

لمحاضرة الثامنة

المشتقة والدوال الضمنية

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القوانين الرئيسية:

$$1 - Y = C$$

$$\dot{Y} = 0$$

$$2 - Y = X^N \text{ ----- } \bar{Y} = nX^{n-1}$$

$$3 - Y = CX^N \text{ ----- } \bar{Y} = C.nX^{n-1}$$

$$4 - Y = U^n \text{ ----- } \bar{Y} = n(U)^{n-1} \frac{du}{dx}$$

$$5 - Y = X^2 + X^4 - 3X \text{ ----- } \bar{Y} = 2X + 4X^3 - 3$$

$$6 - Y = X^2(X-2)^3 \text{ ----- } \bar{Y} = X^2.3(X-2)^2 + (X-2)^3.2X$$

$$7 - Y = \frac{X^2}{(X^2-4)} \text{ ----- } \bar{Y} = \frac{(X^2-4).2X - X^2.2X}{(X^2-4)^2}$$

الاختبار القبلي

$$1 - Y = 5X^8 \text{ ----- } \bar{Y} = 40X^7$$

$$2 - Y = \frac{3}{X^2} \text{ ----- } \bar{Y} = -6X^{-3}$$

$$3 - Y = \frac{3X}{(X^2+4)} \text{ ----- } \bar{Y} = \frac{(X^2+4).3 - 3X(2X)}{(X^2+4)^2}$$

$$4 - X^2 + Y^2 = 1 \text{ ----- } -2X + 2Y \frac{dy}{dx} = 0$$

$$\therefore 2Y \frac{dy}{dx} = -2X \text{ ----- } \therefore \frac{dy}{dx} = \frac{-2X}{2Y} = \frac{-X}{Y}$$

مسائل تطبيقية

أوجد مشتقة الدوال التالية :

$$1-Y = \frac{2X+5}{3X-2} \text{-----} \bar{Y} = \frac{(3X-2)(2)-(2X+5)(3)}{(3X-2)^2}$$

$$2-Y = \sqrt{X^2-4X+1} = (X^2-4X+1)^{\frac{1}{2}} \text{-----} \bar{Y} = \frac{1}{2}(X^2-4X+1)^{-\frac{1}{2}}(2X-4)$$

$$3-Y = \sqrt{X} + \frac{1}{\sqrt{X}} \text{-----} \bar{Y} = \frac{1}{2}X^{-\frac{1}{2}} + (-\frac{1}{2}X^{-\frac{3}{2}})$$

$$4-Y = 2X^2 + 3 + 4X^{-1} + 5X^{-2}$$

$$\bar{Y} = 4X + 0 - 4X^{-2} - 10X^{-3}$$

$$5-Y = (3X-2)(2X^5+1) \text{-----}$$

$$\bar{Y} = (3X-2).10X + (2X^5+1).3$$

$$6-Y = 5X\sqrt{(X^2+1)} \text{-----} \bar{Y} = 5X.\frac{1}{2}(X^2+1)^{-\frac{1}{2}} + (X^2+1)^{\frac{1}{2}}.5$$

مشتقة الدوال المثلثية

القوانين الرئيسية:

$$1 - \frac{d\sin u}{dx} = \cos u \frac{du}{dx}$$

$$4 - \frac{d}{dx} \sec u = \sec u \cdot \tan u \cdot \frac{du}{dx}$$

$$2 - \frac{d\cos u}{dx} = -\sin u \frac{du}{dx}$$

$$5 - \frac{d}{dx} \cot u = -\csc^2 u \frac{du}{dx}$$

$$3 - \frac{d}{dx} \tan u = \sec^2 u \frac{du}{dx}$$

$$6 - \frac{d}{dx} \csc u = -\csc u \cdot \cot u \cdot \frac{du}{dx}$$

مسائل تطبيقية

EX :

$$1 - Y = \sin 2X \dots\dots\dots \bar{Y} = \cos 2x \cdot 2$$

$$2 - Y = \sin 5X^3 \dots\dots\dots \bar{Y} = \cos 5X^3 \cdot 15X^2$$

$$3 - Y = \sin(5X^2 + 4) \dots\dots\dots \bar{Y} = \cos(5X^2 + 4) \cdot 10X$$

$$4 - Y = X^2 \cdot \sin 4X^2 \dots\dots\dots \bar{Y} = X^2 \cdot \cos 4X^2 \cdot 8X + \sin 4X^2 \cdot 2X$$

الدوال الضمنية

القوانين الرئيسية:

الدالة الصريحة هي :

$$Y = x^2 + 3x - 5$$

الدالة الضمنية هي :

$$x^2y + y^2x = 6$$

$$x^2 \frac{dy}{dx} + 2xy + y^2 + 2xy \frac{dy}{dx} = 0$$

المشتقة هي :

مسائل تطبيقية

أوجد مشتقات $\left(\frac{dy}{dx} \right)$ الدوال التالية :

$$1- \quad x^2 + y^2 = 1$$

$$2x + 2y \cdot \frac{dy}{dx} = 0$$

$$\therefore 2y \frac{dy}{dx} = -2x$$

$$\therefore \frac{dy}{dx} = \frac{-2x}{2y} = -\frac{x}{y}$$

$$2- \quad -x^3 - xy + y^3 = 1$$

$$\dots\dots\dots 3x^2 - (x \cdot y \frac{dy}{dx} + y \cdot 1) + 3y^2 \frac{dy}{dx} = 0$$

$$\dots\dots\dots \frac{dy}{dx} (-x + 3y) = y - 3x^2$$

$$\therefore \frac{dy}{dx} = \frac{y - 3x^2}{3y^2 - x}$$

مسائل تطبيقية

$$1 - x = \sin y$$

$$1 = \cos y \frac{dy}{dx}$$

$$\therefore \frac{dy}{dx} = \frac{1}{\cos y} = \sec y$$

$$3 - x^2 y + y^{\frac{2}{3}} = 1$$

$$x^2 \frac{dy}{dx} y \cdot 2x + \frac{2}{3} y^{\frac{-1}{3}} \frac{dy}{dx} = 0$$

$$(x^2 + \frac{2}{3} y^{\frac{-1}{3}}) \frac{dy}{dx} = -2xy$$

$$\frac{dy}{dx} = \frac{-2xy}{x^2 + \frac{2}{3} y^{\frac{-1}{3}}}$$

$$2 - x^2 y - y^2 x = 6$$

$$\dots\dots x^2 \cdot 1 \cdot \frac{dy}{dx} + y \cdot 2x - (y^2 \cdot 1 + x \cdot 2y \frac{dy}{dx}) = 0$$

$$\dots\dots x^2 \frac{dy}{dx} + 2xy - y^2 - 2xy \frac{dy}{dx} = 0$$

$$\dots\dots \frac{dy}{dx} (x^2 - 2xy) = y^2 - 2xy$$

$$\dots\dots \frac{dy}{dx} = \frac{y^2 - 2xy}{x^2 - 2xy}$$

مسائل تطبيقية

$$1 - (y - x)^2 = 2x + 4$$

$$2(y - x) \cdot (\frac{dy}{dx} - 1) = 2 + 0$$

$$(\frac{dy}{dx} - 1) = \frac{2}{2(y - x)}$$

$$\frac{dy}{dx} = \frac{1}{(y - x)} + 1$$

$$3 - \sin x + \cos y = 3$$

$$\cos x \cdot 1 - \sin y \cdot \frac{dy}{dx} = 0$$

$$\cos x = \sin y \frac{dy}{dx}$$

$$\therefore \frac{dy}{dx} = \frac{\cos x}{\sin y}$$

$$2 - (3x + 7)^5 = 2y^3$$

$$5(3x + y)^4 \cdot 3 = 6y^2 \frac{dy}{dx}$$

$$\therefore \frac{dy}{dx} = \frac{15(3x + y)^4}{6y^2}$$

$$4 - x \cdot \tan y = y \cdot \tan x$$

$$x \cdot \sec^2 y \frac{dy}{dx} + \tan y \cdot 1 = y \cdot \sec^2 x \cdot 1 + \tan x \cdot \frac{dy}{dx}$$

$$x \sec^2 y \frac{dy}{dx} - \tan x \frac{dy}{dx} = y \sec^2 x - \tan y$$

$$\therefore \frac{dy}{dx} (x \sec^2 y - \tan x) = y \sec^2 x - \tan y$$

$$\therefore \frac{dy}{dx} = \frac{y \sec^2 x - \tan y}{x \sec^2 y - \tan x}$$

مسائل تطبيقية

$$1 - Y = \cos 3X \dots\dots\dots \bar{Y} = -\sin 3X \cdot 3 \dots\dots\dots \bar{Y} = 3\sin 3X$$

$$2 - Y = \cos(3X + 7) \dots\dots\dots \bar{Y} = -\sin(3X + 7) \cdot 3 \dots\dots\dots = 3\sin(3X + 7)$$

$$3 - Y = \cos^4 5X^2 \dots\dots\dots \bar{Y} = 4 \cdot \cos^3 5X^2 \cdot (-\sin 5X^2) \cdot 10X = -40X \cdot \sin 5X^2 \cos^3 5X^2$$

$$4 - Y = \sqrt{\sin 10X} = (\sin 10X)^{\frac{1}{2}} \dots\dots\dots \bar{Y} = \frac{1}{2} (\sin 10X)^{-\frac{1}{2}} \cdot \cos 10X \cdot 10 = 5 \frac{\cos 10X}{\sqrt{\sin 10X}}$$

$$5 - Y = 3\sin 2X + 5\cos 3X \dots\dots\dots \bar{Y} = 6\cos 2X - 15\sin 3X$$

$$6 - Y = \sin^3 5X \dots\dots\dots \bar{Y} = 3\sin^2 5X \cdot \cos 5X \cdot 5 = 15\sin^2 5X \cdot \cos 5X$$

أوجد مشتقة الدوال التالية :

$$1 - Y = \tan 3X^2 \dots\dots\dots \bar{Y} = \sec^2 3X^2 \cdot 6X = 6X \cdot \sec^2 3X^2$$

$$2 - X \tan Y = Y \tan X$$

$$X \cdot \sec^2 Y \cdot \frac{dy}{dx} + \tan Y (1) = Y \cdot \sec^2 X + \tan X \cdot \frac{dy}{dx}$$

$$X \cdot \sec^2 Y \cdot \frac{dy}{dx} - \tan X \cdot \frac{dy}{dx} = Y \cdot \sec^2 X - \tan Y$$

$$\therefore \frac{dy}{dx} (X \cdot \sec^2 Y - \tan X) = Y \cdot \sec^2 X - \tan Y$$

$$\therefore \frac{dy}{dx} = \frac{Y \cdot \sec^2 X - \tan Y}{X \cdot \sec^2 Y - \tan X}$$

$$3 - x = \sin y$$

$$1 = \cos y \frac{dy}{dx}$$

$$\therefore \frac{dy}{dx} = \frac{1}{\cos y} = \sec y$$

$$4 - Y = \sec^3 3X$$

$$\bar{Y} = 3\sec^2 3X \cdot \sec 3X \cdot \tan 3X \cdot 3$$

$$\dots = 9\sec^3 3X \cdot \tan 3X$$

$$4 - \sin x + \cos y = 3$$

$$\cos x \cdot 1 - \sin y \cdot \frac{dy}{dx} = 0$$

$$\cos x = \sin y \frac{dy}{dx}$$

$$\therefore \frac{dy}{dx} = \frac{\cos x}{\sin y}$$

$$5 - Y = \csc 5X \dots\dots\dots \bar{Y} = -\csc 5X \cdot \cot 5X \cdot 5$$