Northern Technical University

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Power Mechanics Technologies

Refrigeration and Air Conditioning Branch

HEAT TRANSFER

Method of Heat Transfer

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Subject Lecture

Second Level

Chapter (1–2)

Introduction

Heat Transfer:- is that science which seeks in the energy transfer between material bodies as a result of a temperature difference and to predict the rate at which the energy transfer will take place . heat is transferred from high temperature body to low temperature body .

Heat:- is the form of energy that can be transferred from one system to another as a result of temperature difference.

Temperature:- is the measure of the amount of molecular energy contained in a substance .

Heat transfer: The science that deals with the determination of the rates of energy transfers as a heat form.

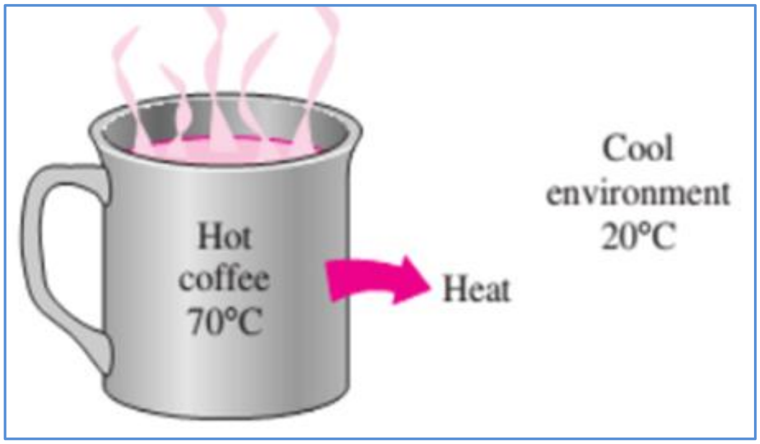


Fig (1-1) Heat flows in the direction of decreasing temperature

Methods of Heat Transfer: - Heat transfer takes place in three ways:

1- Conduction.
2- Convection.
3- Radiation.

The conduction and radiation methods are a basic ways i.e. Each of them can be take place alone but convection is mixture between conduction and radiation.

Thermal Conductivity (K): -

is the rate of heat transfer through a unit thickness of the material per unit area per unit temperature difference. It can be classified into three categories.

a- If the value of (K) is high , this material used for places which needs to transfer high amount of heat (Condenser, evaporator, Radiator) .

b- If the value of (K) about (1w/m.k) like the materials which used in building brick (0.75), juss (0.6 - 0.7), concrete (0.8 - 1).

c- If the value of (K) is low these materials called insulation+ material ex glass wool (0.01 w/m.k) .

Table (1–1) The thermal conductivities of some materials at room temperature

Material	**Thermal conductivity (w/m.℃)**
Diamond	2300
Silver	429
Copper	401
Gold	317
Aluminum	237
Iron	80.2
Mercury (l)	8.54
Glass	0.78
Brick	0.72
Water(l)	0.613
Wood	0.17
Helium(g)	0.152
Glass fiber	0.043
Air (g)	0.026

Heat and other forms of energy : -

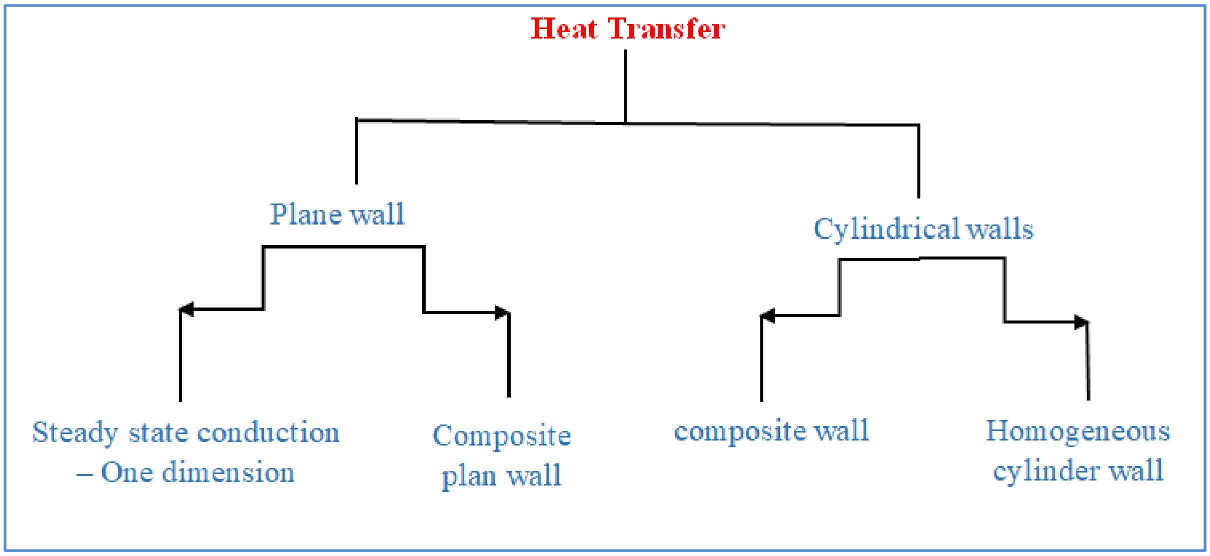
Energy can exist in numerous forms such as thermal, mechanical, kinetic, potential, electrical, magnetic, chemical, the unit of energy is joule (J) or kilojoule (1 kJ =1000 J). In the English system, the unit of energy is the British thermal unit (Btu). Another unit of energy is the calorie (1 cal =4.1868 J).

Calorie:- the energy needed to raise the temperature of (1 gram)of water at (14.5°C) by (1°C).

Thermal Diffusivity: -

The product ρ.Cp , which is means how much energy stored in material per unit volume.

$$\alpha = \frac{k}{\rho . cp} \quad (m^2/s) .$$



1- Conduction Heat Transfer: -

Conduction is an exchange in energy from the high temperature particles of the body to low temperature particles in the same body.The basic law in conduction heat transfer is called **(Fourier's law)** which is states that:

$$q = -KA\frac{\Delta T}{\Delta X}$$

Where:-

(q) = Heat transfer rate (watt) w.

(K) = Thermal conductivity (W/m.℃) .

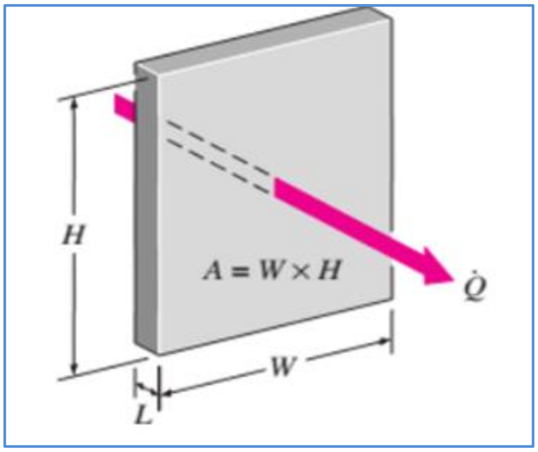
(ΔT) = Temperature difference (K).

(ΔX) = Thickness (m).

The rate of heat conduction through a medium depends on the thickness (Δx), and the material(k), as well as the temperature difference(ΔT) across the medium.

Consider steady heat conduction through a large plane wall of thickness

Δx (or L) and area A, as shown in Figure below



Fig(1-3) In heat conduction analysis ,A, represent the area normal to the direction of heat transfer

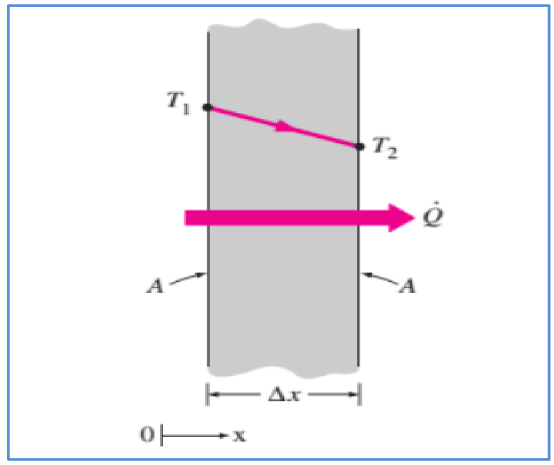
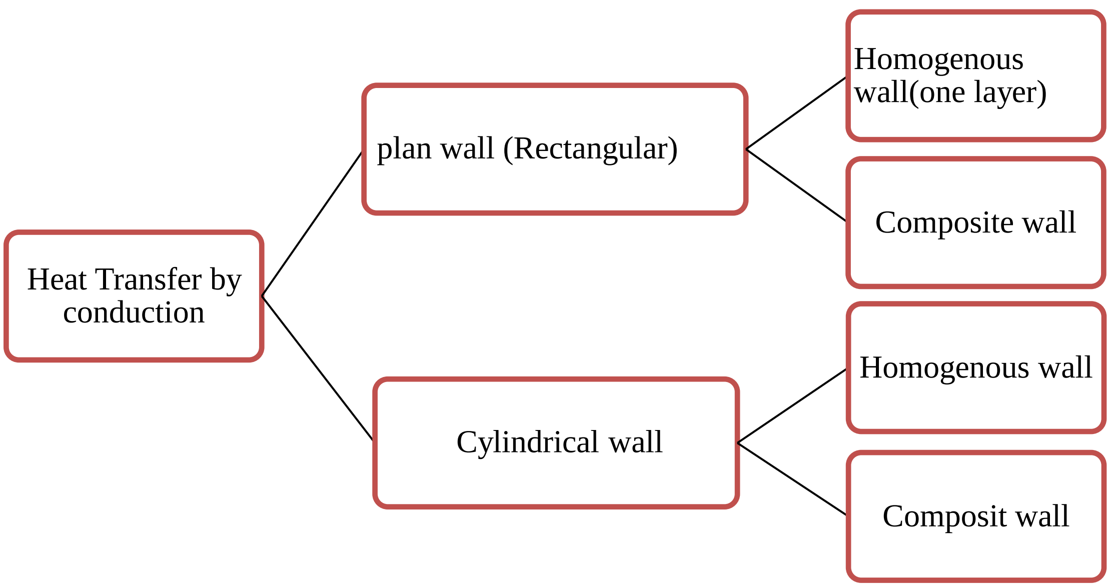


Fig (1-2) Heat Conduction through a large plane wall of thickness ΔX and area A.



1-The Plane Wall: -

First consider the plane wall where a direct application of Fourier's law may be made Integration yields.

$$q = -KA\frac{\Delta T}{\Delta X}$$

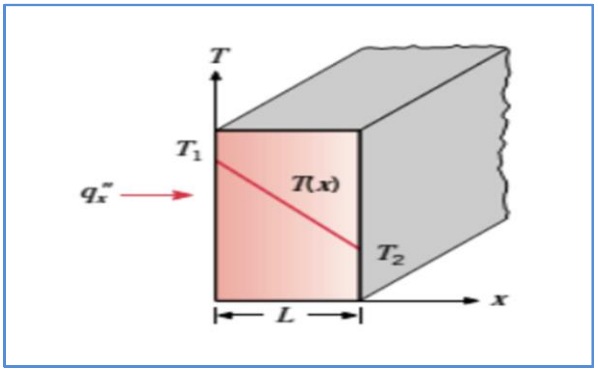


Fig (1- 4) Plane Wall

Example: - The roof of an electrically heated home is 6 m long, 8 m wide, and 0.25 m thick, and is made of a flat layer of concrete whose thermal conductivity is k = 0.8 W/m ·°C (Fig. 1– 4). The temperatures of the inner and the outer surfaces of the roof one night are measured to be 15°C and 4°C, respectively, for a period of 10 hours. Determine the rate of heat loss through the roof that night ?

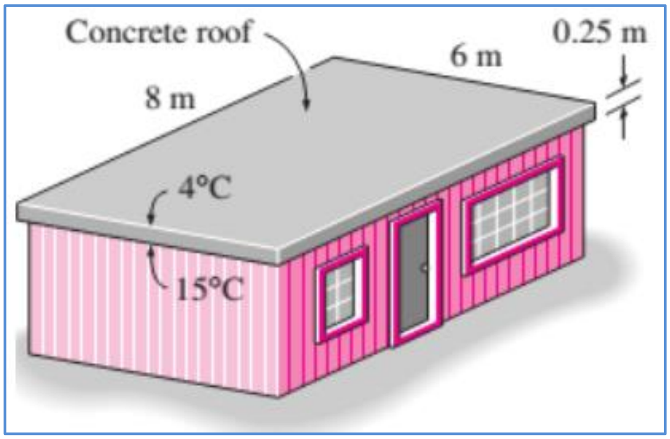


Fig (1-4)

Solution: -

The area of the roof is A = 6 m ×8 m = 48 m^2, the rate of heat transfer (Q) through the roof is determined by:

$q = -KA\frac{\Delta T}{\Delta X}$ A= 6m×8m =48m^2 ΔX= 0.25m   K= 0.8 W/m ·°C

$\therefore q = -(0.8) \times 48 \frac{4-15}{0.25}$ =1690w , 1.69 kw

Example:- if the heat loss per unit area of a furnace wall (36 cm) is (0.5 kw/m^2) .calculate the outside surface temp of the wall if the inside temp is (230 °C) and the thermal conductivity is (0.9 w/m. °C).

Solution: -

0.5 kw/m^2 = $0.5 \times 1000 = 500$ w/m^2 $\Delta x = 36\ cm \rightarrow\rightarrow= \frac{36}{100} = 0.36\ m$

q/A = $K\frac{\Delta T}{\Delta X} = 500 = 0.9 \times \frac{\Delta T}{0.36} \rightarrow\rightarrow\ \Delta T = 200℃\ \therefore\ \Delta T = T_{in} - T_{out}\ \rightarrow\rightarrow\ 200 = 230 - T_{out}$

$T_{out} = 30℃$

Example:- if (3kw) of heat is conducted through a section of insulating material with (0.6 m^2) cross section area and (2.5 cm) thick and (0.2 w/m. ℃) thermal conductivity . compute the temperature difference a cross the material?

Solution: -

q = 3kw $\longrightarrow$ 3000w ΔX= 2.5 cm $\longrightarrow$ ΔX = 0.025 m

$$q = KA\frac{\Delta T}{\Delta X} \rightarrow\rightarrow 3000 = 0.2 \times 0.6 \times \frac{\Delta T}{0.025} \rightarrow\rightarrow \Delta T = 625℃$$

H.M: - One face of Copper plate (K=370 W/m. °C). 3cm thick is maintained at (400 °C) and the other face is maintained at (100 °C). How much heat is transferred through the plate per unit area?

2-The Composite Plane Wall: -

Consider the Composite plane wall and electrical analog as shown in the fig (1-5).

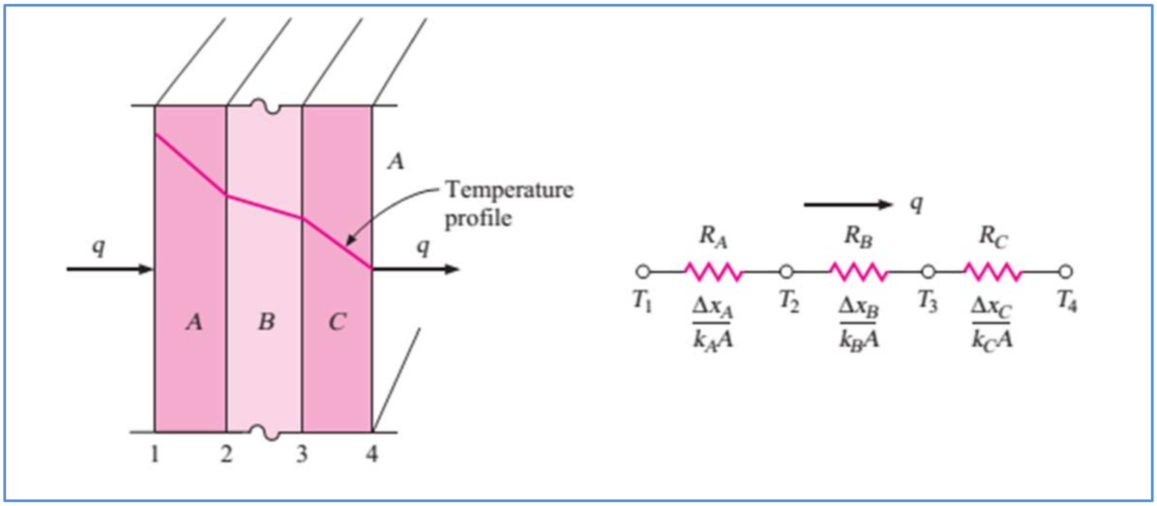


Fig (1-5) Composite plane wall

The temperature gradients in the three materials are shown, and the heat flow may be written:

$q = q_1 = q_2 = q_3$

$$q = -K_A A \frac{(T_2 - T_1)}{\Delta X_A} = -K_B A \frac{(T_3 - T_2)}{\Delta X_B} = -K_C A \frac{(T_4 - T_3)}{\Delta X_C}$$

Solving these three equations simultaneously, the heat flow is written:

$$q = \frac{(T_{in} - T_{out})}{\frac{\Delta X_A}{K_A A} + \frac{\Delta X_B}{K_B A} + \frac{\Delta X_C}{K_C A}} \longrightarrow \therefore q = \frac{\Delta T_{overall}}{\sum R_{th}}$$

Example: -

A house wall consists of an outer layer of common brick (10cm) thick, (K=0.69 W/m.°C) followed. by a (1.25cm) layer of Celotex sheathing (K=0.048 W/m.°C) . A (1.25cm) layer of sheetrock (K=0.744 W/m.°C) forms the inner surface. And the outside brick temperature is (5°C) the inner wall surface is maintained at (20°C). What is the rate of heat loss per unit area of wall?

Solution: -

X_1= (0.1m) k = (0.69 W/m.°C) T_1= (20°C)

X_2 = (0.0125m) k = (0.048 W/m.°C) T_2= (5°C)

X_3 = (0.0125m) k = (0.744 W/m.°C)

$$\frac{q}{A} = ?$$

$$q/A = \frac{(T_{in} - T_{out})}{(\frac{\Delta x_1}{k_1} + \frac{\Delta x_2}{k_2} + \frac{\Delta x_3}{k_3})} = \frac{q}{A} = \frac{(20 - 5)}{(\frac{0.1}{0.69} + \frac{0.0125}{0.048} + \frac{0.0125}{0.744})} = (35.714)\ (\frac{w}{m^2})$$

Example: -

Reactor wall consist of three layers, first layer thickness is (225 mm) of fire brick and second layers' thickness is (120 mm) of insulating brick, and third layers' thickness is (225mm) 0f building brick and the inside and outside surface temperature of the wall are (1200 k and 330 k) respectively. If the thermal conductivity for three layers are (1.4,0.2 and 0.7 w/m.k) respectively . Calculate: -

1- The heat loss per ($1m^2$).
2- The contact temperature.

Solution: -

ΔX_1= (0.225m), ΔX_2 = (0.12m), ΔX_3= (0.225m)

k_1= (1.4 w/m. k), k_2 = (0.2w/m. k), k_3= (0.7w/m. k)

1- $$q = A\frac{\sum \Delta T}{\sum \frac{\Delta X}{K}} = \frac{T_{in} - T_{out}}{\frac{\Delta x_1}{K_1 A} + \frac{\Delta x_2}{K_2 A} + \frac{\Delta x_3}{K_3 A}} = \frac{1200 - 330}{\frac{0.225}{1.4\times1} + \frac{0.12}{0.2\times1} + \frac{0.255}{0.7\times1}} = (804.810\ w)$$

2- $$q = K_1 A\frac{\Delta T}{\Delta X_1} \longrightarrow\longrightarrow 804.810 = 1.4\times1\frac{1200 - T_2}{0.225} \longrightarrow\longrightarrow T_2 = (1071K)$$

Example: -

A furnace wall consist of three layers the first (0.09 m) thick of fire brick the second is (0.06 m) thick of insulating brick and third layer is (0.04 m) of building brick .the thermal conductivity are (1, 0.08 ,0.07) (w/m.k) respectively and the inside and outside temperature are (1250 , 300) k Calculate ?

1- The amount of heat loss per meter squire of furnace ?
2- The contact temperature between the layers ?

Solution: -

$$q = \frac{\Delta T_{overall}}{\sum R_{th}} = \frac{T_i - T_4}{\frac{\Delta X_1}{K_1 A} + \frac{\Delta X_2}{K_2 A} + \frac{\Delta X_3}{K_3 A}} \rightarrow\rightarrow {}^{q}/_{A} = \frac{1250-300}{\frac{0.09}{1} + \frac{0.06}{0.08} + \frac{0.04}{0.07}} \rightarrow\rightarrow {}^{q}/_{A} = 673.281 \ \text{w/m}^2$$

$${}^{q}/_{A} = K_1 \frac{T_1 - T_2}{\Delta X_1} \rightarrow\rightarrow 673.281 = 1\times \frac{1250 - T_2}{0.09} \rightarrow\rightarrow T_2 = 1189 \text{ K}$$

$$q/A = K_2 \frac{T_2 - T_3}{\Delta X_2} \rightarrow\rightarrow 673.281 = 0.08 \times \frac{1189 - T_3}{0.06} \rightarrow\rightarrow \therefore T_3 = 684\text{K}$$

H.M: -Two layers of rectors wall (0.2m) thick and (5.9 w/m. k) of inside layer (0.1m) thick and (0.5 w/m. k) of outside layer. The temperature of inside and outside surface is (900 k and 325k) respectively. Calculation the lose heat per hour through (10 m^2), surface area of wall, and estimate the interface temperature?

1- Cylindrical walls: -

Consider along cylinder of inside radius (r_i), outside radius (r_o) and length (L) such as the one shown in the following fig.

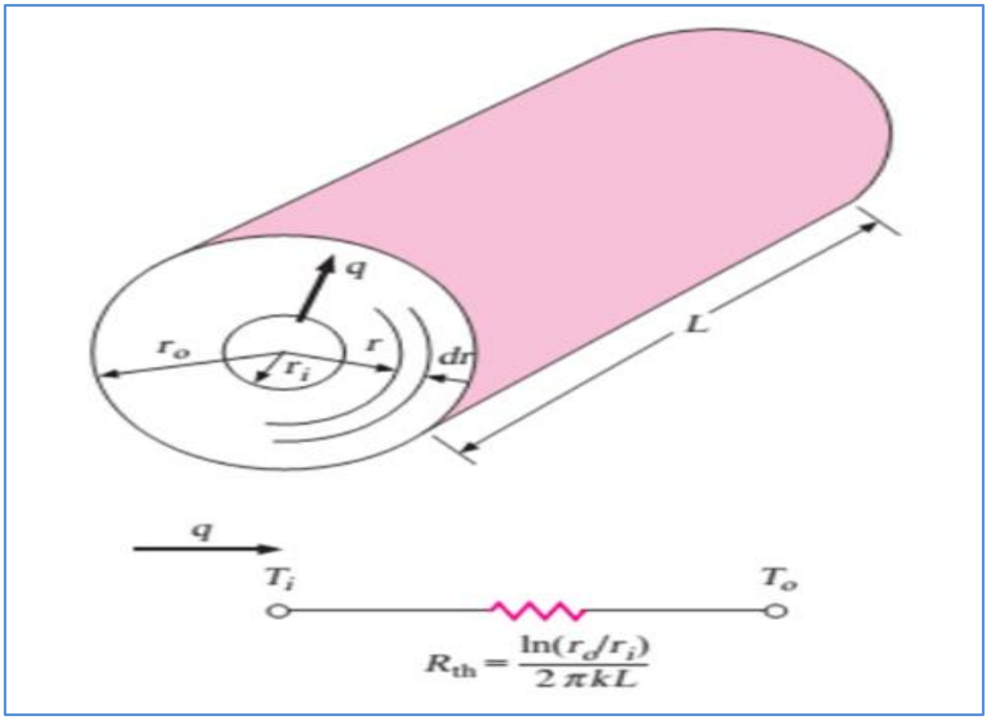


Fig (2-3) One dimensional heat flow through a hollow

$$A_r = 2\pi rL$$

So that Fourier's Law is written: -

$$q = \frac{T_i - T_o}{\frac{\ln(\frac{ro}{ri})}{2\pi KL}}$$

and the thermal resistance in this case is: -

$$R_{th} = \frac{\ln(\frac{ro}{ri})}{2\pi KL}$$

4- Composite cylindrical wall:-

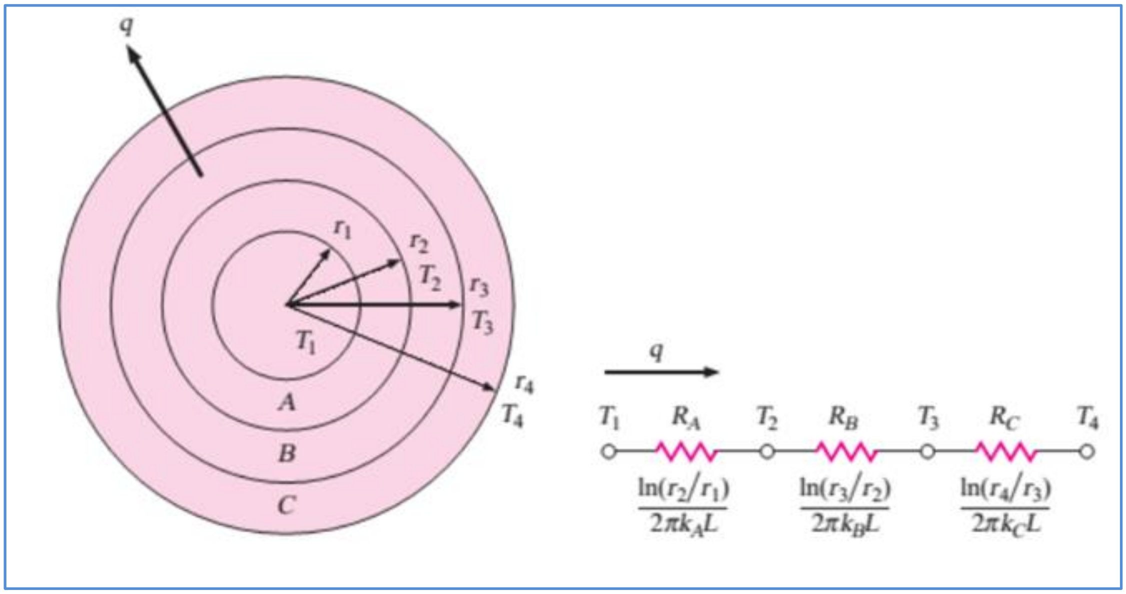


Fig (2-4) One dimensional heat through multiple cylindrical sections and electrical

عدة طبقات وكما مبين في الشكل التالي فأن تحليلاً مشابها لتحليل الحائط المستوي

واذا كان الجدار الاسطواني مركباً من عدة طبقات وكما مبين في الشكل التالي فأن تحليلاً مشابها لتحليل الحائط المستوي المركب يؤدي الى نتيجة مماثلة وهي

$$q=\frac{T_i-T_O}{\frac{\ln(\frac{r_1}{r_i})}{2\pi K_1 L}+\frac{\ln(\frac{r_2}{r_1})}{2\pi K_2 L}+\frac{\ln(\frac{ro}{r_2})}{2\pi K_3 L}} \qquad = \frac{T_i-T_o}{\sum R_{th}}$$

Example: -

Calculate the heat loss of tube if L= (10 m) and inside diameter (6 cm), shell of insulating thickness is (5 cm) and thermal conductivity coefficient k = (0.055 w/m. k), The inside and outside surface temperature for insulating are (467 k and 299 k) respectively?

Solution: -

L = (10 m), k = (0.055 w/m. k), d_1= (6 cm), r_1= (0.03 m), thi = (5 cm), r_2 = (0.03 + 0.05 = 0.08m)

T_i = (467 k), T_o = (299 k)   q = ?

$$\therefore\ q=\frac{T_i-T_o}{\frac{\ln(\frac{r_o}{r_i})}{2\pi KL}} \longrightarrow q=\frac{(467-299)}{\frac{\ln(\frac{0.08}{0.03})}{2\pi(0.055\times10)}} \longrightarrow$$

q = 592 w

Example: -

A thick – walled tube of stainless steel (K= 19 W/m.℃) with (2 cm inside diameter and outer diameter is (4cm) , is covered with a (3cm) layer of asbestos insulation (K = 0.2 W/m. °C) . If the inside wall temperature of the pipe is maintained at (T_i = 600°C) and the outside of the insulation at (T_o = 100°C) . Calculate the heat loss per meter of length?

Solution: -

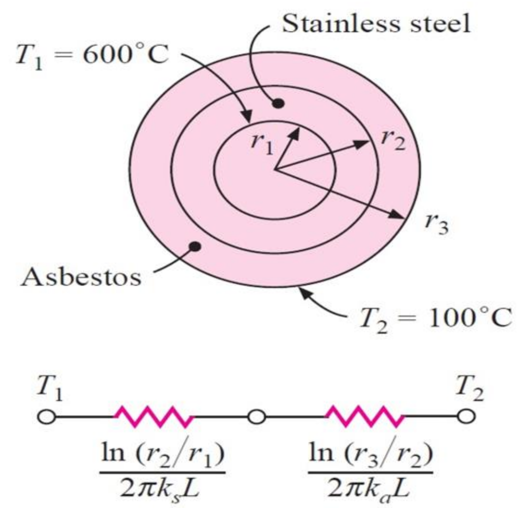
K_1 = (19 w/m.°C) , K_2 = (0.2 w/m.°C)

d_1 = (2 cm), r_1 = (0.01 m)

d_2 = (4 cm), r_2 = (0.02 m)

$r_3 = r_2 + 0.03m \longrightarrow r_3 = (0.05\ m)$

T_i = (600°C) , T_o = (100 °C) .



$$\frac{q}{L}=\frac{(T_i-T_o)}{\frac{\ln(\frac{r_2}{r_1})}{2\pi k_1}+\frac{\ln(\frac{r_3}{r_2})}{2\pi k_2}} \rightarrow\rightarrow \frac{q}{L}=\frac{(600-100)}{\frac{\ln(\frac{0.02}{0.01})}{2\times\pi\times(19)}+\frac{\ln(\frac{0.05}{0.02})}{2\times\pi\times(0.2)}} \rightarrow\rightarrow$$

$$\frac{q}{L}=(680w/m)$$

H.W: -

A wrought iron pipe (K = 55 W/m. °C) with (r_1= 5.113 cm) inside radius , and (r_2= 5.715 cm) outside radius is covered with (2.5 cm) of magnesia insulation (K =0.071 W/m. °C) . If the inside pipe wall temperature is maintained at (T_i = 150°C) and outer insulation surface temperature is maintained at (T_o = 30 °C) . Find the heat loss per meter of pipe length? and the contact temperature between layers (T_2)?

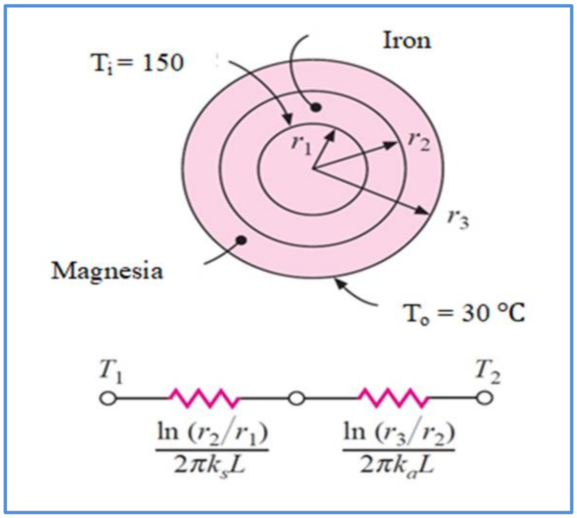
Solution: -

K_1= (55 W/m.°C) , K_2= (0.071W/m. °C)

r_1= (0.05113 m), r_2 = (0.05715 m)

r_3= r_2 + 0.025m ⟶ r_3 = (0.08215 m)

T_i = (150 °C) , T_o = (30 °C)



$$1- \frac{q}{L} = \frac{T_i - T_o}{\frac{\ln(\frac{r_2}{r_1})}{2\pi K_1} + \frac{\ln(\frac{r_3}{r_2})}{2\pi K_2}} \rightarrow \frac{q}{L} = \frac{(150-30)}{\frac{\ln(\frac{0.05715}{0.05113})}{2\times3.14\times55} + \frac{\ln(\frac{0.08215}{0.05715})}{2\times3.14\times0.071}} \rightarrow \boxed{\frac{q}{L} = (147.5\frac{w}{m})}$$

$$\frac{q}{L} = \frac{(T_1 - T_2)}{\frac{\ln(\frac{r_2}{r_1})}{2\pi K_1}} \rightarrow (147.5) = \frac{150 - T_2}{\frac{\ln(\frac{0.05715}{0.05113})}{2 \times 3.14 \times 55}} \rightarrow T_2 = (149.9\ °C)$$

Chapter (3)

Convection

Heat Convection: - Convection is the mode of energy transfer between a solid surface and the liquid or gas that is in motion.

There are two types of convection natural and force convection.

Forced convection if the fluid is forced to flow over the surface by external means such as a fan, pump, or the wind. natural (or free) convection if the fluid motion is caused by buoyancy forces by density differences due to the variation of temperature in the fluid.

$$q = h\,A\,(T_S - T_\infty) \ldots\ldots \textbf{(Newton's Law)}$$

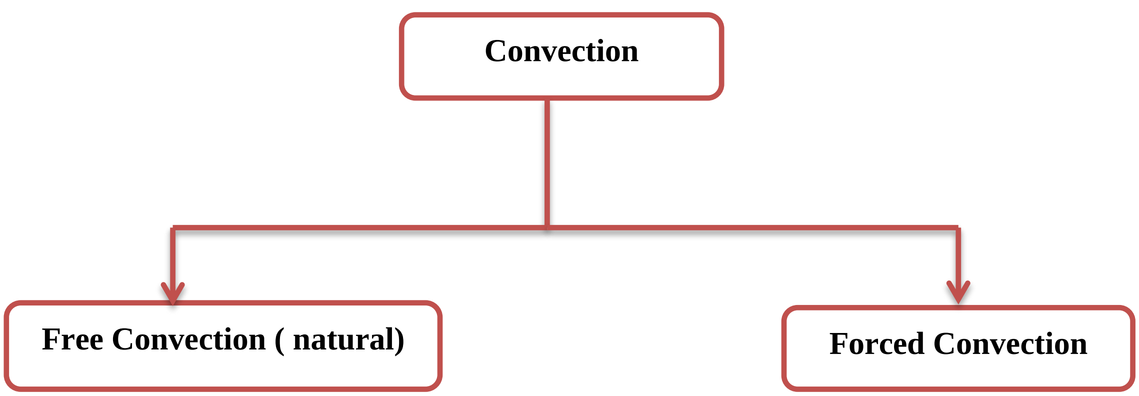
Where: -

(h) = Convection heat transfer coefficient ($W/m^2.\ °C$).

(A) = Surface area (m^2).

(T_s) = Surface temperature (°C) or (K).

(T_∞) = Fluid temperature (°C) or (K) .



Film temperature: - It is the average temperature of surface temperature & fluid temperature

$$T_{Film}=\frac{T_{surface}+T_{fluid}}{2}$$

Film layer: - It is thin fluid layer palpation the surface.

There are two general types of fluid in pipes: -

Laminar flow (viscosity flow): - Where the fluid particles, flow in stream line parallel to the axis at the pipe. Show fig (3-1(a)).

Fig (3-1a) Laminar flow

Turbulent flow: - The fluid particles flow in stream line parallel to the pipe axis but with radial Show fig(3-1(b)).

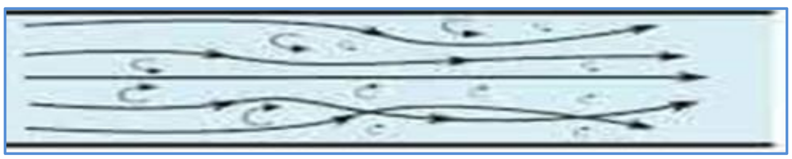


Fig (3-1b) Turbulent flow

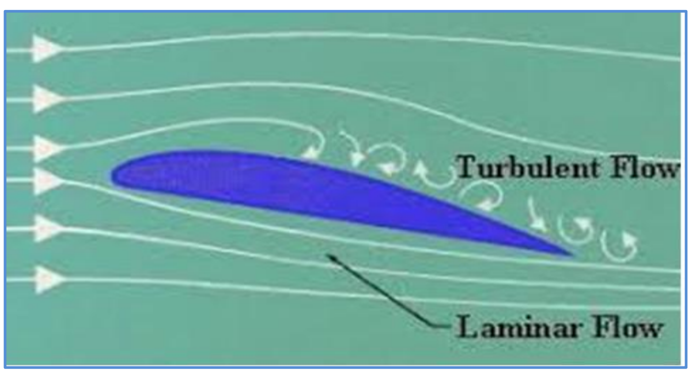


Fig (3-1) Laminar and Turbulent Flow

Dime nationless group: -

1- Reynolds number: -

$$N_{Re} = \frac{V \times D}{\upsilon}$$

Where: -

V = fluid velocity (m/sec).

D = pipe dimeter (m).

υ = kinematic viscosity (m^2/sec).

$$\therefore \upsilon = \frac{\mu}{\rho}$$

$$N_{Re} = \frac{V \times D \times \rho}{\mu}$$

Where: -

ρ = fluid density (kg/m^3).

μ= dynamic viscosity ($\frac{kg}{m.sec}$) .

For flow in pipes: -

The flow laminar when (Re < 2000) and the flow become Turbulent when (Re > 2000).

For flow over flat plate: -

The flow laminar when ($Re < 5\times 10^5$)

And becomes turbulent when ($Re > 5 \times 10^5$)

2-Prandtl Number (Pr): -

A dimensional number that links the properties of heat propagation and momentum propagation

$$P_r = \frac{\mu C_P}{K}$$

Where: -

μ = dynamic viscosity of the fluid (kg / m.sec).

C_p = specific heat of the fluid (J /kg. k).

K= thermal conductivity (w / m.k).

3- Grashof number(Gr): -

The Grashof number is defined as:

$$G_r = \frac{g \times \beta \times (T_S - T_\infty) L^3}{\upsilon^2}$$

Where: -

g = gravity acceleration.

β = the coefficient of volume expansion

For ideal gas ($\beta = \frac{1}{T}$) Where T= absolute temperature in (1/k)

T_S = Surface temperature (k).

T_∞= Fluid temperature(k).

L = Characteristic dimension.

4-Nusselt Number (Nu): -

$$Nu = \frac{hD}{K} = \frac{hL}{k}$$

Where :-

h = film coefficient (w/m^2.k) . L = plate length (m) .

D = pipe diameter (m) . k = thermal conductivity of fluid (w/m.k).

EX (1): - Prove that (N_{Re}) is dime nationless group.

$$N_{Re}=\frac{V.D}{\upsilon} \Rightarrow N_{Re}=\frac{\frac{m}{sec.m}}{\frac{m^2}{sec}}=1$$

$$N_{Re}=\frac{\frac{L}{T.L}}{\frac{L^2}{T}}=1$$

EX (2): - Prove that (N_{Nu}) is dime nationless group.

$$N_{Nu}=\frac{h.D}{K} \Rightarrow N_{Nu}=\frac{\frac{w}{m^2.k}\times m}{\frac{w}{m.k}} \Rightarrow N_{Nu}=\frac{\frac{J}{sec.m^2}\times k\times m}{\frac{J}{sec.m.k}}=1$$

Heat transfer by free convection: -

The motion of fluid particles is due to different in fluid densities (Ex: - hot air rises) The general Nusselt Number relation of free convection

$$N_{NU}=C\,(N_{Gr}\times N_{Pr})^m$$

Where: -

N_{NU} = Nusselt number .

N_{Gr}= Grashof number.

N_{Pr}= Prandtl number .

C & m = Constant depends on the flow type.

1- Vertical Plane Surfaces (Plate & Cylinder): -

Turbulent flow ($10^9 < Ra < 10^{12}$)

$$N_u = 0.13\,(R_a)^{1/3}$$

Laminar flow ($10^4 < R_a < 10^9$)

$$N_u = 0.59\,(R_a)^{1/4}$$

Rayleigh number (R_a): -

$$R_a = G_r \times P_r$$

لحساب عدد رالي من خلال الجداول مباشرة نتبع مايلي :-

$$T_{Film} = \frac{T_{surface} + T_{fluid}}{2}$$

Where: -

T= °C + 273 = K°

$$G_r \times P_r = \frac{L^3 \times \rho^2 \times \beta \times \Delta T \times g}{\mu^2} \times \frac{\mu \times c_p}{K}$$

$$= L^3 \times \Delta T \left(\frac{\rho^2 \times g \times \beta \times c_p}{\mu \times k}\right) \Longrightarrow L^3 \times \Delta T \times Z$$

Z = هي كافة خواص المائع ووحداتهُ ($^{1}/_{m^3.k}$)

***All properties are evaluated at film temperature :-**

$$T_f = \frac{T_s + T_\infty}{2} = (°C) + 273 = K$$

K= → μ, ρ, C_p, K, P_r

$$\beta = \frac{1}{T_f}$$

Geometry	(Gr Pr)	C	m	Nu
Vertical plates and cylinders				
Laminar	$10^4 - 10^9$	0.59	1/4	$0.59\,(Gr\,Pr)^{1/4}$
Turbulent	$10^9 - 10^{13}$	0.13	1/3	$0.13\,(Gr\,Pr)^{1/3}$
Horizontal cylinders				
Laminar	$10^4 - 10^9$	0.53	1/4	$0.53\,(Gr\,Pr)^{1/4}$
Turbulent	$10^9 - 10^{12}$	0.13	1/3	$0.13\,(Gr\,Pr)^{1/3}$
Horizontal plates				
Laminar (heated surface up or cooled surface down)	$10^4 - 10^7$	0.54	1/4	$0.54\,(Gr\,Pr)^{1/4}$
Turbulent (heated surface up or cooled surface down)	$10^7 - 10^{11}$	0.15	1/3	$0.15\,(Gr\,Pr)^{1/3}$
Laminar (heated surface down or cooled surface up)	$10^5 - 10^{11}$	0.27	1/4	$0.27\,(Gr\,Pr)^{1/4}$

Example: -

Estimate the film coefficient for free convection from a vertical heated surface (54 cm) height at (90 °C) to still air at (14 °C) take k of air = (0.033 w/m.k) .

Solution: -

L = (0.54 m) , K= (0.033 W/m.k) , T_{fluid} = (14 °C) ,$T_{surface}$ (90 °C)

$$T_{Film} = \frac{T_{surface}+T_{fluid}}{2} \Rightarrow T_{Film} = \frac{90+14}{2} \Longrightarrow T_{Film} = (52°C) + 273 = (325k)$$

$$\therefore R_a = \left(0.758\times10^9\right)(\Longrightarrow \text{Turbulent flow})$$

$$\therefore N_u = 0.13\,(0.756\times10^9)^{1/3} \Longrightarrow N_u = (118.5) \Longrightarrow N_u = \frac{h.L}{K} \Longrightarrow h = \frac{N_u.k}{L}$$

$$h = \frac{118.5\times0.033}{0.54} \Longrightarrow h = (7.24)(w/m^2, k).$$

Example: -

A vertical hot oven door (L = 0.5 m) high, Is at (200°C) and is exposed to atmospheric pressure air at (20 °C) . Estimate the average heat transfer coefficient at the surface of the door?

Solution: -

L= (0.5 m) , $T_{surface}$ = (200°C) , T_{fluid} = (20°C)

$$T_{Film} = \frac{T_{surface}+T_{fluid}}{2} \Longrightarrow T_{Film} = \frac{(200+20)}{2} = (110°C)$$

$$\Longrightarrow \textit{from tables given}$$

$\upsilon = (24.1\times10^{-6}\ m^2/s)$, k = (0.03194 w/m. °C) , P_r = (0.704)

$$\beta = \frac{1}{T_f} \Longrightarrow \beta = \frac{1}{(110+273)} = \left(\frac{1}{383.}\right) k^{-1} \text{ (ideal gas)}$$

$$G_r=\frac{g.\beta.(T_S-T_\infty)L^3}{\upsilon^2}\Rightarrow G_r=\frac{(9.8).(\frac{1}{383.})(200-20)(0.5)^3}{(24.1\times10^{-6})^2}$$

$G_r=(9.9\times10^8)\Rightarrow R_a=G_r\,.\,P_r=(9.9\times10^8)(0.704)$

$R_a=(6.97\times10^8)$

لكونها اقل من (10^9)

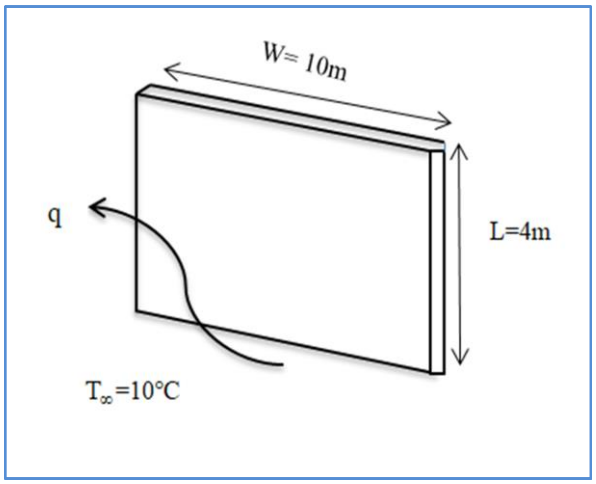
$\therefore$ *Laminar flow*

$N_u=0.59\,(R_a)^{1/4}\Rightarrow N_u=0.59\,(6.97\times10^8)^{1/4}=(95.86)$

$$N_u=\frac{h.L}{K}\Rightarrow h=\frac{(N_u)(k)}{L}\Rightarrow h=\frac{(95.86)(0.03194)}{(0.5)}=(6.12)(w/m^2.°C)$$

Example: -

A large vertical plate (4m) high is maintained at (60 °C) and exposed to atmospheric air at (10°C) . Calculate the heat transfer if the plate is (10 m) wide



Solution: -

L = (4 m) ,T surface = (60 °C) , $T\infty=(10°C)$

$$T_{Film}=\frac{T_{surface}+T_\infty}{2}\Rightarrow T_{Film}=\frac{(60+10)}{2}=(35°C\quad,308\,K)$$

From tables give

$\upsilon=\left(16.5\times10^{-6}\ m^2/s\right)$, (K = 0.02685w/m. k), (Pr = 0.7)

$$\beta=\frac{1}{T_f}\Rightarrow\beta=\frac{1}{308}=(3.25\times10^{-3})\,k^{-1}$$

$$G_r=\frac{g.\beta.(T_s-T_\infty)L^3}{\upsilon^2}\Rightarrow G_r=\frac{(9.8)(3.25\times10^{-3})(60-10)(4)^3}{(16.5\times10^{-6})^2}=(3.743\times10^{11})$$

$R_a=G_r\,.\,P_r\Rightarrow R_a=(3.743\times10^{11}).(0.7)\Rightarrow R_a=(2.62\times10^{11})\Rightarrow$(Turbulent flow)

$N_u=0.13.R_a^{1/3}\Rightarrow N_u=(0.13)(2.62\times10^{11})^{1/3}\Rightarrow N_u=(831.7)$

$$N_u=\frac{h.L}{K}\Rightarrow h=\frac{(N_u)(K)}{(L)}\Rightarrow h=\frac{(831.7)(0.02685)}{4}\Rightarrow h=(5.58)(\frac{w}{m^2.°C})$$

$q = h.A\,(T_s - T_\infty) \Longrightarrow q = (5.58)\,(4 \times 10)(60 - 10) \Longrightarrow q = (11160)w$

H.W:-

A vertical plate (L= 4m) high , has a surface temperature of (93.3 °C) . It is placed in a quiescent fluid at (4.4 °C) . Find the free convective surface heat transfer coefficient. and how much heat is transferred through the plate per unit area?

Solution: -

$L = (4\ m)$, $T_{surface} = (93.3\ °C)$, $T_\infty = (4.4\ °C)$

$$T_{Film} = \frac{T_{surface} + T_{fluid}}{2} \Rightarrow T_{Film} = \frac{(93.3 + 4.4)}{2} = (48.85\ °C)$$

From tables give

$$\upsilon = \left(1.781 \times 10^{-5}\ m^2/s\right) , (K = 0.0277 w/m.\ °C) , (P_r = 0.709)$$

$$\beta = \frac{1}{T_f} \Rightarrow \beta = \frac{1}{48.85 + 273} = (3.1 \times 10^{-3}) k^{-1}$$

$$G_r = \frac{g.\beta.(T_s - T_\infty) L^3}{\upsilon^2} \Rightarrow G_r = \frac{(9.8)(3.1 \times 10^{-3})(93.3 - 4.4)(4)^3}{(1.781 \times 10^{-5})^2} = (3.1 \times 10^{10})$$

$$R_a = G_r \,.\, P_r \Rightarrow R_a = (3.1 \times 10^{10}).\ (0.709) \Rightarrow R_a = (2 \times 10^{10}) \Rightarrow (\mathit{Turbulent\ flow})$$

$$N_u = 0.13.R_a^{1/3} \Rightarrow N_u = (0.13)\left(2 \times 10^{10}\right)^{1/3} \Rightarrow N_u = (160.048)$$

$$N_u = \frac{h.L}{K} \Rightarrow h = \frac{(N_u)(K)}{(L)} \Rightarrow h = \frac{(160.048)(0.0277)}{4} \Rightarrow h = (\ 1.1083)\left(\frac{w}{m^2.°C}\right)$$

$$q = h.\left(T_s - T_\infty\right) \Rightarrow q = (1.1083)\left(93.3 - 4.4\right) \Rightarrow q/A = (98.527)\ w/m^2 .$$

2- Horizontal plates: -

Turbulent flow ($10^7 < Ra < 10^{10}$)

$$N_u = 0.14\,(R_a)^{1/3}$$

Laminar flow ($10^5 < R_a < 10^7$)

$$N_u = 0.54\,(R_a)^{1/4}$$

اذا كان المائع تحت السطح المسخن او فوق السطح المبرد فان العلاقة المعتمدة هي :-

$$N_u = 0.27\,R_a^{1/4} \Longrightarrow (\,10^5 < R_a < 10^{10}\,)$$

Example: -

Calculate the heat losses for free convection from a heated square plate (18 cm) on a side at (149°C) facing upward to the still air of a room at (27°C) take (K = 0.033 w/m.k) .

Solution: -

L= (0.18 m) , $T_{surface}$ = (149 °C) , T_{fluid} = (27 °C) , k = (0.033 w/m.k)

$$T_{Film} = \frac{T_{surface} + T_{fluid}}{2} \Longrightarrow T_{Film} = \frac{(149+27)}{2} \Longrightarrow T_{Film} = (88\ ^\circ C)$$

$$T_f = 88 + 273 \Longrightarrow T_f = (361\ k)$$

From chart find ($Z = 38.2\times 10^6$) $\Longrightarrow \therefore G_r.P_r = L^3 .\ \Delta T\ .\ Z$

$$R_a = (0.18)^3 .\ (149 - 27).\ 38.2\times 10^6 \quad \Longrightarrow \quad R_a = (2.72 \times 10^7)$$

∴Turbulent flow

$$N_u = 0.14\,R_a^{1/3} \Longrightarrow N_u = (0.14)\left(2.72 \times 10^7\right)^{1/3} \Longrightarrow N_u = (42.10)$$

$\therefore N_u = \frac{h.L}{k} \Rightarrow h = \frac{(N_u)(k)}{L} \Rightarrow h = \frac{(42.10)(0.033)}{(0.18)} \Rightarrow$

$h = (7.71)(^{W}/_{m^2.k})$

$q = h.\ A.\ \Delta\ T \Rightarrow q = (7.71).(0.18)^2\ .\ (149 - 27) \Rightarrow q = (30.47)\ w$

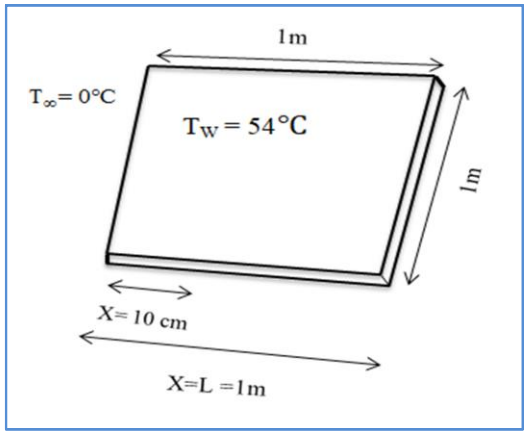
Example: -

A $(1\times 1)\ m^2$ flat plate is positioned horizontally and maintained at (54°C) . The plate is exposed to air at (1 atm) and (0°C) . Calculate the heat transfer over the first (10 cm) of the plate, and also over the entire length of the plate for the upper facing of heated plate.

Solution: -

$A = 1m^2$ $T_{surface} = (54°C)$ $T_{fluid} = (0°C)$ X=10cm $\therefore X = 0.1m$

$T_{Film} = \frac{T_{surface} + T_{fluid}}{2} \Rightarrow T_{Film} = \frac{(54+0)}{2} \Rightarrow T_{Film} = (300k)$



From tables give

$\upsilon = \left(15.69\times10^{-6}\ ^{m^2}/_{s}\right)$, (K = 0.02624w/m. °C) , (P_r = 0.708)

$\beta = \frac{1}{T_f} \Rightarrow \beta = \frac{1}{300} = (3\times10^{-3})k^{-1}$

For X=10 cm $\therefore X = 0.1m$

$G_r = \frac{g.\beta.(T_s - T_\infty)X^3}{\upsilon^2} \Rightarrow G_r = \frac{(9.8)(3\times10^{-3})(54 - 0)(0.1)^3}{(15.69\times10^{-6})^2} = (6.4\times10^6)$

$R_a = G_r\ .\ P_r \Rightarrow R_a = (6.4\times10^6)\ .\ (0.709) \Rightarrow R_a = (4.56\times10^6) \Rightarrow$ (Laminar flow)

$N_u = 0.54.(R_a)^{1/4} \Rightarrow N_u = (0.54)(4.56\times10^6)^{1/4} \Rightarrow N_u = (24.953)$

$N_u = \frac{h.L}{K} \Rightarrow h = \frac{(N_u)(K)}{(X)} \Rightarrow h = \frac{(24.953)(0.0262)}{0.1} \Rightarrow h = (6.537)(\frac{w}{m^2.°C})$

$q = h.A\ (T_s - T_\infty) \Rightarrow q = (6.537)(1\times0.1)(54 - 0) \Rightarrow q = (35.259)w$.

For X = L =1 m   ∴L= 1m

$$G_r = \frac{g.\beta.(T_s - T_\infty)L^3}{\upsilon^2} \Rightarrow G_r = \frac{(9.8)(3\times10^{-3})(54-0)(1)^3}{(15.69\times10^{-6})^2} = (6.4\times10^6)$$

$$R_a = G_r . P_r \Rightarrow R_a = (6.4\times10^6).(0.709) \Rightarrow R_a = (4.56\times10^9) \Rightarrow (\text{Turbulent flow})$$

$$N_u = 0.14.R_a^{1/3} \Rightarrow N_u = (0.14)(4.56\times10^9)^{1/3} \Rightarrow N_u = (215.569)$$

$$N_u = \frac{h.L}{K} \Rightarrow h = \frac{(N_u)(K)}{(L)} \Rightarrow h = \frac{(215.569)(0.0262)}{1} \Rightarrow h = (5.649)(\frac{w}{m^2.°C})$$

$$q = h.A(T_s - T_\infty) \Rightarrow q = (5.649)(1\times1)(54-0) \Rightarrow q = (305.046)w.$$

Example: -

A horizontal pipe of (0.3m) diameter is maintained at (250 °C) in a room of (15 °C) ambient air . Calculate the free convection heat loss per meter of length .

Solution: -

$d = (0.3m)$ $T_{surface} = (250°C)$ $T_{fluid} = (15°C)$

$$T_{Film} = \frac{T_{surface} + T_{fluid}}{2} \Rightarrow T_{Film} = \frac{(250+15)}{2} \Rightarrow T_{Film} = (405.5k)$$

From tables give

$$\upsilon = \left(26.54\times10^{-6}\ m^2/s\right),\ (K = 0.03406\ w/m.°C),\ (P_r = 0.687)$$

$$\beta = \frac{1}{T_f} \Rightarrow \beta = \frac{1}{405.5} = (2.46\times10^{-3})k^{-1}$$

$$G_r = \frac{g.\beta.(T_s - T_\infty)d^3}{\upsilon^2} \Rightarrow G_r = \frac{(9.8)(2.46\times10^{-3})(250-15)(0.3)^3}{(26.54\times10^{-6})^2} = (2.1\times10^8)$$

$$R_a = G_r . P_r \Rightarrow R_a = (2.1\times10^8).(0.687) \Rightarrow R_a = (1.49\times10^8) \Rightarrow (\text{Turbulent flow})$$

$$N_u = 0.14.(R_a)^{1/4} \Rightarrow N_u = (0.14)(1.49\times10^8)^{1/4} \Rightarrow N_u = (69.70)$$

$$N_u = \frac{h_d.d}{K} \Rightarrow h_d = \frac{(N_u)(K)}{(d)} \Rightarrow h_d = \frac{(69.70)(0.03406)}{0.3} \Rightarrow h_d = (7.91)(\frac{w}{m^2.°C})$$

$q = h_d.A\,(T_s - T_\infty) \Rightarrow q = (7.91)(\pi\, d\, L)(250 - 15) \Rightarrow q/L = (1751)\,W/m$.

Example: -

A (2cm) diameter horizontal heater is maintained at a surface temperature of (38 °C) and submerge in water at (27 °C) . Calculate the free convection heat loss per meter of length of heater .

Solution: -

$k = (0.63\ w/m.\ ^\circ C)$ $T_{surface} = (38^\circ C)$ $T_{fluid} = (27^\circ C)$ $d = 2cm \longrightarrow d = (0.02m)$

$$T_{Film} = \frac{T_{surface} + T_{fluid}}{2} \Rightarrow T_{Film} = \frac{(38 + 27)}{2} \Rightarrow T_{Film} = (305.5k)$$

From tables give

$(K = 0.63\ w/m.\ ^\circ C)$, $Z = 2.48\times10^{10}\ (1/_{m^3.k})$

$G_r\, P_r = d^3.\Delta T.Z \Rightarrow G_r p_r = (0.02)^3(38 - 27).(2.48\times10^{10}) \Rightarrow G_r p_r = (2.18\times10^6)$

$R_a = G_r\,.\,P_r \Rightarrow R_a = (2.18\times10^6) \Rightarrow \Rightarrow$(Laminar flow)

$N_{ud} = 0.54.R_a^{1/4} \Rightarrow N_u = (0.54)(2.18\times10^6)^{1/4} \Rightarrow N_u = (20.72)$

$$N_u = \frac{h_d.d}{K} \Rightarrow h_d = \frac{(N_u)(K)}{(d)} \Rightarrow h_d = \frac{(20.72)(0.63)}{0.02} \Rightarrow h_d = (652.6)\left(\frac{w}{m^2.^\circ C}\right)$$

$q = h_d.A\,(T_s - T_\infty) \Rightarrow q/L = (652.6)(\pi\, d\,)(38 - 27) \Rightarrow$

$q/L = (652.6)\times\pi\times(0.02)(38 - 27) \Rightarrow \therefore\ q/L = (450.8\ W/m)$.

(Heat Transfer by Forced Convection)

It is the most important method of heat transfer in engineering . It is used in at most in every type of heat exchanger for one fluid and often for both. The general expression of (N_U) in forced convection is:-

$N_U = C\,(R_e)^m (P_r)^n$

Where:-

N_U = Nusselt nomber.

P_r = Prandtl noumber.

C,m = Constant depended on the type of the flow .

n = Constant depends on the fluid if it is being heated or cooled .

a – Flow over Flat Plate :-

1- Laminar flow over flat plate : $R_e < 5\times10^5$

* Laminar , Local , and $T_W = \text{Constant} \Longrightarrow Nu_X = \frac{h_x \cdot X}{K} = 0.332\,(R_{ex})^{1/2}\,(P_r)^{1/3}$

* Laminar , Local , and $q_w = \text{Constant} \Longrightarrow Nu_X = \frac{h_x \cdot X}{K} = 0.453\,(R_{ex})^{1/2}\,(P_r)^{1/3}$

* Laminar , average , $T_W = \text{Constant} \Longrightarrow \overline{N}u_L = \frac{h_L \cdot L}{K} = 0.664\,(R_{eL})^{1/2}\,(P_r)^{1/3}$

2-Turbulent flow over flat plate $10^7 > R_e > 5\times 10^5$

* Turbulent , Local , $T_W = \text{Constant} \Longrightarrow Nu_X = \frac{h_x \cdot X}{K} = 0.0296\,(R_{ex})^{0.8}\,(P_r)^{1/3}$

* Turbulent , Local , $q_W = \text{Constant} \Longrightarrow Nu_X = \frac{h_x \cdot X}{K} = 1.04\,(R_{ex})^{0.8}\,(P_r)^{1/3}$

* Turbulent , average , T_W = Constant $\Rightarrow \bar{N}u_X = \frac{\bar{h}_x \, . \, L}{K} = 0.037\,(R_{ex})^{0.8}\,(P_r)^{1/3}$

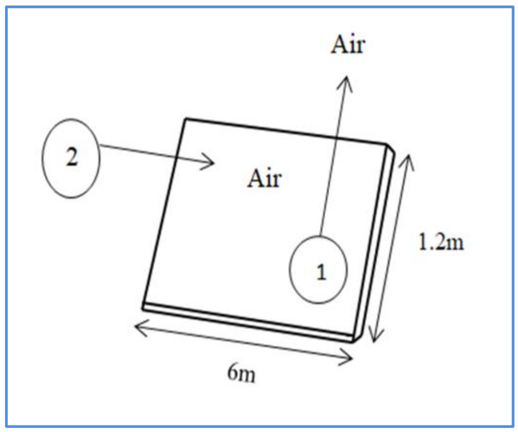
Example: -

The local atmospheric pressure in a certain site is (96.25 Kpa) and the temperature is (27 °C) , and this air is flowing over a flat plate ($6 \times 1.2\ m^2$) with a velocity equal to (8 m/sec) . if the surface temperature of plate is maintained at (127 °C) . Determine the rate of heat transfer if :

1- Air is blowing parallel to (1.2m) side .
2- Air is blowing parallel to (6m) .

Solution: -

P = (96.25 Kpa) , $T_{surface}$= (127°C) , T_{fluid}= (27°C) , A= ($6 \times 1.2\ m^2$) , V= (8 m/sec)



$$T_f = \frac{T_w + T_\infty}{2} = \frac{127+27}{2} \Rightarrow T_f = (\,350\ k)$$

$$\rho = \frac{P}{RT} = \frac{(96.25\times 10^3)}{(287)(350)} \Rightarrow \rho = (0.9582\ \ Kg/m^3)$$

from table (A-5)

$R = 287\ \ J/kg.k$

$\mu = 2.075\times 10^{-5}\ Kg/m.s$

$k = 0.03003\ \ w/m.°C$

$P_r = 0.697$

1- $R_{eL} = \frac{\rho V_\infty L}{\mu} = \frac{(0.9582)(8)(1.2)}{(2.075\times 10^{-5})} \Rightarrow R_{eL} = (4.43\times 10^5)$ $< 5\times 10^5$ $\therefore$ Laminar flow

$$\therefore \bar{N}_{uL} = 0.664(R_{eL})^{1/2}\,(P_r)^{1/3} \Rightarrow \bar{N}_{uL} = 0.664(4.43\times 10^5)^{1/2}\ \ (0.697)^{1/3} \Rightarrow \bar{N}_{uL} = (392)$$

$$\bar{N}u_L = \frac{h_L\ .\ L}{K} \Rightarrow h_L = \frac{\bar{N}u_L.K}{L} \Rightarrow h_L = \frac{(392)(\,0.03003\,)}{1.2} \Rightarrow h_L = (9.81\ w/m^2.k\,)$$

$$q = hA(T_{surface} - T_\infty) \Rightarrow q = 9.81(6\times 1.2)\,(127\text{-}27) \Rightarrow\ \therefore q = (7063.2\ w)$$

2- $R_{eL} = \frac{\rho V_\infty L}{\mu} = \frac{(0.9582)(8)(6)}{(2.075\times 10^{-5})} \Rightarrow R_{eL} = (2.22\times 10^6)$ $> 5\times 10^5$ $\therefore$Turbulent

$\therefore \bar{N}_{uL} = 0.037(R_{eL})^{0.8} (P_r)^{1/3} \Rightarrow \bar{N}_{uL} = 0.037(2022\times10^6)^{0.8} (0.697)^{1/3} \Rightarrow \bar{N}_{uL} = (3918)$

$$\bar{N}u_L = \frac{h_L \cdot L}{K} \Rightarrow h_L = \frac{\bar{N}u_L.K}{L} \Rightarrow h_L = \frac{(3918)(0.03003)}{6} \Rightarrow h_L = (19.61\ w/m^2.k)$$

$q = hA(T_{surface} - T_\infty) \Rightarrow q = 19.61(6\times1.2)(127\text{-}27) \Rightarrow \therefore q = (14119\ w)$

Example: -

Air at (7kpa) and (35°C) flows a cross a (30 cm) square flat plate at (7.5 m/sec) . The plate is maintained at (65 °C) Estimate the heat loss from the plate ?

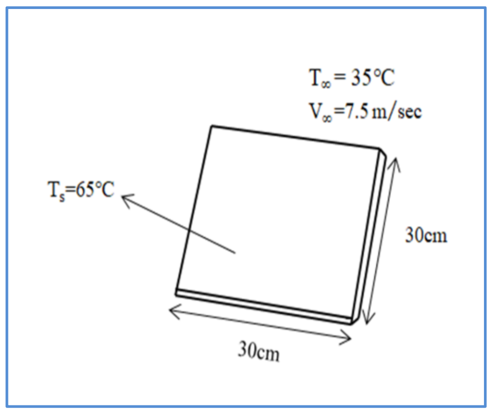
Solution: -

P = 7kpa , $T_{surface} = (65°C)$, $T_{fluid} = (35°C)$, V= (7.5 m / sec) , L= 30cm L= 0.3m

R = 287 J/kg.k

$$T_f = \frac{T_{surface} + T_\infty}{2} = \frac{65+35}{2} \Rightarrow T_f = (323\ k)$$

$$\rho = \frac{P}{RT} = \frac{(7\times10^3)}{(287)(323)} \Rightarrow \rho = (0.0755\ Kg/m^3)$$



The properties of air at 323k are

$\mu = 2.025\times10^{-5}\ Kg/m.s$

$k = 0.02798\ w/m.°C$

$P_r = 0.7$

$$R_{eL} = \frac{\rho V_\infty L}{\mu} = \frac{(0.0755)(7.5)(0.3)}{(2.025\times10^{-5})} \Rightarrow R_{eL} = (8388.8) \quad < 5\times10^5 \quad \therefore \text{Laminar}$$

$\therefore \bar{N}_{uL} = 0.664(R_{eL})^{1/2} (P_r)^{1/3} \Rightarrow \bar{N}_{uL} = 0.664\ (8388.8)^{1/2}\ (0.7)^{1/3} \Rightarrow \bar{N}_{uL} = (54.063)$

$$\bar{N}u_L = \frac{h_L \cdot L}{K} \Rightarrow h_L = \frac{\bar{N}u_L.k}{L} \Rightarrow h_L = \frac{(54.063)(0.02798)}{0.3} \Rightarrow h_L = (5.042\ w/m^2.k)$$

$q = hA(T_{surface} - T_\infty) \Rightarrow q = 5.042\ (0.3\times0.3)\ (65 - 35) \Rightarrow \therefore q = (13.613w)$

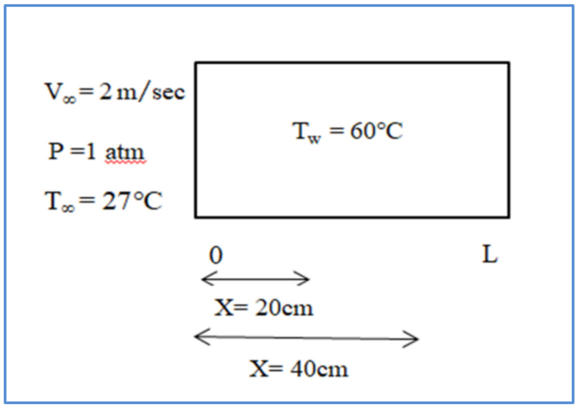
Example: -

Air is flow over flat plate as show in fig . Calculate the heat transfer rate in

1- the first (20cm)of plat.
2- The first (40cm) of plat .

When:- V= (2m/sec) , p = (1 atm) , $T_{surface}$= (60°C) , T_{fluid}= (27°C)

Solution: -



$$T_f = \frac{T_{surface} + T_\infty}{2} = \frac{60 + 27}{2} \Rightarrow T_f = (316.5\ k)$$

From table (A-5)

$\upsilon = 17.36 \times 10^{-6}\ m^2/s$

$k = 0.02749\ \ w/m.°C$

$P_r = 0.7$

1-$R_{ex} = \frac{V_\infty X}{\upsilon} = \frac{(2)(0.2)}{(17.36 \times 10^{-6})} \Rightarrow R_{ex} = (23041) \quad < 5\times 10^5 \quad \therefore$ Laminar

$\therefore \bar{N}_{ux} = 0.332 (R_{eL})^{1/2} (P_r)^{1/3} \Rightarrow \bar{N}_{ux} = 0.332\ (23041)^{1/2}\ (0.7)^{1/3} \Rightarrow \bar{N}_{ux} = (44.79)$

$$\bar{N}u_x = \frac{h_x .\ X}{K} \Rightarrow h_x = \frac{\bar{N}u_x.k}{X} \Rightarrow h_x = \frac{(44.79)(\ 0.02749\ )}{0.2} \Rightarrow h_x = (6.15\ w/m^2.k)$$

$q = hA(T_{surface} - T_\infty) \Rightarrow q = 6.15\ (0.2\times 1)\ (60 - 27) \Rightarrow\ \therefore q = (40.59w)$

2 - $R_{ex} = \frac{V_\infty X}{\upsilon} = \frac{(2)(0.4)}{(17.36 \times 10^{-6})} \Rightarrow R_{ex} = (46082) \quad < 5\times 10^5 \quad \therefore$ Laminar

$\therefore \bar{N}_{ux} = 0.332 (R_{eL})^{1/2} (P_r)^{1/3} \Rightarrow \bar{N}_{ux} = 0.332\ (46082)^{1/2}\ (0.7)^{1/3} \Rightarrow \bar{N}_{ux} = (63.35)$

$$\bar{N}u_x = \frac{h_x .\ X}{K} \Rightarrow h_x = \frac{\bar{N}u_x.k}{X} \Rightarrow h_x = \frac{(63.35)(\ 0.02749\ )}{0.4} \Rightarrow h_x = (4.35\ w/m^2.k)$$

$q = hA(T_{surface} - T_\infty) \Rightarrow q = 4.35\ (0.4\times 1)\ (60 - 27) \Rightarrow\ \therefore q = (57.4w)$

b – Flow in pipes :-

1- For Fully developed Laminar flow $R_{ed} < 2300$

* $N_{ud} = 1.86\,(R_{ed}.P_r)^{1/3}\,(\frac{d}{L})^{1/3}\,(\frac{\mu}{\mu_w})^{0.14}$

2- For Fully developed Turbulent flow $R_{ed} > 2300$

* $N_{ud} = 0.023\,(R_{ed})^{0.8}\,.(P_r)^n$

n = 0.4 for heating of fluid ($T_s > T_b$)

n = 0.3 for cooling of fluid ($T_s < T_b$)

$T_b = \frac{T_i + T_o}{2}$ (bulk mean fluid temperature)

$R_{ed} = \frac{V_\infty\, d}{\upsilon} = \frac{\rho.V_\infty.d}{\mu}$

$N_{ud} = \frac{h.d}{K}$

$q = h\,A(T_s - T_b)$

$A = \pi\, d\, L$ (المساحة السطحية)

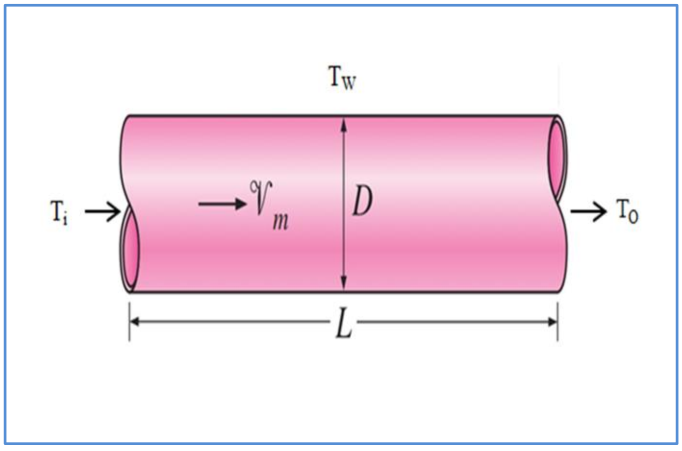
$q = \dot{m}\, C_P\, (T_i + T_o)$

$\dot{m} = \rho\, v_\infty\, A_o = \rho \dot{v}$

Where:-

$\dot{m}$ = mass flow rate (kg/sec)

$\dot{v}$ = volumetric flow rate (m^3/sec)



A_o= cross section area $(\frac{\pi}{4})d^2 \Rightarrow A = (\pi r^2)$

Example: -

Air at (2atm) and (220°C)is heated as it flow through a tube with a diameter of (2.54 cm) at a velocity of (10 m/sec). Calculate the heat transfer per unit length of tube if the tube wall temperature is (20°C) above the air temperature, (assume air exit at 180°C)?

Solution: -

p=(2atm), T_{in}= (220°C) T_{out}= (180°C) , V= (10 m/sec) ,T_{wall}= (20°C) d =(2.54 cm) $\Rightarrow$ d = (0.0254m)

$$T_b = \frac{T_{in}+T_{out}}{2} = \frac{220+180}{2} \Rightarrow T_b = (473k)$$

$$\therefore \rho = \frac{p}{RT} = \frac{(2)(1.0132\times10^5)}{(287)(473)} \Rightarrow \rho = (1.493\ kg/m^3)$$

From tables , the properties of air at a bulk temperature (T_b) of (473k)are :-

$\mu = (2.57\times10^{-5}\ kg/m.s)$, (K = 0.0386 w/m. °C) , (P_r = 0.681)

$$R_e = \frac{\rho VD}{\mu} \Rightarrow R_e = \frac{(1.493)(10)(0.0254)}{(2.57\times10^{-5})} \Rightarrow R_e = (14756) > 2300 \therefore \text{Turbulent flow}$$

$$N_{ud} = 0.023(R_e)^{0.8}(P_r)^{0.4} \Rightarrow N_{ud} = (0.023)(14756)^{0.8}(0.681)^{0.4} \Rightarrow N_{ud} = (42.67)$$

$$N_{ud} = \frac{h_d.d}{K} \Rightarrow h_d = \frac{(N_{ud})(K)}{(d)} \Rightarrow h_d = \frac{(42.67)(0.0386)}{0.0254} \Rightarrow h_d = (64.84)(\frac{w}{m^2.°C})$$

$$q/L = h_d.A\,(T_s - T_b) \Rightarrow q/L = (64.84)(\pi d)(240-200) \Rightarrow q/L = (207) W/m .$$

Example: -

Water at (60°C) enters a tube of (2.54 cm) diameter at mean velocity of (2 m/sec) . Calculate the exit water temperature if the tube is (3m) long and the wall temperature is constant at (80°C) .

Solution: -

$d = (2.54 cm) \Rightarrow d = (0.0254\ m)$, $L = (3m)$, $V = (2\ m/sec)$

ملاحظة :- في حالة عدم معرفة درجة حرارة الدخول أو الخروج تقوم بالاعتماد على الحرارة المعطى في السؤال للمائع في ايجاد الخواص الفيزياوية يعني نعتبره (T_b) .

$T_b = T_i = 60°C$ the properties of water at ($T_b = 60°C$) are :-

$(\rho = 985\ kg/m^3)$

$(\mu = 4.71 \times 10^{-4}\ kg/m.sec)$

$(k = 0.651\ w/m.°C)$

$(P_r = 3.02)$

$(C_p = 4.18\ kj/kg.°C)$

$$R_e = \frac{\rho VD}{\mu} = \frac{(985)(2)(0.0254)}{(4.71\times10^{-4})} \Longrightarrow R_e = (1062) < 2300 \quad \therefore \text{So the flow is laminar}$$

$$\therefore N_{ud} = 1.86\,(R_{ed}.P_r)^{1/3}\left(\frac{d}{L}\right)^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14} \Longrightarrow N_{ud} = 1.86\left(\frac{R_e.P_r.d}{L}\right)^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14}$$

$$\therefore N_{ud} = 1.86\left(\frac{(1062).(3.02)(0.0254)}{(3)}\right)^{1/3}\left(\frac{4.71\times10^{-4}}{3.55\times10^{-4}}\right)^{0.14} \Longrightarrow N_{ud} = (5.816)$$

$$N_{ud} = \frac{h.D}{K} \Longrightarrow h = \frac{N_{ud}.k}{d} \Rightarrow \frac{(5.816)(0.651)}{(0.0254)} \Rightarrow \therefore h = (149.1\ w/m^2.°C)$$

$$\dot{m} = \rho.V.A = \rho.V.\left(\frac{\pi.D^2}{4}\right)$$

$$\dot{m} = (985)(2)(\pi)\left(\frac{(0.0254)^2}{4}\right) \Rightarrow \dot{m} = (9.977\times10^{-3}\ kg/sec)$$

Inserting the value of (h), ($\dot{m}$) , (T_{b1}) , and ($T_s = 80\ °C$) into the energy balance eq

$$q = h.A\,(T_s - T_b) = \dot{m}\,C_P\,(T_o - T_i)$$

$$q = (149.1)(\pi)(0.0254)(3)\left[80 - \frac{(60+T_0)}{2}\right] = (9.977\times10^{-3})(4.18\times10^3)(T_0 - 60)$$

$$35.693\left[80 - \left(\frac{60+T_0}{2}\right)\right] = 41.725(T_0 - 60)$$

$$80 - \left(\frac{60 + T_0}{2}\right) = 1.169\ (T_0 - 60)$$

$$80 - 30 - 0.5T_0 = 1.169T_0 - 70.14$$

$$T_o = 71.98\ ℃$$

(The Overall Heat Transfer Coefficient)

Consider the plane wall shown in the following figure exposed to a hot fluid (A) on one side and a cooler fluid (B) on the other side. The heat-transfer process may be represented by the resistance network in following figure.

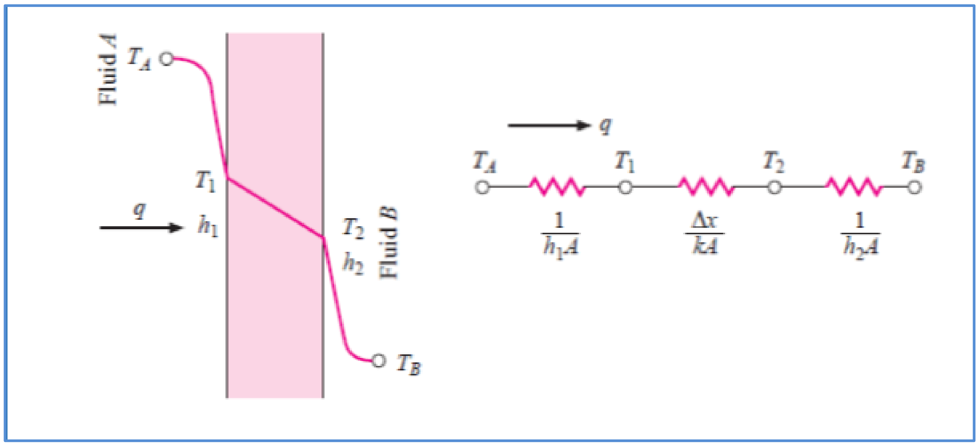


Fig (2-5) Overall heat transfer through a plane wall.

The heat transfer is expressed by:

$$q = h_1 A(T_A - T_1) \quad = \quad KA\frac{(T_1 - T_2)}{L} \quad = \quad h_2 A(T_2 - T_B)$$

Convection   Conduction   Convection

The overall heat transfer is calculated as the ratio of the overall temperature difference to the sum of the thermal resistances:

$$q = \frac{(T_A - T_B)}{\frac{1}{h_1 A} + \frac{L}{KA} + \frac{1}{h_2 A}}$$

The overall heat transfer by combined conduction and convection is frequently expressed in terms of an overall heat-transfer coefficient (U) defined by the relation:

$$q = UA(T_A - T_B)$$

∴ The overall heat transfer coefficient would be:

$$U = \frac{1}{\frac{1}{h_1} + \frac{L}{K} + \frac{1}{h_2}} \quad (w/m^2.°C)$$

*For a hollow cylinder bodies (all heat exchangers that having tubes) . when it exposed to fluids have different temperatures (hot fluid inside it) and (cold fluid outside it) . The overall heat transfer rate can be calculated from .

$$q = h_a A_i(T_A - T_i) = \frac{T_i - T_o}{\frac{\ln\frac{r_o}{r_i}}{2\pi kl}} = h_b A_o (T_o - T_B)$$

$$q = \frac{\Delta T_{overall}}{\sum R_{th}} \Rightarrow q = \frac{T_A - T_B}{\frac{1}{h_A A_i} + \frac{\ln\frac{r_o}{r_i}}{2\pi kl} + \frac{1}{h_B A_o}}$$

$$A_i = \pi d_i L = 2\,\pi\, r_i L$$

$$A_o = \pi d_o L = 2\,\pi\, r_o L$$

$$\therefore q = U_i A_i \Delta T_{overall} = U_o A_o \Delta T_{overall}$$

$$U_i = \frac{1}{\frac{1}{h_i} + \frac{2\,\pi\, r_i L \ \ln\frac{r_o}{r_i}}{2\,\pi\, k\, L} + \frac{2\,\pi\, r_i\, L}{2\,\pi\, r_o\, L\, h_o}} = \frac{1}{\frac{1}{h_i} + \frac{r_i \ln\frac{r_o}{r_i}}{k} + \frac{r_i}{r_o h_o}}$$

$$U_o = \frac{1}{\frac{2\,\pi\, r_o\, L}{2\,\pi\, r_i\, L\, h_i} + \frac{2\,\pi\, r_o L \ln\frac{r_o}{r_i}}{2\,\pi\, k\, L} + \frac{1}{h_o}} = \frac{1}{\frac{r_o}{r_i h_i} + \frac{r_o \ln\frac{r_o}{r_i}}{k} + \frac{1}{h_o}}$$

q = Overall heat transfer rate (w)

U = Overall heat transfer Coefficient ($w/m^2 °C$)

U_i = Overall heat transfer Coefficient based on inside area of pipe $(w/m^2°C)$

U_o = Overall heat transfer Coefficient based on outside area of pipe $(w/m^2°C)$

A= heat transfer area (m^2)

A_i= inside area of pipe (m^2)

A_o= outside area of pipe (m^2)

$\Delta T_{overall}$ = overall temperature difference between the inside and outside fluid (k,°C)

*ملاحظة :- بالنسبة للأشكال المضلعة (الجدار) يمكن ان يطلب (q/A) فيكون كالاتي :-

$$q/A = U\Delta T_{overall} = \frac{\Delta T_{overall}}{\frac{1}{h_A} + \frac{\Delta x}{k} + \frac{1}{h_B}}$$

*ملاحظة :- بالنسبة للأشكال الاسطوانية (الانابيب) ممكن ان يطلب (q/L) فتصبح المعادلة :-

$$q/L = U_i A_i \Delta T_{overall} = \frac{\Delta T_{overall}}{\frac{1}{h_i} + \frac{r_i \ln\frac{r_o}{r_i}}{k} + \frac{r_i}{r_o h_o}}$$

$$U_o\ A_o\ \Delta T_{overall} = \frac{\Delta T_{overall}}{\frac{r_o}{r_i h_i} + \frac{r_o \ln\frac{r_o}{r_i}}{K} + \frac{1}{h_o}}$$

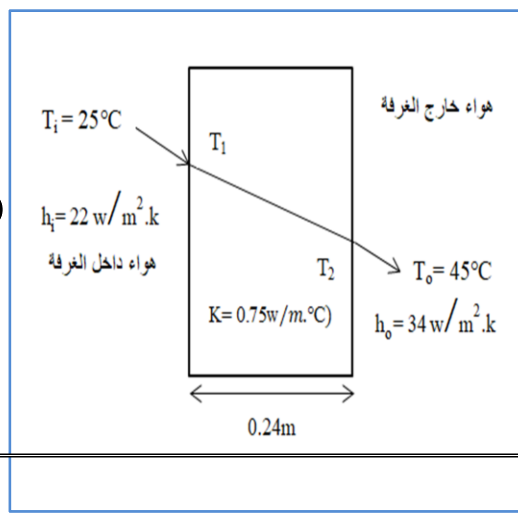
Example: -

The building wall shown in below figure , has a (24 cm) thickness and (k= 0.75thermal conductivity. If the inside and outside heat transfer coefficients are ($h_i = 22\ w/m^2.k$) and ($h_o = 34\ w/m^2.k$) and temperatures .of Air inside and outside are (T_i= 25°C , T_o= 45 °C).Calculate the overall heat transfer coefficient and the rate heat transfer per unit area ?

Solution: -

X = 24 cm ⟹X= (0.24 m) , k = (0.75 $w/m^2.°C$) , h_i = (22 $w/m^2.k$)

h_o= (34 $w/m^2.k$) T_i= (25°C) , T_o= (45 °C).



$$q = UA\,\Delta T_{overall}$$

$$U = \frac{1}{\frac{1}{h_i}+\frac{\Delta x}{k}+\frac{1}{h_o}} \quad U = \frac{1}{\frac{1}{22}+\frac{0.24}{0.75}+\frac{1}{34}} \Rightarrow \therefore U = (2.53\ w/m^2.k)$$

$$q/A = U\Delta T_{overall} \Rightarrow q/A = 2.53\times(45-25) \Rightarrow q/A = (50.6\ w/m^2)$$

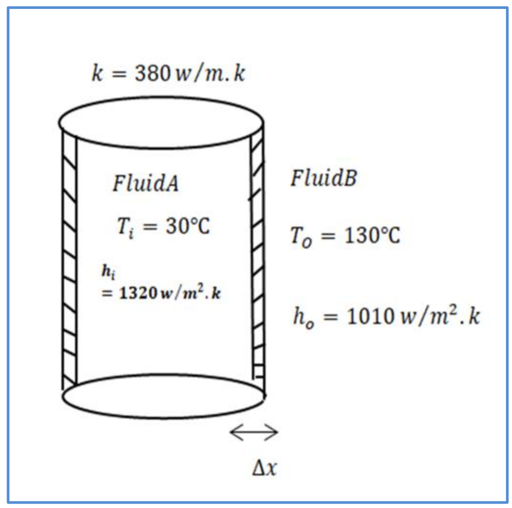
Example: -

Find the overall heat transfer Coefficient based on the outside area (U_o) and the overall heat transfer rate (q) for the pipe shown in bellow figure per unit length .

Solution: -

$k = (380\ w/m.k)$, $T_i = (30°C)$, $T_o = (130°C)$, $h_i = (132\ w/m.k)$, $h_o = (10101\ w/m.k)$

$r_i = (0.023\ m)$, $r_o = (0.025\ m)$, $\Delta x = \left(r_o - r_i = 0.002\ m\right)$.



$$U_o = \frac{1}{\frac{r_o}{r_i\,h_i}+\frac{r_o \ln\frac{r_o}{r_i}}{k}+\frac{1}{h_o}} =$$

$$U_o = \frac{1}{\frac{0.025}{0.023\times1320}+\frac{0.025\ln\frac{0.025}{0.023}}{380}+\frac{1}{10101}} \Rightarrow U_o = (1077.6\ w/m^2.k)$$

$$q = U_o A_o\,\Delta T_{overall} \Rightarrow q = U_o\times(2\pi\,r_o\,L)\times(T_o - T_i)$$

$$\therefore\ q/L = 1077.6\times(2\pi\times0.025)\times\left(130-30\right) \Rightarrow q/L = (16918.3\ w/m)\ .$$

Example: -

The masonry wall of a building consists of an outer layer of facing brick (K_1 = 1.32 W/m.°C) and (L_1 = 10cm) thick followed by a (L_2=15cm) thick layer of common brick (K_2 = 0.69 W/m.°C), followed by a (L_3 = 1.25cm) layer of gypsum plaster (K_3= 0.48 W/m.°C) , an inside and outside convection heat transfer coefficient of (h_i = 30 W/m^2.°C) , (h_o = 8 W/m^2.°C) respectively .What will be the rate of heat gain per unit area , when the inside and outside temperature is (35°C),(22 °C) respectively ?

Solution: -

L_1 = (0.1m), L_2 = (0.15m) , L_3= (0.0125m) , k_1= (1.32 w/m.°C) , k_2 = (0.69 w/m.°C)

k_3 = (0.48 w/m.°C) , h_1 =(30 w/m^2. °C) , h_2 = (8 w/m^2. °C), T_1= (35 °C) , T_2 = (22 °C).

$$q/A = U(T_o - T_i) \Longrightarrow q/A = (T_o - T_i)\left(\frac{1}{\frac{1}{h_1}+\frac{L_1}{k_1}+\frac{L_2}{k_2}+\frac{L_3}{k_3}+\frac{1}{h_2}}\right)$$

$$\Longrightarrow q/A = (35 - 22)\left(\frac{1}{\frac{1}{30}+\frac{0.1}{1.32}+\frac{0.15}{0.69}+\frac{0.0125}{0.48}+\frac{1}{8}}\right) \Longrightarrow q/A = (35 - 22)\ (2.09)$$

$$q/A = (27.2\ w/m^2)$$

Chapter (4)

Heat Exchangers

Heat Exchangers:-

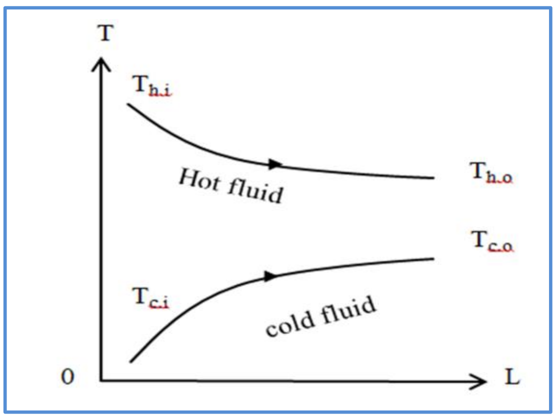
Is a device that is used to transfer thermal energy between two or more fluids at different temperatures . the exchangers are used in many applications such as petroleum , power plant food industries and so on .

Types of Heat Exchangers

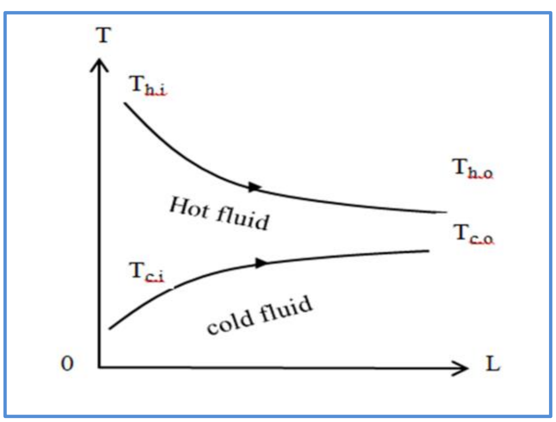
1- Parallel Flow Heat Exchanger:-

In this type, both fluids (hot & cold) flow parallel to each other in the same direction .

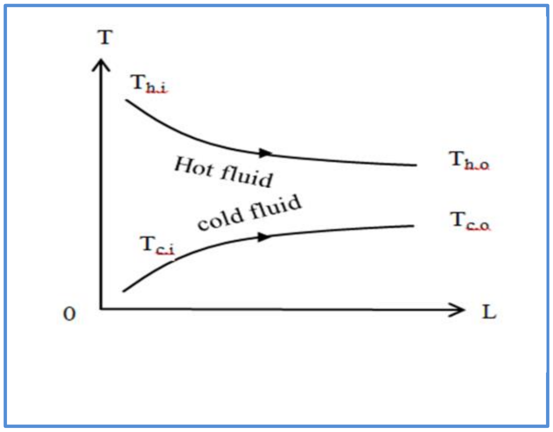
a - $(\dot{m}C_p)h > (\dot{m}C_p)c$



b - $(\dot{m}C_p)h < (\dot{m}C_p)c$



c - $(\dot{m}C_p)h = (\dot{m}C_p)c$

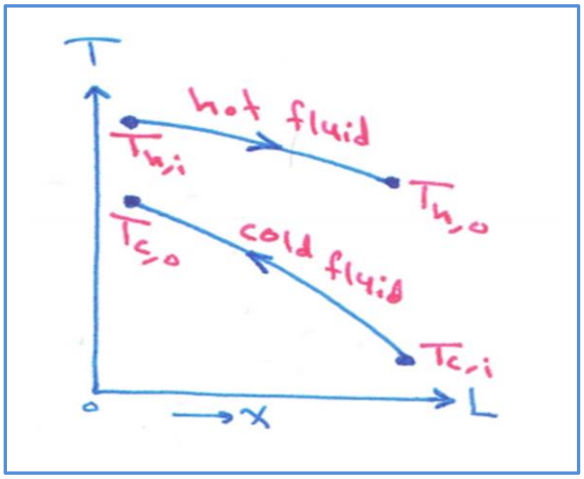


ملاحظة :- المائع الذي يمتلك قيمة ($\dot{m}C_p$) اكبر يكون تغير درجة حرارته أقل والمائع الذي يمتلك ($\dot{m}C_p$) صغير فأن مقدار تغير درجة حرارته كبيرة .

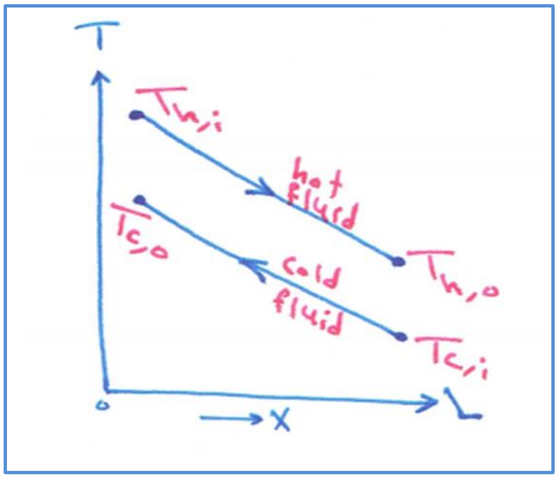
2- Counter Flow Heat Exchanger:-

In this type, both fluids (hot & cold) flow parallel to each other but in opposite direction .

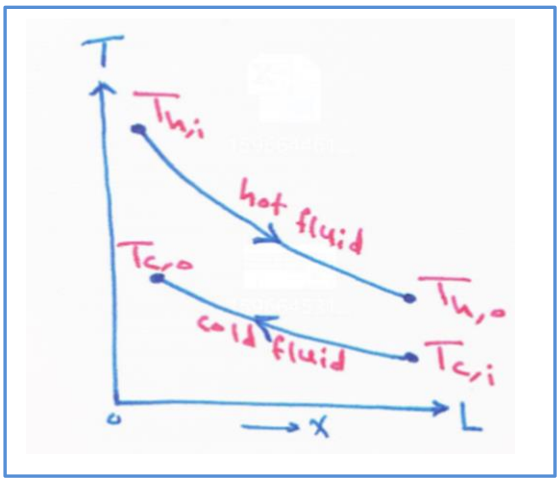
a - $(\dot{m}C_p)h > (\dot{m}C_p)c$



b - $(\dot{m}C_p)h = (\dot{m}C_p)c$



c - $(\dot{m}C_p)h < (\dot{m}C_p)c$

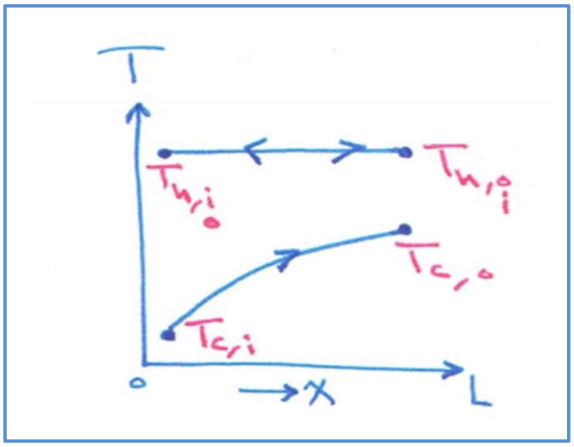


3- Cross Flow Heat Exchanger:-

In this type ,the two fluids flow in directions normal to each other (perpendicular flow) .

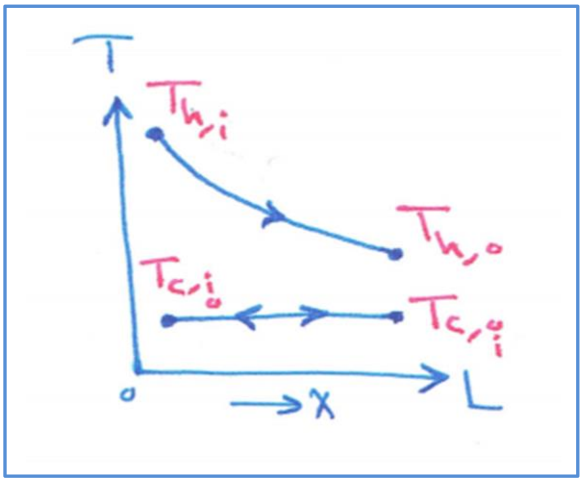
4- Condensing Heat Exchanger:-

In this type ,the hot fluid has a constant temperature and the cold fluid temperature will increases



5- Evaporating Heat Exchanger :-

In this type, the cold fluid has a constant temperature and the hot fluid temperatures will decreases .



Log Mean Temperature Difference

This term (or this parameter) is used in heat exchanger calculations instead of ($\Delta T_{overall}$) because that the temperature difference between the hot and cold fluids is a long the heat exchanger .

$$q = UA\Delta T_{overall} \Longrightarrow q = UA\Delta T_{Lm}$$

$$\therefore LMTD = \Delta T_{Lm} = \frac{\Delta T_I - \Delta T_{II}}{\ln \frac{\Delta T_I}{\Delta T_{II}}}$$

Where :-

ΔT_I= temperature difference between two fluids at the first side of Heat Exchanger (Counter or parallel)

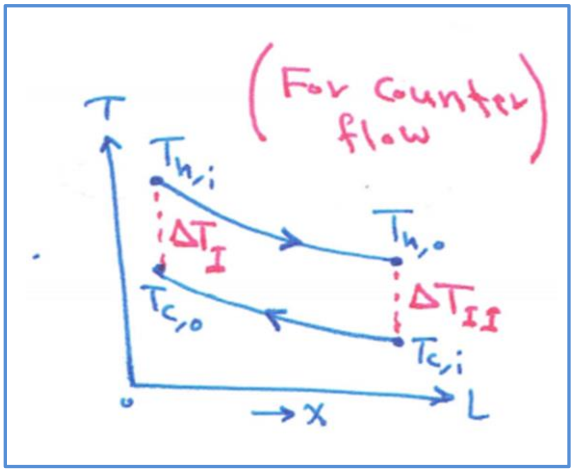
$\therefore \Delta T_I = T_{h,i} - T_{c,i}$ (for parallel flow).

$\therefore \Delta T_I = T_{h,i} - T_{c,o}$ (for counter flow).

ΔT_{II} = temperature difference between fluids at second side of Heat Exchanger (Counter or parallel)

$\therefore \Delta T_{II} = T_{h,o} - T_{c,o}$ (for parallel flow).

$\therefore \Delta T_{II} = T_{h,o} - T_{c,i}$ (for counter flow).

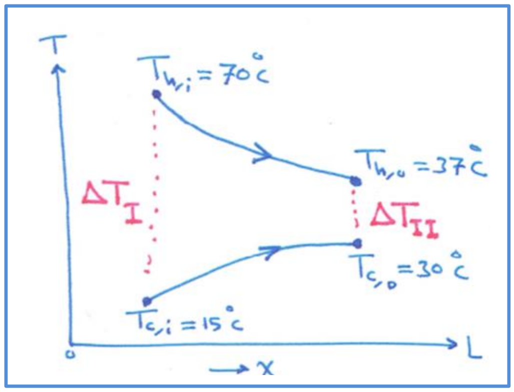


Example: -

In a double pipe heat exchanger water is used as a cooling fluid (where in enters at 15°C and leaves at 30°C) .The oil is used as a hot fluid (where it enters at 70°C and leaves at 37°C) Find the (LMTD) for both

1-Parallel flow   2- Counter flow ?

Solution: -

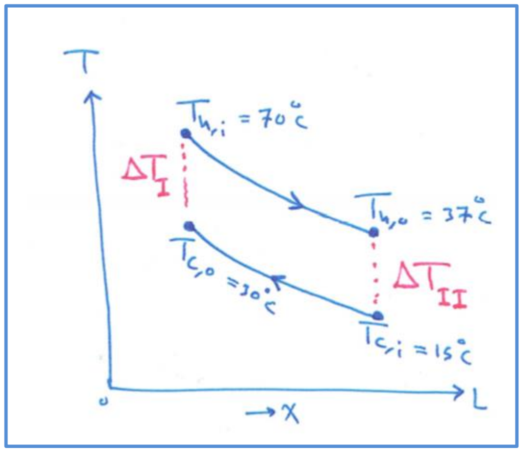


1- for parallel flow

$\Delta T_I = T_{h,i} - T_{c,i} \Longrightarrow\Longrightarrow \Delta T_I = 70 - 15 \Longrightarrow\Longrightarrow \Delta T_I = (55°C)$.

$\Delta T_{II} = T_{h,o} - T_{c,o} \Longrightarrow\Longrightarrow \Delta T_{II} = 37 - 30 \Longrightarrow\Longrightarrow \Delta T_{II} = (7°C)$.

$$\therefore LMTD = \frac{\Delta T_I - \Delta T_{II}}{\ln\frac{\Delta T_I}{\Delta T_{II}}} \Rightarrow LMTD = \frac{55 - 7}{\ln\frac{55}{7}} \Rightarrow LMTD = \frac{48}{2.061} \Rightarrow \therefore LMTD = (23.3°C)$$



2 - for Counter flow

$\Delta T_I = T_{h,i} - T_{c,o} \Longrightarrow\Longrightarrow \Delta T_I = 70 - 30 \Longrightarrow\Longrightarrow \Delta T_I = (40°C)$.

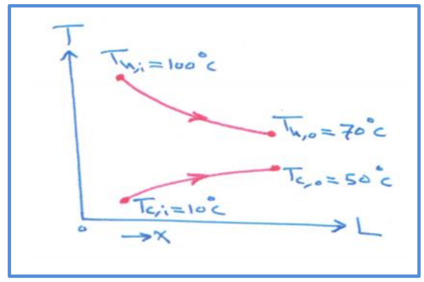
$\Delta T_{II} = T_{h,o} - T_{c,i} \Longrightarrow\Longrightarrow \Delta T_{II} = 37 - 15 \Longrightarrow\Longrightarrow \Delta T_{II} = (22°C)$.

$$\therefore LMTD = \frac{\Delta T_I - \Delta T_{II}}{\ln\frac{\Delta T_I}{\Delta T_{II}}} \Rightarrow LMTD = \frac{40 - 22}{\ln\frac{40}{22}} \Rightarrow LMTD = \frac{18}{0.597} \Rightarrow \therefore LMTD = (30.1°C)$$

Example: -

For the following arrangements in the below figures find the heat transfer rate . Take the overall heat transfer coefficient ($U = 100\ w/m^2.°C$) and the heat transfer area ($A = 1\ m^2$).

Solution: -

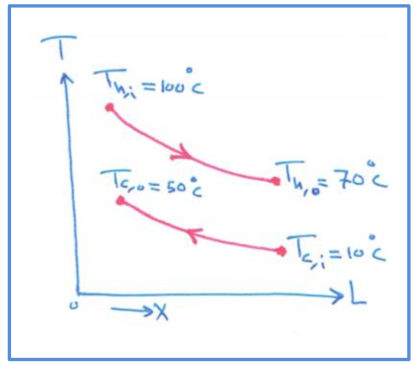


$\Delta T_I = T_{h,i} - T_{c,i} \Longrightarrow \Delta T_I = 100 - 10 \Longrightarrow \Delta T_I = (90°C)$.

$\Delta T_{II} = T_{h,o} - T_{c,o} \Longrightarrow \Delta T_{II} = 70 - 50 \Longrightarrow \Delta T_{II} = (20°C)$.

$$\therefore \Delta T_{Lm} = \frac{\Delta T_I - \Delta T_{II}}{\ln\frac{\Delta T_I}{\Delta T_{II}}} \Rightarrow \Delta T_{Lm} = \frac{90 - 20}{\ln\frac{90}{20}} \Rightarrow \Delta T_{Lm} = \frac{70}{1.504} \Rightarrow \therefore \Delta T_{Lm} = (46.67°C)$$

$q = UA\Delta T_{Lm} \Longrightarrow q = 100 \times 1 \times 46.67 \Longrightarrow q = (4667w)$



$\Delta T_I = T_{h,i} - T_{c,o} \Longrightarrow \Delta T_I = 100 - 50 \Longrightarrow \Delta T_I = (50°C)$.

$\Delta T_{II} = T_{h,o} - T_{c,i} \Longrightarrow \Delta T_{II} = 70 - 10 \Longrightarrow \Delta T_{II} = (60°C)$.

$$\therefore \Delta T_{Lm} = \frac{\Delta T_I - \Delta T_{II}}{\ln\frac{\Delta T_I}{\Delta T_{II}}} \Rightarrow \Delta T_{Lm} = \frac{50 - 60}{\ln\frac{50}{60}} \Rightarrow \Delta T_{Lm} = \frac{-10}{-0.182} \Rightarrow \therefore \Delta T_{Lm} = (54.84°C)$$

$q = UA\Delta T_{Lm} \Longrightarrow q = 100 \times 1 \times 54.84 \Longrightarrow q = (5484w)$

Heat Exchanger Effectiveness (ε)

It is defined as the ratio between the actual heat transfer rate to the possible maximum heat transfer rate .

$$\text{Effectiveness } (\boldsymbol{\varepsilon}) = \frac{\text{actual heat transfer } (q_{act})}{\text{maximum possible heat transfer } (q_{max})}$$

$$\therefore q_{act} = \dot{m}_h C_h \Delta T_h = \dot{m}_h C_h (T_{h,i} - T_{h,o})$$ الحرارة المفقودة من المائع الحار

$$q_{act} = \dot{m}_c C_c \Delta T_c = \dot{m}_c C_c (T_{c,o} - T_{c,i})$$ الحرارة المكتسبة من المائع البارد

حسب قانون حفظ الطاقة ، فأن الحرارة المفقودة من المائع الحار يجب ان تساوي الحرارة المكتسبة من قبل المائع البارد .

$$\therefore \dot{m}_h C_h (T_{h,i} - T_{h,o}) = \dot{m}_c C_c (T_{c,o} - T_{c,i})$$

$$\begin{cases} \text{if } \Delta T_h > \Delta T_c \Rightarrow (\dot{m}_c)_h < (\dot{m}_c)_c \\ \text{if } \Delta T_h < \Delta T_c \Rightarrow (\dot{m}_c)_h > (\dot{m}_c)_c \end{cases}$$

المائع الذي يمتلك (ΔT) اكبر لدية ($\dot{m}_c$) صغير والعكس بالعكس

$$\therefore q_{max} = (\dot{m}_c)_{min} \Delta T_{max}$$

$$\therefore q_{max} = (\dot{m}_c)_{min} (T_{h,i} - T_{c,i})$$

Where :-

$\dot{m}_h$, $\dot{m}_c$ = mass flow rate for hot and cold fluid (kg/sec)

C_h , C_c = Specific heat for hot and cold (J/kg. k)

$(\dot{m}_c)_h$, $(\dot{m}_c)_c$: Heat Capacity rate for hot and cold fluid (w/k)

$$\therefore \varepsilon_h = \frac{(\dot{m}_c)_h (T_{h,i} - T_{h,o})}{(\dot{m}_c)_{min} (T_{h,i} - T_{c,i})}$$

If the hot fluid is the minimum fluid

$$\varepsilon_h = \frac{(T_{h,i} - T_{h,o})}{(T_{h,i} - T_{c,i})}$$

$$\therefore \varepsilon_c = \frac{(\dot{m}_c)_c (T_{c,o} - T_{c,i})}{(\dot{m}_c)_{min} (T_{h,i} - T_{c,i})}$$

If the cold fluid is the minimum fluid

$$\varepsilon_c = \frac{(T_{c,o} - T_{c,i})}{(T_{h,i} - T_{c,i})}$$

In general way ,the effectiveness of heat exchanger is expressed as

$$\varepsilon = \frac{\Delta T_{min\ .\ fluid}}{\Delta T_{max}}$$

Example: -

In a shell and tube heat exchanger , the hot oil is enter at (75°C) and leaves at (50°C) , and the cold water is enter at (25°C) and leaves at (40°C) . take the specific heat for oil and water as (C_{oil} =1900 J/kg . k) , (C_{water}= 4180 J/kg . k) find the ?

1- mass flow rate of water ($\dot{m}_w$) required to cooling of (5 kg/sec) of oil .

2-The effectiveness of the heat exchanger based on the hot and cold fluid (ε_h & ε_c and also (ε)).

3-The heat transfer rate (q) .

Solution:-

By applying the energy balance :-

$$\dot{m}_h C_h \left(T_{h.i} - T_{h,o}\right) = \dot{m}_c C_c \left(T_{c,o} - T_{c,i}\right)$$

$$(5)(1900)\ (75 - 50) = \dot{m}_c\ (4180)\ (40 - 25)$$

$\therefore \dot{m}_c = (3.788\ \ kg/sec)$ (for cold water required)

2- To find the minimum fluid we must find >

$$\dot{m}_h C_h = (5 \times 1900) \Longrightarrow \dot{m}_h C_h = (9500\ \ w/k)$$

$$\dot{m}_c C_c = (3.788 \times 4180) \Longrightarrow \dot{m}_c C_c = (15834\ \ w/k)$$

$\therefore (\dot{m}_h)_h < (\dot{m}_c)_c \Longrightarrow$ the hot fluid is minimum fluid

$$\therefore \varepsilon_h = \frac{(\dot{m}_h)_h\ \Delta T_h}{(\dot{m}_h)_{min}\ \Delta T_{max}} = \frac{(9500)\ (75 - 50)}{(9500)\ (75 - 25)} \Longrightarrow \varepsilon_h = (0.5)$$

$$\therefore \varepsilon_c = \frac{(\dot{m}_c)_c\ \Delta T_c}{(\dot{m}_h)_{min}\ \Delta T_{max}} = \frac{(15834)\ (40 - 25)}{(9500)\ (75 - 25)} \Longrightarrow \varepsilon_c = (0.5)$$

$$\therefore \varepsilon = \frac{\Delta T_{min.\ fluid}}{\Delta T_{max}} = \frac{T_{h,i} - T_{h,o}}{T_{h,i} - T_{c,i}} \Rightarrow \varepsilon = \frac{(75 - 50)}{(75 - 25)} \therefore\Rightarrow \varepsilon = (0.5)$$

3- $\therefore q = \dot{m}_h C_h (T_{h,i} - T_{h,o})$

$q = (5 \times 1900)(75 - 50) \Rightarrow q = (237500\ w)$

or

$q = \dot{m}_c C_c (T_{c,o} - T_{c,i})$

$q = (3.788 \times 4180)(40 - 25) \Rightarrow q = (23750\ w)$

Chapter (5)

Fins

In heat transfer applications ,we use the fines to enhance the heat transfer (due to the increase of heat transfer area) there are several cases may be considered in order to calculating the heat transfer rate if the fins are existent :-

Case 1 :-

The fin is very long and the temperature at the end of fin is equal to the surrounding fluid temperature :-

$$q = \sqrt{h\,p\,k\,A \times}\ (T_o - T_\infty)$$

Case 2 :-

The fin has finite length and loses heat from its end :-

$$q = \sqrt{h\,p\,k\,A}\ (T_o - T_\infty) \times \frac{\sinh\,(mL) + \left(\frac{h}{mk}\right)\cosh\,(mL)}{\cos h\,(mL) + \left(\frac{h}{mk}\right)\sin h(mL)}$$

Case 3 :-

The end of the fin is insulated :-

$$q = \sqrt{h\,p\,k\,A}\ (T_o - T_\infty) \times \tan h\,(mL)$$

Where:-

q = heat transfer rate from the fin (w).

h = heat transfer coefficient $(w/m^2.k)$.

p = perimeter of fin (m).

k = thermal conductivity of fin metal ($w/m.°C$).

A = Cross-sectional area of fin (m^2) .

L = fin length (m).

$m = \text{Constant} \Rightarrow m = \sqrt{\frac{h\,p}{k\,A}}$

T_o= Temperature of fin at the base(°C).

T_∞= Surrounding flu temperature (°C).

$L = L_c = L + \frac{t}{2}$ تستخدم في الحالة (2,3)

• Fin Efficiency (η_f)

It is the ratio between the actual heat transfer rate to the heat transfer rate if the entire fin area are at the base temperature :-

$$\therefore \eta_f = \frac{q_{actual}}{q_{ideal}}$$

Where :-

q_{actual} = احد الحالات الثلاثة للزعانف

$q_{ideal} = h\,p\,L\,(T_o - T_\infty)$

Example:-

An aluminum fin (k = 200 w/m.°C) with (10 cm)long and (7.5cm) width and (5mm thick) is exposed to ambient air (at 50 °C) and h = (10 w/m^2.°C). Calculate the heat loss from the fin if the fin base temperature is (300°C) assume(Case 1& Case 3) ?

Solution :-

$$m = \sqrt{\frac{h\,p}{k\,A}}$$

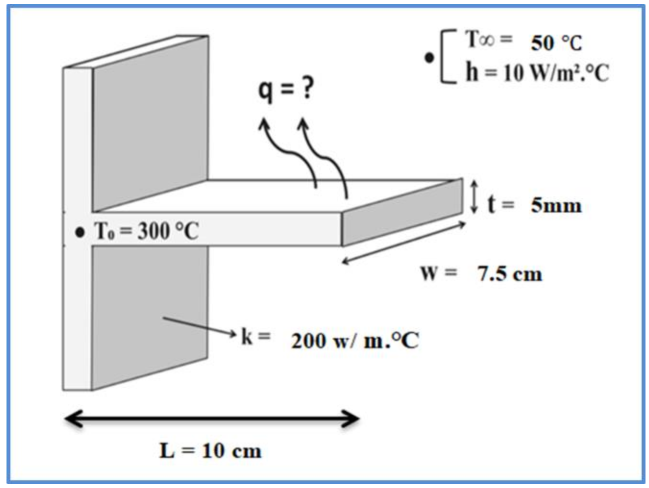
$P = 2w + 2t \Rightarrow p = (2 \times 0.075) + (2 \times 0.005) \Rightarrow \therefore p = (0.16m)$

$A = wt \Longrightarrow A = (0.075 \times 0.005)$

$\therefore A = (0.000375 m^2)$

$$\therefore m = \sqrt{\frac{h\,p}{k\,A}} \Longrightarrow m = \sqrt{\frac{(10)(0.16)}{(200)(0.000375)}}$$

$\therefore m = (4.618\ m^2)$



∴ For Case 1

$$q = \sqrt{h\,p\,k\,A} \times (T_o - T_\infty)$$

$$q = \sqrt{(10)(0.16)(200)(0.000375)} \times (300 - 50)$$

$q = (86.6w)$

∴ For Case 3

$$q = \sqrt{h\,p\,k\,A}\ (T_o - T_\infty) \times \tanh\,(m\,L)$$

$$q = \sqrt{(10)(0.16)(200)(0.000375)}\ (300 - 50) \times \tanh\left[(4.618)(0.1 + \frac{0.005}{2}\right]$$

$$q = (86.6) \times \tanh\left[(4.618)(0.1 + \frac{0.005}{2})\right] \Longrightarrow\Longrightarrow q = (38.1w)$$

Chapter (6)

Heat Transfer by Radiation

It is the transfer of heat by electromagnetic radiations that emitted from a body as a result of its temperature .it don't need a medium to transfer of heat because of the high speed for the waves .

Stefan- Boltzmann Law

The total energy emitted from the radiant body can be calculated by this law :-

$E_b = \sigma T^4$

Where:-

E_b= emissive power from a black body (w/m^2).

T= Surface temperature of a black body (k).

σ = Stefan Boltzmann constant $\left(5.67\times10^{-8}\right) w/m^2 k^4$.

$\therefore q = E A = \sigma A T^4$ $\Longleftarrow\Longleftarrow$(Heat transfer rate for black body)

The ratio of emissive power of a body to the emissive of a black body is called the emissivity (ϵ).

$\epsilon = \frac{E}{E_b} \Rightarrow E = \epsilon E_b$

ϵ = 1 for black body

$q = \epsilon E_b A \Rightarrow q = \epsilon \sigma A T^4$ $\Longleftarrow\Longleftarrow$ (Heat transfer rate for a gray body of real body)

ملاحظة :-

1- إذا ذكر في السؤال جسم اسود (Black body) فلا يذكر لك الانبعاثية (ϵ) لان قيمتها تساوي (1) .

2-إذا كان السؤال حول جسم رمادي أو حقيقي (real or gray body) فيجب أن يذكر لك الانبعاثية (ϵ) .

3- للأجسام السوداء فأن ($\alpha = \epsilon$) الانبعاثية = الامتصاصية .

Absorptivity , Reflectivity , Transmissivity
(α) (ρ) (τ)

When the radiant energy is incident upon any surface part may be absorbed , part may be reflected , and part may be transmitted through the receiving body .

Thus:-

α = part of incident radiation absorbed (absorptivity).

ρ = part of incident radiation reflected (reflectivity).

τ = part of incident radiation transmitted (transmissivity).

$$\alpha + \rho + \tau = (1)$$

The gases generally reflect very little radiant energy ($\rho = 0$)

$$\therefore \alpha + \tau = 1 \text{ (For gases)}$$

The Unitrans parent solids do not transmit radiation ($\tau = 0$)

$$\therefore \alpha + \rho = 1 \text{ (For solids)}$$

Configuration Factor (F)

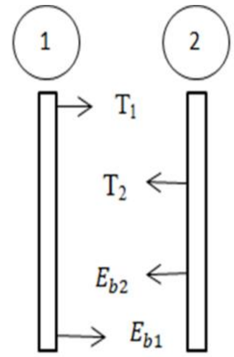
It is defined as the fraction of radiant energy leaving one surface and strikes a second surface directly . there are other names for configuration factor (F) like (shape factor , view factor , angle factor) .

For :-
- Very large surfaces (bodies)
- Infinite surfaces (bodies)
- Tow concentric spheres

} F = 1

Radiation Exchange

Consider the simplest configuration (infinite , parallel) tow black bodies maintained at different temperature (T_1,T_2) as shown in figure the net exchange of radiation energy between surface (1 and 2) is .



$q_{1-2} = \bar{q}_{1-2} - \bar{q}_{2-1} = \alpha_2 E_{b1} A_1 - \alpha_1 E_{b2} A_2$

$\therefore$For black bodies $\Longrightarrow (\alpha_1 = \alpha_2 = 1)$ and $\left(E_{b1} = \sigma T^4 \right)$

1- $q_{1-2} = A_1 \, \sigma \left(T_1^4 - T_2^4 \right)$ ⟵ [Net exchange radiant energy between two black bodies Infinite surfaces in parallel]

or

$q_{1-2}/A = \sigma \left(T_1^4 - T_2^4 \right)$ ⟵ [Net exchange radiant energy per unit area between two black bodies]

Infinite :- $F_{1-2} = F_{2-1} = 1$

For limited surfaces (finite surfaces) of black bodies :-

$q_{1-2} = \bar{q}_{1-2} - \bar{q}_{2-1} = A_1 F_{1-2} E_{b1} - A_2 F_{2-1} E_{b2}$

$= A_1 F_{1-2} \sigma T_1^4 - A_2 F_{2-1} \sigma T_2^4$

According to the reciprocity theorem (حسب نظرية التقابل)

$A_1 F_{1-2} = A_2 F_{2-1}$

2- $q_{1-2} = A_1 F_{1-2} \sigma \left(T_1^4 - T_2^4 \right)$ ⟵ [Net radiant exchange for finite black bodies in parallel]

$= - q_{2-1} = A_2 F_{2-1} \sigma \left(T_2^4 - T_1^4 \right)$

3- For gray bodies :-

$$q_{1-2}/A = \frac{\sigma (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$ ⟵ [The net radiant exchange for finite parallel two gray bodies per unit area]

4- $$q_{1-2} = \frac{\sigma (T_1^4 - T_2^4)}{\frac{1-\epsilon_1}{\epsilon_1 A_1} + \frac{1}{A_1 F_{1-2}} + \frac{1-\epsilon_2}{\epsilon_2 A_2}}$$ ⟵ [The net radiant exchange for finite parallel two gray bodies]

Example:-

A temperature of black body is constant and equal to (350k) . Calculate the amount of radiant heat per unit area ?

Solution :-

$q = \sigma A T^4$ $\qquad E_b = \sigma T^4$

$$\therefore {}^{q}/_{A} = (5.67\times10^{-8})\ (350)^4 \Rightarrow {}^{q}/_{A} = (850.84\ \ w/m^2)$$

Example:-

Determine the total emissive power of a body at (1000 °C) if it assumed

1- Black body .
2- Gray body .with ($\epsilon = 0.8$)

Solution :-

1- For black body :- $E_b = \sigma T^4 \Rightarrow E_b = (5.67\times10^{-8})(1273)^4$ $\quad \therefore \Rightarrow E_b = (148900.6\ \ w/m^2)$
2- For gray body :- $E = \epsilon E_b \Rightarrow (0.8)(148900.6)$ $\quad \therefore\Rightarrow E = (119120.48\ \ w/m^2)$

Example:-

For a ((2m × 2m) wall maintained at (200°C) find the

1- Emissive power of wall.
2- Amount of radiant heat from the wall (Tack the emissivity of wall equal to 0.85) .

Solution :-

1- $E = \in E_b \Longrightarrow E = \in \sigma T^4 \Longrightarrow E = (0.85)\left(5.67\times10^{-8}\right)(473)^4 \Longrightarrow \therefore E = (2412.38\ w/m^2)$

2- $q = \in E_b A \Longrightarrow q = \in E_b A \Longrightarrow q = \in \sigma A T^4 \Longrightarrow \therefore q = (0.85)(5.67\times10^{-8})(4)(473)^4$

$\therefore q = (9649.52\ w)$

Example:-

Two infinite parallel walls maintained at (1000 k and 800 k) assuming that both the walls have (3m×3m) area Calculate the net radiant exchange between the walls if it considered as :-

1- Black bodies .
2- Gray bodies with ($\in_1 = 0.9$, $\in_2 = 0.8$)

Solution :-

1- Two infinite and black bodies ($F_{1-2} = F_{2-1} = 1$) , ($\in_1 = 1, \in_2 = 1$)

$\therefore q_{1-2} = \sigma A\left(T_1^4 - T_2^4\right) \Longrightarrow q_{1-2} = (5.67\times10^{-8})(3\times3)\left[(1000)^4 - (800)^4\right]$

q = (301281.2 w).

2- Two infinite and gray bodies (($F_{1-2} = F_{2-1} = 1$) , ($\in = 0.9$ $\in_2 = 0.8$)

$$\therefore q_{1-2} = \frac{\sigma A\,(T_1^4 - T_2^4)}{\frac{1}{\in_1} + \frac{1}{\in_2} - 1}$$

$$q = \frac{(5.67\times10^{-8})(3\times3)[(1000)^4 - (800)^4]}{\frac{1}{0.9}+\frac{1}{0.8}-1} \Rightarrow q = (221349.6w)$$

Example :-

Two black concentric hollow spherical walls are maintained at (300°C)for outer wall and (500°C) for inner wall if the inner and outer radiuses are (3 & 5 m) respectively , find the net radiant heat exchanged between inner and outer walls .

Solution:-

Black walls and concentric .

$(F_{1-0} = F_{0-1} = 1)$, $(\epsilon_i = \epsilon_o = 1)$

$T_i = 500 + 273 = 773k$

$T_o = 300+273=573k$

$q_{i-o} = A_i \sigma (T_i^4 - T_o^4)$

$= (4\pi r_i^2)(5.67 \times 10^{-8})[(773)^4 - (573)^4]$

$\therefore q = (4 \times \pi \times (3)^2)(5.67 \times 10^{-8})[(773)^4 - (573)^4] \Rightarrow q = (1597477.2\ w)$

تم بحمد الله