

# Electric Circuits

## Part one

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### Reference:

- 1- Charles K. Alexander, Mathew N. O. Sadiku "Fundamental of electric circuit", 3rd.
- 2- Road M. Rasheed, "Lectures electric circuit", Part1.

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# Chapter One

## Basics of electrical circuits

### 1.1 Electrical Quantities and Units:

**Table (1.1): Electrical quantities and units**

Electrical Quantity	Symbol	Measuring Unit	Symbol
Voltage	V or E	Volt	v
Current	I or i	Ampere	A
Resistance	R	Ohm	$\Omega$
Conductance	G	Siemen	S or $\mathcal{O}$
Capacitance	C	Farad	F
Inductance	L	Henry	H
Power	P	Watts	w
Impedance	Z	Ohm	$\Omega$
Frequency	f	Hertz	Hz

### 1.2 Multiple and Submultiple of the Internarial System Units (SI):

Here is a huge range of values encountered in electrical and electronic engineering between a maximum value and a minimum value of a standard electrical unit. For example, resistance can be lower than  $0.001\Omega$  or higher than  $1,000,000\Omega$ . By using multiples and submultiples of the standard unit we can avoid having to write too many zeros to define the position of the decimal point. The table (1.2) shows the Multiple and Submultiple Internarial System Units (SI).

**Table (1.2) Internarial System Units (SI)**

Prefix	Symbol	Multiplier and Submultiple	Power of Ten
kilo	k	1,000	$10^3$
Mega	M	1,000,000	$10^6$
Giga	G	1,000,000,000	$10^9$
Terra	T	1,000,000,000,000	$10^{12}$
centi	c	0.01	$10^{-2}$
milli	m	0.001	$10^{-3}$
micro	$\mu$	0.000001	$10^{-6}$
nano	n	0.000000001	$10^{-9}$
pico	p	0.000000000001	$10^{-12}$

**Example1.1:** Show multiples or submultiple of Internarial System units for each one of the following:

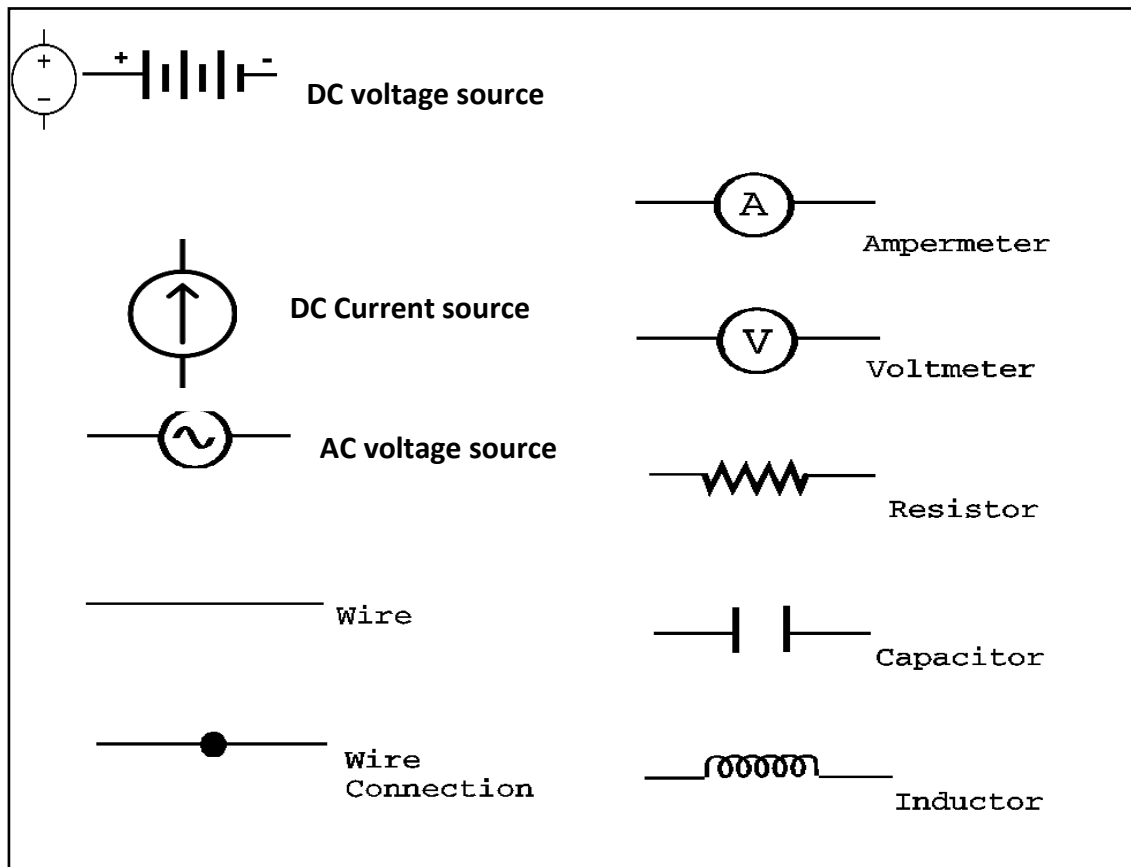
- $V = 1\text{kv} = 1 \cdot 10^3 = 1 \cdot 1000 = 1,000 \text{ v.}$
- $I = 3\text{mA} = 3 \cdot 10^{-3} = 3 \cdot 0.001 = 0.003\text{A.}$
- $A = 5\text{mm}^2 = 5 \cdot (10^{-3})^2 = 5 \cdot 10^{-6} = 0.000005\text{m}^2$

## Homework

**H.W1.1:** Show multiples or submultiple of Internarial System units for each one of the following:

- $C = 100\mu\text{F}$
- $P = 1\text{pw}$
- $F = 1\text{MHz}$
- $V = 7\text{cm}^3$

## 1.3 Electrical Circuit Components



## 1.4 Ohm's law

Ohm's law states that the voltage  $v$  across a resistor is directly proportional to the current  $i$  flowing through the resistor.

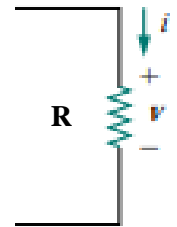
$$V \propto I$$

$$V = R I$$

V: voltage (volt)

I: current (Ampere)

R: Resistance (ohm)



### Ohm's Law Triangle

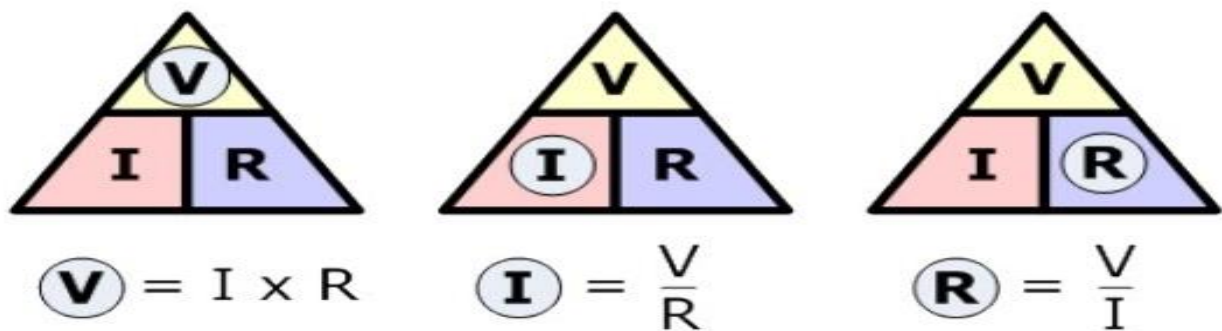
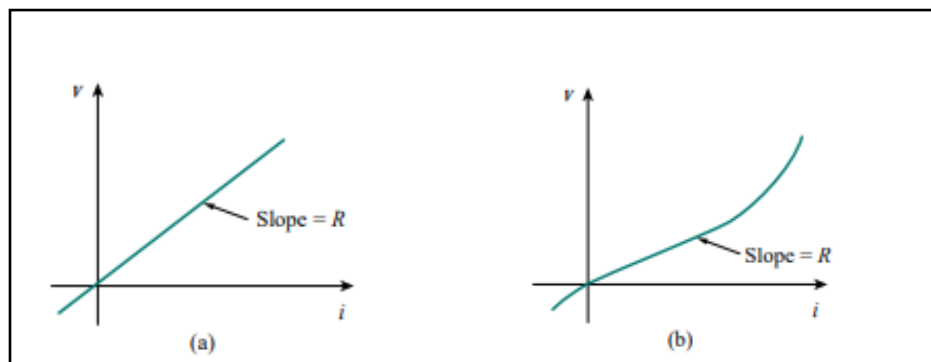


Figure (1.1) shows the relation between the voltage and current of resistance which there are two types of resistances:

- 1- linear resistance (ohm's law)
- 2- non-linear resistance.



**Figure (1.1): The  $i$ - $v$  characteristic of: (a) a linear resistor, (b) a nonlinear resistor.**

-Most of conductors are linear resistance so can be used the ohm's law while in non-linear resistance cannot use ohm's law such as semi-conductor.

-The inverse of the resistance is often useful. It is called conductance and quantity symbol is G. The unit of conductance is Siemen (S) or (v).

$$G = \frac{1}{R}$$

### **1.5 Electrical Power:**

Power (P) in the electrical circuit is given by product the voltage (V) and the current (I). The unit of the power is watt (w).

$$P = V * I$$

**Example1.2:** The current flowing through a resistance is 0.8 A. when the voltage source 20v is applied. Find

- 1) The resistance
- 2) the conductance
- 3) The total power

**Solution:**

$$1) R = \frac{v}{I} = \frac{20}{0.8} = 25\Omega$$

$$2) G = \frac{1}{R} = \frac{1}{25} = 0.04 S$$

$$3) P = V * I = 20 * 0.8 = 16w$$

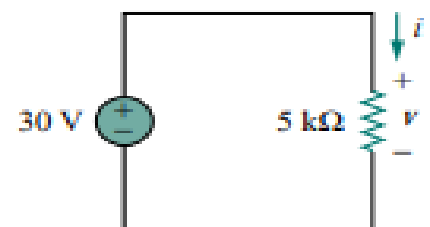
**Example1.3:** In the circuit shown in figure (1.2), calculate the current, the conductance, and the power.

**Solution:**

$$\text{the current is } i = \frac{V}{R} = \frac{30}{5 * 10^3} = 6 \text{ mA}$$

$$\text{The conductance is } G = \frac{1}{R} = \frac{1}{5 * 10^3} = 0.0002 S = 0.2 \text{ mS}$$

$$P = V * I = 30 * (6 \times 10^{-3}) = 180 \text{ mW.}$$



**Figure (1.2)**

**H.W 1.2:** For the circuit shown in Figure (1.3), calculate the voltage  $v$ , the conductance  $G$ , and the power  $p$ . (Answer: 20 V, 100  $\mu\text{S}$ , 40 mW)

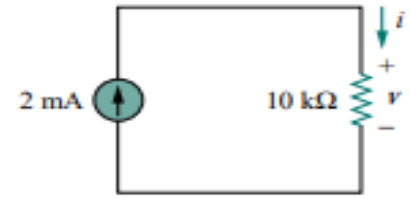


Figure (1.3)

## 1.6 Resistor Power Absorption:

There are two equations:

$$V = I * R \dots\dots\dots (1)$$

$$P = V * I \dots\dots\dots (2)$$

When substituting Eq. (1) in Eq. (2)

$$P = I * R * I = I^2 * R$$

Or

$$P = v * \frac{v}{R} = \frac{v^2}{R}$$

The resistor power absorption is

$$P = I^2 * R \quad \text{Or} \quad P = \frac{v^2}{R}$$

## 1.7 Resistance and Resistivity

The resistance of the conductor of uniform cross section is depend on:

- 1) The type of material (Resistivity).
- 2) Length
- 3) Cross section area
- 4) The temperature.

Hence:

$$\text{At } (20\text{C}^0) \quad R = \rho \frac{l}{A}$$

R: Resistance ( $\Omega$ )

$\rho$ : Resistivity ( $\Omega \cdot \text{m}$ )

$l$ : length (m)

A: cross section Area ( $\text{m}^2$ )

- Resistivity is a fundamental property of material, when the value of the resistivity is very small that means the material is good conductor (conductor material), but when the value of resistivity is very big that means the material is low conductor (isolator material). Table (1.3) represents the resistivity of different materials.

**Table (1.3) represents the resistivity of materials.**

Resistivities at 20°C	
Material	Resistivity ( $\Omega \cdot m$ )
Aluminum	$2.82 \times 10^{-8}$
Copper	$1.72 \times 10^{-8}$
Gold	$2.44 \times 10^{-8}$
Nichrome	$150. \times 10^{-8}$
Silver	$1.59 \times 10^{-8}$
Tungsten	$5.60 \times 10^{-8}$

**Example 1.4:** Find the resistance of 20 C<sup>0</sup> of annealed copper bar 3mm in length and 1.5cm<sup>2</sup> in rectangular cross section area. ( Resistivity =  $1.72 \times 10^{-8} \Omega \cdot m$ )

**Solution:**

$$R = \rho \frac{l}{A}$$

$$R = 1.72 \times 10^{-8} \frac{3 \times 10^{-3}}{1.5 \times (10^{-2})^2}$$

$$R = 3.44 \times 10^{-7} \Omega$$

### Homework

**H.W1.3:** Find the cross-section area of the aluminum wire at 20C<sup>0</sup> that has length of 1Km and the resistance is 0.002  $\Omega$ . ( Resistivity =  $2.82 \times 10^{-8} \Omega \cdot m$ ). (answer: **A=0.0141m<sup>2</sup>**)

### **1.8 Resistor temperature coefficient:**

The resistance of most good conducting material increases almost linearly with temperature over range of normal operating temperature.

$$R_2 = R_1(1 + \alpha_0(T_2 - T_1))$$

$$R_0 = R_T(1 + \alpha_T T)$$

$$R_T = R_0(1 + \alpha_0 T)$$

$$\alpha_T = \frac{\alpha_0}{1 + \alpha_0 * T}$$

$R_1$ : resistance at lower temperature  $T_1$

$R_2$ : resistance at higher temperature  $T_2$

$R_T$ : resistance at temperature  $T$

$R_0$ : resistance at zero temperature 0C<sup>0</sup>

$\alpha_0$ : resistor temperature coefficient at 0C<sup>0</sup>

$\alpha_T$ : resistor temperature coefficient at TC<sup>0</sup> ( $\Omega/C^0$ )



**Example1.5:** coil resistance is  $3.146\ \Omega$  at  $40^\circ\text{C}$  and  $3.717$  at  $100^\circ\text{C}$ . Find

- 1) Resistor temperature coefficient at  $0^\circ\text{C}$ .
- 2) Coil resistance at  $0^\circ\text{C}$ .
- 3) Resistance temperature coefficient at  $40^\circ\text{C}$ .

**Solution:**

$$\begin{aligned}(1) \quad R_2 &= R_1(1 + \alpha_0(T_2 - T_1)) \\ 3.717 &= 3.146(1 + \alpha_0(100 - 40)) \\ 3.717 &= 3.146(1 + \alpha_0 * 60) \\ 3.717 &= ((3.146 * 1) + (3.146\alpha_0 * 60)) \\ 3.717 &= 3.146 + 188.76\alpha_0 \\ 3.717 - 3.146 &= 188.76\alpha_0 \\ 0.571 &= 188.76\alpha_0 \\ \alpha_0 &= \frac{0.571}{188.76} = 0.003\ \Omega/^\circ\text{C}\end{aligned}$$

$$\begin{aligned}(2) \quad R_T &= R_0(1 + \alpha_0 T) \\ 3.146 &= R_0(1 + 0.003 * 40) \\ 3.146 &= 1.12 R_0 \\ R_0 &= \frac{3.146}{1.12} = 2.825\ \Omega\end{aligned}$$

$$\begin{aligned}(3) \quad \alpha_T &= \frac{\alpha_0}{1 + \alpha_0 * T} \\ \alpha_{40} &= \frac{0.003}{1 + 0.003 * 40} = 0.0026\ \Omega/^\circ\text{C}\end{aligned}$$

### Homework

**H.W1.4:** Copper coil resistance is  $100\ \Omega$  at  $20^\circ\text{C}$ . Calculate it is resistance at  $100^\circ\text{C}$ . If the resistance temperature coefficient at  $20^\circ\text{C}$  is  $0.004\ \Omega/^\circ\text{C}$ . (answer:  $R_2 = 129.6\ \Omega$ )

## 1.9 Branches, nodes and loops:

- A branch represents a single element such as a voltage source or a resistor.
- A node is the point of connection between two or more branches.
- A loop is any closed path in a circuit.
- Independent loops (Mesh) is a special case of loop do not have any other loops within it.

**Example1.6:** In the circuit shown in Figure (1.4). Determine:

- the number of branches
- the number of nodes
- the number of independent loop (Mesh).

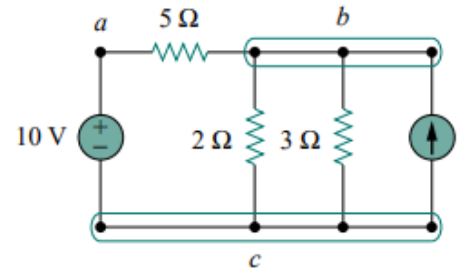
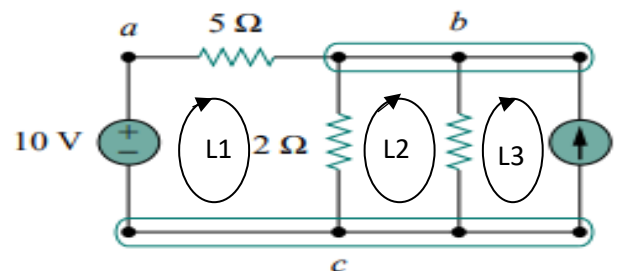


Figure (1.4)

Solution:

- the number of branches (b)=5
- the number of nodes= 3
- the number of loop (Mesh)=3



للتأكيد:

$$b = l + n - 1$$

$$b = 3 + 3 - 1 = 5$$

## Homework

**H.W1.5:** In the circuit shown in Figure (1.5). Determine:

- the number of branches
- the number of nodes
- the number of independent loop (Mesh).

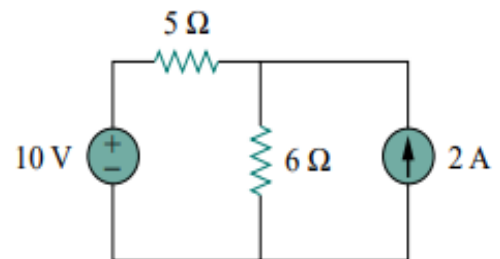
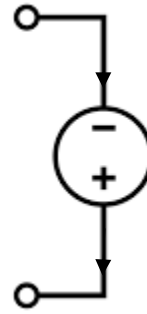
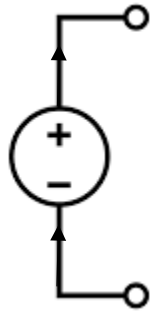


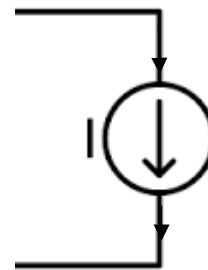
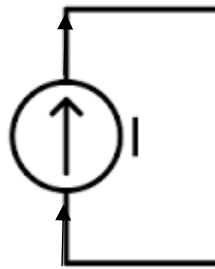
Figure (1.5)

## 1.10 The direction of the current:

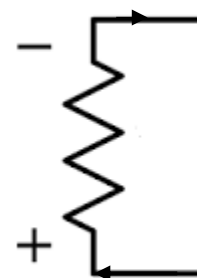
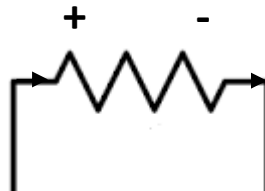
### 1) Voltage source



### 2) Current source



### 3) Resistance



## 1.11 Short circuit and open circuit:

A short circuit is a circuit element with resistance approaching zero.

An element with  $R = 0$  is called a short circuit, as shown in Figure (1.6a).

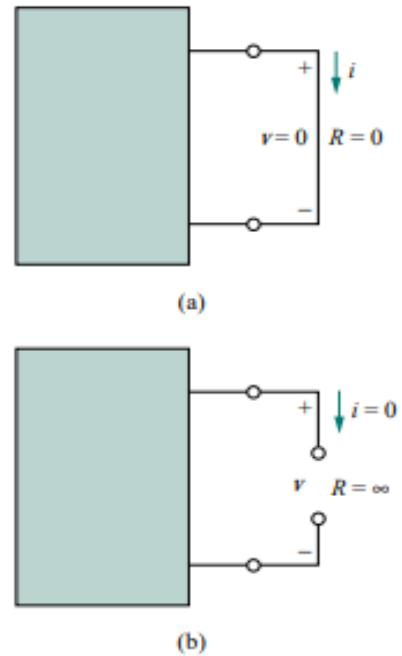
For a short circuit,

$$v = I R = 0$$

An open circuit is a circuit element with resistance approaching infinity.

An element with  $R = \infty$  is known as an open circuit, as shown in Figure (1.6 b).

$$i = \lim_{R \rightarrow \infty} \frac{v}{R} = 0$$



**Figure (1.6): (a) Short circuit ( $R = 0$ ),  
(b) Open circuit ( $R = \infty$ ).**

## 1.12 Kirchhoff's laws:

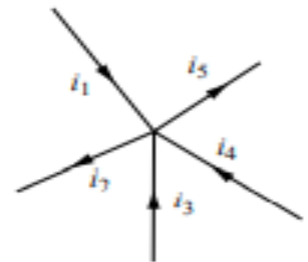
### 1) Kirchhoff's current law

Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node is zero. using KCL for the circuit as shown in figure (1.7) will be given

$$\sum I_{\text{entering and leaving}} = 0 \quad \text{at node}$$

$$i_1 - i_2 + i_3 + i_4 - i_5 = 0$$

التيارات الداخلة اشارتها موجبة  
التيارات الخارج اشارتها سالبة



**Figure (1.7)**

Or the he sums of the currents entering a node is equal to the sum of the currents leaving the node

$$\sum I_{\text{entering}} = \sum I_{\text{leaving}} \quad \text{at node}$$

$$i_1 + i_3 + i_4 = i_2 + i_5$$

2) Kirchhoff's voltage law (KVL):

Kirchhoff's voltage law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero.

For the circuit as shown figure (1.8) using KVL of the loop (Clockwise direction) will be given:

$$\sum V = 0 \quad \text{in loop (Mesh)}$$

$$-V_1 + V_2 + V_3 = 0$$

$$-V_1 + I \cdot R_1 + I \cdot R_2 = 0$$

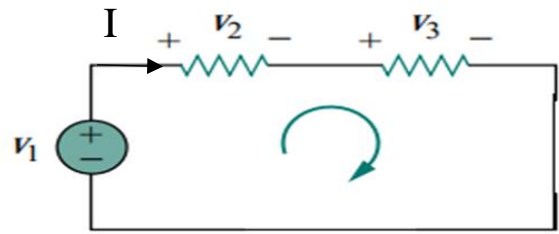


Figure (1.8)

**Example 1.7:** For the circuit as shown figure (1.9) find current  $I_0$  by using KCL.

**Solution:**

$$2 - 10 + 4 - I_0 = 0$$

$$I_0 = -4A$$

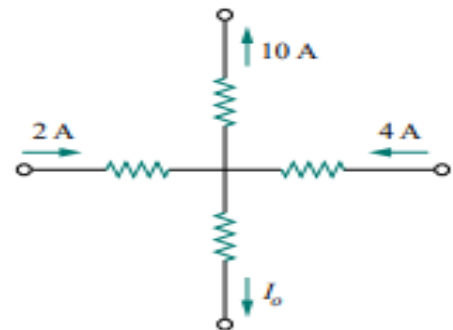


Figure (1.9)

**Example 1.8:** For the circuit as shown figure (1.10) find  $V$  by using KVL.

**Solution:**

loop CW direction

$$-12 + 10 + 8 - V = 0$$

$$V = 6V$$

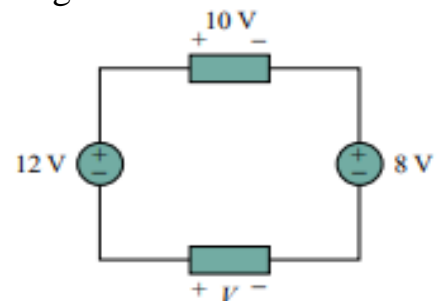


Figure (1.10)

## Homework

**H.W1.6:** For the circuit as shown figure (1.11) find  $V$  by using KVL. (answer:  $V_1=14\text{v}$ )

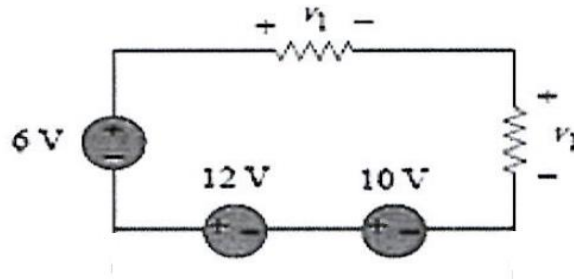


Figure (1.11)

**H.W1.7:** For the circuit as shown figure (1.12) find  $i_1$  and  $i_2$  by using KCL. (answer:  $i_1=7\text{A}$ ,  $i_2=-5$ )

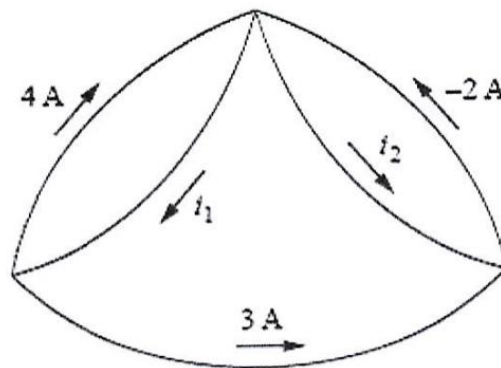


Figure (1.12)

## Chapter 2

### Series and Parallel Circuits

#### 2.1 Series Circuit

As shown in figure (2.1), the two resistors are in series which has characteristic as following:

1) The  $R_1$  and  $R_2$  have the same current and equal  $i$

2)

$$V_1 = i * R_1$$

$$V_2 = i * R_2$$

3) By using KVL to loop (Clockwise direction) we have

$$-V + V_1 + V_2 = 0$$

$$V = V_1 + V_2$$

4) The equivalent resistance:

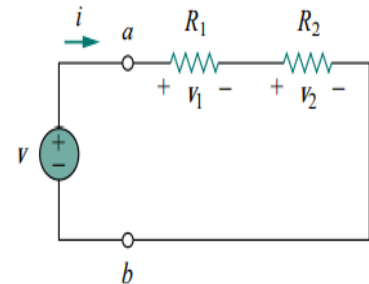
$$R_{eq} = R_1 + R_2$$

For N resistor in Series

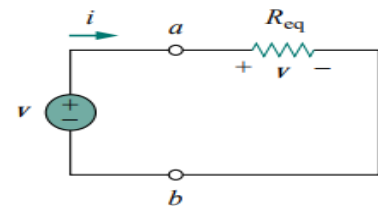
$$R_{eq} = R_1 + R_2 + R_3 + R_4 + \dots + R_N$$

5)

$$V = R_{eq} * i \text{ as shown in figure (2.2).}$$



**Figure (2.1)**



**Figure (2.2): Equivalent circuit**

#### 2.1.1 Voltage divider's law:

$$V_1 = I * R_1 \text{ -----(1)}$$

$$V = R_{eq} * i \text{ -----(2)}$$

$$R_{eq} = R_1 + R_2 \text{ -----(3)}$$

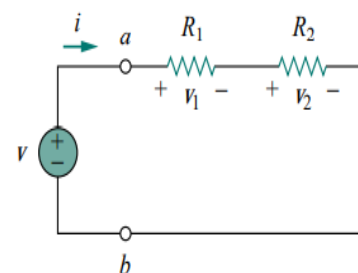
Substituting Eq. (2) and Eq. (3) in Eq. (1)

$$V_1 = \frac{V}{R_1 + R_2} * R_1$$

$$V_2 = \frac{V}{R_1 + R_2} * R_2$$

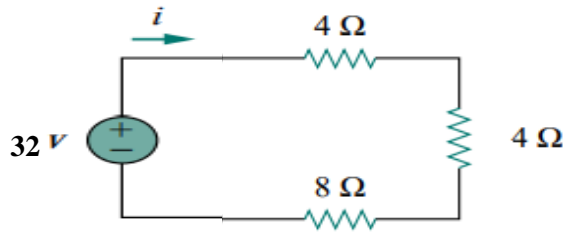
$$V_N = \frac{V}{R_{eq}} * R_N$$

N: represent the resistance



**Example2.1:** From the circuit shown in figure (2.3), find

- 1) The equivalent resistance.
- 2) the current through each resistance
- 3) the voltage of  $8\Omega$  using voltage divider's law.



**Figure (2.3)**

**Solution:**

- 1)  $R_{eq} = R_1 + R_2 + R_3$   
 $R_{eq} = 4 + 4 + 8 = 16\ \Omega$
- 2)  $V = R_{eq} * i$   
 $32 = 16 * i$   
 $i = \frac{32}{16}$   
 $i = 2A$  which  $i = I_{4\Omega} = I_{4\Omega} = I_{8\Omega}$  (Series Connection)
- 3)  $V_N = \frac{V}{R_{eq}} * R_N$   
 $V_{8\Omega} = \frac{32}{4 + 4 + 8} * 8 = 16v$

**Example2.2:** For the circuit shown in figure (2.4), find the voltage source v.

**Solution:**

$$V = R_{eq} * i$$

$$R_{eq} = R_1 + R_2 + R_3$$

$$R_{eq} = 3 + 4 + 5 = 12\ \Omega$$

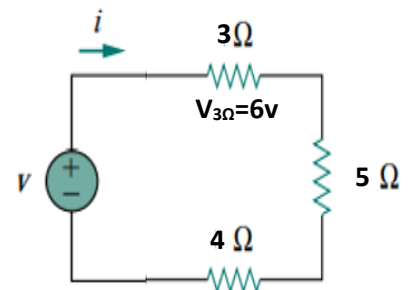
$$V_{3\Omega} = i * R_{3\Omega}$$

$$6 = i * 3$$

$$i = 2A$$

$$V = R_{eq} * i$$

$$V = 12 * 2 = 24v$$



**Figure (2.4)**



## Homework

**H.W2.1:** As shown in figure (2.5), calculate the value of resistance  $R_1$  and the voltages of each resistances **using voltage divider's law**. (answer:  $R_1=11\ \Omega$ ,  $V_1=30\text{v}$ ,  $V_2=15\text{v}$ ,  $v=55\text{v}$ )

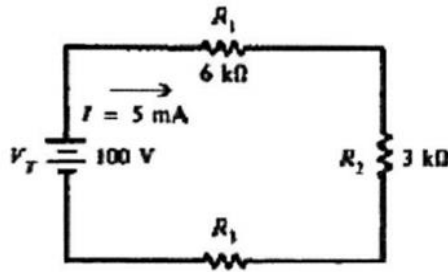


Figure (2.5)

## 2.2 Parallel circuit

As shown in figure (2.6), the two resistors are in parallel which has characteristic as following:

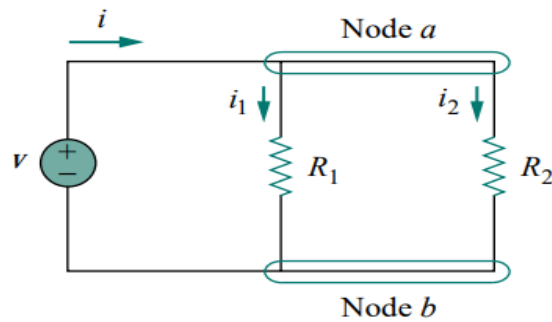


Figure (2.6)

- 1) The  $R_1$  and  $R_2$  have the same voltage and equal  $V$  ( $V$  = Voltage source).

$$V = V_{R1} = V_{R2}$$

$$2) \begin{aligned} i_1 &= \frac{V}{R_1} \\ i_2 &= \frac{V}{R_2} \end{aligned}$$

- 3) By using KCL at node a (Clockwise direction) we have

$$i = i_1 + i_2$$

- 4) The equivalent resistance of two resistance:

$$\begin{aligned} \frac{1}{R_{eq}} &= \frac{1}{R_1} + \frac{1}{R_2} \\ \frac{1}{R_{eq}} &= \frac{R_2 + R_1}{R_1 R_2} \end{aligned}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

For N resistor in parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \dots \dots \dots \frac{1}{R_N}$$

- 5)  $V = R_{eq} * i$  as shown in figure (2.7).

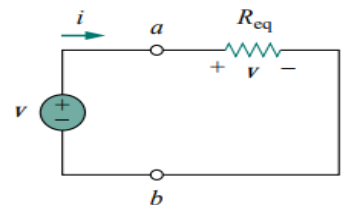


Figure (2.7): Equivalent circuit

### 2.2.1 Current divider's law:

As shown in figure (2.6)

$$V_1 = I * R_1 \text{ -----(1)}$$

$$V = R_{eq} * i \text{ -----(2)}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \text{ -----(3)}$$

Substituting Eq. (2) and Eq. (3) in Eq. (1)

$$i_1 = \frac{i}{R_1 + R_2} * R_2$$
$$i_2 = \frac{i}{R_1 + R_2} * R_1$$

**Example 2.3:** for the circuit in figure (2.8), find

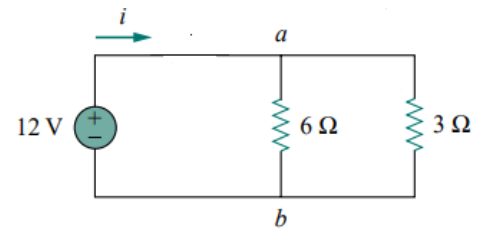
- 1) The equivalent resistance
- 2) Total current
- 3) The current of each resistance

**Solution:**

$$1) R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$
$$R_{eq} = \frac{3 * 6}{3 + 6}$$
$$R_{eq} = \frac{18}{9} = 2\Omega$$

$$2) V = R_{eq} * i$$
$$12 = 2 * i$$
$$i = 6A$$

$$3) i_1 = \frac{V}{R_1}$$
$$i_1 = \frac{12}{6} = 2A$$
$$i_2 = \frac{V}{R_2}$$
$$i_2 = \frac{12}{3} = 4A$$



**Figure (2.8)**

**Example 2.4:** Find  $V$  (voltage source) of the circuit shown in figure (2.9), if the  $i_3=0.2A$ , then find total current  $I$  and the equivalent resistance.

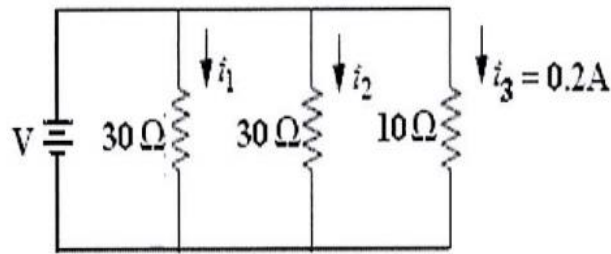


Figure (2.9)

**Solution:**

$$i_3 = \frac{V}{R_3}$$

$$0.2 = \frac{V}{10}$$

$$V = 0.2 \times 10 = 2V$$

$$i_1 = \frac{V}{R_1}$$

$$i_1 = \frac{2}{30} = 0.067 A$$

$$i_2 = \frac{V}{R_2}$$

$$i_2 = \frac{2}{30} = 0.067 A$$

$$i = i_1 + i_2 + i_3$$

- total current

$$i = 0.067 + 0.067 + 0.2 = 0.334 A$$

- equivalent resistance

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_{eq}} = \frac{1}{30} + \frac{1}{30} + \frac{1}{10}$$

$$\frac{1}{R_{eq}} = \frac{1}{30} + \frac{1}{30} + \frac{2}{30}$$

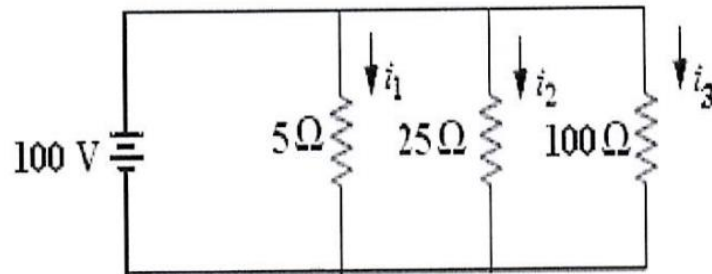
$$\frac{1}{R_{eq}} = \frac{4}{30} = \frac{2}{15}$$

$$R_{eq} = 7.5 \Omega$$

## Homework

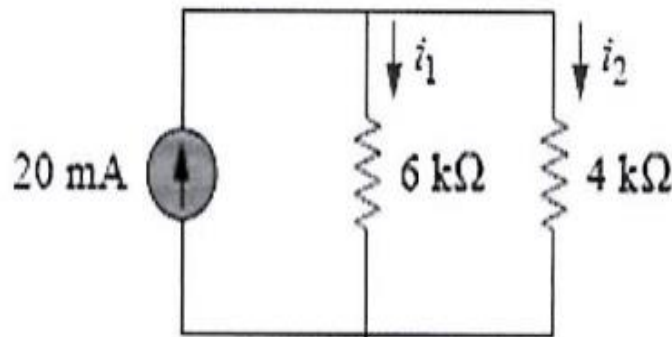
**H.W 2.2:** for the circuit shown in figure (2.10), calculate:

- 1) equivalent resistance
- 2) currents of each resistance ( $i_1$ ,  $i_2$ , and  $i_3$ )
- 3) total current



**Figure (2.10)**

**H.W 2.3:** for the circuit shown in figure (2.11), calculate the currents of each resistance ( $i_1$ ,  $i_2$ ) using current divider's law.



**Figure (2.11)**

## Chapter three

### Series-Parallel combination

In the Series -parallel combination circuit, the circuit components are series connected in same part and parallel in other part. We won't be able to apply a single set of the rules to every of the circuit. Instead, we will have to identify which parts of the circuit are series and which parts are parallel, then selectivity applies series and parallel rules as necessary to determine what is happening.

**Example3.1:** For the circuit shown in figure (3.1), determine:

- 1) The total current
- 2) The voltage and currents of each resistance

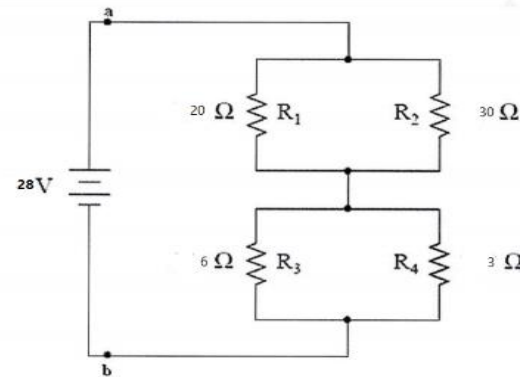
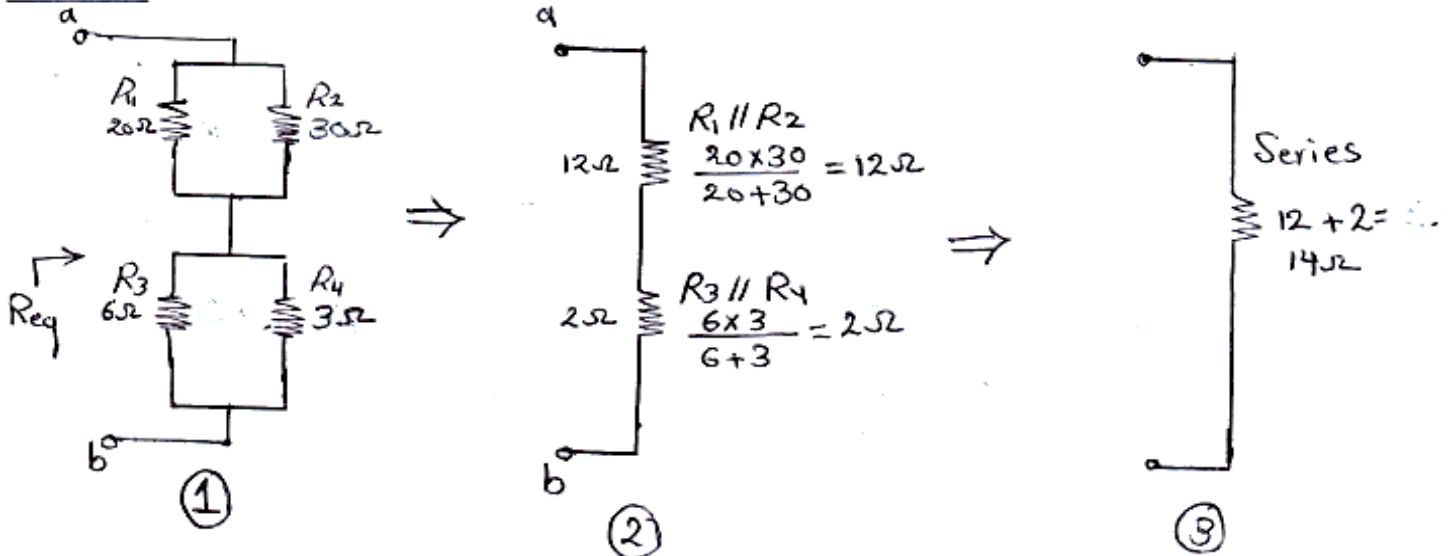


Figure (3.1)

**Solution: 1)**



$$\therefore R_{eq} = 14\Omega$$

$$I = \frac{V}{R_{eq}} = \frac{28}{14} = 2A$$

2) Current-divider's law:-

$$I_1 = \frac{2}{20+30} * 30 = 1$$

$$I_2 = \frac{2}{20+30} * 20 = 0.8A$$

$$V_1 = I_1 * R_1 = 1.2 * 20 = 24V$$

$$V_2 = I_2 * R_2 = 0.8 * 30 = 24V$$

$$V_1 = V_2 = 24V \text{ [because } R_1 \text{ and } R_2 \text{ is parallel]}$$

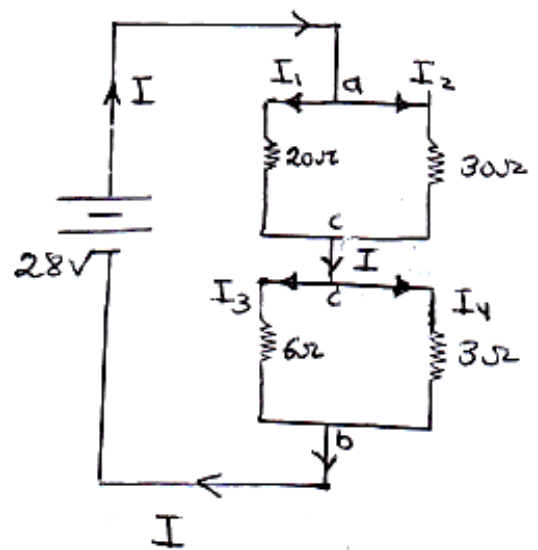
$$I_3 = \frac{2}{6+3} * 3 = 0.667A$$

$$I_4 = \frac{2}{6+3} * 6 = 1.333$$

$$\therefore V_3 = I_3 * R_3 = 0.667 * 6 = 4V$$

$$V_4 = I_4 * R_4 = 1.333 * 3 = 4V$$

$$V_3 = V_4 = 4V \text{ [because } R_3 \text{ and } R_4 \text{ is parallel]}$$



**Example 3.2:** Find the equivalent resistance of the circuit shown in figure (3.2).

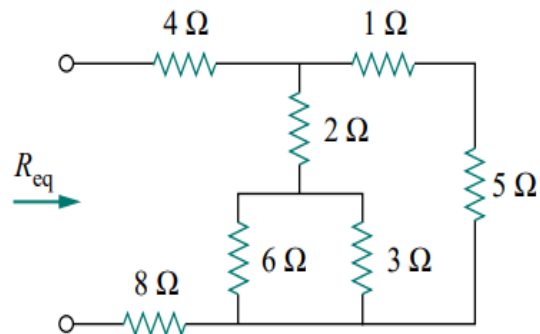
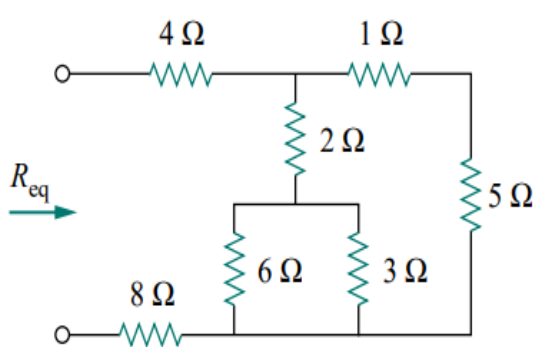
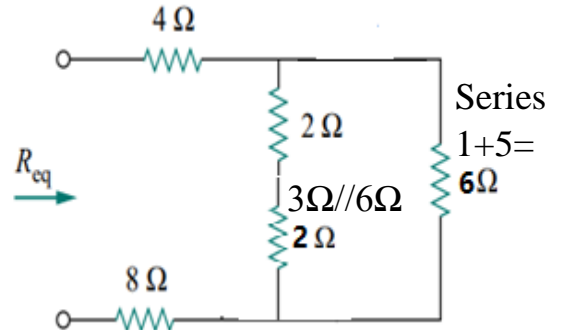


Figure (3.2)

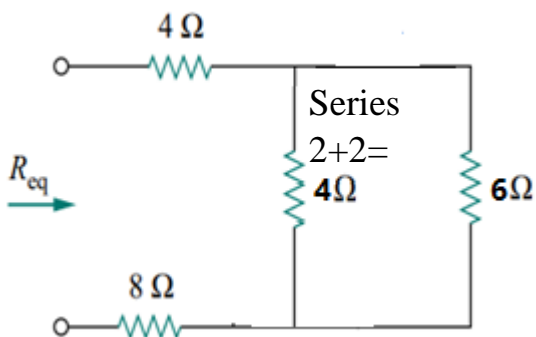
**Solution:**



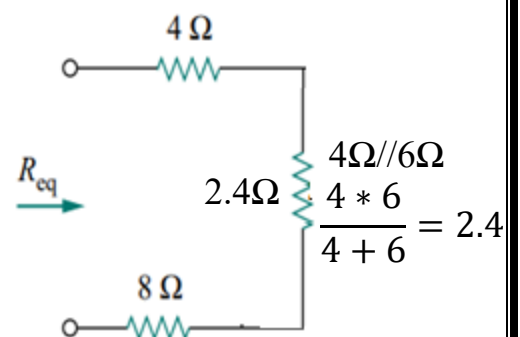
(1)



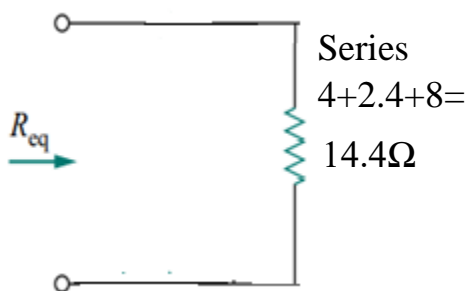
(2)



(3)



(4)



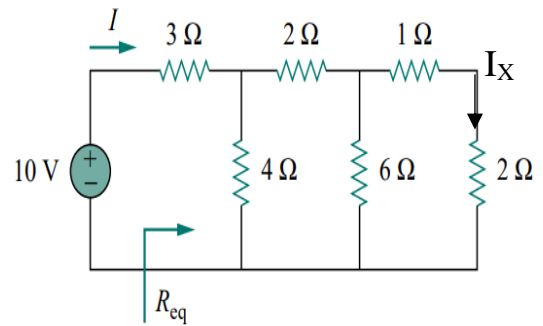
(5)

$$R_{eq} = 14.4\Omega$$



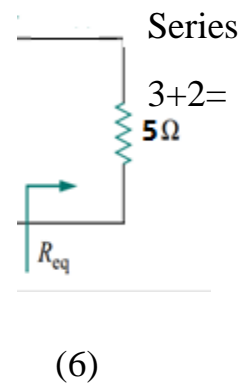
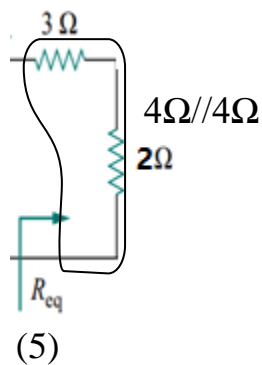
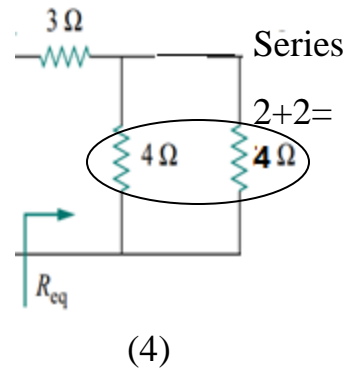
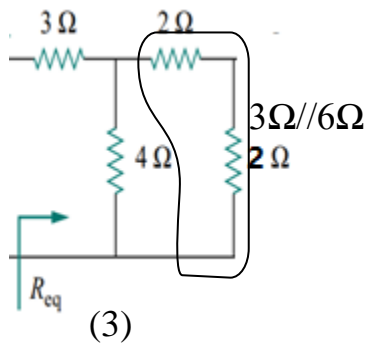
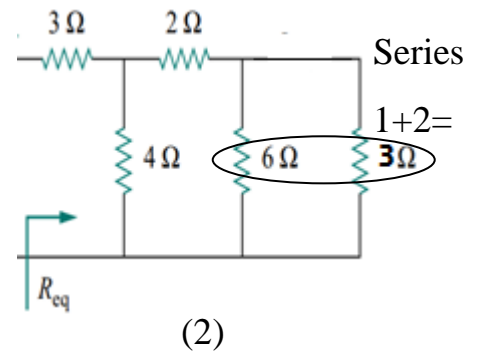
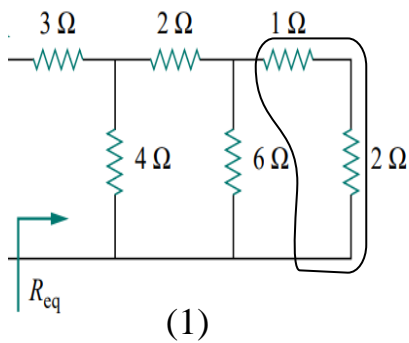
**Example 3.3:** For the circuit shown in figure (3.3), Find:

- 1) The total current  $I$ .
- 2) The value of  $I_x$ .



**Figure (3.3)**

**Solution:**



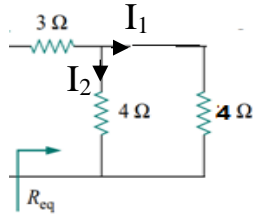
$$R_{eq} = 5\Omega$$

$$I = \frac{10}{5} = 2A$$

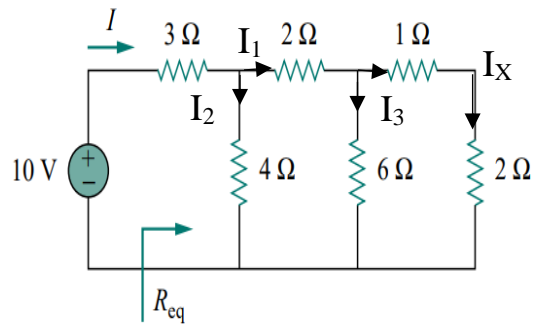
2)by using current divider's law

From figure (4) can be given  $I_1$ :

$$I_1 = \frac{2}{4+4} * 4 = 1A$$



$$I_x = \frac{1}{6+3} * 6 = 0.667A$$



**Example3.4:** For the circuit shown in figure (3.4), Find the value of  $V_0$  (the voltage of  $9K\Omega$ )

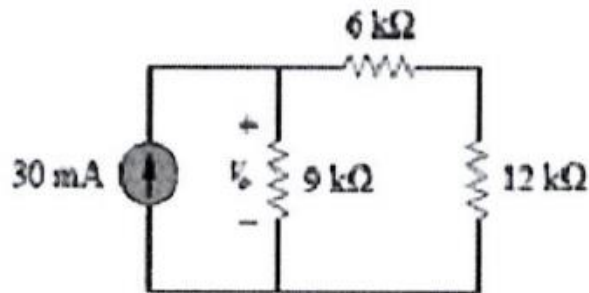


Figure (3.4)

**Solution:**

$6K\Omega$  and  $12K\Omega$  is series

$$6K+12K=18K\Omega$$

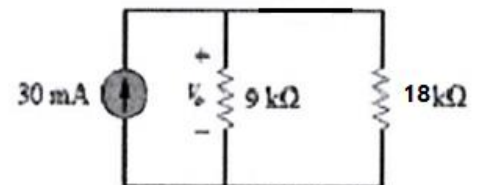
Can be using current divider's law

$$I_0 = \frac{30 \cdot 10^{-3}}{(9 \cdot 10^3 + 18 \cdot 10^3)} * (18 * 10^3) = 0.02A$$

$$V_0 = I_0 * R_{9K\Omega}$$

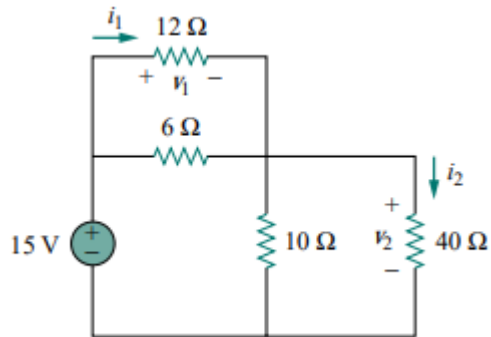
$$V_0 = 0.02 * 9 * 10^3$$

$$V_0 = 180V$$



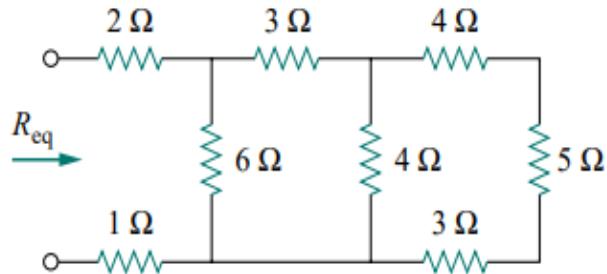
## Homework

**H.W3.1:** Find  $i_1$  and  $i_2$  in the circuit shown in Figure (3.5). Also calculate  $v_1$  and  $v_2$ . (Answer:  $v_1 = 5\text{ V}$ ,  $i_1 = 0.416\text{ A}$ ,  $v_2 = 10\text{ V}$ ,  $i_2 = 0.250\text{ A}$ .)



**Figure (3.5)**

**H.W3.2:** Find equivalent resistance in the circuit shown in Figure (3.6). (Answer:  $R_{eq} = 6\Omega$ .)



**Figure (3.6)**

## Chapter four

### Wye-delta transformations

#### 4.1 Wye-delta transformations

There are some cases often a rise in circuit analysis, when resistors are neither in parallel nor in series. For example, consider the circuit shown in figure (4.1).

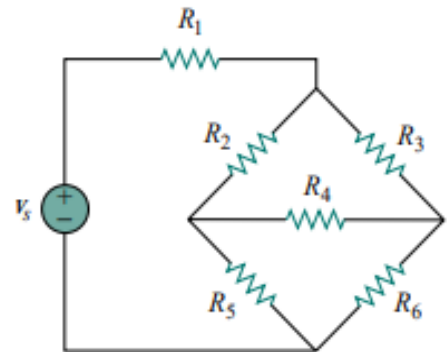
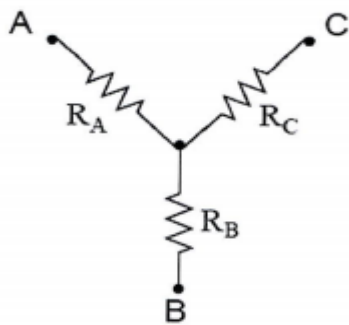
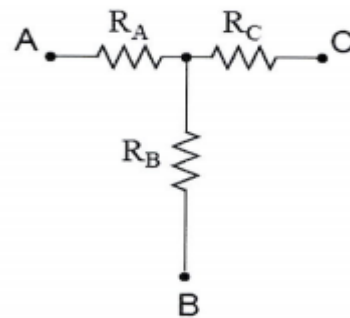


Figure (4.1)

#### Star (wye or T) connection:

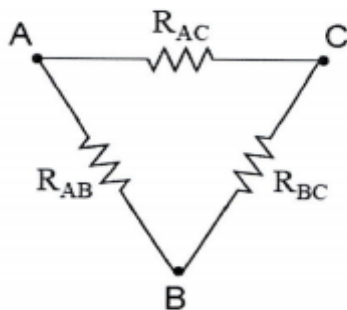


Y-Network

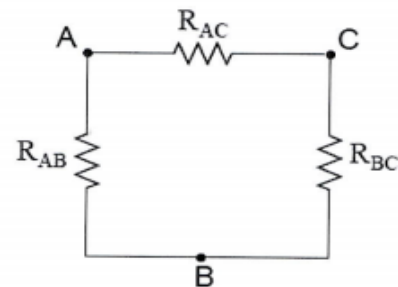


T-Network

#### Delta ( $\Delta$ or $\pi$ ) connection:



$\Delta$ -Network



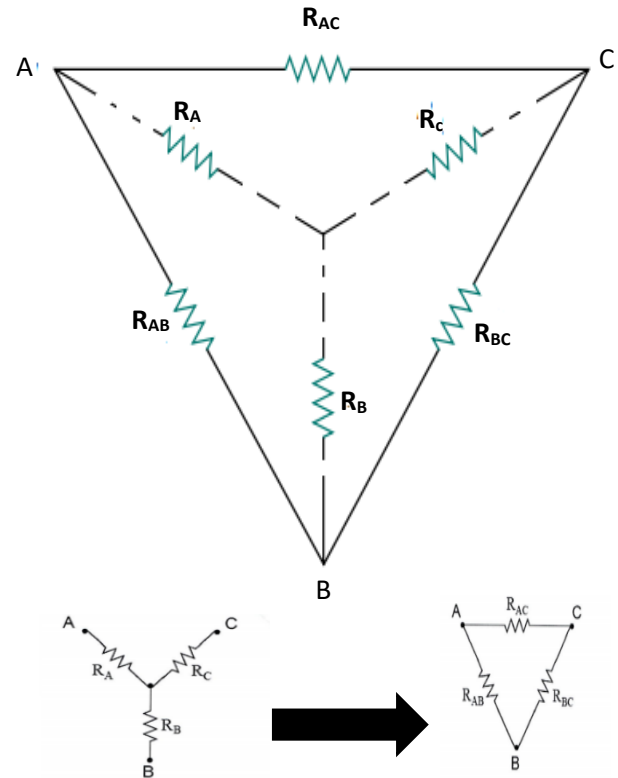
$\pi$ -Network

#### 4.1.1 To convert a Delta ( $\Delta$ ) to Wye (Y):

$$R_A = \frac{R_{AB} R_{AC}}{R_{AB} + R_{AC} + R_{BC}}$$

$$R_B = \frac{R_{AB} R_{BC}}{R_{AB} + R_{AC} + R_{BC}}$$

$$R_C = \frac{R_{AC} R_{BC}}{R_{AB} + R_{AC} + R_{BC}}$$

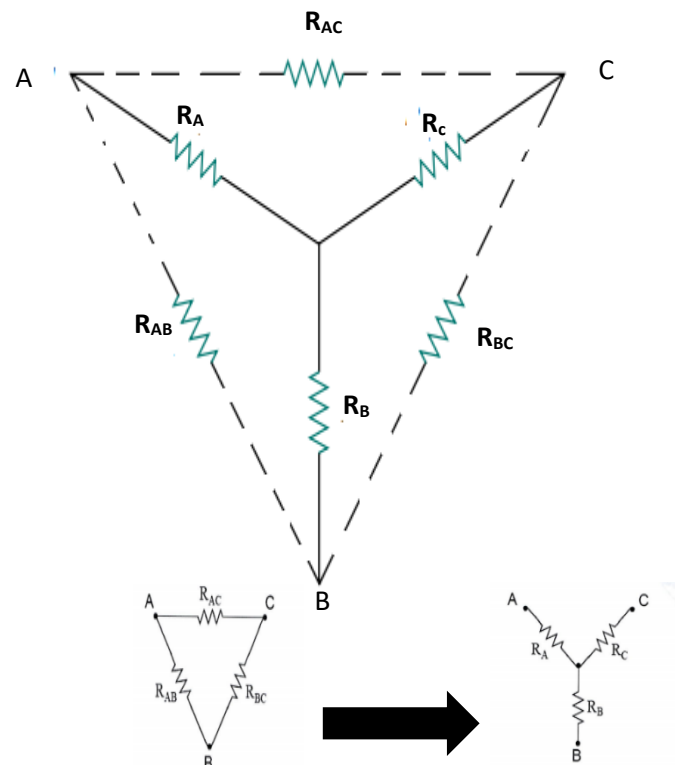


#### 4.1.2 To convert a Wye (Y) to Delta ( $\Delta$ ):

$$R_{AB} = R_A + R_B + \frac{R_A R_B}{R_C}$$

$$R_{BC} = R_B + R_C + \frac{R_B R_C}{R_A}$$

$$R_{AC} = R_A + R_C + \frac{R_A R_C}{R_B}$$



**Example 4.1:** Using ( $\Delta$ -Y) transformations to find the equivalent resistance and total current of the circuit shown in figure (4.2) (let voltage source  $V = 10\text{V}$ ).

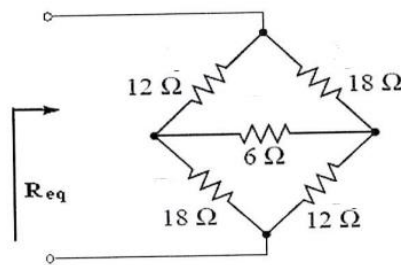
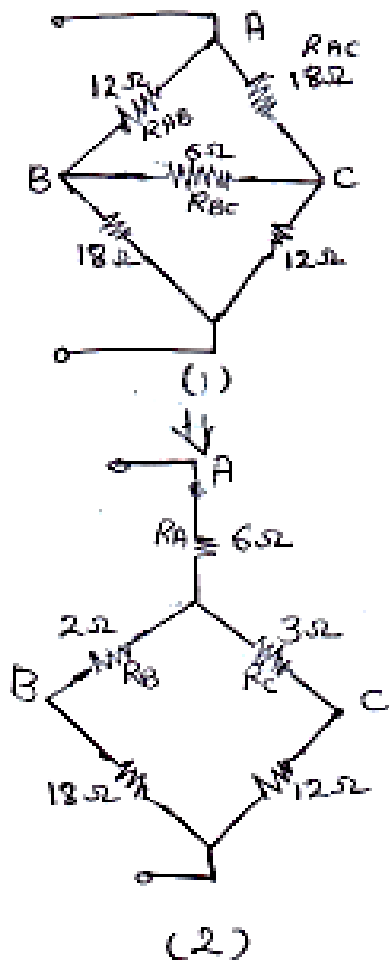


Figure (4.2)

**Solution:**



transform  $\Delta \rightarrow Y$   
 $\therefore R_{AB} = 12\Omega$  &  $R_{AC} = 18\Omega$ ,  $R_{BC} = 6\Omega$

$$R_A = \frac{R_{AB} R_{AC}}{R_{AB} + R_{AC} + R_{BC}}$$

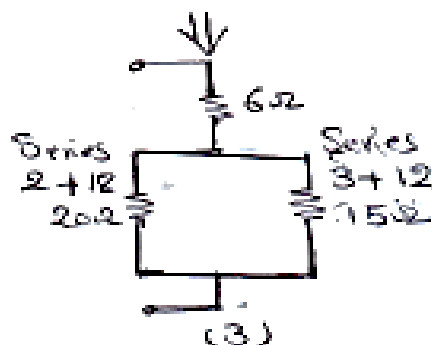
$$= \frac{12 \times 18}{12 + 18 + 6} = \frac{216}{36} = 6\Omega$$

$$R_B = \frac{R_{AB} R_{BC}}{R_{AB} + R_{AC} + R_{BC}}$$

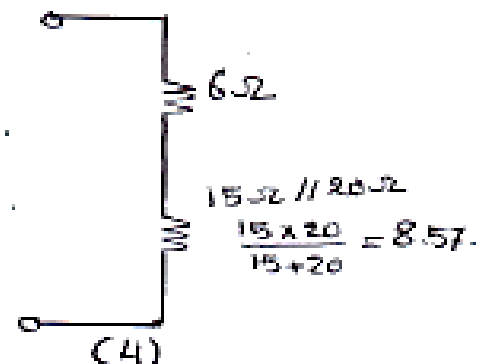
$$= \frac{12 \times 6}{12 + 18 + 6} = \frac{72}{36} = 2\Omega$$

$$R_C = \frac{R_{AC} R_{BC}}{R_{AB} + R_{AC} + R_{BC}}$$

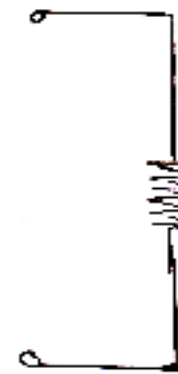
$$= \frac{18 \times 6}{12 + 18 + 6} = \frac{108}{36} = 3\Omega$$



$\Rightarrow$



$$\therefore R_{eq} = 14.57 \Omega$$



(5)

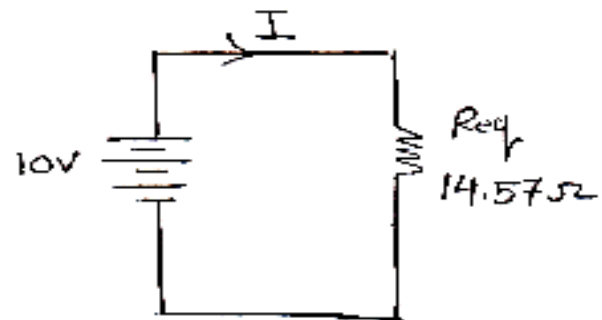
$$\begin{aligned} \text{Series} \\ 6 + 8.57 &= \\ 14.57 \Omega \end{aligned}$$

➤ Total current (Voltage source (10v))

$$I = \frac{V}{R_{eq}}$$

$$I = \frac{10}{14.57}$$

$$I = 0.686 \text{ A}$$



**Example 4.2:** Convert a Wye (Y) Network to equivalent Delta ( $\Delta$ ) Network for the circuit shown in figure (4.3).

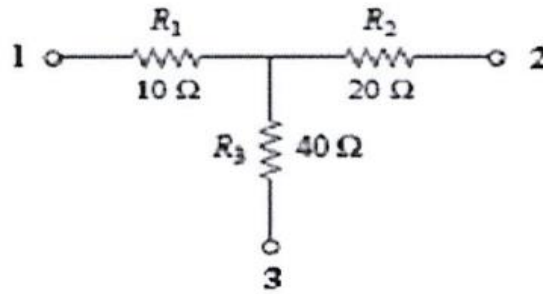


Figure (4.3)

**Solution:**

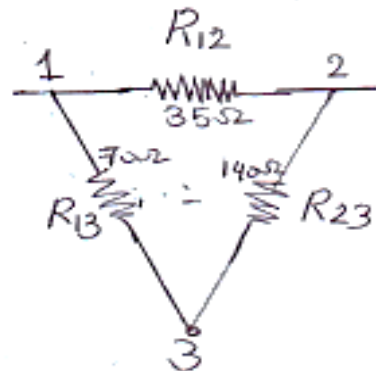
transform  $Y \rightarrow \Delta$  :-

$$R_1 = 10\ \Omega, \quad R_2 = 20\ \Omega, \quad R_3 = 40\ \Omega$$

$$\begin{aligned} R_{12} &= R_1 + R_2 + \frac{R_1 R_2}{R_3} \\ &= 10 + 20 + \frac{10 \times 20}{40} \\ &= 35\ \Omega \end{aligned}$$

$$\begin{aligned} R_{23} &= R_2 + R_3 + \frac{R_2 R_3}{R_1} \\ &= 20 + 40 + \frac{20 \times 40}{10} \\ &= 140\ \Omega \end{aligned}$$

$$\begin{aligned} R_{13} &= R_1 + R_3 + \frac{R_1 R_3}{R_2} \\ &= 10 + 40 + \frac{10 \times 40}{20} \\ &= 70\ \Omega \end{aligned}$$





**Example 4.3:** Find the equivalent resistance and total current of the circuit shown in figure (4.4) by Using (Y-Δ) transformations of the nodes a, b and c.

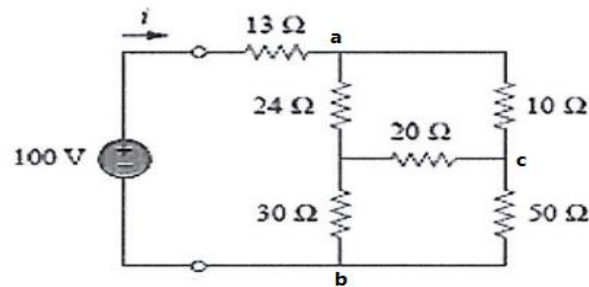
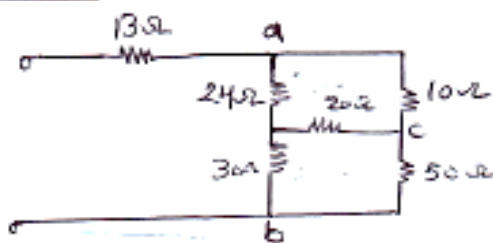
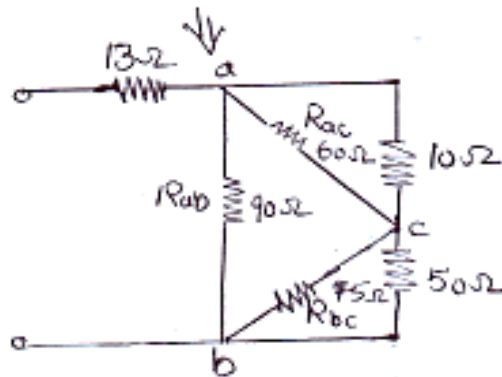


Figure (4.4)

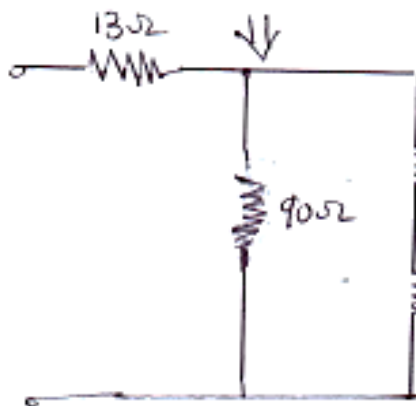
**Solution:**



(1)



(2)



(3)

transform  $\text{Y} \rightarrow \Delta$

$$R_a = 24\Omega, R_b = 30\Omega, R_c = 20\Omega$$

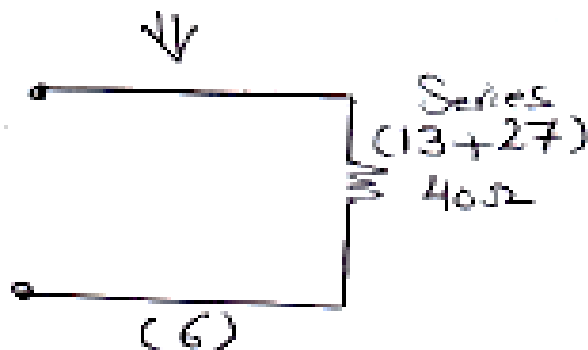
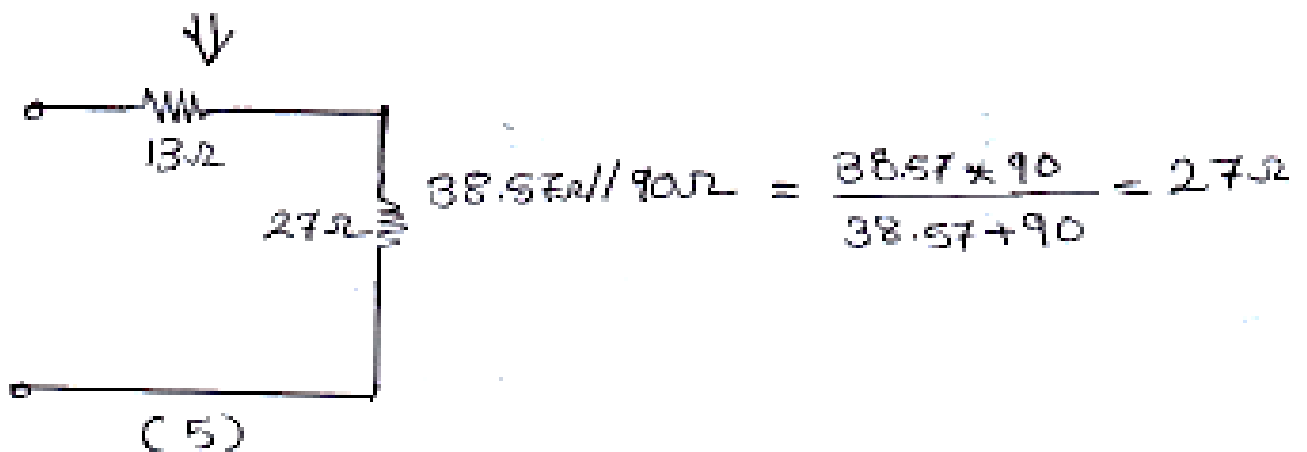
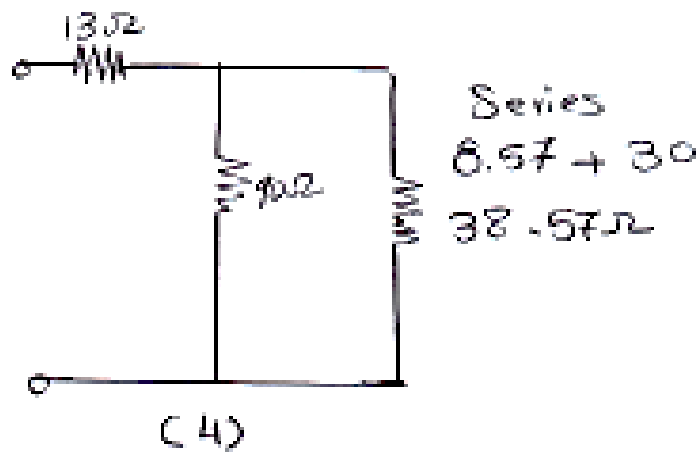
$$R_{ab} = R_a + R_b + \frac{R_a \cdot R_b}{R_c} \\ = 24 + 30 + \frac{24 \times 30}{20} = 90\Omega$$

$$R_{bc} = R_b + R_c + \frac{R_b \cdot R_c}{R_a} \\ = 30 + 20 + \frac{30 \times 20}{24} = 75\Omega$$

$$R_{ac} = R_a + R_c + \frac{R_a \cdot R_c}{R_b} \\ = 24 + 20 + \frac{24 \times 20}{30} = 60\Omega$$

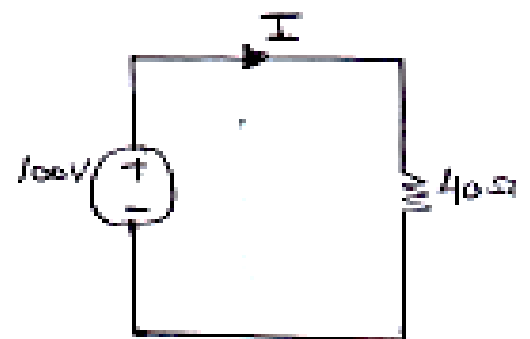
$$10\Omega \parallel 60\Omega = \frac{10 \times 60}{10 + 60} = 8.57\Omega$$

$$50\Omega \parallel 75\Omega = \frac{50 \times 75}{50 + 75} = 30\Omega$$



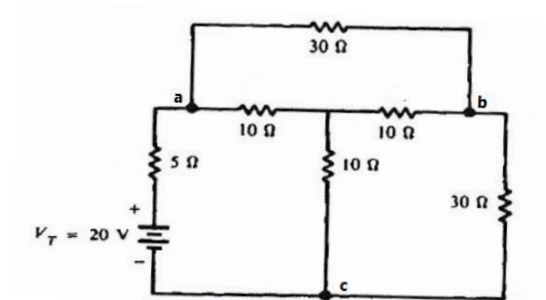
$$I = \frac{V}{R_{eq}}$$

$$I = \frac{100}{40} = 2.5A$$



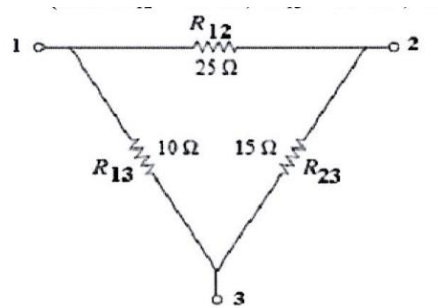
## Homework

**H.W4.1:** Find the equivalent resistance and total current of the circuit shown in figure (4.5) by Using (Y- $\Delta$ ) transformation of the nodes a, b and c. (answer:  $20\Omega$ )



**Figure (4.5)**

**H.W4.2:** Convert Delta ( $\Delta$ ) Network a to equivalent Wye (Y) Network for the circuit shown in figure (4.6).



**Figure (4.6)**

# Chapter five

## DC Circuit Analysis

### 5.1 Kirchhoff's law method (Branch current method)

In This method, we assume direction of the current in network, then write equations describing their relationships to each other through Kirchhoff's and ohm's laws. Once we have one equation for every unknown current, we can solve the simultaneous equation and determine all the current and therefore all voltage drops in the network.

**Example 5.1:** from the circuit shown in figure (5.1) find all the currents in the resistance by **using Kirchhoff's law method.**

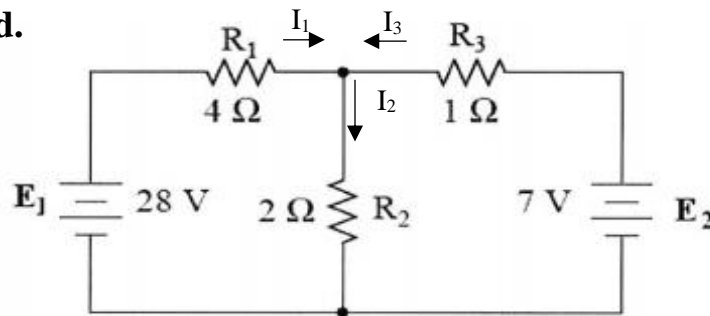
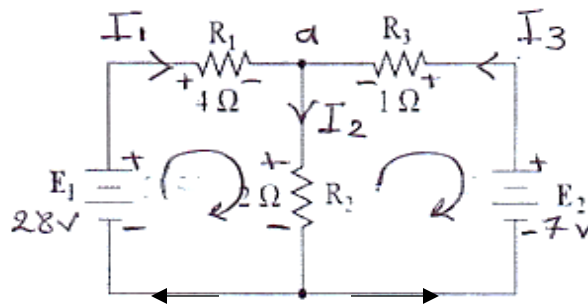


Figure (5.1)

**Solution:**



- KCL at node (a)

$$I_1 - I_2 + I_3 = 0$$

$$\therefore I_1 = I_2 - I_3 \quad \text{----- (1)}$$

- KVL for Loop(1):

$$-28 + 4I_1 + 2I_2 = 0$$

$$4I_1 + 2I_2 = 28 \quad \text{----- (2)}$$

### KVL For Loop (2)

$$-2I_2 - I_3 + 7 = 0$$

$$-2I_2 - I_3 = -7 \quad \text{---- (3)}$$

نعوض المعادلة رقم (1) في المعادلة رقم (3)

$$4I_1 - 2I_2 = 28$$

$$4(I_2 - I_3) + 2I_2 = 28$$

$$4I_2 - 4I_3 + 2I_2 = 28$$

$$6I_2 - 4I_3 = 28 \quad \text{---- (4)}$$

لنجمع معادلة رقم (3) مع معادلة رقم (4) وحيد المجهول

$$[-2I_2 - I_3 = -7] \times 3 \quad \text{---- (3)}$$

$$6I_2 - 4I_3 = 28 \quad \text{---- (4)}$$

$$\hline -6I_2 - 3I_3 = -21 \quad \text{---- (3)}$$

$$6I_2 - 4I_3 = 28 \quad \text{---- (4)}$$

$$\hline +7I_3 = 7$$

$$\therefore \underline{\underline{I_3 = -1A}}$$

نعوض  $I_3$  في المعادلة رقم (3) أو (4) لإيجاد  $I_2$

$$-2I_2 - I_3 = -7 \quad \text{---- (3)}$$

$$-2I_2 - (-1) = -7 \Rightarrow -2I_2 = -7 - 1$$

$$-2I_2 = -8$$

$$\therefore \underline{\underline{I_2 = \frac{-8}{-2} = 4A}}$$

$$\underline{\underline{I_1}} = I_2 - I_3 \Rightarrow I_1 = 4 - (-1) = \underline{\underline{5A}}$$

**Example 5.2:** Find the voltage of resistances to the circuit shown in figure (5.2) using **Branch current method**.

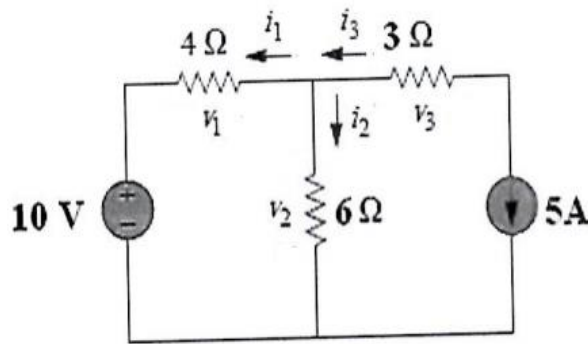


Figure (5.2)

**Solution:**

-هناك مصدر التيار في احد اطراف الدائرة اي ان احد تيارات معلوم وهو  $i_3$

لان الاتجاه المفروض عكس اتجاه المصدر  $i_3 = -5$

**KCL at node a**

$$-i_1 - i_2 + i_3 = 0$$

$$i_1 = -i_2 + i_3$$

$$i_1 = -i_2 - 5 \quad \dots\dots\dots (1)$$

**KCL at node a**

$$-10 - 4i_1 + 6i_2 = 0 \quad \dots\dots\dots (2)$$

نعوض المعادلة (١) في معادلة (٢)

$$-10 - 4(-i_2 - 5) + 6i_2 = 0$$

$$-10 + 4i_2 + 20 + 6i_2 = 0 \quad \Longrightarrow \quad 10 + 10i_2 = 0 \quad \Longrightarrow \quad i_2 = -10/10 \quad \Longrightarrow \quad \boxed{i_2 = -1A}$$

نعوض  $i_2$  في معادلة (١)

$$i_1 = -i_2 - 5 \quad \Longrightarrow \quad i_1 = -(-1) - 5 \quad \Longrightarrow \quad \boxed{i_1 = -4A}$$

$$V_1 = i_1 * R_{4\Omega} = -4 * 4 = -16V$$

$$V_2 = i_2 * R_{6\Omega} = -1 * 6 = -6V$$

$$V_3 = i_3 * R_{3\Omega} = -5 * 3 = -15V$$

**Example 5.3:** Find the voltage of resistances to the circuit shown in figure (5.3) using **Branch current method**.

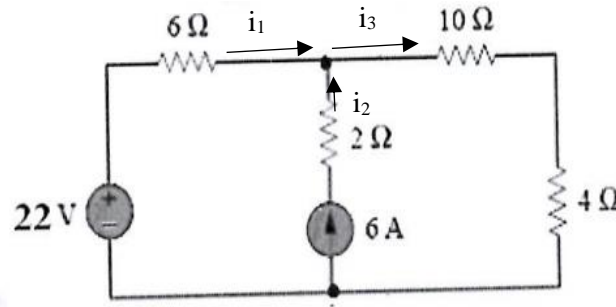


Figure (5.3)

**Solution:**

-هناك مصدر التيار في وسط ال loops اي ان احد تيارات معلوم وهو  $i_2$

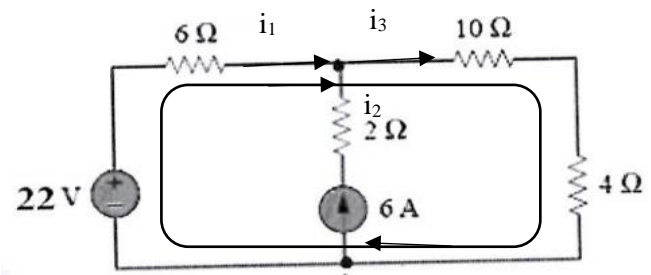
$$i_2 = 6A$$

**KCL at node a**

$$i_1 + i_2 - i_3 = 0$$

$$i_1 = -i_2 + i_3$$

$$i_1 = -6 + i_3 \quad \dots\dots\dots (1)$$



**KVL for super loop**

$$-22 + 6i_1 + 10i_3 + 4i_3 = 0$$

$$-22 + 6i_1 + 14i_3 = 0 \quad \dots\dots\dots (2)$$

نعوض المعادلة (١) في معادلة (٢)

$$-22 + 6(-6 + i_3) + 14i_3 = 0$$

$$-22 + (-6 \cdot 6) + 6i_3 + 14i_3 = 0 \implies -22 + 36 + 20i_3 = 0 \implies i_3 = -58 / -20 \implies \boxed{i_3 = 2.9A}$$

نعوض  $i_3$  في معادلة (١)

$$i_1 = 6 - i_3 \implies i_1 = 6 - (2.9) \implies \boxed{i_1 = 3.1A}$$

$$V_{6\Omega} = i_1 \cdot R_{6\Omega} = 3.1 \cdot 6 = 18.6$$

$$V_{2\Omega} = i_2 \cdot R_{2\Omega} = 6 \cdot 2 = 12v$$

$$V_{4\Omega} = i_3 \cdot R_{4\Omega} = 2.9 \cdot 4 = 11.6v$$

$$V_{10\Omega} = i_3 \cdot R_{10\Omega} = 2.9 \cdot 10 = 29v$$

## Homework

**H.W5.1:** From the circuit shown in figure (5.4) find all the currents in the resistance by **using Kirchhoff's law method**. (answer:  $I_1=5\text{mA}$ ,  $I_2=0\text{A}$ ,  $I_3=5\text{mA}$ )

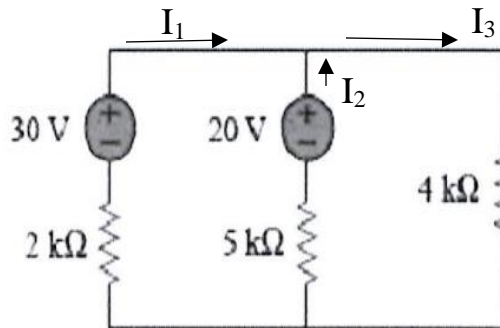


Figure (5.4)

**H.W5.2:** From the circuit shown in figure (5.5), find the  $v_1$ ,  $v_2$  and  $v_3$  of the resistance **by using Kirchhoff's law method**. (answer:  $i_1=3\text{A}$ ,  $i_2=2\text{A}$ ,  $i_3=1\text{A}$ )

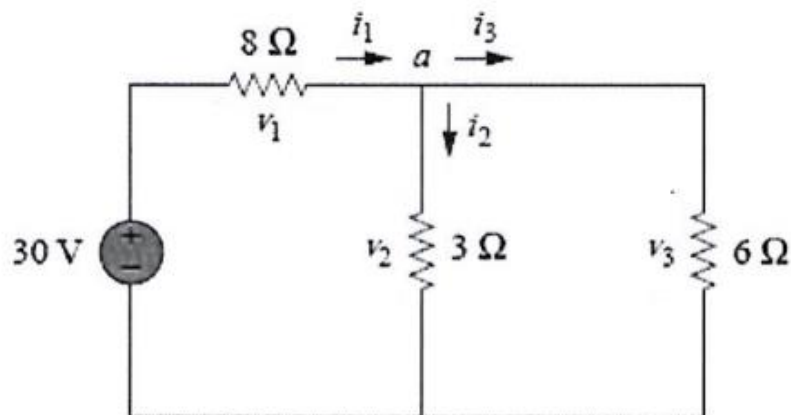


Figure (5.5)



## 5.2 Mesh method (Maxwell current loop method)

Mesh analysis provides another general procedure for analyzing circuits, using mesh currents as the circuit variables. Using mesh currents instead of element currents as circuit variables is convenient and reduces the number of equations that must be solved simultaneously. Recall that a loop is a closed path with no node passed more than once. A mesh is a loop that does not contain any other loop within it. Mesh analysis is also known as loop analysis or the mesh-current method. Mesh method applies KVL only to find unknown voltages or current in a given circuit, while KCL not applies.

**Example 5.4:** From the circuit shown in figure (5.6), find all the currents in the resistance by using Mesh method.

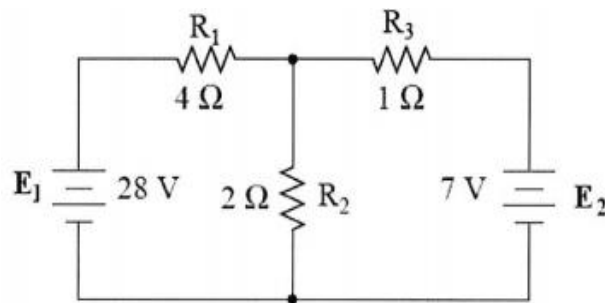


Figure (5.6)

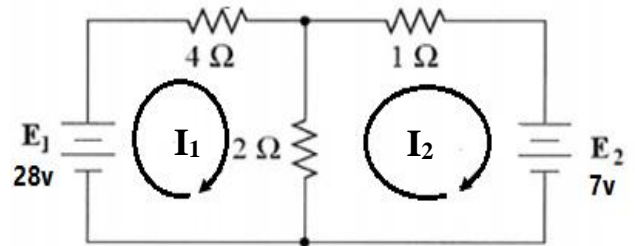
**Solution:**

Loop(1) KVL :-

$$\begin{aligned} -28 + 4I_1 + 2(I_1 - I_2) &= 0 \\ -28 + 4I_1 + 2I_1 - 2I_2 &= 0 \\ \therefore 6I_1 - 2I_2 &= 28 \quad \text{--- (1)} \end{aligned}$$

Loop(2) KVL :-

$$\begin{aligned} 2(I_2 - I_1) + I_2 + 7 &= 0 \\ 2I_2 - 2I_1 + I_2 + 7 &= 0 \\ \therefore 3I_2 - 2I_1 &= -7 \quad \text{--- (2)} \end{aligned}$$



$$6I_1 - 2I_2 = 28 \quad \text{--- (1)}$$

$$[-2I_1 + 3I_2 = -7] \times 3 \quad \text{--- (2)}$$

$$6I_1 - 2I_2 = 28 \quad \text{--- (1)}$$

$$-6I_1 + 9I_2 = -21 \quad \text{--- (2)}$$

$$7I_2 = 7 \Rightarrow I_2 = 1A$$

$$\therefore \boxed{I_2 = 1A = I_{R_3}}$$

$$6I_1 - 2I_2 = 28 \quad \text{--- (1)}$$

$$6I_1 - 2 \times 1 = 28 \Rightarrow 6I_1 = 28 + 2$$

$$\therefore I_1 = \frac{30}{6} = 5A$$

$$\boxed{I_1 = I_{R_1} = 5A}$$

$$I_{R_2} = I_{2R}$$

$$I_{R_2} = I_1 - I_2 = 5 - 1 = 4A$$

$$\therefore \boxed{I_{R_2} = 4A}$$

**Example 5.5:** Find the voltage of resistances to the circuit shown in figure (5.7) using Maxwell current loop method.

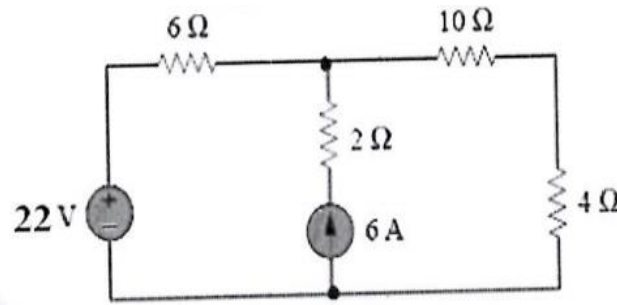
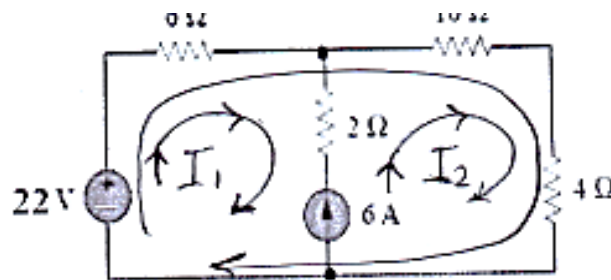


Figure (5.7)

**Solution:**



حسب التيار في رسله ال (Loops)

$$I_2 - I_1 = 6A$$

$$I_2 = 6 + I_1 \quad \text{--- (1)}$$

KVL Super Loop

$$-22 + 6I_1 + 10I_2 + 4I_2 = 0$$

$$-22 + 6I_1 + 14I_2 = 0 \quad \text{--- (2)}$$

نحوين معادله رسم (1) في معادله رسم (2)

$$-22 + 6I_1 + 14(6 + I_1) = 0$$

$$-22 + 6I_1 + 84 + 14I_1 = 0 \Rightarrow 62 = -20I_1 \Rightarrow$$

$$I_1 = -3.1A \quad \boxed{I_1 = I_{6\Omega} = -3.1A}$$

$$I_2 = 6 + I_1 \Rightarrow I_2 = 6 + (-3.1) = 2.9A$$

$$\boxed{I_2 = I_{10\Omega} = I_{4\Omega} = 2.9A}$$

$$\boxed{I_{2\Omega} = 6A}$$

$$V_{6\Omega} = I_1 * R_{6\Omega} = 3.1 * 6 = 18.6V$$

$$V_{2\Omega} = 6 * R_{2\Omega} = 6 * 2 = 12V$$

$$V_{10\Omega} = I_2 * R_{10\Omega} = 2.9 * 10 = 29V$$

$$V_{4\Omega} = I_2 * R_{4\Omega} = 2.9 * 4 = 11.6V$$

**Example 5.6:** From the circuit shown in figure (5.8) find all the currents in the resistance by using Mesh method.

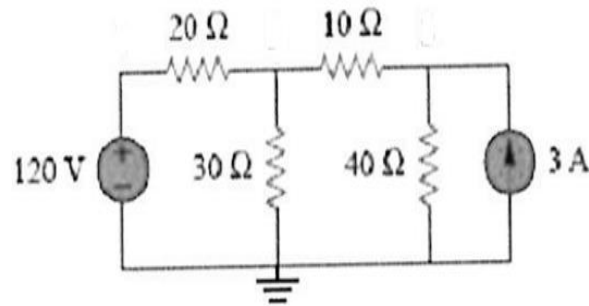
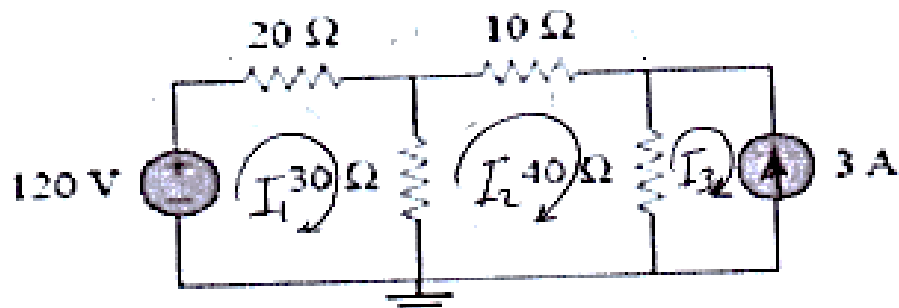


Figure (5.8)

**Solution:**



$$I_3 = -3A$$

Loop(1) KVL

$$-120 + 20 I_1 + 30(I_1 - I_2) = 0$$

$$-120 + 50 I_1 - 30 I_2 = 0$$

$$50 I_1 - 30 I_2 = 120 \quad \text{----- (1)}$$

Loop(2) KVL

$$30(I_2 - I_1) + 10 I_2 + 40(I_2 - I_3) = 0$$

$$80 I_2 - 30 I_1 - 40 I_3 = 0$$

$$80 I_2 - 30 I_1 - 40 (-3) = 0$$

$$80 I_2 - 30 I_1 = -120 \quad \text{--- (2)}$$

$$[ 50 I_1 - 30 I_2 = 120 ] * 8 \quad \text{--- (1)}$$

$$[ -30 I_1 + 80 I_2 = -120 ] * 3 \quad \text{--- (2)}$$

$$400 I_1 - 240 I_2 = 960 \quad \text{--- (1)}$$

$$-90 I_1 + 240 I_2 = -360 \quad \text{--- (2)}$$

$$310 I_1 = 600$$

$$\boxed{I_1 = 1.935 \text{ A}}$$

$$I_1 = I_{20\Omega} = 1.935 \text{ A}$$

$$50 I_1 - 30 I_2 = 120 \quad \text{--- (1)}$$

$$50 * 1.935 - 30 I_2 = 120$$

$$\boxed{I_2 = -0.775 \text{ A}}$$

$$I_2 = I_{10\Omega} = -0.775 \text{ A}$$

$$I_{30\Omega} = I_1 - I_2 = 1.935 - (-0.775) = 2.71 \text{ A}$$

$$I_{40\Omega} = I_2 - I_3 = -0.775 - (-3) = 2.225 \text{ A}$$

## Homework

**H.W5.3:** Find  $v_1$  and  $v_2$  of the circuit shown in figure (5.9) by **using Mesh method**. (answer:  $v_1 = 18\text{ V}$ ,  $v_2 = 26\text{ V}$ )

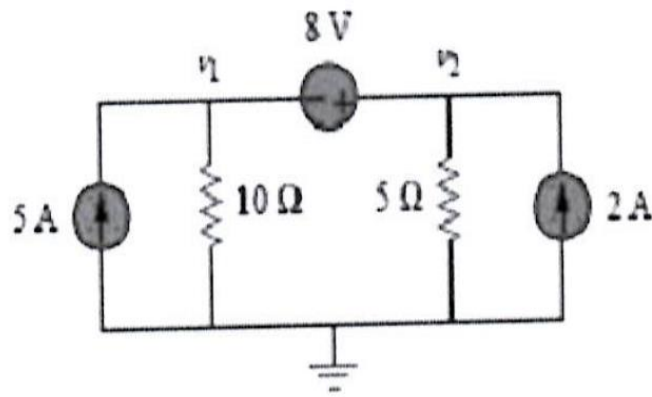


Figure (5.9)

# Chapter six

## Circuits theorems

### **6.1 Superposition theorem:**

The superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

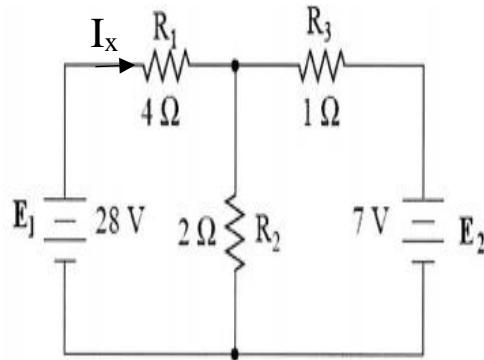
The principle of superposition helps us to analyze a linear circuit with more than one independent source by calculating the contribution of each independent source separately. However, to apply the superposition principle.

We consider one independent source at a time while all other independent sources are turned off. This implies that we replace every voltage source by 0 V (or a short circuit), and every current source by 0 A (or an open circuit). This way we obtain a simpler and more manageable circuit. Other terms such as killed, made inactive, deadened, or set equal to zero are often used to convey the same idea.

### **Steps to Apply Superposition Principle:**

1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source.
2. Repeat step 1 for each of the other independent sources.
3. Find the total contribution by adding algebraically all the contributions due to the independent sources

**Example 6.1:** Use the superposition theorem to find  $I_x$  for  $4\Omega$  in the circuit in Figure (6.1).



**Figure (6.1)**

**Solution:**

**$E_1$  is On and  $E_2$  is Off**

**$E_2$  is short circuit (because voltage source)**

$$R'_{eq} = (1\Omega // 2\Omega) + 4\Omega$$

$$R'_{eq} = \left(\frac{1 \cdot 2}{1+2}\right) + 4 = 0.667 + 4 = 4.667$$

$$I'_x = \frac{V}{R'_{eq}} = \frac{28}{4.667} = 6A$$

**$E_1$  is Off and  $E_2$  is On**

**$E_1$  is short circuit (because voltage source)**

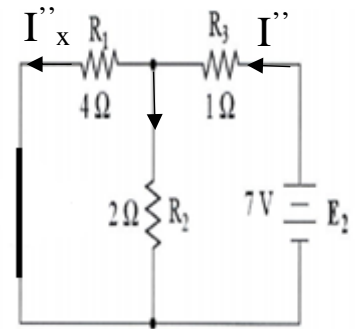
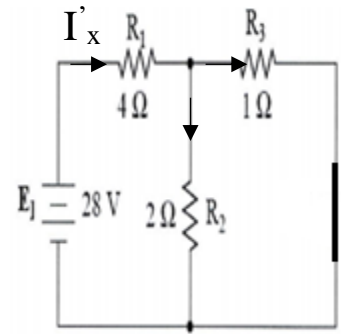
$$R''_{eq} = (4\Omega // 2\Omega) + 1\Omega$$

$$R''_{eq} = \left(\frac{4 \cdot 2}{4+2}\right) + 1 = 0.332 + 1 = 1.332$$

$$I'' = \frac{V}{R'_{eq}} = \frac{7}{1.332} = 3A$$

$$I''_x = \frac{3}{4+2} \cdot 2 = \frac{6}{6} = 1A \text{ (current divider's law)}$$

$$I_X = I'_x + I''_x = 6 - 1 = 5A$$





**Example 6.2:** Find  $v$  in the circuit in Figure (6.2) by using the superposition theorem.

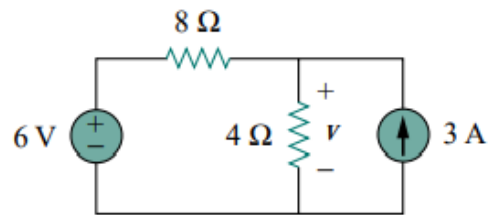


Figure (6.2)

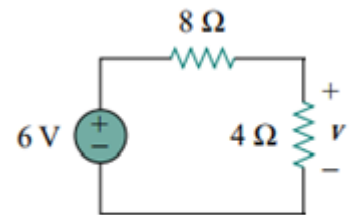
**Solution:**

**6v is On and 3A is Off**

**3A is open circuit (because is current source)**

$$I' = \frac{V}{R'_{eq}} = \frac{6}{8+4} = 0.5 \text{ A}$$

$$V' = 0.5 * 4 = 2V$$

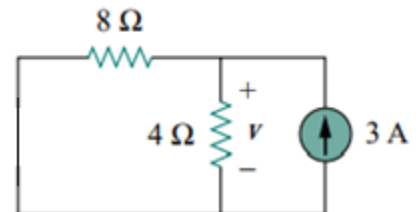


**6v is Off and 3A is On**

**6v is short circuit (because is voltage source)**

$$I'' = \frac{3}{8+4} * 8 = 2A \text{ (current divider's law)}$$

$$V'' = 2 * 4 = 8V$$



$$V = V' + V'' = 2 + 8 = 10$$

## Homework

**H.W6.1:** Using the superposition theorem, find  $V_o$  in the circuit in Figure (6.3). (answer:  $V_o=12\text{V}$ )

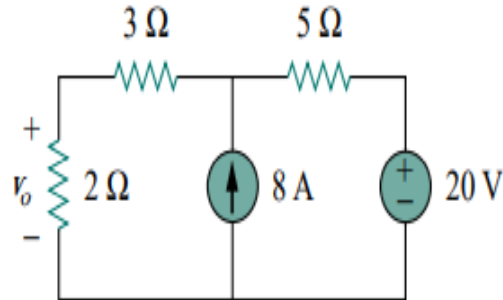


Figure (6.3)

**H.W6.2:** determine the value  $i$  in the circuit in Figure (6.4) by Using the superposition theorem. (answer:  $i=3.5\text{A}$ )

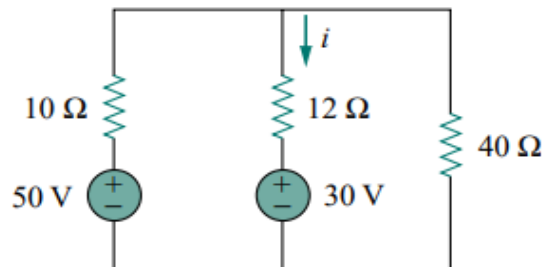


Figure (6.4)

## 6.2 Thevenin's theorem

Thevenin's theorem is a way to reduce a network to equivalent circuit composed of a single voltage source, series resistance and load.

Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source  $V_{Th}$  in series with a resistor  $R_{Th}$ , where  $V_{Th}$  is the open-circuit voltage at the terminals and  $R_{Th}$  is the input or equivalent resistance at the terminals when the independent sources are turned off.

### Steps to follow for Thevenin's theorem

- 1) Find the Thevenin source by remove the load resistance from the original circuit and calculating voltage across the open connection points where the load resistor used to be.
- 2) Find the Thevenin resistance by removing all power sources in original circuit (the voltage source is short circuit and the current source open circuit) and calculating total resistance between the open connection.
- 3) Draw the Thevenin equivalent circuit composed of a single voltage source, series resistance and load as shown in figure (6.5).
- 4) Analyze voltage and current for the load resistor following rules for series connection such as current of load resistance  $I_L$ .

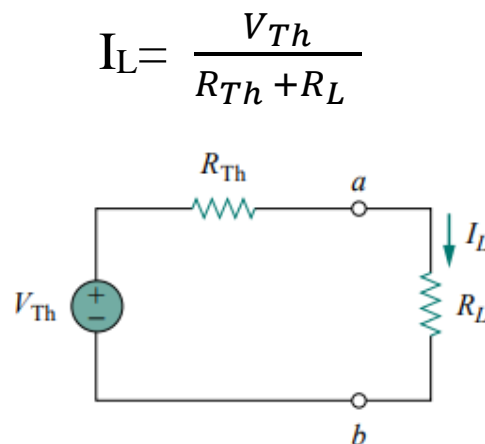


Figure (6.5)

**Example 6.3:** Find the load current of resistance in the circuit figure (6.6) by using Thevenin's theorem

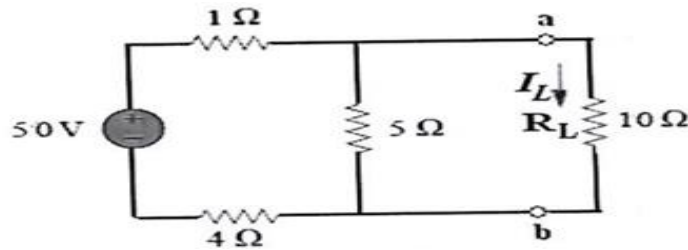


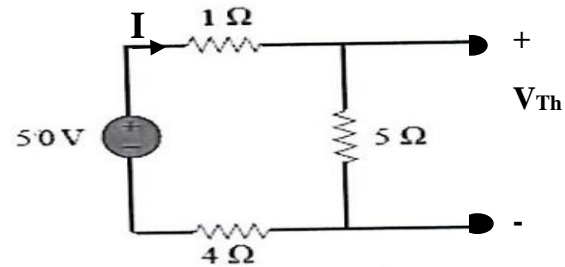
Figure (6.6)

**Solution:**

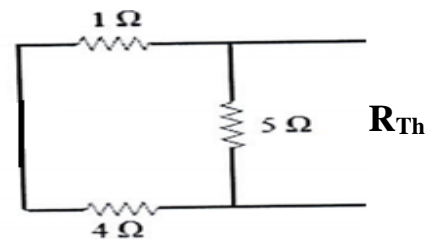
$V_{Th}$

$$I = \frac{50}{1+5+4} = 5A$$

$$V_{Th} = V_{5\Omega} = 5 * 5 = 25 \text{ v}$$

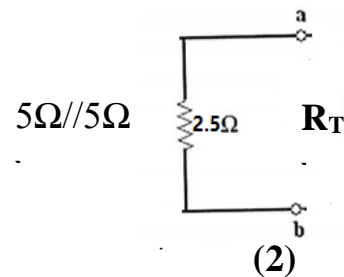
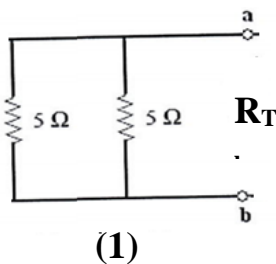


$R_{Th} = 2.5\Omega$



series

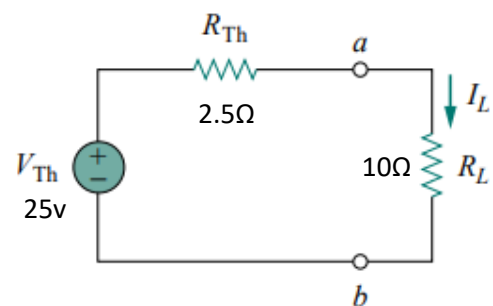
$$1+4=5\Omega$$



$R_{Th} = 2.5\Omega$

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

$$I_L = \frac{25}{2.5 + 10} = 1.5A$$



**Example 6.4:** Find the load current of resistance in the circuit figure (6.7) by using Thevenin's theorem.

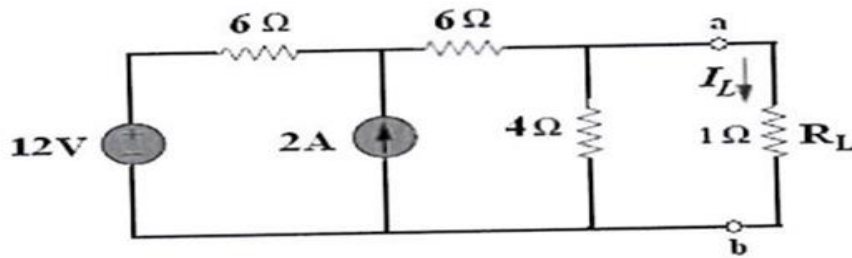
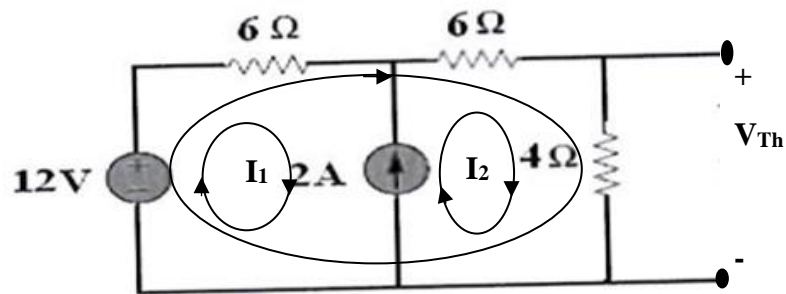


Figure (6.7)

**Solution:**



Using Mesh Method

$$I_2 - I_1 = 2$$

$$I_2 = 2 + I_1 \dots \dots (1)$$

**super loop**

$$-12 + 6I_1 + 6I_2 + 4I_2 = 0$$

$$-12 + 6I_1 + 10I_2 = 0 \dots \dots (2)$$

$$-12 + 6I_1 + 10(2 + I_1) = 0$$

$$-12 + 6I_1 + 20 + 10I_1 = 0$$

$$16 I_1 = -8$$

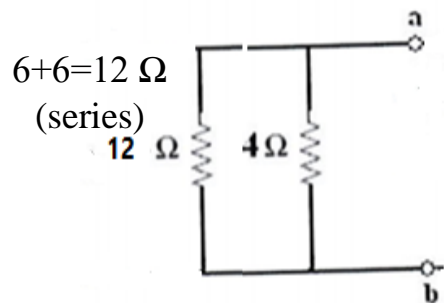
$$I_1 = \frac{-8}{16} = -0.5A$$

$$I_2 = 2 + I_1$$

$$I_2 = 2 + (-0.5) = 1.5$$

$$V_{Th} = V_{4\Omega} = (4 \times 1.5) = 6v$$

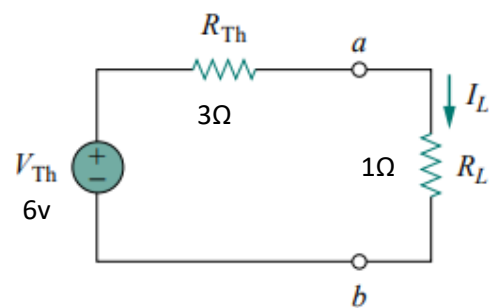
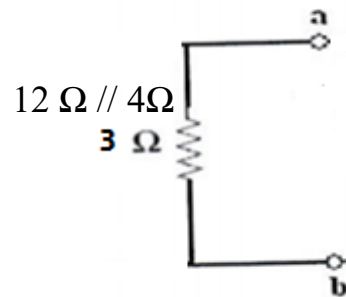
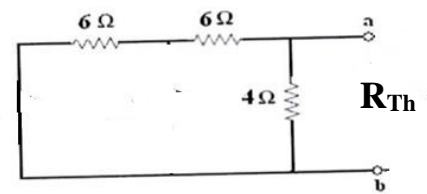
$R_{Th}$



$$R_{Th} = 3\Omega$$

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

$$I_L = \frac{6}{3 + 1} = 1.5A$$



**Example 6.5:** Find the load current of resistance in the circuit figure (6.8) by using Thevenin's theorem

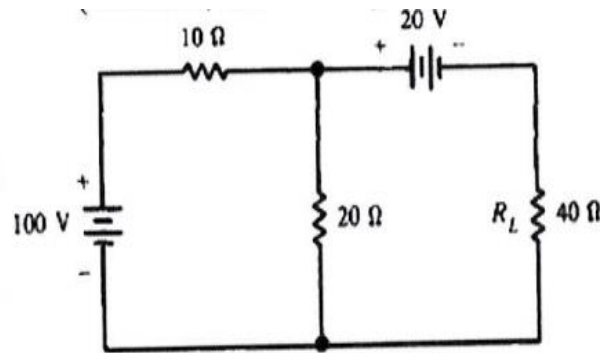


Figure (6.8)

**Solution**

$$I = \frac{100}{10+20} = 3.33A$$

**KVL:**

$$-20I + 20 + V_{Th} = 0$$

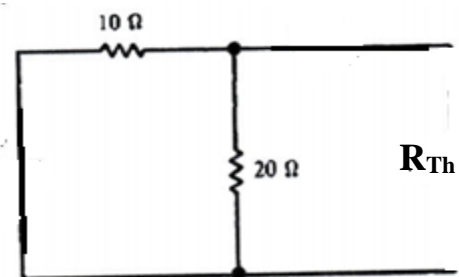
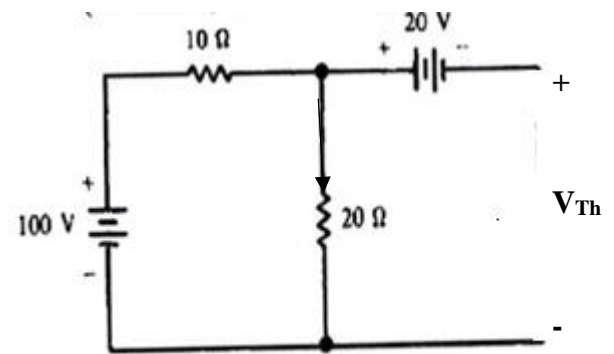
$$-20 * 3.33 + 20 + V_{Th} = 0$$

$$-66.6 + 20 + V_{Th} = 0$$

$$V_{Th} = 46.6v$$

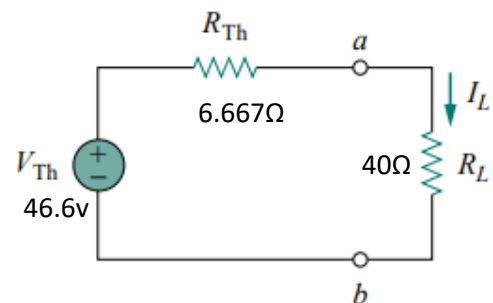
$$R_{Th} = 10\Omega // 20\Omega$$

$$R_{Th} = \frac{10*20}{10+20} = 6.667\Omega$$



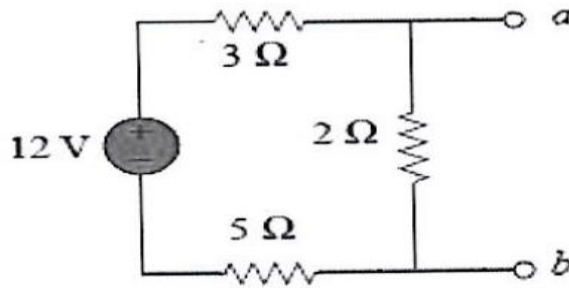
$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

$$I_L = \frac{46.6}{6.67 + 40} = 1A$$



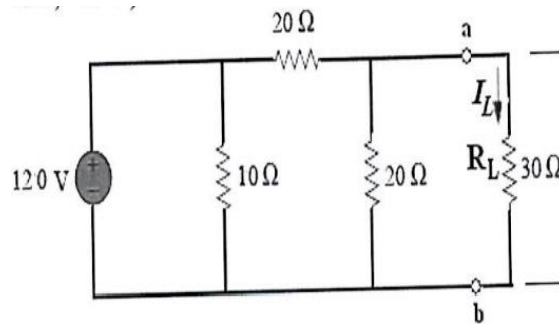
## Homework

**H.W 6.3:** Find the Thevenin equivalent of the circuit figure (6.9). (answer:  $V_{Th}=2.4\text{V}$ ,  $R_{Th}=1.6\Omega$ )



**Figure (6.9)**

**H.W 6.4:** Find the load current of resistance in the circuit figure (6.10) by using Thevenin's theorem. (answer:  $V_{Th}=60\text{V}$ ,  $R_{Th}=10\Omega$ ,  $I_L=1.5\text{A}$ )



**Figure (6.10)**



### 6.3 Norton's Theorem

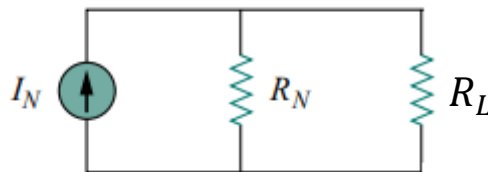
Norton's theorem is a way to reduce a network to equivalent circuit composed of a single current source, parallel resistance and load.

Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source  $I_N$  in parallel with a resistor  $R_N$ , where  $I_N$  is the short-circuit current through the terminals and  $R_N$  is the input or equivalent resistance at the terminals when the independent sources are turned off.

#### Steps to follow for Norton's theorem

- 1) Find the Thevenin source by remove the load resistance from the original circuit and calculating voltage across the open connection points where the load resistor used to be.
- 2) Find the Norton resistance by removing all power sources in original circuit (the voltage source is short circuit and the current source open circuit) and calculating total resistance between the open connection.
- 3) Draw the Norton equivalent circuit composed of a single current source, parallel resistance and load as shown in figure (6.11).
- 4) Analyze voltage and current for the load resistor following rules for parallel connection such as current of load resistance  $I_L$ .

$$I_L = \frac{I_N}{R_N + R_L} * R_N$$



**Figure (6.11)**

**Example 6.6:** Find the load current of resistance in the circuit figure (6.12) by using Norton's theorem.

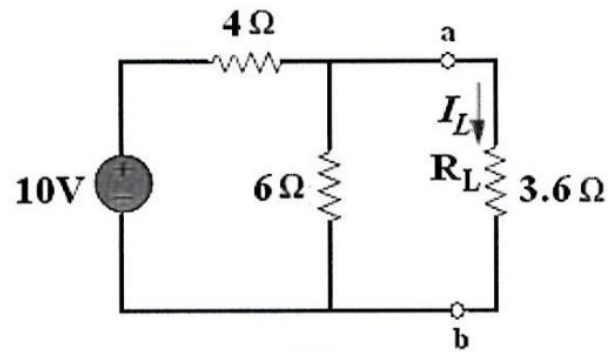
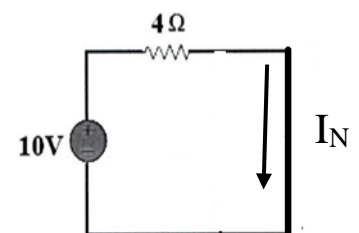
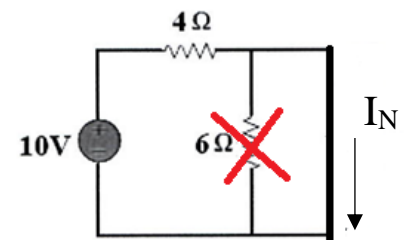


Figure (6.12)

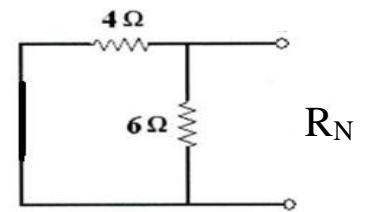
**Solution:**

$$I_N = \frac{10}{4} = 2.5\text{A}$$



Find  $R_N$

$$4\Omega // 6\Omega = \frac{4 \cdot 6}{4 + 6} = 2.4\Omega$$

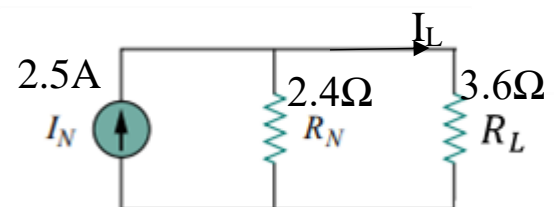


Norton Equivalent

$$I_L = \frac{I_N}{R_N + R_L} * R_N$$

$$I_L = \frac{2.5}{2.4 + 3.6} * 2.4$$

$$I_L = 1\text{A}$$



**Example 6.7:** In the circuit figure (6.13) Find Norton equivalent circuit.

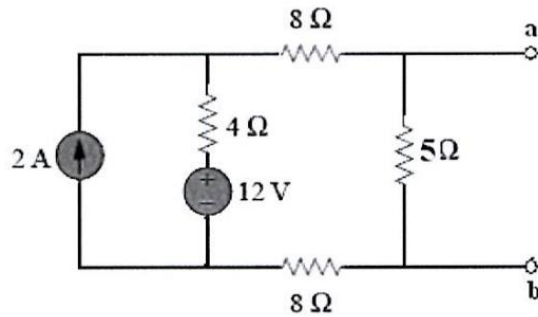


figure (6.13)

**Solution:**

$$I_1 = 2A$$

$$-12 + 4(I_2 - I_1) + 8I_2 + 8I_2 = 0 \quad (\text{loop 2})$$

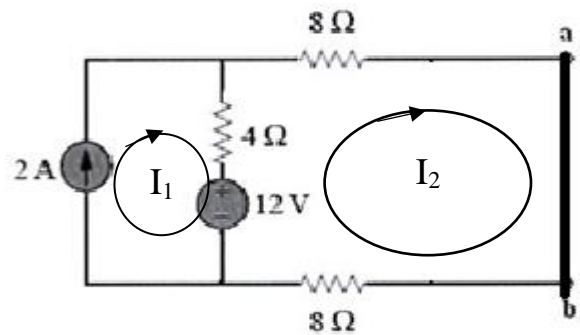
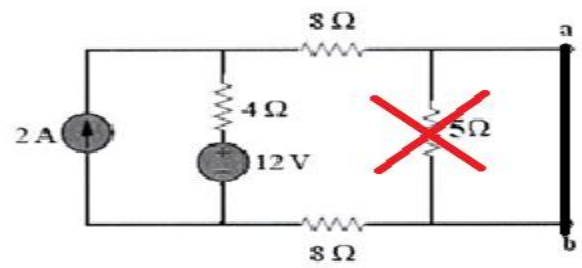
$$-12 + 4I_2 - 4I_1 + 16I_2 = 0$$

$$-12 - (4 \cdot 2) + 20I_2 = 0$$

$$-20 + 20I_2 = 0$$

$$I_2 = 1A$$

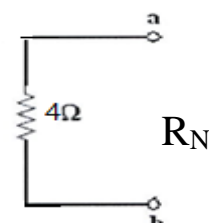
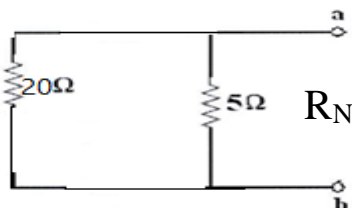
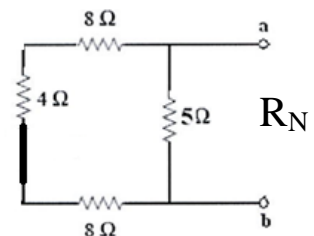
$$I_2 = I_N = 1A$$



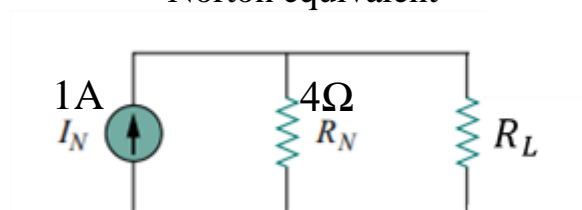
$$R_N$$

$$4 + 8 + 8 = 20\Omega$$

$$20\Omega \parallel 5\Omega = 4\Omega$$



Norton equivalent



## Homework

**H.W 6.5:** Find the current of load resistance in the circuit figure (6.14) using Norton's theorem.  
(answer:  $I_N=8A$   $R_N=5\Omega$ ,  $I_L=4A$ )

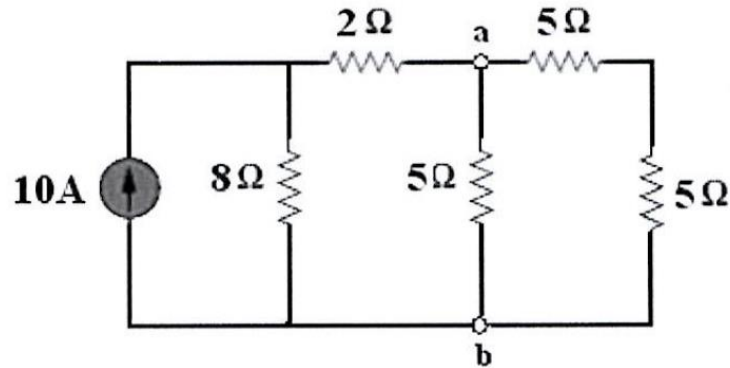


Figure (6.14)