

# المحاضرة التاسعة

## التكامل (INTEGRATION)

### القوانين والاشتقاقات:

$$1 - \int dx = x + c$$

$$2 - \int dy = y + c$$

$$3 - \int a dx = a \int dx = ax + c$$

$$4 - \int n^n dx = \frac{n^{n+1}}{n+1} + c$$

### مسائل تطبيقية ١

أوجد تكاملات الدوال التالية:

$$1 - \int X^5 dx = \frac{X^6}{6} + c$$

$$2 - \int (X^5 + 4X^3 + 3) dx = \frac{X^6}{6} + \frac{4X^4}{4} + 3X + c$$

$$3 - \int (X+1)(X-2)dx = \int (X^2 - X - 2)dx = \frac{X^3}{3} - \frac{X^2}{2} - 2X + c$$

## مسائل تطبيقية ٢

$$1 - \int \frac{dx}{(3x+2)^4} = \int (3x+2)^{-4} dx = \frac{1}{3} \int (3x+2)^{-4} 3dx \Rightarrow \Rightarrow \frac{1}{3} \frac{(3x+2)^{-3}}{-3} + c$$

$$2 - \int \sqrt{X^2 + 2X^4} dx \Rightarrow \Rightarrow \int \sqrt{X^2(1+2X^2)} dx \Rightarrow \int X(1+2X^2)^{\frac{1}{2}} dx$$

$$3 - \int \frac{(z+1)dz}{\sqrt{z^2 + 2z + 5}} \Rightarrow \int (Z^2 + 2Z + 5)^{-\frac{1}{2}} (Z+1)dz \Rightarrow \frac{1}{2} \int (Z^2 + 2Z + 5)^{-\frac{1}{2}} 2(Z+1)dz$$

$$= \frac{1}{2} \frac{(Z^2 + 2Z + 5)^{\frac{1}{2}}}{\frac{1}{2}} + c \Rightarrow \Rightarrow \sqrt{Z^2 + 2Z + 5} + c$$

$$4 - \int \left( \frac{1}{\sqrt{X}} - \frac{1}{\sqrt{X+1}} \right) dx \Rightarrow \int X^{-\frac{1}{2}} dx - \int (X+1)^{-\frac{1}{2}} dx \Rightarrow$$

$$\frac{X^{\frac{1}{2}}}{\frac{1}{2}} - \frac{(X+1)^{\frac{1}{2}}}{\frac{1}{2}} + c = 2\sqrt{X} - 2\sqrt{X+1} + c$$

## مسائل تطبيقية ٣

$$1 - \int \left( \frac{X}{\sqrt{X^2 + 4}} - 1 \right) dx = \int \frac{Xdx}{\sqrt{X^2 + 4}} - \int dx$$

$$= \int (X^2 + 4)^{-\frac{1}{2}} Xdx - \int dx = \frac{1}{2} \int (X^2 + 4)^{-\frac{1}{2}} 2Xdx - \int dx$$

$$= \frac{1}{2} \frac{(X^2 + 4)^{\frac{1}{2}}}{\frac{1}{2}} - X + C = \sqrt{X^2 + 4} - X + C$$

$$\begin{aligned}
 2 - \int X^2 \sqrt{1 - X^3} dx &= \int X^2 (1 - X^3)^{\frac{1}{2}} dx \\
 &= \frac{-1}{3} \frac{(1 - X^3)^{\frac{3}{2}}}{\frac{3}{2}} + C = -\frac{2}{9} (1 - X^3)^{\frac{3}{2}} + C
 \end{aligned}$$

4

$$\begin{aligned}
 3 - \int \frac{3X dx}{\sqrt{1 - X^2}} &= \int 3X (1 - X^2)^{-\frac{1}{2}} dx \Rightarrow -\frac{1}{2} \int 3X (1 - X^2)^{-\frac{1}{2}} (-2) dx \\
 &= \frac{-3}{2} \frac{(1 - X^2)^{\frac{1}{2}}}{\frac{1}{2}} + c \Rightarrow -3\sqrt{1 - X^2} + c
 \end{aligned}$$

$$\begin{aligned}
 4 - \int \frac{(X^2 - 2)^2}{X^2} dx &\Rightarrow \frac{X^4 - 4X^2 + 4}{X^2} dx \Rightarrow \int \frac{X^4}{X^2} dx - \int \frac{4X^2}{X^2} dx + \int \frac{4}{X^2} dx \\
 &= \int X^2 dx - \int 4 dx + \int 4X^{-2} dx = \frac{X^3}{3} - 4X + \frac{4}{X} + c
 \end{aligned}$$

## تکامل الدوال المثلثية

$$1 - \int \sin u du \Rightarrow -\cos u + C$$

$$2 - \int \cos u du \Rightarrow \sin u + C$$

$$3 - \int \sec^2 u du \Rightarrow \tan u + C$$

$$4 - \int \csc^2 u du \Rightarrow -\cot u + C$$

$$5 - \int \sec u \tan u du \Rightarrow \sec u + C$$

$$6 - \int \csc u \cot u du \Rightarrow -\csc u + C$$

# مسائل تطبيقية ٤

أوجد تكاملات الدوال المثلثية التالية :

$$1 - \int \sin 3X dx \dots \dots \dots u = 3x \dots \dots \dots du = 3dx$$

$$= \frac{1}{3} \int \sin 3X \cdot 3dx = -\frac{1}{3} \cos 3X + C$$

$$2 - \int \sin(7X + 5) dx \dots \dots \dots u = (7X + 5) \dots \dots \dots du = 7dx$$

$$= \frac{1}{7} \int \sin(7X + 5) \cdot 7dx = -\frac{1}{7} \cos(7X + 5) + C$$

$$3 - \int \cos 6X dx \dots \dots \dots u = 6X \dots \dots \dots du = 6dx$$

$$= \frac{1}{6} \int \cos 6X \cdot 6dx = \frac{1}{6} \sin 6X + C$$

$$6 - \int \frac{\cos \frac{X}{2}}{\sin^3 \frac{X}{2}} dx \dots \dots \dots u = 2X^2 \dots \dots \dots du = 4XdX$$

$$= \frac{1}{4} \int \sin^2 2X^2 \cdot 4XdX = -\frac{1}{4} \cos 2X^2 + C$$

$$= 2 \int \sin^{-3} \frac{X}{2} \cos \frac{X}{2} \cdot \frac{1}{2} dx \dots \dots \dots = 2 \frac{\sin^{-2} \frac{X}{2}}{2} + C \dots \dots \dots = \frac{-1}{\sin^2 \frac{X}{2}} + C$$

$$= \frac{2}{9} \int \cos 3X^3 \cdot 9X^2 dx = \frac{2}{9} \sin 3X^3 + C$$

$$7 - \int \sin^5 5X \cos 5X dx \dots \dots \dots u = \sin 5X \dots \dots \dots du = \cos 5X \cdot 5dx$$

$$= \frac{1}{5} \int \sin^5 5X \cos 5X \cdot 5dx \dots \dots \dots = \frac{1}{5} \frac{\sin^6 5X}{6} + C = \frac{1}{30} \sin^6 5X + C$$

## مسائل تطبيقية ٥

$$\begin{aligned}
 1 - \int (\sec^2 3X - 1) dx \\
 = \frac{1}{3} \int 3 \sec^2 3X dx - \int dx \dots \dots \dots u = \sec^2 3X \dots \dots \dots du = \tan 3X \cdot 3 dx \\
 = \frac{1}{3} \tan 3X - X + C
 \end{aligned}$$

$$\begin{aligned}
 2 - \int \csc^2 2\theta \cdot d\theta \dots \dots \dots u = 2\theta \dots \dots \dots du = 2 d\theta \\
 = \frac{1}{2} \int \csc^2 2\theta \cdot 2 d\theta \dots \dots \dots = -\frac{1}{2} \cot 2\theta + C
 \end{aligned}$$

$$\begin{aligned}
 3 - \int (3 \sin 2X + 4 \cos 3X) dx \\
 = \frac{3}{2} \int \sin 2X \cdot dx + \frac{4}{3} \int \cos 3X dx \\
 = -\frac{3}{2} \cos 2X + \frac{4}{3} \sin 3X + C
 \end{aligned}$$

$$4 - \int \frac{d\theta}{\cos^2 5\theta} = \int \sec^2 5\theta \cdot d\theta \dots \dots \dots = \frac{1}{5} \tan 5\theta + C$$

$$5 - \int \sec 5X \cdot \tan 5X \cdot dx \dots \dots \dots = \frac{1}{5} \sec 5X + C$$

$$6 - \int \csc 3X \cdot \cot 3X dx \dots \dots \dots = -\frac{1}{3} \csc 3X + C$$

$$\begin{aligned}
 7 - \int \cos 2X \sqrt{4 - \sin 2X} \cdot dx \dots \dots \dots u = \sqrt{4 - \sin 2X} \dots \dots \dots du = -\cos 2X \cdot 2 dx \\
 = -\frac{1}{2} \int (4 - \sin 2X)^{\frac{1}{2}} \cdot \cos 2X \cdot 2 dx \\
 = -\frac{1}{2} \cdot \frac{(4 - \sin 2X)^{\frac{3}{2}}}{\frac{3}{2}} \dots \dots \dots = -\frac{1}{3} (4 - \sin 2X)^{\frac{3}{2}} + C
 \end{aligned}$$

$$\begin{aligned}
 8 - \int \frac{\sec^2 5X}{\sqrt{(\tan 5X + 1)}} dx \dots \dots \dots u = \tan 5X + 1 \dots \dots \dots du = \sec^2 5X \cdot 5 dx + 0 \\
 = \frac{1}{5} \int (\tan 5X + 1)^{-\frac{1}{2}} \sec^2 5X \cdot 5 dx \dots \dots \dots = \frac{1}{5} \frac{(\tan 5X + 1)^{\frac{1}{2}}}{\frac{1}{2}} \dots \dots = \frac{2}{5} \sqrt{\tan 5X + 1} + C
 \end{aligned}$$