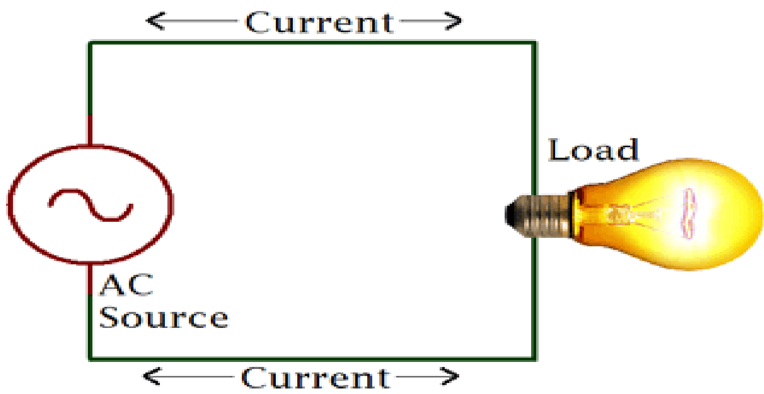


دوائر تيار متناوب

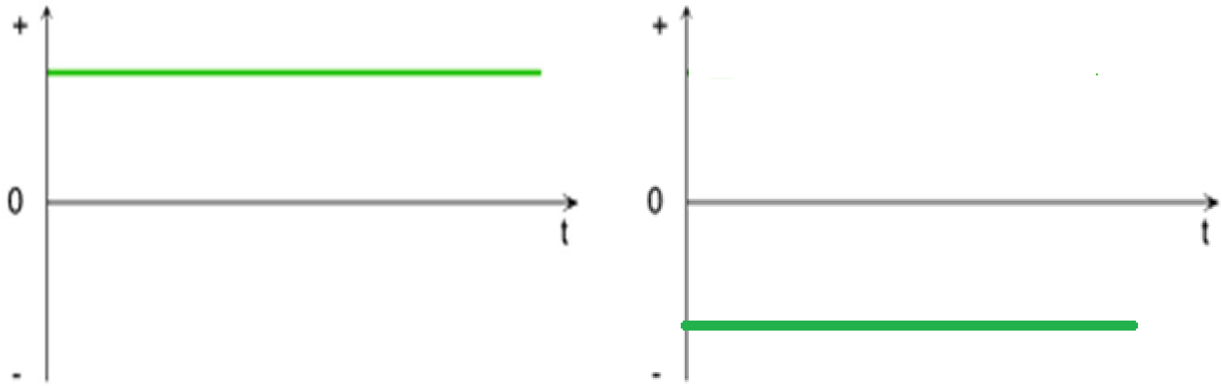
AC Circuits

سماء خالد غانم

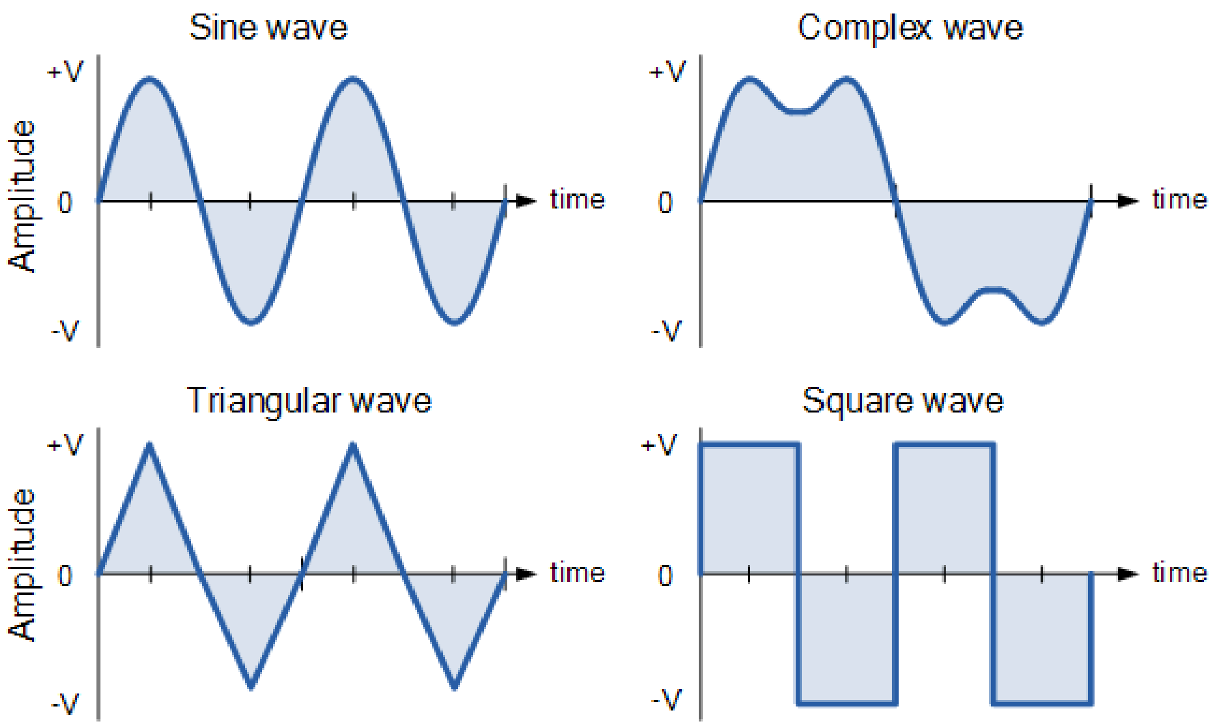
2022-2023

Waveform

a. **Direct current (DC)**



b. **Alternating current (AC)**



Chapter one
Sinewave function and phasor

1.1 Sinewave function in time domain:

A sinusoid is a signal that has the form of the sine or cosine function. A sinusoidal current is usually referred to as *alternating current* (*ac*). Such a current reverse at regular time intervals and has alternately positive and negative values as shown in figure (1.1). Circuits driven by sinusoidal current or voltage sources are called *ac circuits*. Consider the sinusoidal voltage

$$v(t) = V_m \sin \omega t$$

V_m = the *amplitude* of the sinusoid

ω = the *angular frequency* in radians/s

ωt = the *argument* of the sinusoid

$$W=2\pi f$$

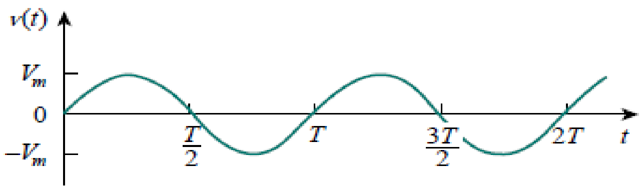


Figure (1.1) Sinewave in time domain

1.1.1Frequency, Period, and wavelength:

The amount of time for completion of 1 cycle is the peroid. It is inducated by the symbol T and the unit is expressed in second (sec).

$$T = \frac{1}{f}$$

The number of cycle per second is called frequency(f). It is inducated by the symbol F and the unit is expressed in Hertz (Hz).

$$f = \frac{1}{T}$$

Wavelength (λ) is length of the one complete wave or cycle. It os depend on the frequency of the periodic variation and the velocity of transmission. Expressed as a formula:

$$\lambda = \frac{velocity}{frequency} = \frac{C}{f}$$

λ: Wavelength(m)

C: speed of light(m/sec)

f: frequency, Hz

1.1.2 Angular measurement

The one cycle of sinewave is 360^0, one half cycle is 180^0.Aquarter tune is 90^0. Degrees also expressed in radians (rad). A complete cycle is 2π in radius.

$$360^0=2\pi$$

$$1^0= \frac{\pi}{180}$$

$$\frac{\emptyset degree}{180} = \frac{\emptyset radians}{\pi}$$

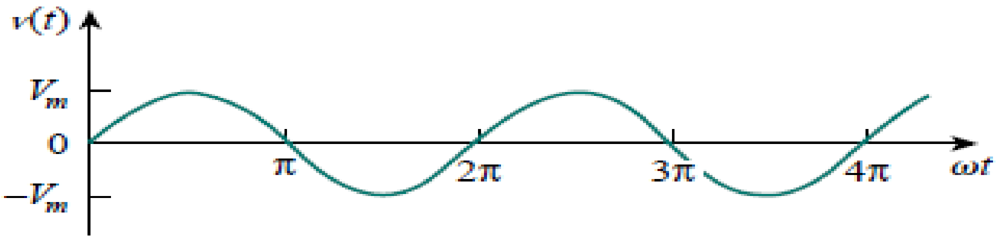


Figure (1.2) Two cycle of alternating voltage generated by rotating loop

Example(1.1): how many radians are there in 30^0?

Solution:

$$\frac{\emptyset degree}{180} = \frac{\emptyset radians}{\pi}$$

$$\frac{30}{180} = \frac{\emptyset radians}{\pi}$$

$$\emptyset radians = \frac{\pi}{6}$$

***H.W(1.1)*:** how many degree are there in $\frac{\pi}{3}\ rad$? answer(60^o)

1.1.3 Characteristic value of voltage and current of sinewave

Figure (1.3) shows the characteristic value of voltage and current of sinewave which represents:

1. **Peak value** is the maximum value of the sinewave.it applied to either the positive or negative peak.
2. **Peak to peak** value is double the peak value when the positive and negative peaks are symmetrical.
3. **The average value** is the arithmetic average of all values in a sine wave for 1 half-cycle.
4. **Root mean square (effective value)** is 0.707 times the peak value.

Sinewave in voltage	*Sinewave in current*
$V(t) = V_P \sin(wt)$	$I(t) = I_P \sin(wt)$
$V_{p\text{-}p} = 2 * V_p$	$I_{p\text{-}p} = 2 * I_p$
$V_{av} = 0.637 * V_p$	$I_{av} = 0.637 * I_p$
$V_{r.m.s} = 0.707 * V_p$	$I_{r.m.s} = 0.707 * I_p$

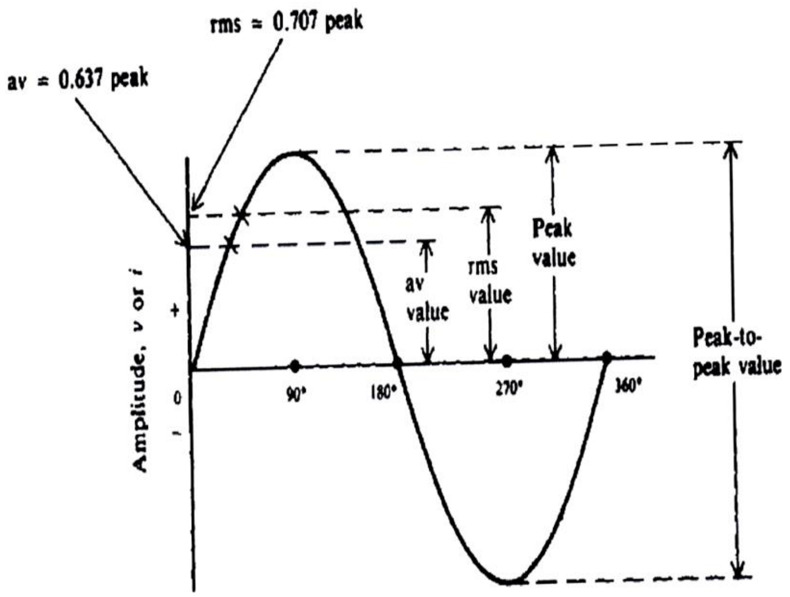


Figure (1.3): Characteristic value of voltage of sinewave

1.1.4 Form Factor and Peak Factor of sinewave:

Form factor= $\frac{RMS\ value}{Avarage\ value} = \frac{0.707Vp}{0.637Vp}$=1.11

Peak Factor= $\frac{Maximum\ value}{RMS\ value} = \frac{Vp}{0.707Vp} = 1.414$

1.2 Phase angle:

When we have two waves having the same frequency, the time relation between waves called phase and the angular relation between them called phase angle. As an example, the phase angle between waves A and B is 90° in shown figure (1.4). Which B stars at maximum value and reduce to zero in 90°, while wave A star zero value and increase to maximum value at90°. So wave B is leads wave A by 90° or wave A lags wave B by 90°.

$A=V_m \sin(wt)$ *(reference)*
$B= V_m \sin(wt+90)$) $v(t)=V_m \sin(wt+\emptyset)$

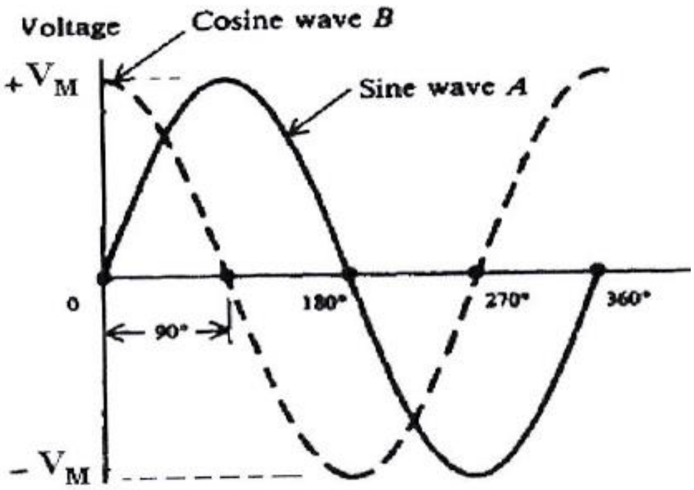


Figure (1.4) wave B is leads wave A by phase angle 90°

1.2.1 Lead or lag terms

We have three waves A, B and C is the same frequency as shown in figure (1.5). The time relation between them are:

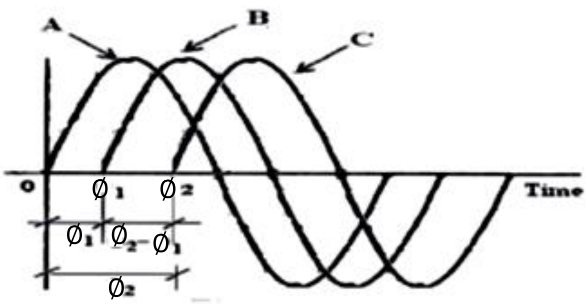


Figure (1.5)

A is reference	B is reference	C is reference
B lags A by φ_1	A leads B by φ_1	A leads C by φ_2
C lags A by φ_2	C lags B by $\varphi_2 - \varphi_1$	B leads C by $\varphi_2 - \varphi_1$

Example (1.2): find the relation (lead or lag) and the phase angle which the V_1 is reference:

$V_1=10\sin wt$

$V_2=20\sin(wt+\pi/3)$

$V_3=25\sin(wt-\pi/6)$

Solution:

V_2 leads V_1 by $\pi/3$ $\varphi=\pi/3$

V_3 lags V_1 by $\pi/6$ $\varphi=-\pi/6$

Example(1.3): As shown in figure (1.6) find the relation (lead or lag) and the phase angle which the v is reference.

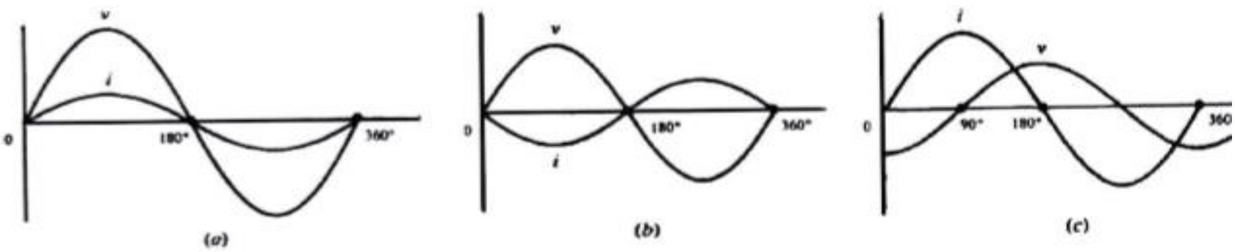


figure (1.6)

Solution:

(a)i and V in same phase $\varphi=0$

(b) i lags V $\varphi=-180^0$

(c)i leads V $\varphi=90^0$

H.W(1.3): As shown in figure (1.7) find the relation (lead or lag) and the phase angle which the v is reference.

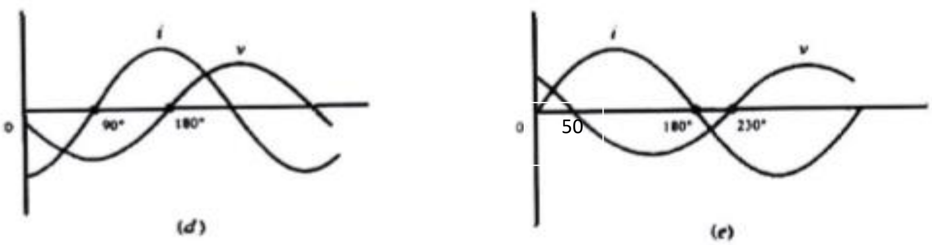


figure (1.7)

Example(1.4): If v(t)=10 sin $(20\pi t+\frac{\pi}{3})$, find:

(1) Peak voltage, the average of the voltage, Root Mean Square of voltage
(2) The angular frequency, frequency, and periodic of the waveform
(3) The wavelength of the waveform
(4) The phase shift of the waveform and explain the waveform is leading or lagging?
(5) Form factor and Peak factor
(6) The value of the v(t) when t=5sec.

Solution:

(1) $V_p=10v$ $v(t)=V_p \sin(wt+\emptyset)$

$V_{av}=0.637*V_p=0.637*10=6.37v.$

$V_{r.m.s}=0.707*V_p=0.707*10=7.07v.$

(2) angular frequency (w)=20π rad/sec.

$w=2\pi f \Longrightarrow 20\pi=2\pi f \Longrightarrow f=10$ Hz

$T=1/f \Longrightarrow T=1/10 \Longrightarrow T=0.1$ sec

(3) $\lambda= \frac{C}{f} = \frac{3*10^8}{10} = 0.3 * 10^8\ m$

(4) $\varphi= \frac{\pi}{3}rad$ leading because positive.

(5) Form factor= $\frac{RMS\ value}{Avarage\ value} = \frac{7.07}{6.37}=1.11$

Peak Factor= $\frac{Maximum\ value}{RMS\ value} = \frac{10}{7.07} = 1.414$

(5) $v(t)=10 \sin(20\pi t+\frac{\pi}{3})$

$v(t)=10 \sin(20\pi*5+\frac{\pi}{3})$

$v(t)=10 \sin(301\pi/3)$

$v(t)=10*0.866=8.66v$

H.W(1.4): If I(t)=5 cos $(10\pi t-\frac{\pi}{6})$, find:

(1) Peak current, the average of the current, Root Mean Square of current.
(2) The angular frequency, frequency, and periodic of the waveform.
(3) The wavelength of the waveform
(4) The phase shift of the waveform and explain the waveform is leading or lagging?
(5) Form factor and Peak factor
(6) The value of the I(t) when t=2msec.

1.3 <u>Phasor diagram representation of the voltage and current in sinewave:</u>

$V(t)=V_m \sin(wt)$ $I(t)=I_m \sin(wt+\emptyset)$

$V(t)=V_m \angle 0$ $I(t)=I_m \angle\emptyset$

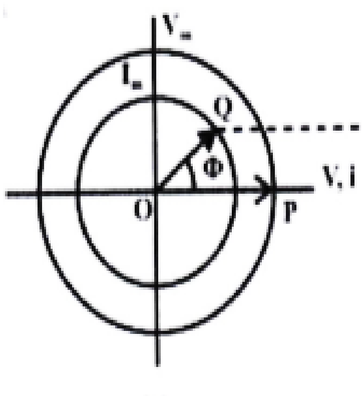


Figure (1.8) Phasor representation of sine wave of the voltage and current

<u>Example(1.5):</u> Transform these sinusoids to phasors:

(a) $v = 4 \sin(30t + 50)$

(b) $i = 6 \sin(50t - 40)$

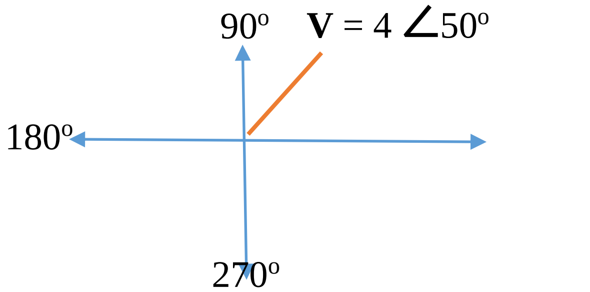
<u>Solution:</u>

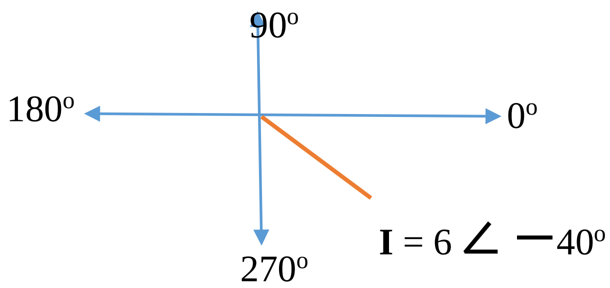
(a) $v = 4 \sin(30t + 50)$

$\mathbf{V} = 4 \angle 50^o$

(b) $i = 6 \sin(50t - 40)$ has the phasor

$\mathbf{I} = 6 \angle -40^\circ$





<u>H.W(1.4):</u> Express these sinusoids as phasors:

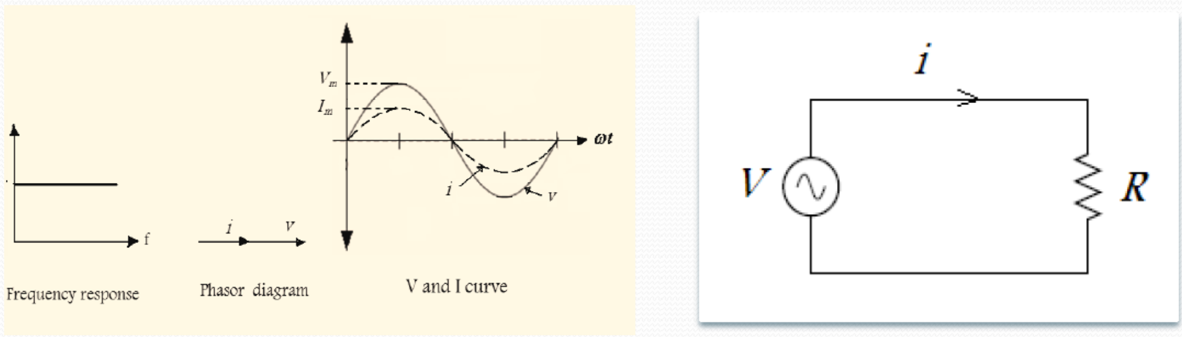
(a) $v = -7 \sin(2t + 40^o)$

(b) $i = 4 \sin(10t - 100^o)$

Chapter two

AC circuits

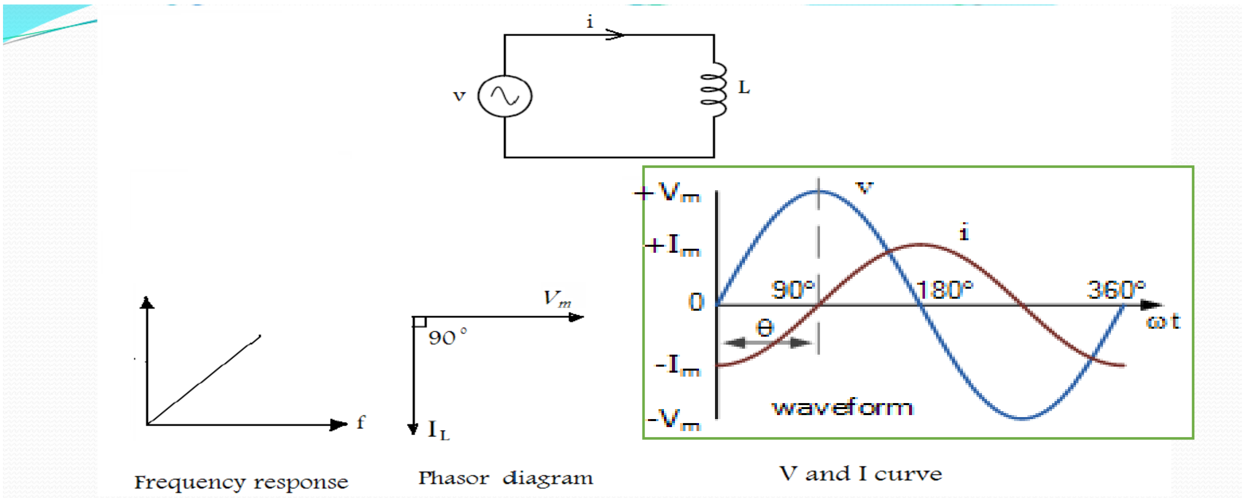
2.1 Purely resistive circuit:



$$\left.\begin{aligned} v &= V_m \sin\omega t \\ i &= I_m \sin\omega t \end{aligned}\right\} v \ and \ i \ in \ phase$$

$$I_R = I_m = \frac{V_m}{R}$$

2.2 Purely inductive circuit

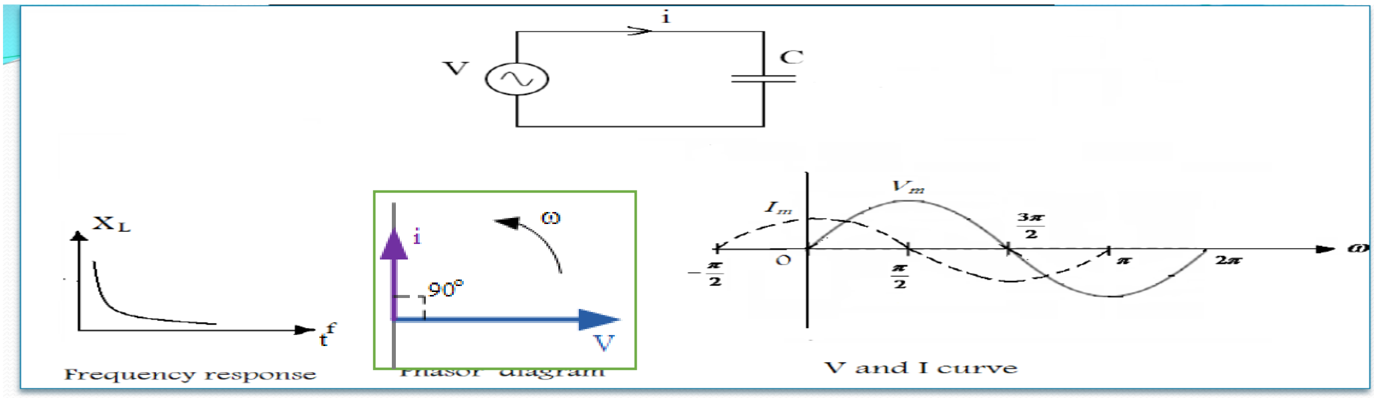


$$\left.\begin{aligned} v &= V_m \sin\omega t \\ i &= I_m \sin(\omega t - 90) \end{aligned}\right\} i \ lags \ v \ 90^\circ$$

$$I_L = I_m = \frac{V_m}{X_L}$$

X: inductive reactance (Ω)

2.3 Purely capacitive circuit



$$\left.\begin{aligned} v &= V_m \sin \omega t \\ i &= I_m \sin(\omega t + 90) \end{aligned}\right\} i \ \ leads \ \ v \ \ 90^\circ$$

$$I_c = I_m = \frac{V_m}{X_c}$$

X_C: capacitive reactance (Ω)

Example (2.1): Find the inductive reactance X_l , the current through the inductor to the circuit, and timing equation in voltage and current in figure (2.1) draw the voltage phasor diagram.

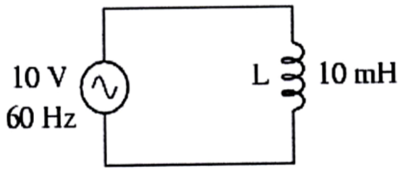


Figure (2.1)

Solution:

$X_l = 2\pi fL = 2*\pi*60*10*10^{-3} = 3.7699\ \Omega$

$I_L = \frac{v}{Xl} = \frac{10}{3.7699} = 2.6A$

V=Vm sin(2πft)

V=10 sin(2π*60*t)

V=10 sin(120πt)

I=Im sin(2πft-90 °)

I=2.6 sin(2π*60*t-90 °)

I=2.6sin(120πt-90°)

Example (2.2): Find the capacitive reactance, the current through the capacitor to the circuit and timing equation in voltage and current in figure (2.2) draw the voltage phasor diagram.

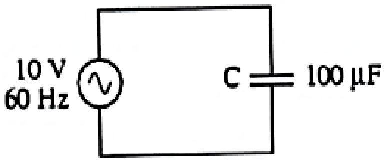


Figure (2.2)

Solution

$$Xc = \frac{1}{2\pi f C} = \frac{1}{2 * \pi * 60 * 100 * 10^{-6}} = 26.5\Omega$$

$$I_c = \frac{v}{Xc} = \frac{10}{26.5} = 0.377A$$

V=Vm sin(2πft)

V=10 sin(2π*60*t)

V=10 sin(120πt)

I=Im sin(2πft+90 º)

I=0.377sin(2π*60*t+90 º)

I=0.377sin (120πt+90º)

H.W (2.1): Find the value of capacitor to the circuit in figure (2.3).

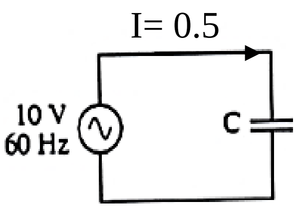
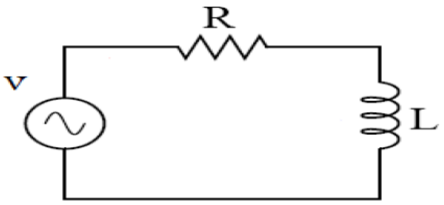
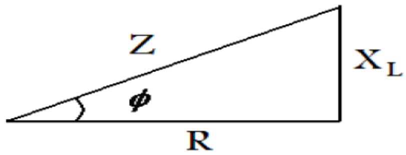


Figure (2.3)

2.4 (RL) Series circuit:





Where Z is the impedance in ohms (Ω)

$$Z = \sqrt{R^2 + X_L^{\,2}}$$

$X_L = 2\pi f L$

$\phi = \tan^{-1}\frac{X_L}{R}$ $\qquad \phi$ phase angle with axis

$I_T = \frac{V}{Z}$ $\qquad$ Z: impedance (Ω)

V_R= I *R

V_L= I *X_L

$V_{T}= \sqrt{V_R^{\,2} + V_L^{\,2}}$

Example (2.3): Find the V_R and V_L for the circuit shown in figure (2.4)

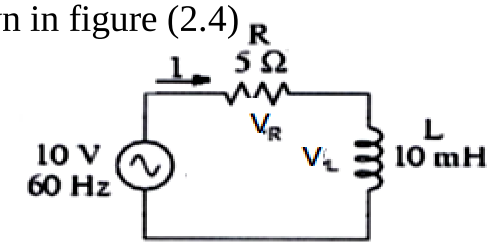


Figure (2.4)

Solution:

$X_l = 2\pi fL = 2*\pi*60*10*10^{-3} = 3.769\Omega$

$Z_{T}= \sqrt{R^2 + X_l^{\,2}}$

$Z_{T}= \sqrt{5^2 + 3.769^2}$

$Z_T = 6.26\ \Omega$

$\emptyset = \tan^{-1}(\frac{Xl}{R})$

$\emptyset = \tan^{-1}(\frac{3.769}{5}) = 37^{o}$

$I = \frac{V}{Z} = \frac{10}{6.26} = 1.59A$

V_R= I *R= 1.59 * 5= 7.95v

V_L= I *X_L= 1.59 *3.769=5.99 v

H.W2.2: find the value of inductive as shown in figure (2.5)

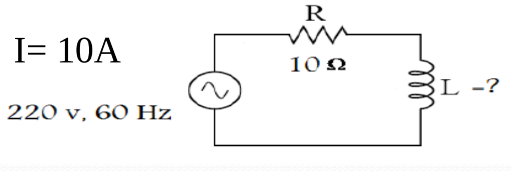
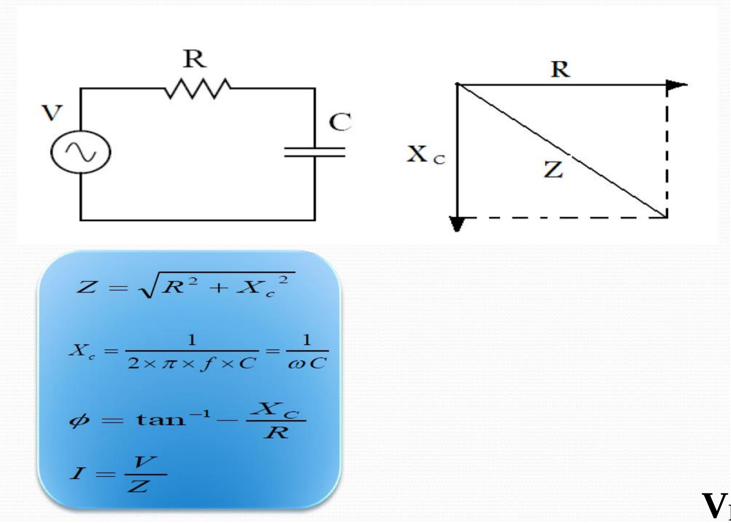


figure (2.5)

2.5 (RC) Series circuit:



V_R= I *R

V_c= I *X_c

$V_{T=}\sqrt{{V_R}^2 + {V_c}^2}$

Example (2.4) Find the total current and V_R and V_C to the circuit in figure (2.6). Then draw the voltage phasor diagram.

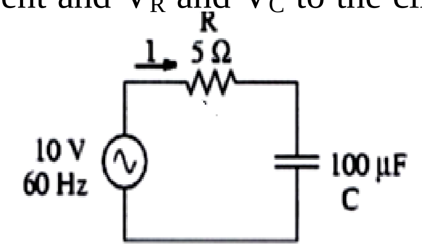


Figure (2.6)

Solution:

$$Xc = \frac{1}{2\pi f C} = \frac{1}{2 * \pi * 60 * 100 * 10^{-6}} = 26.52\Omega$$

$Z_{T=}\sqrt{R^2 + {X_c}^2}$

$Z_{T=}\sqrt{5^2 + 26.52^2}$

$Z_{T=}$ 26.98 Ω

$\theta = -\tan^{-1}(\frac{Xc}{R})$

$$\theta = -\tan^{-1}\left(\frac{26.52}{5}\right) = -79.32$$

$$I = \frac{V}{Z_T} = \frac{10}{26.98} = 0.37A$$

$$V_R = I * R = 0.37 * 5 = 1.85v$$

$$V_L = I * X_L = 0.37 * 26.52 = 9.8124v$$

H.W2.3: find the total voltage of the circuit as shown in figure (2.7)

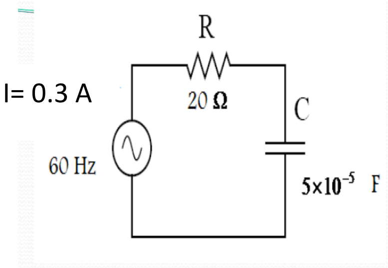
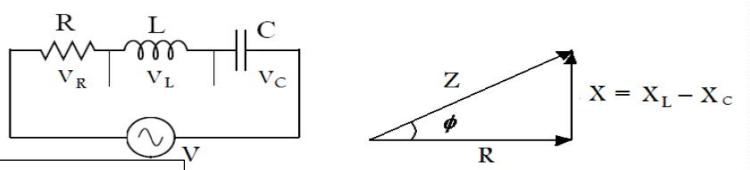


figure (2.7)

2.6 (R-L-C) Series circuit:



$X_L - X_c$ or $X = X_C - X_L$

$$Z = \sqrt{R^2 + X^2}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\phi = \tan^{-1}\frac{X}{R} \qquad I = \frac{V}{Z}$$

$V_X = V_L - V_C$ if $V_L > V_C$

$V_X = V_c - V_L$ if $V_C > V_L$

$V_{T} = \sqrt{{V_R}^2 + {V_X}^2}$

$V_R = I * R$

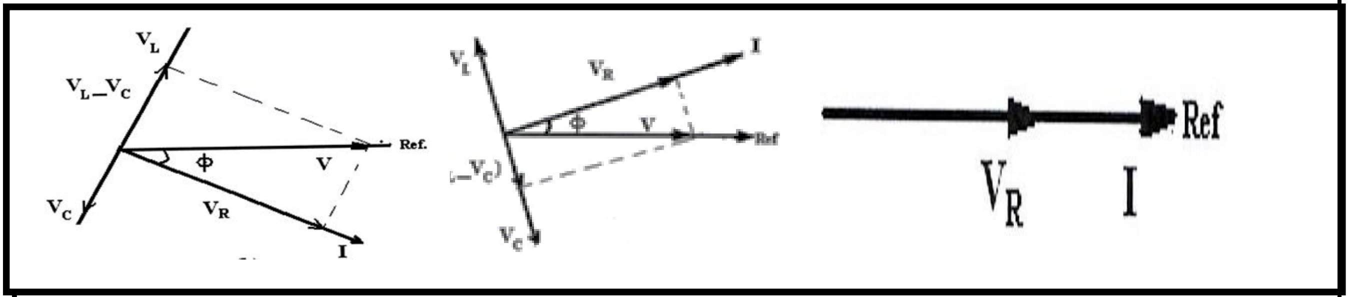
$V_L = I * X_L$

$V_c = I * X_c$

2.4.1 RCL Series appear three cases:

1. **Inductive characteristic**
   In this case the circuit is inductive characteristic which the total reactance is positive because $X_L > X_c$. The current lags the voltage by φ (taken as positive) with the voltage phasor taken as reference as shown in figure(2.8(a)).

2. **Capacitive characteristic.**
   In the second case the circuit is Capacitive characteristic which the total reactance is negative because $X_c > X_L$. The current leads the voltage by φ (taken as negative) with the voltage phasor taken as reference as shown in figure(2.8(b)).
3. **Resistive characteristic**
   In the three case the circuit is resistive characteristic which the total reactance is zero because $X_c = X_L$. Total impedance Z=R The current and the voltage are in phase as shown in figure(2.8(c)).



(a) **(b)** **(c)**

Figure(2.8)RLC Series circuit (a) Inductive circuit
(b) Capaitive circuit
(c) Resisttive circuit

Example (2.5): find the V_R,V_L and V_c for the circuit shown in figure (2.9), then draw the voltage phasor diagram.

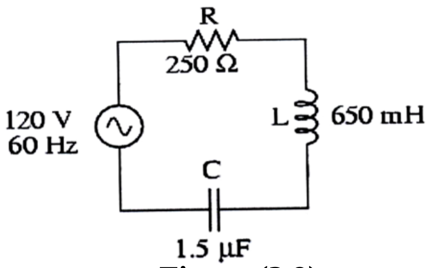


Figure (2.9)

Solution:

R=250Ω

$X_L = 2\pi fL = 2*\pi*60*650*10^{-3} = 245.04\Omega$

$$Xc = \frac{1}{2\pi fC} = \frac{1}{2*\pi*60*1.5*10^{-6}} = 1768.3\Omega$$

$X = X_C - X_l$

$X = 1768.3 - 245.04$

X=1523.26

$Z_T = \sqrt{R^2 + X^2}$

$Z_T = \sqrt{250^2 + 1523.26^2}$

$Z_T = 1543.63\ \Omega$

$\theta = -\tan^{-1}(\frac{X}{R})$

$$\theta = -\tan^{-1}\left(\frac{1523.26}{250}\right) = -80.67^0$$

$$I = \frac{V}{Z_T} = \frac{120}{1543.63} = 0.077A$$

$$V_R = I * R = 0.077 * 250 = 19.25v$$

$$V_L = I * X_L = 0.077 * 245.04 = 18.86v$$

$$V_C = I * X_C = 0.077 * 1768.3 = 136.15v$$

Capacitive characteristic because ($X_c > X_L$) so the current leads the voltage

H.W (2.4): Find the V_R,V_L and V_c and voltage source Vfor the circuit shown in figure (2.10), then draw the voltage phasor diagram.

$I = 0.5A$

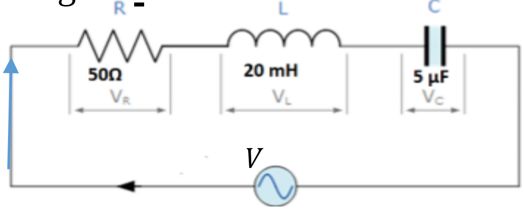
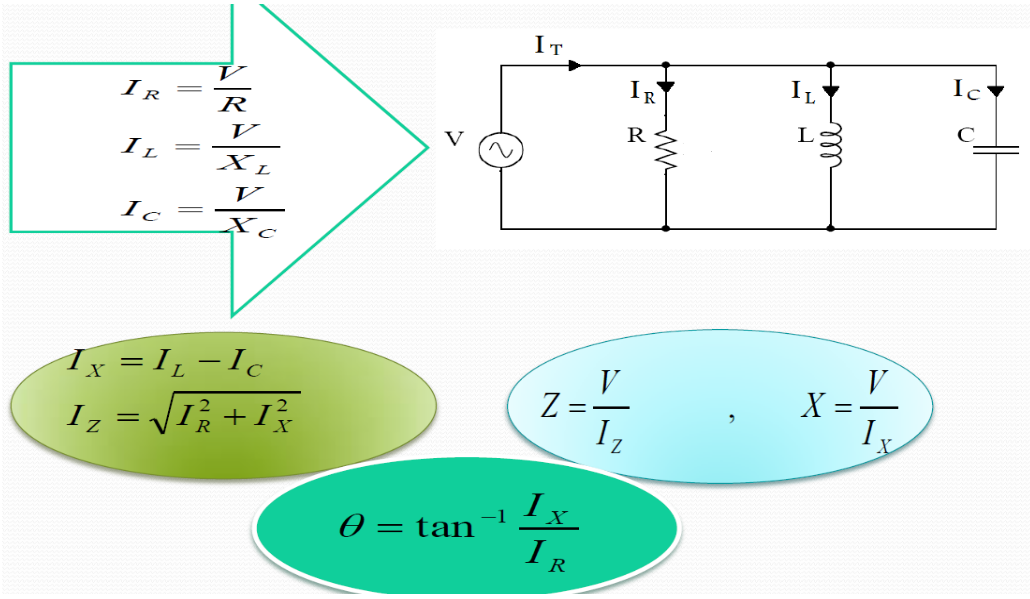


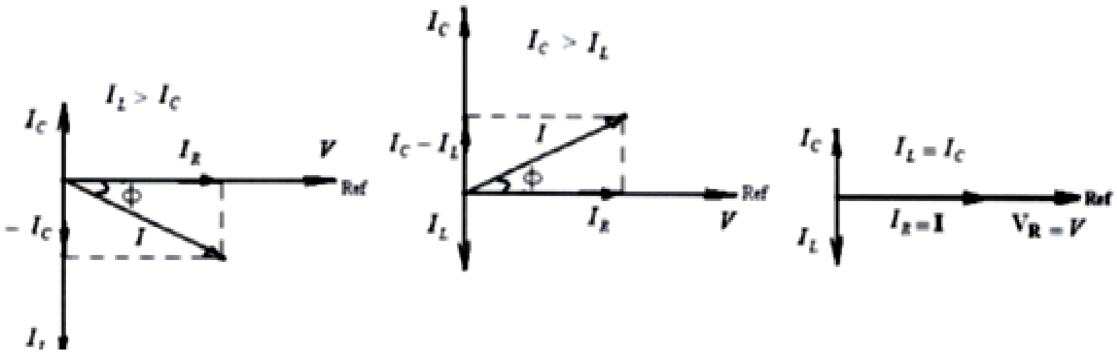
Figure (2.10)

2.7 R-L-C parallel circuit:



2.5.1 RCL Parallel appear three cases:

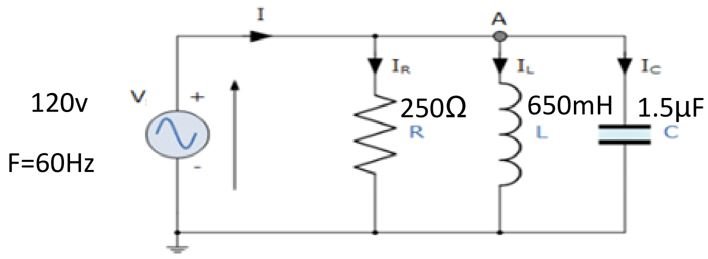
1. **Inductive characteristic**
   In this case the circuit is Inductive characteristic, which the current lags the voltage by φ (taken as positive) with the voltage phasor taken as reference because $I_L > I_c$ as shown in figure (2.11(a)).
2. **Capacitive characteristic**.
   In the circuit, The current leads the voltage by φ (taken as negative) with the voltage phasor taken as reference because $I_c > I_L$ as shown in figure (2.11 (b)).
3. **Resistive characteristic**
   In the three case is resistive characteristic because $I_c = I_L$ which the current and the voltage are in phase and total current (I)= I_R as shown in figure(2.11 (c)).



(a) (b) (c)

Figure (2.11) (a) Inductive circuit
(b) Capaitive circuit
(c) Resisttive circuit

Example(2.6): find the I_R,I_L, I_c and the total impedance for the circuit as shown in figure (2.12).



Solution:

Figure (2.12)

$X_L=2\pi fL=2\pi*60*650*10^{-3}=245.04\Omega$

$$X_C=\frac{1}{wc}=\frac{1}{2\pi fC}=\frac{1}{2\pi*60*1.5*10^{-6}}=1768.3K\Omega$$

$$I_R=\frac{V}{R}=\frac{120}{250}=0.48A$$

$$I_L=\frac{V}{X_L}=\frac{120}{245.04}=0.489A$$

$$I_C=\frac{V}{X_C}=\frac{120}{1768.3}=0.0678A$$

$I_x=I_L-I_c = 0.489 - 0.0678 = 0.4212$ A

$$I_{T=}\sqrt{{I_R}^2+{I_X}^2}$$

$$I_{T=}\sqrt{0.48^2+0.4212^{\ 2}}$$

$I=0.638A$

$$Z_T=\frac{V}{I}=\frac{120}{0.638}=188.08\Omega$$

H.W(2.5):

1- find the current through the capacitor and the resistor and the total impedance as shown in figure in the figure (2.13).

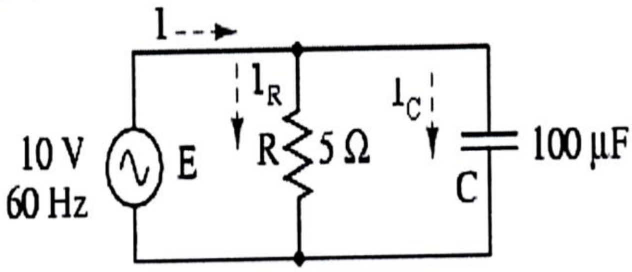


Figure (2.13)

2-Find the total current and the voltage drop across R and C to the circuit in figure (2.14). Then draw the voltage phasor diagram.

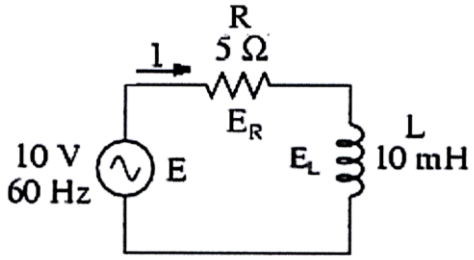


Figure (2.14)

3- Find I_R, I_L, Ic, I_T , Z for the circuit as shown in figure (2.15)

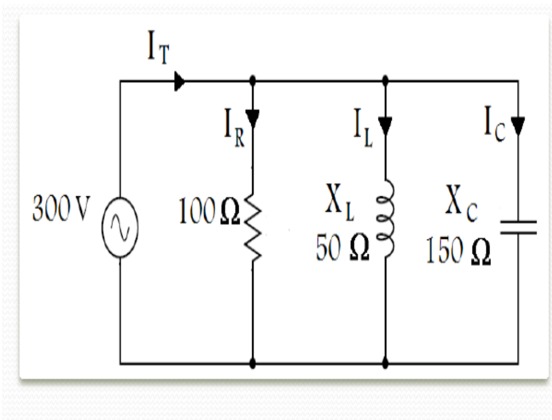
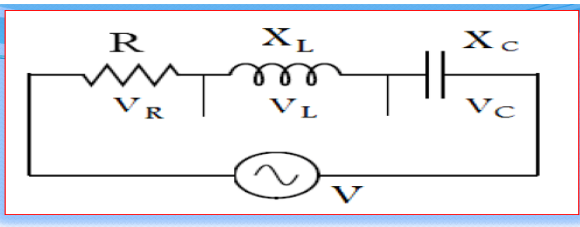


Figure (2.15)

Chapter three

Resonance circuit

At resonance

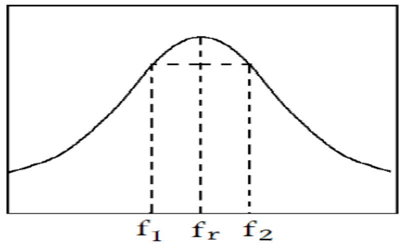


$$X_L = X_C$$

$$Z = \sqrt[2]{R^2 + (X_L - X_c)^2}$$

$$Z = R$$

Resonance Frequency



Resonanc Curve

$$X_L = X_C$$

$$2 \times \pi \times f \times L = \frac{1}{2 \times \pi \times f \times C}$$

$$f^2 = \frac{1}{4 \times \pi^2 \times LC}$$

$$f = \frac{1}{2\pi\sqrt{LC}}$$

Quality Factor (Q)

The *Q, quality factor,* of a resonant circuit is a measure of the goodness. or quality of a resonant circuit.

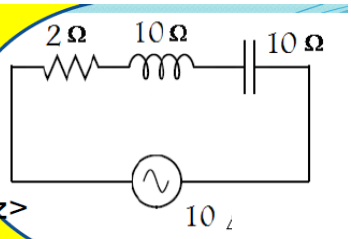
$$Q = \frac{V_L}{V_R} = \frac{I_L \times X_L}{I_R \times R} = \frac{X_L}{R}$$

$$Bw = f_2 - f_1 = \frac{f_r}{Q}$$

Example 3.1

For the resonance circuit shown below find:

- I, V_R, V_L, V_C in polar form
- The quality factor
- the bandwidth Bw if the resonance frequency 5000 Hz>



$$X_L = X_C \quad (Resonance)$$

$$Z_T = R = 2$$

$$I = \frac{V}{Z_T} = \frac{10\angle}{2\angle} = 5\angle \quad A$$

$$V_R = I_R \times R = 5 \times 2 = 10 \quad V$$

$$V_L = I_L \times X_L = 5\angle 0° \times 10\angle = 50 \quad V$$

$$V_C = I_C \times X_C = 5 \times 10 = 50 \quad V$$

$$Q = \frac{X_L}{R} = \frac{10}{2} = 5$$

$$Bw = \frac{f_r}{Q} = \frac{5000}{5} = 1000 \quad Hz$$

Example 3.2

The bandwidth of a series resonance circuit is 400 Hz if the resonance 4000 Hz, R= 10Ω find: Q, X_L, L, C.

$$Q = \frac{f_r}{Bw} = \frac{4000}{400} = 10$$

$$Q = \frac{X_L}{R} \Rightarrow X_L = Q \times R = 10 \times 10 = 100 \quad \Omega$$

$$X_L = 2 \times \pi \times f \times L \Rightarrow L = \frac{X_L}{2 \times \pi \times f} = \frac{100}{2 \times 3.14 \times 4000} = 0.0039 \quad H$$

$$in\ resonance \quad X_L = X_C$$

$$X_C = \frac{1}{2 \times \pi \times f \times C} \Rightarrow C = \frac{1}{X_C \times 2 \times \pi \times f} = \frac{1}{100 \times 2 \times 3.14 \times 4000} = 0.00000039 \quad F$$