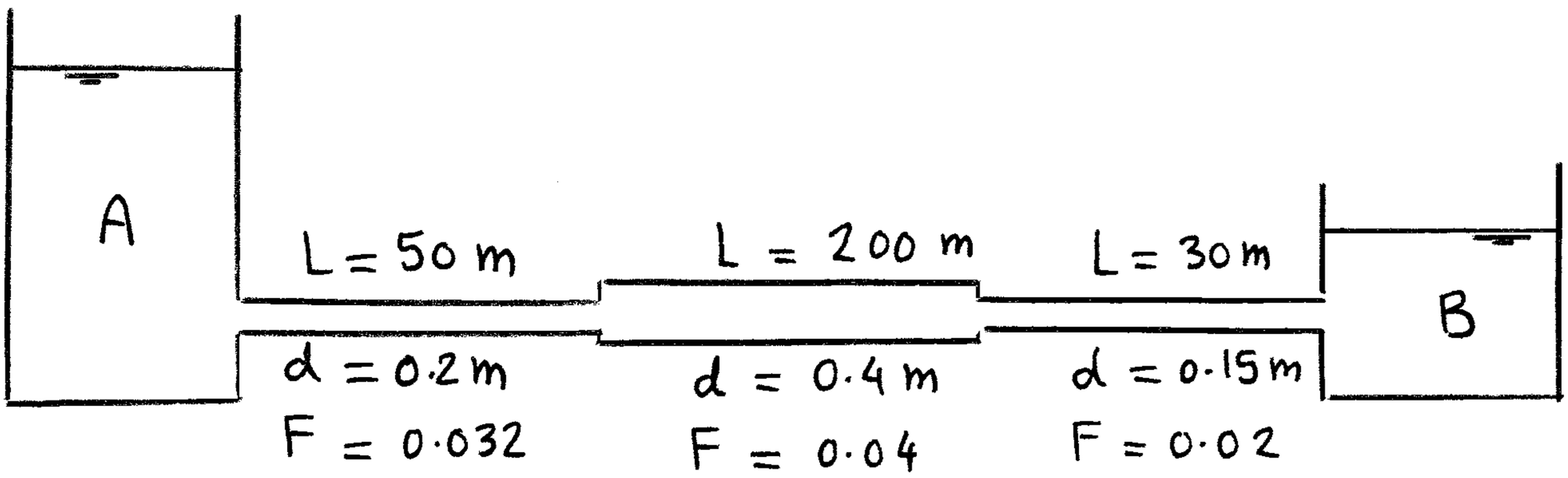
Pipes in parallel



$$Q = Q_1 + Q_2 + Q_3$$

$$H = h_{f_1} = h_{f_2} = h_{f_3}$$

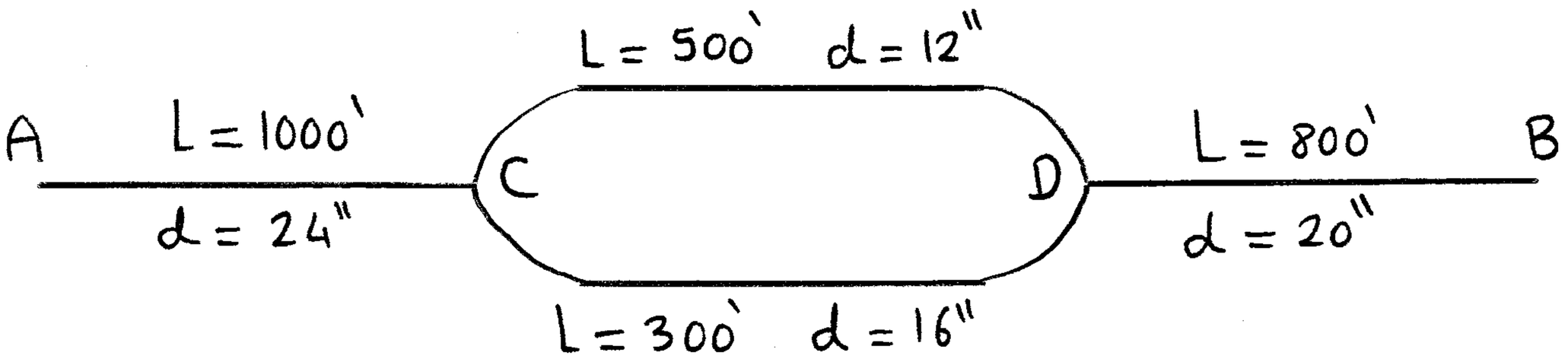
A Two reservoirs are connected by 3 pipes in series as shown in Fig.1. The velocity of water in the first pipe is 5.73 m/s. Considering minor losses, calculate the difference in levels of the water in the two reservoirs. Sketch the T.E.L and the H.G.L. showing all vertical dimensions. K_c=0.5



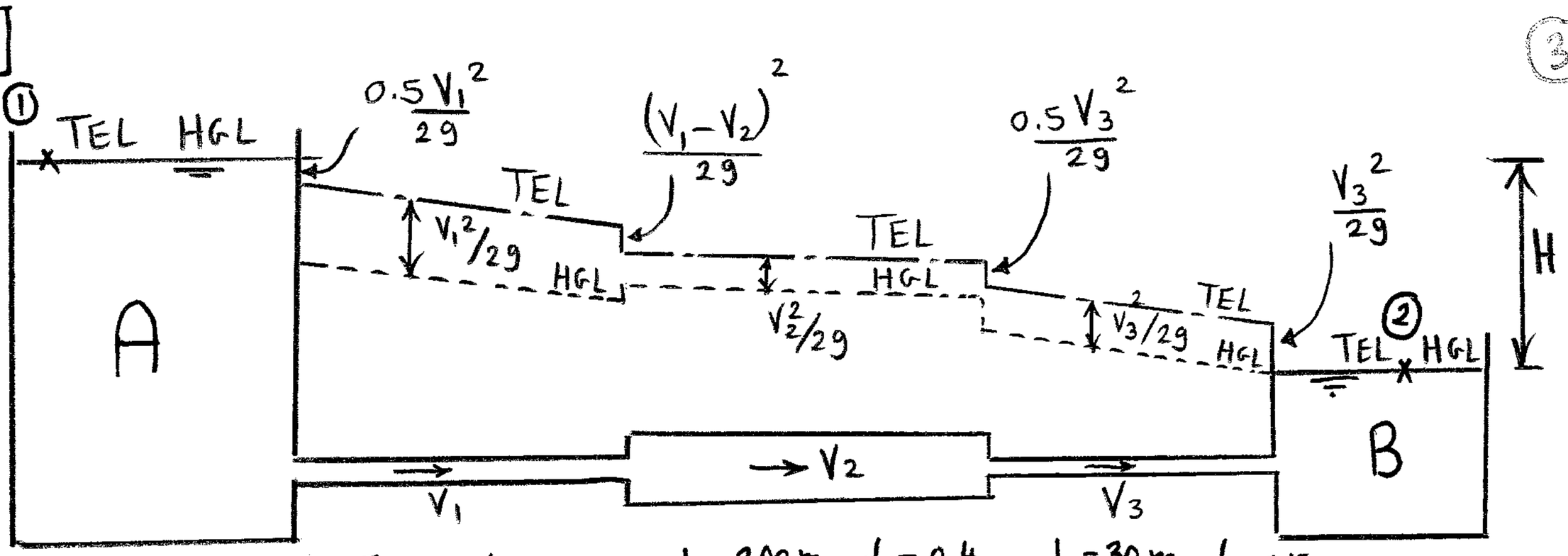
B Two pipes are connected in parallel to each other between two reservoirs with length L_1=2400m, D_1=1.20m, F_1=0.024; L_2=2400m, D_2=1.0m, and F_2=0.02. Find the total flow, if the difference in elevation is 25m.

C Reservoir A is connected to reservoir B through a pipeline having a length 64 km. The elevation of reservoir A is 180 m higher than that of reservoir B. The pipeline was designed to convey a flow rate of 28,000 m^3/day. It was decided to raise the flow rate from reservoir A to reservoir B by laying an identical pipe connected in parallel to the original pipe for a length of 32 km. Find the new flow rate from reservoir A to reservoir B. (F = 0.016 for all pipes).

13 For the system shown in the Fig
What flow will occur when the drop in the hydraulic grade line from A to B is 200 ft. (take F=0.04 for all pipes)
What length of 20-in pipe is equivalent to A-B



A



$V_1 = 5.73\ m/s \qquad A_1 = \frac{\pi d_1^2}{4} = \frac{\pi (0.2)^2}{4} = 0.0314 \qquad Q = A_1 V_1 = 0.18 \frac{m^3}{s}$

$A_2 = \frac{\pi (0.4)^2}{4} = 0.1257\ m^2 \qquad V_2 = \frac{Q}{A_2} = 1.433\ m/s$

$A_3 = \frac{\pi (0.15)^2}{4} = 0.0177\ m^2 \qquad V_3 = \frac{Q}{A_3} = 10.16\ m/s$

Applying Bernoulli equation between ①, ② Datum at ②

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + \sum h_L$$

$$0 + 0 + H = 0 + 0 + 0 + \sum h_{L_{friction}} + \sum h_{L_{secondary}}$$

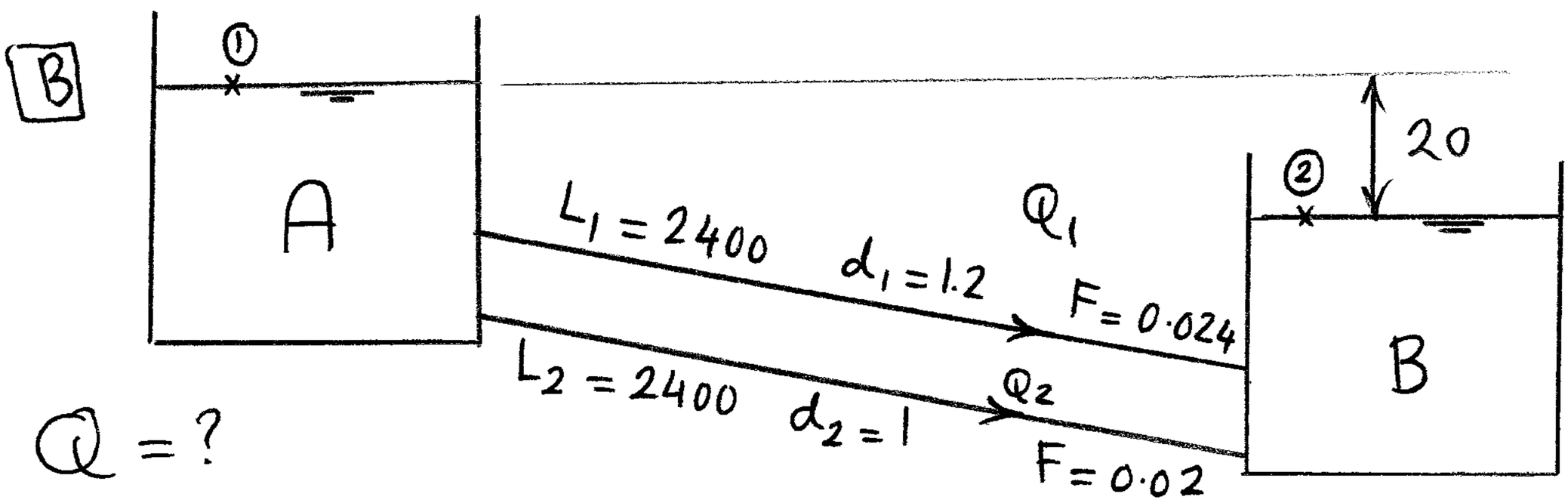
$$H = \frac{F_1 L_1 V_1^2}{2g d_1} + \frac{F_2 L_2 V_2^2}{2g d_2} + \frac{F_3 L_3 V_3^2}{2g d_3} + 0.5\frac{V_1^2}{2g} + \frac{(V_1 - V_2)^2}{2g}$$

$$+ 0.5\frac{V_3^2}{2g} + \frac{V_3^2}{2g}$$

$$H = \frac{0.032(50)(5.73)^2}{2 * 9.81 * 0.2} + \frac{0.04(200)(1.433)^2}{2 * 9.81 * 0.4} + \frac{0.02(30)(10.16)^2}{2 * 9.81 * 0.15}$$

$$+ \frac{0.5(5.73)^2}{2(9.81)} + \frac{(5.73 - 1.433)^2}{2(9.81)} + 1.5\frac{(10.16)^2}{2(9.81)} = \boxed{46.2\ m}$$

B



$Q = ?$

Apply Bernoulli equation between ① , ②

$\therefore \quad h_L = 20$

$\because$ the two pipes are in parallel

$\Rightarrow \quad h_{L_1} = h_{L_2} = 20$

$$\because \quad h_L = \frac{8 F L Q^2}{\pi^2 g d^5}$$

Pipe ①

$$20 = \frac{8 (0.024)\ 2400\ Q_1^2}{\pi^2 (9.81)\ (1.2)^5} \qquad \Rightarrow Q_1 = \underline{3.23\ m^3/s}$$

Pipe ②

$$20 = \frac{8 (0.02)\ 2400\ Q_2^2}{\pi^2 (9.81)\ (1)^5} \qquad \Rightarrow Q_2 = \underline{2.25\ m^3/s}$$

$$Q = Q_1 + Q_2 = \underline{5.48\ m^3/s}$$

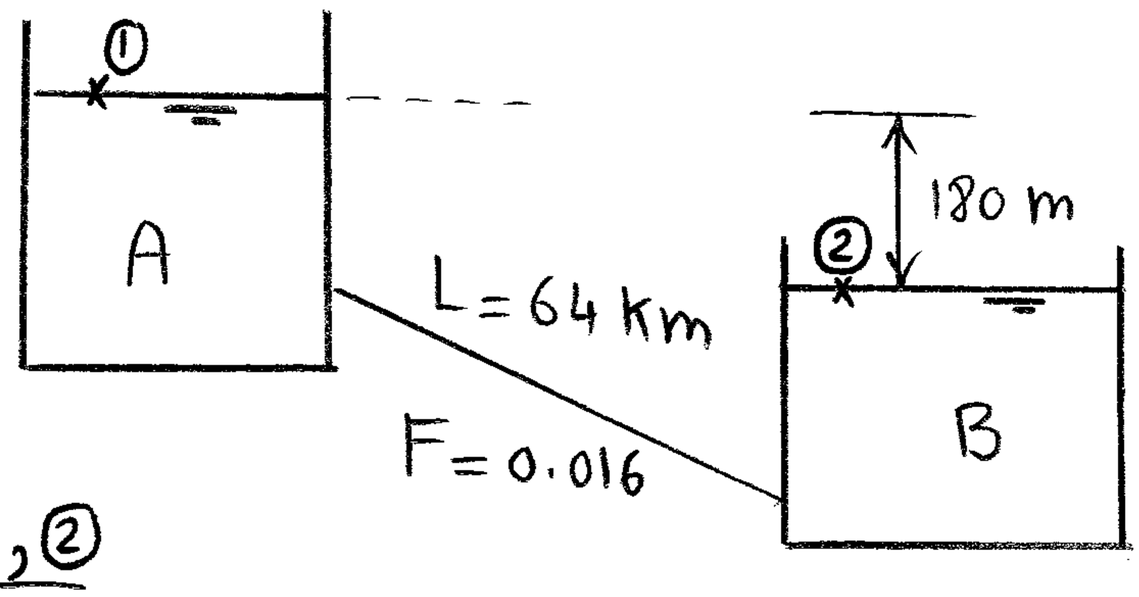
C

Case ① Before laying an identical pipe

Jan 2004

$Q = 28{,}000\ m^3/day$

$= \frac{28{,}000}{24*60*60} = 0.324\ m^3/s$



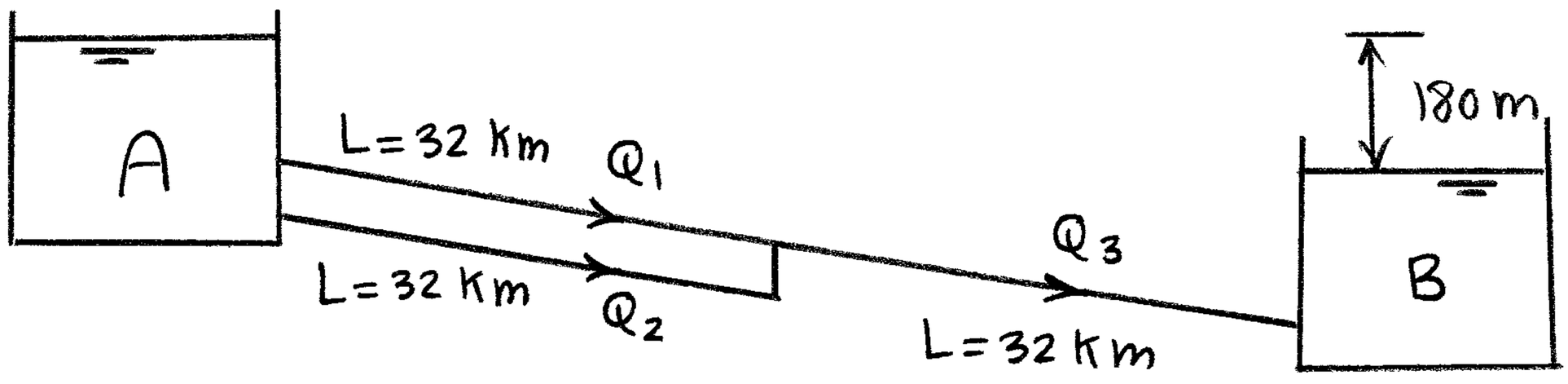
Apply Bernoulli eqn between ①, ②

$\therefore h_L = 180$

$\therefore h_L = \frac{8 F L Q^2}{\pi^2 g d^5}$

$180 = \frac{8 (0.016) (64*10^3) (0.324)^5}{\pi^2 (9.81) d^5} \Rightarrow d = \underline{0.548}$

Case ② After laying an identical pipe L = 32 km



∵ Pipes ① and ② are connected in parallel

$\therefore h_{L_1} = h_{L_2}$

$\frac{8 F L Q_1^2}{\pi^2 g d^5} = \frac{8 F L Q_2^2}{\pi^2 g d^5}$

$\therefore Q_1 = Q_2 \rightarrow ①$

$$\therefore \quad Q_3 = Q_1 + Q_2 \qquad \rightarrow (2)$$

$$\therefore \quad \boxed{Q_3 = 2Q_1}$$

$\therefore$ Pipes (1) and (3) are connected in series

$$\therefore \quad h_{L_1} + h_{L_3} = H$$

$$\frac{8 F L Q_1^2}{\pi^2 g d^5} + \frac{8 F L Q_3^2}{\pi^2 g d^5} = 180 \qquad \rightarrow (3)$$

$$\frac{8 F L}{\pi^2 g d^5} \left(Q_1^2 + (2Q_1)^2 \right) = 180$$

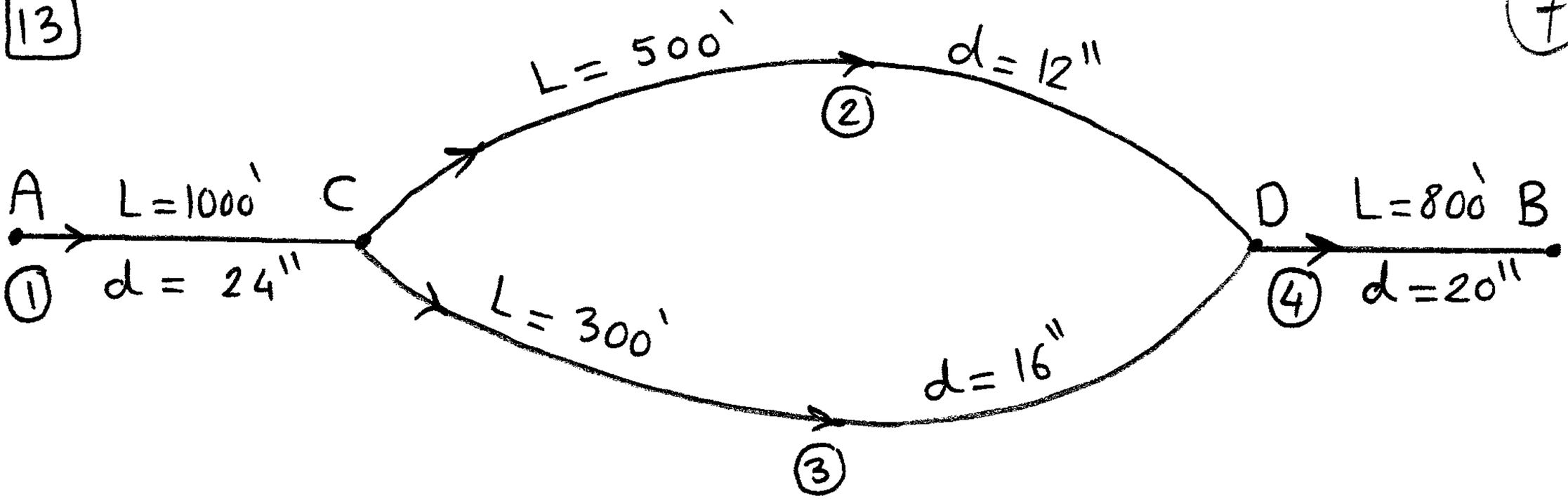
$$\frac{8 (0.016)\ 32 * 10^3}{\pi^2 (9.81)\ (0.548)^5} \left[5 Q_1^2 \right] = 180$$

$$\Rightarrow \quad Q_1 = 0.205 \ m^3/s$$

$$Q_3 = 2 Q_1 = 0.41 \ m^3/s$$

$\therefore$ new flow rate $= 0.41 * (24 * 60 * 60) = \underline{\underline{35\,420}} \ m^3/day$

13



$h_{L_{A\to B}} = 200$ ft $F = 0.04$ $Q = ?$

Solution

$$h_L = \frac{8FL}{\pi^2 g d^5} Q^2 = kQ^2$$

get $K = \frac{8FL}{\pi^2 g d^5}$ for each pipe

K_1	K_2	K_3	K_4
0.0314	0.503	0.0717	0.0626

$h_{L_{CD}} = h_{L_2} = h_{L_3}$ (Parallel pipes)

$K_2 Q_2^2 = K_3 Q_3^2$ → ①

$0.503\, Q_2^2 = 0.0717\, Q_3^2$

$Q_3 = 2.65\, Q_2$

$\because\ Q_1 = Q_2 + Q_3$ → ②

$\therefore\ Q_1 = Q_2 + 2.65\, Q_2 = 3.65\, Q_2$

$$\therefore \quad h_{L_{A\to B}} = 200 \text{ ft}$$

$$\therefore \quad h_{L_{AC}} + h_{L_{CD}} + h_{L_{DB}} = 200 \text{ ft}$$

$$K_1 Q_1^2 + K_2 Q_2^2 + K_4 Q_4^2 = 200$$

$$0.0314\, Q_1^2 + 0.503\, Q_2^2 + 0.0626\, Q_1^2 = 200$$

$$0.0314\,(3.65 Q_2)^2 + 0.503\, Q_2^2 + 0.0626\,(3.65\, Q_2)^2 = 200$$

$$1.75\, Q_2^2 = 200 \quad \Rightarrow \quad Q_2 = 10.67 \text{ ft}^3/\text{s}$$

$$\therefore \quad Q_1 = 3.65\, Q_2 = \underline{\underline{38.96 \text{ ft}^3/\text{s}}}$$

---

b) $d = 20''$ $\qquad L_{eq} = ?$

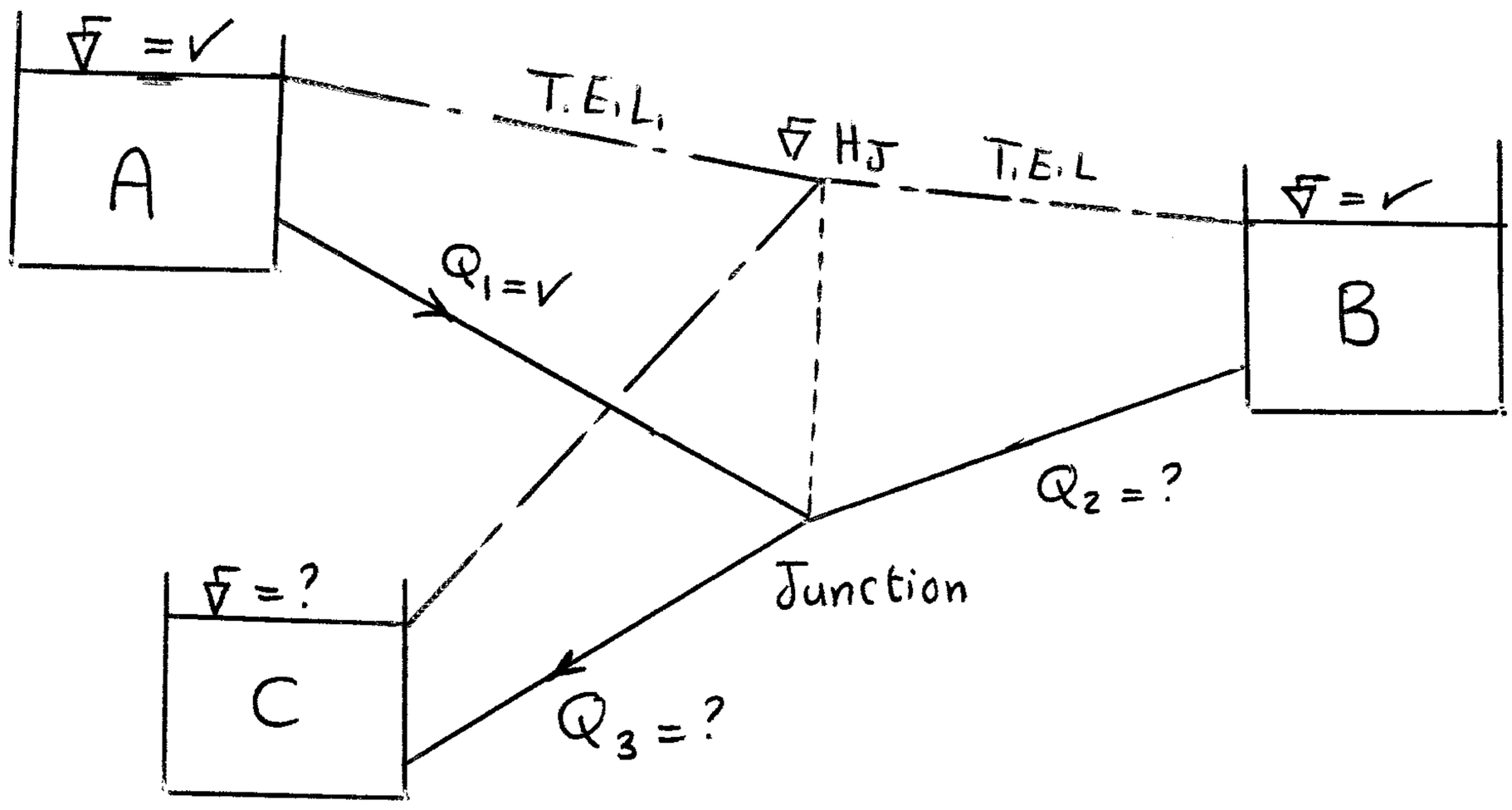
$$\therefore \quad h_L = \frac{8 F L_{eq} Q^2}{\pi^2 g d^5}$$

$$200 = \frac{8(0.04)\, L_{eq}\,(38.96)^2}{\pi^2 (32.2)\,(20/12)^5}$$

$$\therefore L_{equivalent} = 1682 \text{ ft}$$

Three tank problem

(a) Q given



Given: F, L, d for each pipe

Elevation of A, B , Flow from A → J

Required: Elevation of C

Let $H_A = \frac{P_A}{\gamma} + \frac{V_A^2}{2g} + Z_A$ = T.E. at A

Get $K = \frac{8FL}{\pi^2 g d^5}$ for each pipe ∴ $K_1 = ✓$, $K_2 = ✓$, $K_3 = ✓$

Apply Bernoulli equation Between A, J

$$\left(\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + Z_A\right) = \left(\frac{P_J}{\gamma} + \frac{V_J^2}{2g} + Z_J\right) + h_{L_{A\to J}}$$

$$H_A = H_J + K_1 Q_1^2 \quad \Rightarrow \quad H_J = ✓$$

(✓ under H_A; ? under H_J; ✓ ✓ under $K_1 Q_1^2$)

If $H_J < H_B \Rightarrow$ flow from B → J

If $H_J > H_B \Rightarrow$ flow from J → B

Say $H_J > H_B$ $\therefore$ flow from J → B

Apply Bernoulli equation between J, B

$$H_J = H_B + K_2 Q_2^2 \quad \Rightarrow \quad Q_2 = \checkmark$$

$$\checkmark \qquad \checkmark \qquad \checkmark \quad ?$$

If $Q_1 > Q_2$ $\therefore Q_1 = Q_2 + Q_3$ $H_J > H_C$

If $Q_1 < Q_2$ $\therefore Q_1 + Q_3 = Q_2$ $H_C > H_J$

Say $Q_1 > Q_2$ $\therefore$ flow from J → C

$$\therefore \quad Q_1 = Q_2 + Q_3 \quad \Rightarrow \quad Q_3 = \checkmark$$

$$\checkmark \qquad \checkmark \qquad ?$$

Apply Bernoulli eqn between J, C

$$H_J = H_C + K_3 Q_3^2 \quad \Rightarrow H_C = \checkmark$$

$$\checkmark \qquad ? \qquad \checkmark \ \checkmark$$

If $Q_1 < Q_2$ $\therefore$ flow from C → J

$$\therefore \quad Q_1 + Q_3 = Q_2 \quad \Rightarrow \quad Q_3 = \checkmark$$

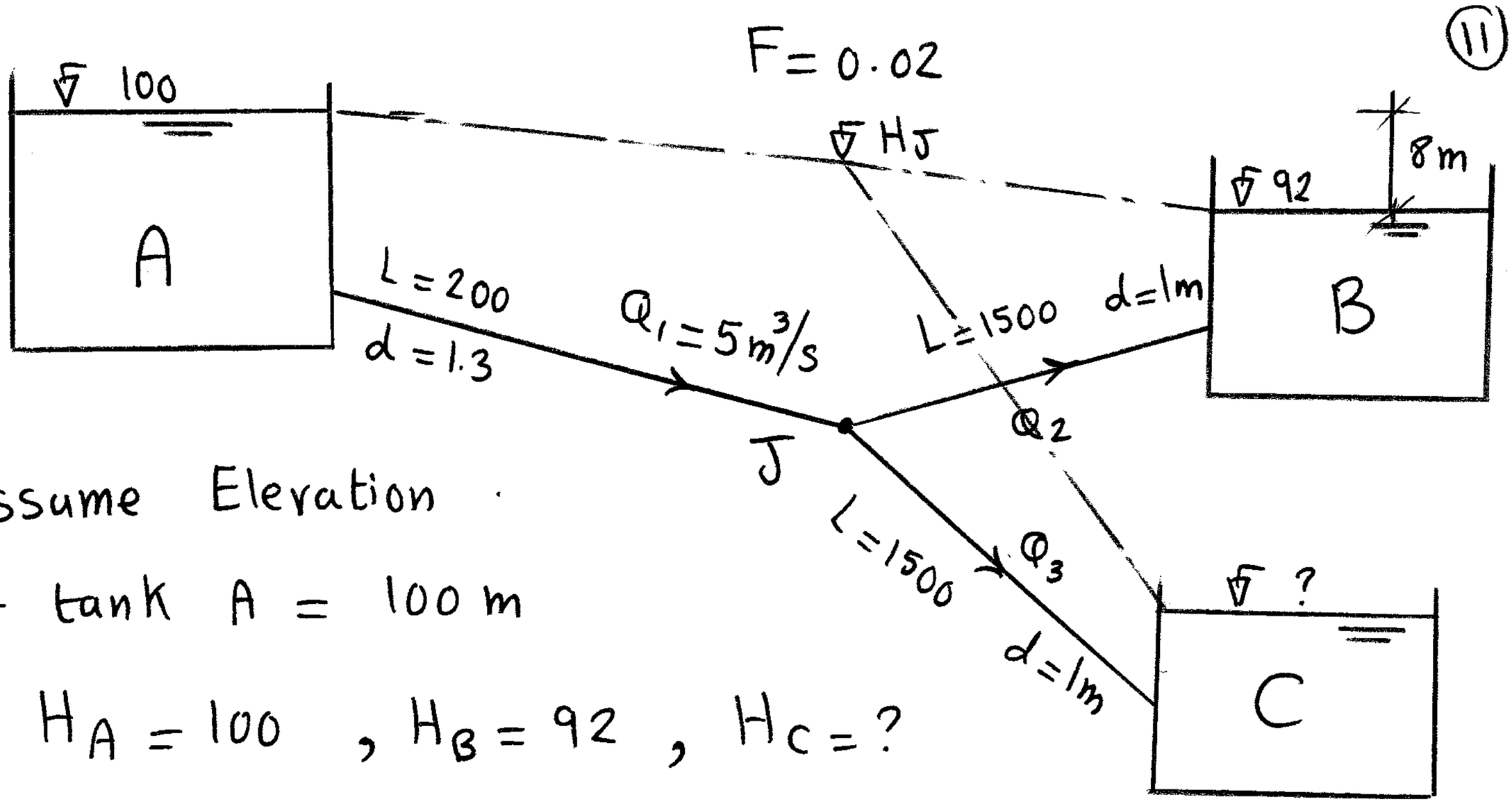
$$\checkmark \qquad ? \qquad \checkmark$$

Apply Bernoulli eqn between C, J

$$H_C = H_J + K_3 Q_3^2 \quad \Rightarrow \quad H_C = \checkmark$$

$$? \qquad \checkmark \qquad \checkmark \ \checkmark$$

15



Assume Elevation of tank A = 100 m

$\therefore H_A = 100 \;, H_B = 92 \;, H_C = ?$

$$K = \frac{8FL}{\pi^2 g d^5}$$

no	pipe	F	L	d	K
1	AJ	0.02	200	1.3	0.089
2	JB	0.02	1500	1	2.48
3	JC	0.02	1500	1	2.48

Apply Bernoulli eqn between A, J

$$H_A = H_J + h_{L_{A \to J}}$$

$$H_A = H_J + K_1 Q_1^2$$

$$100 = H_J + 0.089(5)^2 \quad \Rightarrow H_J = 97.78\,m$$

$\therefore H_J > H_B \Rightarrow$ water discharges from J to B

Bernoulli between J, B

$$H_J = H_B + K_2 Q_2^2$$

$$97.78 = 92 + 2.48\, Q_2^2 \quad \Rightarrow Q_2 = 1.53\ m^3/s$$

$\because Q_1 > Q_2$ water flows from J to C

$$\therefore Q_1 = Q_2 + Q_3$$

$$5 = 1.53 + Q_3 \quad \Rightarrow Q_3 = 3.47\ m^3/s$$

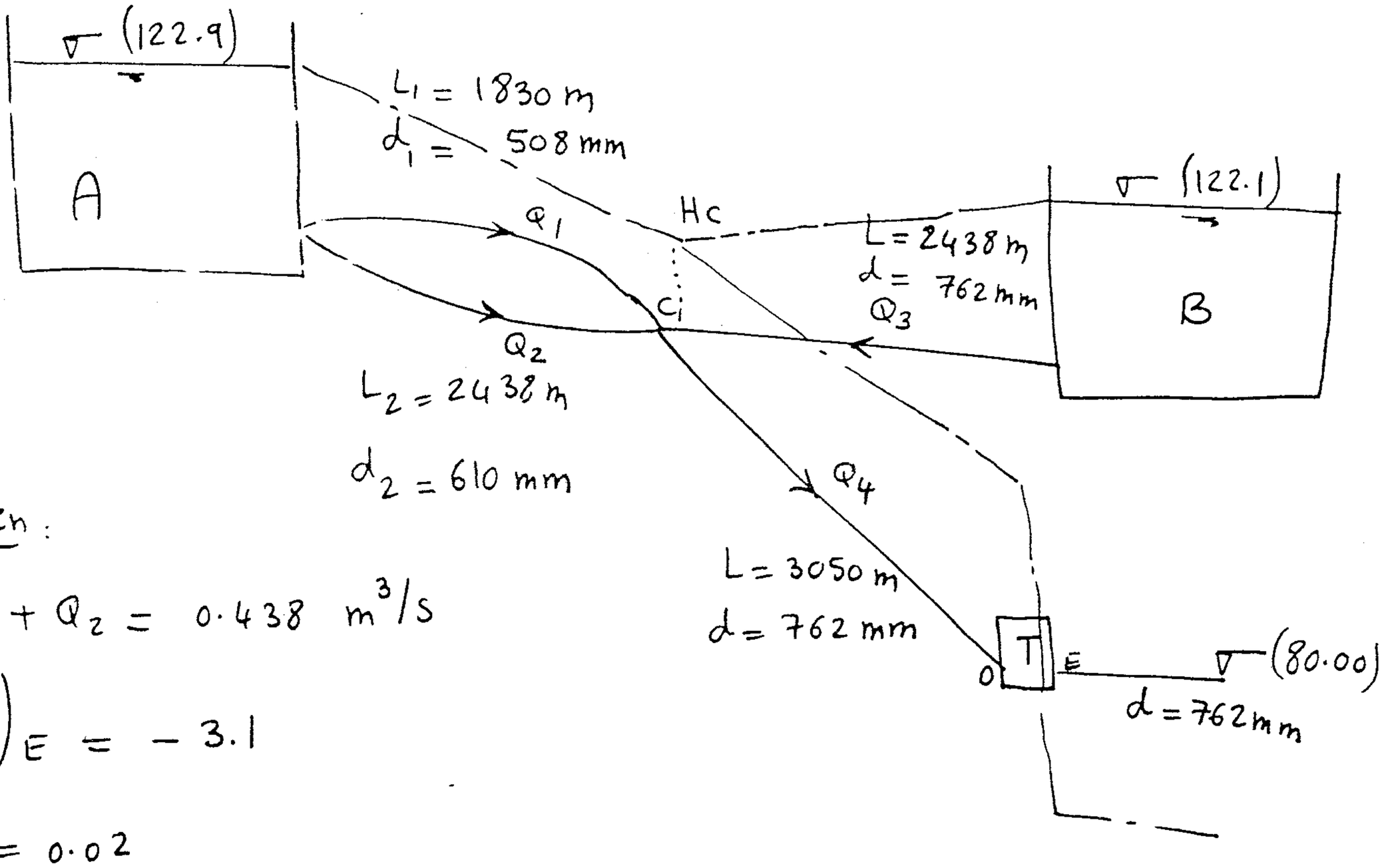
Bernoulli between J, C

$$H_J = H_C + K_3 Q_3^2$$

$$97.78 = H_C + 2.48\,(3.47)^2$$

$$\Rightarrow H_C = \underline{\underline{67.92\ m}}$$

D The flow from reservoir A is $0.438\ m^3/s$. Determine the power extracted by turbine DE, if the pressure head at E is -3.1 m. $F = 0.02$ (13)



Given:

$Q_1 + Q_2 = 0.438\ m^3/s$

$\left(\frac{P}{\gamma}\right)_E = -3.1$

$F = 0.02$

Sol

Head at A $H_A = \frac{P}{\gamma} + \frac{V^2}{2g} + Z$

$\therefore H_A = 122.9$

$H_B = 122.1$

$\because h_L = \frac{8FLQ^2}{\pi^2 g d^5} = kQ^2$

$\therefore k = \frac{8FL}{\pi^2 g d^5}$

$k_1 = \frac{8FL_1}{\pi^2 g d_1^5}$, $k_2 = \frac{8FL_2}{\pi^2 g d_2^5}$, $k_3 = \frac{8FL_3}{\pi^2 g d_3^5}$, $k_4 = \frac{8FL_4}{\pi^2 g d_4^5}$

$\because$ Pipe 1, 2 are in parallel

$\therefore \quad h_{f_1} = h_{f_2}$

$$K_1 Q_1^2 = K_2 Q_2^2 \quad \rightarrow (1)$$

$$\therefore \quad Q_1 + Q_2 = 0.438\ m^3/s \quad \rightarrow (2)$$

from (1), (2)

$\Rightarrow Q_1 = \checkmark \qquad Q_2 = \checkmark$

Bernoulli A, C

$$H_A = H_C + K_1 Q_1^2$$

$$122.9 = (H_C)_? + \underset{\checkmark}{K_1}\ \underset{\checkmark}{Q_1^2}$$

$\Rightarrow H_C = \checkmark$ Say $90 < 120$

$\therefore$ Flow from B $\rightarrow$ C

Bernoulli B, C

$$H_B = H_C + K_3 Q_3^2$$

$$122.1 = 90 + \underset{\checkmark}{K_3}(Q_3^2)_?$$

$\Rightarrow Q_3 = \checkmark$

$$\therefore Q_4 = Q_1 + Q_2 + Q_3 = \checkmark$$

Bernoulli C, E

$$H_C = H_E + h_{L_{C\rightarrow E}} + h_T$$

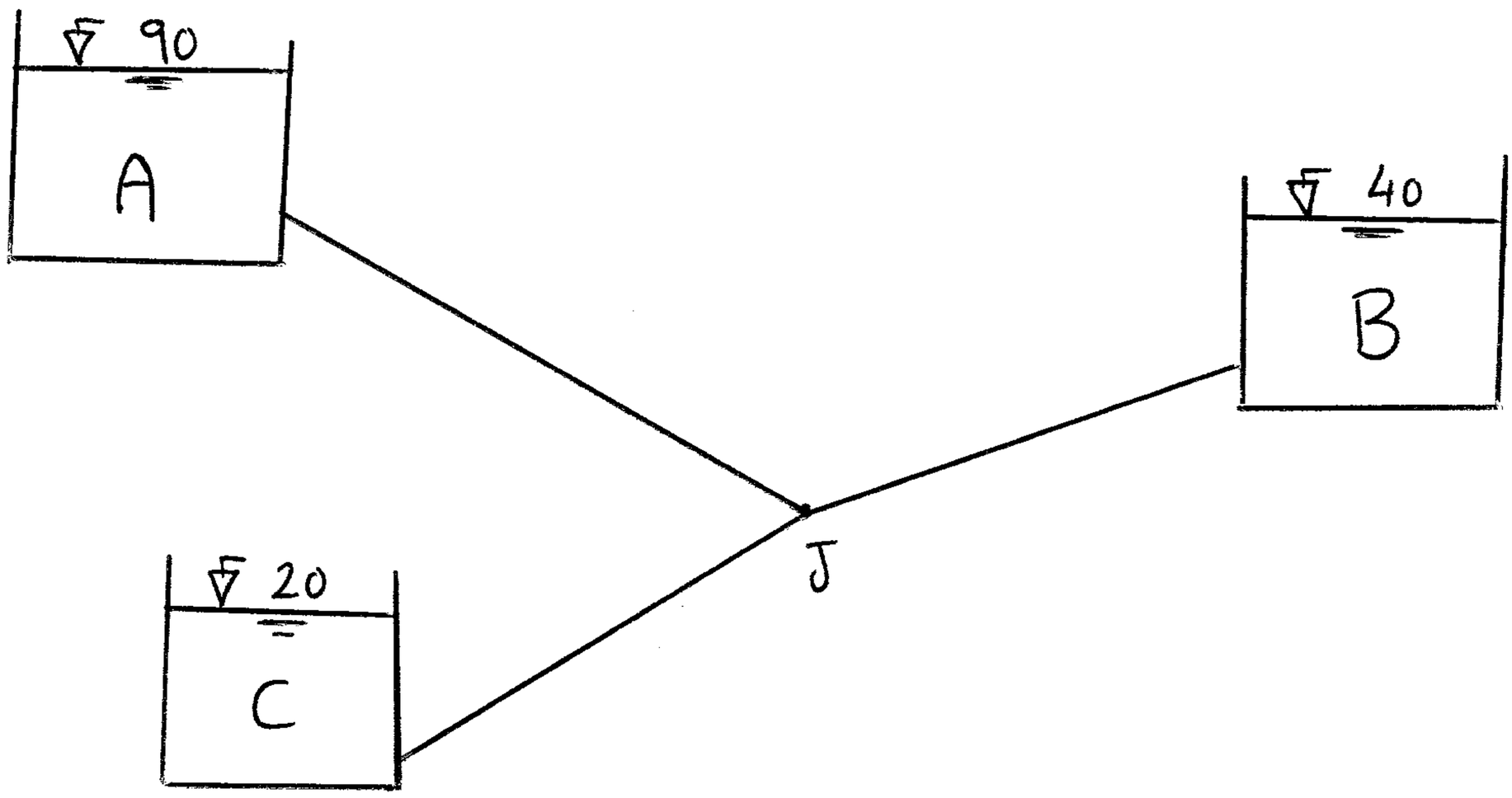
$$90 = \left(\underset{-3.1}{\frac{P_1}{\gamma}} + \cancel{\frac{V_1^2}{2g}} + \underset{80}{z_E}\right) + \underset{\checkmark}{K_4}\ \underset{\checkmark}{Q_4^2} + (h_T)_?$$

$\Rightarrow h_T = \checkmark$

$$\therefore \text{Power}_{\text{Turbine}} = \gamma Q_4 h_T = \checkmark$$

Three tank problem

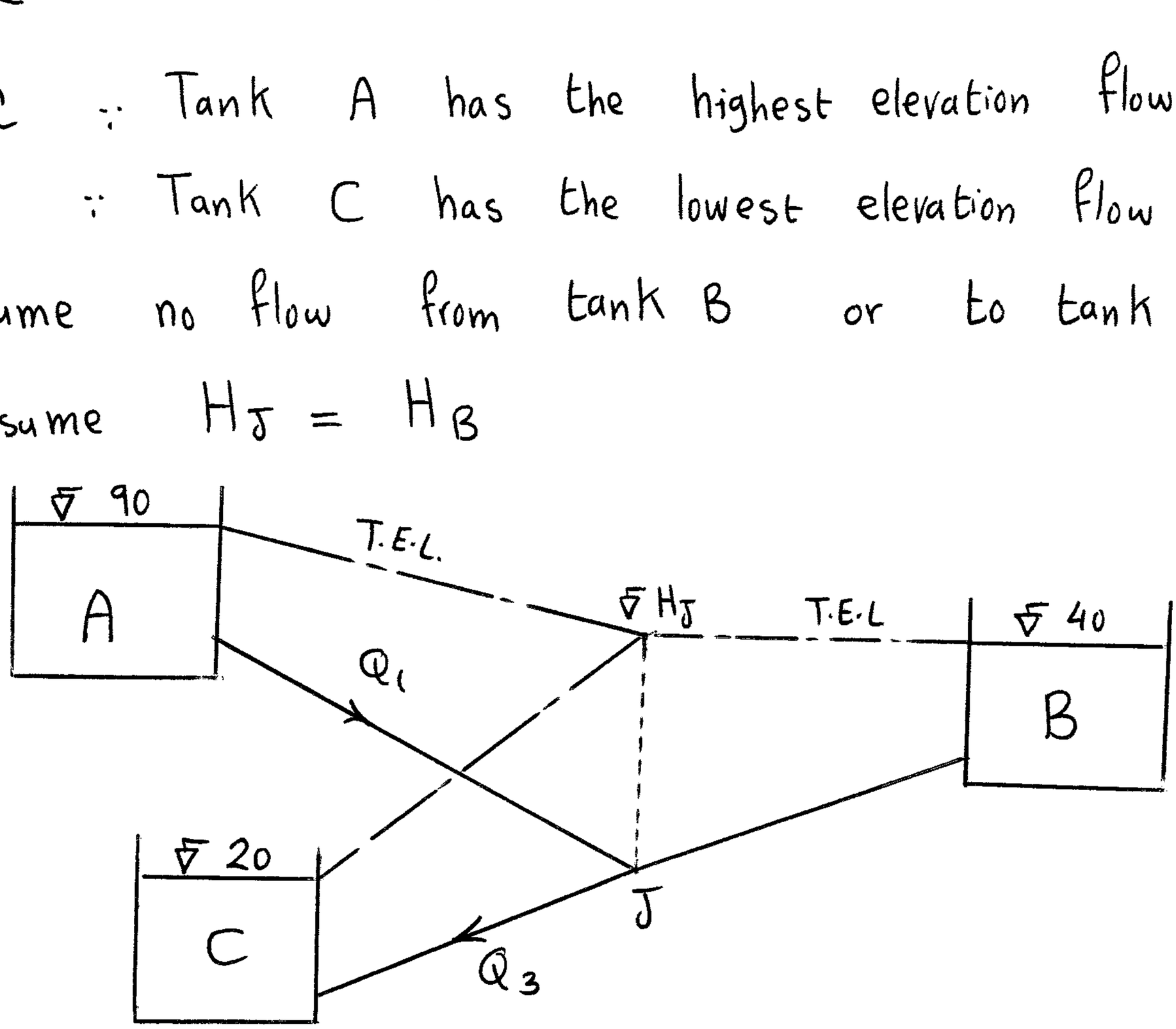
(b) Q not given



Given : F, L, d for each pipe
Elevation of each tank

Required : Q_1 , Q_2 , Q_3

Solution ∴ Tank A has the highest elevation flow from A
∴ Tank C has the lowest elevation flow to B
Assume no flow from tank B or to tank B
∴ Assume $H_J = H_B$



Pipe	F	L	d	$K = 8FL/\pi^2 g d^5$
①				$K_1 = \checkmark$
②				$K_2 = \checkmark$
③				$K_3 = \checkmark$

Assume $H_J = H_B = 40$ $\Rightarrow$ $Q_2 = 0$

Apply Bernoulli eqn between A, J

$$H_A = H_J + h_{L\,A \to J}$$

$$90 = 40 + K_1 Q_1^2 \qquad \Rightarrow Q_1 = \checkmark\ 4$$

Apply Bernoulli eqn between J, C

$$H_J = H_C + h_{L\,J \to C}$$

$$40 = 20 + K_3 Q_3^2 \qquad \Rightarrow Q_3 = \checkmark\ 1$$

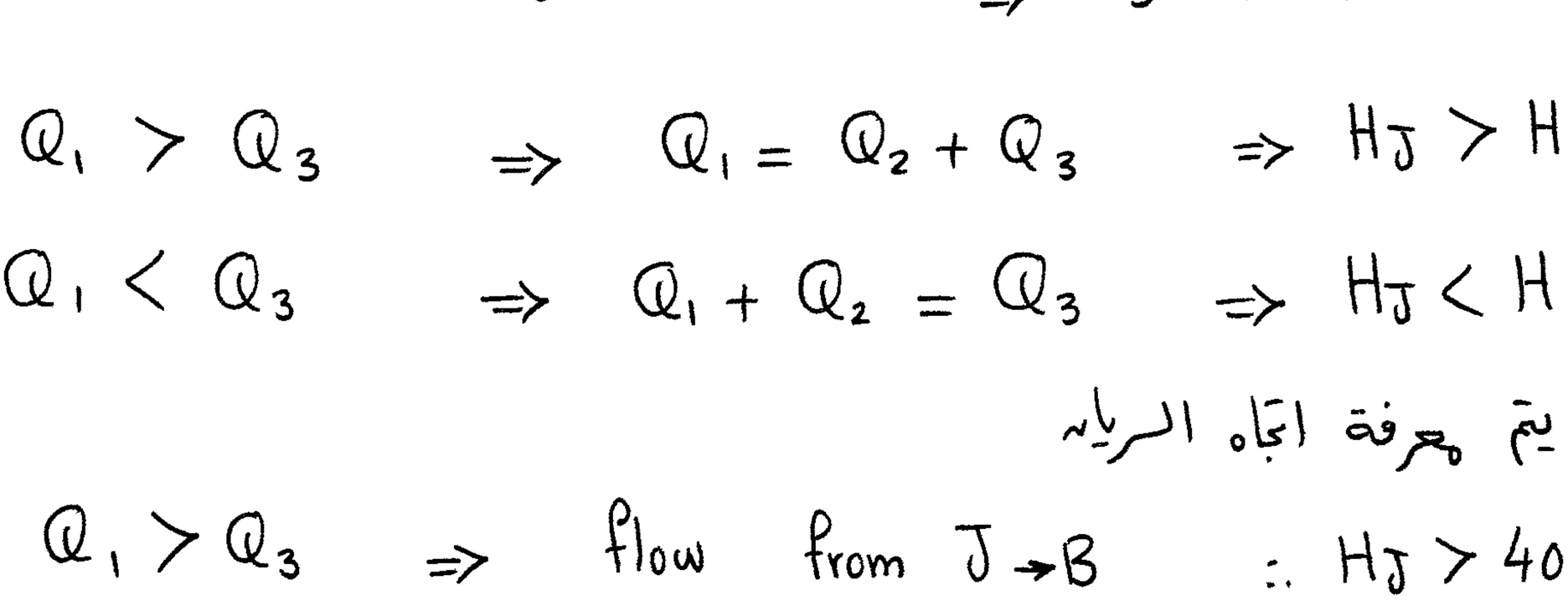
If $Q_1 > Q_3 \Rightarrow Q_1 = Q_2 + Q_3 \Rightarrow H_J > H_B$

If $Q_1 < Q_3 \Rightarrow Q_1 + Q_2 = Q_3 \Rightarrow H_J < H_B$

⇐ يتم معرفة اتجاه السريان

Say $Q_1 > Q_3 \Rightarrow$ flow from $J \to B$ $\therefore H_J > 40$

Assume $H_J = 60$



Second trial

$$Q_1 = Q_2 + Q_3 \quad \rightarrow (1)$$

$$H_A = H_J + h_{L_{A\rightarrow J}} \quad \rightarrow (2)$$

$$90 = 60 + K_1 Q_1^2 \quad \Rightarrow Q_1 = \checkmark\ 2$$

$$H_J = H_B + h_{L_{J\rightarrow B}} \quad \rightarrow (3)$$

$$60 = 50 + K_2 Q_2^2 \quad \Rightarrow Q_2 = \checkmark\ 1$$

$$H_J = H_C + h_{L_{J\rightarrow C}} \quad \rightarrow (4)$$

$$60 = 20 + K_3 Q_3^2 \quad \Rightarrow Q_3 = \checkmark\ 3$$

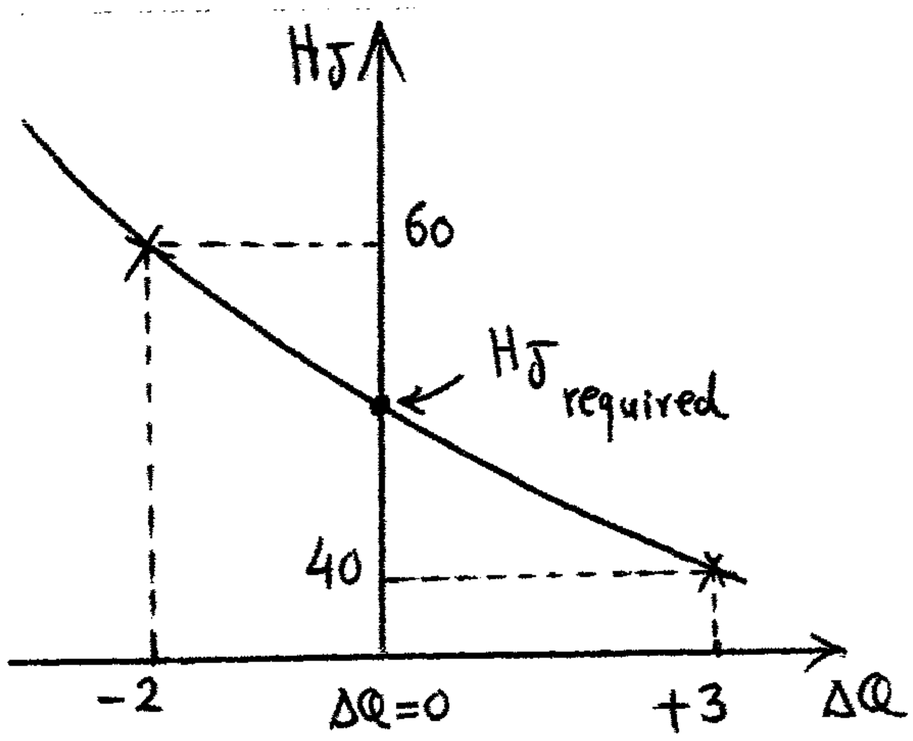
$$\therefore \Delta Q_{II} = Q_{in} - Q_{out}$$

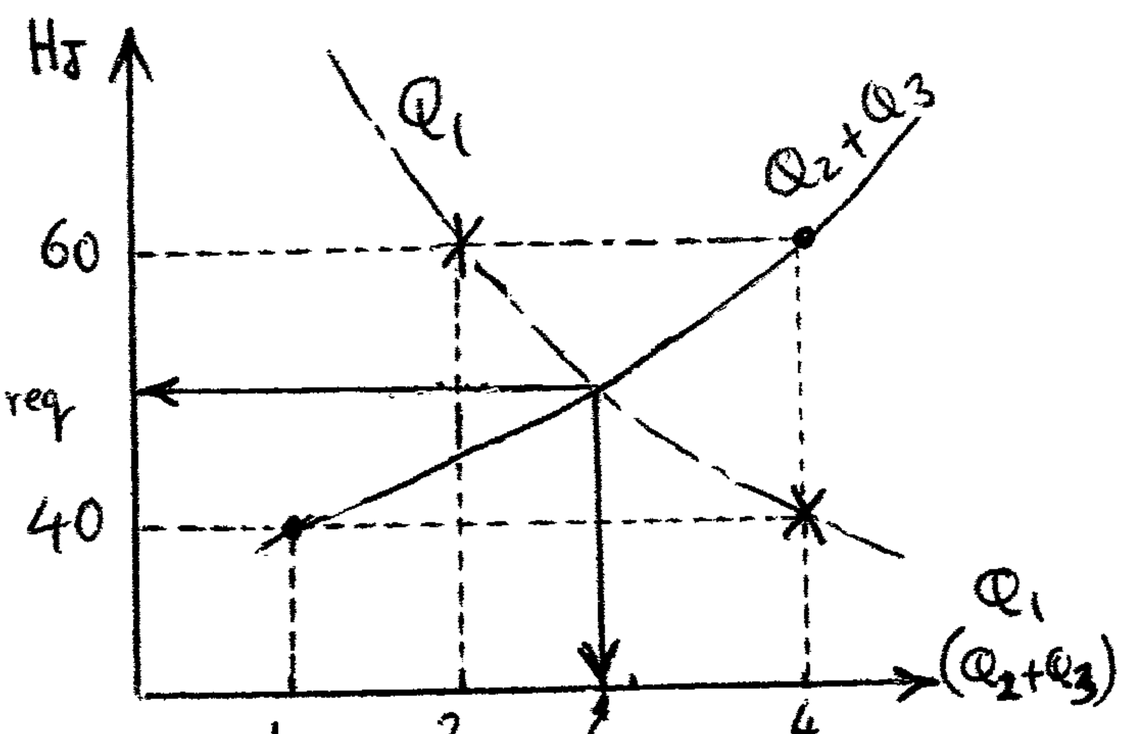
$$= (Q_1) - (Q_2 + Q_3)$$

$$= 2 - (1+3) = -2$$

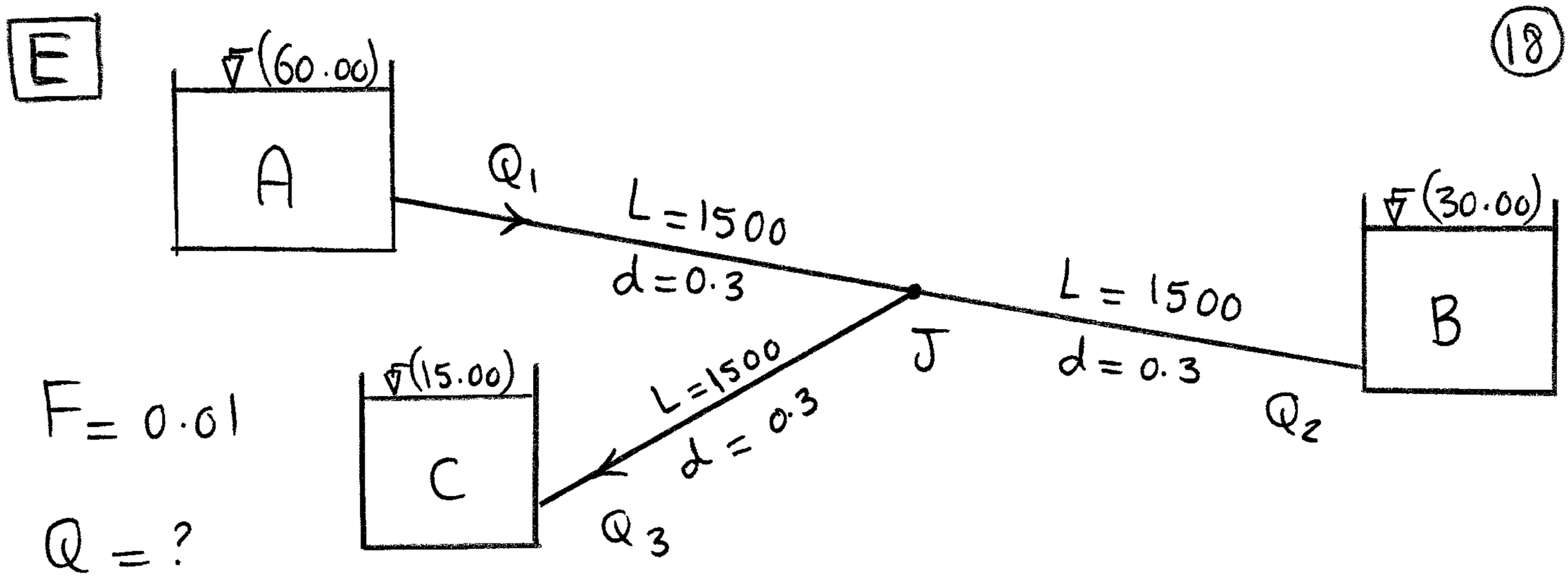
عندما تقترب $\Delta Q \simeq 0$ يتم ايقاف المحاولات أو يتم عمل جدول أو عمل أي من المنحنيين

H_J	Q_1	Q_2	Q_3	Q_2+Q_3	ΔQ
40	4	0	1	1	+3
60	2	1	3	4	−2
50					





E



$F = 0.01$

$Q = ?$

$$K_1 = K_2 = K_3 = \frac{8FL}{\pi^2 g d^5}$$

$$K = \frac{8\,(0.01)\,1500}{\pi^2 (9.81)(0.3)^5} = 510$$

Assume $H_J = H_B = 30\,m$

Bernoulli A, J

$$H_A = H_J + K_1 Q_1^2$$

$$60 = 30 + 510\,Q_1^2 \quad \Rightarrow \quad Q_1 = 0.242\ m^3/s$$

Bernoulli J, C

$$H_J = H_C + K_3 Q_3^2$$

$$30 = 15 + 510\,Q_3^2 \quad \Rightarrow \quad Q_3 = 0.17\ m^3/s$$

$\because Q_1 > Q_3 \quad \Rightarrow \quad Q_1 = Q_2 + Q_3 \qquad H_J > 30$

$$\Delta Q = Q_1 - (Q_2 + Q_3) = 0.07$$

Assume $H_J = 40$

* $H_A = H_J + K_1 Q_1^2$

$60 = 40 + 510\, Q_1^2$ $\Rightarrow Q_1 = 0.198\ m^3/s$

* $H_J = H_B + K_2 Q_2^2$

$40 = 30 + 510\, Q_2^2$ $\Rightarrow Q_2 = 0.14\ m^3/s$

* $H_J = H_C + K_3 Q_3^2$

$40 = 15 + 510\, Q_3^2$ $\Rightarrow Q_3 = 0.22\ m^3/s$

$\Delta Q = Q_1 - (Q_2 + Q_3) = -0.16$

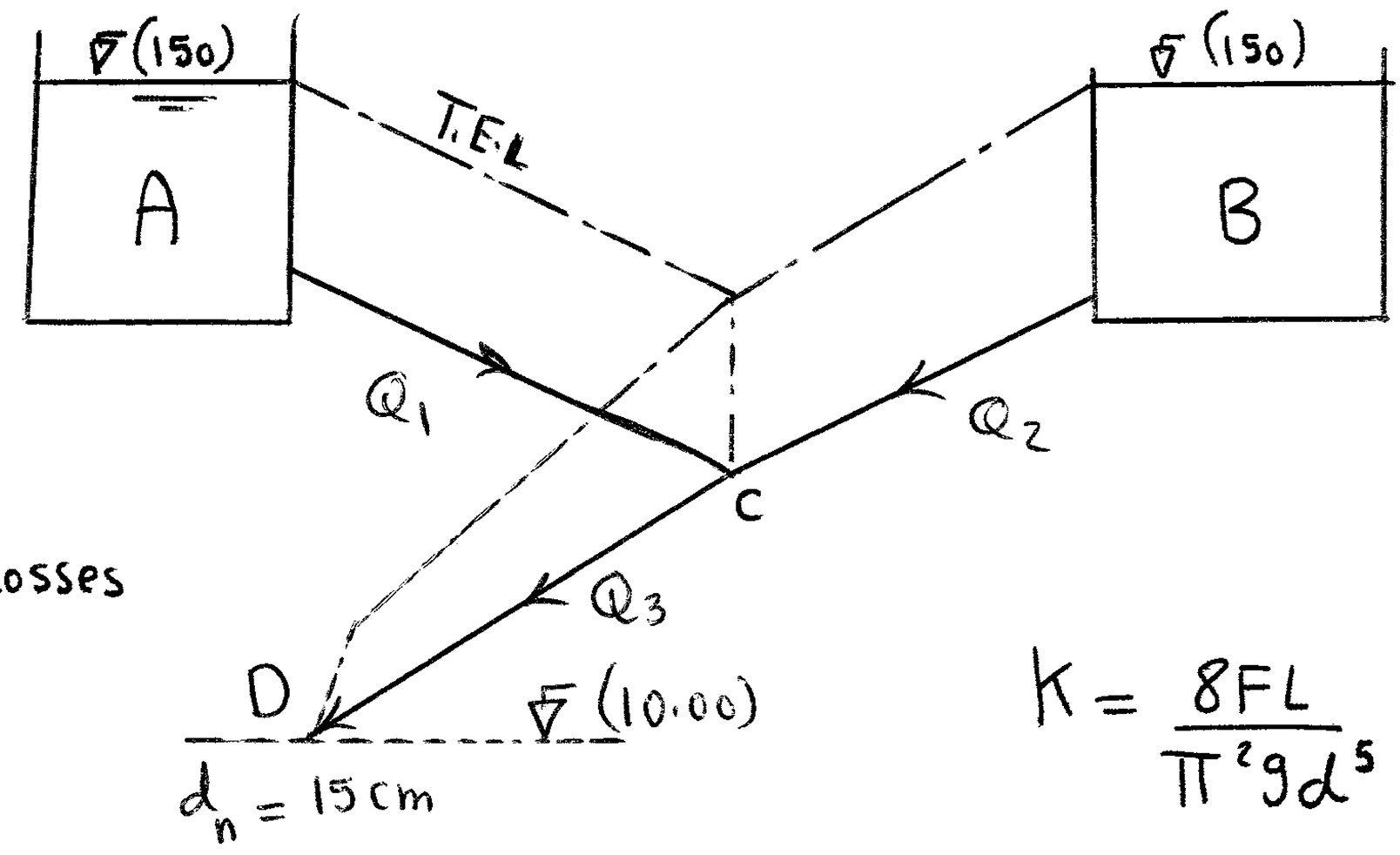
H_J	Q_1	Q_2	Q_3	ΔQ
30	0.242	0	0.17	0.07
40	0.198	0.14	0.22	−0.16
35	0.221	0.1	0.198	−0.075
34	0.226	0.088	0.193	−0.055
32	0.234	0.0626	0.182	−0.01
31	0.238	0.044	0.177	0.017
31.6	0.236	0.056	0.18	0

16

$Q = ?$

neglect $V^2/2g$

neglect minor losses



$$K = \frac{8FL}{\pi^2 g d^5}$$

Line	L (m)	D (m)	F	K
AC	40 000	0·50	0·02	2115
BC	20 000	0.25	0·02	33844
CD	60 000	0·75	0·02	417.8

assume $H_c = 100\ m$

$$H_A = H_c + K_1 Q_1^2$$

$$150 = 100 + 2115\ Q_1^2 \quad \Rightarrow Q_1 = 0.154$$

$$H_B = H_c + K_2 Q_2^2$$

$$150 = 100 + 33844\ Q_2^2 \quad \Rightarrow Q_2 = 0.038$$

$$H_c = H_D + K_3 Q_3^2$$

$$100 = 10 + 417.8\ Q_3^2 \quad \Rightarrow Q_3 = 0.464$$

$$(Q_1 + Q_2) < Q_3 \quad \Rightarrow H_c < 100$$

$$\Delta Q = Q_{in} - Q_{out} = (Q_1 + Q_2) - (Q_3)$$

$$= (0.154 + 0.038) - (0.464) = -0.272$$

H_c	Q_1	Q_2	Q_3	Q_1+Q_2	ΔQ
100	0.154	0.038	0.464	0.192	-0.272
80	0.182	0.0454	0.41	0.23	-0.182
60	0.206	0.052	0.346	0.258	-0.088
40	0.228	0.057	0.268	0.285	$+0.017$
45	0.223	0.055	0.289	0.278	-0.01
44	0.224	0.056	0.285	0.28	$-5.4 * 10^{-3}$
43	0.225	0.056	0.281	0.281	$+1 * 10^{-4} \simeq 0$

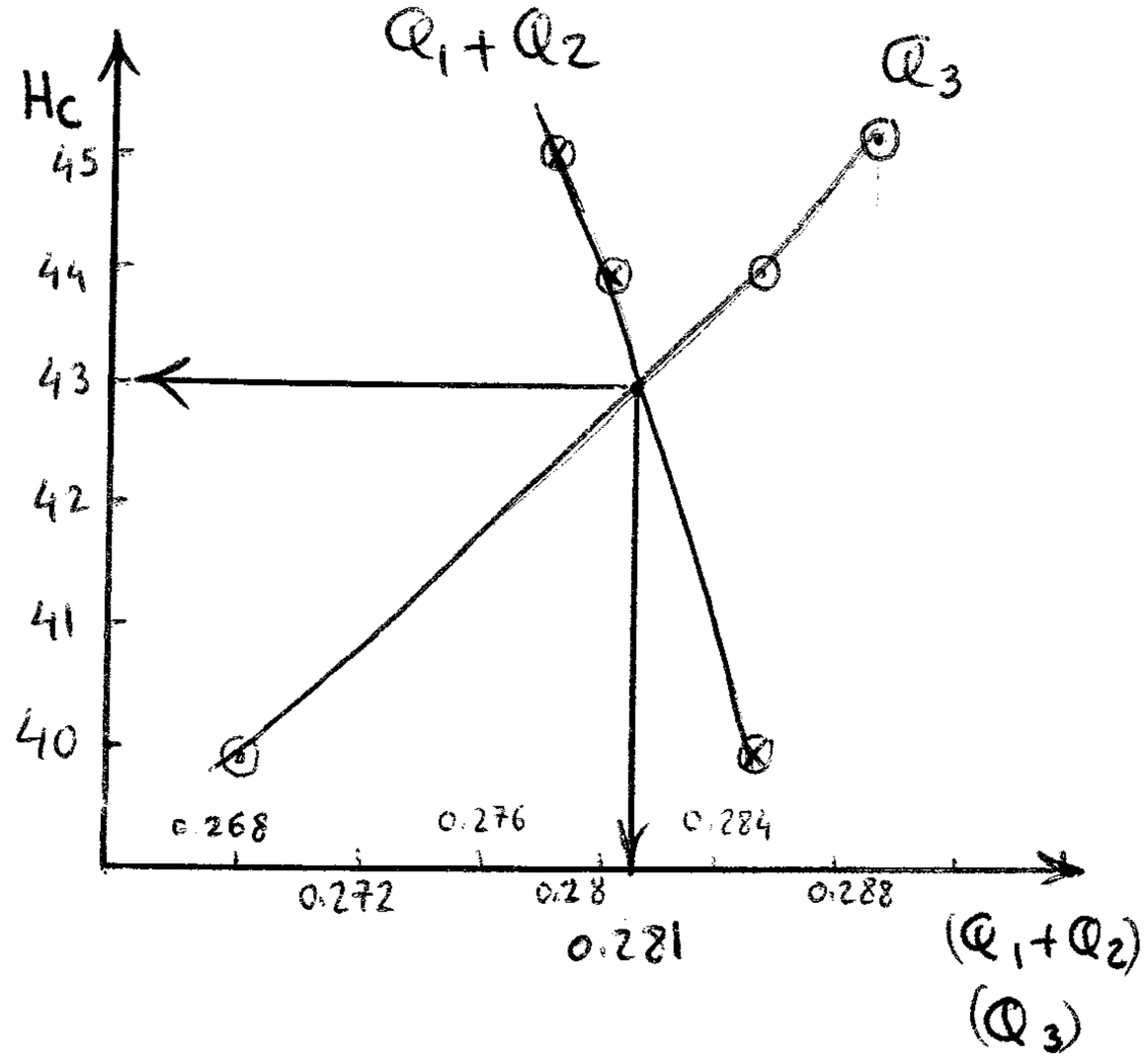
$$\therefore \quad Q_1 = 0.225 \;,\; Q_2 = 0.056 \;,\; Q_3 = 0.281$$

$$V_n = \frac{Q_3}{A} = \frac{0.281}{\pi \frac{(0.15)^2}{4}} = 15.9 \text{ m/s}$$

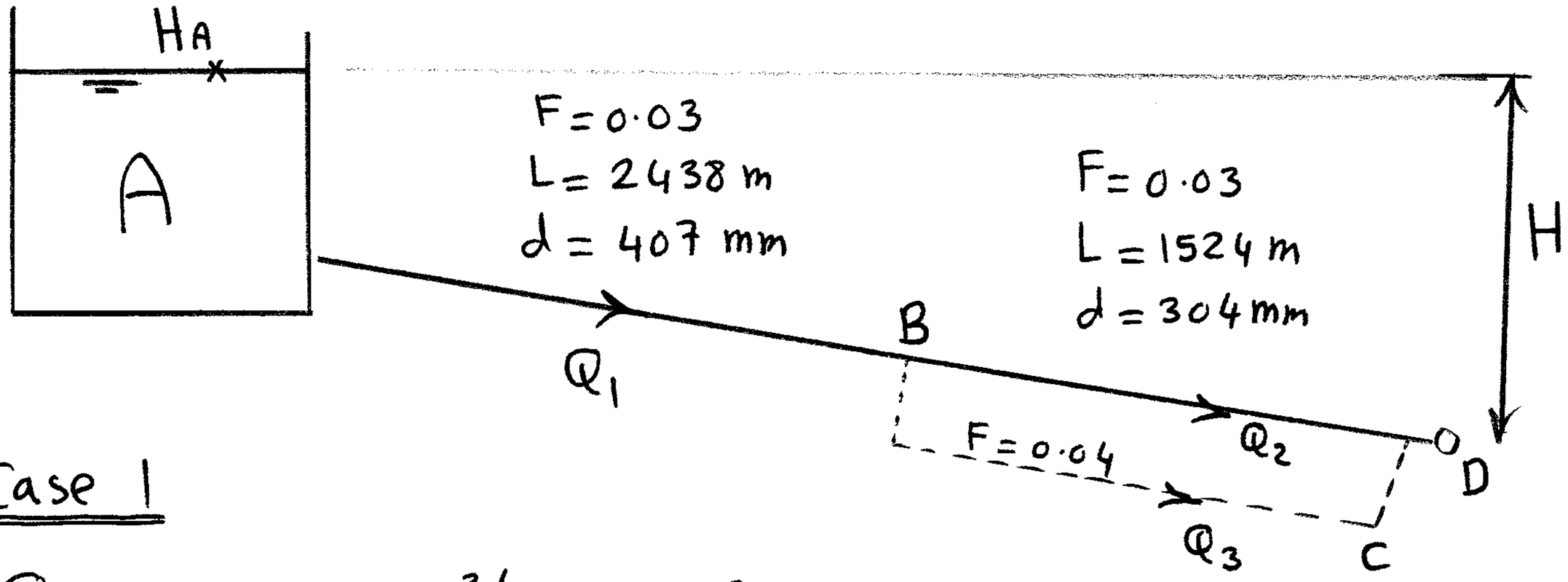
$$\text{Power} = \frac{\gamma Q_3 \left(\frac{V_n^2}{2g}\right)}{k}$$

$$= \frac{9810\,(0.281)\,\frac{(15.9)^2}{2\,(9.81)}}{735}$$

$$= 48.3 \text{ horse}$$



10. For the system shown when the flow from reservoir A to main D is 0.1424 m^3/s, the pressure at D is 1.38 bar. The flow to D must be increased to 0.186 m^3/s with the pressure 2.76 bar. What is size of a pipe 1524 m long, which should be laid from B to C parallel to the exiting 304 mm to accomplish this result.



Case 1

$$Q = 0.1424 \ m^3/s \ , \quad P_0 = 1.38 \ bar$$

$$V_2 = \frac{Q_2}{A_2} = \frac{0.1424}{\frac{\pi(0.304)^2}{4}} = 1.962 \ m/s$$

Let $H = \frac{P}{\gamma} + \frac{V^2}{2g} + z_A$ (T.E.L. at any Point)

Let $K = \frac{8FL}{\pi^2 g d^5}$

$$\therefore K_1 = \frac{8F_1L_1}{\pi^2 g d_1^5} = \frac{8(0.03)\,2438}{\pi^2(9.81)(0.407)^5} = 541.1$$

$$K_2 = \frac{8F_2L_2}{\pi^2 g d_2^5} = \frac{8(0.03)\,1524}{\pi^2(9.81)(0.304)^5} = 1455$$

Bernoulli eqn between A, D Datum at D

$$H_A = H_D + \Sigma h_L$$

$$H_A = \frac{P_D}{\gamma} + \frac{V_D^2}{2g} + z_D + K_1 Q_1^2 + K_2 Q^2$$

$$H_A = \frac{1.38*10^5}{9810} + \frac{(1.962)^2}{2(9.81)} + 0 + 541.1(0.1424)^2 + 1455(0.1424)^2 = \underline{54.74}$$

Case 2 $\quad Q = 0.186\ m^3/s \quad P_D = 2.76\ bar$

Bernoulli eqn between A, D

$$H_A = H_D + h_{L\,A\to D}$$

$$H_A = \frac{P_D}{\gamma} + \frac{V_D}{2g} + \cancel{Z_D}^{=0} + K_1 Q_1^2 + K_2 Q_2^2$$

$$54.74 = \frac{2.76 * 10^5}{9810} + \frac{Q_2^2}{2\,9.81\,\frac{\pi(0.304)^2}{4}} + 541.1\,(0.186) + 1455\,Q_2^2$$

$$\Rightarrow Q_2 = 0.073$$

$$\because Q_1 = Q_2 + Q_3$$

$$0.186 = 0.073 + Q_3 \quad \Rightarrow Q_3 = 0.113\ m^3/s$$

$\because$ Pipes ② and ③ are laid in Parallel

$$\because h_{L_2} = h_{L_3}$$

$$\therefore K_2 Q_2^2 = K_3 Q_3^2$$

$$1455\,(0.073)^2 = K_3\,(0.113)^2$$

$$\Rightarrow K_3 = 607.2$$

$$\because K = \frac{8 F L}{\pi^2 g d^5}$$

$$607.2 = \frac{8\,(0.04)\,(1524)}{\pi^2\,(9.81)\,d^5} \quad \Rightarrow d = 0.384\ m$$

$$= \underline{\underline{384\ mm}}$$

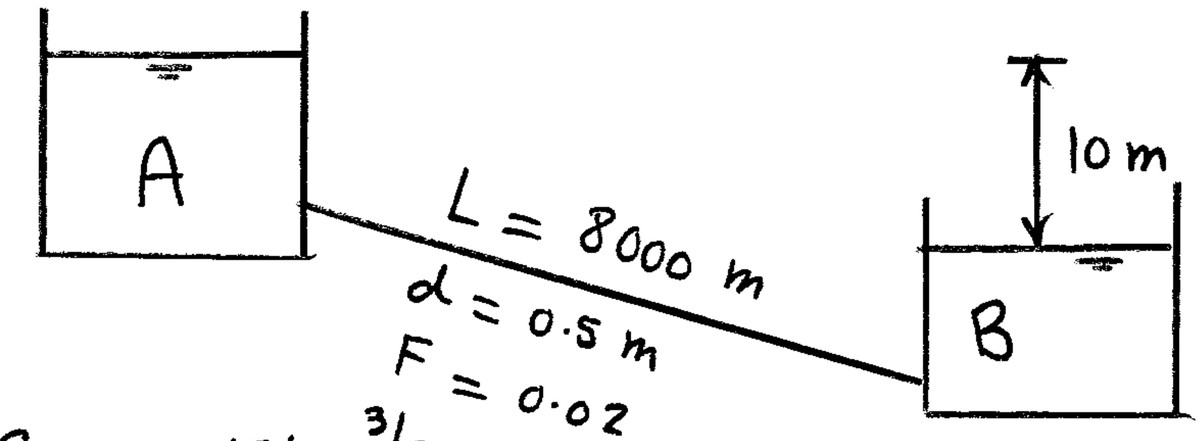
14 Two reservoirs whose surface level differ by 10 m, are connected by a pipe 0.5 m in diameter and 8000 m long. Calculate the discharge passing to the lower reservoir. It is required to increase the discharge by 50%, by the following two choices

a) Installing a pump on the line,

b) Adding another pipe, of the same diameter and of a certain length, parallel to the original pipe. at distance L. Which solution will be preferable if the meter length of the new pipe costs 20 L.E. and the pump costs 80 L.E per HP. The coefficient of friction for all pipes is 0.02 and the efficiency of the pump is 80%.

$Q_1 = ?$

$$h_f = \frac{8 F L Q_1^2}{\pi^2 g d^5}$$



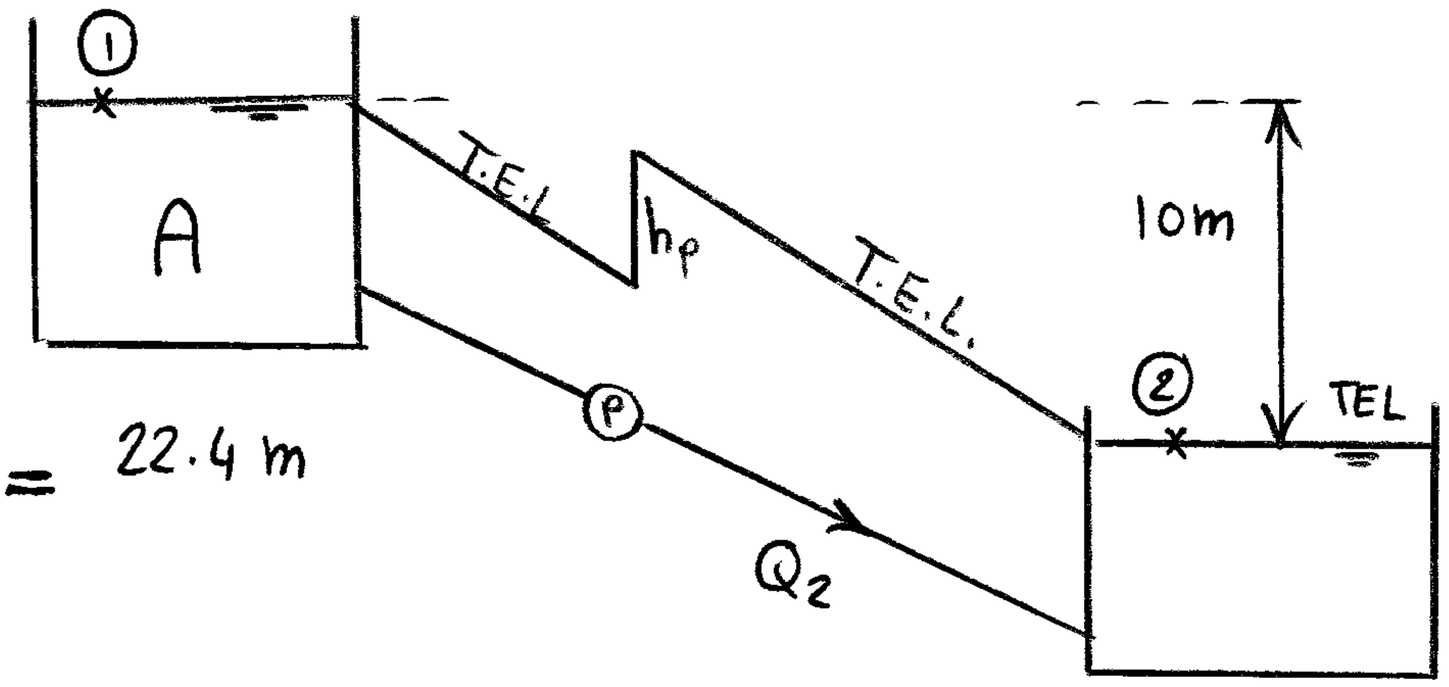
$$10 = \frac{8(0.02)\,8000\,Q_1^2}{\pi^2 (9.81)(0.5)^5} \Rightarrow Q_1 = \underline{\underline{0.154\ m^3/s}}$$

$$Q_2 = 1.5\,Q_1 = 0.23\ m^3/s$$

<u>1st using a pump</u>

$$h_f = \frac{8 F L Q_2^2}{\pi^2 g d^5}$$

$$= \frac{8(0.02)(8000)(0.23)^2}{\pi^2 (9.81)(0.5)^5} = 22.4\ m$$



<u>Applying Bernoulli eqn between ①, ②</u>

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L$$

$$0 + 0 + 10 + h_p = 0 + 0 + 0 + 22.4$$

$$\therefore h_p = 12.4\ m$$

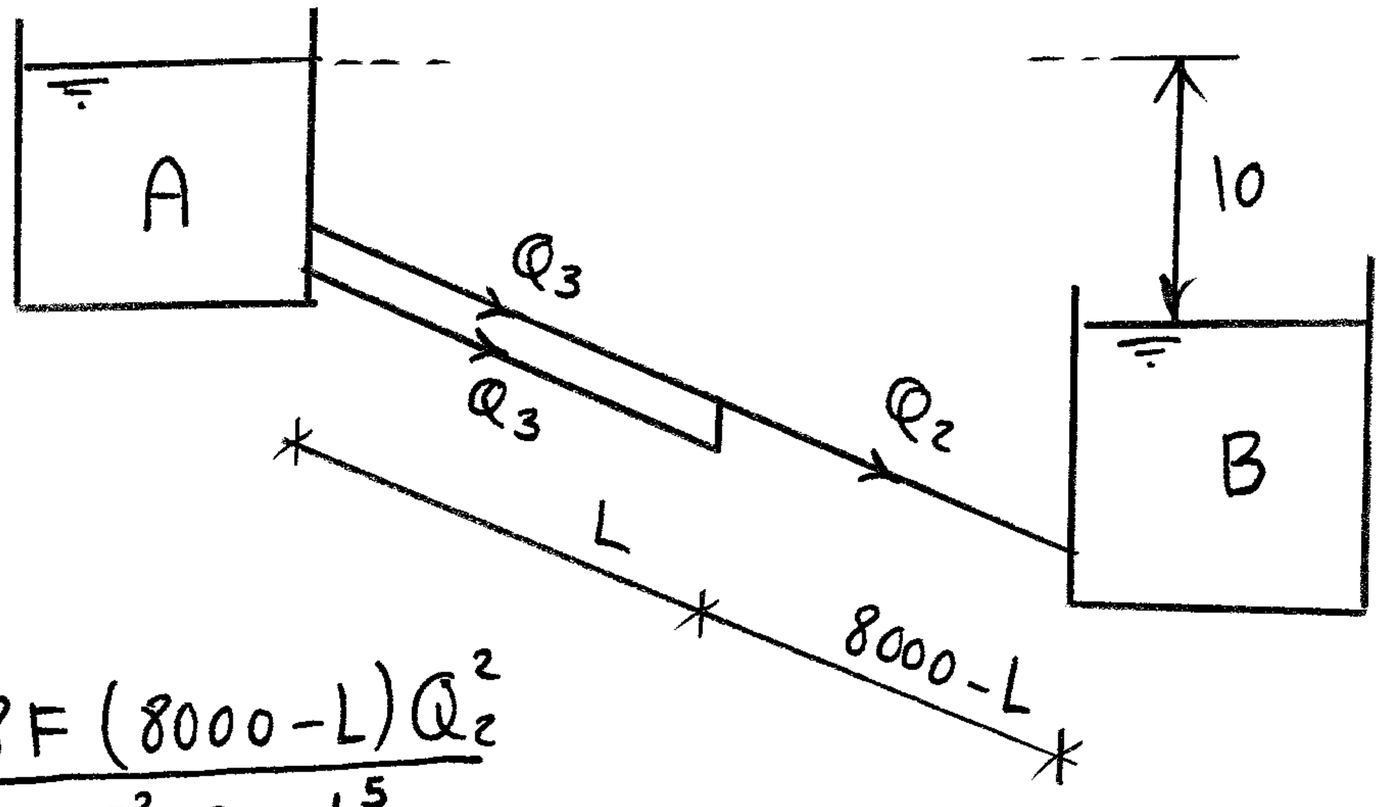
$$HP = \frac{\gamma Q h_p}{K \eta} = \frac{9810 * 0.23 * 12.4}{735 * 0.8} = 47.6 \text{ H.P}$$

$$\text{Cost} = 80 * 47.6 = 3808 \text{ L.E.}$$

<u>2nd using 2 pipes</u>

$$Q_2 = 2Q_3 = 0.23 \ m^3/s$$

$$Q_3 = \frac{0.23}{2} = 0.115 \ m^3/s$$



$$hf_T = hf_1 + hf_2$$

$$10 = \frac{8FLQ_3^2}{\pi^2 g d^5} + \frac{8F(8000-L)Q_2^2}{\pi^2 g d^5}$$

$$10 = \frac{8(0.02)L(0.115)^2}{\pi^2(9.81)(0.5)^5} + \frac{8(0.02)(8000-L)(0.23)^2}{\pi^2(9.81)(0.5)^5}$$

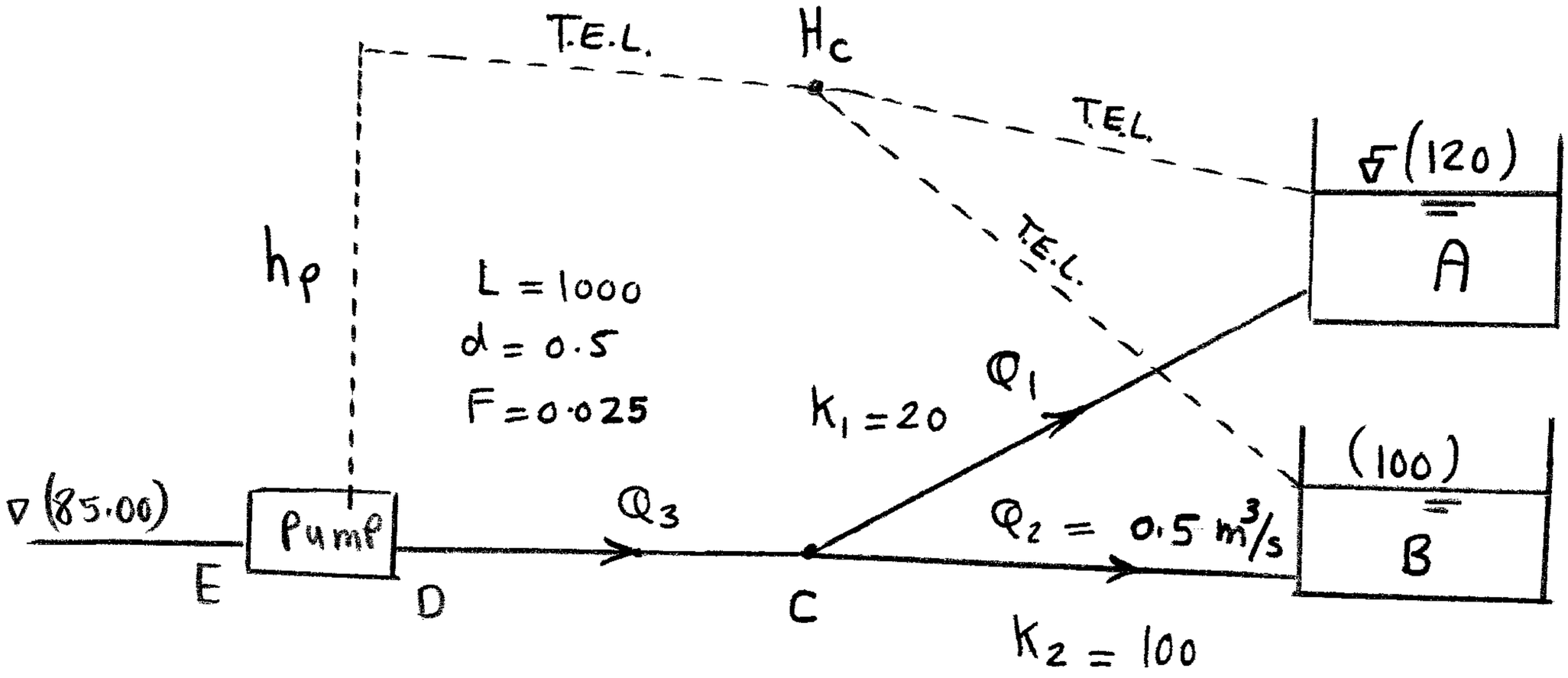
$$10 = 7*10^{-4} L + 2.8*10^{-3}(8000-L)$$

$$\therefore L = 5904$$

$$\text{Cost} = 20 * 5904 = 118095 \text{ L.E.}$$

The pump alternative is more preferable as its much more cheaper.

F In the Fig. Determine the head of the pump to feed the pipe line system if the pressure head at E is zero m. Take $K_1 = 20$, $K_2 = 100$, $Q_2 = 0.5\ m^3/s$, $L_3 = 1000\ m$, $d_3 = 0.5\ m$, $F = 0.025$ (26)



Bernoulli eqn between C, B

$$H_C = H_B + h_{L_{CB}}$$

$$H_C = 100 + K_2 Q_2^2$$

$$= 100 + 100(0.5)^2 = 125\ m$$

* long Pipes
L/d > 1000
neglect $V^2/2g$ | neglect sec losses | TEL = HGL

Bernoulli eqn between C, A

$$H_C = H_A + K_1 Q_1^2$$

$$125 = 120 + 20\ Q_1^2 \Rightarrow Q_1 = 0.5\ m^3/sec$$

$$\therefore Q_3 = Q_1 + Q_2 = 0.5 + 0.5 = 1\ m^3/s$$

Bernoulli eqn between D, C

$$H_D = H_C + \frac{8FLQ_3^2}{\pi^2 g d^5}$$

$$H_D = 125 + \frac{8(0.025)(1000)(1)^2}{\pi^2 (9.81)(0.5)^5}$$

$$= 191.1\ m$$

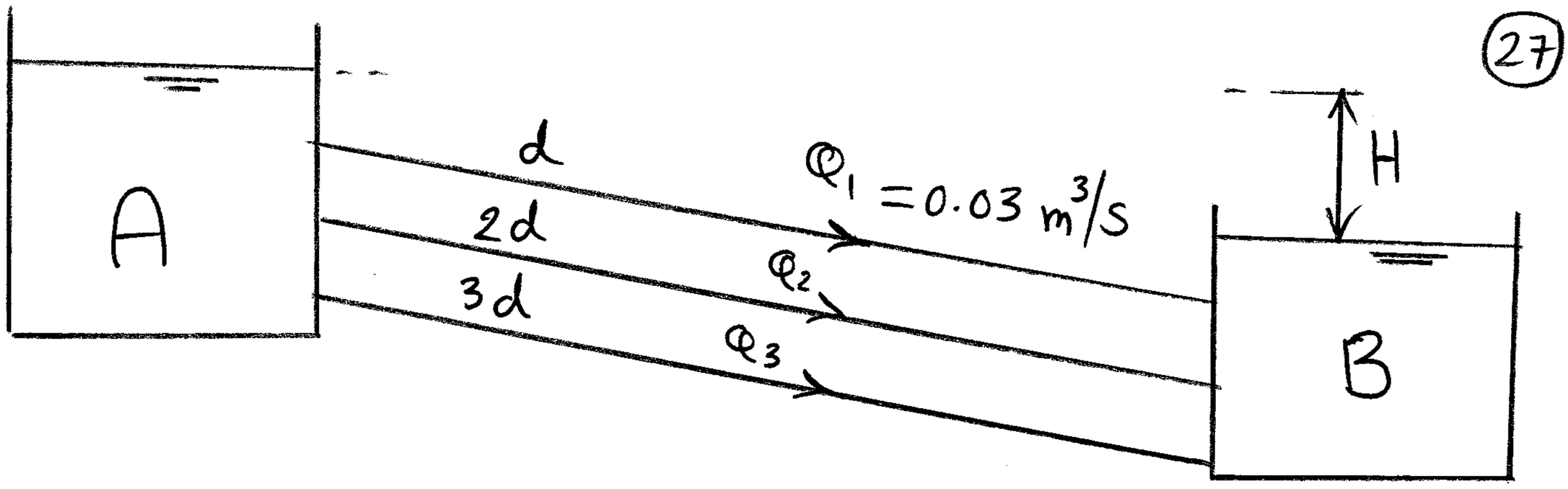
$\because H_E + h_p = H_D$

$$\frac{p}{\gamma} + \frac{V^2}{2g} + Z_E + h_p = H_D$$ ($\frac{V^2}{2g} \approx 0$)

$$0 + 0 + 85 + h_p = 191.1$$

$$h_p = \underline{106.1\ m}$$

8



$\therefore$ the 3 pipes are in parallel

$$\therefore \quad h_{L_1} = h_{L_2} = h_{L_3}$$

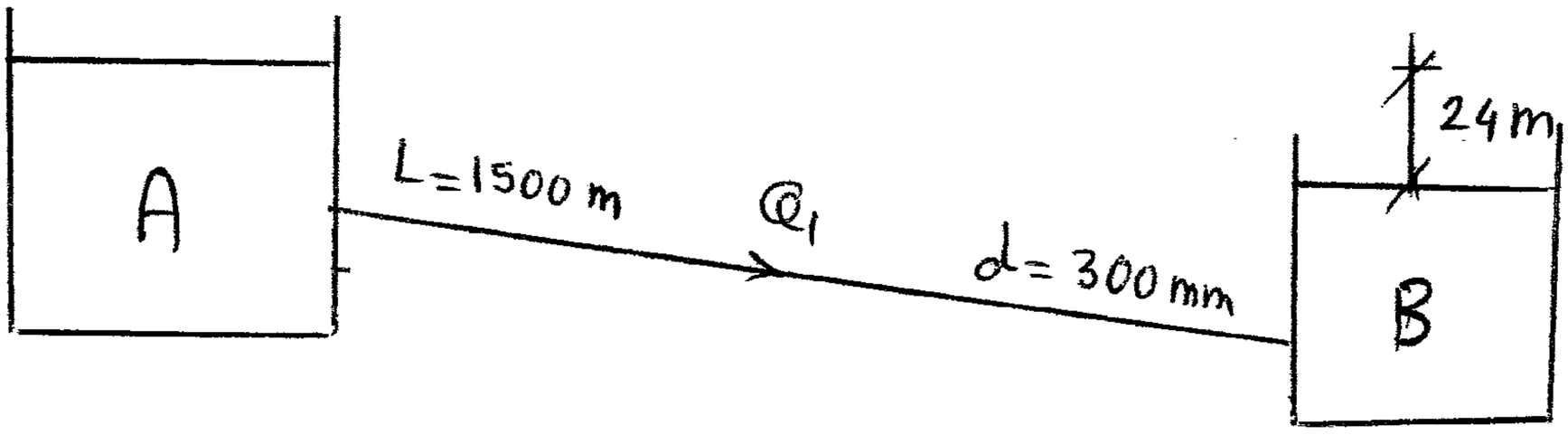
$$\frac{8 f L Q_1^2}{g d_1^5} = \frac{8 f L Q_2^2}{\pi^2 g d_2^5} = \frac{8 f L Q_3^2}{\pi^2 g d_3^5}$$

$$\frac{Q_1^2}{d_1^5} = \frac{Q_2^2}{d_2^5} = \frac{Q_3^2}{d_3^5}$$

$$\frac{(0.03)^2}{d^5} = \frac{Q_2^2}{(2d)^5} = \frac{Q_3}{(3d)^5}$$

$$\Rightarrow \quad Q_2 = 0.17 \quad m^3/s$$

$$\Rightarrow \quad Q_3 = 0.467 \quad m^3/s$$



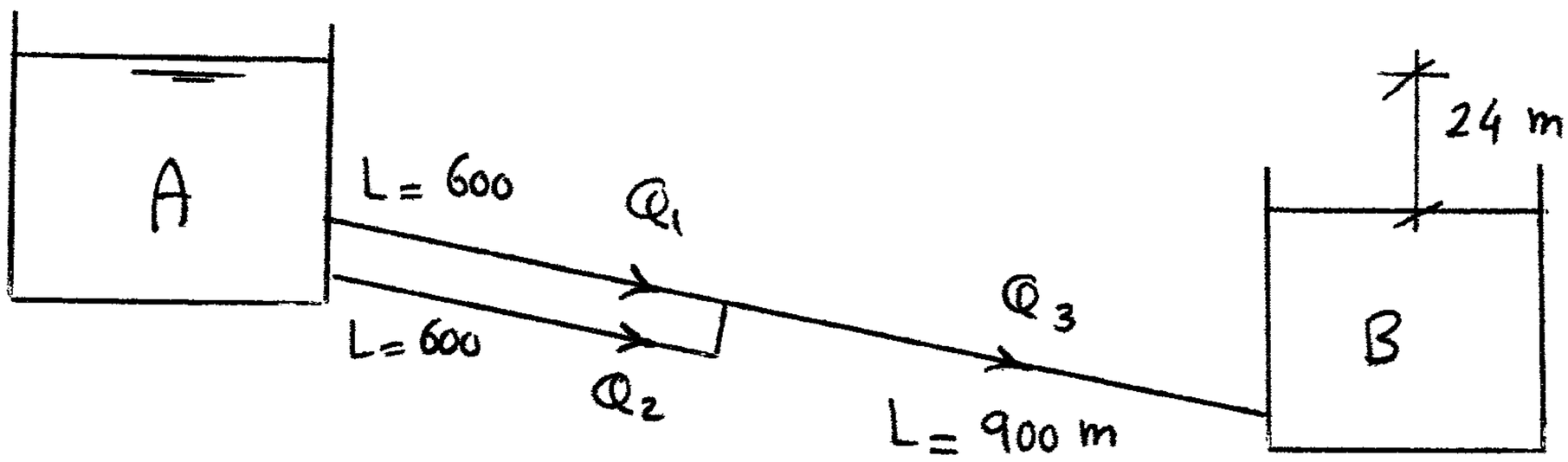
Case 1 one pipe used $L = 1500$ m

$Q_1 = 0.15 \ m^3/s$

$$h_f = \frac{8 F L Q_1^2}{\pi^2 g d^5}$$

$$24 = \frac{8 F (1500)(0.15)^2}{\pi^2 \ 9.81 \ (0.3)^5} \Rightarrow F = \underline{\underline{0.209}}$$

Case 2 After Connecting a 600 m pipe in parallel



∵ Pipes ①, ② Connected in parallel

$$\therefore h_{L_1} = h_{L_2}$$

$$\frac{8 F L Q_1^2}{\pi^2 g d^5} = \frac{8 F L Q_2^2}{\pi^2 g d^5}$$

$$Q_1 = Q_2 \quad \rightarrow ①$$

$$\therefore \quad Q_3 = Q_1 + Q_2 \quad \rightarrow (2)$$

$$\therefore \quad \boxed{Q_3 = 2Q_1}$$

$\because$ pipes (1), (3) Connected in series

$$\therefore \quad h_{L_1} + h_{L_3} = H$$

$$\frac{8FLQ_1^2}{\pi^2 g d^5} + \frac{8FLQ_3^2}{\pi^2 g d^5} = 24 \quad \rightarrow (3)$$

$$\frac{8(0.209)\,600\,Q_1^2}{\pi^2\,9.81\,(0.3)^5} + \frac{8(0.209)\,900\,(2Q_1)^2}{\pi^2\,9.81\,(0.3)^5} = 24$$

$$\therefore \quad Q_1 = 0.0896$$

$$\Rightarrow \quad Q_3 = 2Q_1 = 0.179$$

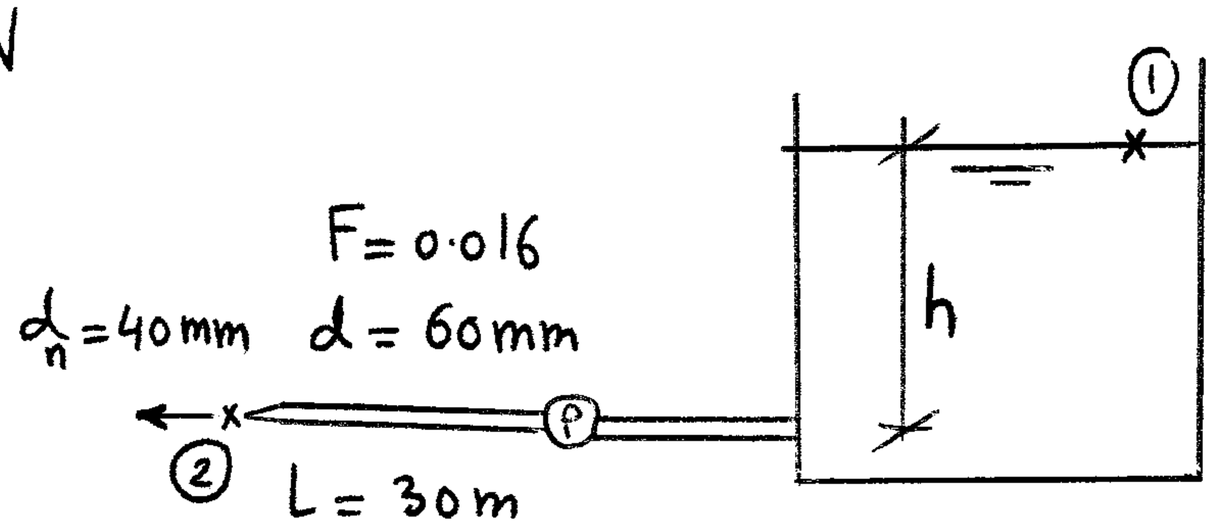
$$\% \text{ increase in } Q = \frac{Q_{new} - Q_{old}}{Q_{old}} * 100$$

$$= \frac{0.179 - 0.15}{0.15} * 100 = 19.4\ \%$$

12

Power = 45 KW

$Q = 0.04\ m^3/s$



Power $= \gamma Q h_p$

$45 * 10^3 = 9810\,(0.04)\,h_p$

$\Rightarrow h_p = 114.68$

$V_{pipe} = \dfrac{Q}{A_{pipe}} = \dfrac{0.04}{\frac{\pi (0.06)^2}{4}} = 14.15$

$V_{nozzle} = \dfrac{Q}{A_{nozzle}} = \dfrac{0.04}{\frac{\pi (0.04)^2}{4}} = 31.83$

Case 1 $h = ?$

apply Bernoulli eqn between ①, ②

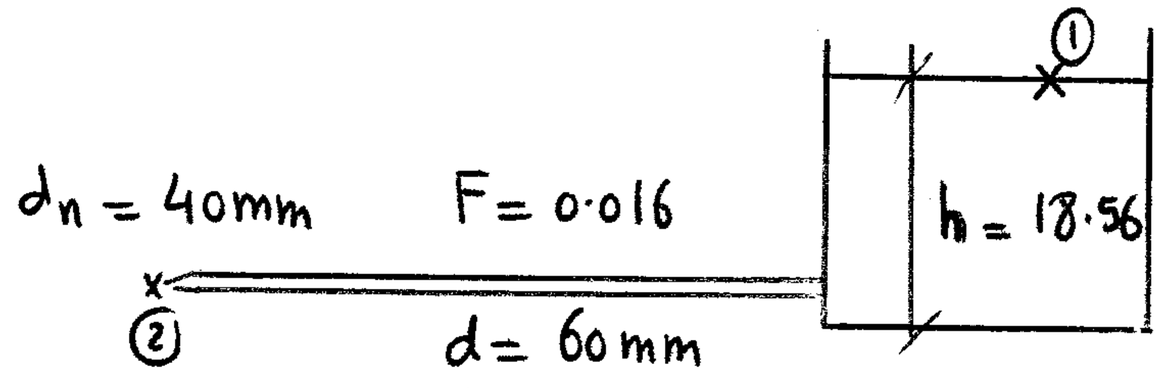
$$0 + 0 + h + h_p = 0 + \frac{V_n^2}{2g} + 0 + h_L \qquad h_L = \frac{8FLQ^2}{\pi^2 g d^5}$$

$$h + 114.68 = \frac{(31.83)^2}{2(9.81)} + \frac{8(0.016)\,30\,(0.04)^2}{\pi^2 (9.81)(0.06)^5}$$

$\Rightarrow h = \underline{\underline{18.56}}$

Case 2 Pump removed $Q = ?$

Bernoulli eqn bet ①, ②



$$0 + 0 + h = 0 + \frac{V_n^2}{2g} + 0 + \frac{8FLQ^2}{\pi^2 g d^5}$$

$$18.56 = \frac{Q^2}{2(9.81)\left[\frac{\pi (0.04)^2}{4}\right]^2} + \frac{8(0.016)\,30\,Q^2}{\pi^2 (9.81)(0.06)^5}$$

$\Rightarrow Q = \underline{\underline{0.019\ m^3/sec}}$

15 Dimension Analysis

Definition

It is a method to get the relation of some variables in the form of an exponential equation which must be dimensionally homogenous.

Dimensional Homogeneity

An equation is called dimensionally homogenous (متجانس) if the fundamental dimensions (M, L, T) have the same power (أس) on both sides. (units left = units right)

Ex

$$\text{Discharge} = \text{Area} * \text{Velocity}$$

$$L^3 T^{-1} = L^2 * L T^{-1}$$

$$\therefore L^3 T^{-1} = L^3 T^{-1}$$

∴ The equation is dimensionally homogenous

Ex

$$\text{Power} = \gamma Q h$$

$$F \frac{L}{T} = \frac{F}{L^3} \cdot \frac{L^3}{T} \cdot L$$

$$F L T^{-1} = F L T^{-1} \quad \text{(Homogenous)}$$

Uses of dimensional analysis

1-Converting units of a system to another
2-Checks the dimensional homogeneity of an equation
3-Simplifies a theoretical equation
4-Reduces no of variables to 3 (M, L, T)
5-Derivation of equations and design of curves and models

Example

The Reynolds number R_n is a function of density ρ, dynamic viscosity μ, velocity of a fluid v, and the diameter d, of a pipe. Use the dimensional analysis to establish Reynolds number.

$$R_n = f(\rho, \mu, V, d)$$

Dimensionally

$$M^\circ L^\circ T^\circ = (ML^{-3})^a (ML^{-1}T^{-1})^b (LT^{-1})^c L^d$$

Equating exponents of M, L, T we obtain

$$M^\circ \quad \therefore \quad 0 = a + b \quad \Rightarrow \quad a = -b$$

$$T^\circ \quad \therefore \quad 0 = -b - c \quad \Rightarrow \quad c = -b$$

$$L^\circ \quad \therefore \quad 0 = -3a - b + c + d$$

$$0 = -3(-b) - b - b + d \quad \Rightarrow \quad d = -b$$

$$\therefore R_n = \rho^{-b} \mu^b V^{-b} d^{-b} = \left(\frac{\mu}{\rho V d}\right)^b$$

$$\therefore R_n = \frac{\rho V d}{\mu}$$

بعمل تجارب وُجد أن **b = -1**

Types of forces Concerning fluid flow

a- Internal forces

1- Inertia forces Newton's eqn $\sum Forces = m.a$

$F = mass * acceleration$

$$F_I = \rho \forall * a = \rho L^3 \frac{V^2}{L}$$

$$\boxed{F_I = \rho L^2 V^2}$$

2- Surface tension

$$\because \sigma = \frac{F}{L} \Rightarrow \boxed{F_T = \sigma L}$$

3- Viscosity

$$\because \tau = \mu \frac{V}{y}$$

$$F_V = \mu \frac{V}{y} A = \mu \frac{V}{L} \cdot L^2 = \mu V L$$

$$\boxed{F_V = \mu V L}$$

4- Elasticity

$$K = \frac{Stress}{Strain} = \frac{\frac{F}{A}}{\frac{\Delta \forall}{\forall}} = \frac{\frac{F}{L^2}}{\frac{L^3}{L^3}}$$

$$\boxed{F_e = K L^2}$$

b- external forces

5- Pressure

$$P = \frac{\text{Force}}{\text{Area}} = \frac{F}{L^2} \Rightarrow \boxed{F_\rho = P.L^2}$$

6- Gravity

$$F = mg = \rho L^3 g \Rightarrow \boxed{F_g = \rho g L^3}$$

Dimensionless numbers

∵ Inertia forces always exist in any problem of fluids

∴ Force ratio are found out with respect to inertia forces

1- Reynolds no.

It is the ratio of the inertia force to Viscous force

$$\frac{F_I}{F_V} = \frac{\rho L^2 V^2}{\mu V L} = \frac{\rho V L}{\mu} = R_n$$

$$\boxed{R_n = \frac{Vd}{\nu}}$$

$R_n < 2000$ laminar flow

$R_n > 4000$ turblent flow

2- Froude no.

It is the ratio of the inertia forces to gravity forces

$$\frac{F_I}{F_g} = \frac{\rho L^2 V^2}{\rho g L^3} = \frac{V^2}{gL}$$

$$\boxed{F_n = \frac{V}{\sqrt{gL}}}$$

$F_n < 1$ Subcritical flow

$F_n > 1$ Supercritical flow

3- Weber no.

It is the ratio of inertia forces to Surface tension forces

$$\frac{F_I}{F_T} = \frac{\rho L^2 V^2}{\sigma L} = \frac{\rho L V^2}{\sigma} = \frac{V^2}{\sigma/\rho L}$$

$$\boxed{W_n = \frac{V}{\sqrt{\sigma/\rho L}}}$$

4- Mach no.

It is the ratio of inertia forces to elastic forces

$$\frac{F_I}{F_e} = \frac{\rho L^2 V^2}{K L^2} = \frac{\rho V^2}{K} = \frac{V^2}{K/\rho}$$

$$\boxed{M_n = \frac{V}{\sqrt{K/\rho}}}$$

$M_n < 1$ Subsonic

$M_n > 1$ Supersonic

سرعة الصوت : $\sqrt{K/\rho}$

5- Euler's no.

It is the ratio of inertia forces to pressure forces

$$\frac{F_I}{F_P} = \frac{\rho L^2 V^2}{P L^2} = \frac{\rho V^2}{P} = \frac{V^2}{P/\rho}$$

$$\boxed{E_n = \frac{V}{\sqrt{\frac{2\Delta P}{\rho}}}}$$

Quantities used in fluid mechanics and their dimensions

	Quantity	F L T	M L T
	a- Fundamental quantities		
1	Mass M	$F L^{-1} T^{2}$	M
2	Length L	L	L
3	Time T	T	T
	b- Geometric quantities (L)		
4	Area A	L^{2}	L^{2}
5	Volume V	L^{3}	L^{3}
6	Moment of Inertia I	L^{4}	L^{4}
	c- Kinematic quantities (L , T)		
7	Linear velocity v	$L T^{-1}$	$L T^{-1}$
8	Angular velocity ω	T^{-1}	T^{-1}
9	Acceleration a	$L T^{-2}$	$L T^{-2}$
10	Gravity g	$L T^{-2}$	$L T^{-2}$
11	Discharge Q	$L^{3} T^{-1}$	$L^{3} T^{-1}$
	d- Dynamic quantities (M , L , T)		
12	Force F	F	$M L T^{-2}$
13	Pressure p, Shear stress τ	$F L^{-2}$	$M L^{-1} T^{-2}$
14	Momentum	F T	$M L T^{-1}$
15	Work, Energy, Torque	F L	$M L^{2} T^{-2}$
16	Power	$F L T^{-1}$	$M L^{2} T^{-3}$
	e- Fluid properties		
17	Density ρ	$F L^{-4} T^{2}$	$M L^{-3}$
18	Specific weight γ	$F L^{-3}$	$M L^{-2} T^{-2}$
19	Dynamic viscosity μ	$F L^{-2} T$	$M L^{-1} T^{-1}$
20	Kinematic viscosity ν	$L^{2} T^{-1}$	$L^{2} T^{-1}$
21	Surface tension σ	$F L^{-1}$	$M T^{-2}$
22	Bulks modulus k	$F L^{-2}$	$M L^{-1} T^{2}$

Buckingham π-theorem

تعريف

أى ظاهرة هيدروليكية يؤثر عليها عدة متغيرات (n variables)

وأى متغير لابد وأن يكون له dimensions

R/n , F/n , C/d

فإذا وُجدت علاقة تربط هذه المتغيرات مع بعضها $f(A_1, A_2, A_3, \ldots\ldots) = 0$

and the number of fundamental dimensions is m

Fundamental dimensions in fluid (M, L, T) $\Rightarrow m = 3$

فإنه يمكن التعبير عن هذه المتغيرات بدلالة أخرى function

The relation is arranged in (m-n) dimensionless (π-terms)

$$\Phi (\pi_1, \pi_2, \pi_3, \ldots\ldots, \pi_{n-m}) = 0$$

Procedure خطوات الحل

1-Divide the variables to the following groups

a- Fluid properties (ρ, γ, μ, ν, σ, k)

b- Geometric variables (L, d, h, b, ε, A, ∀, I)

c- Kinematic and (v, a, g, ω, Q)

dynamic variables (F, P, τ, Momentum, Work, Power)

↖ Pressure

2-Select one variable from each group to be used as a repeated variable (A_1, A_2, A_3)

يمكن تحديدهم من البداية بمعرفة صورة المطلوب اثباته

N.B. no of repeated variables = m m = 3 غالباً

3- Write a group of π-terms such that each π-term must contain the repeated variables in an exponential form (a, b, c مرفوعة لأس) and one of the remaining variables.

ثم نحل لنوجد a, b, c لكل π-term

4- Substitute the values of a, b, c to get each π-term.

5- Write the relation as a function of π-terms

$$f(\pi_1, \pi_2, \pi_3, \ldots\ldots \pi_{n-m}) = \text{constant}$$

Properties of π-terms

Any π-term can be replaced by any power because it is dimensionless

1- π can be replaced by π^2 or π^3 or π^{-1} or $\sqrt{\pi}$

2- $\pi_6 = \pi_1 * \pi_2$ $\pi_7 = \pi_1 / \pi_2$ $\pi_8 = \frac{\pi_6}{\pi_5} * \pi_4$

3- $\pi_1 = \pi_1 / 2 = -5\,\pi_1 = 20\,\pi_1$ وهكذا

4- $\pi_1 = f(\pi_2, \pi_3, \ldots\ldots \pi_{n-m})$

Example 1

Obtain an expression for the pressure gradient in a circular pipeline of effective roughness ε, conveying in a an incompressible fluid of density ρ, dynamic viscosity μ at mean velocity v, as a function of non-dimensionless groups.

By comparison with the Darcy-Weisbach equations, show that the friction factor is a function of relative roughness and Reynolds no.

$$f\left(\frac{\Delta P}{L}, d, v, \rho, \mu, \varepsilon\right) \qquad \Rightarrow \quad n = 6$$

1- Fluid properties : ρ, μ (ρ)

Geometric Variables : d, ϵ (d)

Dynamic Variables : $\frac{\Delta p}{L}$, V (V)

2- Repeated Variables ρ, V, d $\quad m = 3$

No of Π terms $= n - m = 6 - 3 = 3$

3- $\Pi_1 = \rho^{a_1} V_1^{b_1} d^{c_1} \frac{\Delta P}{L}$

$$(ML^{-3})^{a_1} (LT^{-1})^{b_1} L^{c_1} \quad ML^{-2}T^{-2} = M^0 L^0 T^0$$

for [M] $\quad a_1 + 1 = 0 \quad \Rightarrow a_1 = -1$

[T] $\quad -b_1 - 2 = 0 \quad \Rightarrow b_1 = -2$

[L] $\quad -3a_1 + b_1 + c_1 - 2 = 0 \quad \Rightarrow c_1 = 1$

$$\Pi_1 = \rho^{-1} V^{-2} d^1 \frac{\Delta P}{L}$$

$$\Rightarrow \boxed{\Pi_1 = \left(\frac{d}{\rho V^2}\right) \cdot \frac{\Delta P}{L}}$$

$$\Pi_2 = \rho^{a_2} V^{b_2} d^{c_2} \mu$$

$$(ML^{-3})^{a_2} (LT^{-1})^{b_2} L^{c_2} \; ML^{-1}T^{-1} = M^0 L^0 T^0$$

for [M] $a_2 + 1 = 0 \Rightarrow a_2 = -1$

[T] $-b_2 - 1 = 0 \Rightarrow b_2 = -1$

[L] $-3a_2 + b_2 + c_2 - 1 = 0 \Rightarrow c_2 = -1$

$$\Pi_2 = \rho^{-1} V^{-1} d^{-1} \mu$$

$$\boxed{\Pi_2 = \frac{\mu}{\rho V d}}$$

$$\Pi_3 = \rho^{a_3} V^{b_3} \underline{\underline{d}}^{c_3} \underline{\underline{\epsilon}}$$

$\because$ d and ϵ have the same dimensions

$\therefore$ $\boxed{\Pi_3 = \frac{\epsilon}{d}}$

<u>Proof</u> $(ML^{-3})^{a_3} (LT^{-1})^{b_3} L^{c_3} \; L = M^0 L^0 T^0$

for [M] $a_3 = 0$

[T] $-b_3 = 0$

[L] $-3a_3 + b_3 + c_3 + 1 = 0 \Rightarrow c = -1$

$\therefore \Pi_3 = \frac{\epsilon}{d}$

$$\therefore \quad \phi\left(\Pi_1, \Pi_2, \Pi_3\right) = 0$$

$$\therefore \quad \phi\left(\frac{d}{\rho V^2} \cdot \frac{\Delta P}{L}, \frac{\mu}{\rho V d}, \frac{\epsilon}{d}\right) = 0$$

$$\frac{d}{\rho V^2} \cdot \frac{\Delta P}{L} = \phi\left(\frac{\rho V d}{\mu}, \frac{\epsilon}{d}\right)$$

$$\boxed{\frac{\Delta P}{L} = \frac{\rho V^2}{d}\, \phi\left(R_n, \text{relative roughness}\right)}$$

$\because$ hydraulic gradient $\frac{\Delta P}{L} = \frac{\gamma \Delta H}{L} = \frac{\gamma h_L}{L} = \rho g \frac{h_L}{L}$

$$\rho g \frac{h_L}{L} = \frac{\rho V^2}{d}\, \phi\left(R_n, \epsilon/d\right)$$

$$h_L = \frac{L V^2}{g d}\, \phi\left(R_n, \epsilon/d\right)$$

Comparing with darcy $h_L = \frac{F L V^2}{2 g d}$

$$\therefore \quad \boxed{F = \phi\left(R_n, \epsilon/d\right)}$$

Example 2

Show that the discharge, Q, of liquid of density, ρ, dynamic viscosity, μ, and surface tension, σ, over a V notch under a head, H, may be expressed in the form:

$$Q = g^{0.5} H^{2.5} \Phi\left(\frac{\rho g^{0.5} H^{1.5}}{\mu}, \frac{\rho g H^{2}}{\sigma}, \theta \right)$$

Where θ is the notch angle, and hence define the parameters upon which the discharge coefficient is dependent.

$f(\rho, g, H, Q, \mu, \sigma, \theta) \quad \Rightarrow \quad n = 7$

1- Fluid properties : ρ, μ, σ (ρ)

Geometric Variables : H, θ (H)

Kinematic Variables : g, Q (g)

في حالة إعطاء الصورة النهائية فإن أى متغير متكرر (**repeated**) يؤخذ متغير متكرر

2- Repeated Variables $\rho, H, g \quad \Rightarrow \quad m = 3$

no of Π-terms $= n - m = 7 - 3 = 4$

3- $\Pi_1 = \rho_1^{a_1} g^{b_1} H^{c_1} Q$

$(ML^{-3})^{a_1} (LT^{-2})^{b_1} L^{c_1} L^3 T^{-1} = M^0 L^0 T^0$

for [M] $a_1 = 0 \quad \Rightarrow a_1 = 0$

[T] $-2b_1 - 1 = 0 \quad \Rightarrow b_1 = -\frac{1}{2}$

[L] $-3a_1 + b_1 + c_1 + 3 = 0 \quad \Rightarrow c_1 = -\frac{5}{2}$

$\Pi_1 = g^{-\frac{1}{2}} H^{-\frac{5}{2}} Q \quad \Rightarrow \quad \boxed{\Pi_1 = \frac{Q}{g^{0.5} H^{2.5}}}$

$$\Pi_2 = \rho^{a_2}\, g^{b_2}\, H^{c_2}\, \mu$$

$$(ML^{-3})^{a_2} (LT^{-2})^{b_2} L^{c_2}\, ML^{-1}T^{-1} = M^0 L^0 T^0$$

for [M] $a_2 + 1 = 0 \Rightarrow a_2 = -1$

[T] $-2b_2 - 1 = 0 \Rightarrow b_2 = -\frac{1}{2}$

[L] $-3a_2 + b_2 + c_2 - 1 = 0 \Rightarrow c_2 = -\frac{3}{2}$

$$\Pi_2 = \rho^{-1} g^{-\frac{1}{2}} H^{-\frac{3}{2}} \mu$$

$$\boxed{\Pi_2 = \frac{\mu}{\rho g^{0.5} H^{1.5}}}$$

$$\Pi_3 = \rho^{a_3}\, g^{b_3}\, H^{c_3}\, \sigma$$

$$(ML^{-3})^{a_3} (LT^{-2})^{b_3} L^{c_3}\, MT^{-2} = M^0 L^0 T^0$$

for [M] $a_3 + 1 = 0 \Rightarrow a_3 = -1$

[T] $-2b_3 - 2 = 0 \Rightarrow b_3 = -1$

[L] $-3a_3 + b_3 + c_3 = 0 \Rightarrow c_3 = -2$

$$\Pi_3 = \rho^{-1} g^{-1} H^{-2} \sigma$$

$$\boxed{\Pi_3 = \frac{\sigma}{\rho g H^2}}$$

$$\Pi_4 = \rho^{a_4}\ g^{b_4}\ H^{c_4}\ \theta \qquad (14)$$

Hint : If a quantity is dimensionless $\Rightarrow$ it is a Π-term

$\because$ θ is dimensionless $\Rightarrow$ $\Pi_4 = \theta$

Proof

$$(ML^{-3})^{a_4}(LT^{-2})^{b_4}\ L^{c_4} = M^{\circ}L^{\circ}T^{\circ}$$

for [M] $a_4 = 0$ $\Rightarrow a_4 = 0$

[T] $-2b_4 = 0$ $\Rightarrow b_4 = 0$

[L] $-3a_4 + b_4 + c_4 = 0$ $\Rightarrow c_4 = 0$

$\therefore$ $\boxed{\Pi_4 = \theta}$

5- $\phi(\Pi_1, \Pi_2, \Pi_3, \Pi_4) = 0$

$$\phi\left(\frac{Q}{g^{0.5}H^{2.5}}, \frac{\mu}{\rho g^{0.5}H^{1.5}}, \frac{\sigma}{\rho g H^2}, \theta\right)$$

$$\Pi_2 = \frac{1}{\Pi_2}, \quad \Pi_3 = \frac{1}{\Pi_3}$$

$$\frac{Q}{g^{0.5}H^{2.5}} = \phi\left(\frac{\rho g^{1/2} H^{3/2}}{\mu}, \frac{\rho g H^2}{\sigma}, \theta\right)$$

$$\boxed{Q = g^{0.5}H^{2.5}\,\phi\left(\frac{\rho g^{0.5}H^{1.5}}{\mu}, \frac{\rho g H^2}{\sigma}, \theta\right)}$$

$$Q = g^{0.5} H^{2.5} \phi \left(\frac{\rho g^{0.5} H^{1.5}}{\mu}, \frac{\rho g H^2}{\sigma}, \theta \right) \quad \rightarrow (1)$$

From V notch

$$Q = \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2} H^{5/2} \quad \rightarrow (2)$$

$$C_d = \phi \left(\frac{\rho g^{0.5} H^{1.5}}{\mu}, \frac{\rho g H^2}{\sigma} \right)$$

$$\frac{\rho g^{0.5} H^{1.5}}{\mu} = \frac{\rho}{\mu} (L T^{-2})^{0.5} L^{1.5} = \frac{\rho}{\mu} L^2 T^{-1}$$

$$= \frac{\rho}{\mu} L (L T^{-1}) = \frac{\rho L V}{\mu} = R_n$$

$$\frac{\rho g H^2}{\sigma} = \frac{\rho}{\sigma} (L T^{-2}) L^2 = \frac{\rho (L^2 T^{-2}) L}{\sigma}$$

$$= \frac{\rho}{\sigma} (L T^{-1})^2 L = \frac{V^2}{\sigma/\rho L} = W_n^2$$

weber no.

$$\therefore \boxed{C_d = \phi (R_n, W_n)}$$

Surface tension effects represented by Weber number may become important at low discharges.